

FOURIER TRANSFORM

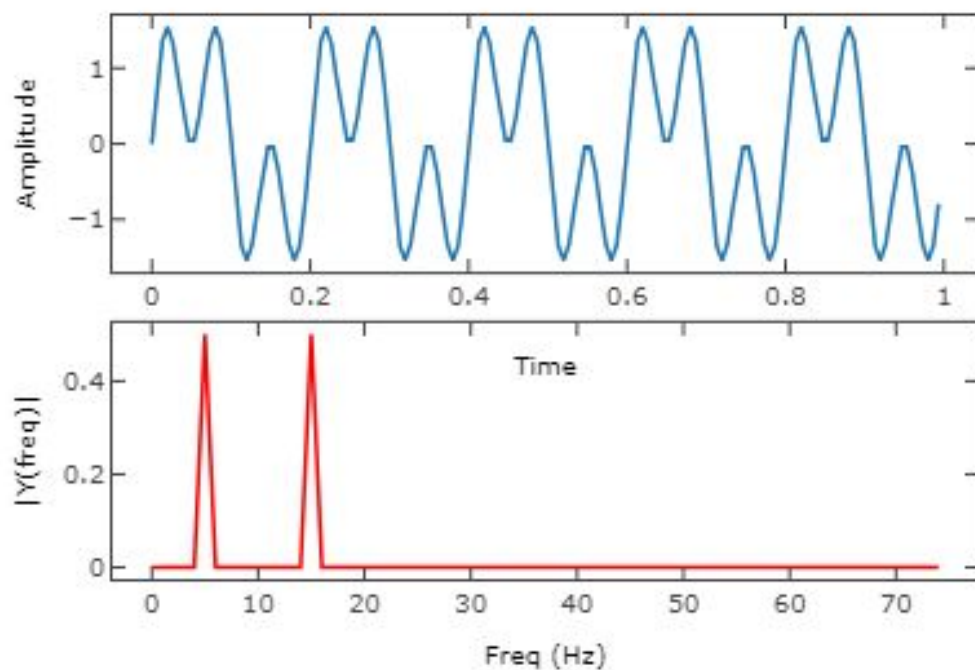
Maria Kesa

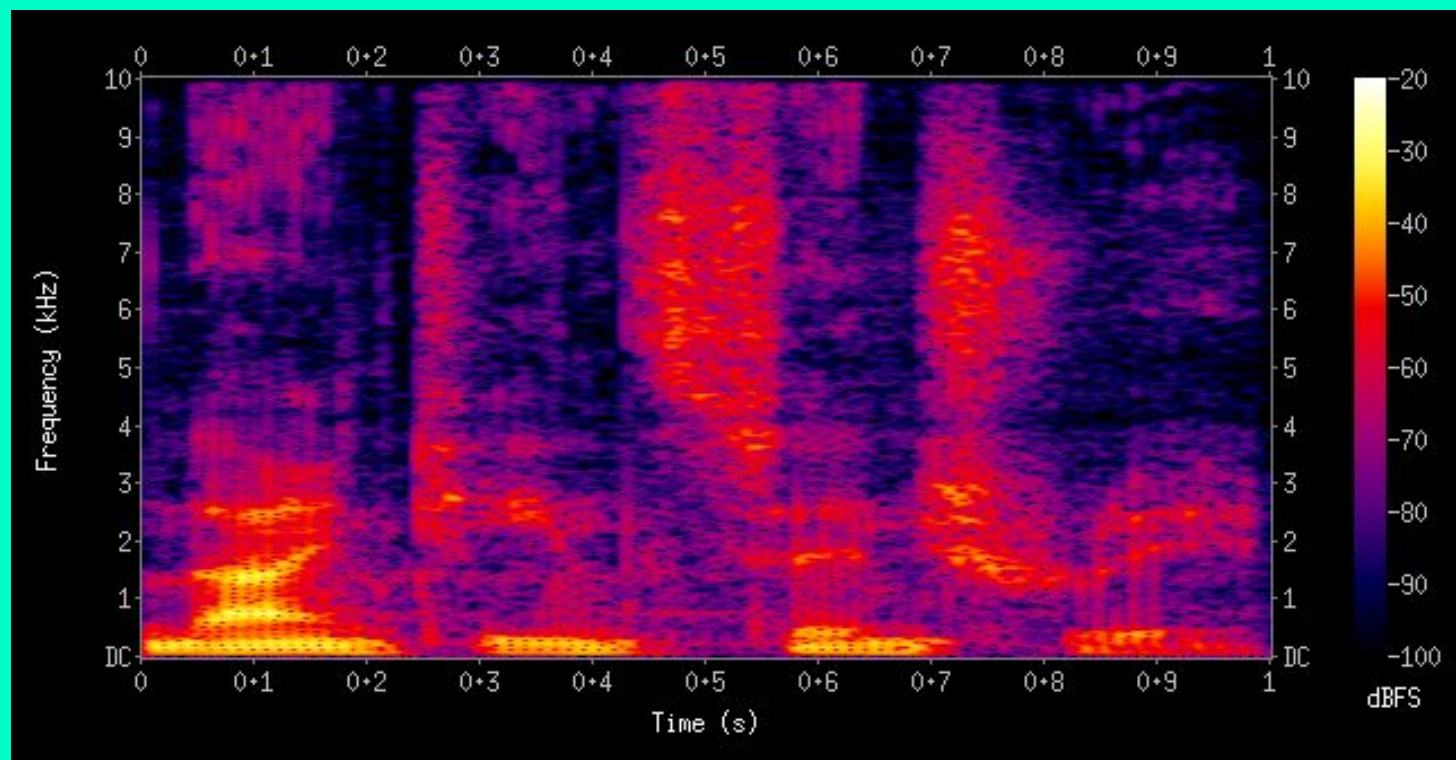


THE POINT OF THE FOURIER TRANSFORM

If we have a time series of events, we may want to know its spectrum-- what frequencies are present. This is called **analysis**, because we decompose the observed time series into a sum of **components**.

In the Fourier transform these **components** are sines and cosines, which are connected together in the complex exponential through the Euler formula (see next slides).







EULER FORMULA

Euler's Formula

$$e^{i\phi} = \cos \phi + i \sin \phi$$

Euler's identity

$$e^{i\pi} + 1 = 0$$

NUMBERS OF THE FORM
 $n\sqrt{-1}$ ARE "IMAGINARY,"
BUT CAN STILL BE USED
IN EQUATIONS.

OKAY.

AND $e^{\pi\sqrt{-1}} = -1$.

NOW YOU'RE JUST
FUCKING WITH ME.



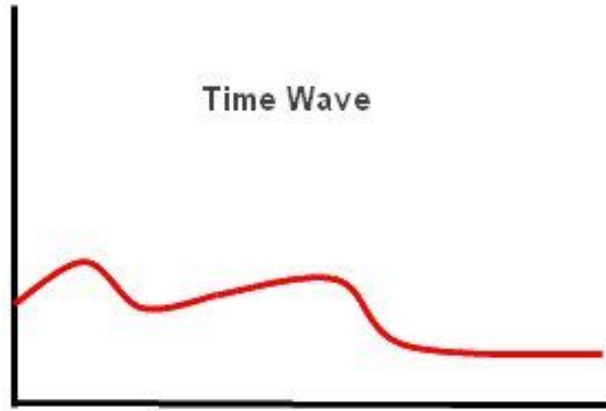
TO TAKE THE FOURIER TRANSFORM WE CORRELATE COMPLEX EXPONENTIALS WITH DIFFERENT FREQUENCIES WITH THE SIGNAL TO GET THE FOURIER COEFFICIENTS WHICH REPRESENT HOW MUCH OF THAT FREQUENCY IS IN THE SIGNAL.

