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Business Location Theories & Spatial Autocorrelation

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26/04/2020

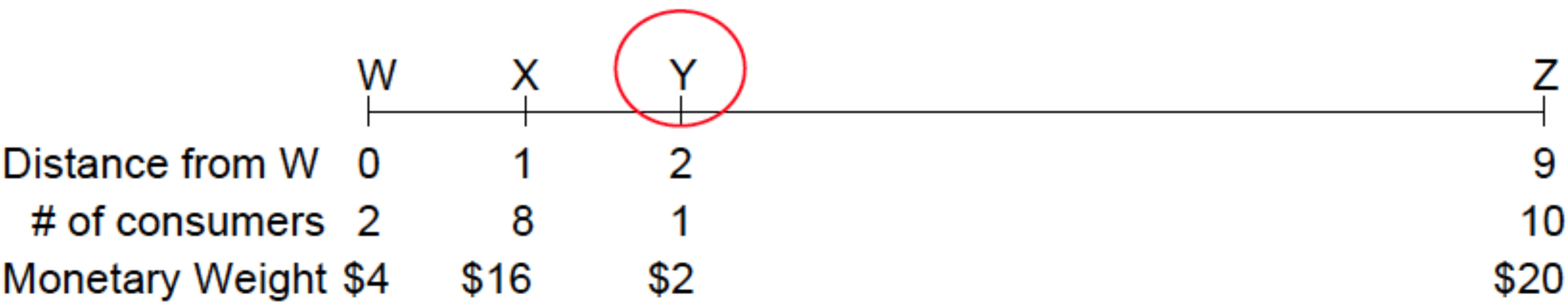
Location on a Line: Principle of Median Location

- Median Location has ½ of the monetary weight to the left and ½ to the right.
- Optimal location?

	W	X	Y	Z
Distance from W	0	1	2	9
# of consumers	2	8	1	10
Monetary Weight	$4	$16	$2	$20

Principle of Median Location

- Why not move from Y to X?
  - Total TC at X: \$4×1 + \$16×0 + \$2×1 + \$20×8 = \$168
  - Total TC at Y: \$4×2 + \$16×1 + \$2×0 + \$20×7 = \$164
- What if 4 new consumers move to W?



Location on a Line:
Weber Linear Model

- Nomenclature
  - S: Location at the Source
  - M: Location at the Market
  - t: Transportation costs per unit
  - T: Total transportation costs
  - X: input weight
  - x: output weight
  - D: total distance between S and M
  - d: unit of distance
  - AC: Assembly Costs (Same as RMC before)
  - DC: Distribution Costs

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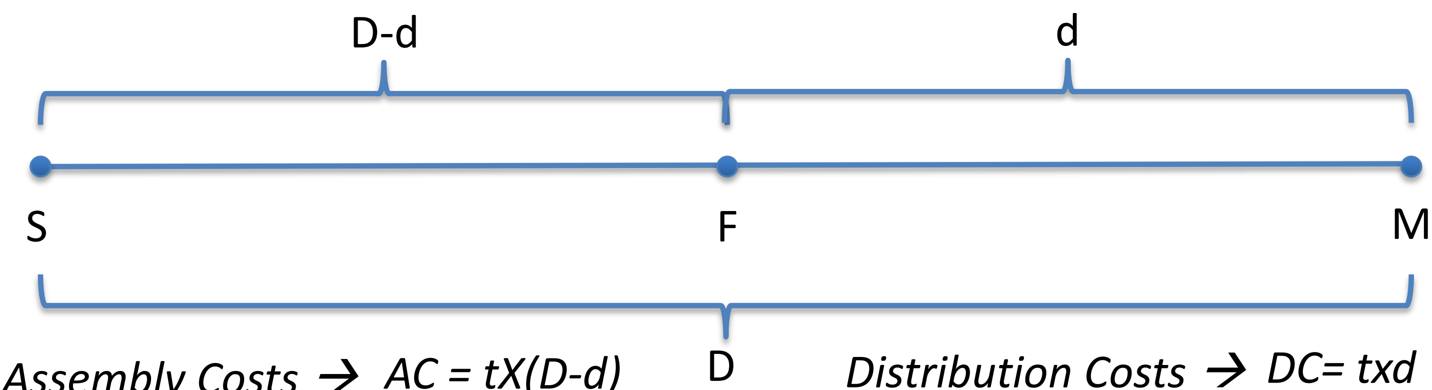
Weber Linear Model

