Stone skipping

Numerical Analysis Course - Midterm Project

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Physical behaviour of Stone Skipping

Equations of Motion (ODEs):

•Position equations:

dx/dt = vx

dy/dt = vy

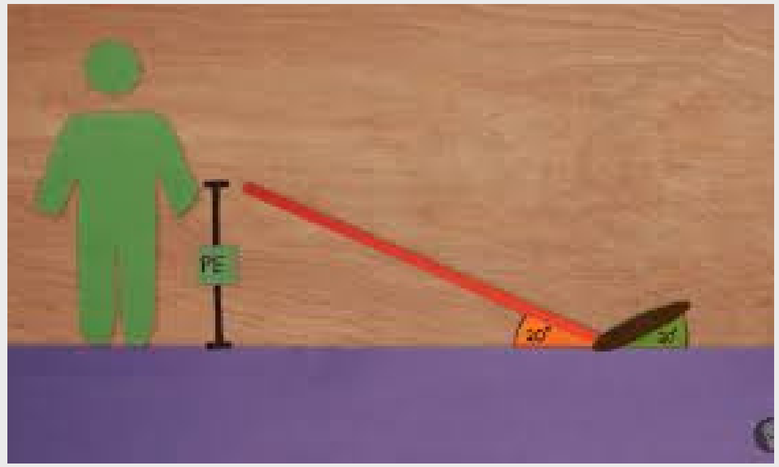
dz/dt = vz

•Velocity equations:

dvx/dt = -cd*vx/m

dvy/dt = -cd*vy/m

dvz/dt = -g - cd*vz/m



Key Factors Affecting Stone Skipping

1) Initial Velocity (Launch Speed) : Higher speed gives more energy, helping the stone to bounce off the water surface multiple times (at least 6–8 m/s for a successful skip but too high a speed can cause the stone to dig into the water).

2) Launch Angle (Elevation Angle) : Optimal angle: 10–20 degrees above horizontal.
Too steep: the stone dives into the water.
Too shallow: it doesn’t get lift to bounce properly. Perfect angle: 15° to 25°

3) Azimuth Angle (Direction) : The stone’s direction over the horizontal plane. Important for accuracy or hitting a target, but doesn’t affect the physics of skipping much unless wind or current are involved.

Key Factors Affecting Stone Skipping

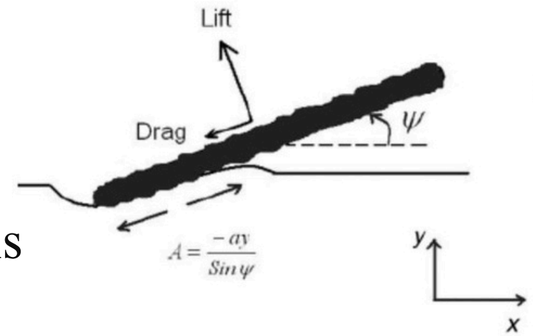
4) Spin (Angular Velocity) : A spinning stone is more stable and resists flipping.
Acts like a gyroscope, keeping the flat surface oriented toward the water.
Without spin, the stone may tumble and sink quickly.

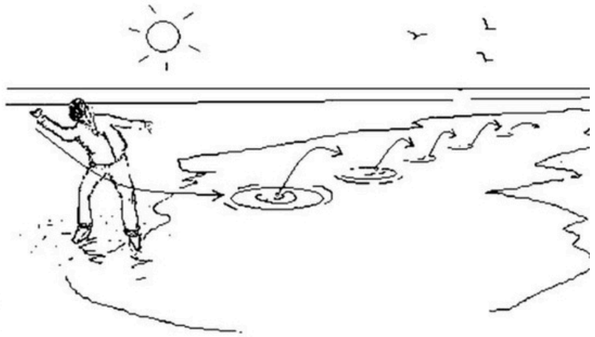
5) Water Surface Conditions
Calm water = more consistent skips.
Choppy water = unpredictable, fewer skips.

6) Restitution ‘r’ (Energy Loss on Impact) : Describes how much energy is retained after each bounce. Lower restitution = more energy lost = fewer skips.

7) Lift and Drag Forces : As the stone hits the surface, lift pushes it up, enabling the skip.
Drag slows it down — especially over many skips.

Mathematical Modeling & RK4 Method





- The stone's motion is modeled using differential equations based on Newton's laws.
- We account for gravity and air resistance 'cd' in 3D space.
- The motion is complex especially with impacts and energy loss, so we use a numerical method to solve it.

Why RK4 ?

- RK4 is a method used to simulate motion accurately.
- It calculates the next position of the stone using four intermediate steps, which makes it more accurate than simple methods like Euler's.

