Expressing ignorance or indifference – Modal implicatures in Bi-directional OT

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Abstract The article presents a formal analysis in the framework of bi-directional optimality theory of the ignorance and indifference implicatures conveyed by the use of indefinite expressions or disjunctions. Ignorance is expressed by standard means of epistemic logic. To express indifference we use Groenendijk and Stokhof's question meanings. To derive implicature, Grice's conversational maxims are taken as violable constraints used to select optimal candidates out of a set of alternative sentence-context pairs. Intuitively, if a sentence-context pair (ϕ, C) is optimal, a speaker in C can utter ϕ with a minimal violation of the conversational maxims. The implicatures of an utterance of ϕ are then defined as the sentences which are entailed by any optimal context for ϕ (but not by ϕ itself). Modal implicatures of disjunctions and indefinites will follow, but also scalar implicatures and exhaustification.

1 Background and motivations

The article proposes a formal analysis of the ignorance and indifference effects conveyed by the use of disjunctions or indefinite pronouns. As an illustration consider the German prefixed indefiniteness marker *irgend* in examples (1) from Haspelmath (1997) and (2) from Kratzer and Shimoyama (2002):[1]

(1) a. *Irgend jemand* hat angerufen. (# – Wer war es?)
‘Someone (I dont' know/care who) has called. (# – Who was it?)'

b. Jemand hat angerufen. (– Wer war es?)
‘Somebody has called. (– Who was it?)'

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[1]For ignorance effects see also, for example, the *to*-series in Russian (Haspelmath, 1997), for indifference and free choice readings the Italian *uno qualsiasi* (Chierchia, 2004).

(2) Mary musste *irgendeinen* Mann heiraten.

'Mary had-to marry irgend-one man.'

a. There is some man Mary had to marry, the speaker doesn't know or care who it was. (ignorance or indifference)

b. Mary had to marry a man, any man was a permitted marriage option for her. (free choice)

In (1), by using *irgend*, the speaker conveys that she doesn't know or care about who called. So the hearer cannot ask who it was. Example (2) is ambiguous between a specific reading (2a) conveying an ignorance or indifference meaning and a non-specific reading (2b) conveying a free choice effect.

Disjunction gives rise to similar effects as shown in the following examples:

(3) a. Ron is a movie star or a politician. (ignorance or indifference)

b. Have you ever kissed a Russian or an American? (indifference)

c. Ron must go to Tbilisi or Batumi. ($\Rightarrow$ Ron may go to Tbilisi and may go to Batumi) (free choice)

Following Schulz (2003) and Kratzer and Shimoyama (2002), we assume that ignorance, indifference and free choice effects are not part of the meaning of the *irgend*-indefinites or disjunction, rather they have the status of an implicature. An indication that this is indeed so comes from the fact that these effects disappear in the scope of downward entailing contexts (cf. Gazdar 1979):

(4) a. Ron isn't a movie star or a politician.

b. Niemand musste *irgend jemand* einladen. 'Noone had to invite anyone'

If modal effects were part of the meaning of the sentence, (4a) could be true in a situation where Ron is a movie star or a politician and the speaker knows or cares about which of the two. And (4b) 'could be true in a situation where people had to invite a particular person, hence weren't given any options. This is clearly not so.' [Kratzer and Shimoyama (2002), p.14]

Ignorance implicatures (of indefinites and disjunctions) have received a lot of attention in the literature (e.g. Gazdar 1979). Free choice effects have also been largely discussed (e.g. Kratzer and Shimoyama 2002 and Schulz 2003). In this article I will extend these approaches with an explicit analysis of indifference implicatures. To express indifference I will use Groenendijk and Stokhof's question meanings, that is, equivalence relations on the logical space. The intuitions is that, if two possible worlds are related by such relation, then their differences are irrelevant to the subject under discussion. Ignorance will be expressed by standard means of epistemic logic. To derive implicatures, pragmatic reasonings will be formalized in the framework of bi-directional optimality theory. Grice's conversational maxims are formulated as violable constraints used to select optimal candidates out of a set of alternative sentence-context pairs.

Intuitively, if a sentence-context pair (ϕ, C) is optimal, a speaker in C can utter ϕ with a minimal violation of the conversational maxims. The implicatures of an utterance of ϕ are then defined as the sentences which are entailed by any optimal context for ϕ (but not by ϕ itself). Modal implicatures of disjunctions and indefinites will follow, but also scalar implicatures and exhaustification (cf. Spector, 2003 and Schulz and van Rooij 2004). As we will see, the advantage of a formalization in the framework of bi-directional optimality theory is that it gives us a perspicuous account, for each implicature, of the principles and the complexity of the reasoning required for its derivation.

The rest of the article is structured as follows. The next section introduces Grice's theory of implicatures. Section 3 proposes an account of quantity implicatures in the framework of bi-directional optimality theory. Section 4 shows how ignorance, indifference and free choice implicatures follow from such an analysis. Section 5 extends the analysis to other quantity implicatures, in particular scalar implicatures and exhaustification. Section 6 draws conclusions and describes some further lines of research.

2 Grice's theory of implicature

Conversational implicatures are inferences which arise from interplay of basic semantic content and general principles of social interaction. The key ideas about implicatures have been proposed by Grice who identified four of such principles.

(5) Quantity (i) Make your contribution as informative as is required for the current purposes of the exchange; (ii) Do not make your contribution more informative than is required;

Quality Make your contribution one that is true.

Manner Be perspicuous.

Relation Be relevant.

The main goal of this article is to show that modal implications of indefinites and disjunctions can be derived from the assumption that speakers satisfy these maxims. On the present account all indefinite expressions implicate speaker's ignorance or indifference. An open question concerns then the difference between *irgend*-indefinites and plain indefinites. The implicatures of the latter have clearly a conversational nature. The implicatures of the former, instead, seem to have a double nature. On the one hand, as we will see, they are derivable by the Gricean maxims like standard *conversational* implicatures. On the other, like *conventional* implicatures (e.g. those of *therefore* or *but*), they are hard to cancel, and somehow seem to be part of the lexical meaning of the pronoun.

2.1 Modal implications as quantity implicatures

In the present analysis, ignorance and indifference implications follow from the following intuitive reasoning where the first Quantity submaxim plays a crucial role. For ease of exposition we restrict ourselves to the case of disjunction (existential sentences can be seen as generalized disjunctions).

(6) a. The speaker S said 'A or B', rather than the more informative 'A'. Why?

b. Suppose 'A' were relevant to the current purposes of the exchange, and S had the information that A. Then S should have said so. [QUANTITY]

c. Therefore,

(i) Either 'A' is irrelevant; (indifference)

(ii) Or S has no evidence that 'A' holds. (ignorance)

d. Parallel reasoning for 'B'.

Indifference readings of 'A or B' arise in situations were both 'A' and 'B' are irrelevant, but their disjunction is not. In case both disjuncts are relevant, ignorance readings arise. The speaker knows that 'A or B' is true but has no evidence that 'A' holds or 'B' holds. Therefore, both options are epistemically possible.

Free choice effects of necessity statements follow by the same reasoning under the assumption that the speaker is *maximally informed* about the specific modality involved (in section 5 more on this assumption).

(7) a. S said '$\Box$(A or B)', rather than the more informative '$\Box$A'. Why?

b. Suppose '$\Box$A' were relevant to the current purposes of the exchange, and S had the information that $\Box$A. Then S should have said so.

c. Therefore,

(i) Either '$\Box$A' is irrelevant; (indifference)

(ii) Or S has no evidence that '$\Box$A' holds. (ignorance)

d. Parallel reasoning for '$\Box$B'.

If both '$\Box$A' and '$\Box$B' are relevant, we can conclude that the speaker does not know $\Box$A and does not know $\Box$B. Under the assumption that the speaker is *maximally informed* we can conclude that '$\Box A$' and '$\Box B$' are both false. This fact, in combination with the original sentence, implies the free choice implication '$\Diamond A$ and $\Diamond B$'.

In what follows, I will formalize these Gricean reasonings in the framework of bi-directional optimality theory. The main motivation for assuming such a framework concerns the free choice implications of possibility statements as in the following example.

(8) John may go to Tbilisi or Batumi. $\Rightarrow$ John may go to Tbilisi and John may go to Batumi.

We would like to derive from $\Diamond(A \vee B)$ the conjunction $\Diamond A \wedge \Diamond B$. It is easy to see, however, that if we apply the reasoning illustrated above, assuming as alternatives to $\Diamond(A \vee B)$ the natural candidates $\Diamond A$ and $\Diamond B$, we do not obtain the desired free choice effects.[2] One might conclude that free choice effects of possibility statements are not QUANTITY implicatures after all (e.g. Aloni 2002). In what follows, however, I will show that these free choice implications can be derived from QUANTITY (with a natural choice of alternatives) if we assume a more complex reasoning than the one illustrated in (6) and (7), a reasoning involving competition and blocking between different forms for different contents, i.e. the sort of reasoning formalized in bi-directional optimality theory (henceforth BiOT).

3 Conversational implicature in BiOT

In optimality theory ranked constraints are used to select a set of optimal candidates from a larger set of candidates. In our analysis, the constraints are given by the Gricean maxims, and the competing candidates are pairs consisting of a sentence and a context. Intuitively, if a sentence-context pair (ϕ, C) is optimal, a speaker in C can utter ϕ with a minimal violation of the maxims. Optimal pairs are defined by Blutner and Jäger's notion of weak optimality (see Blutner, 2000):

(11) A candidate (FORM, CONTENT) is *weakly optimal* iff there are no other better *weakly optimal* pairs (FORM$'$, CONTENT) or (FORM, CONTENT$'$).

As standard in OT, a candidate α is at least as good as α' iff α's constraint violations are no more severe than α''s, where single violations of a higher ranked constraint override in severity multiple violations of lower ranked constraints. In the following paragraphs we give a precise definition of the competing candidates and of the adopted constraints.

[2] A different, but less natural choice of alternatives would give us better results. Kratzer and Shimoyama (2002), for example, seem to assume the following set of alternatives for the modal cases (cf. also Chierchia 2004) which indeed would give us the desired free choice implicatures:

(9) a. $\text{Alt}(\Diamond(A \vee B)) = \{\Diamond A \wedge \neg\Diamond B, \Diamond B \wedge \neg\Diamond A\}$

b. $\text{Alt}(\Box(A \vee B)) = \{\Box A \wedge \neg\Box B, \Box B \wedge \neg\Box A\}$

However, firstly this choice of alternatives cannot be defined compositionally, and secondly, it does not generalize to the case of plain disjunction. If $\text{Alt}(A \vee B) = \{A \wedge \neg B, B \wedge \neg A\}$, then $A \vee B$ would implicate $A \wedge B$.

Aloni & van Rooij (2004) assume the following compositionally defined set of alternatives, which also would give us the right results:

(10) a. $\text{Alt}(A \vee B) = \{A, B\}$ closed under Boolean operators

b. $\text{Alt}(\Box\phi) = \{\Box\psi \mid \psi \in \text{Alt}(\phi)\}$

c. $\text{Alt}(\Diamond\phi) = \{\Box\psi \mid \psi \in \text{Alt}(\phi)\}$

Note however that here $\Diamond$ requires an *ad hoc* move, which is hard to justify. In this article I will show that free choice implications of possibility statements can be derived from QUANTITY given a natural choice of alternatives, if we assume bi-directional reasonings.

Competing candidates: sentences and contexts Let W be a set of worlds and V a valuation function which assigns in each world a truth value to each propositional letter. Then a context C is a pair (s, Q) where s is a state (a subset of W), and Q is an issue (an equivalence relation over W). States represent what the speaker believes. Issues represent what the speaker cares about (c.f. Groenendijk, 1999). For example, a speaker in (W, W^2) knows and cares about nothing, a speaker in $(W, \{(w, v) \in W^2 \mid w = v\})$ knows nothing and cares about everything, and finally a speaker in $(\{w\}, W^2)$ knows everything and cares about nothing. Intuitively, if two worlds are related by Q, then their differences are irrelevant to the speaker. So indifference wrt p can be represented by an equivalence relation connecting p-worlds with not p-worlds.

We will say that a context (s, Q) entails $\heartsuit?\phi$ to be read as 'I care whether ϕ' iff Q entails $?\phi$ according to the standard Groenendijk and Stokhof's notion of entailment between questions; and, as standard in update semantics, a context (s, Q) entails $\Diamond/\Box\phi$, to be read epistemically, iff s is consistent with/entails ϕ (see Veltman, 1996). Here are more detailed definitions of these notions in terms of an update semantics. The language under consideration is that of modal propositional logic with the addition of the sentential operator '$\heartsuit?$'.

Definition 1 [Updates]

- $C[p] = C'$ iff $s_{C'} = \{w \in s_C \mid V(p)(w) = 1\}$ & $Q_{C'} = Q_C$
- $C[\neg\phi] = C'$ iff $s_{C'} = s_C \setminus s_{C[\phi]}$ & $Q_{C'} = Q_C$
- $C[\phi \wedge \psi] = C[\phi][\psi]$
- $C[\Box\phi] = C'$ iff $C' = C$, if $C[\phi] = C$, & $C' = (\emptyset, Q_C)$, otherwise
- $C[\heartsuit?\phi] = C'$ iff $C' = C$, if $C[?\phi] = C$, & $C' = (\emptyset, Q_C)$, otherwise
  where $C[?\phi] = C'$ iff $s_{C'} = s_C$ & $Q_{C'} = \{(w, v) \in Q_C \mid (\{w\}, Q_C)[\phi] = (\{w\}, Q_C)$ iff $(\{v\}, Q_C)[\phi] = (\{v\}, Q_C)\}$

Disjunction, implication and possibility are defined as standard in terms of conjunctions, negation and necessity. Entailment is defined as follows.

Definition 2 [Entailment] $C \models \phi$ iff $C[\phi] = C$

All clauses in definition 1 are standard in update semantics, except that for $\heartsuit?\phi$. $\heartsuit?\phi$ is, like $\Box\phi$, a test returning either the original context (if updating with $?\phi$ does not bring anything new) or the absurd state (otherwise). An update with $?\phi$ can only modify the issue parameter. In most cases the output issue is the intersection between the input issue and the partition assigned to $?\phi$ by Groenendijk and Stokhof's standard theory of questions. So, for example, $[?\phi] = [?\neg\phi]$, and, therefore, $\heartsuit?\phi$ iff $\heartsuit?\neg\phi$. The only difference with the standard partition theory concerns the epistemic cases $?\Box/\Diamond\phi$. On the present account, $[?\phi] = [?\Box/\Diamond\phi]$. Therefore, we obtain that whenever $\heartsuit?\phi$ holds $\heartsuit?\Box/\Diamond\phi$ holds as well. Finally note that $\heartsuit?$ can be iterated, but its iteration yields a tautology. $\heartsuit?\heartsuit?\phi$ is true in any context. The intuition is that disregarding whether you care or not whether ϕ, you always care whether you care whether ϕ.

Syntactic alternatives One of the ideas at the heart of BiOT is that the evaluation of a sentence does not occur in isolation, but depends on a number of alternative forms the speaker might have used. One of the problems of BiOT analyses is that it is not always clear, for a given sentence, which alternative forms should also be taken into consideration: should we consider all the other sentences of the language? or only those up to a certain length? The notion of a syntactic alternative defined below give us a precise answer to these questions.

Definition 3 [Syntactic alternatives]

- $Alt(p) = \{p, q, r, s, ...\}$ closed under $\vee$ and $\wedge$
- $Alt(\neg\phi) = \{\neg\psi \mid \psi \in Alt(\phi)\}$
- $Alt(\phi \vee \psi) = \{\phi' \vee \psi' \mid \phi' \in Alt(\phi)\ \&\ \psi' \in Alt(\psi)\}$
- $Alt(\Box\phi) = \{\Box\psi \mid \psi \in Alt(\phi)\} \cup \{\Diamond\phi\}$
- $Alt(\Diamond\phi) = \{\Diamond\psi \mid \psi \in Alt(\phi)\} \cup \{\Box\phi\}$

This choice of alternatives remains somehow stipulative, but is not without intuitive appeal. For example, if $Alt(A) = Alt(B) = \{A, B, A \vee B, A \wedge B\}$, we then obtain:

(12) a. $Alt(A \vee B) = \{A, B, A \vee B, A \wedge B\}$

b. $Alt(\Box(A \vee B)) = \{\Box A, \Box B, \Box(A \vee B), \Box(A \wedge B), \Diamond(A \vee B)\}$

c. $Alt(\Diamond(A \vee B)) = \{\Diamond A, \Diamond B, \Diamond(A \vee B), \Diamond(A \wedge B), \Box(A \vee B)\}$

The notion of a syntactic alternative gives us a way to determine while evaluating a sentence ϕ what other sentences will count as competitors, namely all syntactic alternatives called up by ϕ or by any of its alternatives. For example, in the evaluation of $\Diamond(A \vee B)$ all the alternatives in (12c) will be competing, but also those in (12b). The former because called up directly by the sentence, the latter because called up by a direct alternative to the sentence, namely $\Box(A \vee B)$. We have then a difference between direct and indirect alternative to a sentence. For example $\Box(A \vee B)$ or $\Diamond A$ are direct alternative to $\Diamond(A \vee B)$; $\Box A$, instead, is only an indirect alternative. Direct and indirect alternatives will all have the power to block less preferred interpretations. Only direct alternatives, however, will play a role in the evaluation of the quantity constraint, as we will see in the following paragraph.

Constraints On the present account, Grice's maxims are formulated as properties of sentence-context pairs (ϕ, C), and are ordered, according to their relative degree of violability. We consider for the moment only three constraints, ordered as follows:

(13) Quality, Relation > Quantity

QUANTITY formalizes only the first submaxim of Grice's original principle. The second submaxim is covered by RELATION. The maxim of MANNER is missing in the formalization.

Definition 4 [Constraints]

QUALITY $C \models \Box\phi$

RELATION $C \models \heartsuit?\phi$

QUANTITY $\forall\psi \in Alt(\phi) : C \models \heartsuit?\psi \ \&\ \phi \not\models \psi \Rightarrow C \not\models \psi$

For a candidate (ϕ, C), QUALITY holds iff the context C entails the sentence ϕ; RELATION holds iff C entails $?\phi$; and QUANTITY iff none of the non-trivial direct alternatives to ϕ which are relevant according to Q_C are entailed by s_C.[3] Notice that while QUALITY depends solely on the state-parameter s (unless ϕ is a question), and RELATION only on the issue-parameter Q, QUANTITY depends on both s and Q, capturing Grice's original insight that speakers should make their contribution as *informative* as is required for the current *purposes* of the exchange.

To sum up, we have formalized Gricean maxims as property of sentence-context pairs. The maxims select for each sentence ϕ a set of optimal contexts. The implicatures of ϕ can then be defined as what must hold in all these optimal contexts. Let $opt(\phi)$ be the set of contexts C such that (ϕ, C) is optimal.

Definition 5 [Implicatures] ϕ implicates ψ, $\phi \approx\!\!\!| \psi$ iff $\forall C : C \in opt(\phi)$: $C \models \psi$ & $\phi \not\models \psi$

4 Applications

The BiOT analysis presented in the previous section makes the following predictions.

(14) a. $\phi_1 \vee \phi_2 \approx\!\!\!| \neg\Box\phi_i \vee \neg\heartsuit?\phi_i$ $\quad\forall i \in \{1, 2\}$

b. $\Box(\phi_1 \vee \phi_2) \approx\!\!\!| \neg\Box\phi_i \vee \neg\heartsuit?\phi_i$

c. $\Diamond(\phi_1 \vee \phi_2) \approx\!\!\!| \neg\Box\phi_i \vee \neg\heartsuit?\phi_i$

A speaker using a (modal) disjunction implicates that for each disjunct ϕ_i either she doesn't believe that it is true or she doesn't care whether it is true.

Ignorance and free choice implicatures are obtained if we restrict competition to contexts in which the speaker cares about both disjuncts. In these cases, uses of (modal) disjunctions implicate that both disjuncts are epistemically possible.

[3]The reason why here we consider all non-trivial alternatives rather than only the stronger ones has to do, as most of the complications of this system, with the case of possibility statements. In order to derive free choice implicatures of possibility disjunction we cannot have $\Diamond A \wedge \Diamond B$ as alternative to $\Diamond(A \vee B)$ or $\Diamond A$. But then $\Diamond B$ must count as an alternative when evaluating quantity for $\Diamond A$, otherwise, for example, $\neg\Diamond B$ would not be derivable as an implicature from the sentence.

(15) a. $\phi_1 \vee \phi_2 \approx_{?\phi_1,?\phi_2} \Diamond\phi_1 \wedge \Diamond\phi_2$

b. $\Box(\phi_1 \vee \phi_2) \approx_{?\Box\phi_1,?\Box\phi_2} \Diamond\phi_1 \wedge \Diamond\phi_2$

c. $\Diamond(\phi_1 \vee \phi_2) \approx_{?\Diamond\phi_1,?\Diamond\phi_2} \Diamond\phi_1 \wedge \Diamond\phi_2$ (needs bi-directional reasoning)

Result (15c) requires the notion of weak optimality. As we will see in section 5, results (14b)-(15b) and (14c)-(15c) can be extended to non- epistemic modals $\Box'/\Diamond'$ if we restrict competition to contexts in which the following principles hold: $\neg\Box\Box'\phi \rightarrow \neg\Box'\phi$ and $\neg\Box\Diamond'\phi \rightarrow \neg\Diamond'\phi$ (cf. Zimmermann 2000 and Schulz 2003). Since existential statements can be seen as generalized disjunctions all these results extend to the case of indefinite expressions. In what follows we have a closer look at these results. We start with the ignorance and indifference implicatures of plain disjunctions.

Plain disjunction Any context C resulting optimal for $A \vee B$ according to the discussed ranked constraints, entails for each disjunct that either it is not believed to be true by the speaker or it is irrelevant (see (14a)). This result which does not require bi-directional reasoning follows by the optimization illustrated by the following tableau (a large number of uninteresting candidates have been omitted). In the table, by $\mathbf{Q}_{(?\phi)?\psi}$ I denote the partition expressed by (the conjunction of $?\phi$ and) $?\psi$. By $[w', w'']$, the state consisting of the two worlds w' and w''. For simplicity, we are considering only four worlds, where each world is indexed with the atomic propositions holding in it. For example, in w_A, only A holds, in w_{AB} only A and B hold and in $w_\emptyset$, no atomic proposition holds. Finally, '⇒' indicates an optimal candidate, '!*' a crucial constraint violation.

		QUAL	REL	QUAN
	$A \vee B$ - $(\mathbf{Q}_{?A}, [w_A, w_B, w_{AB}, w_\emptyset])$	!*	!*	
a. ⇒	$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_A, w_B, w_{AB}])$			
b. ⇒	$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_A, w_B])$			
	$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_A])$			!*
	$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_B])$			!*
	$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_{AB}])$			!***
c. ⇒	$A \vee B$ - $(\mathbf{Q}_{?(A\vee B)}, [w_A, w_B, w_{AB}])$			
d. ⇒	$A \vee B$ - $(\mathbf{Q}_{?(A\vee B)}, [w_A, w_B])$			
e. ⇒	$A \vee B$ - $(\mathbf{Q}_{?(A\vee B)}, [w_A])$			
f. ⇒	$A \vee B$ - $(\mathbf{Q}_{?(A\vee B)}, [w_B])$			
g. ⇒	$A \vee B$ - $(\mathbf{Q}_{?(A\vee B)}, [w_{AB}])$			
h. ⇒	$A \vee B$ - $(\mathbf{Q}_{?A?(A\vee B)}, [w_A, w_B, w_{AB}])$			
i. ⇒	$A \vee B$ - $(\mathbf{Q}_{?A?(A\vee B)}, [w_A, w_B])$			
	$A \vee B$ - $(\mathbf{Q}_{?A?(A\vee B)}, [w_A])$			!*
l. ⇒	$A \vee B$ - $(\mathbf{Q}_{?A?(A\vee B)}, [w_B])$			
	$A \vee B$ - $(\mathbf{Q}_{?A?(A\vee B)}, [w_{AB}])$			!*

We have three types of optimal contexts for $A \vee B$. In the first type (candidates (a) and (b)), both disjuncts are relevant, Q_C entails $?A$ and $?B$. To satisfy QUANTITY such contexts must entail that none of the disjuncts is known, which by QUALITY entails that both disjuncts are epistemically possible.

(16) $A \vee B \mathrel{|\!\!\approx}_{?A?B} \Diamond A \wedge \Diamond B$ (ignorance)

The second type is a context in which none of the disjuncts are relevant, but the disjunction is, Q_C entails $?(A \vee B)$, but it does not entail $?A$ or $?B$. These contexts (candidates (c)–(g)) model the indifference reading, where it matters whether the disjunction is true, but the differences between the disjuncts are irrelevant.

(17) $A \vee B \mathrel{|\!\!\approx}_{?(A \vee B)} \neg \heartsuit A \wedge \neg \heartsuit B \wedge \heartsuit(A \vee B)$ (indifference)

Note that in these contexts no conclusion can be drawn about the speaker's epistemic attitude towards the two disjuncts, beyond the fact that at least one of the two must be true (by QUALITY). This is because QUANTITY does not play any role here, since none of the non-trivial syntactic alternatives to the sentence are relevant in these contexts.

There is also a third option, in which only one of the disjuncts is relevant beside the disjunction itself, e.g. candidates (h)–(l) where Q_C entails $?(A \vee B)$ and $?A$, but it does not entail $?B$. An interesting candidate is (l). It formalizes a situation like the following. Suppose you are expecting Ann's call ($C \models \heartsuit A$). Instead Bill calls ($C \models B$), of whom you don't care ($C \not\models \heartsuit B$). We correctly predict then that in this situation you can say (18) signaling that you don't care of that particular person who called, namely Bill, that he called.

(18) Irgend iemand hat angerufen. 'Irgend-one has called'

Epistemic Modals Our BiOT analysis predicts a difference between necessity and possibility with respect to the complexity of the reasoning leading to their modal implicatures. Modal implicatures of $\Box(A \vee B)$ follow directly from our constraints, just as in the case of plain disjunction discussed above. Those of $\Diamond(A \vee B)$, instead, require a bi-directional notion of optimality.

For simplicity just consider, contexts in which all syntactic alternatives to $\Diamond(A \vee B)$, namely $\Diamond A$, $\Diamond B$ $\Diamond(A \wedge B)$ and $\Box(A \vee B)$ are relevant. As the following tableau illustrates, if we assume a uni-directional perspective, that is, we just compare different contexts for the given sentence, our constraints predict as optimal for $\Diamond(A \vee B)$ two contexts, context (a), where only $\Diamond A$ is entailed, and context (b) where only $\Diamond B$ is entailed. These contexts have no interesting entailment in common. Therefore no implicatures can be derived.

	$\Diamond(A \vee B)$	QUAL	REL	QUAN
$\Rightarrow$	a. $(\mathbf{Q}_{?A?B}, [w_A, w_\emptyset])$			*
$\Rightarrow$	b. $(\mathbf{Q}_{?A?B}, [w_B, w_\emptyset])$			*
	c. $(\mathbf{Q}_{?A?B}, [w_A, w_B, w_\emptyset])$			!**
	d. $(\mathbf{Q}_{?A?B}, [w_A])$			!**
	e. $(\mathbf{Q}_{?A?B}, [w_A, w_B, w_{AB}, w_\emptyset])$			!***
	f. $(\mathbf{Q}_{?A?B}, [w_\emptyset)$	!*		

In the following tableau, instead, we adopt a bi-directional perspective. That is, we compare not only different contexts for our sentence, but also different sentences for a given context.

				QUAL	REL	QUAN
	a. $\Diamond(A \vee B)$	-	$(\mathbf{Q}_{?A?B}, [w_A, w_\emptyset])$			!*
$\Rightarrow$	$\Diamond A$	-	$(\mathbf{Q}_{?A?B}, [w_A, w_\emptyset])$			
	b. $\Diamond(A \vee B)$	-	$(\mathbf{Q}_{?A?B}, [w_B, w_\emptyset])$			!*
$\Rightarrow$	$\Diamond B$	-	$(\mathbf{Q}_{?A?B}, [w_B, w_\emptyset])$			
$\Rightarrow$	c. $\Diamond(A \vee B)$	-	$(\mathbf{Q}_{?A?B}, [w_A, w_B, w_\emptyset])$			**
	$\Diamond A$	-	$(\mathbf{Q}_{?A?B}, [w_A, w_B, w_\emptyset])$			!*
	$\Diamond B$	-	$(\mathbf{Q}_{?A?B}, [w_A, w_B, w_\emptyset])$			!*
	d. $\Diamond(A \vee B)$	-	$(\mathbf{Q}_{?A?B}, [w_A])$			!**
$\Rightarrow$	$\Box A$	-	$(\mathbf{Q}_{?A?B}, [w_A])$			

Here the contexts in (a) and (b) are blocked for $\Diamond(A \vee B)$ by the existence of alternative sentences which would be more appropriate choice for a speaker there, namely $\Diamond A$ and $\Diamond B$ respectively. For $\Diamond(A \vee B)$ remains then as unique weakly optimal contexts the one in (c) which entails indeed the free choice implication $\Diamond A \wedge \Diamond B$.

5 Adding competence

Building on (Spector 2003) and (Schulz and van Rooij 2004), this final section introduces a new constraint formalizing the assumption, often implicit in Gricean reasonings, that the speaker is maximally informed with respect to the subject matter and applies it to derive scalar and other quantity implicatures. At last, it discusses to which extend this constraint can be used to account for free choice implicatures of non-epistemic modal sentences.

5.1 Other quantity implicatures

Given our choice of alternatives for possibility statements, our OT analysis easily captures the following standard quantity implicatures:

(19) a. John may be a gorilla $\Rightarrow$ John may also not be a gorilla.

b. It is allowed to offend. $\Rightarrow$ It is not obligatory to offend.

Other well-known quantity implicatures are the scalar implicature illustrated in (20), and the exhaustivity implicature illustrated in (21) (see Spector, 2003 and Schulz and van Rooij, 2004).

(20) Ron is a movie star or a politician. ($\Rightarrow$ not both)

(21) Q: Who smokes? A: Ann ($\Rightarrow$ not Bill)

Obviously, exhaustivity implicatures depend on the question under discussion which determines what are the relevant alternatives.

(22) Q: Does Ann smoke? A: Yes ($\not\Rightarrow$ not Bill)

But also scalar implicatures depend on the issue under discussion. Interestingly, as shown by (23), they do not arise on an indifference reading of disjunction.

(23) Q: Have you ever kissed a Russian or an American? A: Yes. ($\not\Rightarrow$ not both)

In order to account for (**??**)-(23) we would like to derive (24), but, so far, we only have (25):

(24) a. $A \vee B \mathrel{|\!\!\approx}_{?A?B} \neg(A \wedge B)$ b. $A \mathrel{|\!\!\approx}_{?A?B} \neg B$

(25) a. $A \vee B \mathrel{|\!\!\approx}_{?A?B} \neg\Box(A \wedge B)$ b. $A \mathrel{|\!\!\approx}_{?A?B} \neg\Box B$

What is missing in our OT analysis so far is a formalization of the leap of faith implicit in Gricean derivations from 'the speaker does not believe that ϕ' to 'it is not the case that ϕ'. As an illustration, consider the following standard derivation of a scalar implicature:

(26) a. S said $A \vee B$ rather than $A \wedge B$, which would have also been relevant.

b. So, by Quantity S has no evidence that $A \wedge B$ holds.

c. S is well informed (about the issue under discussion)

d. Therefore it is not the case that $A \wedge B$ holds.

Our formalization so far captures the step from (a) to (b). The missing step from (b) to (d) relies on the assumption that the speaker is well informed or competent about the issue under discussion. To formalize this assumption we can add to our BiOT analysis the following principle as the lowest ranking constraint:

Maximal Information (Max) $\forall\psi \in Alt(\phi) : C \models \heartsuit\psi \;\&\; \phi \not\models \psi \Rightarrow C \models \neg\psi$

Like quantity, this principle depends on the syntactic alternatives of the given sentence. While quantity was satisfied if none of the relevant alternatives was entailed in the given context, Max requires each of these alternatives to be false in the given context.

I will denote by $\mathrel{|\!\!\approx}^{max}$ the implicature relation derived by assuming this extra constraint. As it is easy to see, by this notion we can account now for scalar implicatures, and exhaustification.

(27) a. $A \vee B \mathrel{|\!\!\approx}^{max}_{?A?B} \neg(A \wedge B)$ b. $A \mathrel{|\!\!\approx}^{max}_{?A?B} \neg B$

The following tableaux illustrate the role of Max in the two derivations.

	QUAL	REL	QUAN	MAX
$A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_A, w_B, w_{AB}])$				!***
$\Rightarrow$ $A \vee B$ - $(\mathbf{Q}_{?A?B}, [w_A, w_B])$				**

	QUAL	REL	QUAN	MAX
A - $(\mathbf{Q}_{?A?B}, [w_A, w_{AB}])$				!**
$\Rightarrow$ A - $(\mathbf{Q}_{?A?B}, [w_A])$				

Note that the implicatures in (27) arise only under the assumptions that $A \wedge B$ and B are relevant. This is correct as shown by the contrast between (21) and (22) and between (20) and (23). In case it is not known what is relevant, MAX unsurprisingly predicts a preference for the indifference reading of disjunctions, rather than the ignorance one, only the former being compatible with a maximally informed speaker. Whether this prediction is sustained by the facts remains an open question.

5.2 Non-epistemic modals

The principle of maximal information has no effect on the implicatures of epistemic modal sentences. This is not surprising because if ϕ is an epistemic sentence then the principle $\neg\Box\phi \rightarrow \neg\phi$, which MAX formalizes, is already satisfied. Indeed, epistemic modals in general and in our formalization satisfy the principles of positive and negative introspection.

(28) a. $\neg\Box\Box\phi \rightarrow \neg\Box\phi$ (positive introspection)

b. $\neg\Box\Diamond\phi \rightarrow \neg\Diamond\phi$ (negative introspection)

When we turn to non-epistemic modals $\Box'/\Diamond'$ though, we immediately see that the following principles (corresponding to the ones in (28) but involving now different sorts of modality) are not in general satisfied:

(29) a. $\neg\Box\Box'\phi \rightarrow \neg\Box'\phi$

b. $\neg\Box\Diamond'\phi \rightarrow \neg\Diamond'\phi$

For example, consider deontic modals. If you don't believe that it is obligatory/allowed to do something, in general it does not mean that it is not obligatory/allowed to do something. Unless we assume that you are maximally informed about the subject matter. In this case the implications in (29) are intuitively warranted. MAX, which formalizes this assumption, indeed, makes a difference for the case of non-epistemic modals. As it is easy to see, the assumption of MAX is crucial in the derivation of the free choice effects of non-epistemic necessity sentences like $\Box'(A \vee B)$.

(30) $\Box'(A \vee B) \;\approx^{max}_{?\Box' A, ?\Box' B}\; \Diamond' A \wedge \Diamond' B$

This result is obtained because MAX rules out as optimal candidates all contexts in which $\Box' A$ or $\Box' B$ are not false, for example context (a) in the following tableau that without MAX would have been an optimal choice for the given sentence (by w_{w_A,w_B} I denote a world from which both worlds w_A and w_B are accessible).

		QUAL	REL	QUAN	MAX
	a. $\Box'(A \vee B)$ - $([?\Box' A ?\Box' B], [w_{w_A}, w'_{w_B}])$				!**
$\Rightarrow$	b. $\Box'(A \vee B)$ - $([?\Box' A ?\Box' B], [w_{w_A,w_B}, w'_{w_A,w_B}])$				
	c. $\Box'(A \vee B)$ - $([?\Box' A ?\Box' B], [w_{w_A}, w'_{w_A}])$			!*	*

That the assumption of maximal information is needed for deriving free choice effects of non-epistemic modals has been shown by Zimmermann (2000) who observed that when, as in (31), a speaker explicitly reveals that she is not maximally informed about the subject matter no free choice effects arise.

(31) You must go to Tbilisi or Batumi, but I don't remember which.

While computing the implicatures of (31), MAX should not apply. Candidate (a) in the tableau above would then be optimal for the sentence.

Let us now turn to possibility sentences. Unfortunately, the result in (30) does not extend to possibility sentences which again require an *ad hoc* move.

(32) $\Diamond'(A \vee B) \;\not\approx^{max}_{?\Diamond' A, ?\Diamond' B}\; \Diamond' A \wedge \Diamond' B$

The following tableau illustrates why MAX is not sufficient here to rule out contexts like the one in (a) which do not satisfy the principles of maximal information. Although candidate (a) violates MAX twice, it is still optimal because all other candidates involve more severe constraint violations. But the context in (a) clearly does not entail $\Diamond' A \wedge \Diamond' B$.

	QUAL	REL	QUAN	MAX
$\Rightarrow$ a. $\Diamond'(A \vee B)$ - $([?\Diamond' A ?\Diamond' B], [w_{w_A, w_\emptyset}, w'_{w_B, w_\emptyset}])$				**
b. $\Diamond'(A \vee B)$ - $([?\Diamond' A ?\Diamond' B], [w_{w_A, w_B, w_\emptyset}])$			!**	**
c. $\Diamond'(A \vee B)$ - $([?\Diamond' A ?\Diamond' B], [w_{w_A, w_\emptyset}])$			!*	*

I am not sure how to solve this problem. One possibility consists in restricting the OT competition for non-epistemic modal sentences to contexts which do satisfy the principles in (29). Contexts like (a) would then be ruled out from the beginning. Maybe this extra complication indicates once again that free choice effects of possibility statements are not quantity implicatures after all.

6 Conclusion

We have presented a formal analysis of quantity implicatures in the framework of Bi-directional OT, and we have applied it to explain modal implicatures of disjunctions and indefinite expressions, but also scalar implicatures and exhaustification. A large number of further questions arise. The most urgent concerns the exact relation between different kinds of indefinite pronouns (see Haspelmath, 1997). We have further observed a difference in complexity between free choice implicatures of necessity statements and possibility statements. The latter cases are somehow much harder to derive. Interestingly, our prediction can be tested empirically. Children are very bad with implicatures and are even worst with bi-directional reasoning (according to a number of studies, e.g. de Hoop and Krämer (2004), they start reasoning bi-directionally only at age 6). Suppose children turn out to be very good in deriving free choice effects of possibility statements. This could be taken as evidence that these effects have more a semantic nature than it has been recognized in this article.

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