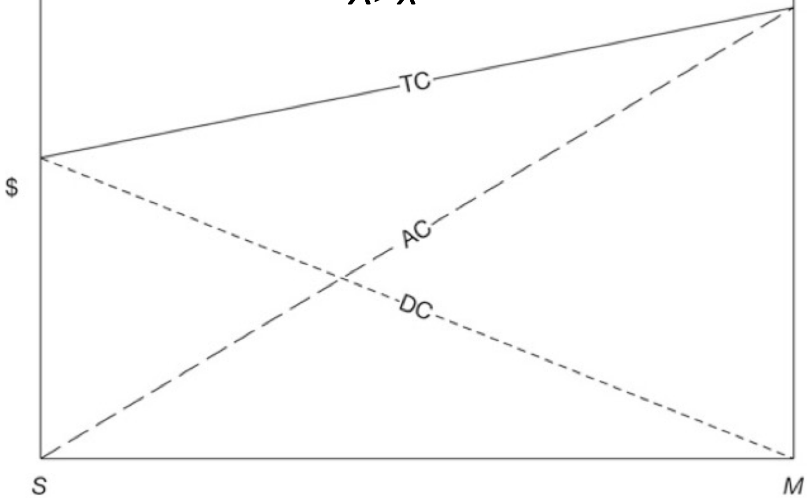


Assembly Costs → $AC = tX(D-d)$

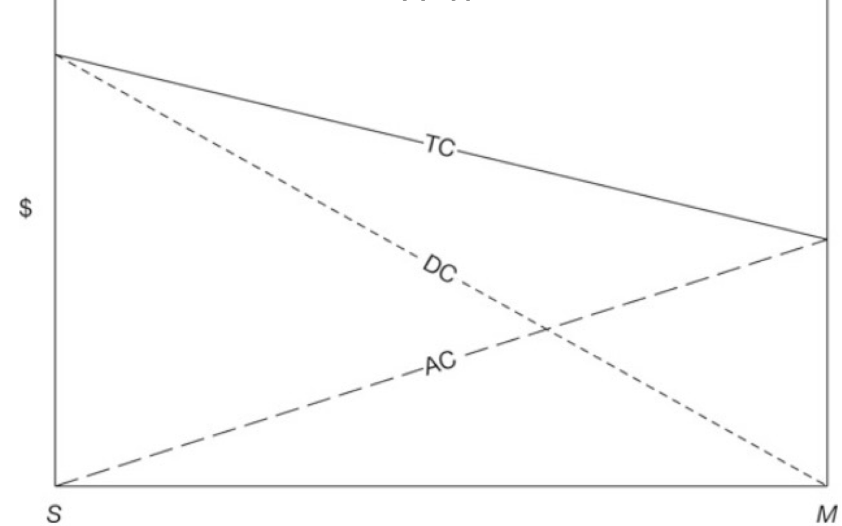
Distribution Costs → $DC = txd$

Total Transp. Costs → $T = tX(D-d) + txd$

Resource Oriented Firm
$X>x$

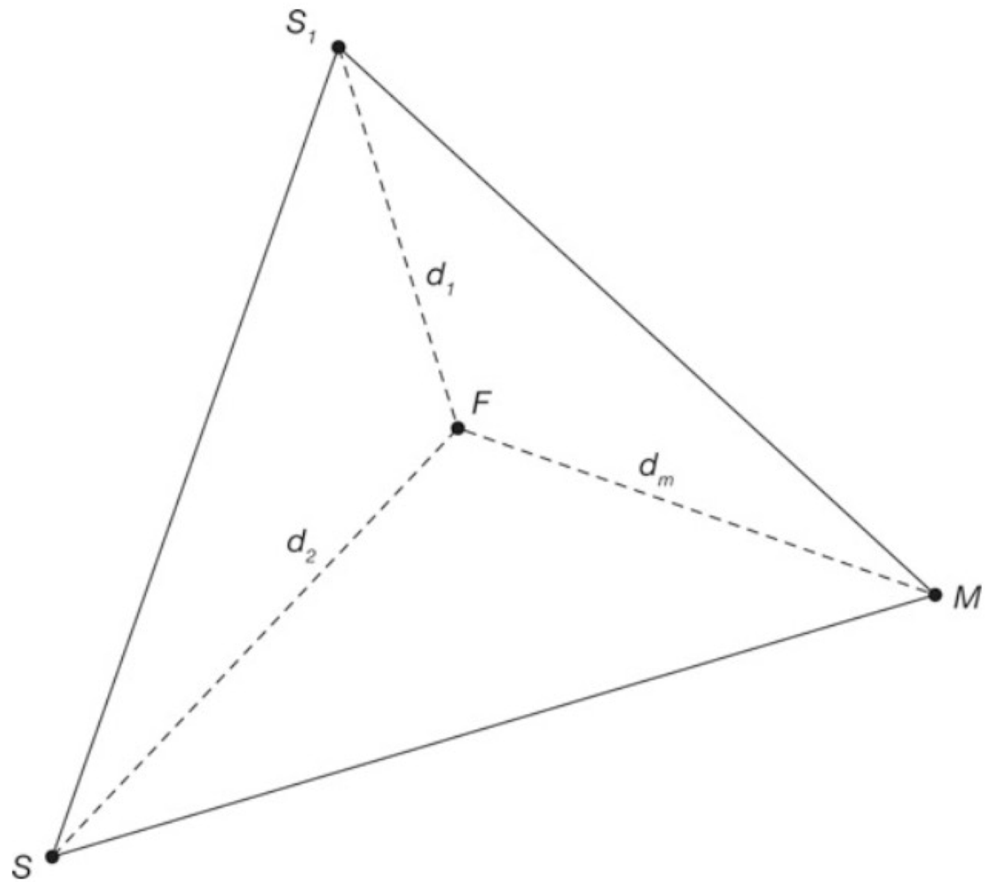


Market Oriented Firm
$X<x$



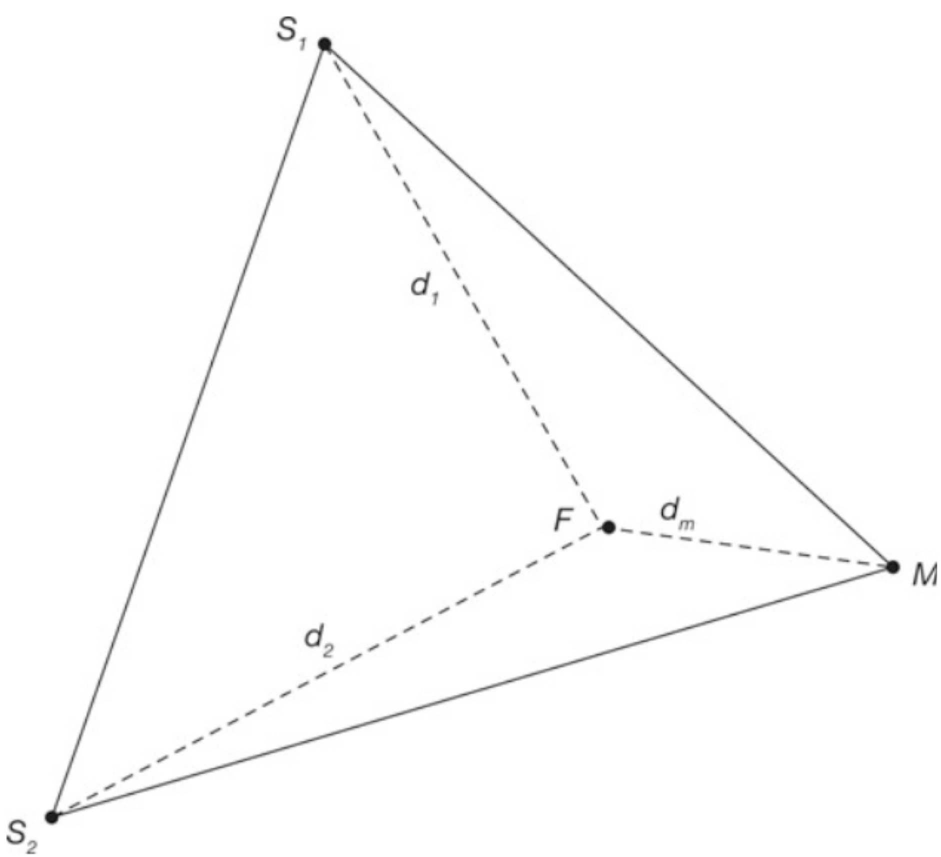
Location on a Plane: Weber Triangular Model