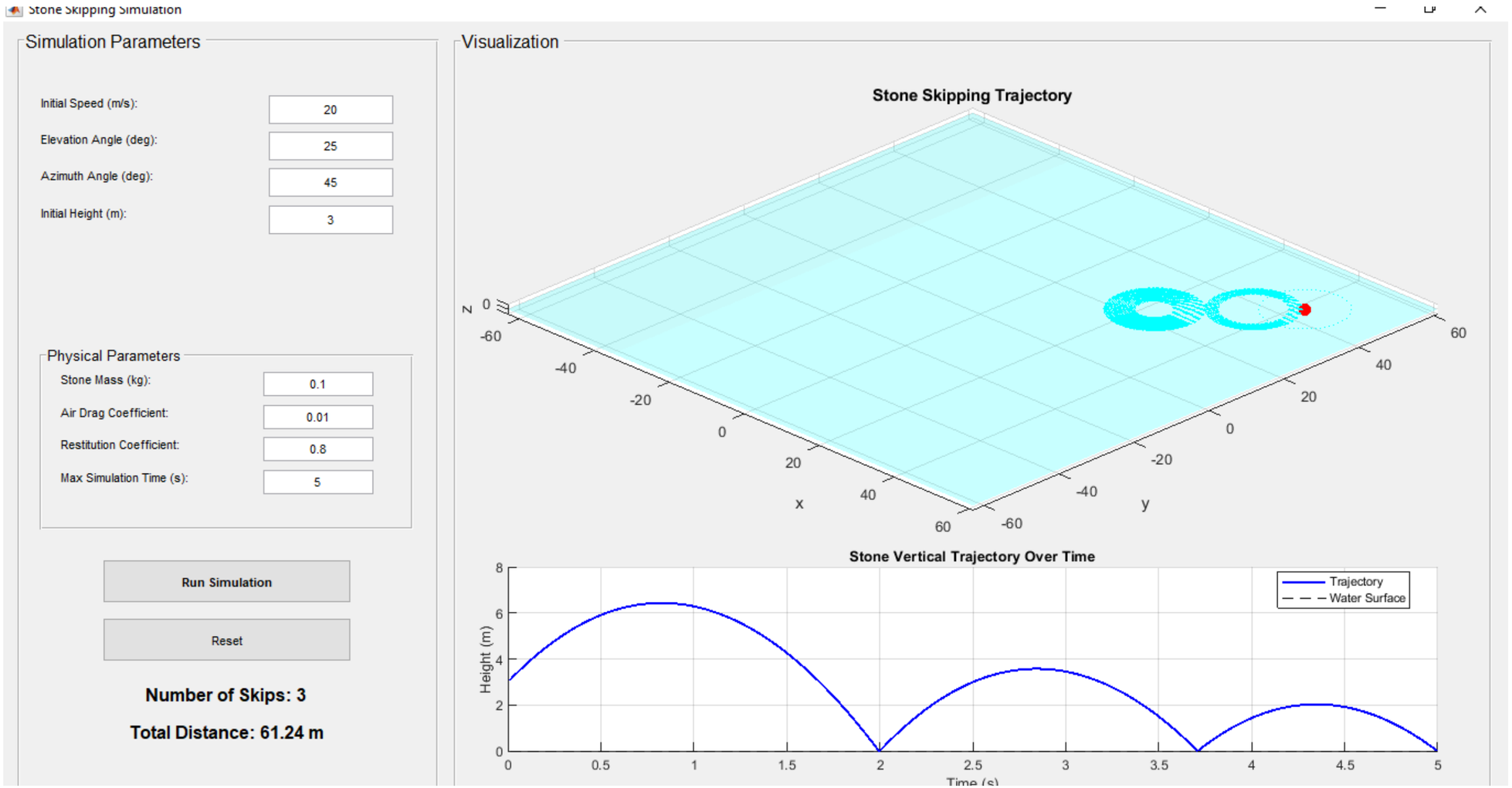
WORLD RECORD ×

Record Holder: Kurt Steiner (USA) 🎗️
Distance: 88 skips on a single throw
Year: 2013
Achieved using a highly tuned technique with optimal speed, angle, and spin.

MATLAB Code

1. Purpose of the Code: This code simulates the motion of a skipping stone in 3D using physics-based equations and visualizes the skips in real time.
2. What It Does: It takes user inputs like speed, angle, and height, and models how the stone flies, interacts with water, and skips using numerical integration.
3. Physics Inside:
It considers gravity ‘g’, air resistance (drag) ‘cd’, and energy loss on each skipvia restitution coefficient ‘r’.
4. Method: we used the RK4 (Runge-Kutta 4th Order) method to numerically solve the equations of motion accurately over time.
5. Visual Output: It shows the stone's trajectory, marks each skip, and draws water ripples at impact points — helping us understand how conditions affect skipping.
6. Extra Features: The water surface expands dynamically, and there's a second plot that shows how the height changes over time.

Simulation


Stone Skipping Simulation
Simulation Parameters
Initial Speed (m/s):
20
Elevation Angle (deg):
25
Azimuth Angle (deg):
45
Initial Height (m):
3
Physical Parameters
Stone Mass (kg):
0.1
Air Drag Coefficient:
0.01
Restitution Coefficient:
0.8
Max Simulation Time (s):
5
Run Simulation
Reset
Number of Skips: 3
Total Distance: 61.24 m
Visualization
Stone Skipping Trajectory
z
x
y
Stone Vertical Trajectory Over Time
Trajectory
Water Surface
Height (m)
Time (s)

THANK YOU

For more informations:
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