Pragmatic enrichments in state-based modal logic

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Truthmakers Semantics Workshop
Amsterdam, 11 December 2019

State-based modal logic

$[M = \langle W, R, V \rangle]$

Formulas evaluated wrt info states rather than possible worlds

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- State-based modal logic: (support in info states)

$$M, s \models \phi, \text{ where } s \subseteq W$$

- Partial nature: although state-based logical consequence can be classical, we can have states where neither p nor $\neg p$ is supported:

$$\begin{array}{rcl} M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\ M, s \models \neg p & \text{iff} & \forall w \in s : V(w, p) = 0 \end{array}$$

- Info states: less determinate entities than worlds, just like
  - truthmakers, situations, possibilities, ...
- Technically:
  - Truthmakers: points in a partially ordered set
  - Info states: sets of worlds (also elements of a partially ordered set)

State-based modal logic: applications

Partial nature makes state-based systems particularly suitable to capture phenomena at the semantics-pragmatics interface

Epistemic contradictions

(1) #It might be raining and it is not raining. (Veltman, Yalcin)

- Challenge: $\Diamond p \wedge \neg p \models \bot$, while $\Diamond p \not\models p$
- Main idea: (1) captured via *state-sensitive* constraint on epistemic accessibility relation assuming a classical notion of modality

Pragmatic enrichment

- Free choice inference (today's focus)
- Ignorance triggered by *or* and *at least* (vOrmondt & MA):

(2) a. ?I have two or three children. (Grice)
b. ?I have at least two children. (Nouwen & Geurts)

(3) a. Every woman in my family has two or three children.
b. Every woman in my family has at least two children.

Crucial ingredient: *split disjunction* from team logic (Y&V, H&S-T)

State-based modal logic

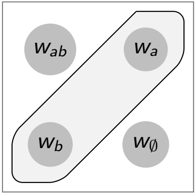
Three notions of disjunction (Aloni 2016)

$M, s \models \phi \vee_1 \psi$ iff $\forall w \in s : M, \{w\} \models \phi$ or $M, \{w\} \models \psi$ (classical disjunction)

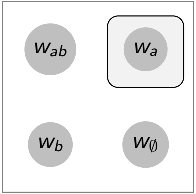
$M, s \models \phi \vee_2 \psi$ iff $\exists t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$ (split disjunction)

$M, s \models \phi \vee_3 \psi$ iff $M, s \models \phi$ or $M, s \models \psi$ (inquisitive/truthmaker disjunction)

- $\vee_1$ and $\vee_2$ equivalent in distributive systems where they behave classically, while $\vee_3$ leads to violation of LEM;
- $\vee_{1/2}$ allow a direct account of indeterminacy of disjunction, while $\vee_3$ produces inquisitive content;
- Today: $\vee_2$ in a non-distributive system.



(a) $\models$ $(a \vee_{1/2/*3} b)$;
and $\models$ $(a \vee_2 b)^+$



(b) $\models$ $(a \vee_{1/2/3} b)$;
but $\not\models$ $(a \vee_2 b)^+$

The challenge of free choice (FC)

- Classical examples of FC inferences:

(4) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(5) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

- Logical rendering of FC inferences:

(6) $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ (NB: $\Diamond\alpha \wedge \Diamond\beta \neq \Diamond(\alpha \wedge \beta)$)

Is this inference valid in classical modal logic? No.

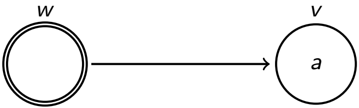


Figure: $M, w \models \Diamond(a \vee b)$, but $M, w \not\models \Diamond a \wedge \Diamond b$

The paradox of free choice

- Free choice permission in natural language:

(7) You may (A or B) $\rightsquigarrow$ You may A

- But (8) not valid in standard deontic logic (von Wright 1968):

(8) $\Diamond(\alpha \vee \beta) \to \Diamond\alpha$ [Free Choice Principle]

- Plainly making the Free Choice Principle valid, for example by adding it as an axiom, would not do (Kamp 1973):

(9)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by classical reasoning]
3. $\Diamond b$ [from 2, by free choice principle]

- The step leading to 2 in (9) uses the classically valid (10):

(10) $\Diamond\alpha \to \Diamond(\alpha \vee \beta)$ [Modal Addition]

- Natural language counterpart of (10), however, seems invalid:

(11) You may A $\not\rightsquigarrow$ You may (A or B) [Ross's paradox]

$\Rightarrow$ Intuitions on natural language in direct opposition to the principles of classical logic

