

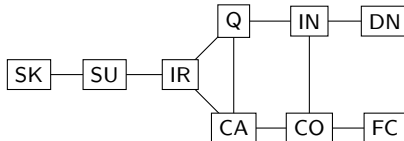
Epistemic indefinites and methods of identification

Maria Aloni (joint work with Angelika Port)

University of Amsterdam, ILLC

Workshop on Epistemic Indefinites
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Haspelmath Map



► **9 main functions** (context/meaning) **for indefinites:**

- (1)
 - a. *Someone* called. Guess who? [Specific Known]
 - b. I heard *someone*, but I couldn't tell who it was. [Sp. Unkn]
 - c. ...
 - d. In Freiburg the weather is nicer than *anywhere* else. [CO]
 - e. John can kiss *anybody*. [Free Choice]

- **Epistemic indefinites:** indefinites that have the specific unknown (SU) function, but no specific known (SK) function.

Epistemic Indefinites

German *irgendein*:

- (2) *Irgendein* Student hat angerufen. #Rat mal wer?
 Some student has called. Guess prt who?
 'Some student called. Guess who?' [SK]
- (3) Ich habe *irgendein* Geräusch gehört, ich weiss nicht was es war.
 I have some noise heard, I know not what it was
 'I heard some noise, but I don't know what it was' [SU]

Epistemic Indefinites

Italian *un qualche*

- (4) Ha sposato un qualche professore. #Indovina chi?
She-has married a some professor. Guess who?
'She married some professor. Guess who?' [SK]
- (5) Ho sentito un qualche rumore, ma non saprei dire quale.
I-have heard a some noise, but not I-could say which.
'I heard some noise, but I could not say which' [SU]

Qualche vs *un qualche* (Zamparelli 2007)

- (6) a. Ho sposato qualche ragazza \Rightarrow I am polygamous
b. Ho sposato una qualche ragazza. \Rightarrow I don't know who I married

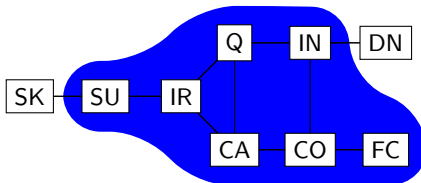
Introduction

Overview

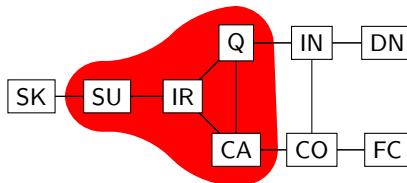
1. The variety of epistemic indefinites: German *irgendein* and Italian *un qualche*
2. The pragmatic approach and its problems
3. Alternative dynamic account using conceptual cover (CC) and its potential problems
4. Conclusion

Distribution of *irgendein* and *un qualche*

- *Irgendein* (Port 2010)



- *Un qualche* (Zamparelli 2007)



Modal Variation effect in specific uses (spMV)

► Ignorance effect in episodic sentences:

- (7) Irgendein Student hat angerufen, (#nämlich Peter).
 Some student has called (#namely Peter)
 'Some student called, I don't know who'
- (8) Maria ha sposato un qualche professore, (#cioè Vito).
 Maria has married a some professor (#namely Vito)
 'Maria married some professor, I don't know who'

Indifference reading also possible, but disregarded in this talk.

► Modal Variation (MV) effect rather than Free Choice (FC):

- (9) Hide-and-seek situation (M&O 2010): we don't know where John is, but we know that he is not in the bedroom or in the bathroom
- Gianni è in una qualche stanza della casa.
 - Hans ist in irgendeinem Zimmer im Haus.
 - John is in some room of the house.
- (10) a. MV: I don't know where $\mapsto \neg \exists x \Box \phi$
 b. FC: He might be anywhere $\mapsto \forall x \Diamond \phi$

Modal Variation under Epistemic Modals (epiMV)

► Ignorance effect under epistemic modals:

- (11) Maria muss irgendeinen Doktor geheiratet haben.
Maria must some doctor married have
'Maria must have married some doctor, I don't know who'
- (12) Maria deve/?può aver sposato un qualche professore.
Maria must/?may have married a some professor
'Maria must have married some professor, I don't know who'

► Modal variation effect rather than free choice:

- (13) Hide-and-seek situation (M&O 2010):
- Gianni deve essere in una qualche stanza della casa.
 - Hans muss in irgendeinem Zimmer im Haus sein.
 - John must be in some room of the house.

