Tree-Based Methods

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Outline

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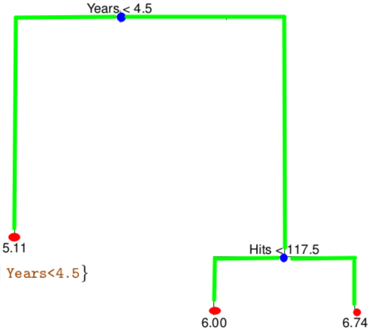
- To make a prediction, we use the mean or the mode for the training observations in the region to which it belongs.
- Let's begin with an example...
- Let's predict a baseball player's **Salary** based on the **years** that he has played in the major leagues and the **hits** that he made in the previous year.

Introduction



$R_1 = \{X \mid$ `Years<4.5`$\}$

$R_2 = \{X \mid$ `Years>=4.5`, `Hits<117.5`$\}$ $R_3 = \{X \mid$ `Years>=4.5`, `Hits>=117.5`$\}$

- Terminal nodes or leaves.
- Internal nodes.
- Branches.

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- One way to solve this is consider a very large tree T_0 , and then prune it back in order to obtain a subtree that leads to the **lowest test error rate**.
- **Cost complexity pruning** —also known as *weakest link pruning*—gives us a way to do just this.

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  - α controls a trade-off between the subtree's complexity and its fit to the training data.

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- As α increases, the quantity (1) will tend to be minimized for a smaller subtree.
- We can select a value of α using cross-validation as is described next.

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   Average the results for each value of α, and pick α to minimize the average error.
3. Return the subtree that corresponds to the chosen value of α.

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    - In practice two other measures are preferable.

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   - In this sense, Entropy is also referred as a measure of *node purity*.

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- Any of these three approaches might be used when **pruning** the tree, but CER is preferable if **prediction accuracy** of the final pruned tree is the goal.

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The Basics of Decision Trees

Advantages and Disadvantages of Trees

Advantages

- ✓ Trees can be displayed graphically, and are easily interpreted.
- ✓ Can handle qualitative predictors without the need to create dummy variables.

Disadvantages

- × Do not have the same level of predictive accuracy as some methods seen before.
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Note

By aggregating many decision trees, using methods like bagging, random forests, and boosting, the predictive performance of trees can be substantially improved.

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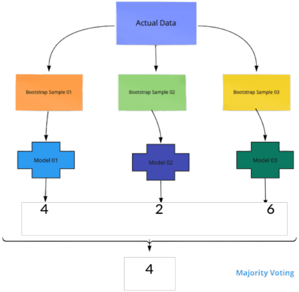

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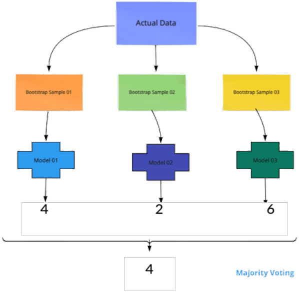

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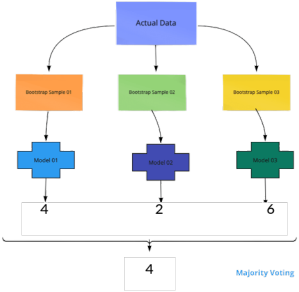

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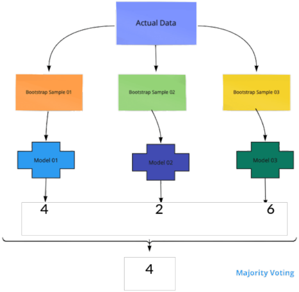

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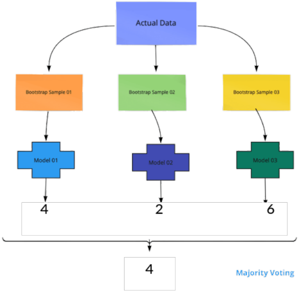

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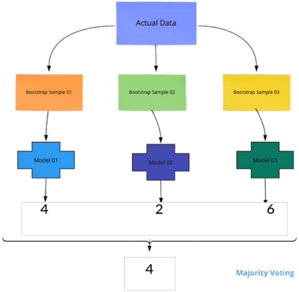

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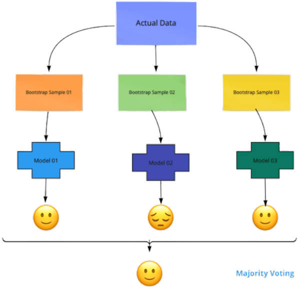

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- The resulting OOB error is a valid estimate of the test error for the bagged model.