$$T = X_1 d_1 t + X_2 d_2 t + x d_m t$$

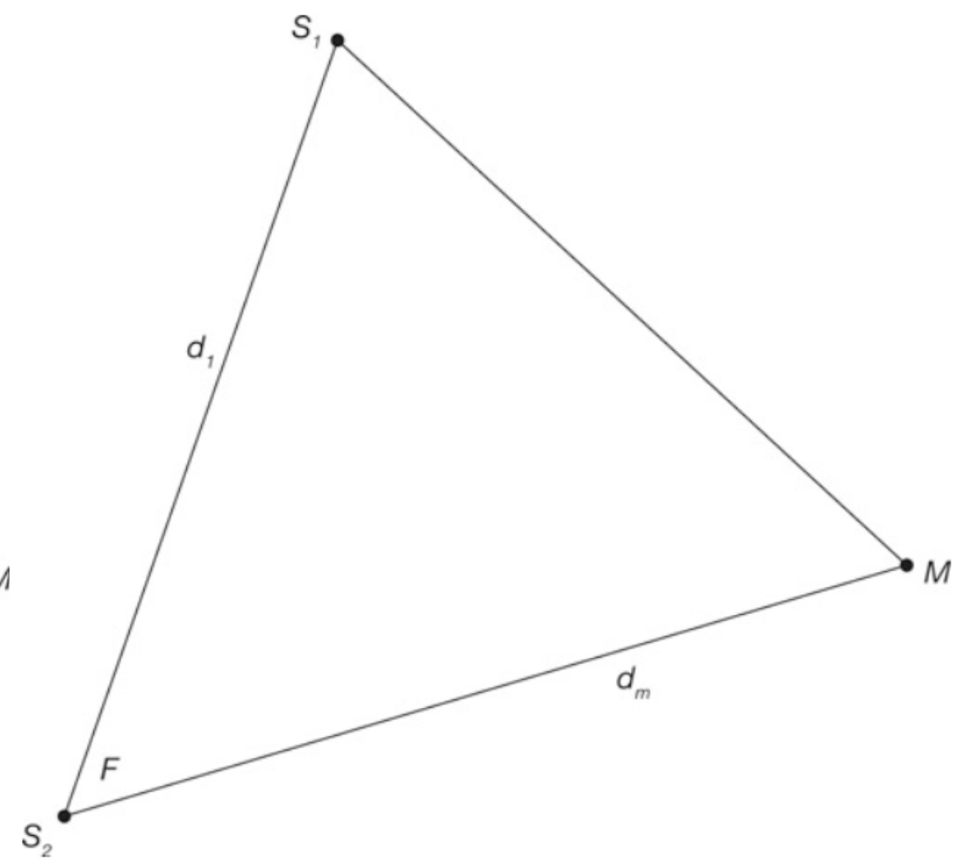


Location on a Plane: Weber Triangular Model

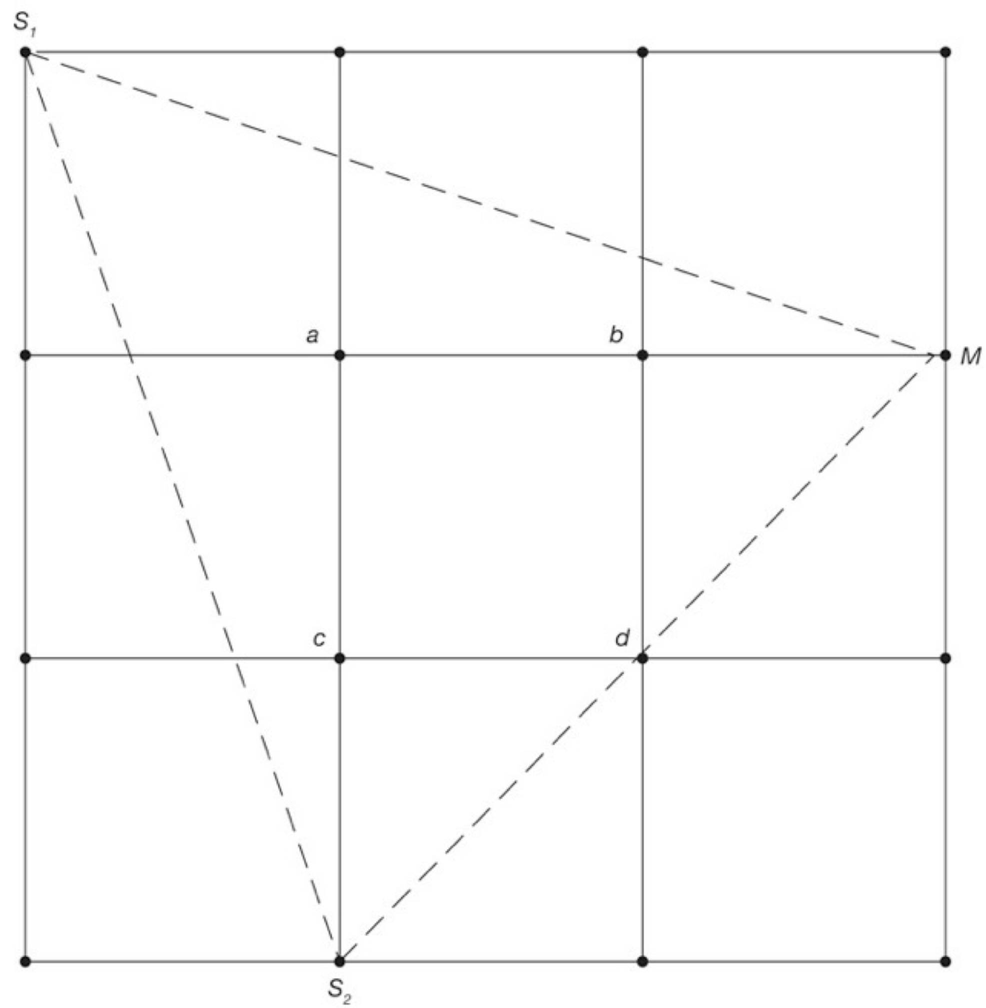
X1=X2 and x>X
but x < X1 + X2



X2 > X1 + x

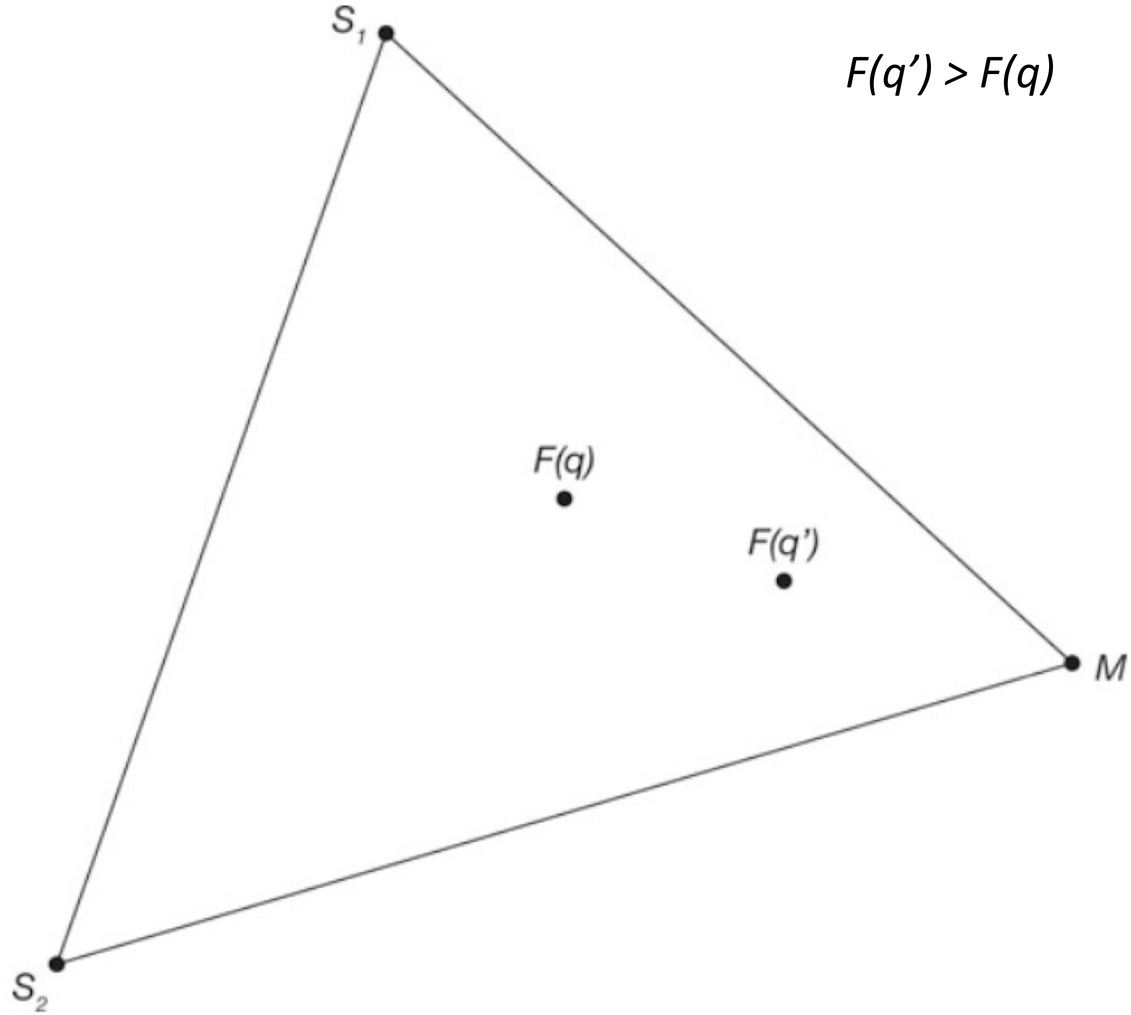


Location on a Network: Weber Triangular Model in a Network

