Inquisitive Semantics and Dialogue Pragmatics

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1 Inquisitive Syntax

First we give the language for which we give an inquisitive semantics and pragmatics.

Definition 1 (Inquisitive Propositional Syntax) Let $\wp$ be a finite set of propositional variables. The sentences of $L_\wp$ is the smallest set such that:

1. If $p \in \wp$, then $p \in L_\wp$
2. $\bot \in L_\wp$
3. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \to \psi) \in L_\wp$
4. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \wedge \psi) \in L_\wp$
5. If $\varphi \in L_\wp$ and $\psi \in L_\wp$, then $(\varphi \vee \psi) \in L_\wp$

The language contains questions, they can be defined in terms of disjunction.

Definition 2 (Notation Conventions) Let $\varphi \in L_\wp$.

1. $\neg\varphi = (\varphi \to \bot)$
2. $\top = \neg\bot$
3. $!\varphi = \neg\neg\varphi$
4. $?\varphi = (\varphi \vee \neg\varphi)$

2 Inquisitive Semantics

We will define the semantics in the format of an update semantics. Sentences will be interpreted as functions from states to states.

Definition 3 (States)

1. The set of *indices* I for $L_{\wp}$ is the set of functions i such that for all $p \in \wp$: $i(p) \in \{0, 1\}$.
2. A *state* s is a reflexive and symmetric relation on a subset of I.

The relation on indices embodied by a state is a *relation of indifference*. When two indices i and j are related in s, i.e., $\langle i, j \rangle \in s$, the state is not interested in the differences between i and j.

The operation of indifferentiation ignores the current issues in the state.

Definition 4 (Indifferentiation) $s^{\star} = \{\langle i, j \rangle \mid \langle i, i \rangle \in s \ \&\ \langle j, j \rangle \in s\}$

We will use this operation when we want to change the current issue.

Definition 5 (Three Special States)

1. The *initial state* $\imath$ is the identity relation on I.
2. The *state of ignorance and indifference* $\omega = \iota^{\star}$, the total relation on I.
3. The *inconsistent absurd state* is $\emptyset$.

We can distinguish possibilities in a state, which are largest sets of indices such that all them are related to each other in the state.

Definition 6 (Possibilities in States) Let s be a state.

ρ is a *possibility* in s iff ρ is a set of indices such that:

(a) for all $i, j \in \rho$: $\langle i, j \rangle \in s$; and
(b) there is no set of indices $\rho' \supset \rho$ such that ρ' satisfies (a)

The possibilities in a state may overlap. States need not be partitions.

Definition 7 (Inquisitive Propositional Update Semantics)

1. $s[p] = \{\langle i, j\rangle \in s \mid i(p) = 1 \;\&\; j(p) = 1\}$, for $p \in \wp$
2. $s[\bot] = \emptyset$
3. $s[\varphi \to \psi] = \{\langle i, j\rangle \in s \mid \forall \iota \in \{i, j\}^2\colon \iota \in s[\varphi] \Rightarrow \iota \in s[\varphi][\psi]\}$
4. $s[\varphi \vee \psi] = s[\varphi] \cup s[\psi]$
5. $s[\varphi \wedge \psi] = s[\varphi][\psi]$

Definition 8 (Inquisitiveness and Informativeness) Let $\varphi \in L$.

1. φ *is inquisitive* iff $\exists i, j\colon \langle i, i\rangle \in \omega[\varphi]$ & $\langle j, j\rangle \in \omega[\varphi]$, & $\langle i, j\rangle \notin \omega[\varphi]$
2. φ *is informative* iff $\exists i, j\colon \langle i, i\rangle \in \omega[\varphi]$ & $\langle j, j\rangle \notin \omega[\varphi]$

Questions and assertions are not syntactic categories, but can be distinguished in semantic terms. A third semantic category are hybrids.

Definition 9 (Questions, Assertions, and Hybrids) Let $\varphi \in L$.

1. φ is a *question* iff φ is not informative.
2. φ is an *assertion* iff φ is not inquisitive.
3. φ is a *hybrid* iff φ is inquisitive and informative.

Fact 1 (Hybrid Disjunctions) $p \vee q$ is a hybrid.

There are two overlapping possibilities in $\omega[p \vee q]$.