Agent-oriented epistemic effects (epiMV)

► Agent-oriented epistemic effects under propositional attitude verbs:

- (14) Andy glaubt, dass Maria irgendeinen Doktor geheiratet hat.
Andy believes that Maria some doctor married had
- a. 'Andy believes that Maria married some doctor, I don't know who' [spMV]
 - b. 'Andy believes that Maria married some doctor, *Andy* doesn't know who' [agent-oriented epiMV]
- (15) Antonio crede che Maria abbia sposato un qualche professore.
Antonio believes that Maria has_{subj} married a some professor
- a. 'Antonio believes that Maria married some professor, I don't know who' [spMV]
 - b. 'Antonio believes that Maria married some professor, *Antonio* doesn't know who' [agent-oriented epiMV]

Negative polarity uses (NPI)

- *Irgendein*: narrow scope existential meaning in negative contexts:

(16) Niemand hat irgendeine Frage beantwortet. [NPI]
Nobody has some question answered.
'Nobody answered any question'

- *Un qualche*: deviant in negative contexts:

(17) ??Nessuno ha risposto a una qualche domanda. [#NPI]
Nobody has answered to a some question.
'Nobody answered any question'

Free Choice uses (deoFC)

- *Irgendein*: Free choice effect under deontic modals

- (18) Maria muss/darf *irgendeinen* Professor heiraten.
Maria must/can some professor marry.
- a. 'There is some professor Maria must/can marry, I don't know who' [spMV]
 - b. 'Maria must/can marry a professor, any professor is a permissible option' [deoFC]

- *Un qualche*: no free choice effects under deontic modals

- (19) Maria deve/può sposare un qualche professore.
Maria must/can marry a some professor.
- a. 'There is some professor Maria must/can marry, I don't know who' [spMV]
 - b. # 'Maria must/can marry a professor, any professor is a permissible option' [#deoFC]

Variety of epistemic indefinites

- ▶ Four main functions (context/meaning) for epistemic indefinites:
 - ▶ spMV: ignorance (MV) effect in specific uses
 - ▶ epiMV: ignorance (MV) effect under epistemic modals
 - ▶ NPI: narrow scope existential meaning in DE contexts
 - ▶ deoFC: free choice effect under deontic modals
- ▶ Variety of epistemic indefinites:

	spMV	epiMV	NPI	deoFC
<i>irgendein</i>	yes	yes	yes	yes
<i>algun</i>	yes	yes	yes	no
<i>un qualche</i>	yes	yes	no	no
<i>si</i>	yes	no	no	no
<i>vreun</i>	no	yes	yes	no
<i>any</i>	no	no	yes	yes

- ▶ Alternative definition of epistemic indefinites: indefinite with spMV or epiMV function

Pragmatic analyses of epistemic indefinites

- ▶ Main idea: MV and FC effects in EIs are conversational implicatures:
 - ▶ Derivable by Gricean reasoning
 - ▶ Defeasible/Reinforceable
 - ▶ Non-detachable
- ▶ Defended in various forms:
 - ▶ Kratzer & Shimoyama, 2002, Kratzer 2005, Chierchia 2006
 - ▶ Alonso-Ovalle & Menéndez-Benito 2009, 2010
 - ▶ Schulz 2005, Aloni 2007, Aloni and van Rooij 2007
- ▶ Parsimonious, but doubts on defeasibility and reinforceability.
- ▶ Non-detachable: inference based on meaning rather than form ⇒ different semantics must be given for different EIs

Doubts on defeasibility and reinforceability (Schulz)

- ▶ Reinforceability: implicatures can be made explicit without that the utterance loses acceptability (O&M)

- (20) a. I ate some of the cookies, *but* not all of them.
b. This table is round, (*#but*) it doesn't have corners.
- (21) a. *Irgendjemand* hat angerufen, (?but) I don't know who.
b. Bea esce con *un qualche* studente, (?but) I don't know who.
c. Bea sale con *algún* estudiante, but I don't know who.

- ▶ Defeasibility: implicatures can be defeated by information to the contrary.

