Fourier transform

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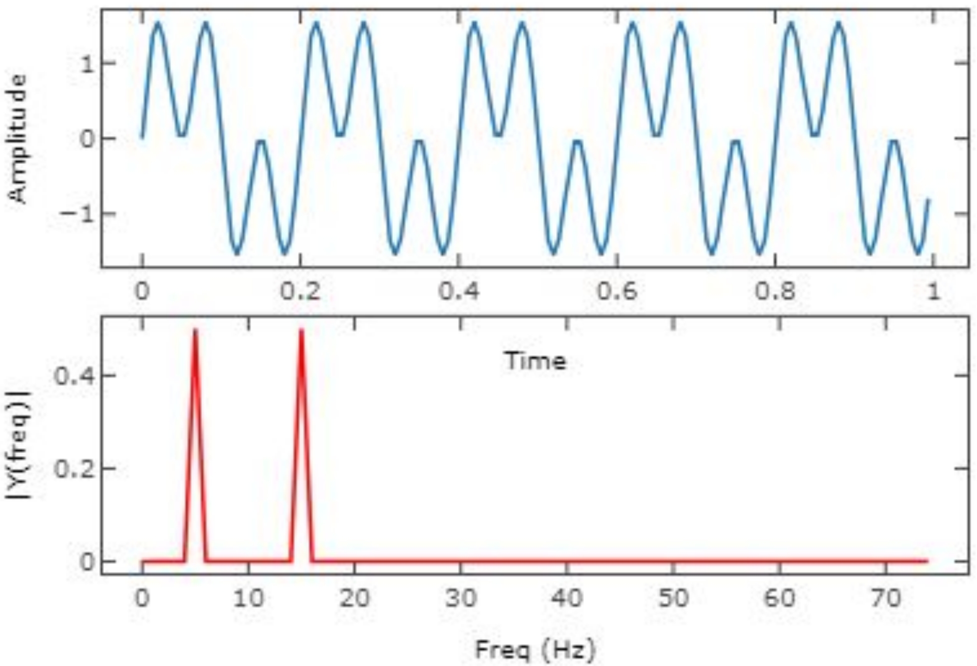
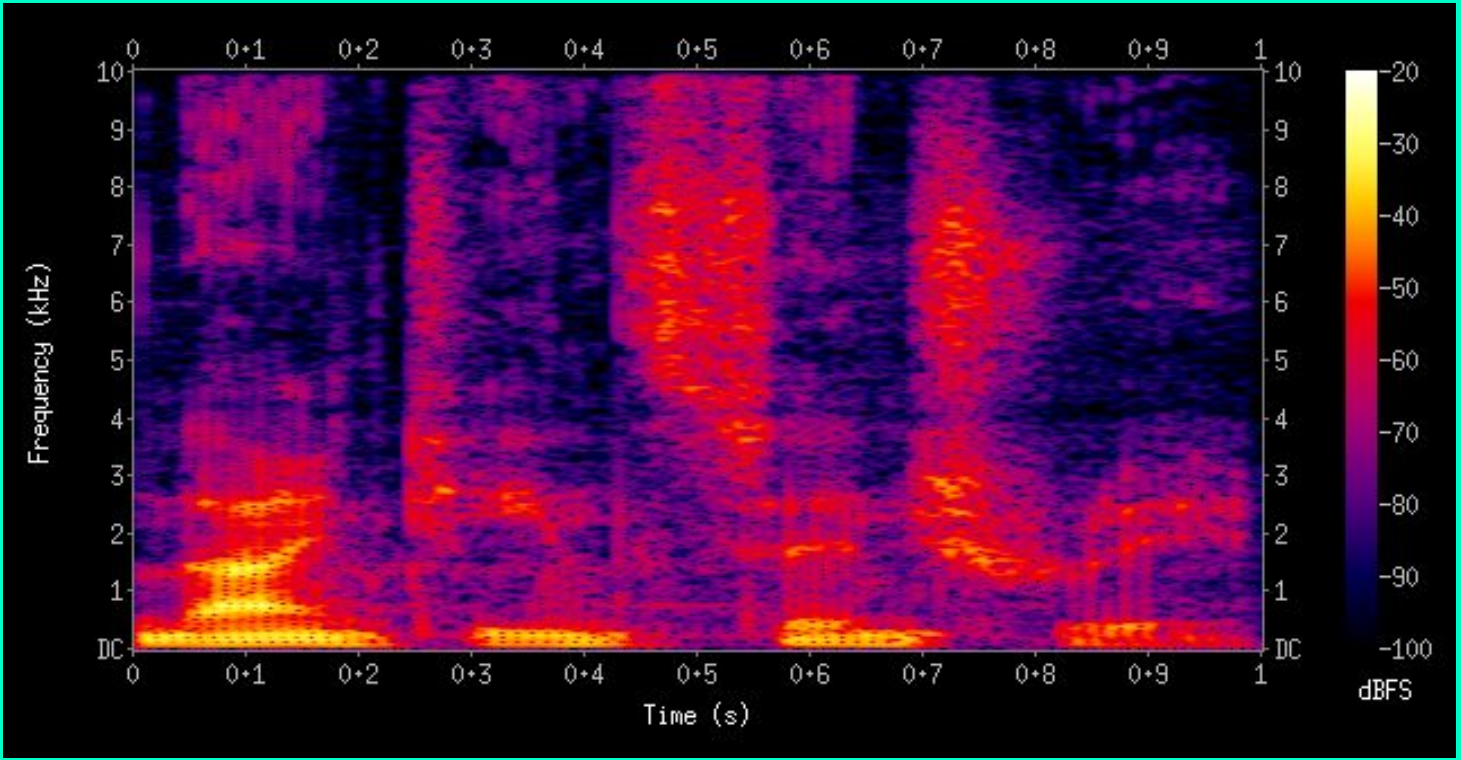
The point of the Fourier transform