Fact 2 (Questions and Assertions) For all sentences φ

1. $?\varphi$ is a question
2. $\neg\varphi$ is an assertion, and hence $!\varphi$ is an assertion as well
3. $??\varphi$ is equivalent with $?\varphi$
4. $\neg?\varphi$ is equivalent with $\bot$
5. φ is equivalent with $?\varphi \wedge !\varphi$ ($?\varphi$ is the *theme*, $!\varphi$ the *rheme* of φ)

3 Dialogue Pragmatics

3.1 Common Ground

We consider dialogues with two participants: the *stimulator* and the *responder*. The central pragmatic notion is that of *the common ground*.

Definition 10 (Common Ground) Let c and s and r be states.

> c is a *common ground for s and r* iff $s^\star \subseteq c^\star$ & $r^\star \subseteq c^\star$.

Only the data in the states play a role in determining whether there is a common ground, not the issues.

Fact 3 (Indifferentation and the Common Ground) If c is a common ground for s and r, then $c^\star$ is a common ground for s and r.

The most fundamental pragmatic dialogue rule reads as follows:

Definition 11 (First Dialogue Principle) *Maintain a common ground!*

- We will define *dialogue management rules* that determine the effects of utterances and reactions to them on the common ground. When the rules are followed, a common ground is always maintained.

Given our semantics, the following holds without exception:

Fact 4 (Common Ground and Simultaneous Update) If c is a common ground for s and r, then $c[\varphi]$ is a common ground for $s[\varphi]$ and $r[\varphi]$.

However, it will not always be possible to *accept* a proposed update.

Definition 12 (Second Dialogue Principle) *Keep your state consistent! Publicly announce* cancelling *a proposed update to maintain consistency.*

- We will model the common ground as a *stack of states* to manage the *uptake* of a proposed update, and the effects of publicly announced *cancellation, acceptance* and *support.*

3.2 Compliance

The inquisitive pragmatic version of the Maxims of Relation and Quantity:

Definition 13 (Third Dialogue Principle) Other things being equal,
Be as compliant as you can to the current state of the common ground!

Compliance will involve the notions of *relatedness and homogeneity* of utterances relative to the current state of the common ground. We define two *dialogue oriented relations between states* that correspond to these notions.

Definition 14 (Relatedness) *r is related to s*, $r \propto s$ iff

> every possibility in r is the union of a set of possibilities in s.

(Unions of) possibilities in a state s correspond to propositions that would (partially) resolve the issue in s. If r is related to s, r embodies *not more and not less than* a partial resolution and/or a subissue of the issue in s.

- The dialogue rules will guarantee that in the current state of the common ground there is always an issue to relate to.

Definition 15 (Continuity and Homogeneity)

1. *r is continuous to s* iff $r^\star \subseteq s^\star$ (Guaranteed for subsequent states in stack)
2. *r is homogeneous to s*, $r \succeq s$ iff r is continuous to s and

$$\forall i, j \colon \langle i, i\rangle \in r \;\&\; \langle j, j\rangle \in r \;\&\; \langle i, j\rangle \notin r \Rightarrow \langle i, j\rangle \notin s$$

If r is homogeneous to s than r contains as least as much information (continuity) and at most as much of an issue as s.

Definition 16 (Compliance: Relatedness and Homogeneity)

1. φ *is related to s* iff $s^\star[\varphi] \propto s$.
2. φ *is homogeneous to s* iff $s[\varphi] \succeq s$.
3. φ is *compliant to s* iff φ is related to s and φ is homogeneous to s.

We can compare homogeneity of alternative compliant utterances.

Definition 17 (Comparative Homogeneity)

> φ is at least as homogeneous to s, as ψ iff $s[\varphi] \succeq s[\psi]$

Other things being equal, comparative homogeneity prefers *more informative* and *less inquisitive* reponses.

- Compliance gives rise to implicatures. If ψ is an obvious compliant alternative to φ, and ψ is more homogeneous than φ, because ψ is less inquisitive, then it is implicated that φ is not inquisitive at all.

We can raise the contextual notions of compliance and comparative homogeneity to general relations between sentences.