- (22) (wife to husband, after talking to somebody at the door)
#Irgendeiner unserer Söhne möchte mit dir sprechen.
'Irgendeiner of our sons wants to talk to you'

- (23) *#Ho* sposato una qualche ragazza.
'I have married una qualche girl'

(Zamparelli)

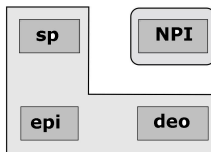
Focus on Menendez-Benito & Alonzo-Ovalle

- ▶ Wrong predictions wrt MV and FC:
 - ▶ *irgendein* \mapsto maximal domain widening \Rightarrow FC
 - ▶ specific \mapsto FC: $\forall x \Diamond \phi$ (too strong!)
 - ▶ under epistemic modal \mapsto FC: $\forall x \Diamond \phi$ (too strong!)
 - ▶ under deontic modal \mapsto FC: $\forall x \Diamond \phi$
 - ▶ under negation: no effect
 - ▶ *algún* \mapsto minimal domain widening \Rightarrow MV
 - ▶ specific \mapsto MV: $\neg \exists x \Box \phi$
 - ▶ under epistemic modal \mapsto MV: $\neg \exists x \Box \phi$
 - ▶ under deontic modal \mapsto MV: $\neg \exists x \Box \phi$
 - ▶ under negation: no effect
- ▶ Potential problems with assumption that episodic sentences are implicitly modalized: *vreun* sensitive to overttness of its licensing operator?

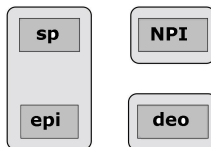
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<i>algun</i>	yes	yes	yes	no
<i>un qualche</i>	yes	yes	no	no
<i>si</i>	yes	no	no	no
<i>vreun</i>	no	yes	yes	no

Summary

- ▶ Pragmatic approach: parsimonious, but empirical problems



- ▶ Desiderata:



- ▶ Our proposal: Dynamics with Conceptual Covers:
 - ▶ $sp \equiv epi \mapsto$ obligatory ignorance (MV) effects (via CC-shift)
 - ▶ $epi \not\equiv deo$ (via dynamic analysis of modality)
 - ▶ deoFC (and indifference): still unaccounted

An alternative analysis for epistemic indefinites

- ▶ Epistemic indefinites \mapsto existentials with two characteristics:
[cf. Kadmon & Landman 1993]
 1. **Domain Shift**: induce an obligatory domain shift
 2. **Felicity Condition**: express conditions on the input context that must be satisfied for the indefinite to be felicitous
- ▶ Modal Variation effect as result of lexically encoded felicity condition rather than Gricean reasoning (cf. dynamics of presupposition)
 \Rightarrow ??defeasible, ??reinforceable
- ▶ MV as fossilized implicature: inferences, pragmatic in origin, now part of lexically encoded meaning \Rightarrow derivable by Gricean means
- ▶ Difference between different indefinites in terms of different domain shifts they can induce \Rightarrow variety of *Els*

Domain shift triggered by epistemic indefinites

- ▶ Epistemic indefinites *block context induced domain selections*:
[Zamparelli 2007]
- ▶ Two ways in which context determine quantificational domains:
 - ▶ Contextual domain restriction (Westerståhl 1984):

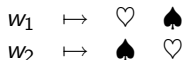
(24) Everybody passed the exam. [e.g. everybody in my class]
Blocking \mapsto **domain widening** (DW)
 - ▶ Pragmatic selection of a method of identification (Aloni 2001):

(25) **The card scenario:** Two face-down cards, the ace of hearts and the ace of spades. You know that the winning card is the ace of hearts, but you don't know whether it's the card on the left or the one on the right.