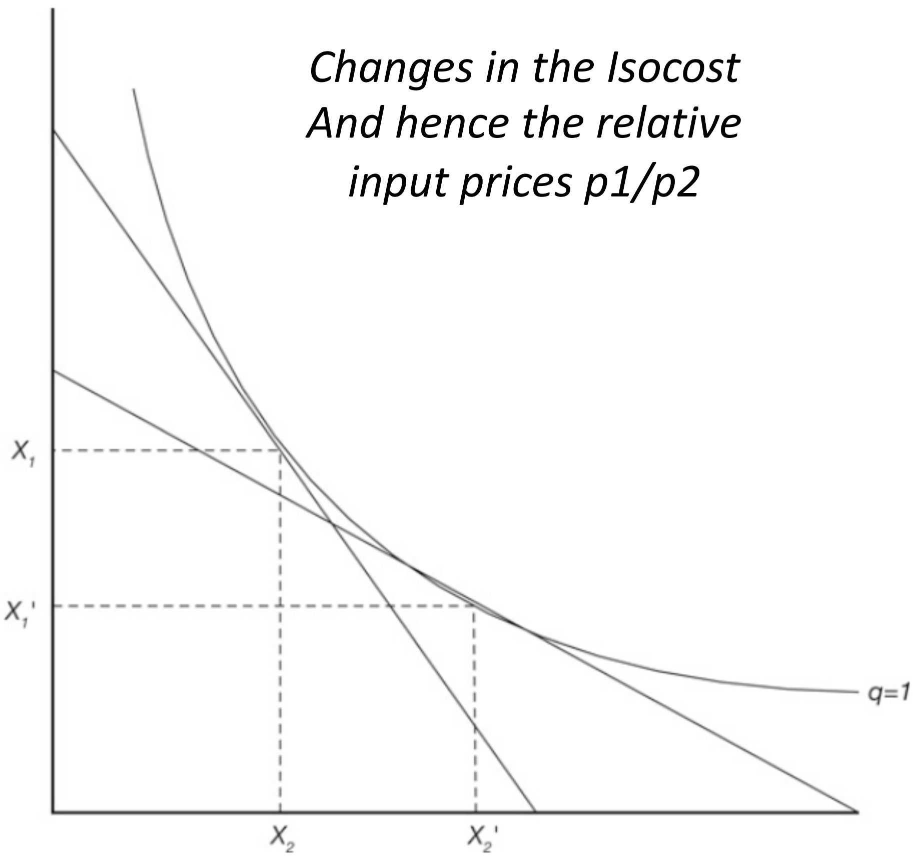


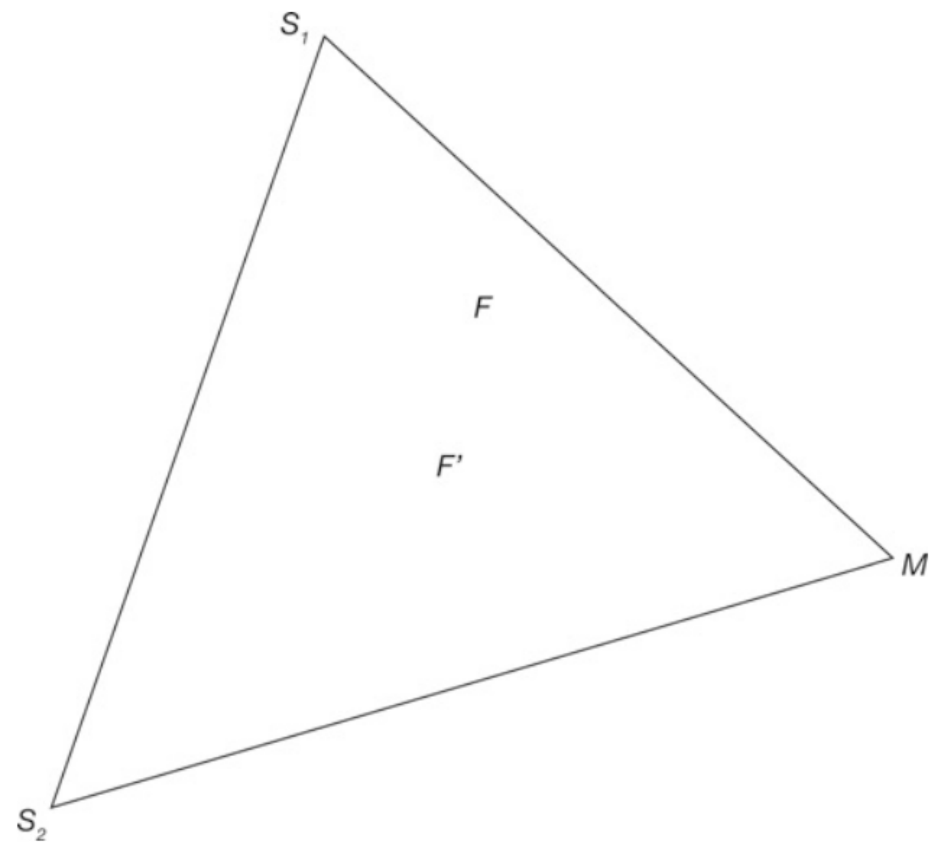
Weber Triangular Model with Economies of Scale

F(q') > F(q)

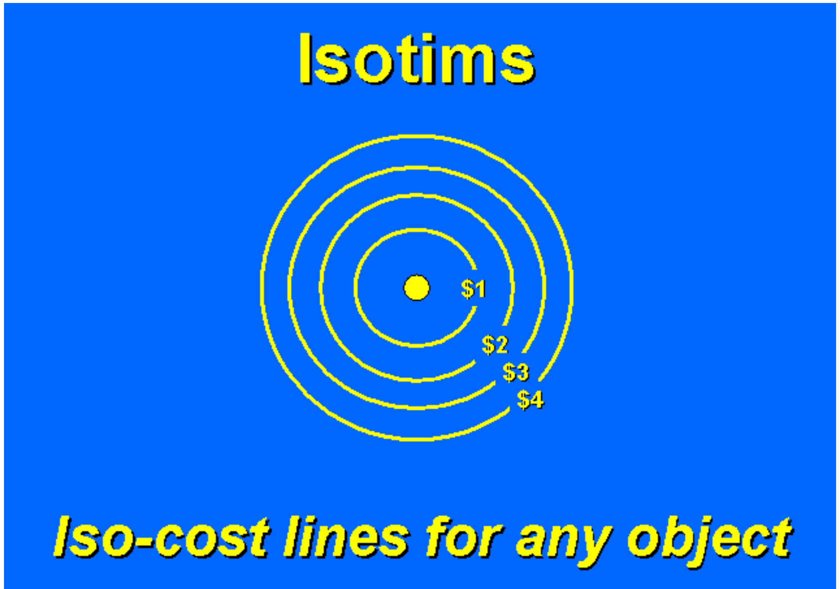


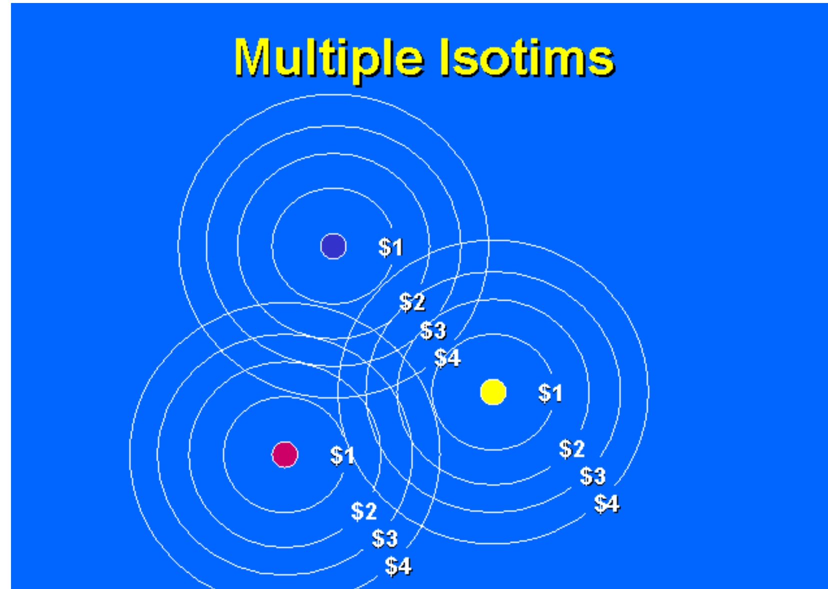
Weber Triangular Model with Economies of Scale

