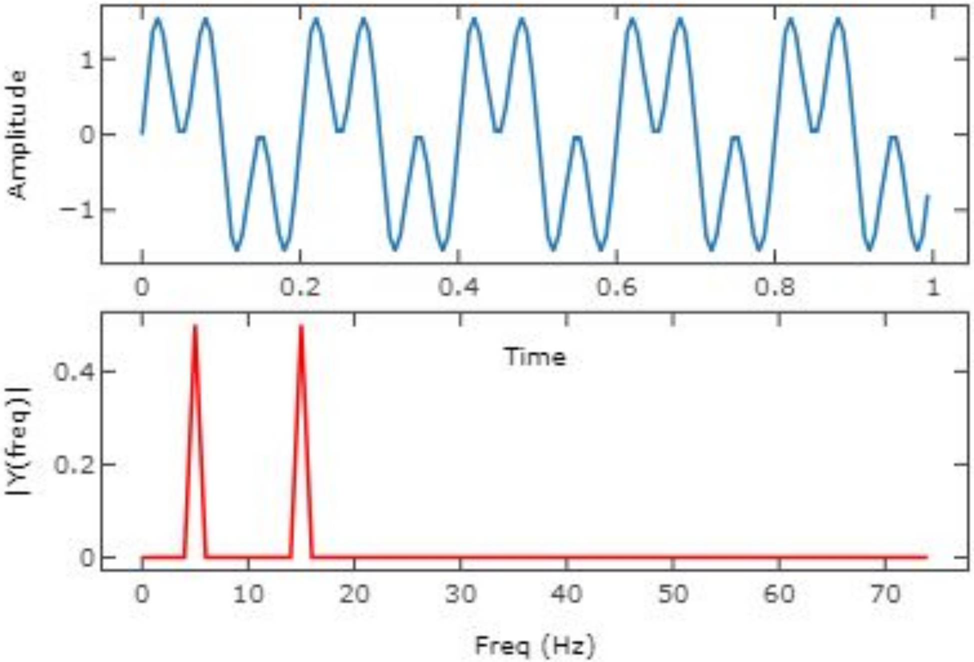
Fourier transform

Spring Style 20XX

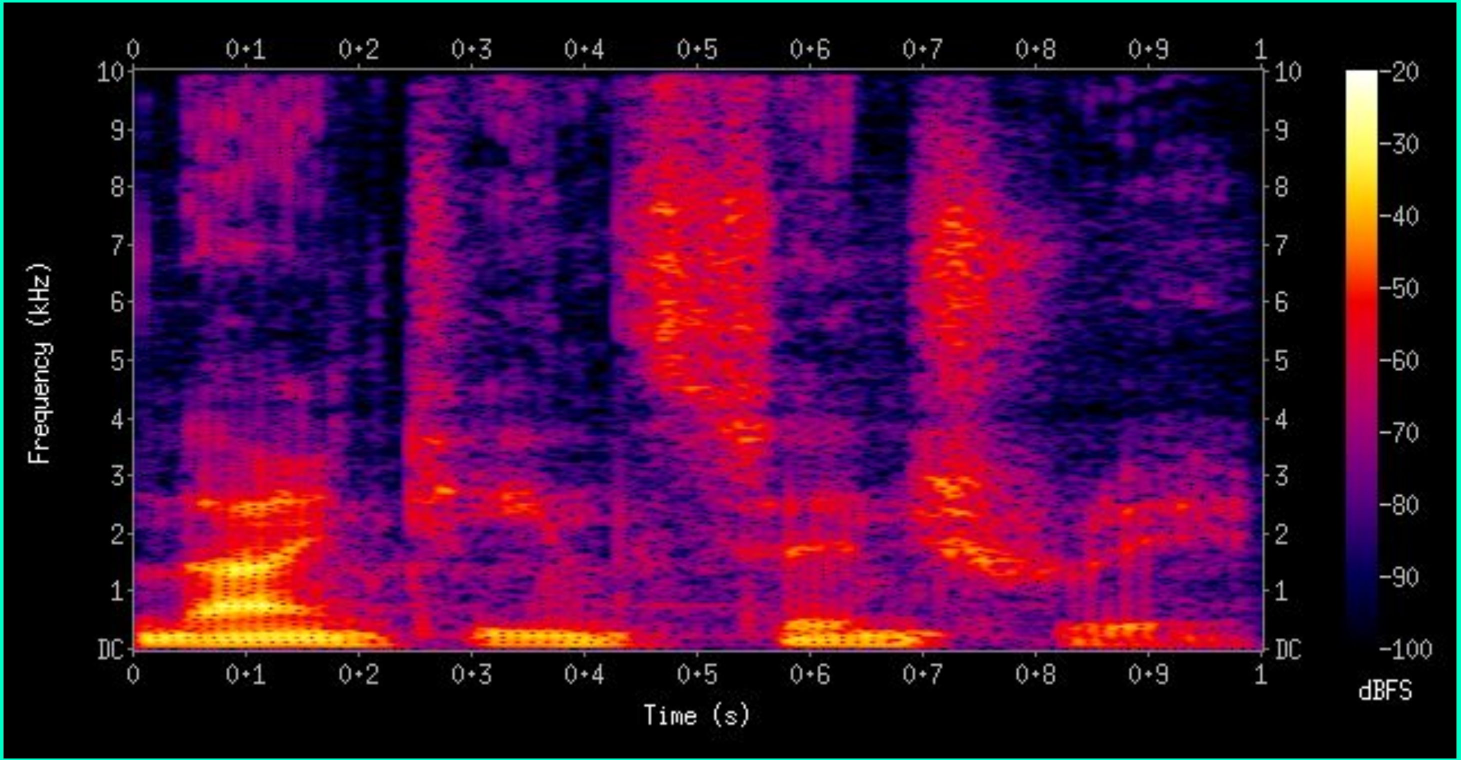
The point of the Fourier transform

If we have a time series of events, we may want to know it's spectrum-- what frequencies are present. This is called analysis, because we decompose the observed time series into a sum of components.

In the Fourier transform these components are sines and cosines, which are connected together in the complex exponential through the Euler formula (see next slides).



Amplitude
1
0
−1
0
0.2
0.4
0.6
0.8
1
Time
|Y(freq)|
0.4
0.2
0
0
10
20
30
40
50
60
70
Freq (Hz)



Frequency (kHz)
Time (s)
dBFS
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1
DC
10
-20
-30
-40
-50
-60
-70
-80
-90
-100

Euler formula

Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