Reactions to paradox

- Paradox of Free Choice Permission:

(12)
1. $\Diamond a$ [assumption]
2. $\Diamond(a \vee b)$ [from 1, by modal addition]
3. $\Diamond b$ [from 2, by FC principle]

- Pragmatic solutions [⇒ keep logic as is]
  - FC inferences are conversational implicatures, i.e. pragmatic inferences derived as the product of rational interactions between cooperative language users (+ classical logic meanings)
  - ⇒ step leading to 3 is unjustified
- Semantic solutions [⇒ change the logic]
  - FC inferences are semantic entailments
  - ⇒ step leading to 3 is justified, but step leading to 2 is no longer valid
- Free choice: semantics or pragmatics? My view:
  - FC inferences: neither purely semantic nor purely pragmatic
  - derivable by conversational principles but lacking other defining properties of gricean inferences
- Proposal: a logic-based account of FC inferences beyond canonical semantics vs pragmatics divide

Argument against semantic accounts of FC

Free choice effects systematically disappear in negative contexts:

(13) Dual Prohibition (Alonso-Ovalle 2005)

a. You are not allowed to eat the cake or the ice-cream.
$\leadsto$ You are not allowed to eat either one.

b. $\neg\Diamond(\alpha \vee \beta) \leadsto \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Argument against pragmatic accounts of FC

Free choice effects embeddable under universal quantification:

(14) Universal FC (Chemla 2009)

a. All of the boys may go to the beach or to the cinema.
$\leadsto$ All of the boys may go to the beach and all of the boys may go to the cinema.

b. $\forall x\Diamond(\alpha \vee \beta) \leadsto \forall x(\Diamond\alpha \wedge \Diamond\beta)$

Argument against most accounts (including localist view)

Free choice effects also arise with wide scope disjunctions:

(15) Wide Scope FC (Zimmermann 2000)

a. Mr. X might be in Victoria or he might be in Brixton. $\leadsto$ Mr. X might be in Victoria and might be in Brixton.

b. $\Diamond\alpha \vee \Diamond\beta \leadsto \Diamond\alpha \wedge \Diamond\beta$

Free choice: summary data and predictions

(16) a. $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Narrow Scope FC]
b. $\neg\Diamond(\alpha \vee \beta) \rightsquigarrow \neg\Diamond\alpha \wedge \neg\Diamond\beta$ [Dual Prohibition]
c. $\forall x\Diamond(\alpha \vee \beta) \rightsquigarrow \forall x(\Diamond\alpha \wedge \Diamond\beta)$ [Universal FC]
d. $\Diamond\alpha \vee \Diamond\beta \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$ [Wide Scope FC]

	N Scope FC	Dual Prohibition	Universal FC	W Scope FC
Semantic	yes	no	yes	no*
Pragmatic	yes	yes	no	no

Free choice: semantics or pragmatics?

- A purely semantic or pragmatic approach cannot account for this complex pattern of inference
- I propose a hybrid approach where
  - FC inference derived by allowing pragmatic principles intrude in the recursive process of meaning composition
- Pragmatic intrusion captured in a bilateral state-based modal logic which models assertion/rejection conditions rather than truth

Bilateral state-based modal logic

- Classical modal logic: (truth in worlds)

$$M, w \models \phi, \text{ where } w \in W$$

- State-based modal logic: (support in info states)

$$M, s \models \phi, \text{ where } s \subseteq W$$

- Bilateral state-based modal logic:

$$\begin{array}{ll} M, s \models \phi, \text{ ``}\phi \text{ is assertable in } s\text{''}, & \text{with } s \subseteq W \\ M, s \mathrel{=\!\!|} \phi, \text{ ``}\phi \text{ is rejectable in } s\text{''}, & \text{with } s \subseteq W \end{array}$$

$[M = \langle W, R, V \rangle]$

Pragmatic intrusion in state-based modal logic

- Conversation is ruled by a principle that prescribes to avoid contradictions ('avoid $\bot$') [follows from QUALITY]
- Proposal: FC inferences follow from the systematic "intrusion" of 'avoid $\bot$' into the recursive process of meaning composition

Implementation

To model such intrusion we need a way to formally represent 'avoid $\bot$':

- In classical logic no non-trivial way to do it: $\neg\bot = \top$

In a state-based semantics:

- $\emptyset \mapsto$ state of logical insanity, supports everything including contradictions: $\emptyset \models \phi \wedge \neg\phi$
- But then we can represent 'avoid $\bot$' by means of a constant, NE, which requires the supporting state to be non-empty ($\neq \emptyset$)

$$M, s \models \mathrm{NE} \quad \text{iff} \quad s \neq \emptyset$$
$$M, s \mathrel{=\!\!|} \mathrm{NE} \quad \text{iff} \quad s = \emptyset$$

