A modal logic for imperatives

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Commands and permissions

- Imperatives come with a puzzling range of illocutionary potential: *commands*, warnings, ... but also *permissions*.
- Necessity imperatives $\Box$:

(1) a. Kill Bill! COMMAND
    b. Keep away from fire! WARNING
    c. ...

- Possibility imperatives $\Diamond$: (Hamblin 1987, Schwager 2005)

(2) a. Come earlier if you like! PERMISSION
    b. (How could I save money?)
    Stop smoking for example!

- Challange: default $\Box$, sometimes (when modified) $\Diamond$.

Free choice effects

- Parallelism *any* and *or*: whenever *any* is licensed, *or* gives rise to non-standard inference pattern (Horn 1972, Kamp 1973)

(3) a. #Joe is anywhere. (episodic)
b. Joe is in Paris or in London. $\not\Rightarrow$
Joe is in Paris and Joe is in London.

(4) a. Joe may be anywhere. (possibility)
b. Joe may be in Paris or in London. $\Rightarrow$
Joe may be in Paris *and* may be in London.

(5) a. #Joe must be anywhere. (necessity)
b. Joe must be in Paris or in London. $\not\Rightarrow$
Joe must be in Paris and must be in London.

Free choice in imperatives

- Imperatives pattern with possibility statements:

(6) To continue, push any key!

(7) a. Post this letter! $\not\Rightarrow$ b. Post this letter or burn it!
c. $\phi \models \phi \vee \psi$ (Ross 1941)

- Double nature of disjunctive and *any*-imperatives:

(8) Push any x! $\Rightarrow$
a. You must push at least one x!
b. For each x: you may push x!

(9) Do a or do b! $\Rightarrow$
a. You must do a or b!
b. You may do a & you may do b!

Examples

(10) GRANDMA: Take any card!
KID GETS UP TO GET A CARD.
GRANDMA: (?) Don't you dare take the ace!

(11) MOTHER: Do your homework or help Dad in the kitchen!
SON GOES TO THE KITCHEN.
FATHER: Do your homework!
SON: But, Mom told me I could also help you in the kitchen!

Imperatives in modal logic

- The challenge:
  - (i) reconcile command ($\Box$) and permission ($\Diamond$) usages of imperatives;
  - (ii) account for free choice effects.
- In **previous attempts**, imperatives as:
  - necessity statements: $\Box$ (e.g. Schwager 05a)
  - possibility statements: $\Diamond$ (e.g. Schwager 05b)
  - underspecified: $\Box/\Diamond$ (e.g. Platzack & Rosengren 97)
  - other notions (e.g. Hausser 80, Portner 05, Mastop 05)

My proposal

- Imperatives as ∇ statements:

(12) $\nabla(\phi_1, ..., \phi_n) := \Diamond\phi_1 \wedge ... \wedge \Diamond\phi_n \wedge \Box(\phi_1 \vee ... \vee \phi_n)$

- Main characteristics of ∇:
  - ∇ operates over sets of propositions:
  - Possibility $\Diamond$ and necessity $\Box$ can be expressed using ∇:

(13) a. $\Diamond\phi \equiv \nabla(\phi, T)$
b. $\Box\phi \equiv \nabla(\phi) \vee \nabla\emptyset$ $\quad$ ($\equiv \nabla(\phi)$ in serial frames)

First applications

- Necessity imperatives: (singleton sets)

(14) Do a! $\mapsto$ $\nabla(a) \equiv \Box a$

- Possibility imperatives: (alternative sets)

(15) Do a, if you like/for example! $\mapsto$

a. $\nabla(a, T) \equiv \Diamond a$, or
b. $\nabla(a, \neg a) \equiv \Diamond a \wedge \Diamond \neg a$, or
c. $\nabla(a, ...) \models \Diamond a$, but $\not\models \Box a$

- Free choice imperatives: (alternative sets)

(16) Do *a* or *b*! $\mapsto$ $\nabla(a, b) \equiv \Diamond a \wedge \Diamond b \wedge \Box(a \vee b)$

(17) Push any *x*! $\mapsto$

$$\nabla(\phi(d), \phi(e), \phi(f), ...) \equiv \forall x \Diamond \phi(x) \wedge \Box(\exists x \phi(x))$$

- Next issue: how/where do these alternative sets originate?

Origin of alternative sets: strategy

- All possibility imperatives reduced to disjunctive imperatives:

(18) a. 'Do *a*, for example' := 'Do *a* or something else'
b. 'Do *a*, if you like' := 'Do *a* or don't do *a*'

- Account of alternative sets generated by disjunction:

(19) '*a* or *b*' generates (*a*, *b*)

- Generalization to the existential case (*any*-imperatives).

(20) 'any x ϕ' generates (ϕ(d), ϕ(e), ϕ(f), ...)