(26) You know which card is the winning card. [True or false?]
Blocking \mapsto Shift of identification method or **conceptual cover shift** (CC-shift)

Conceptual Covers

- ▶ Identification methods formalized as *conceptual covers* [Aloni 2001]
- ▶ A conceptual cover CC is a set of concepts such that in each world, every individual instantiates exactly one concept in CC .
- ▶ The card scenario



Only two covers definable in this model:

$$(27) \quad \{\lambda w. \llbracket \iota x. \text{ON-THE-LEFT}(x) \rrbracket_w, \lambda w. \llbracket \iota x. \text{ON-THE-RIGHT}(x) \rrbracket_w\}$$

$$(28) \quad \{\lambda w. \llbracket \iota x. \text{ACE-OF-SPADES}(x) \rrbracket_w, \lambda w. \llbracket \iota x. \text{ACE-OF-HEARTS}(x) \rrbracket_w\}$$

$$(29) \quad \#\{\lambda w. \llbracket \iota x. \text{ON-THE-LEFT}(x) \rrbracket_w, \lambda w. \llbracket \iota x. \text{ACE-OF-HEARTS}(x) \rrbracket_w\}$$

Context Dependence

- ▶ Interpretation often depends on the assumed method of identification.
- ▶ Different methods of identification selected at different occasions:

(30) **Context:** A is compiling a bibliography:

A: Who_n is the author of *Semantic Structures*? [$?x_n.x_n = \alpha$]
 B: Lee Jackson. [$n \mapsto \text{Naming}$]
 B: ??That lady over there. [$n \mapsto \text{Ostension}$]

(31) **Context:** A wants to meet the author of *Semantic Structures*:

A: Who_n is the author of *Semantic Structures*? [$?x_n.x_n = \alpha$]
 B: ??Lee Jackson. [$n \mapsto \text{Naming}$]
 B: That lady over there. [$n \mapsto \text{Ostension}$]

(32) A: Who_n is Lee Jackson? [$?x_n.x_n = \alpha$]
 B: ??Lee Jackson. [$n \mapsto \text{Naming}$]
 B: That lady over there. [$n \mapsto \text{Ostension}$]

Epistemic indefinites & identification methods

- ▶ Puzzle of specific unknown uses:

(33) Devo incontrare un qualche professore.
I-must meet a some professor
'I must meet a certain professor, but I don't know who he is.'

- ▶ Specific: speaker has someone in mind \Rightarrow speaker can identify
- ▶ But unknown: speaker doesn't know who \Rightarrow speaker cannot identify
- ▶ Different identification methods are at play:
 - ▶ Speaker can identify on one method (e.g. naming) (specific)
 - ▶ But not on another (e.g. ostension) (unknown)
- ▶ Main intuition: Referents of EIs typically identified via a method different from the one required for knowledge

Workshop scenario: the case of *irgendein*

- (34) a. Ich muss irgendjemand hier treffen. Sie heisst Regine Eckardt, aber ich weiss nicht wer sie ist.
I have to meet somebody. Her name is Regine Eckardt, but I don't know who she is.
b. *Speaker-can-identify* \mapsto [Naming], *unknown* \mapsto [Ostension]
- (35) a. Maria hat vorgeschlagen, dass ich einen Artikel von irgendjemandem hier lesen sollte. Von der Frau da drueben. Weisst Du wer das ist?
Maria has suggested that I should read an article from somebody here. That lady over there. Do you know who she is?
b. *Speaker-can-identify* \mapsto [Ostension], *unknown* \mapsto [Naming]

Workshop scenario: the case of *un qualche*

- (36) a. Devo leggere un articolo di un qualche professore. ??È quella signora seduta lì, ma non so come si chiama.
I have to read an article of some professor. ??It's that lady over there, but I don't know her name.
b. ??*Speaker-can-identify* \mapsto [Ostension], *unknown* \mapsto [Naming]
- (37) a. Devo incontrare un qualche professore. Si chiama Regine Eckardt, ma non so che aspetto abbia.
I have to meet some professor. Her name is Regine Eckardt, but I don't know how she looks like.
b. *Speaker-can-identify* \mapsto [Naming], *unknown* \mapsto [Ostension]
- (38) a. Devo incontrare un qualche professore. È il capo del dipartimento di filosofia, ma non so come si chiama.
I have to meet some professor. She is the Head of the Philosophy Department, but I don't know her name.
b. *Speaker-can-identify* \mapsto [Description], *unknown* \mapsto [Naming]

Els & identification methods: Romance vs Germanic

- ▶ Ranking on methods of identification (Aloni 2001):

(39) ostension > naming > description

- ▶ Hypothesis:

(40) In Romance, but not in German, identification method required for knowledge must be higher in order than identification method required for specific Els