Pragmatic intrusion in state-based modal logic

Pragmatic enrichment

- Pragmatically enriched formulas ϕ^+ come with the requirement to satisfy $\textsc{ne}$ ('avoid $\bot$') distributed along each of their subformulas:

$$\begin{array}{rcl} p^+ & = & p \wedge \textsc{ne} \\ (\neg\phi)^+ & = & \neg\phi^+ \wedge \textsc{ne} \\ (\phi \vee \psi)^+ & = & (\phi^+ \wedge \textsc{ne}) \vee (\psi^+ \wedge \textsc{ne}) \\ \ldots & & \end{array}$$

Main result

- By pragmatically enriching every formula, we derive:
  - Narrow scope $\textsc{fc}$: $\Diamond(\alpha \vee \beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope $\textsc{fc}$: $(\Diamond\alpha \vee \Diamond\beta)^+ \models \Diamond\alpha \wedge \Diamond\beta$ (with restrictions)
  - Universal $\textsc{fc}$: $\forall x\Diamond(\alpha \vee \beta)^+ \models \forall x(\Diamond\alpha \wedge \Diamond\beta)$
  - Distribution: $\forall x(\alpha \vee \beta)^+ \models \exists x\alpha \wedge \exists x\beta$ and more
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $\neg\Diamond(a \vee b)^+ \models \neg\Diamond a \wedge \neg\Diamond b$
- Subtle predictions wrt wide scope $\textsc{fc}$ confirmed by pilot experiment

Propositional Modal Logic of Pragmatic Intrusion

Language

$$\phi \quad := \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \Diamond\phi \mid \mathrm{NE}$$

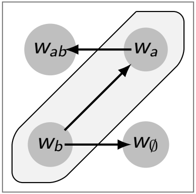
where $p \in A$.

Models and States

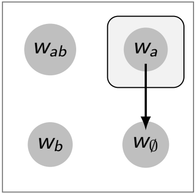
- Classical Kripke models: $M = \langle W, R, V \rangle$
- States: $s \subseteq W$, sets of worlds in a Kripke model

Examples

for $A = \{a, b\}$



(a) $\not\models a$; $\models \Diamond a$



(b) $\models a$; $\not\models \Diamond a$

Semantic clauses

$[M = \langle W, R, V\rangle;\ s, t, t' \subseteq W]$

$$
\begin{array}{rll}
M, s \models p & \text{iff} & \forall w \in s : V(w, p) = 1 \\
M, s \dashv p & \text{iff} & \forall w \in s : V(w, p) = 0 \\
M, s \models \neg\phi & \text{iff} & M, s \dashv \phi \\
M, s \dashv \neg\phi & \text{iff} & M, s \models \phi \\
M, s \models \phi \wedge \psi & \text{iff} & M, s \models \phi \ \&\ M, s \models \psi \\
M, s \dashv \phi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \dashv \phi \ \&\ M, t' \dashv \psi \\
M, s \models \phi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \ \&\ M, t \models \phi \ \&\ M, t' \models \psi \\
M, s \dashv \phi \vee \psi & \text{iff} & M, s \dashv \phi \ \&\ M, s \dashv \psi \\
M, s \models \Diamond\phi & \text{iff} & \forall w \in s : \exists t \subseteq R^{\rightarrow}(w) : t \neq \emptyset \ \&\ t \models \phi \\
M, s \dashv \Diamond\phi & \text{iff} & \forall w \in s : R^{\rightarrow}(w) \dashv \phi \\
M, s \models \mathrm{NE} & \text{iff} & s \neq \emptyset \\
M, s \dashv \mathrm{NE} & \text{iff} & s = \emptyset
\end{array}
$$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

Pragmatic intrusion

$$\begin{array}{rcl} p^+ & = & p \wedge \mathrm{NE} \\ (\neg\phi)^+ & = & \neg\phi^+ \wedge \mathrm{NE} \\ (\phi \vee \psi)^+ & = & (\phi^+ \wedge \mathrm{NE}) \vee (\psi^+ \wedge \mathrm{NE}) \\ (\phi \wedge \psi)^+ & = & (\phi^+ \wedge \mathrm{NE}) \wedge (\psi^+ \wedge \mathrm{NE}) \\ (\Diamond\phi)^+ & = & \Diamond\phi^+ \wedge \mathrm{NE} \\ \mathrm{NE}^+ & = & \mathrm{NE} \end{array}$$

