Sensitivity analysis of stochastic polynomials and its application to epidemic forecasts

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Rationale:

Incorporating heterogeneity in disease dynamics

Model with a distribution of secondary infections, specifically probability generating functions (PGFs)

Incorporating error into data into a model

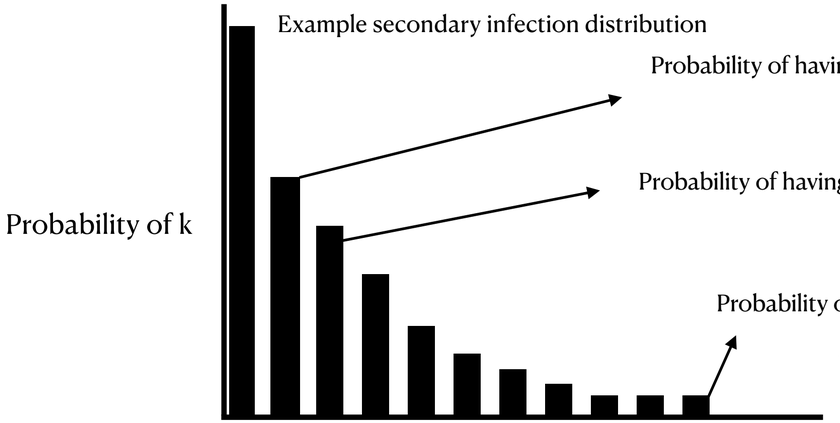
Perturb degree distribution

Solving PGF with perturbed degree distribution

Determine the sensitivity of certain degree distributions when solving for outbreak sizes

C. Dye, and N. Gay, Modeling the SARS epidemic. *Science* 300.5627 (2003): 1884-1885.

How we do this:



Determine polynomial coefficients from distribution

Network degree distribution PGF

$$G_0(x) = \sum_{k=0}^{\infty} p_k x^k$$

Take derivative

Transmission (excess) degree distribution PGF

$$G_1(x) = \sum_{k=0}^{\infty} \frac{k p_k}{\langle k \rangle} x^{k-1} = \sum_{j=0}^{\infty} q_j x^j$$

Add error to coefficients

$$G_1(x) = \sum_{j=0}^{20} q_j(1 + ERROR)x^j$$
$$= \sum_{j=0}^{20} (a_j + \delta a_j)x^j$$

Solve the self consistent equation

$$G_1(u) = u$$
$$G_1(u) - u = 0$$

Solve for final outbreak size

Final outbreak size $= 1 - u$

Do this over and over again for differing ERROR distributions

M.E. Newman, S.H. Strogatz, and D.J. Watts, Random graphs with arbitrary degree distributions and their applications, *Physical Review E* 64, 026118 (2001).