⇒ if referent identified by ostension, Els infelicitous in Romance

- ▶ Lambada example [Alonso-Ovalle & Menendez-Benito 2003]:

(41) a. Look! Some/Irgendein professor is dancing on the table!
b. *Speaker-can-identify* \mapsto [Ostension], *unknown* \mapsto [Naming]

(42) a. Look! ??Un qualche/algún professor is dancing on the table!
b. ??*Speaker-can-identify* \mapsto [Ostension], *unknown* \mapsto [Naming]

Proposal

► Epistemic indefinites: existentials with two characteristics:

1. Induce obligatory domain-shift ($D \rightarrow D'$):

- *un qualche*: CC-shift
- *irgendein*: CC + DW

2. Are felicitous in context σ iff domain-shift is for a reason:

(i) CC-shift \mapsto NECESSARY WEAKENING:

$$(43) \quad \sigma \models \dots \exists x_{D'} \dots, \text{ but } \sigma \not\models \dots \exists x_D \dots \quad [\text{Quality}]$$

CC-shift justifies only if otherwise speaker state would not support statement

(ii) DW \mapsto STRENGTHENING:

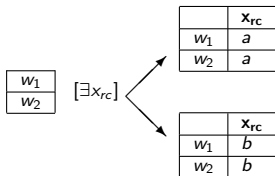
$$(44) \quad \dots \exists x_{D'} \dots \models \dots \exists x_D \dots \quad [\text{Quantity}]$$

► Predictions of implementation in Dynamic Semantics:

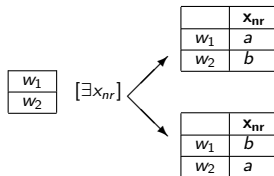
	spMV	epiMV	NPI	deoFC
<i>irgend</i>	yes	yes	yes	no (wrong!)
<i>un qualche</i>	yes	yes	no	no

Epistemic Indefinites in Dynamic Semantics with CC

- ▶ Specific uses of indefinites introduce discourse referents
- ▶ In dynamic semantics with CC, discourse referents are elements of a pragmatically determined conceptual cover



rigid cover (rc)



non-rigid cover (nr)

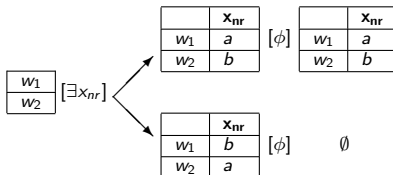
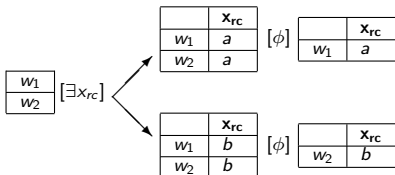
Simplifying:

- ▶ Assume knowing who requires rigid identification
- ▶ Epistemic indefinites signal obligatory shift to a non-rigid cover (CC-shift) \mapsto introduce non-rigid discourse referents
- ▶ If CC-shift is not trivial, use of indefinite entails not knowing who (MV)

Deriving Modal Variation

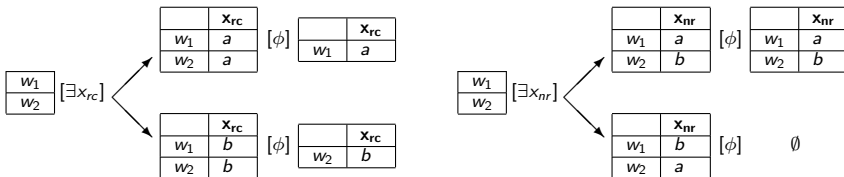
- ▶ Modal Variation effect whenever CC-shift is for a reason.
- ▶ **Felicity Condition:**
CC-shift for a reason only if otherwise speaker state would not support statement
- ▶ **Fact:** Necessary weakening \Rightarrow Modal Variation

$$(45) \quad \sigma \not\models \exists x_{rc} \phi, \text{ but } \sigma \models \exists x_{nr} \phi$$



Support versus truth

- ▶ **Truth:** $\sigma \vdash \phi$ iff $\forall i \in \sigma : \exists \sigma' (\sigma[\phi]\sigma' \ \& \ i \prec \sigma')$
- ▶ **Support:** $\sigma \models \phi$ iff $\exists \sigma' (\sigma[\phi]\sigma' \ \& \ \forall i \in \sigma : i \prec \sigma')$
- ▶ Support \Rightarrow truth, but not \Leftarrow . E.g. $\sigma \vdash \exists x_{rc} \phi$, but $\sigma \not\models \exists x_{rc} \phi$:



- ▶ Only support is a CC-sensitive notion, e.g. $\sigma \not\models \exists_{rc} \phi$, but $\sigma \models \exists_{nc} \phi$
- ▶ **Necessary weakening**, and **epistemic modals** defined in terms of support
- ▶ Other modals (notably **deontic**) defined in terms of truth

Un qualche (only CC-shift): spMV

- ▶ Assume knowledge requires rigid covers:
 - (46) a. Speaker does not know who Maria married.
 - b. $\neg \exists y_m \Box \phi(y_m)$ m must be rigid
- ▶ *Un qualche*-indefinites induce shift to a non-rigid cover (CC-shift):
 - (47) a. Maria married un qualche professor.
 - b. $\exists x_n \phi(x_n)$ n must be non-rigid
- ▶ Whenever CC-shift is for a reason, we predict an ignorance effect
- ▶ Technically: Modal variation as *pragmatic entailment* (cf. Strawson entailment):
 - (48) a. Maria married un qualche professor \Rightarrow S does not know who
 - b. $\exists x_n \phi(x_n) \models_P \neg \exists y_m \Box \phi(y_m)$
 - c. $\phi \models_P \psi$ iff $\forall \sigma: \phi, \psi$ felicitous in σ & $\sigma \models \phi \Rightarrow \sigma \models \psi$

Un qualche (only CC-shift): epiMV

- ▶ epiMV speaker-oriented:

- (49)
- a. Maria deve aver sposato un qualche professore.
 - b. Maria must have married some professor \Rightarrow
Speaker doesn't know who.
 - c. $\Box \exists x_n \phi(x_n) \models_P \neg \exists y_m \Box \phi(y_m)$
 - d. $\sigma[\Box \phi] \{i \in \sigma \mid \sigma \models \phi\}$ [V]

[Veltman 1997]

- ▶ epiMV agent-oriented:

- (50)
- a. Antonio crede che Maria abbia sposato un qualche professore.
 - b. Antonio believes that Maria married some professor \Rightarrow *Antonio* doesn't know who
 - c. $\Box_a \exists x_n \phi(x_n) \models_P \neg \exists y_m \Box_a \phi(y_m)$
 - d. $\sigma[\Box_a \phi] \{i \in \sigma \mid F(i)_a \models_P \phi\}$

Un qualche (only CC-shift): #NPI and #deoFC

- ▶ CC-shifts are trivial in negative and deontic contexts:

$$(51) \quad \begin{array}{ll} \text{a.} & \forall n, m: \neg \exists x_n \phi \equiv \neg \exists x_m \phi \\ \text{b.} & \forall n, m: \Delta \exists x_n \phi \equiv \Delta \exists x_m \phi \end{array}$$

- ▶ We correctly predict #NPI & #deoFC (no reason here for CC-shift):

$$(52) \quad \begin{array}{ll} \text{a.} & ??\text{Non ho risposto a una qualche domanda.} \quad [\# \text{NPI}] \\ \text{b.} & \# \text{'I didn't answer any question'} \\ \text{c.} & \neg \exists x_n \phi \\ \text{d.} & \sigma[\neg \phi] \{i \in \sigma \mid \neg \exists \sigma' : \sigma[\phi] \sigma' \ \& \ i \prec \sigma'\} \end{array}$$

$$(53) \quad \begin{array}{ll} \text{a.} & \text{Maria deve sposare un qualche professore.} \quad [\# \text{deoFC}] \\ \text{b.} & \# \text{'Maria must marry a professor, any professor is a} \\ & \text{permissible option'} \\ \text{c.} & \Delta \exists x_n \phi \\ \text{d.} & \sigma[\Delta \phi] \{i \in \sigma \mid F(i)_D \vdash \phi\} \end{array}$$

The case of *irgend*-indefinites: CC+DW

- ▶ spMV, epiMV: as for *un qualche*
- ▶ NPI: via DW + STRENGTHENING:

- (54)
- a. Niemand hat *irgendjemanden* angerufen.
 - b. 'Nobody called anybody'
 - c. $\neg \exists x_m \exists x_n \phi$
 - d. Prediction: *irgend* felicitous, no epistemic effect

- ▶ DeoFC: problem!