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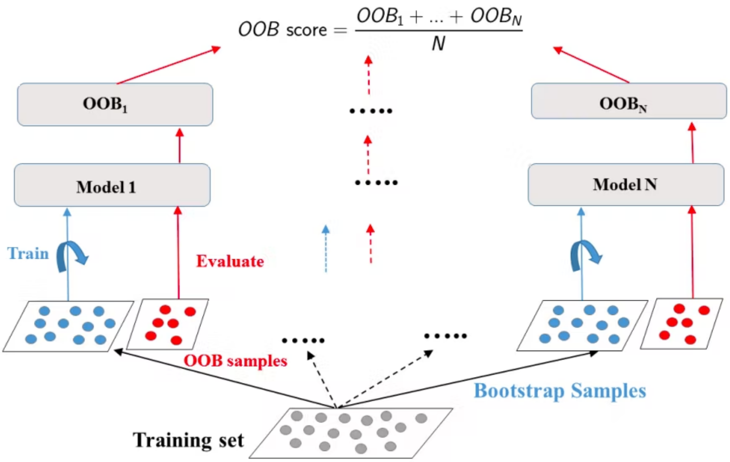

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- On average $(p - m)/p$ of the splits will not even consider the strong predictor, and so other predictors will have more of a chance.

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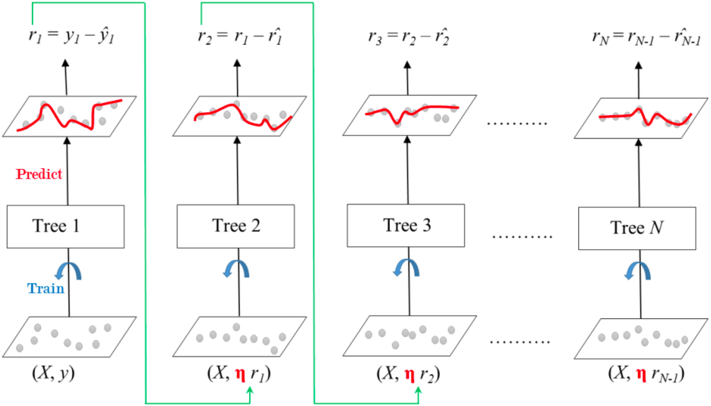
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Predict
Tree 1
Tree 2
Tree 3
Tree N
Train
(X, y)
$(X, \eta\, r_1)$
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$(X, \eta\, r_{N-1})$

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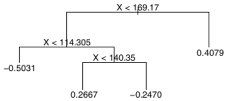
What is the random perturbation?

1. We may change the structure of the tree by adding or pruning branches.
2. We may change the prediction in each terminal node of the tree.

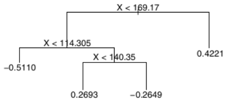
Bayesian Additive Regression Trees (BART)

(a): $\hat{f}_k^{b-1}(X)$



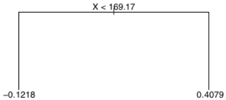
The kth tree at the $(b-1)$*st iteration,*

(b): Possibility #1 for $\hat{f}_k^{b}(X)$



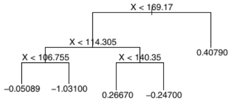
same structure as $\hat{f}_k^{b-1}(X)$*, but with different predictions at the terminal nodes.*

(c): Possibility #2 for $\hat{f}_k^{b}(X)$



pruning $\hat{f}_k^{b-1}(X)$.

(d): Possibility #3 for $\hat{f}_k^{b}(X)$



$\hat{f}_k^{b}(X)$ *may have more terminal nodes than* $\hat{f}_k^{b-1}(X)$.

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  4. **BART**: we once again only make use of the original data, and we grow the trees successively. However, each tree is perturbed.

Thank you!

Any question?