Definition 18 (Compliance and Comparative Homogeneity)

1. φ *is compliant to* ψ iff φ *is compliant to* $\omega[\psi]$.
2. φ is at least as homogeneous to ψ as χ iff φ is at least as homogeneous to $\omega[\psi]$ as χ

Fact 5 (Compliance)

1. $\perp$ and $\top$ are compliant to φ
2. φ is compliant to φ; if φ is compliant to χ, and χ to ψ, then φ is compliant to ψ
3. Let $\psi \in L_{\wp}$. φ is compliant to ψ iff $\exists \varphi' \in L_{\wp}\colon \varphi \Leftrightarrow \varphi'$ & φ' is compliant to ψ.
4. For all $\varphi \in L_{\{p,q\}}\colon \varphi$ is compliant to $?p \wedge ?q$.
5. If φ is compliant to ψ, then $!\varphi$ is compliant to ψ.
6. Apart from what can be obtained from (1)-(5), or what can be obtained from (a)-(e) by using (1)-(5):
   (a) Only $!\varphi$ is compliant to $!\varphi$
   (b) Only p and $\neg p$ are compliant to $?p$.
   (c) Only $p \rightarrow q$ and $p \rightarrow \neg q$ are compliant to $p \rightarrow ?q$
   (d) Only p and q are compliant to $p \vee q$
   (e) Only $(\neg)p \vee (\neg)q$ are compliant to $?p \vee ?q$

4 Inquisitive Dialogue Management

The dialogue principles dictate that the stimulator and the responder try to obtain a more homogeneous state of the common ground. The relation of homogeneity between states tells us:

Fact 6 (Minimal and Maximal Homogeneity) $\forall s\colon \emptyset \succeq s \succeq \imath$

Given this, the natural *initial common ground* is the initial state $\imath$, where the following holds:

Fact 7 (Initial Compliance) $\forall s\colon s \subseteq \imath \Rightarrow \forall \varphi\colon \varphi$ is compliant to s.

The general aim of a dialogue is eliminating possibilities from $\imath$, where indices and possibilities are exchangeable. We generally do so by focussing on some subissue (of a subissue...). Once a subissue is resolved we are back at a subset of the possibilities in $\imath$, we can focus on a new subissue, etc., etc.

4.1 Inquisitive Stacks

We define stacks in the following way:

Definition 19 (Stacks) The set of stacks is the smallest set such that:

1. $\langle\ \rangle$ is a stack.
2. If s is a state, and σ is a stack, then $\langle \sigma, s\rangle$ is a stack.

Starting rom the empty stack $\langle\ \rangle$, comes handy in defining operations on stacks that percolate down the stack.

Definition 20 (Percolation) Let $[\cdot]$ be an operation on states.

1. $\langle\ \rangle^{\triangleleft}[\cdot] = \langle\ \rangle$
2. $\langle \sigma, s\rangle^{\triangleleft}[\cdot] = \langle \sigma^{\triangleleft}[\cdot], s[\cdot]\rangle$

For example, the operation ${}^{\triangleleft}[\varphi]$ percolates the update of a state with φ down through all the elements of a stack, until it is halted by $\langle\ \rangle$.

4.2 Operations on Stacks

In our dialogue setting there are three subsequent operations on stacks.

1. Primary uptake of the *semantic content* of an utterance
2. Secundary uptake of *compliance implicatures* of the utterance
3. Absorbing the *reactions* to the utterance

All operations on stacks will preserve continuity. Inquisitive dialogue pragmatics prefers homogeneous stacks.

Definition 21 (Continuous and Homogeneous Stacks)
We stipulate: $\langle\langle\ \rangle, s\rangle$ is homogenious (and hence continuous)

1. $\langle\langle\sigma, s\rangle, t\rangle$ is *continuous* iff $\langle\sigma, s\rangle$ is continuous and $t^\star \subseteq s^\star$.
2. $\langle\langle\sigma, s\rangle, t\rangle$ is *homogeneous* iff $\langle\sigma, s\rangle$ is homogeneous and $t \succeq s$.

Compliance implicatures will have the effect of enhancing homogeneity.

4.3 Primary Uptake: Thematize and Assume

The primary uptake of a sentence φ in a stack goes in two steps: to thematize φ and to assume φ. To assume φ may be forbidden by the principle: keep your state consistent. Then cancellation of the assumption is publicly announced. We pop to the thematization of φ, and critical responses to φ are compliant.