- (55)
- a. Marie muss *irgendeinen* Doktor heiraten.
 - b. 'Mary has to marry *irgend*-one doctor'
 - c. $\exists x_n \Delta \phi$ \Rightarrow [spMV]
 - d. $\Delta \exists x_n \phi$ (neither CC+WE nor DW+ST)
 - e. Prediction: spMV, #deoFC

The case of *irgend*-indefinites: CC+DW

Predictions

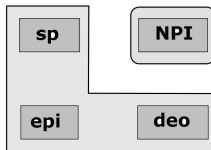
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<i>un qualche</i>	yes	yes	no	no

Possible solutions

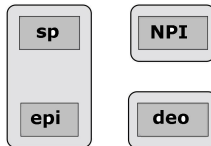
- ▶ Kratzer & Shimoyama's anti-exhaustivity inference:
 - ▶ FC inference as 'pragmatic' effect
 - ▶ Felicity: add new option in DW-felicity condition, e.g. avoidance false exhaustivity inference
 - ▶ Problem: FC inference not defeasible
- ▶ Performative analysis of deontic modals (Lewis 79, Veltman 09):
 - ▶ FC inference as semantic entailment
 - ▶ Felicity via strengthening
 - ▶ Problem: what about non-performative cases, and #deoFC for *algún*
- ▶ Condoravdi's local discharge?

Conclusions

- ▶ Pragmatic approach: parsimonious, but empirical problems



- ▶ CC-dynamic approach:



- ▶ **Future plans**

- ▶ deoFC, and indifference
- ▶ $sp \neq epi$: the case of Czech *-si*, and Romanian *vreun*

Appendix

► The Semantics (building on Aloni 2001, chapter 3)

$$\begin{aligned}
 \sigma[Rt_1, \dots, t_n]\sigma' & \text{ iff } \sigma' = \{i \in \sigma \mid \langle i(t_1), \dots, i(t_n) \rangle \in i(R)\} \\
 \sigma[\neg\phi]\sigma' & \text{ iff } \sigma' = \{i \in \sigma \mid \neg\exists\sigma'' : \sigma[\phi]\sigma'' \ \& \ i \prec \sigma''\} \\
 \sigma[\phi \wedge \psi]\sigma' & \text{ iff } \exists\sigma'' : \sigma[\phi]\sigma''[\psi]\sigma' \\
 \sigma[\exists x_n\phi]\sigma' & \text{ iff } \sigma[x_n/c][\phi]\sigma' \text{ for some } c \in C(n) \\
 \sigma[\Box\phi]\sigma' & \text{ iff } \sigma' = \{i \in \sigma \mid \sigma \models \phi\} \\
 \sigma[\Box_a\phi]\sigma' & \text{ iff } \sigma' = \{i \in \sigma \mid F(i)_a \models_{(P)} \phi\} \\
 \sigma[\Box_D\phi]\sigma' & \text{ iff } \sigma' = \{i \in \sigma \mid F(i)_D \vdash \phi\}
 \end{aligned}$$

where $F(\langle g, w \rangle)_x = \{\langle g, w' \rangle \mid wR_x w'\}$

Appendix

► Support:

$$\sigma \models \phi \quad \text{iff} \quad \exists \sigma' : \sigma[\phi]\sigma' \ \& \ \forall i \in \sigma : i \prec \sigma'$$

$$\sigma \models_P \phi \quad \text{iff} \quad \sigma \models \phi \ \& \ \phi \text{ felicitous in } \sigma$$

► Truth:

$$\sigma \vdash \phi \quad \text{iff} \quad \forall i \in \sigma : \exists \sigma' : \sigma[\phi]\sigma' \ \& \ i \prec \sigma'$$

► Entailment:

$$\phi \models \psi \quad \text{iff} \quad \forall \sigma : \sigma \models \phi \Rightarrow \sigma \models \psi$$

$$\phi \models_P \psi \quad \text{iff} \quad \forall \sigma : \phi \ \& \ \psi \text{ felicitous in } \sigma : \sigma \models \phi \Rightarrow \sigma \models \psi$$