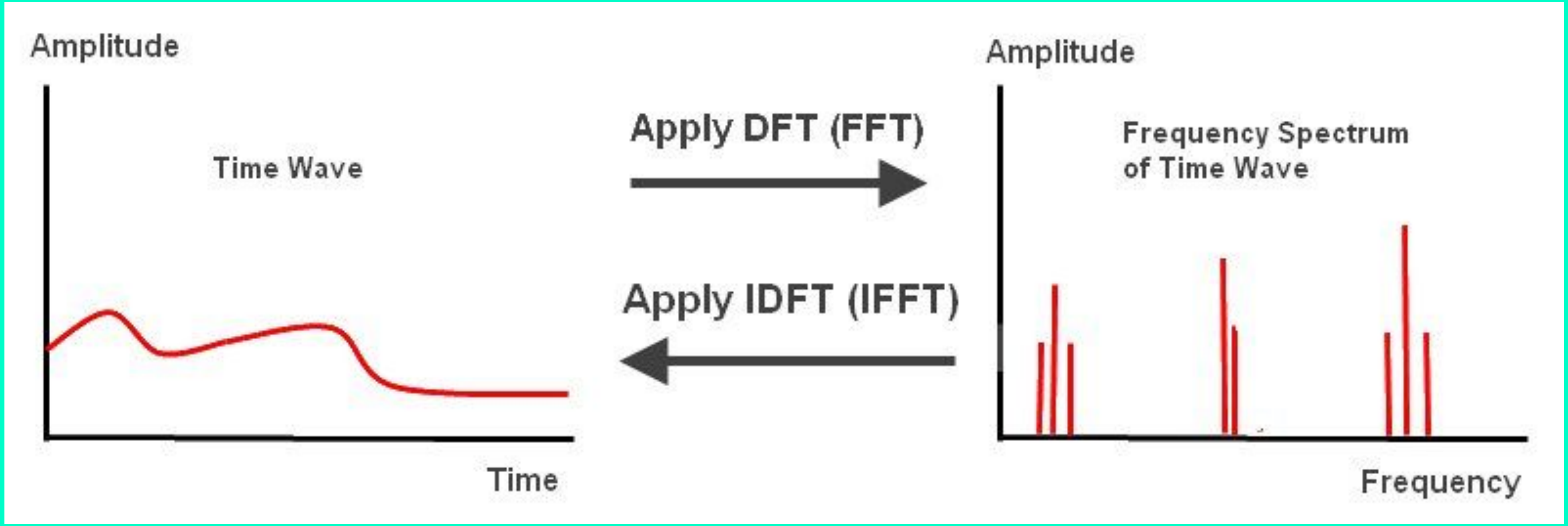
Euler's identity

$$e^{i\pi} + 1 = 0$$

To take the fourier transform we correlate complex exponentials with different frequencies with the signal to get the fourier coefficients which represent how much of that frequency is in the signal.

$$x[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

Reverse Fourier transform



FAST FOURIER TRANSFORM (FFT)

The fast Fourier transform is one of the most important algorithms in this world. It uses a divide and conquer technique to compute the Fourier transform efficiently. Because it is fast (O(n*logn)), it can be used as a building block in efficient numerical analysis of large signals.
We will see how the FFT is used to compute the cross-correlation of two spike trains.