Definition 22 (Uptake: Thematize and Assume)

1. $\langle\sigma, s\rangle[\varphi]^? = \langle\langle\sigma, s\rangle, s \cup s^\star[?\varphi]\rangle$
2. $\langle\sigma, s\rangle[\varphi]^\uparrow = \langle\langle\sigma, s\rangle, s[\varphi]\rangle$
3. $\langle\sigma, s\rangle[\varphi]^\Uparrow = \langle\sigma, s\rangle[\varphi]^?[\varphi]^\uparrow$

4.3.1 Thematizing

In thematizing φ in $\langle\sigma, s\rangle$, we add the theme of φ to the current issue in s by taking $s \cup s^\star[?\varphi]$. If the current issue in s can be written as $?\psi$, then $s = s^\star[?\psi]$ and $s \cup s^\star[?\varphi]$ can be written as $s^\star[?\psi \vee ?\varphi]$. Obviously:

Fact 8 (Homogeneity of Thematizing) $s \cup s^\star[?\varphi]$ is homogeneous to s.

(1) $s^\star[p \to ?q] \cup s^\star[?(p \to q)] = s^\star[p \to ?q]$

(2) $s^\star[p \vee q] \cup s^\star[?p] = s^\star[p \vee q]$

(3) $s^\star[?p \wedge ?q] \cup s^\star[?p] = s^\star[?p] \propto s^\star[?p \wedge ?q]$

(4) $s^\star[?p] \cup s^\star[?q] = s^\star[?p \vee ?q] \not\propto s^\star[?p]$

The thematization of a question $?\psi$ leads to $s \cup s^\star[??\psi] = s \cup s^\star[?\psi]$.

Fact 9 (Thematizing Compliant Questions)

If $?\psi$ is compliant to s, then $s \cup s^\star[?\psi] = s^\star[?\psi]$.

4.3.2 Thematize and Assume

The effect of to assume φ after thematizing φ results in:

$$\langle\sigma, s\rangle[\varphi]^\Uparrow = \langle\langle\sigma, s\rangle, s \cup s^\star[?\varphi]\rangle[\varphi]^\uparrow = \langle\langle\langle\sigma, s\rangle, s \cup s^\star[?\varphi]\rangle, (s \cup s^\star[?\varphi])[\varphi]\rangle$$

We can economize this result:

Fact 10 (Primary Uptake) $\langle\sigma, s\rangle[\varphi]^\Uparrow = \langle\langle\langle\sigma, s\rangle, s \cup s^\star[?\varphi]\rangle, s^\star[\varphi]\rangle$

Combining Fact 9 with Fact 10:

Fact 11 (Primary Uptake of Compliant Questions)

If $?\varphi$ is compliant to s, then $\langle\sigma, s\rangle[?\varphi]^\Uparrow = \langle\langle\langle\sigma, s\rangle, s^\star[?\varphi]\rangle, s^\star[?\varphi]\rangle$

Although thematization always preserves homogeneity, the uptake of a sentence does not.

Fact 12 (Homogeneity of Uptake)

If φ is compliant to s, then $\langle\sigma, s\rangle[\varphi]^\Uparrow$ is homogeneous.

See the appendix for some worked out examples.

4.4 Secundary Uptake: Compliance Implicatures

4.4.1 Suggestions and Implicatures of Disjunction

Definition 23 (Implicatures and Suggestions)

1. An *implicature* of a sentence φ is a piece of information that is provisionally added to the common ground by a secundary uptake on top of the primary uptake of φ.

2. A *suggestion* of a sentence φ has no immediate effect on the common ground, but drives the responder to φ in a direction where a compliant response ψ has an implicative uptake effect on the common ground.

Like any other piece of information provisionally added by the uptake of a sentence, implicatures may be cancelled by the responder. And a responder can refrain from following a suggestion by giving a non-compliant response.

The cases we will deal with:

- An alternative question $?(p \vee q)$ *implicates the exclusion of* $\neg p \wedge \neg q$.
- Both a hybrid disjunction $p \vee q$, and an alternative question $?(p \vee q)$, *suggest the exclusion of* $p \wedge q$.