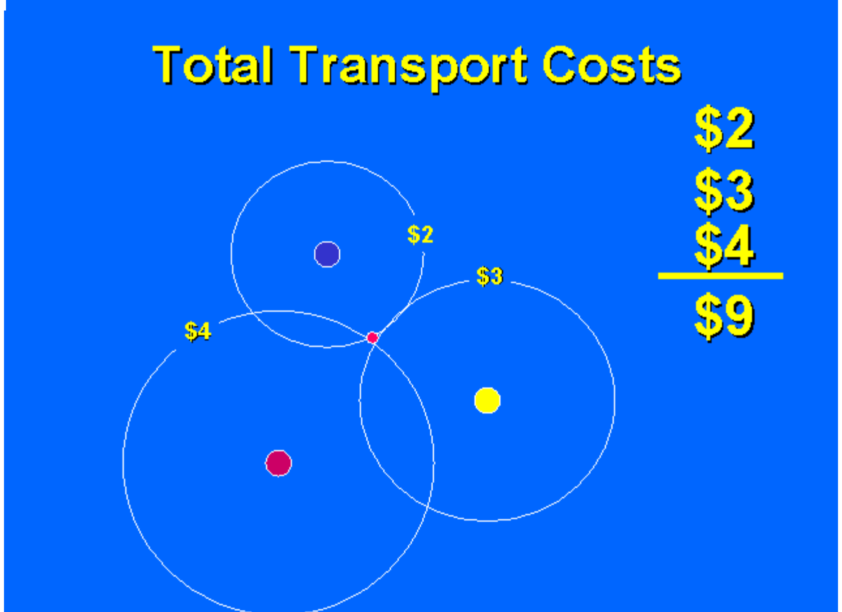


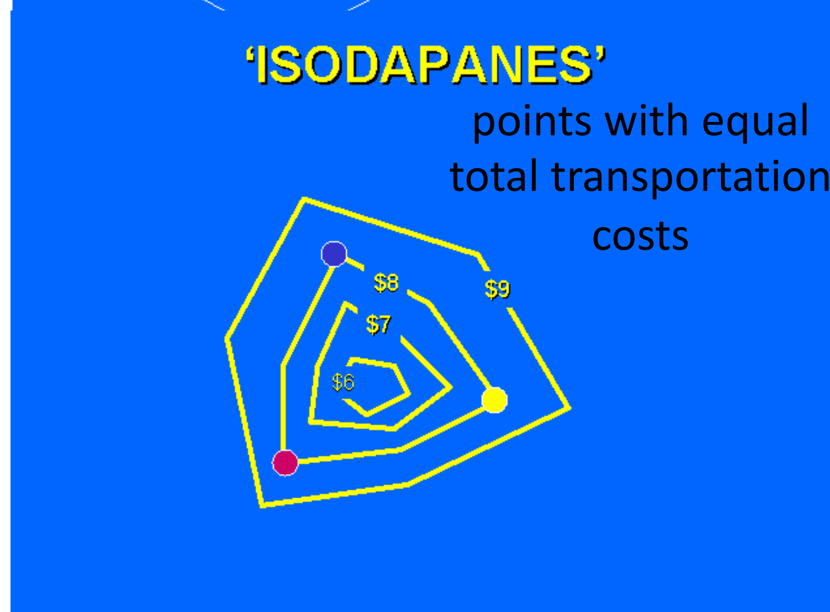


Isotims and Isodapanes









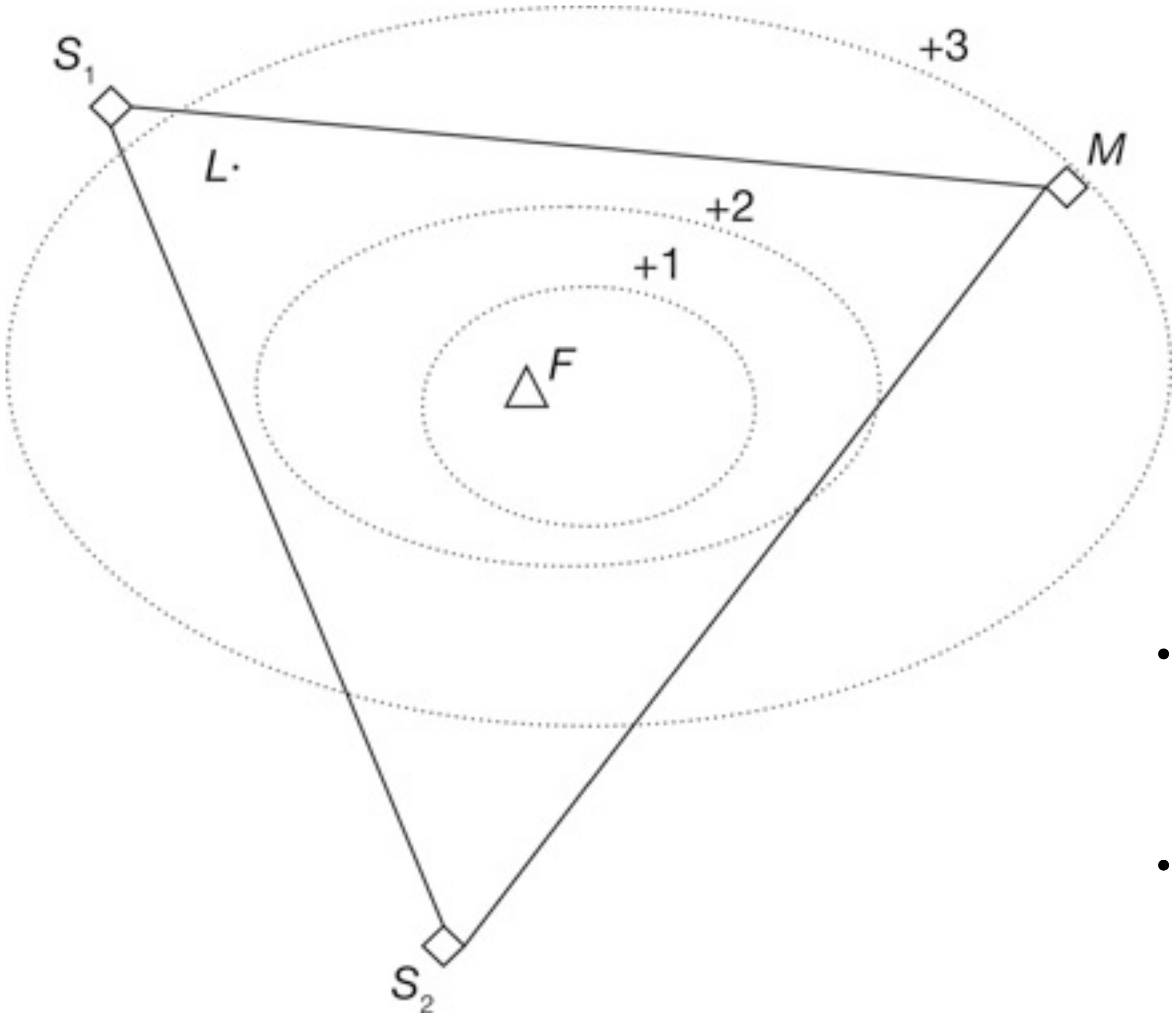
Weber Linear Model (Simple)

- Labor cost Differ across space
  - S: Location at the Source
  - M: Location at the Market
  - t: Transportation costs
  - X: input weight
  - x: output weight
  - D: total distance between S and M
  - d: unit of distance

Weber Linear Model + Labor Costs

- Labor cost Differ across space
  - S: Location at the Source
  - M: Location at the Market
  - t: Transportation costs
  - X: input weight
  - X: output weight
  - D: total distance between S and M
  - d: unit of distance
  - w_s and w_m, wages at source and market respectively

Weber Linear Model + Labor Costs

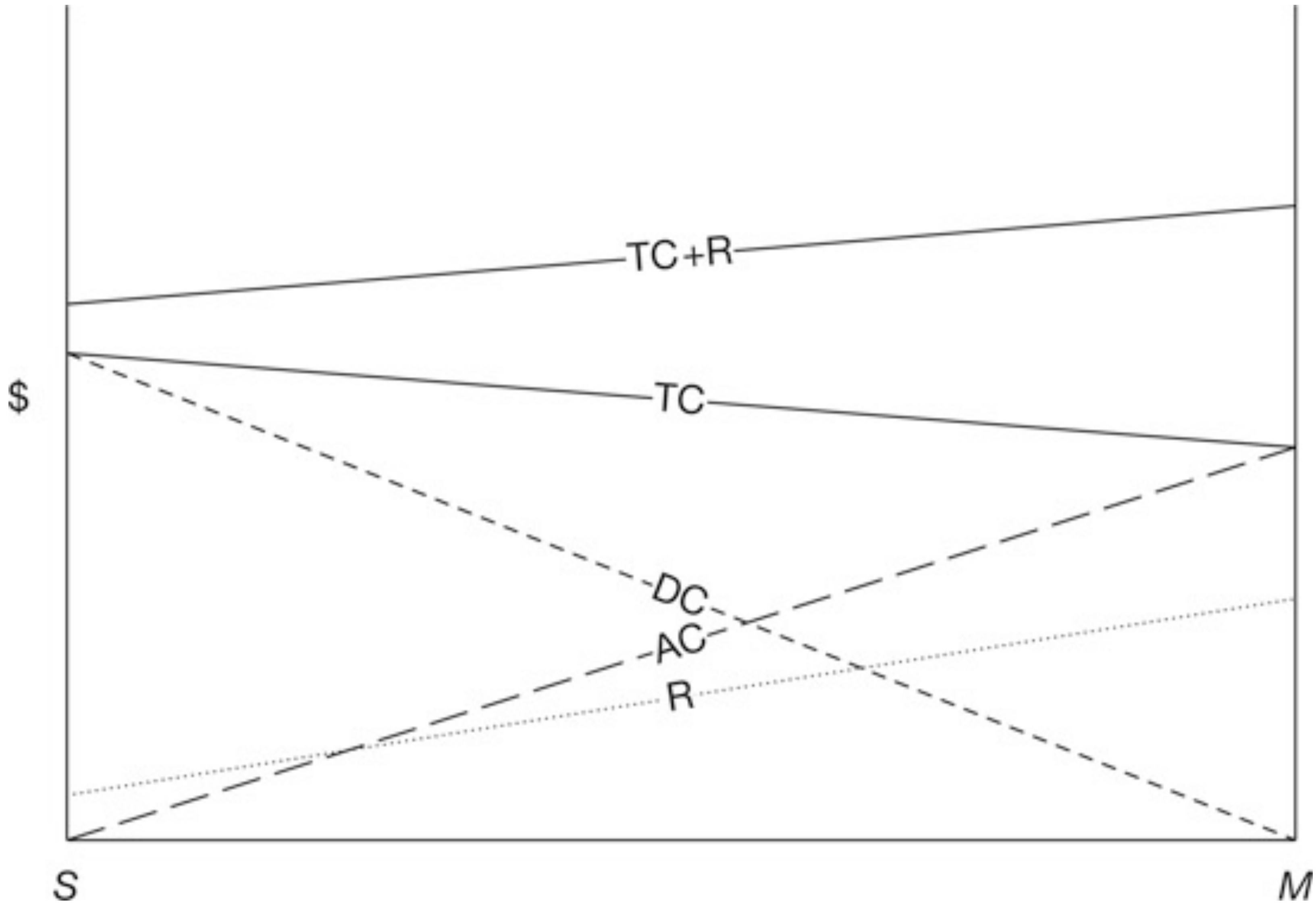


- Assume labor costs are the same except at *L*
  - *Unemployment*
  - *Wage subsidy*
- Pick *L* or *F*?
  ➔ *depends of the size of the labor savings at L*