If we have a time series of events, we may want to know it's spectrum-- what frequencies are present. This is called analysis, because we decompose the observed time series into a sum of components.

In the Fourier transform these components are sines and cosines, which are connected together in the complex exponential through the Euler formula (see next slides).



Amplitude
1
0
−1
0
0.2
0.4
0.6
0.8
1
Time
|Y(freq)|
0.4
0.2
0
0
10
20
30
40
50
60
70
Freq (Hz)


Frequency (kHz)
Time (s)
dBFS
0
0•1
0•2
0•3
0•4
0•5
0•6
0•7
0•8
0•9
1
DC
1
2
3
4
5
6
7
8
9
10
-20
-30
-40
-50
-60
-70
-80
-90
-100

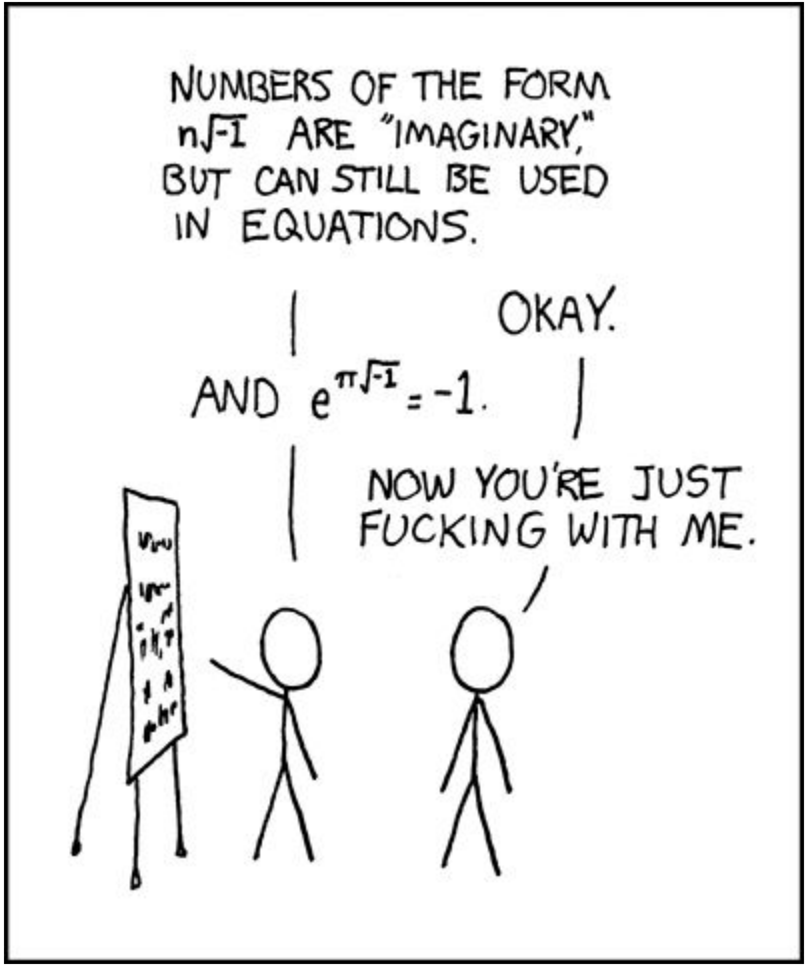
Euler formula

Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

Euler's identity

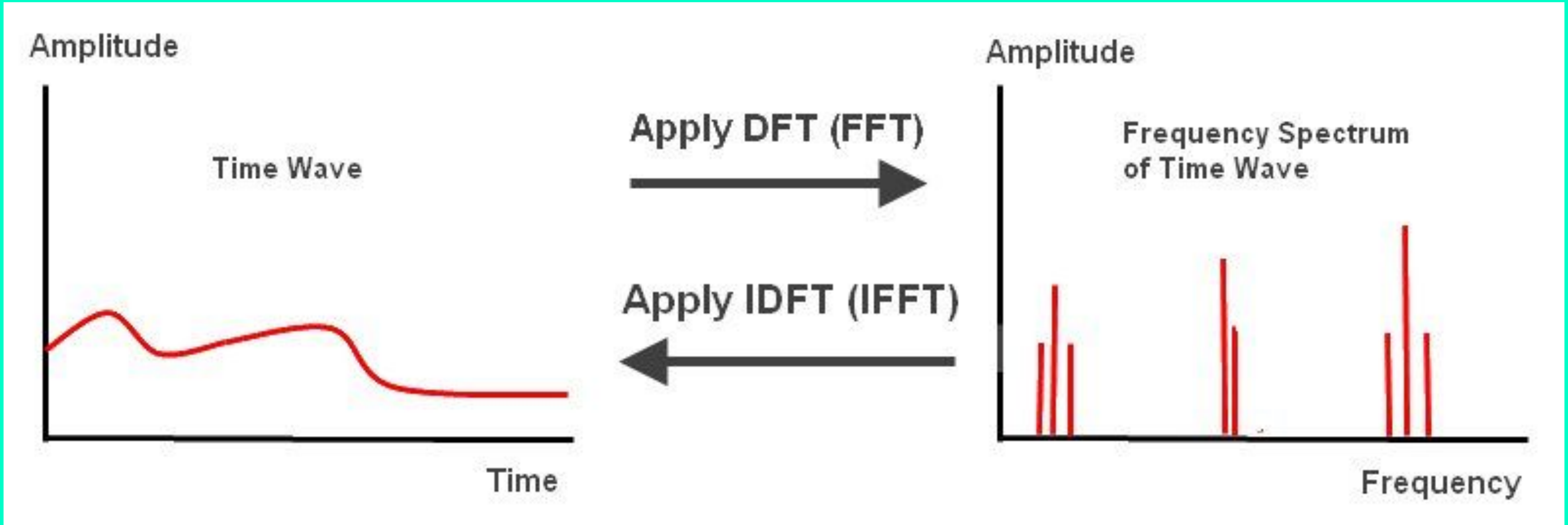
$$e^{i\pi} + 1 = 0$$


NUMBERS OF THE FORM $n\sqrt{-1}$ ARE "IMAGINARY," BUT CAN STILL BE USED IN EQUATIONS.
OKAY.
AND $e^{\pi\sqrt{-1}} = -1$.
NOW YOU'RE JUST FUCKING WITH ME.

TO TAKE THE FOURIER TRANSFORM WE CORRELATE COMPLEX EXPONENTIALS WITH DIFFERENT FREQUENCIES WITH THE SIGNAL TO GET THE FOURIER COEFFICIENTS WHICH REPRESENT HOW MUCH OF THAT FREQUENCY IS IN THE SIGNAL.

$$x[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

Reverse Fourier transform



Fast Fourier Transform (FFT)

The fast Fourier transform is one of the most important algorithms in this world. It uses a divide and conquer technique to compute the Fourier transform efficiently. Because it is fast (O(n*logn)), it can be used as a building block in efficient numerical analysis of large signals. In particular FFT and reverse FFT can be used to efficiently compute convolution-- you just multiply two signals in the Fourier domain and do a reverse FFT on the product.
We will see how the FFT is used to compute the cross-correlation of two spike trains.

For beautiful Fourier videos YouTube

"3blue1brown Fourier"