Note the following:

- *On the responder accepting* the exclusion implicature of an alternative question $?(p \vee q)$, its effects on the common ground become the same as those of the hybrid disjunction $p \vee q$.
- *On the responder accepting* the exclusion suggestion of an alternative question $?(p \vee q)$ or a hybrid disjunction $p \vee q$ its effects on the common ground become those of an exclusive disjunction.

The source for the exclusion implicature of alternative questions is:

Fact 13 $\forall s$: if $?(p \vee q) \propto s$, then $?!(p \vee q) \propto s$, and $s[?!(p \vee q)] \succeq s[?(p \vee q)]$

Comparative homogeneity will invariably prefer the polar question $?!(p \vee q)$ over the alternative question $?(p \vee q)$, because it is *less inquisitive*. The dialogue principle: be as compliant to the common ground as you can, is flouted, *unless we assume* the less inquisitive question $?!(p \vee q)$ not to be inquisitive at all. We can only make sense of that if we assume that the possibility that $\neg p \wedge \neg q$ apparently does not count.

The source for the suggestion of exclusiveness of $p \vee q$ and $?(p \vee q)$ is:

Fact 14 $p \wedge q$ is not compliant to $p \vee q$ or $?(p \vee q)$, and $p \wedge q$ is more homogeneous to $p \vee q$ and $?(p \vee q)$ than either p or q

The option to respond to $p \vee q$ or $?(p \vee q)$ with $p \wedge q$ is blocked by relatedness, whereas at the same time $p \wedge q$ would be more homogeneous, because it is more informative than either p or q. We can only make sense of that if we assume that the possibility that $p \wedge q$ apparently does not count.

If the responder goes along with that suggestion and reponds with one of the possibilities, say p, she thereby implicates that the *alternative* possibility q is excluded.

4.4.2 Possibilities and Alternatives

Alternatives can be detected in the common ground stack.

Definition 24 (Alternatives) Let $\sigma = \langle\langle\langle\sigma', s\rangle, t\rangle, u\rangle$, and $u \propto s$ & $u \succeq s$.

1. The *alternatives* in σ are the possibilities in t which are not possibilities in u.
2. A^σ is the union of the alternatives in σ if the precondition of the definition is met, else $A^\sigma = \emptyset$.

Let $s = \omega[p \vee q]$; t the effect of thematizing p in $\omega[p \vee q]$, i.e., $t = \omega[p \vee q]$ as well; u the effect of assuming p: $u = \omega[p]$. The precondition of the definition is met, p is compliant to $p \vee q$. The possibilities in $t = \omega[p \vee q]$ are the possibility that p: $\{i \in I \mid i(p) = 1\}$ and the possibility that q: $\{i \in I \mid i(q) = 1\}$. The only possibility in $u = \omega[p]$ is the possibility that p. The only alternative in this situation is the possibility that q. Hence, $A^\sigma = \{i \in I \mid i(q) = 1\}$.

Note that not only the possibility that q counts as an alternative in a situation like the one above, where we were dealing with overlapping possibilities. According to the definition the possibility that q also counts as an alternative after the answer p to the yes/no question $?p$, and the possibility that $p \to \neg q$ counts as an alternative after the answer $p \to q$ after the conditional question $p \to ?q$. Such non-overlapping alternatives play no role.

The secundary uptake of alternative exclusion is defined as follows:

Definition 25 (Alternative Exclusion)

$$\langle\langle\sigma, s\rangle, t\rangle[\textsc{ExclA}] = \langle\langle\langle\sigma, s\rangle, t\rangle, \{\langle i, j\rangle \in t \mid i, j \notin A^{\langle\langle\sigma,s\rangle,t\rangle}\}\rangle.$$

Continuing on our example. The new top added to the stack is:

$$\{\langle i, j\rangle \in \omega[p] \mid i, j \notin \{i \in I \mid i(q) = 1\}\} = \omega[p \wedge \neg q]$$

Note that alternative exclusion has no effect when there are no alternatives and the union of the alternatives is the empty set. Neither will alternative exclusion have effect in case there are no overlapping possibilities in s.

4.4.3 Excluding Blocks

In stating the operation that will take care of the exclusion implicature of alternative questions the following notion is used.