$$x[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

Reverse Fourier transform

Amplitude

Time Wave



Time

Apply DFT (FFT)

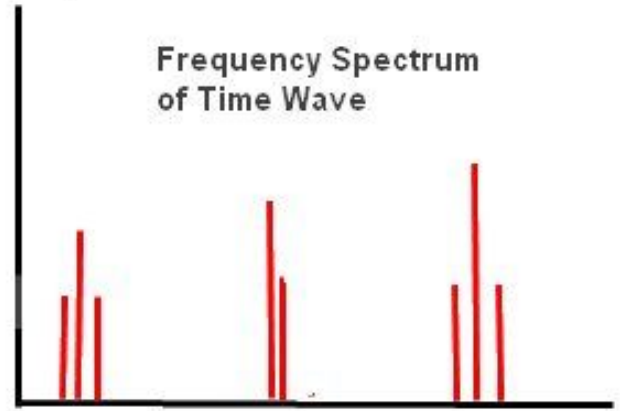


Apply IDFT (IFFT)



Amplitude

Frequency Spectrum
of Time Wave



Frequency



FAST FOURIER TRANSFORM (FFT)

The fast Fourier transform is one of the most important algorithms in **this world**. It uses a divide and conquer technique to compute the Fourier transform efficiently. Because it is fast ($O(n \log n)$), it can be used as a building block in efficient numerical analysis of large signals. In particular FFT and reverse FFT can be used to efficiently compute **convolution**-- you just multiply two signals in the Fourier domain and do a reverse FFT on the product. We will see how the FFT is used to compute the **cross-correlation** of two spike trains.

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