Weber Linear Model + Rent Costs

- Labor cost Differ across space
  - *S*: Location at the Source
  - *M*: Location at the Market
  - *t*: Transportation costs
  - *x*: input weight
  - *X*: output weight
  - *D*: total distance between *S* and *M*
  - *d*: unit of distance
  - *Rd*: Rent at distance *d*
  - Rm: Rent at Market (highest)
  - *r*: rate at which rent falls per one unit of *d*

Weber Linear Model + Rent Costs



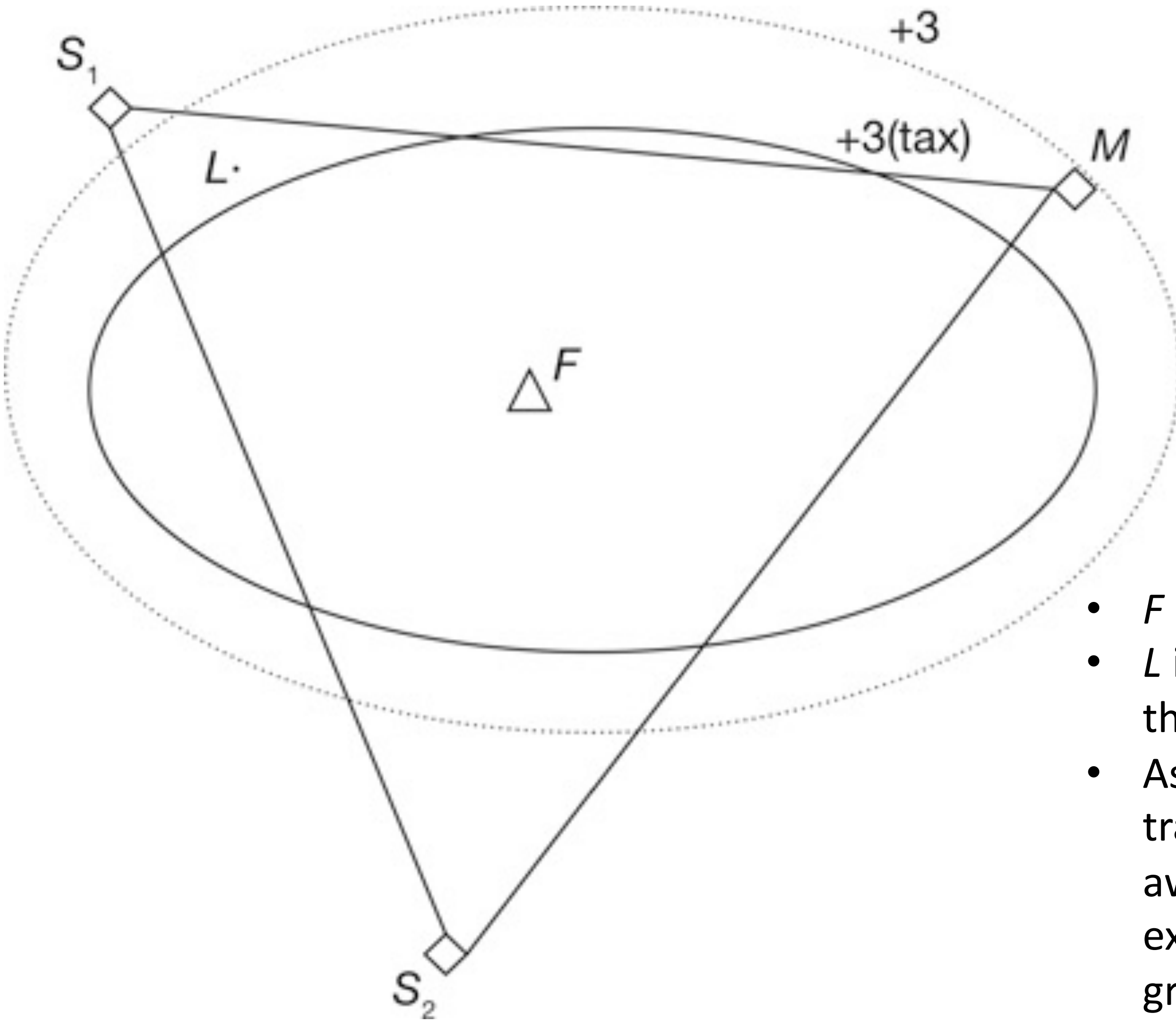
Weber Linear Model + Taxes

- Labor cost Differ across space
  - *S*: Location at the Source
  - *M*: Location at the Market
  - *t*: Transportation costs
  - *x*: input weight
  - *X*: output weight
  - *D*: total distance between *S* and *M*
  - *d*: unit of distance
  - *Φ*: Fixed/constant amount of tax
  - *φ*: Variable tax rate per unit of distance *d*

Weber Linear Model + Taxes

- Case 1: Undifferentiated transportation taxes
  - Fixed amount across space
  - Does NOT affect the decision to locate ($x>X$ → Market | $x<X$ → Source)
- Case 2: Differentiated transportation taxes
  - Variable amount across space
  - Affects the decision to locate depending of the costs of transport
  - ($\Phi x>X$ → Market | $x<\Phi X$ → Source)
  - ($\varphi dtx>\varphi dTX$ → Market | $\varphi dtx<\varphi dTX$ → Source)

Weber Linear Model + Taxes



- *F* is the lowest cost location
- *L* is where labor cost $3 lower than everywhere else
- As taxes are imposed on transportation costs, driving away from F becomes more expensive, hence covering less ground and making the Isodapane smaller

Empirical analysis of the Weber model
“Hub airport location in air cargo system”

Watanabe, et al. (2008), SICE Annual Conference

- Objective: how changes in transportation costs affect a single hub airport location of air cargo in the U.S.?
- Features: non-linear transportation costs taking into consideration the cost of fuel, volume, and distance rates.
- Where the market is located, is an important determinant of hub airport locations.

- The demand is concentrated mainly in the east side of the country, then, the Weber optimum point has moved to the east.

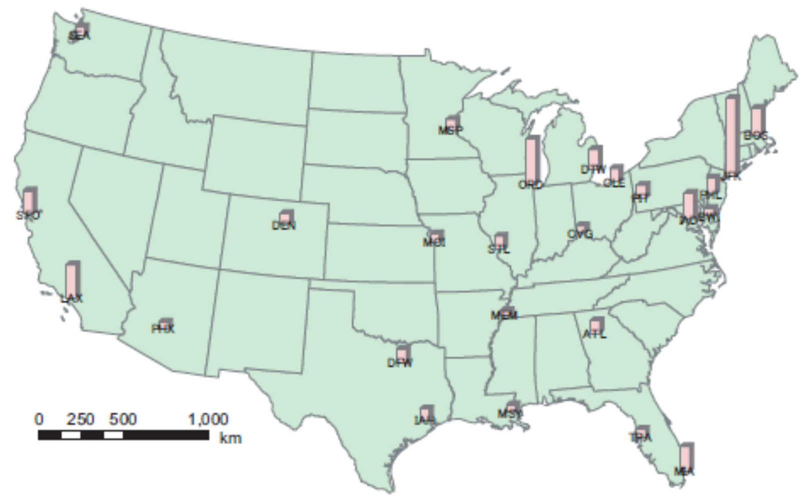


Fig. 6 Distribution of Demand

Empirical analysis of the Weber model
“Hub airport location in air cargo system”