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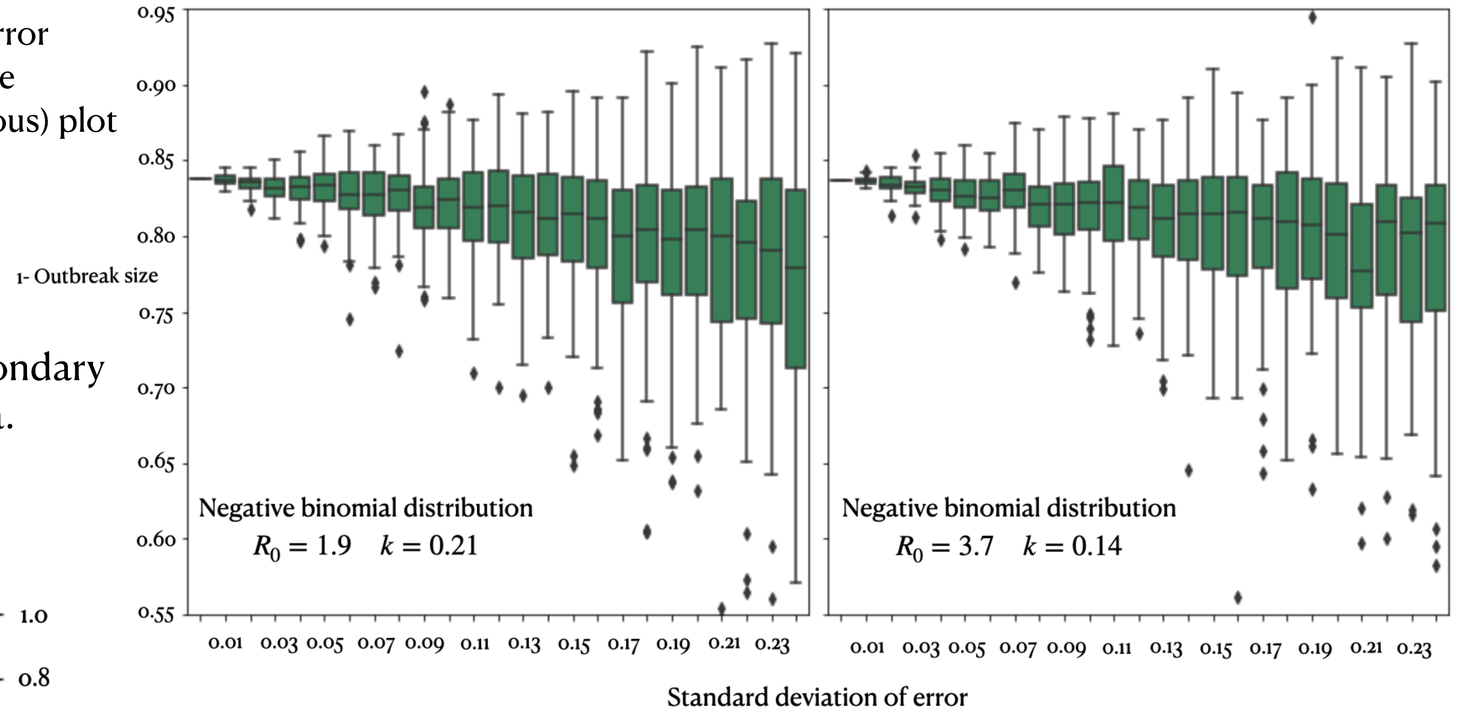
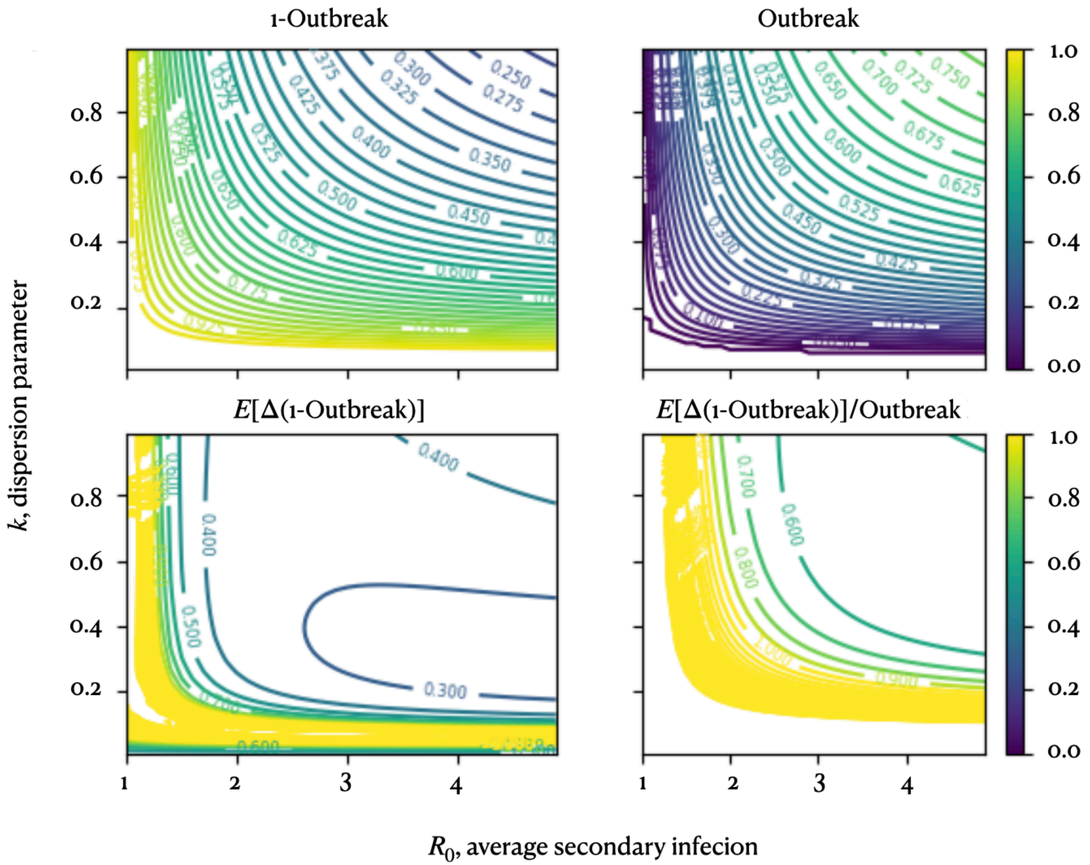
Right: Two systems, homogenous and heterogenous, exhibit how added error create varying final outbreak sizes as more error is added to the system. The box plots grow larger and outbreak sizes vary more on the left (homogeneous) plot compared to the the left (heterogeneous).



Take away: Outbreak sizes can vary with more homogeneous secondary degree networks, meaning they are more sensitive to errors in data.



Left: Given varying average secondary infection rates, R_0, and dispersion parameters, k, PGF roots (1-outbreak size), outbreak size, expectation of the change in (1-outbreak) and expectation of the change in (1-outbreak) over the outbreak are calculated. For the top and bottom row of the figure, each contour line either represents a percentage of the network and percentage of variability respectively.

Take away: When keeping outbreak size constant, the expectation of the change in (1-outbreak) is more drastic and sensitive for smaller outbreaks.

J.O. Lloyd-Smith, S. Schreiber, P. Kopp, *et al.*, Superspreading and the effect of individual variation on disease emergence. *Nature* 438.7066 (2005): 355-359.