- Two possibilities for (19) and (20):
  - Kratzer and Shimoyama's (02) *Hamblin semantics*
  - *Logic of alternatives* (Aloni 02,03), i.e. modal logic with prop. quantifiers (Fine 70) + witness sequences (e.g. Dekker 94)

Logic of Alternatives

- $\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \Diamond\phi \mid \phi = \psi \mid \exists p \phi \mid$
- $M = (W, R, P, V)$, where (W, R, P) is a general frame.
- Satisfaction wrt M, w and s, a sequence of witnesses from P:

$$
\begin{array}{rcl}
M, w, s \models p & \text{iff} & V(p)(w) = 1 \\
M, w, s \models \neg\phi & \text{iff} & M, w, cs \models \phi \text{ for no } c \in P^{n(\phi)} \\
M, w, cs \models \phi \wedge \psi & \text{iff} & M, w, s \models \phi \ \&\ M, w, cs \models \psi, \text{ with } c \in P^{n(\psi)} \\
M, w, s \models \Diamond\phi & \text{iff} & \exists w' : wRw' \ \&\ M, w', cs \models \phi, \text{ with } c \in P^{n(\phi)} \\
M, w, s \models \phi = \psi & \text{iff} & \forall w' : M, w', s \models \phi \text{ iff } M, w', s \models \psi \\
M, w, qs \models \exists p \phi & \text{iff} & M', w, s \models \phi \ \&\ V'(p) = q
\end{array}
$$

- $n(\phi)$ number of (surface) existential quantifiers in ϕ.

(21) $n(\exists p \phi) = n(\phi) + 1$ & $n(\phi \wedge \psi) = n(\phi) + n(\psi)$; 0, otherwise.

Core idea

- Sentences mapped to **structured propositions** (or dynamic info states, or files), rather than propositions:

(22) a. structured propositions : $\lambda s \lambda w : M, w, s \models \phi$
b. propositions: $\lambda w : M, w \models \phi$

- Extra structure used to derive:
  - **anaphora**
  - **proposition sets** (for imperatives, but also questions)
  - ...

Anaphora

- If we add 'anaphoric pronouns' $p_1, p_2, ..$ (Dekker 1994):

(23) $M, w, s \models p_i$ iff $s_i(w) = 1$

- We obtain dynamic binding:

(24) a. $\exists p \phi \wedge \psi(p_1) \equiv \exists p(\phi \wedge \psi[p_1/p])$
b. $\exists p \phi \rightarrow \psi(p_1) \equiv \forall p(\phi \rightarrow \psi[p_1/p])$

(25) a. Yesterday I dreamt something. It was a nightmare.
b. If I dream something, it is a nightmare.

Proposition sets

From structured propositions ($\lambda s \lambda w[\phi]_{M,w,s}$) we can derive:

- Alternative sets (+/- Hamblin 73)
- Partitions (Groenendijk & Stokhof 84)

Illustration:

- Atoms: q

(26) a. $q / \exists p(p \wedge p = q)$ b. alt: $\boxed{q}$ & part: $\boxed{q \mid \textit{not } q}$

- Disjunctions: a or b

(27) a. $a \vee b / \exists p(p \wedge p = (a \vee b))$

b. alt: $\boxed{a \vee b}$ & part: $\boxed{a \vee b \mid \textit{not}(a \vee b)}$

(28) a. $\exists p(p \wedge (p = a \vee p = b))$

b. alt: $\boxed{a \mid b}$ & part: $\boxed{\textit{none} \mid \textit{only } a \mid \textit{only } b \mid \textit{both}}$

Polar/alternative questions

(29) a. Is 4 an odd number? Yes/No. (polar question)

b. $?a$ c. alt: $\boxed{a}$ & part: $\boxed{a \mid \textit{not } a}$

(30) Is 4 odd or even?

a. polar reading (expected answers: *yes*/*no*):

a'. $?(a \vee \neg a)$ alt & part: $\boxed{T}$

b. alternative reading (expected answers: *odd*/*even*):

b'. $?\exists p(p \wedge (p = a \vee p = \neg a))$ alt & part: $\boxed{a \mid \textit{not } a}$

- Same *partition* for (29) & (30b) $\mapsto$ same *embedded question*:

(31) a. I wonder whether 4 is odd $\equiv$

b. I wonder whether 4 is odd or even.

- But different *alternative sets* $\mapsto$ different *expected answers*.

A modal logic for imperatives

- Logic of Alternatives + $!\phi$.
- $!\phi$ is true iff all propositions in $alt(\phi)$ are possible & their union is necessary:

(32) $M, w, s \models\ !\phi$ iff $M, w, s \models \nabla(alt(\phi))$

- $alt(\phi)$ represents the compliance conditions of $!\phi$
  - singleton sets $\mapsto$ no-choice imperatives
  - genuine alternatives $\mapsto$ choice-offering imperatives

First application: disjunctive imperatives

- Disjunctive imperatives ambiguous (cf. Åquist 1965):

(33) Do *a* or *b*!

a. $!\exists p(p \wedge (p = a \vee p = b))$ (choice-offering)

$\Rightarrow \ \Diamond a, \Diamond b, \Box(a \vee b)$ alt: | *a* | *b* |

b. $!(a \vee b)$ (alternative-presenting)

$\not\Rightarrow \ \Diamond a, \Diamond b$ alt: $\boxed{a \vee b}$

- An alternative-presenting imperative (Rescher & Robison 64)

(34) TEACHER: John, stop that foolishness or leave the room!
JOHN GETS UP AND STARTS TO LEAVE.
TEACHER: Don't you dare leave this room!

Second application: existential imperatives

- Variety of indefinites (Haspelmath): *any, a, some*...

(35) a. Take any card! (only free choice)
b. Take some/a card! (only existential/both)

- Two possible analyses:

(36) a. $!\exists p(p \wedge \exists x(p = \phi(x)))$ (free choice)
$\Rightarrow \forall x \Diamond \phi, \Box \exists x \phi$ alt $=$ | $\phi(a)$ | $\phi(b)$ | ... |
b. $!\exists x \phi(x)$ (purely existential)
$\not\Rightarrow \forall x \Diamond \phi(x)$ alt $= \boxed{\exists x \phi(x)}$

- Why (36-b) never available for (35-a)? Possible answer: (i) *Any* induces domain widening (DW) & DW should be for a reason (Kadmon & Landman 1993) (ii) Only in (36-a) enough reason for DW to occur (in (36-b) DW would weaken the statement)

- New challenges from *any*:

(37) a. To continue, push any key! $\Rightarrow \forall x \Diamond \phi$, $\Box \exists x \phi$
b. Take any candy! $\Rightarrow \forall x \Diamond \phi$, ???$\Box \exists x \phi$
c. Confiscate any gun! $\Rightarrow \Box \forall x \phi$

- *Any* triggers *exhaustification* which can apply at two levels:
  (i) IP/CP level producing **partitions**:

a. Take any x $= \nabla(exh[\text{any } x, \lambda x P x])$

b.

no x is taken	only *a* is taken	...	every x is taken

c. $\not\Rightarrow \ \Box \exists x \phi$

  (ii) DP level producing **maximal sets of individuals**:

a. Take any x (you find) $= \nabla(\lambda x P x(exh[\text{any } x, \lambda x Q x]))$

b. $\boxed{\text{all/the found x are taken}}$

c. $\Rightarrow \ \Box \forall x \phi$

Third application: possibility imperatives

(38) a. Come earlier, if you like — alt: $\boxed{a \mid \textit{not } a}$

b. $!\exists p(p \wedge (p = a \vee p = \neg a))$ — $\Rightarrow \Diamond a \wedge \Diamond \neg a$

(39) a. Stop smoking, for example — alt: $\boxed{a \mid \ldots}$

b. $!\exists p(p \wedge (p = a \vee \exists q(q \neq a \wedge p = q)))$ — $\Rightarrow \Diamond a$

$\not\Rightarrow \Box a$

A note on ‘if you like’

- Not a command conditionalized on the addressee’s wishes, but a genuine permission (Hamblin):

(40) a. Do a, if you like (not $q \rightarrow \Box a$)
b. Do a (if you like) or don’t (if you don’t like)
c. $\nabla(a, \neg a) \equiv \Diamond a \wedge \Diamond \neg a$

- Only a pragmatic function for the if-clause. Cf. anankastic conditionals (vFintel & Iatridou, vStechow et al)

(41) a. If you want sugar in your soup, you should call the waiter. (not $q \rightarrow \Box p$)
b. (If you want sugar in your soup), you should call the waiter to get the sugar.

(42) (If you want to know), 4 isn’t a prime number.

Conclusion and further work

- Challenges: (i) reconcile command and permission usages of imperatives; (ii) account for free choice effects.
- Proposal: imperatives as ∇ statements:

(43) $!\phi = \nabla(\mathsf{alt}(\phi))$

  - Obligations $\mapsto$ singleton sets ($|\mathsf{alt}(\phi)| = 1$)
  - Permissions and free choice $\mapsto$ alternative sets ($|\mathsf{alt}(\phi)| > 1$)

- Next:
  - Axiomatization (with Balder ten Cate);
  - Semantics/pragmatics interface;
  - ...

Appendix

From structured propositions ($\lambda s \lambda w[\phi]_{M,w,s}$) we can derive:

(44) **Alternative sets**: (+/- Hamblin 73)

$alt(\phi) = \{\lambda w[\phi]_{M,w,s} \mid s \in P^n(\phi)\}$

(45) **Partitions**: (Groenendijk & Stokhof 84)

$part(\phi) = \{\lambda v[\forall s \in P^{n(\phi)} : [\phi]_{M,v,s} = [\phi]_{M,w,s}] \mid w \in W\}$