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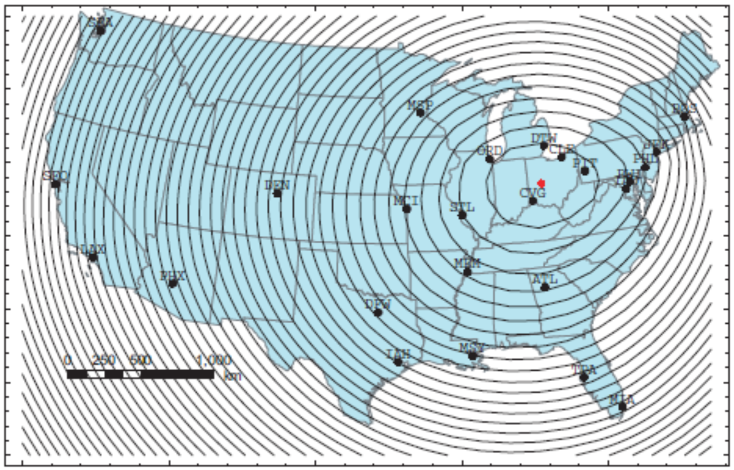

Fig. 7 Weber point

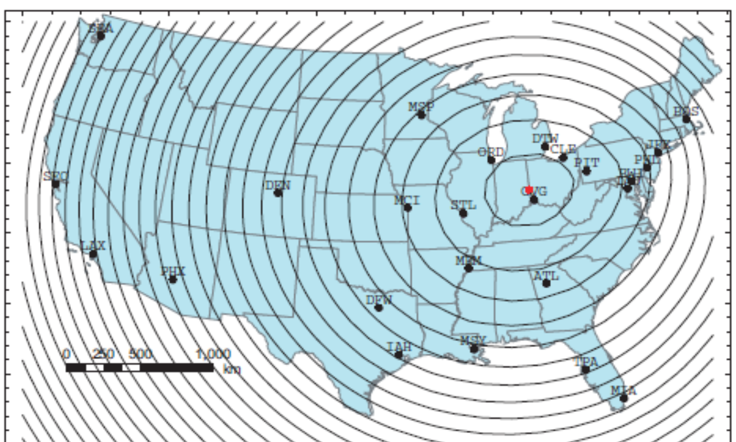

Fig. 8 Optimal point in 1999

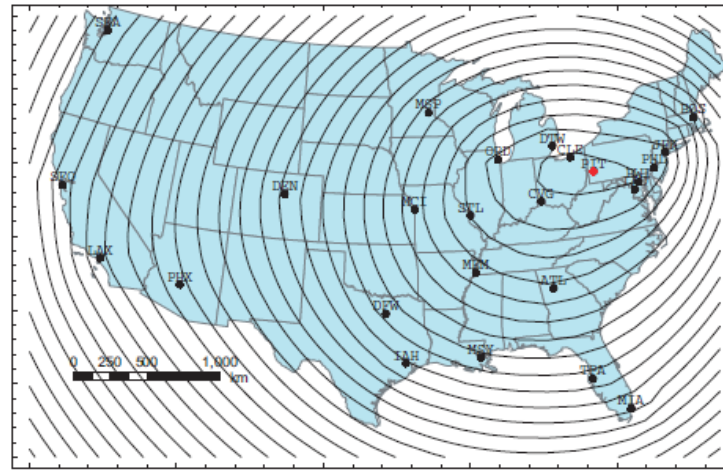

Fig. 9 Optimal point in 2002

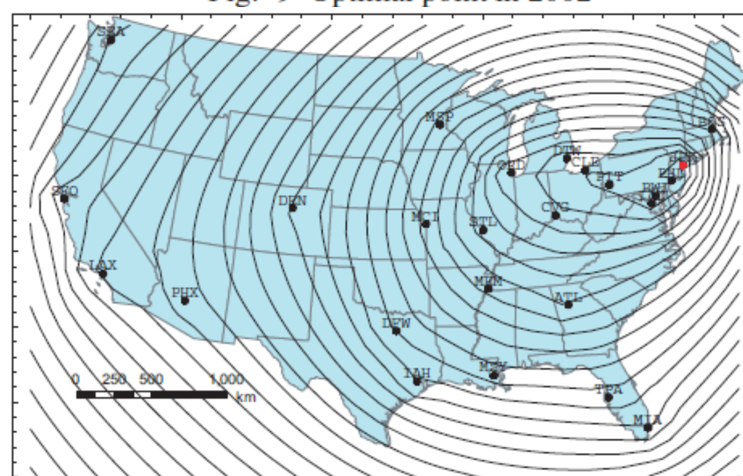

Fig. 10 Optimal point in 2007

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GLOBAL AUTOCORRELATION

Slides inspired on Luc Anselyn's Lectures on Spatial Data, Spatial Analytics, and Spatial Data Science
Watch Luc's video on: https://youtu.be/MmCYeJ27DsA

Spatial Autocorrelation

- Main idea: Combining…
  - Variable/attribute similarity
    - Cross products, squared differences, etc.
  - Locational similarity
    - Spatial Weight matrices
- Two main types:
  - Global autocorrelation
    - Big picture of spatial interaction – clustering
      - One statistic for all - regression diagnostics
  - Local autocorrelation
    - Local picture of spatial interaction – clusters
      - hot/cold spots, spatial dynamics

Global Autocorrelation – Moran's I

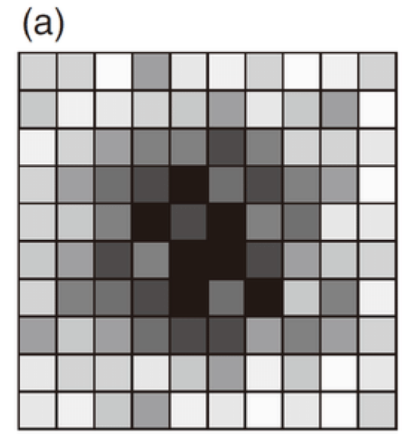
- Most used of many other statistics
- $I = \left(\sum_i \sum_j w_{ij} \frac{z_i \cdot z_j}{S_0}\right) / \left(\sum_i z_i^2 / N\right)$
  - where, $z_i = y_i - \bar{y}$ (deviations from the mean)
    - Inference only works if
      - Assumption 1: constant mean? -> take dev. from the mean
      - Assumption 2: constant variance? -> especially important with rates
- $z_i \cdot z_j$ = cross product (correlation like)
  - Measures similarity (not dissimilarity)
- Type of weights change its value
- S_0 = number of actual neighbors given by W
  - Used instead of N^2

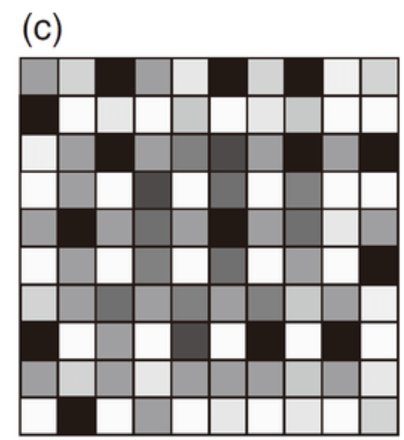
Global Autocorrelation – Moran's I

- How to do Inference?
  - A way assess if value obtained statistic from the data is significantly different than the null (spatial randomness)
- Analytical approach:
  - Assume normal distribution
- Computational approach (permutation)
  - Step 1: create a random variable (reference dist.)
  - Step 2: Compare value to the reference dist.
  - Step 3: randomize again and do step 2 several times
- Must be used with z-values
  - To assure comparability across variables and spatial weights
  - Z-values: z = [Observed I – Mean (I)]/ Std. Dev (I)