Definition 26 (Blocks)

1. The *blocks* in s are those possibilities in s which are also possibilities in the Euclidean closure of s. (Non-overlapping possibilities in s)
2. B^s is the union of the blocks in s, if the union of the blocks in s does not equal $s^\star$, else $B^s = \emptyset$

Consider the example where $s = \omega[?(p \vee q)]$. There are three possibilities in $\omega[?(p \vee q)]$: the possibility that p; the possibility that q and the possibility that $\neg p \wedge \neg q$. The possibilities that p and that q overlap. In the Euclidean closure of $\omega[?(p \vee q)]$, which equals $\omega[?!(p \vee q)]$, the union of the possibilities that p and that q forms a single possibility. The possibility that $\neg p \wedge \neg q$ remains untouched, and is the only possibility in $\omega[?(p \vee q)]$ that survives as such in the Euclidean closure of $\omega[?(p \vee q)]$. Hence, $B^s = \{i \in I \mid i(p) = i(q) = 0\}$.

The secundary uptake of block exclusion is defined as follows:

Definition 27 (Block Exclusion)

$$\langle\sigma, s\rangle[\textsc{ExclB}] = \langle\langle\sigma, s\rangle, \{\langle i, j\rangle \in s \mid i, j \notin B^s\}\rangle$$

Continuing on our example where $s = \omega[?(p \vee q)]$. The new top added to the stack by block exclusion is:

$$\{\langle i, j\rangle \in \omega[?(p \vee q)] \mid i, j \notin \{i \in I \mid i(p) = i(q) = 0\}\} = \omega[p \vee q]$$

For block exclusion to do any work, there should be at least three possibilities in s, one of which is a block and two of which are overlapping possibilities.

4.4.4 Secundary Uptake

Every primary uptake of a sentence is *blindly followed* by the following secundary uptake, which operates purely on the stack produced by the primary uptake as such.

Definition 28 (Secundary Uptake: Exclusion Implicatures)

$$\langle\langle\sigma, s\rangle, t\rangle[\text{EXCL}] = \begin{cases} \langle\langle\sigma, s\rangle, t\rangle[\text{EXCLA}] & \text{if } s^\star \neq t^\star \\ \langle\langle\sigma, s\rangle, t\rangle[\text{EXCLB}] & \text{if } s^\star = t^\star \end{cases}$$

In inspecting whether $s^\star = t^\star$ or $s^\star \neq t^\star$, the operation detects whether the last sentence was informative in the top of σ or not. If it was (the case of p in response to $p \vee q$), alternative exclusion is applied, if it was not (the case of $?(p \vee q)$), block exclusion is applied.

There are many cases where the operation just adds a copy of t to the stack. In cases where there are no exclusion implicatures the effect of the operation is void. E.g., if $?p$ was a compliant question, we find (at least) three identical states $s^\star[?p]$ after each other on top of the stack. There is no harm in this, the third type of operations on stacks which absorb reactions to the last sentence uttered and its exclusion implicatures, will deal with all such 'redundant' copies in one go.

4.5 Absorbing Reactions: Cancel, Accept, and Support

As long as the common ground stack and the stacks of the responder and the stimulator simultaneously undergo the uptake operations, the state on top of the common ground stack is a common ground for the states on top of the stacks of the stimulator and the responder. That deals with the first dialogue principle.

The second dialogue principle involved to maintain consistency of one's state, and to publicly announce if for this reason a provisional update, an informative step in the uptake, is to be *cancelled*. After that the responder can relate to the result of thematizing the stimulus, which allows for critical responses.

Even in happier circumstances, not much has been gained yet. The common ground stack has been expanded, but the common ground as such, the bottom of the stack, has remained untouched. Improving on the common ground requires a signal from the responder that she can *accept*, or perhaps even *supports* the information provided by the stimulus, turning provisional updates into real ones.

We now turn to the operations that absorp the reactions of cancellation, acceptance and support. For the latter two, we first define an auxiliary notion of restricting a state s to the data present in a state t.

Definition 29 (Restriction) $s\lceil t\rceil = \{\langle i,j\rangle \in s \mid \langle i,i\rangle \in t \;\&\; \langle j,j\rangle \in t\}$

Restriction eliminates all pairs of indices $\langle i,j\rangle$ in s, when i or j is not present in t. So, we update s with the information present in t.