Logical consequence

- $\phi \models \psi$ iff for all $M, s : M, s \models \phi \;\Rightarrow\; M, s \models \psi$

State-sensitive constraints on accessibility relation

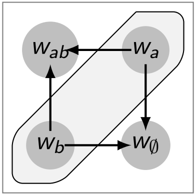
- R is indisputable in (M, s) iff $\forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
- R is state-based in (M, s) iff $\forall w \in s : R^{\rightarrow}(w) = s$

where $R^{\rightarrow}(w) = \{v \mid wRv\}$

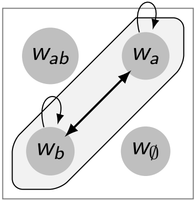
Main ingredients: constraints on accessibility relation

- State-sensitive constraints on accessibility relation:
  - R is indisputable in (M, s) iff $\forall w, v \in s : R^{\rightarrow}(w) = R^{\rightarrow}(v)$
    $\mapsto$ all worlds in s access exactly the same set of worlds
  - R is state-based in (M, s) iff $\forall w \in s : R^{\rightarrow}(w) = s$
    $\mapsto$ all and only worlds in s are accessible within s

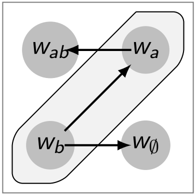
  where $R^{\rightarrow}(w) = \{v \mid wRv\}$



(c) indisputable



(d) state-base (and so also indisputable)

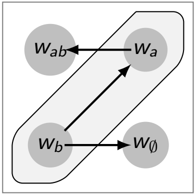


(e) neither

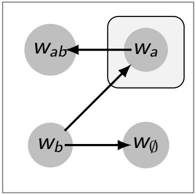
- Difference deontic vs epistemic modals captured by different properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)

Main ingredients: split disjunction

- We adopt a bilateral version of split disjunction:
  - A state s supports $\phi \vee \psi$ iff s can be split into two substates, each supporting one of the disjuncts;
  - A state s rejects $\phi \vee \psi$ iff s rejects ϕ and rejects ψ.
- Pragmatically enriched disjunction:
  - After pragmatic intrusion: $(\phi \vee \psi)^+ =: (\phi^+ \wedge \mathrm{NE}) \vee (\psi^+ \wedge \mathrm{NE})$
  - A state s supports $(\phi \vee \psi)^+$ iff s can be split into two non-empty substates, each supporting one of the disjuncts, e.g.



(f)$\models a \vee b; \models (a \vee b)^+$



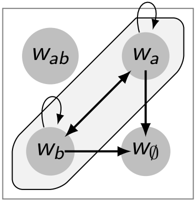
(g)$\models a \vee b; \not\models (a \vee b)^+$

  - Pragmatic enrichment vacuous under negation:
    $\neg(a \vee b)^+ = \neg((a \wedge \mathrm{NE}) \vee (b \wedge \mathrm{NE})) = \neg(a \wedge \mathrm{NE}) \wedge \neg(b \wedge \mathrm{NE}) = \neg a \wedge \neg b = \neg(a \vee b)$

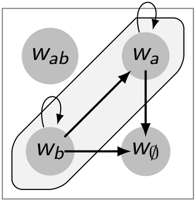
Main ingredients: modals

- A “classical” notion of modality:
  - A state s supports $\Diamond\phi$ iff for all $w \in s$: there is a non-empty subset of the set of worlds accessible from w which support ϕ
  - A state s rejects $\Diamond\phi$ iff for all $w \in s$: the set of worlds accessible from w rejects ϕ

$\Rightarrow$ Free choice effect derived in combination with enriched disjunctions



(h) $\models\ \Diamond(a \vee b)^+$



(i) $\not\models \Diamond(a \vee b)^+$

- Suppose s supports $\Diamond a$ but not $\Diamond b \Rightarrow$ no b-world accessible from some w in $s \Rightarrow (a \vee b)^+$ not supported by any subset of worlds accessible from $w \Rightarrow \Diamond(a \vee b)^+$ not supported in s

Summary of results propositional fragment

Before pragmatic intrusion

- The NE-free fragment of our state-based system is equivalent to classical modal logic;
- But we can capture infelicity of epistemic contradictions (#*It might be raining and it is not raining*) by putting constraints on epistemic accessibility relation:
  1. Epistemic contradiction: $\Diamond a \wedge \neg a \models \bot$ [if R is state-based]
  2. Non-factivity: $\Diamond a \not\models a$