Moran’s I interpretation

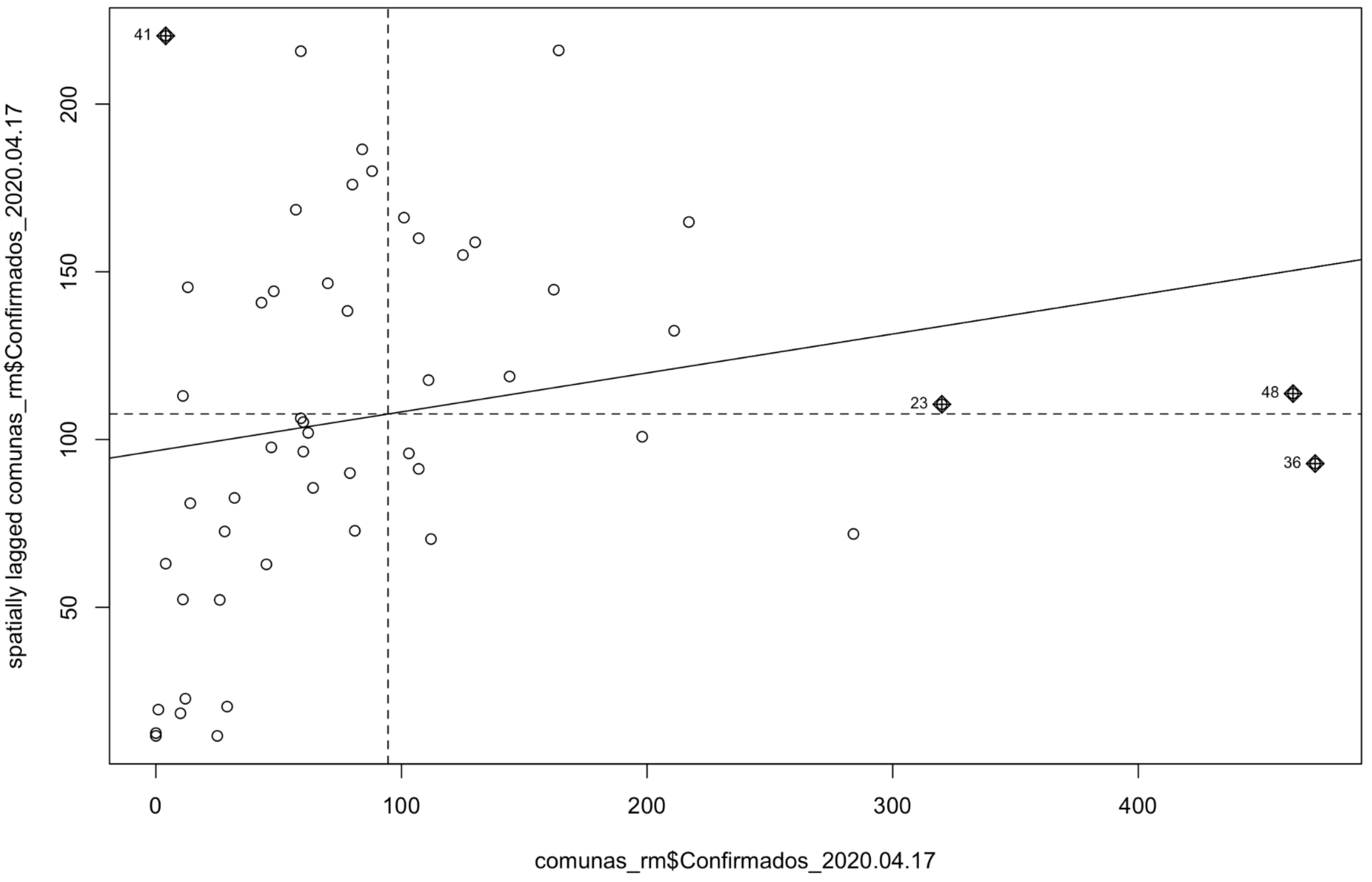
- Sign
  - Theoretical mean:
    - I/(N-1)
    - Zero for very large N
  - Positive and significant
    - Clustering of similar value
    - Not clustering of high/low values (could be either)
  - Negative and significant
    - Alternating values
    - Spatial heterogeneity
  - Global not local pattern!
  - If not significant = Spatial random
    - (doesn’t have a sign!)





Moran's I Scatter Plot

- $I = \left(\sum_i \sum_j w_{ij} \frac{z_i \cdot z_j}{S_0}\right) / \left(\sum_i z_i^2 / N\right)$
  - For row-standardized weights ➔ S_0 = N
- $I = \frac{\left(\sum_i \sum_j w_{ij} z_i \cdot z_j\right)}{\left(\sum_i z_i^2\right)},$
- $I = \frac{\left(\sum_i z_i \left(\sum_j w_{ij} \cdot z_j\right)\right)}{\left(\sum_i z_i^2\right)},$
  - Moran's I is the slope in a regression of $\sum_j w_{ij} \cdot z_j$ on z_i
  - Spatial_lag (y) ~ y ➔ $\sum_j w_{ij} \cdot z_j \sim z_i$
- Scatterplot $[z_i, \sum_j w_{ij} \cdot z_j]$



41
23
48
36
200
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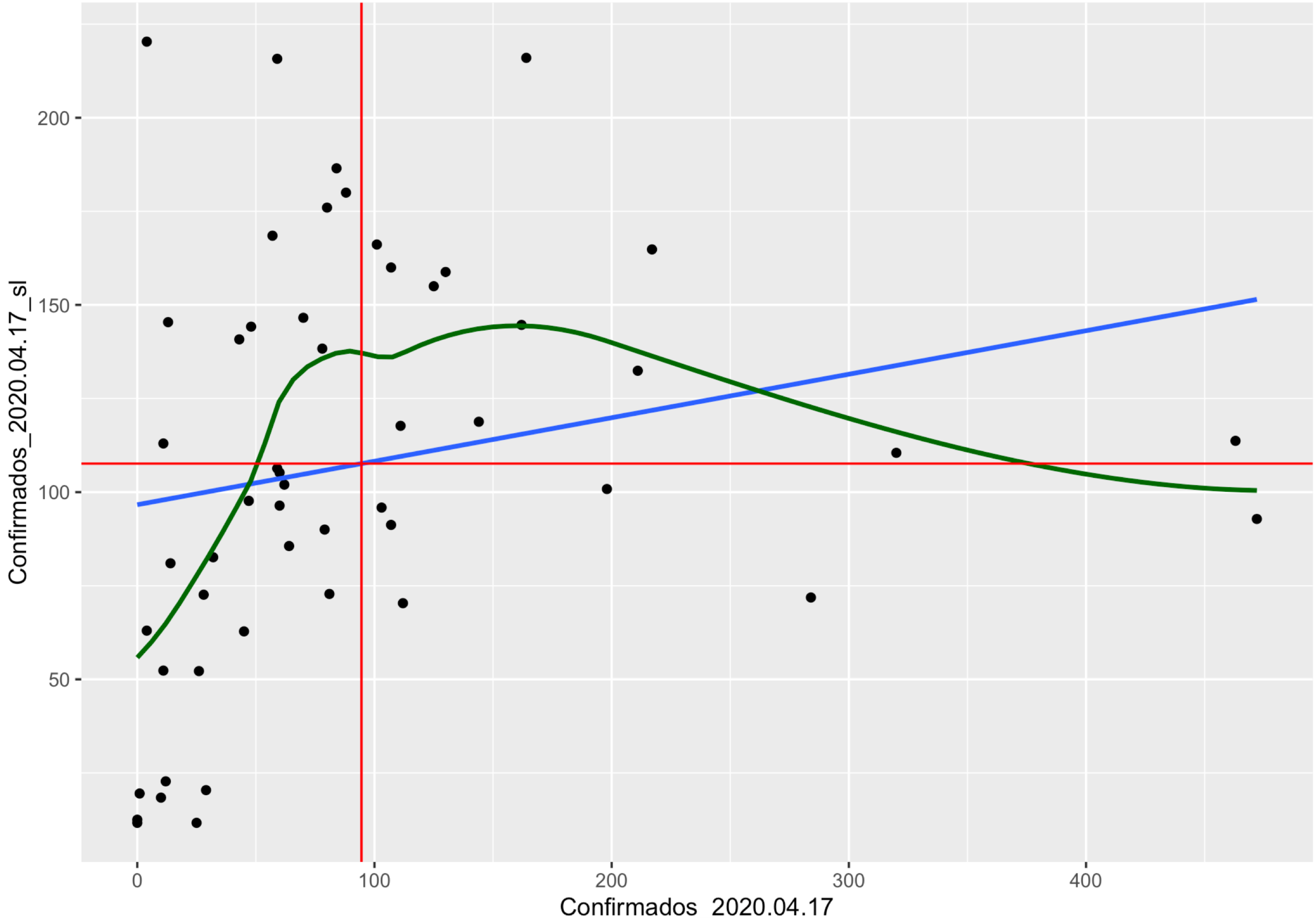

200
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Local Spatial patterns

- Local Indicator's of Spatial Association (LISA)
  - Local spatial autocorrelation
  - Where clusters are?
  - Hot/cold spots
  - Are these clusters significant?

- Global

$$I = \frac{\left(\sum_i z_i \left(\sum_j w_{ij} \cdot z_j\right)\right)}{\left(\sum_i z_i^2\right)} = \frac{\sum_i z_i}{\left(\sum_i z_i^2\right)}\left(\sum_j w_{ij} \cdot z_j\right)$$

Local

$$I_i = \frac{z_i}{\left(\sum_i z_i^2\right)}\left(\sum_j w_{ij} \cdot z_j\right),$$

  - Sum of all I_i divided by N = I
    - *Average of all local*
- Inference:
  - Also based on 'conditional' permutation (for a particular location)
- Result:
  - Local significant Map

Local Moran's I