

The exponential function

Maria Hinge Pedersen

June 22, 2022

1 Introduction of the exponential function

The exponential function is a mathematical function denoted by $f(x) = \exp(x)$ or e^x (where the argument x is written as an exponentiation). It is an important function in both mathematics and physics. It is defined in several different ways, but for this introduction the following definition is used:

$$\exp(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots \quad (1)$$

Its value at 1, $e = \exp(1)$ is a mathematical constant known as Euler's number.¹

To implement the function, the code shown in the figure 1 is used.

If the argument is below 0, the function returns $1/e^x$ which is equivalent to e^{-x} . For relatively large x (i.e. $x > 1/8$) the recursive definition $\exp x = (\exp x/2)^2$ is used, while the Taylor expansion in equation 1 can be used for smaller, but positive values of x .

¹https://en.wikipedia.org/wiki/Exponential_function

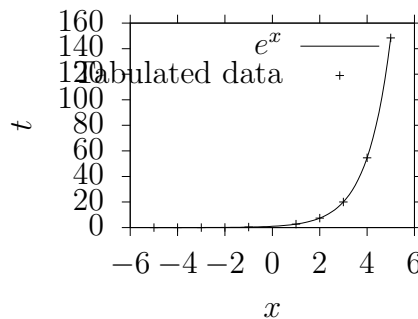


Figure 1: Plot of the exponential function by the code.

The implementation is then tested by plotting it against known table values, which is done in figure ?? . And it works.

```
static double ex(double x){
if(x<0)return 1/ex(-x);
if(x>1.0/8)return Pow(ex(x/2),2);
return 1+x*(1+x/2*(1+x/3*(1+x/4*(1+x/5*(1+x/6*(1+x
/7*(1+x/8*(1+x/9*(1+x/10)))))))));
}
```