The sphere eversion project

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0.1 Generalities

Fix a field K.

Definition 0.1. *A valuation $v\colon K \to \mathbb{Z}$ on K is discrete if it is non-archimedean and surjective.*

We have the

Proposition 0.2 ([**?**, Chap I,§1, Proposition 1])**.** *If K is complete with respect to a discrete valuation v, then its unit ball K° is a discrete valuation ring.*

Lemma 0.3 ([**?**, Chap II,§2, last part of Proposition 3])**.** *If K is complete with respect to a discrete valuation v and if L/K is a finite extension, then L has a discrete valuation $w\colon L \twoheadrightarrow \mathbb{Z}$ inducing v and L is complete with respect to w.*

Moreover,

Proposition 0.4 ([**?**, Chap II,§2, Proposition 3])**.** *If K is complete with respect to a discrete valuation v and if L/K is a finite extension, then the integral closure of K° inside L coincides L° and so, in particular, it is a discrete valuation ring by 0.2. Moreover, L° is a finite, free K°-module of rank $n = [L : K]$.*

0.2 Local Fields

Definition 0.5. *A (nonarchimedean) local field is a field complete with respect to a discrete valuation and with finite residue field.*

Definition 0.6. *A mixed characteristic local field is a finite field extension of the field $\mathbb{Q}_p$ of p-adic numbers, for some prime p.*

Definition 0.7. *An equal characteristic local field is a finite field extension of the field $\mathbb{F}_p(\!(X)\!)$, for some prime p.*

Lemma 0.8. *A mixed characteristic local field is a local field.*

Lemma 0.9. *An equal characteristic local field is a local field.*

0.2.1 Ramification Index

1. Define a local ramification index to get rid of the variables $\mathfrak{p}$ and $\mathfrak{P}$ in the mathlib definition
2. Obtain from there the formula $n = ef$
3. Prove that $\mathbb{Q}_p$ and $\mathbb{F}_p(\!(X)\!)$ are unramified
4. If possible, go through [**?**, Chap. I, §6] (before completion, the results there hold for DVR's).

0.3 Global to Local

Starting with a Dedekind domain R and a non-zero maximal ideal $\mathfrak{p}$, let $K = \mathrm{Frac}(R)$. Then we prove the

Proposition 0.10 ([**?**, Chap. II, §2, Théorème 1])**.** *The completion $K_\mathfrak{p}$ has a discrete valuation extending the valuation $v_\mathfrak{p}$ and such that ${K_\mathfrak{p}}^\circ = R_\mathfrak{p}$. In particular, this localization is a DVR by Proposition 0.2.*

Proposition 0.11. *Let F be a number field and let v be a finite place of residue characteristic p. Then F_v is a mixed characteristic local field with residue characteristic p.*

Proposition 0.12. *Let F be a function field and let v be any place. Then F_v is an equal characteristic local field with residue characteristic p.*

Blablabla