The following fact tells us that restricting s to $t[\varphi]$ is the same as updating s with the rheme $!\varphi$ of φ, as long as $t[\varphi]$ is continuous to s. That holds for all states preceding $t[\varphi]$ in a stack.

Fact 15 (Restriction) $s\lceil t[\varphi]\rceil = s[!\varphi]$, if $t[\varphi]^\star \subseteq s^\star$

The operations to be performed when the need for cancellation, or the possibility for acceptance or support are publicly signalled, are defined as follows.

Definition 30 (Cancellation, Acceptance, and Support)

1. $\langle\langle\sigma,s\rangle,t\rangle\ [\bot] = \begin{cases} \langle\sigma,s\rangle & \text{if } s \text{ is not indifferent} \\ \langle\sigma,s\rangle[\bot] & \text{otherwise}\end{cases}$

2. $\langle\langle\sigma,s\rangle,t\rangle\ [\Diamond] = \begin{cases} \langle\sigma,s\lceil t\rceil\rangle & \text{if } s\lceil t\rceil \text{ is not indifferent} \\ \langle\sigma,s\lceil t\rceil\rangle[\Diamond] & \text{otherwise}\end{cases}$

3. $\langle\langle\sigma,s\rangle,t\rangle\ [\top] = \begin{cases} \langle\sigma^{\triangleleft}\lceil t\rceil, s\lceil t\rceil\rangle & \text{if } s\lceil t\rceil \text{ is not indifferent} \\ \langle\sigma,s\lceil t\rceil\rangle[\top] & \text{otherwise}\end{cases}$

All three operations deconstruct a stack, popping a state from the stack in case of cancellation, and pulling information down the stack in case of acceptance and support, and they keep doing so until they meet a state in which there remains an issue. Even after that, support keeps percolating information all the way down the stack.

So, after one of these operation has been perfomed, there is always an issue to relate to for the next move in the dialogue, until all possible issues have been resolved.

It is not essential in order to maintain a common ground that the current state of the stimulator actually fully supported the information, implicatures and suggestions, signalled by her own utterance. The only thing that matters in order to maintain a common ground, is that *after* the responder has signalled that she accepts them, upon which the common ground is made to accept them, the stimulator performs the same operation signalled by the responder on her own stack as well.

This is particularly relevant for implicatures and suggestions made by the stimulator, which certainly need not concern information strongly supported by her state. But they are pieces of information that the stimulator should be willing to accept once the responder has signalled that she is willing to do so.

5 Loose End

The loose end are explicitly non-exclusive disjunctions and alternative questions, like $p \vee q \vee (p \wedge q)$ and $?(p \vee q \vee (\neg p \wedge \neg q))$. They are semantically fully equivalent to their non-exclusive brethren. One could use Manner to deal with them. They obviously involve more processing effort. Hence, there must be a reason for asking this extra effort, etc.

One can take this to invite an alternative interpretation where, e.g., $p \vee q \vee (p \wedge q)$ is taken to mean $\textsc{only}(p) \vee \textsc{only}(q) \vee (p \wedge q)$. The good news is that our notion of alternative exclusion gives us the tools we need to implement the required interpretation of ONLY.

A more radical solution, not requiring to actually introduce ONLY as an operator in the language, is to declare that disjunction is pragmatically ambiguous between two ways of turning a relation on states into a Euclidean relation. One is standard Euclidean closure where you extend the relation by adding pairs $\langle j, k\rangle$ in case $\langle i, j\rangle$ and $\langle i, k\rangle$, but not $\langle j, k\rangle$. Leading from $p \vee q$ to $!(p \vee q)$.

The other way might be called Euclidean disclosure (or perhaps the other way around), where you remove pairs $\langle j, k\rangle$ in case $\langle i, j\rangle$ and $\langle i, k\rangle$, but not $\langle j, k\rangle$. Leading from $p \vee q$ to $(p \wedge \neg q) \vee (q \wedge \neg p)$.

