New appendix

In this appendix we present a dynamic semantics for questions and their embedding under “know”. The system is one way of formalizing Schaffer’s remarks on the dynamics of conversation, but making explicit the importance of the order in which questions are asked.

Question representation Questions are represented by formulas of the form $?p_1, ..., p_n\ \phi$ where ? is a query-operator, $p_1, ..., p_n$ is a possibly empty sequence of propositional variables, and ϕ is a formula of predicate logic with propositional variables. Polar questions result when the query-operator binds no variable (example (1)-a). Alternative questions are represented by formulas like (1)-b which asks which of the propositions ϕ and ψ is true. Formula (1)-c represents constituent questions. It can be paraphrased as ‘which of the propositions $\phi(d), \phi(d'), ...$ are true?’.

(1)
a. Polar questions: $?\phi$
b. Alternative questions: $?p(p \wedge (p = \phi \vee p = \psi))$
c. Constituent questions: $?p(p \wedge \exists x(p = \phi(x)))$

Question denotation Questions are mapped into sets of pairs $\langle \sigma, w\rangle$ where $\sigma = \alpha_1, ..., \alpha_n$ is a sequence of propositions and w is a possible world. From such sets of pairs, representing the denotation of question Q, we will be able to recover (a) the partition induced by the question and (b) the topics set up by Q. In terms of partitions we will define various notions of answers. Topics will be used to model the contextual restriction at work in Schaffer’s examples.

Let $[\![\phi]\!]_{M,g}$ be the proposition expressed by an indicative sentence ϕ:[1]

(2) $[\![\phi]\!]_{M,g} = \{w \mid\ M, w \models_g \phi\}$.

Questions denotations are then defined as follows, where $\vec{p}$ stands for the sequence $p_1, ..., p_n$, and $\vec{\alpha}$ for the sequence $\alpha_1, ..., \alpha_n$.

(3) $[\![?\vec{p}\ \phi]\!]_{M,g} = \{\langle \vec{\alpha}, w\rangle \mid\ w \in [\![\phi]\!]_{M,g[\vec{p}/\vec{\alpha}]}\}$

The denotation of a polar question $?\phi$ is the set of pairs $\langle \lambda, w\rangle$ such that λ is the empty sequence and w satisfies ϕ. The denotation of an alternative question $?p(p \wedge (p = \phi \vee p = \psi))$ is the set of pairs $\langle p, w\rangle$ such that w satisfies p and either p is the proposition expressed by ϕ or it is the proposition

[1] The satisfaction relation $\models$, assumed in (2), is defined in a standard way. The only clause which deserves some attention is the one for propositional identity:

(i) $M, w \models_g \phi = \psi$ iff $\forall w' : M, w' \models_g \phi$ iff $M, w' \models_g \psi$

expressed by ψ. And finally the denotation of a constituent question represented as $?p(p \wedge \exists x(p = \phi(x)))$ is the set of pairs $\langle p, w \rangle$ such that w satisfies p and $p = [\![\phi]\!]_{M,g[x/d]}$ for some individual d.

Answers The denotation of a question Q determines an equivalence relation (or, equivalently, a partition) $\mathrm{Part}(Q)$ over the set of possible worlds, from which different notions of answers can be defined. For $Q = ?\vec{p}\,\phi$, two worlds w and v are in the same cell of $\mathrm{Part}(Q)$ if for each sequence of propositions $\vec{\alpha}$, $\langle \vec{\alpha}, w \rangle$ belongs to the denotation of Q iff $\langle \vec{\alpha}, v \rangle$ does as well.

(4) $\mathrm{Part}_{M,g}(?\vec{p}\,\phi) = \{\langle w, v \rangle \mid \forall \vec{\alpha} : \langle \vec{\alpha}, w \rangle \in [\![?\vec{p}\,\phi]\!]_{M,g} \text{ iff } \langle \vec{\alpha}, v \rangle \in [\![?\vec{p}\,\phi]\!]_{M,g}\}$

Exhaustive answers to Q correspond to cells in $\mathrm{Part}(Q)$. The *exhaustive true answer* to Q in w is the cell including w in $\mathrm{Part}(Q)$. Finally, *partial answers* (true in w) correspond to non-trivial unions of cells of $\mathrm{Part}(Q)$ (including w), namely unions of cells different from $\mathrm{Part}(Q)$.

Topics In terms of a question denotation we can also define the topics set up by the question as follows:

(5) $\mathrm{Top}_{M,g}(?\vec{p}\,\phi) = \{\vec{\alpha} \mid \exists w : \langle \vec{\alpha}, w \rangle \in [\![?\vec{p}\,\phi]\!] m, g\}$

Topics are sets of (sequences of) propositions. These topics will be used in what follows to define the contextual restriction on the belief state of the subject of knowledge-wh.

Results:

(6) Polar questions ("did John leave?")

a. Representation: $?\phi$
b. Partition: $\{\phi, \neg\phi\}$
c. Topics: $\{\lambda\}$ (the set containing the empty sequence)

(7) Alternative questions ("did John leave, or did Mary leave?")

a. Representation: $?p(p \wedge (p = \phi \vee p = \psi))$
b. Partition: $\{\phi \wedge \neg\psi, \neg\phi \wedge \psi, \neg(\phi \vee \psi), \phi \wedge \psi\}$
c. Topics: $\{\phi, \psi\}$

(8) Constituent questions ("who left?")

a. Representation: $?p(p \wedge \exists x(p = \phi(x)))$
b. Partition: $\{\forall x \neg\phi(x), \forall x(\phi \leftrightarrow x = d), ..., \forall x \phi(x)\}$
c. Topics: $\{\phi(d), \phi(d'), ...\}$

Dynamics A context C is defined as an ordered pair whose first index s_C is an information state (set of worlds), and whose second index i_C is a sequence of question denotations (a sequence of sets of sequence-world pairs) representing the issues under discussion in C. A context C can be updated either by an assertion P, or by the introduction of a new question Q:

(9) a. $C + P = (s_C \cap [\![P]\!], i_C)$
b. $C + Q = (s_C, i_C + [\![Q]\!])$

Updating C with an assertion P only influence the state-index eliminating those worlds in s_C in which P is false. Updating with a question Q extends the issue parameter by adding to i_C the denotation of Q as last issue under discussion.

Knowledge Let $\mathrm{ANS}_w(Q)$ be the true exhaustive answer to Q in w, and $\mathrm{Top}(C)$ denote the union of the topics introduced by all the issues in C, i.e. for $C = (s_c, [\![Q_1]\!], ..., [\![Q_n]\!])$: $\mathrm{Top}(C) = \bigcup_{i \in n} \mathrm{Top}(Q_i) \setminus \{\langle\rangle\}$. We then define knowledge as follows:

(10) "S knows Q" is true in world w with respect to context C iff

a. $K_w \cap \mathrm{Top}(C) \subseteq \mathrm{ANS}_w(Q)$, if $\mathrm{Top}(C) \neq \emptyset$;
b. $K_w \subseteq \mathrm{ANS}_w(Q)$, otherwise.