After pragmatic intrusion

- FC (and ignorance) inferences derived for pragmatically enriched disjunction:
  - Narrow scope FC: $\Diamond(a \vee b)^+ \models \Diamond a \wedge \Diamond b$
  - Wide scope FC: $(\Diamond a \vee \Diamond b)^+ \models \Diamond a \wedge \Diamond b$ (if R is indisputable)
  - Ignorance: $(a \vee b)^+ \models \Diamond a \wedge \Diamond b$ (if R is state-based)
- Only disjunctions in positive environments (and logically equivalent formulas) affected by pragmatic intrusion:
  - Dual prohibition: $\neg\Diamond(a \vee b)^+ \models \neg\Diamond a \wedge \neg\Diamond b$

Applications: epistemic contradiction

Epistemic contradiction and non-factuality

1. $\Diamond a \wedge \neg a \models \bot$ [if R is state-based]
2. $\Diamond a \not\models a$

Epistemics vs deontics

- Differ wrt properties of accessibility relation:
  - Epistemics: R is state-based
  - Deontics: R is possibly indisputable (e.g. in performative uses)
- Epistemic contradiction predicted for epistemics, but not for deontics:

(17) #It might be raining and it is not raining. (Veltman, Yalcin)

(18) You don't smoke but you may smoke.

Applications: epistemic free choice

Narrow scope and wide scope FC

1. $\Diamond(a \vee b)^{+} \models \Diamond a \wedge \Diamond b$
2. $(\Diamond a \vee \Diamond b)^{+} \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Epistemic modals

- R is state-based, therefore always indisputable:

  (19) He might either be in London or in Paris. [+fc, narrow]

  (20) He might be in London or he might be in Paris. [+fc, wide]

- $\Rightarrow$ narrow and wide scope FC always predicted for pragmatically enriched epistemics

Applications: deontic free choice

Narrow scope and wide scope FC

1. $\Diamond(a \vee b)^+ \models \Diamond a \wedge \Diamond b$
2. $(\Diamond a \vee \Diamond b)^+ \models \Diamond a \wedge \Diamond b$ [if R is indisputable]

Deontic modals

- R may be indisputable if speaker is knowledgable (e.g. in performative uses)
- Predictions:
  - $\Rightarrow$ narrow scope FC always predicted for enriched deontics
  - $\Rightarrow$ wide scope FC only if speaker knows what is permitted/obligatory
- Further consequence: all cases of (overt) FC cancellations involve a wide scope configuration

Deontic FC: comparison with localist view

- Current proposal vs Fox (2007)

	NS+K	NS¬K	WS+K	WS¬K
MA	yes	yes	yes	no
Fox (2007)	yes	no	no	no

K $\mapsto$ speaker knows what is permitted/obligatory;

NS $\mapsto$ narrow scope FC; WS $\mapsto$ wide scope FC.

- MA's predictions confirmed by pilot experiment (Cremers et al. 2017)
- Speaker knowledge has effect on FC inference only in wide scope configurations:

(21) We may either eat the cake or the ice-cream. [narrow, +fc]

(22) Either we may eat the cake or the ice-cream. [wide, +/–fc]

Position of *either* favors a narrow scope interpretation in (21), while it forces a wide scope interpretation in (22) (Larson 1985)

Deontic FC: (overt) FC cancellations

- Prediction: all cases of (overt) FC cancellations involve a wide scope configuration
- Sluicing arguably triggers wide scope configuration in (23) but not in (24) (Fusco 2018):

(23) You may either eat the cake or the ice-cream, I don't know which (you may eat). [wide, –fc]

(24) You may either eat the cake or the ice-cream, I don't care which (you eat). [narrow, +fc]

- Wide scope configuration also plausible for (25) (Kaufmann 2016):

(25) You may either eat the cake or the ice-cream, it depends on what John has taken. [wide, –fc]

Conclusions

- Free choice: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived using general principles of conversation
- Standard implementation: two separate components
  - Semantics: classical logic
  - Pragmatics: Gricean reasoning

  Elegant picture, but incorrect for free choice
- My proposal: a state-based modal logic for pragmatic intrusion
  - Free choice derived by letting pragmatic principles intrude into semantic composition;
  - Classical logic can be recovered (as NE-free fragment);
  - Adopted bilateral system defines assertion/rejection conditions rather than truth.
- Future research:
  - Logic: proof-theory; syntactic (via NE) vs semantic (via elimination of empty state) characterisation of pragmatic intrusion
  - Language: testing of predictions (experimental); analysis of overt FC cancellations (theoretical)