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A Examples

(1) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{?} = \langle\langle\langle\ \rangle, \imath\rangle, \imath \cup \imath^{\star}[?(p \vee q)]\rangle = \langle\langle\langle\ \rangle, \imath\rangle, \imath \cup \omega[?(p \vee q)]\rangle =$

$= \langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle$

(2) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow} = \langle\langle\ \rangle, \imath\rangle[p \vee q]^{?}[p \vee q]^{\uparrow} = \langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle[p \vee q]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle$

(3) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond] = \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle[\Diamond] =$

$= \langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\lceil\omega[p \vee q]\rceil\rangle = \langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)][!(p \vee q)]\rangle =$

$= \langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle$

(4) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond][p]^{\Uparrow} = \langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle[p]^{\Uparrow} = \langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle[p]^{?}[p]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q] \cup (\omega[p \vee q])^{\star}[?p]\rangle[p]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q] \cup (\omega[!(p \vee q)][?p]\rangle[p]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q] \cup (\omega[p \vee (\neg p \wedge q)]\rangle[p]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q \vee (\neg p \wedge q)]\rangle[p]^{\uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle[p]^{\uparrow} =$

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, (\omega[p \vee q])^{\star}[p]\rangle =$

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle$

(5) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond][p]^{\Uparrow}[\textsc{Excl}] = \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle[\textsc{Excl}] =$

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle[\textsc{Excl}\mathrm{A}] =$

Since $A^{\langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle} = \{i \in I \mid i(q) = 0]\}$,

and $\omega[p \wedge \neg q] = \{\langle i, j\rangle \in \omega[p] \mid i, j \notin \{i \in I \mid i(q) = 0]\}\}$

$= \langle\langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle, \omega[p \wedge \neg q]\rangle$

(6) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond][p]^{\Uparrow}[\textsc{Excl}][\Diamond] =$

$= \langle\langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle, \omega[p \wedge \neg q]\rangle[\Diamond] =$

Since $\omega[p]\lceil\omega[p \wedge \neg q]\rceil = \omega[p \wedge \neg q]$ is indifferent:

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p \wedge \neg q]\rangle[\Diamond] =$

Since $\omega[p \vee q]\lceil\omega[p \wedge \neg q]\rceil = \omega[p \wedge \neg q]$ is indifferent:

$= \langle\langle\langle\ \rangle, \imath\rangle, \omega[p \wedge \neg q]\rangle[\Diamond] =$

Since $\imath\lceil\omega[p \wedge \neg q]\rceil = \imath[p \wedge \neg q]$ is *not* indifferent:

$= \langle\langle\ \rangle, \imath[p \wedge \neg q]\rangle$

(7) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond][p]^{\Uparrow}[\textsc{Excl}][\bot] =$

$= \langle\langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p]\rangle, \omega[p \wedge \neg q]\rangle[\bot] =$

Since $\omega[p \wedge \neg q]$ and $\omega[p]$ are indifferent and $\omega[p \vee q]$ is not:

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle$

(8) $\langle\langle\ \rangle, \imath\rangle[p \vee q]^{\Uparrow}[\Diamond][p]^{\Uparrow}[\text{Excl}][\bot][p \wedge q]^{\Uparrow}[\Diamond] =$

$=\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle[p \wedge q]^{\Uparrow}[\Diamond] =$

$= \langle\langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p \vee q]\rangle, \omega[p \wedge q]\rangle[\Diamond] =$

$= \langle\langle\ \rangle, \imath[p \wedge q]\rangle$

(9) $\langle\langle\ \rangle, \imath\rangle[?(p \vee q)]^{\Uparrow}[\text{Excl}] = \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle[\text{ExclB}] =$

Since the Euclidean closure of $\omega[?(p \vee q)] = \omega[?!(p \vee q)]$,

and $B^{\omega[?(p \vee q)]} = \{i \in I \mid i(p) = i(j) = 0\}$:

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q) \wedge \neg(\neg p \wedge \neg q)]\rangle =$

$= \langle\langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle, \omega[p \vee q]\rangle$

(10) $\langle\langle\ \rangle, \imath\rangle[?(p \vee q)]^{\Uparrow}[\text{Excl}][\bot] = \langle\langle\ \rangle, \imath\rangle[?(p \vee q)]^{\Uparrow} =$

$= \langle\langle\langle\langle\ \rangle, \imath\rangle, \omega[?(p \vee q)]\rangle, \omega[?(p \vee q)]\rangle$