

# Single-particle radial potentials

Some comments on how the shape of the radial potentials affect the energies.

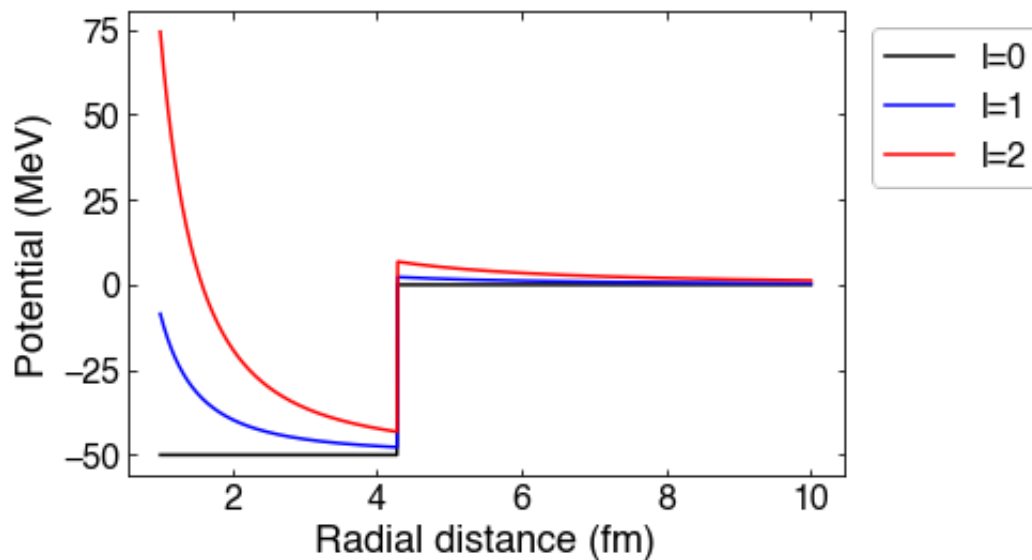
```
In [1]: import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as ss
#from scipy.optimize import curve_fit
plt.rc("font", family=["Helvetica", "Arial"]) # skifter skrifttype
plt.rc("axes", labelsz=18) # skriftstørrelse af `xlabel` og `ylabel`
plt.rc("xtick", labelsz=16, top=True, direction="in") # skriftstørrelse
plt.rc("ytick", labelsz=16, right=True, direction="in")
plt.rc("axes", titlesz=22)
plt.rc("legend", fontsize=16)
```

We first look at neutron eigenstates in a square well with different angular momenta  $l$ . The parameters are inspired by Zelevinsky and Volya section 8.9, and the nucleus is  $^{40}\text{Ca}$ .

```
In [2]: R = 1.25*40**(1/3)
V0 = 50
def Vrad_n(x,l):
    Vneu = np.zeros(len(x))
    for i in range(len(x)):
        if x[i]<R:
            Vneu[i] = -V0 + 197.327**2/(2*939.565*x[i]**2)*l*(l+1)
        else:
            Vneu[i] = 0 + 197.327**2/(2*939.565*x[i]**2)*l*(l+1)
    return Vneu
```

Now plot the neutron potentials for different values of  $l$ . Note how wavefunctions (that preferentially reside at the minima in the potentials) are pushed to larger radial values for increasing  $l$ .

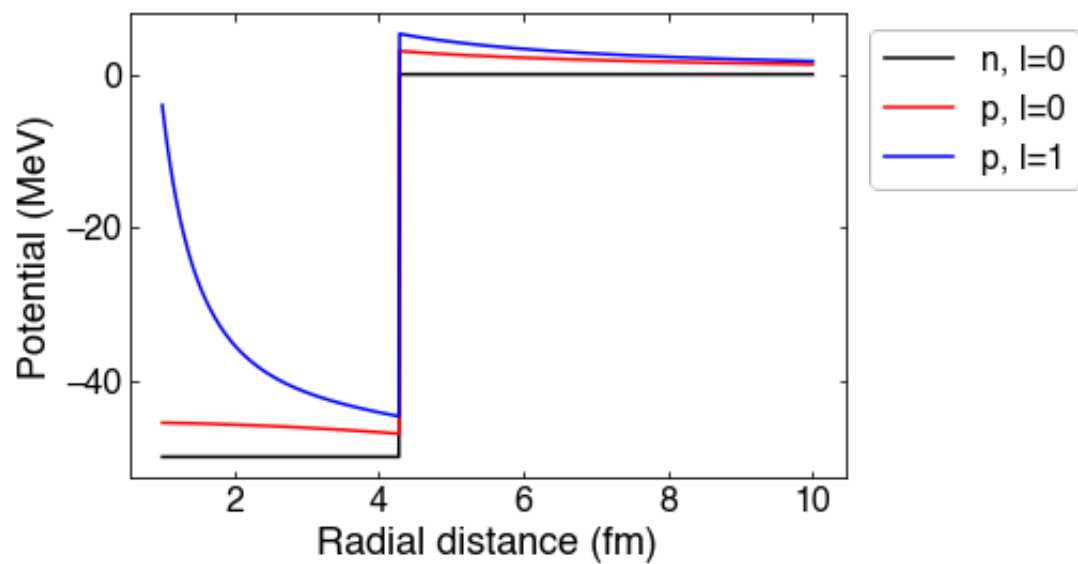
```
In [3]: r_neutron = np.linspace(1.0,10.,900)
plt.plot(r_neutron,Vrad_n(r_neutron,0),"k", label = "l=0")
plt.plot(r_neutron,Vrad_n(r_neutron,1),"b", label = "l=1")
plt.plot(r_neutron,Vrad_n(r_neutron,2),"r", label = "l=2")
plt.xlabel("Radial distance (fm)")
plt.ylabel("Potential (MeV)")
plt.legend(bbox_to_anchor=(1.30, 1))
plt.show()
```



The "push" to higher radii is a general effect of the angular momentum barrier. Compare now the effect of this for the square well, for a harmonic oscillator (h.o.) and a  $1/r$  potential (try perhaps to plot them) - the energy gain for a high  $l$  orbit is clearly largest for the well, still sizeable for the h.o. and much smaller for the  $1/r$  potential. As shown in figure 8.4 in the book the degenerate  $l$  values for the h.o. are therefore split up with highest  $l$  decreasing most. Similarly, the degeneracies for the h.o. and  $1/r$  are completely different.

Define now the proton potentials. For the Coulomb term, use  $\alpha\hbar c$  for the square of the electron charge (the same in SI and cgs units). The Coulomb potential inside the well falls off quadratically and therefore mainly moves the bottom of the well up.

```
In [4]: def Vrad_p(x,l):
    Vpro = np.zeros(len(x))
    for i in range(len(x)):
        if x[i]<R:
            Vpro[i] = -V0 + 197.327**2/(2*938.272*x[i]**2)*l*(l+1) + 197.327
        else:
            Vpro[i] = 0 + 197.327**2/(2*938.272*x[i]**2)*l*(l+1) + 197.327
    return Vpro
r_proton = np.linspace(1.0,10.,900)
plt.plot(r_neutron,Vrad_n(r_neutron,0),"k", label = "n, l=0")
plt.plot(r_proton,Vrad_p(r_proton,0),"r", label = "p, l=0")
plt.plot(r_proton,Vrad_p(r_proton,1),"b", label = "p, l=1")
plt.xlabel("Radial distance (fm)")
plt.ylabel("Potential (MeV)")
plt.legend(bbox_to_anchor=(1.35, 1))
plt.show()
```



In [ ]: