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# Comparison of Pricing in Online and Offline Markets

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## Abstract

In the recent review of Alberto Cavallo (2016), there is evidence that, on average, across countries and industries in 28% cases online and offline prices on the same products diverge. Why does it happen? There is a range of possible explanations: online prices faster adjust to shocks, the decrease in buyers' search costs drives the results, the pricing strategy of the sellers who offer lower prices on some goods, thus capturing the attention of buyers to look on other ones which are worse categorized. This paper presents the model, which states that the nature of the product could explain the difference in pricing in various sectors. Thus, search goods are associated with identical prices in both channels, and experience goods could be priced differently. The difference between goods is exploited by the notion of better «fit». In particular, if it is possible for the producer to divide the online and offline markets between consumers, (s)he sets the higher online price, otherwise it is beneficial to set the higher offline price. The additional finding states that if a producer does not have an online store and compete at the market where rivals have both online and offline stores, it is beneficial for her to open it.

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# 1 Introduction

«If you think online prices are cheaper than offline, think again»<sup>2</sup>. We live in a world where our minds are burden with stereotypes. One of them is that online prices are identical or cheaper in comparison with offline ones. Many people got used to following «show-rooming»: a kind of activity, when you try the clothes in the stores on and then buy them online at a lower price. However, is that result generalizable? Recent research of Roberto Cavallo (2016) seems to test this theory out.

The results of the first large-scale comparison of online and offline prices in 10 countries among 56 of the largest retailers predict that, on average, prices are identical in 72% of cases. However, significant heterogeneity is found across countries, sectors, and retailers. The goal of this paper is to explain the differences in prices among sectors. The highest share of identical prices is found in electronics and clothing with 83% and 92%, respectively, while the lowest one is among drugstores and office-supply retailers with 38% and 25%, respectively. For better illustration, consider Figure 1<sup>3</sup>.

Table 4: Sector - Price Level Differences

Sector	(1) Ret.	(2) Obs	(3) Identical (%)	(4) High On (%)	(5) Low On (%)	(6) Markup (%)	(7) Difference (%)
Food	10	5953	52	32	15	3	1
Clothing	7	2534	92	5	3	3	0
Household	9	7875	79	5	16	-8	-2
Drugstore	4	3053	38	11	52	-5	-3
Electronics	5	3712	83	4	13	-9	-1
Office	2	1089	25	37	38	1	1
Multiple/Mix	18	14149	80	5	15	-9	-2

Figure 1: Sector price level differences

In order to answer the question of why online and offline prices may differ, I develop a theory that challenges to fit the data. I assume that all goods could be divided into two broad categories: experience goods and search goods. The difference between them is our ability to treat their characteristics. Some of them, like technical peculiarities of a notebook, is easy to check online as well as offline. However, it is rather complicated to buy a coat without having tried it on.

<sup>2</sup><https://www.cnet.com/news/if-you-think-online-prices-are-cheaper-than-offline-think-again/>

<sup>3</sup>The figure is originally retrieved from Cavallo, 2016, page 13

The difference in such characteristics is the one that drives the distinction between search and experience goods. This difference plays a leading role in pricing strategies. For experience goods, when there is a significant difference between buying the good in the store or online, online prices could differ a lot in order to segment consumers in various groups. Vice a versa, for search goods, prices should be equal in both channels: if prices in the conventional stores and on the Internet are too different, it provides incentives for arbitrage for some agents at the market. Moreover, here is analyzed the issue of the probability of experience goods to be priced higher offline as well as online. The model differentiates consumers by their reservation prices and how consumers treat the possibility of the so-called «better fit». One of the main results is that if the producer can divide consumers between the online and offline markets, then (s)he will set the higher online price, if not the higher offline price.

The paper is organized as follows. Section 2 touches upon the literature review. Section 3 presents the baseline model: both its environment and main propositions. The main results of the model are presented in section 4. Section 5 discusses the implications of the model. Section 5 proposes possible future extensions of the current research, which would better fit the data. Finally, section 6 concludes.

## 2 Literature Review

The seminal paper of Stahl (1989) states that reduced consumer search costs may be expected to induce stiffer price competition between firms, leading to lower prices and improved consumer welfare. Some papers further supported this notion. Thus, reduced search costs will make it easier for consumers to find low-cost sellers, hence promoting price competition among sellers and, as a result, leads to reduced prices (Bakos, 2001). Further, increased competition makes it more complicated for firms to make profits, especially in the long run because of the Internet being a free-entry market (Liebowitz, 2002). However, Campbell et al.(2005) show that in a dynamic environment, the situation can be reversed. Thus, less costly consumer searches may result in higher prices and, as a result, reduced consumer welfare because of the opportunity of better monitoring each other's prices. Besides, firms may provide better consumer fit, because lower search costs can help firms to reveal qualified consumers. The online channel helps to profile and monitor back such strategies (Lee and Gosain, 2002). Indeed, Kuksov (2004) shows that lower search costs have an ambiguous effect. On the one hand, keeping product differentiation constant will lead to lower equilibrium prices. On the other hand, it also leads to higher incentives for the firms to invest in product design. As a result, this effect offsets to some extent the effect above so that, overall, it may even lead to higher prices. Beyond this, the overall effect of lower search costs on social welfare may be harmful.

Some specific characteristics have also been taken into consideration. Thus, online consumers show higher loyalty, which decreases price sensitivity, and therefore higher online prices are ob-

tainable (Gruber, 2008). Lynch and Ariely (2000) claim that non-price characteristics may be valued by consumers. Thus, convenience leads to some additional markup. Moreover, there exists empirical evidence of higher prices for some products online, indicating consumers' willingness to pay additional funds for convenience (Bailey, 1998). It has been noticed that prices tend to rise faster than to fall, the phenomenon which has been called «rockets and feathers» (Tappata, 2009). The author shows that the essence of equilibrium changes sharply if consumers are imperfectly informed about market prices, and a part of them has positive search costs. Now equilibrium is characterized by price dispersion instead of a single price because producers gain from informational rents. As for the described pattern, he shows that if the producer's marginal costs are high and consumers expect it to remain high, overall, they expect little price dispersion and search very little. If, on the other hand, producer's marginal cost is low and consumers expect it to remain low, so next period price dispersion is expected to be high, consumers search increases, and the prices are increased significantly because of the response on the demand shock.

There was a broad debate on the question of whether the Internet can decrease price competition. The conventional wisdom predicts that lower cost of the distribution for the producer and fewer search costs for the consumer tend to intensify price competition. It may not be the case. Lal and Sarvary (1999) predict that introduction of the Internet may lead to the monopoly pricing in the following cases: the large share of the Internet users, the high relevance of the non-digital characteristics of the goods, the confidence of the consumers in the brand they currently consume and in the case of «destination shopping». The empirical evidence in the insurance markets (Brown and Goolsbee, 2000) provides some controversial results. The initial introduction of the Internet leads to the increase of price dispersion within demographic groups; however, as the use of the Internet spreads, the dispersion falls. Overall the authors claim that prices were reduced by 8-15 %.

The paper, which is the closest one to the question I am interested in (Zettelmeyer, 2000) shows that firms can strategically use the information on multiple channels to achieve finer consumer segmentation. They argue that a variety of channels allows producers to provide selected groups different amounts of information in order to influence their expected utility. Such a policy helps firms to increase their market power.

### **3 The baseline model**

#### **3.1 The story behind**

I will distinguish two types of goods: experience and search goods (Nelson, 1970). The author claims that the consumer has a simple alternative to search: (s)he can use experience. In other words, (s)he can evaluate the quality of brands by purchasing them and trying. As in the seminal paper of Nelson, assume that you would like to choose the best brand of canned tuna fish. The

preferred strategy in this way is to buy several cans, try and after that, using your experience, to buy always your favorite one. Therefore, consumers can prefer information obtained on their own rather than by searching even when this experience is expensive (a priori a consumer buys cans he will not enjoy). So, a search can be even more expensive. Based on this classification, I will assume that «search» goods are the ones which people prefer buying on the Internet. In the case when a buyer additionally needs to try it on, ask advice or obtain information about characteristics that are impossible to obtain through the Internet, the consumer is going to buy the «experience» type of a good. In the model, this story is represented through the reference price  $\alpha_i$  (the price which the consumer has in mind being «fair» for this particular product) and  $q_i$ , which represents the better «fit» to this consumer.

The equilibrium should possess two specific features based on the peculiarities of the data and previous theory:

1) Due to the data in equilibria, prices for search goods in both channels should be identical in most cases, while prices for experience goods may be higher as well as lower online. The possible explanation is as follows. From the producer's perspective, search goods can not be priced too differently because of the threat of arbitrage. However, this issue does not emerge in the case of experience goods. For better understanding, consider the case from Zettelmeyer (2000). Computer companies can not allow luxury to price their products too differently in electronic and conventional channels. It happens because of the existence of numerous firms selling «grey» computer products which have incentives to buy products on the cheaper channel and resell it on the more expensive one.

2) By the construction of search and experience goods I assume that for search goods  $q_1 = q_2 = 0$  while for experience goods  $q_1 > 0$  and  $q_2 > 0$ .

### 3.2 Environment

Let us consider the following framework. Assume that we have one producer and two consumers with different preferences, which are differentiated by  $\alpha_i$  (the reservation price) and  $q_i$ . The producer is a monopolist, and there is a continuum of goods of a specific product (consumers search between different variations/brands of a specific product: say, a coat). It is also assumed that the share of the first type of consumers is  $\gamma$ , and  $1 - \gamma$  is the share of the second type, respectively.

The timing of the model is as follows:

1. a producer sets both online and offline prices;
2. a consumer decides whether to search: if (s)he doesn't search, (s)he either buys online or does not buy. If (s)he decides not to buy, (s)he obtains  $U_j = 0$ ; if (s)he decides to buy online (s)he obtains  $U_j = \alpha_j + q_j E\epsilon_i - p_{on}$ , where:

$\alpha_j$  is a reservation price;

$q_j$  is the value of «better fit» for a consumer;

$\epsilon_i^4$  is «better fit»;

$p_{on}$  is the online price

3. if (s)he decides to search in case of a good search (s)he buys offline, otherwise (s)he either buys online or does not buy (the choice between latter options is connected with no uncertainty, just the relation between parameters). In case of a good search, a consumer obtains  $U_j = \alpha_j + q_j \epsilon_i - p_{off} - s$ . In case of a bad search (s)he obtains  $U_j = \alpha_j + q_j E \epsilon_i - p_{on} - s$  if (s)he decides to buy online even after search and  $U_j = -s$  if in case of a bad search (s)he decides not to buy, where

$p_{off}$  is the offline price;

s- search costs (physical costs of reaching the store and wondering in it while searching)

4. payoffs are realized

The framework above is depicted on the graph for easier understanding (see Figure 2).

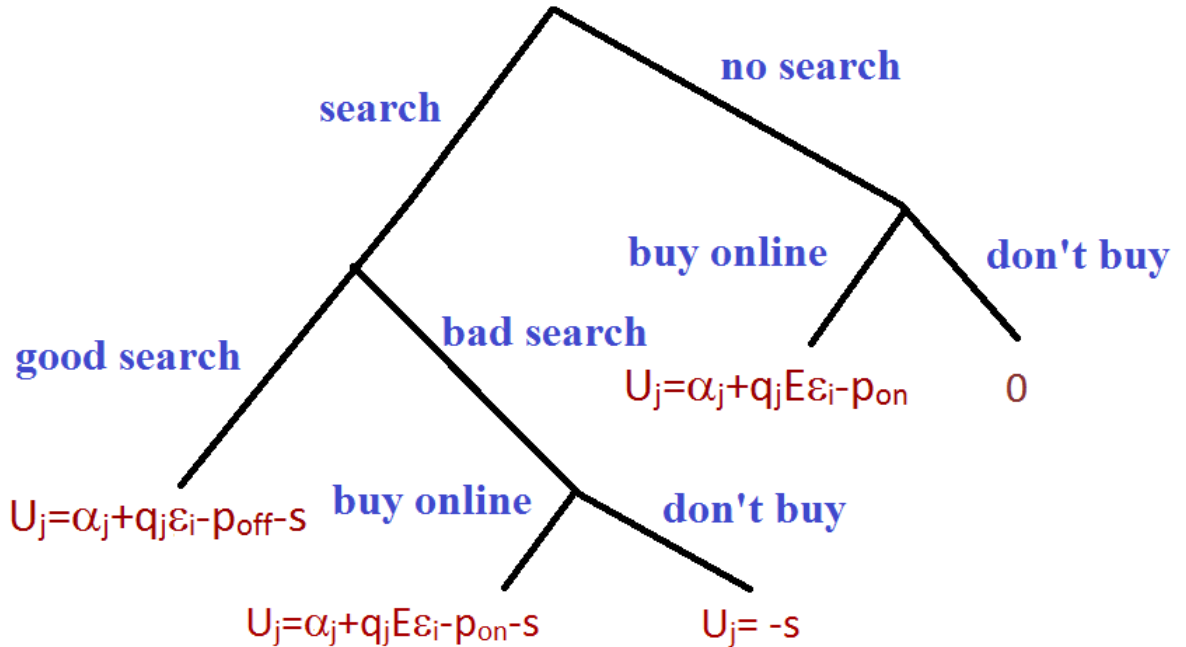


Figure 2: Consumers' strategies and corresponding payoffs

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<sup>4</sup> $\epsilon_i \sim U[0, 1]$



The notions of a good search and bad search are defined as follows. If after a search  $U_{online} > U_{notbuying}$ , the good search is a situation when  $U_{offline} > U_{online}$ . If after search  $U_{online} < U_{notbuying}$ , the good search is when  $U_{offline} > U_{notbuying}$ . The intuition behind this definition states that search should not be a waste of time. In other words, a good search is good when the utility from this option is greater than utility from any other option after search (in particular, greater than the utility from the best outside option after a search).

## 4 Main results

The pairs of consumers' strategies that could construct an equilibrium with the relative correspondence of prices is presented in Table 1. One may find the detailed proof of the existence and non-existence of the equilibria in Appendix 1.

Pairs of consumers' strategies	Existence of equilibrium	$p_{on} > p_{off}$	$p_{off} > p_{on}$	$p_{off} = p_{on}$
<i>nothing-nothing</i>	no	no	no	no
<i>nothing-online</i>	yes	yes	yes	yes
<i>nothing-bad search: online</i>	yes	no	yes	no
<i>nothing-bad search: nothing</i>	no	no	no	no
<i>online-online</i>	yes	yes	yes	yes
<i>online-bad search: online</i>	yes	no	yes	no
<i>online-bad search: nothing</i>	yes	yes	no	yes
<i>bad search: online-bad search- online</i>	yes	no	yes	no
<i>bad search: online-bad search: nothing</i>	yes	no	yes	no
<i>bad search: nothing-bad search: nothing</i>	yes	no	no	no

Table 1: Equilibrium analysis results

What main conclusions can be inferred from Table 1?

**Proposition 1:** In any equilibrium, the online market is served.

This proposition is the direct conclusion from Table 1<sup>5</sup>. Why is it profitable for the producer to always serve the online market, what is the intuition behind this result? If the producer closes

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<sup>5</sup>Any further propositions are made based on the information presented in Table 1, the results of which are proved in Appendix 1

the online market, then if the search of the consumer has resulted in the bad outcome (s)he will lose the consumers who are eager to buy online after bad search even if the online price is higher than the offline price.

Moreover, the indirect conclusion from this statement is that if you sell the product at the market where your competitors sell it in both online and offline channels, and you have only the offline store, it will be beneficial for you to open the online shop. Indeed, in case of absence of the online store your rivals capture some of your potential buyers while having it opened, you can gain (taking into consideration the fact that the costs of serving the online channel are infinitesimal).

Next, let us turn to the analysis of the specific types of goods by which all products according to the assumptions of the model are divided: search goods and experience goods.

#### 4.1 Search goods

Let us start with the analysis of the search goods. For this type of goods, we are interested in the equilibrium where no search occurs. As it has been stated, a search is beneficial for the possible better «fit», but for this type of good, we assume that no search is more profitable since we do not value this better «fit». Once again, in the model, this feature is reflected through setting  $q_1 = q_2 = 0$ . Moreover, we are interested in a type of equilibrium with equal prices so that no arbitrage occurs.

**Proposition 2:** If  $\alpha_1 > 2\alpha_2$ , then  $p_{on} = \alpha_1$  and  $p_{off} \in [p_{on} - s, \infty)$ . Otherwise  $p_{on} = \alpha_2$  and  $p_{off} \in [p_{on} - s, \infty)$ .

The intuition behind it is simple. If there is no additional value of search ( $q_i = 0$ ), then consumers have no incentives to search. If the reservation value of one of the consumers is more than twice the reservation value of another, then it is profitable for the producer to serve only the consumer with the highest value; otherwise (s)he sets the online price in a way that both consumers are served. The good news is that consumers do not search as the assumptions of the proposed model predict. The bad news is that it was expected to see identical prices on both channels. Like Proposition 2 states, there is a whole range of offline prices, which are set only for the sake of the consumers not to deviate from the offline channel, while in reality, offline market is not served. However, one may see from Table 1 that these pairs of strategies are the only ones that yield equal prices in the equilibrium so that no other cases are possible.

Thinking of the natural refinement that one may use in order to understand why similar prices are actually set in reality for this type of goods, we can think about advertising. If you search for a product where better «fit» it is not that important, say new powder, you can use the advice from advertising. In this case, it makes your search costs close to zero. Then, the optimal offline price is equal to<sup>6</sup>  $p_{off} = p_{on} + \frac{q}{4}$ . If the value of «better fit» ( $q$ ) is close to zero, then online and

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<sup>6</sup>For understanding why the optimal offline price is set in such a way, see the analysis of the case «online, bad

offline prices are nearly similar.

## 4.2 Experience goods

The analysis of the case with experience goods is different from above one by three features. First, here I search for the equilibrium, where at least one of the consumers' searches since for this type of good, it is beneficial to have an opportunity of a search for better «fit» ( $q_1 > 0$ ,  $q_2 > 0$ ). Second, I search for equilibrium at different prices because of possible gains from the segmentation of consumers for a producer. Finally, I am interested in two types of equilibria: first, where  $p_{on} > p_{off}$  and, second, where  $p_{on} < p_{off}$ . The main aim of constructing the model is to try to understand why, in some cases, the producer sets the higher offline price and in other ones vice versa.

However, before turning next, I provide a necessary definition.

**Definition:** Non-artificial equilibrium is the one where both online and offline markets are served.

**Proposition 3:** The only non-artificial equilibrium where  $p_{on} > p_{off}$  is the one where one consumer buys online, and another buys nothing in case of a bad search.

What is the intuition behind this equilibrium? It is an equilibrium where the producer indirectly divides two markets between consumers: one always buys online, and another one always buys offline. How is it constructed? For the existence of such an equilibrium, one consumer should have a higher expectation of buying ( $\alpha + 0, 5q$ ) in comparison with the second one relative to search costs and another one should have a higher dispersion of search( $q$ ) relative to search costs. The first condition allows to set online price by the first consumer and prevent the second one from buying online in a case of bad search on the one hand and prevents the first consumer from searching, while the second condition searches an attractive option for the second guy. Being profitable for a producer, it would require a higher share of the guys buying online in order to avoid the deviate with serving only the guy with the high dispersion of search both online and offline. Deviating the first guy is unprofitable because of high search costs.

If you look at another possible equilibria where  $p_{on} > p_{off}$  in Table 1, you will find the equilibria of types «nothing, online» and «online, online». That is in both these equilibria, only the online market is served, strengthening the main argument about what differs this equilibrium from others.

Beyond this, if you look at Table 1, you will realize that if the equilibrium exists, then the relation of prices where  $p_{off} > p_{on}$  can occur in each of the cases. Hence, it is crucial to understand what relation of parameters drives this equilibrium. The next two propositions elaborate on this issue.

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search: online» in Appendix 1

**Proposition 4:** If consumers are homogeneous (have similar reservation prices and valuation of «better fit»), then if the search costs are not high relative to these parameters, both consumers will search and buy online in case of a bad search. If search costs are high, then they both will buy online immediately.

Indeed, since the consumers are similar, it will be beneficial for the producer to serve both consumers: serving only one you will lose at most the same you can earn from another guy, winning only a small gain. In that case, if serving similar consumers is like serving one of them, it is evident that in case of substantial search costs, a consumer will buy online immediately and in case of small search costs, (s)he will search and buy online in case of a bad search.

**Proposition 5:** If consumers are heterogeneous, that is one has considerably lower reservation price and valuation of better «fit», equilibria «nothing, online» and «nothing, bad search: online» are realized. If they have pretty similar reservation prices and one has a considerably higher valuation of «better fit», the equilibrium «online, bad search: online» will be realized. If they have pretty similar valuations of better fit and one has a considerably higher reservation price, the equilibrium «bad search: online, bad search: nothing» will be realized.

Indeed, if one guy has considerably lower reservation price and valuation of «better fit», then it is more beneficial for the producer to serve only the «wealthiest» one so that equilibria «nothing, online» and «nothing, bad search: online» occur.

If they have pretty similar reservation prices, and one has a considerably higher valuation of «better fit», then the equilibrium «online, bad search: online» is realized. It means that the dispersion of search is not enough for one of them in order to start searching.

Finally, if consumers have pretty similar valuations of better fit and one has a considerably higher reservation price, it means that high dispersion of search allows them both to start a search. However, a guy with considerably lower reservation price is not eager to pay a higher online price equal to the willingness to pay of another consumer.

## 5 Discussion

If we return to the Figure 1, where the data about the relation of prices in different industries is presented, it can be noticed that such industries as Electronics and Household price identically their products in the majority cases, which goes in turn with the proposed theory. Indeed, we rarely care about some special mop or prefer buying the notebook on the Net since all the necessary information is presented online so that search is a waste of time in these cases. Such industries as Food in a considerable amount of cases price their production higher online. The proposed model explains this fact in a way that it is possible to divide consumers between these offline and online markets. It seems that it is rather easy to segment consumers in these markets: those who buy online are eager to pay higher money for delivering food right to home (have a higher expectation

of buying in terms of the model above). On the other side, those who buy in conventional stores are eager to spend additional time in order to save money (have a higher dispersion of search).

The major problem happens with clothing. By definition I propose, it is an experience good, and we should see not similar prices, but they coincide in 92% of the cases. The problem is that my model does not capture the vital aspect of this particular industry, which I have mentioned in the introduction: show-rooming. People tend to try some things on in the conventional store and then buy them online at a lower price. That is, conventional stores lose their attractiveness, and the prices tend to equalize. However, such practice happens practically only in this industry and seems to be an exception rather than a rule.

The right question is how to understand whether this particular good is a search one or experience one in reality? My suggestion is that the share of the market should be a good proxy: if the majority of consumers buy this particular good online, then it is a search good, if offline-experience.

## 6 Further directions of research

Even though the paper contributes considerably to our understanding of trends in online and offline pricing, I suggest that concerns similar to the ones below should be incorporated into the discussion in order to focus on the main supporting assumptions with more substantial evidence.

First, all the parameters except prices in the model are exogenous. For example, the reservation price. This is not a realistic simplification which needs to be improved by providing microfoundations. There are two main theories that explain how the reference point is formed. First, the memory-based approach argues that last paid prices influence the current reservation price (Kalyanaram and Little, 1994). The past experience forms the reference scale of the individual, and the best alternative further is treated as an «anchor». This scale becomes the basis for future comparisons. The second framework is based on the current prices of alternative brands. Consumers just compare current prices of alternatives and their reservation price out of that values (Rajendran and Tellis, 1994). The authors argue that consumers do use reference prices. However, they are based on other prices in the store rather than on past prices alone. Besides, they found that the lowest current price seems to be an essential reference point for an immediate choice, within the time the situation is reversed: the brand's own past prices start to be the most important cue.

Second, it is necessary to take into consideration the evolution of consumer preferences over time as a result of past product experience. Especially in the case if a reference point will be modeled based on the previous periods.

Third, it may be a good idea to incorporate into the model the possibility of more than one search (until you find you «dream product»: the one which is beneficial to buy immediately in comparison with a further expectation of search) and possibly introduce competition among producers.

Forth, in order to deal with the case of the Clothing industry, it may be a good idea to incorporate the option to search, return home, and buy the same product.

Finally, it is interesting to understand how this paper is connected with the following particular question:

How does the pricing strategy of Amazon influence the pricing strategies of other retailers such as Walmart? This is a kind of parallel trend to the evolution of consumers' preferences but from the producers' perspective.

## 7 Conclusion

The paper challenges to answer the question of why it may be the case that for the same good online and offline prices are different. I develop a theory of search and experience goods which answers that question. Search goods are the ones that should be priced identically because of the possible arbitrage, while experience goods are the ones where producers may gain from pricing differently because of consumer segmentation. The difference between the two types of goods is introduced through the concept of «fit». If you value high the possibility of «better fit», you prefer to search, otherwise you may buy the product online immediately orienting for the mean of the quality at the market.

One of the additional results is that in equilibrium online market is always served. That is the indirect conclusion of the proposed model states that if the producer competes at the market were both channels are served and does not have an online store, it is beneficial for her to open it.

Furthermore, the model explains that in cases when the producer can segment the consumers to a particular market, it is beneficial for her to set a higher online price. The segmentation is possible if there is a considerable share of consumers for whom expectation from buying is high enough and who has a low dispersion of search, while the low share of consumers with higher dispersion of search. If not, (s)he sets the higher offline price. As you can see, we can end this story by saying: «If you think it is beneficial for the producer always to set online prices lower than offline, think again».

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## Appendix 1

Let us describe the optimal actions of a consumer first. Each of the consumers has four strategies:

1) **Not to buy.**

In this case, a consumer obtains  $U_j = 0$ .

2) **Buy online immediately.**

In this case a consumer obtains<sup>7</sup>  $U_j = \alpha_j + q_j E\epsilon_i - p_{on} = \alpha_j + 0,5q_j - p_{on}$

3) **In case of good search a consumer buys offline, in case of bad search (s)he buys online.**

By definition of good search, the consumer chooses this option if after search  $U_{offline} > U_{online}$  that is

$$\alpha_j + q_j \epsilon_i - p_{off} - s \geq \alpha_j + q_j E\epsilon_i - p_{on} - s$$

That is if

$$E\epsilon_i + \frac{p_{off} - p_{on}}{q_j} < 0, \text{ then } p_{good} = 1$$

$$E\epsilon_i + \frac{p_{off} - p_{on}}{q_j} > 1, \text{ then } p_{good} = 0$$

$$E\epsilon_i + \frac{p_{off} - p_{on}}{q_j} \in [0, 1], \text{ then } p_{good} = \int_{0,5 + \frac{p_{off} - p_{on}}{q_j}}^1 1df = 0,5 - \frac{p_{off} - p_{on}}{q_j}$$

Overall expected utility in this case equals:

$$EU_{expected} = p_{good}(\alpha_j + q_j E\epsilon_i | good - p_{off}) + (1 - p_{good})(\alpha_j + q_j E\epsilon_i - p_{on}) - s$$

3.1 If  $p_{good} = 1$

$$EU_{expected} = \alpha_j + E\epsilon_i | good - p_{off} - s = \alpha_j + 0,5q_j - p_{off} - s, \text{ where}$$

$$E\epsilon_i | good = \int_0^1 \epsilon_i d\epsilon_i = 0,5$$

3.2 If  $p_{good} = 0$

$$EU_{expected} = \alpha_j + q_j E\epsilon_i - p_{on} - s = \alpha_j + 0,5q_j - p_{on} - s$$

3.3 If  $p_{good} \in [0, 1]$

---

<sup>7</sup>As it has been stated in the main part  $\epsilon_i \sim U[0, 1]$

$$E\epsilon_i|good = \int_{0,5+\frac{p_{off}-p_{on}}{q_j}}^1 \frac{\epsilon_i}{p_{good}} d\epsilon_i = \frac{1}{2p_{good}} (1 - (0,5 + \frac{p_{off}-p_{on}}{q_j})^2)$$

$$\begin{aligned} EU_{expected} = & (0,5 - \frac{p_{off}-p_{on}}{q_j})(\alpha_j + \frac{0,5q_j}{0,5 - \frac{p_{off}-p_{on}}{q_j}} (1 - (0,5 + \frac{p_{off}-p_{on}}{q_j})^2) - p_{off}) + \\ & + (0,5 + \frac{p_{off}-p_{on}}{q_j})(\alpha_j + 0,5q_j - p_{on}) - s \end{aligned}$$

4) **In case of good search a consumer buys offline, in case of bad search (s)he buys nothing.**

By definition of good search, the consumer chooses this option if after search  $U_{offline} > U_{notbuying}$  that is

$$\alpha_j + q_j\epsilon_i - p_{off} - s \geq -s$$

That is if

$$\frac{p_{off} - \alpha_j}{q_j} < 0, \text{ than } p_{good} = 1$$

$$\frac{p_{off} - \alpha_j}{q_j} > 1, \text{ than } p_{good} = 0$$

$$\frac{p_{off} - \alpha_j}{q_j} \in [0, 1], \text{ than } p_{good} = \int_{\frac{p_{off}-\alpha_j}{q_j}}^1 1df = 1 - \frac{p_{off} - \alpha_j}{q_j}$$

Overall expected utility in this case equals:

$$EU_{expected} = p_{good}(\alpha_j + q_j E\epsilon_i|good - p_{off}) - s$$

4.1 If  $p_{good} = 1$

$$EU_{expected} = \alpha_j E\epsilon_i|good - p_{off} - s = \alpha_j + 0,5q_j - p_{off} - s, \text{ where}$$

$$E\epsilon_i|good = \int_0^1 \epsilon_i d\epsilon_i = 0,5$$

4.2 If  $p_{good} = 0$

$$EU_{expected} = -s$$

4.3 If  $p_{good} \in [0, 1]$

$$E\epsilon_i|good = \int_{\frac{p_{off}-\alpha_j}{q_j}}^1 \frac{\epsilon_i}{p_{good}} d\epsilon_i = \frac{1}{2p_{good}} (1 - (\frac{p_{off}-\alpha_j}{q_j})^2)$$

$$EU_{expected} = (1 - \frac{p_{off}-\alpha_j}{q_j})(\alpha_j + \frac{0,5q_j}{1 - \frac{p_{off}-\alpha_j}{q_j}}(1 - (\frac{p_{off}-\alpha_j}{q_j})^2) - p_{off}) - s$$

Next, let us define what pairs of consumers' strategies could be in equilibrium.

1) *Nothing, nothing.* That is whether both consumers could buy nothing in equilibrium. Such a pair of strategies can not construct an equilibrium. If producers set the online price by  $\epsilon$  it will be profitable for them in comparison with zero and profitable for consumers to buy online immediately.

2) *Nothing, online.* Such an equilibrium exists. For example, if search costs are considerable (more than  $\max(\alpha_i+0, 5q_i, \alpha_i+0, 5q_i)$ ) and  $\alpha_i+0, 5q_i > 2(\alpha_j+0, 5q_j)$  the producer would serve only the  $i^{th}$  consumer. That is consumer i will be online immediately (without search) and consumer j will buy nothing.

3) *Nothing, bad search: online*

How to set the online price? Let us provide the argument which covers all the cases except the one where  $p_{on} > p_{off}$ , and there are two guys where one chooses to buy online in a case of bad search, and another chooses to buy nothing in a case of bad search.

If online price is set not by the maximum willingness to pay of two guys ( $\max(\alpha_i+0, 5q_i, \alpha_i+0, 5q_i)$ ), there are 2 profitable deviations. Consequently, setting the price equal to the maximum willingness to pay is beneficial except the case mentioned above.

First deviation is as follows: let  $p_{off}$  be fixed and then increase  $p_{on}$  (relative to the old one). For guys who do not buy, nothing happens. Guys who buy immediately online, begin to pay more. For guys who search and in the case of bad search do not buy, nothing has changed. For guys who search and choose in a case of bad search to buy online, if  $p_{on} \leq p_{off}$  then the producer obtains more since after increasing the online price, the probability of good search increases and if guys deviate to offline their price is higher. It is profitable to the producer; if not, then they pay a higher online price in comparison with the old one. In the case when  $p_{on} > p_{off}$ , the deviation doesn't work.

Second deviation is as follows: let increase  $p_{on}$  and  $p_{off}$  simultaneously at the same rate. Then, for those guys who do not buy, nothing has changed. Those who buy online immediately will pay more. For those who choose to search and buy online,  $p_{good}$  has not changed, and they start to pay more in both channels. For guys who choose to buy nothing in a case of bad search, the deviation does not work.

Let us return to our initial case. How to construct such type of equilibrium? In order one of the guys not to buy let's set the online price by the highest guy (say, j) and have the considerable difference between  $\alpha_j+0, 5q_j$ . In particular, for such parameters such an equilibrium exists:  $\alpha_1 = 2$ ,

$q_1 = 3$ ,  $p_{on} = 3,5$ ,  $s = 0,01$ ,  $p_{off} = 4,25$ ,  $\alpha_2 = 0,5$ ,  $q_2 = 1$ ,  $\gamma = 0,5$ . Optimal  $p_{off}$  is set by maximizing

$$p_{off}p_{good} + p_{on}p_{bad} = p_{off}(0,5 - \frac{p_{off} - p_{on}}{q_j}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_j})$$

by  $p_{off}$ . That is we obtain optimal  $p_{off} = p_{on} + \frac{q_j}{4}$ . In this case the expected producer's profit equals

$$\gamma(p_{off}(0,5 - \frac{p_{off} - p_{on}}{q_j}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_j})) = 1,84375$$

Possible deviations? Changing the online price by the reasoning above can only be in a way to set the online price equal to the maximum willingness to pay of the lower guy, in this case offline price is found by the same formula, so that we obtain  $p_{on} = 1$  and  $p_{off} = 1,75$  and expected profit of

$$\gamma(p_{off}(0,5 - \frac{p_{off} - p_{on}}{q_j}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_j})) + (1 - \gamma)p_{on} = 1,09375$$

. Increasing offline price is not beneficial since a guy who buys nothing will not start to buy while the offline one for a guy who searches is the highest since we have maximized by it above. Decreasing offline price may be profitable since we can attract guys who don't buy to search. However, with these parameters it's not possible. The best product low guy could obtain after search gives him  $\alpha_i + q_i = 1 + 0,5 = 1,5$ , setting  $p_{off} = 1,5$  and setting  $p_{on}$  in a way that maximizes  $p_{off}p_{good} + p_{on}p_{bad}$ , so that we obtain from these guys  $\gamma(p_{off}p_{good} + p_{on}p_{bad}) = \gamma(p_{off} + q_j/16) = 0,84375$ . That is we lose from the highest guys  $1,84375 - 0,84375 = 1$  and obtain from the lowest guys at most  $(1 - \gamma) * p_{on} = 0,5 * 1,5 = 0,75$ , which is not a profitable deviation.

Can this pair of strategies construct an equilibrium where  $p_{on} \geq p_{off}$ ? If  $p_{on} > p_{off}$ , we can increase  $p_{off}$  so that for a guy who does not buy, nothing has changed. Another one either stays offline and pays a higher price, which is beneficial for a producer or will go to online where the price is higher, and it is again beneficial for the producer. The same logic corresponds to the case with equal prices in both channels: either the producer obtains a higher offline price or the same online price if the consumer stops to search.

#### 4) *Nothing, bad search: nothing*

Such an equilibrium does not exist. Let us prove it. By the reasoning above the online price (except for the case when  $p_{on} > p_{off}$  and there are two guys where one chooses to buy online in a case of bad search, and another chooses to buy nothing in a case of bad search) is set such that it is equal to maximum willingness to pay of one of the guys. Since one of the guys does not buy, the online price is equal to maximum willingness to pay of the highest guy. Since after search (s)he is indifferent between whether to buy online or does not buy, we assume that a consumer chooses the option which is more beneficial for the producer that is buying online. So the proposed equilibrium does not exist.

### 5) *Online, online*

Such an equilibrium exists and it is easy to construct. Let search costs be higher than the maximum out of  $\alpha_j + 0,5q_j$  and  $\alpha_i + 0,5q_i$ . Then, irrespective of the offline price, consumers will not search. Since both consumers are served, we set the price to the maximum willingness to pay of the lowest of guys. It is profitable to the producer when  $\max(\alpha_j + 0,5q_j, \alpha_i + 0,5q_i) > 2\min(\alpha_j + 0,5q_j, \alpha_i + 0,5q_i)$ . In other words, the equilibria where  $p_{on} > p_{off}$ ,  $p_{off} > p_{on}$  and  $p_{on} = p_{off}$  exist.

### 6) *Online, bad search: online*

Such an equilibrium exists for the following set of parameters:  $\alpha_1 = 1$ ,  $q_1 = 0,1$ ,  $s = 0,05$ ,  $\alpha_2 = 0,1$ ,  $q_2 = 2,2$ ,  $\gamma = 0,9$ . By the reasoning above, the online price is equal to the maximum willingness to pay of one of the guys. Since we are searching for the equilibrium where one of the consumers buys online, we set the online price to the maximum willingness to pay of the lowest guys, which is  $p_{on} = \alpha_1 + 0,5q_1$ . As in the equilibrium *nothing, bad search: online* optimal  $p_{off}$  is set by maximizing

$$p_{off}p_{good} + p_{on}p_{bad} = p_{off}(0,5 - \frac{p_{off} - p_{on}}{q_2}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_2})$$

by  $p_{off}$ . That is we obtain optimal  $p_{off} = p_{on} + \frac{q_2}{4}$ . The corresponding expected profit equals  $\gamma p_{on} + (1 - \gamma)(p_{good}p_{off} + p_{bad}p_{on}) = 1,06375$ . Deviating by  $p_{on}$  we have  $p_{on} = \alpha_2 + 0,5q_2$  and the same  $p_{off} = p_{on} + \frac{q_2}{4}$ , the corresponding profit in this case is 0,13375. So, this is unprofitable deviation since we have a large share of the first type of consumers who leave the market after new  $p_{on}$  is set. Deviating by  $p_{off}$ : increasing  $p_{off}$  is not beneficial since the first type of consumers has not bought by the old offline price and certainly will not do it after the increase. From the perspective of the second type of consumers, it is also unprofitable since we have maximized by them and set the optimal price before the increase. Why don't we decrease the offline price? It can only make sense in order to make the first consumer search. The parameters are set in a way that the expected gain online equals  $\alpha_1 + 0,5q_1 = 1 + 0,5 * 0,1 = 1,05$  while search costs are equal to 0,05 and the best product a consumer can obtain offline will yield him  $1 + 0,01 = 1,01$ , which is the same as the utility online. Consequently, in order to make a consumer search, we need to have  $p_{on} \geq p_{off}$ . The analysis below will show that such a relation of prices is only possible for the pairs of strategies «online, bad search: nothing». In order for the first deviation to exist, we need one guy to have an expectation of utility  $(\alpha_i + 0,5q_i)$  and another one to have high  $q$ . However, we have both higher utility and  $q$  for the second guy, that is such a deviation is not possible.

Above it was shown that the optimal offline price is  $p_{off} = p_{on} + \frac{q}{4}$  that is since for experience goods we assume that  $q > 0$  so that there are no equilibria where  $p_{on} \geq p_{off}$

### 7) *Online, bad search: nothing*

Let's begin the most important case with the analysis of the situation when  $p_{on} > p_{off}$ . Such an equilibrium exists for the following set of parameters:  $\alpha_1 = 1$ ,  $q_1 = 0,1$ ,  $s = 0,2$ ,  $\alpha_2 = 0,01$ ,

$q_2 = 1,9$ . In order to make one guy to buy online immediately and another one to buy nothing in a case of bad search, we should set the online price equal to the maximum willingness to pay of the highest guy that is  $p_{on} = 1 + 0,5 * 0,1 = 1,05$ . The offline price is obtained by maximizing by  $p_{off}$ :

$$p_{off} \left[ 1 - \frac{p_{off} - \alpha_2}{q_2} \right]$$

Then, the optimal offline price in this case is equal to  $p_{off} = \frac{\alpha_2 + q_2}{2} = 0,95$ . What about possible deviations? Setting online price higher is not profitable since first will stop buying, and the second has not wanted with lower  $p_{on}$ . Increasing  $p_{off}$  does not make sense since the first one will not start to search while it was the optimal price for the second guy (we have maximized by him). Decreasing  $p_{off}$  will give the producer less profit from the second guy and is not profitable from the first guy's perspective since now in online (s)he pays higher online in comparison with the offline price. So, the only possible profitable deviation is to decrease online price. In order second guy not to deviate, we should have the following  $\gamma$ . If the online price is decreased to the maximum willingness to pay of the lowest guy (that is  $\frac{0+1,9}{2} = 0,95$ ), then (s)he searches and in a case of bad search begins to buy online, so that new optimal  $p_{off}$  is found by maximizing:

$$(0,5 - \frac{p_{off} - 0,95}{1,9})p_{off} + (0,5 + \frac{p_{off} - 0,95}{1,9})0,95$$

Then, the optimal offline price is 1,425. The producer's expected profit from such prices from the lowest guy is 1,06875. The old expected utility from her equals  $0,95(1 - \frac{0,95-1,05}{1,9}) = 0,525$ . That is, in order the lowest guy not to deviate after decreasing of the online price we need the share of the highest guys to be higher enough:

$$\gamma(1,05 - 0,95) > (1 - \gamma)(1,06875 - 0,525)$$

That is  $\gamma$  should be more than 0,84466. So that we have proved the existence of the equilibrium where  $p_{on} > p_{off}$

Let us now turn to the proof of the non-existence of an equilibrium where  $p_{off} > p_{on}$ . First, notice that optimal  $p_{off} = \frac{\alpha_2 + q_2}{2}$  which is less or equal than  $\alpha_2 + \frac{q_2}{2}$ . Second, we construct such an equilibrium where the second guy in a case of bad search buys nothing, which in particular means that the utility from buying online immediately  $\alpha_2 + 0,5q_2 - p_{on} \leq 0$ . Third, we are searching for the equilibrium where  $p_{on} > p_{off}$ . The system of these three equations is incompatible. Hence, there is no such equilibrium.

Finally, let's prove that the equilibrium where  $p_{on} = p_{off}$  doesn't exist. The system of three equations above shows that the only possible case is when  $p_{off} = \frac{\alpha_2 + q_2}{2}$  is equal to  $p_{on} = \alpha_2 + \frac{q_2}{2}$ . In particular, it means that  $\alpha_1 + \frac{q_1}{2} = \frac{\alpha_2 + q_2}{2} \leq \alpha_2 + \frac{q_2}{2}$ . In other words, if the first guy buys online than the second one in a case of bad search will definitely buy online, so that it is not an

equilibrium.

8) *Bad search: online, bad search: online*

Such an equilibrium exists for the following set of parameters:  $\alpha_1 = 3,8$ ,  $q_1 = 4$ ,  $s = 0,05$ ,  $\alpha_2 = 3,9$ ,  $q_2 = 5$ ,  $\gamma = 0,6$ . As it has been stated above, online price is set by the maximum willingness to pay of one of the guys. Since we are searching for the equilibrium where both guys in a case of bad search choose to buy online, then the producer sets the price to the minimum between  $\alpha_j + 0,5q_j$  and  $\alpha_i + 0,5q_i$ . The offline price is set by maximizing by  $p_{off}$  the expected profit of the producer:

$$\gamma(p_{off}(0,5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_1})) + (1 - \gamma)(0,5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_1})$$

The optimal offline price in this case is equal to:

$$p_{off} = \frac{2\gamma p_{on} q_2 + 2(1 - \gamma)p_{on} q_1 + 0,5q_1 q_2}{2p_{off}((1 - \gamma)q_1 + \gamma q_2)}$$

For these values of parameters expected profit is equal to 6,07173913. What about deviations? Another possible candidate for the online price is the maximum willingness to pay of the highest guy which is equal to 6,4. However, in that case the lowest guy begins to deviate to the option «bad search: nothing» and new offline price is calculated by maximizing by  $p_{off}$ :

$$\gamma(p_{off}(1 - \frac{p_{off} - \alpha_1}{q_1})) + (1 - \gamma)(0,5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0,5 + \frac{p_{off} - p_{on}}{q_1})$$

The optimal offline price, in this case, is equal to:

$$p_{off} = \frac{\gamma(1 + \frac{\alpha_1}{q_1}) + (1 - \gamma)(0,5 + \frac{2p_{on}}{q_2})}{2(\frac{\gamma}{q_1} + \frac{1 - \gamma}{q_2})}$$

In this case, the expected profit is equal to 4,2328; that is, the deviation is not profitable. Decreasing offline price is not beneficial since the consumers are eager to search by higher(current) offline price, while increasing is not beneficial since we have maximized by it, and the current offline price is the maximum one.

The non-existence of the equilibria where  $p_{on} > p_{off}$  is by analogous reasoning as in previous cases.

9) *Bad search: online, bad search: nothing*

First let's search for the equilibrium where  $p_{off} > p_{on}$ . Since we are searching for the equilibrium where one of the consumers buys nothing, then  $p_{off}$  is set by the maximum willingness to pay of the highest guy. Offline price may be obtained by solving the following problem:

$$\gamma(p_{off}(1 - \frac{p_{off} - \alpha_1}{q_1})) + (1 - \gamma)(0, 5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0, 5 + \frac{p_{off} - p_{on}}{q_1})$$

The optimal offline price in this case is equal to:

$$p_{off} = \frac{\gamma(1 + \frac{\alpha_1}{q_1}) + (1 - \gamma)(0, 5 + \frac{2p_{on}}{q_2})}{2(\frac{\gamma}{q_1} + \frac{1-\gamma}{q_2})}$$

The expected profit in this case is equal to 6,1128. Deviation by  $p_{on}$  is equal to the maximum willingness to pay of the lowest guy. It means for her the deviation from the option «bad search: nothing» to «bad search: online» which yields the following maximization by  $p_{off}$ :

$$\gamma(p_{off}(0, 5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0, 5 + p_{off} - p_{on}q_1) + (1 - \gamma)(0, 5 - \frac{p_{off} - p_{on}}{q_1}) + p_{on}(0, 5 + \frac{p_{off} - p_{on}}{q_1})$$

The optimal offline price in this case is equal to:

$$p_{off} = \frac{2\gamma p_{on}q_2 + 2(1 - \gamma)p_{on}q_1 + 0, 5q_1q_2}{2p_{off}((1 - \gamma)q_1 + \gamma q_2)}$$

The expected profit, in this case, equals 6,1049, so that it is not a profitable deviation. Increasing offline price is not beneficial since we have maximized by it, and decreasing offline price will lead to the loss of the profit since now consumers are eager to buy by the current offline price.

It seems that there is no simple argument why, in this case, equilibria where  $p_{on} \geq p_{of}$  does not exist. In Appendix 2, one may find the Matlab simulation code, which shows that for a wide range of parameters, there are no such equilibria.

#### 10) *Bad search: nothing, bad search: nothing*

Such equilibria do not exist. Let us prove it. First, assume that the probability of a good search is less than one. Then, setting the online price equal to the willingness to pay of the highest guy will be profitable for the producer. In a case of bad search, a consumer will start to buy online instead of not buying since (s)he is indifferent and obtains zero utility in both cases. Second, assume that the probability of good search is equal to one. Then, setting  $p_{on} = p_{off} + \epsilon$  will be a profitable deviation. Indeed, if the gain in the case of search for the highest guy is equal to  $\alpha_j + q\epsilon_i$  is less than  $p_{off}$ , then it is profitable for her to deviate to online channel in a case of bad search since  $(\alpha_j + q\epsilon_i - p_{on}) - (\alpha_j + q\epsilon_i - p_{off}) > 0$ .



## Appendix 2

### Matlab code

```
1 clear all;
2 n=200;
3 d=0;
4 for ia=0 : n
5     d
6     a = ia
7     for iq = 1 : n
8         q = iq;
9         for iA=0 : n
10            A = iA;
11            for iQ = 1 : n
12                Q = max(2*a+q-2*A+iQ,iQ);
13                for ig = 1 : n-1
14                    g = ig/n;
15                    poff = (g*a/q+1)/(2*g/q+(1-g)/Q);
16                    if poff/2+Q/4>A+Q/2
17                        pon = A+Q/2;
18                        poff = (g*(1+a/q)+(1-g)*(0.5+2*pon/Q))/(g/q+(1-
19                            g)/Q);
20                    else
21                        pon = poff/2+Q/4;
22                    end
23                    U=g*poff*(1-(poff-a)/q)+(1-g)*(poff*(0.5-(poff-pon)
24                        /Q)+pon*(0.5+(poff-pon)/Q));
25                    if pon>a+q/2 & pon<=A+Q/2 & poff<=pon & poff>a+q/2
26                        & poff<=a+q & pon-poff<=Q/2 & U>= (1-g)*(A+9*Q
27                            /16) & U>= a+q/2+(1-g)*Q/16
28                        d=d+1;
29                        UU(d)=U;
30                        ppoff(d)=poff;
31                        ppon(d)=pon;
32                        aa(d)=a;
33                        AA(d)=A;
34                        qq(d)=q;
35                        QQ(d)=Q;
```

```

32         gg(d)=g;
33     end
34 end
35 end
36 end
37 end
38 end

```