Exploring Political Polarization and Turnout in Two-Party Elections with the Utilitarian Approach*

MARIAM ARZUMANYAN†

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Abstract

Do differentiated policies encourage higher voter turnout than centrist policies? This paper uses the citizen-candidate model and group-based utilitarian approach of costly voting to study how political polarization affects voter turnout under positive voting costs. Two parties enter the electoral competition at no cost with a pre-commitment on the platforms, and the winner implements their ideal policy. Comparative statics results demonstrate that under positive voting costs, the expected turnout is always positive in the presence of political polarization and is only zero when the policies converge. In addition, the expected turnout rises with increased polarization, while it decreases with voting costs. However, parties have higher chances of victory if they choose converging policies.

Keywords: Costly Voting, polarization, voter turnout, utilitarian approach.

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†Ph.D. Candidate, Department of Economics, University of Illinois at Urbana-Champaign. E-mail: mariama2@illinois.edu

1 Introduction

One of the most fundamental questions in the political economy centers around issues raised by positive voter turnout. In large real-world elections, the chance of influencing an election result is negligible for any single individual, and the cost of voting should prevent people from voting; however, we see a significant turnout. Starting with Downs (1957), many attempted to present a model that would exhaustively explain the positive turnout by considering both the strategic voting behavior of citizens and the policy choices of political parties. Most of the literature bypassed either strategic aspects of voting behavior (Wittman (1983), Borgers (2004), Smirnov and Fowler (2007)) or strategic aspects of policy choices (Palfrey and Rosenthal (1983), Palfrey and Rosenthal (1985), Ledyard (1984), Myerson and Weber (1993), Myerson (2000)) or incorporated motivations for citizens' abstention (Adams and Merrill III (2003), Adams et al. (2006), Krasa and Polborn (2010)). The main goal of this paper is to model a large election game with strategic voters and strategic parties to deliver vital comparative statics results on turnout and polarization.

A substantial body of the literature indicates that citizens have incentives outside of simple benefit-cost calculus (Wang (2016), Blais and Achen (2019)). The model presented in this paper is built on a game-theoretic framework introduced by Harsanyi (1977, 1980) and developed further by Feddersen and Sandroni (2006) and Coate and Conlin (2004). Harsanyi (1977) introduced the rule-utilitarian model, assuming that citizens have "ethical" motivations and follow the rule to maximize social welfare. This model was one of the first to explain the stylized facts about voter turnout in large elections depending on the cost. Feddersen and Sandroni (2002) and Coate and Conlin (2004) introduced more diversity into the model, generating robust results explaining turnout. Nevertheless, the above models assume that citizens' support for the candidates is exogenous and given outside the control of their chosen platforms. This paper introduces spatial policy competition between differentiated parties based on Osborne and Slivinski (1996) and Besley and Coate (1997), ensuring that citizens' preferences endogenously depend on the parties' policy choices.

The underlying research problem is understanding voting behavior in political polarization when citizens face positive voting costs. The model is a dynamic election game with two political parties and an infinite number of citizens. Citizens support the party whose proposed policy is closer to their ideal policy. Citizens are strategic, and a fraction of them are influenced by their sense of civic duty, whom we will call the "ethical" citizens based on the previous literature. The party with the majority votes is the election's winner, and the proposed policy will be implemented. The parties may have various motives when choosing their platforms. Two general cases are discussed. Parties cannot credibly change their policy positions away from their ideal policies, and parties are office-motivated.

Citizens' positive voting costs are given by uniform distribution, and a fraction is "ethical" by acting in a group-rule utilitarian fashion. Given the ideal policy position of the citizen, a group rule defines a cost threshold such that only citizens with costs below the threshold vote, and the rest abstain from voting. Ethical citizens receive a strictly positive payoff above their cost when they follow the rule, so they always choose to follow it. Non-ethical citizens do not receive such payoffs; hence they are abstainers. Both parties' supporters determine the cost threshold value by maximizing the social utility of their group. We follow the same consistency requirements defined in the literature: every "ethical" citizen follows the rule and has no incentives to deviate, given that everyone else follows the rule.

This model has the following dynamic aspect: First, parties propose the policies. For each policy pair, citizens decide which party to support based on the proposed policies and their preferences over the proposed policies. Citizens learn their voting costs, then the cost threshold is determined, and ethical citizens follow the voting rule. The party with the most votes wins the election.

The baseline model conveys comparative statics on policies, turnout, the share of votes, probabilities of victory, and the importance of the election. The results complement Feddersen and Sandroni (2006) and Coate and Conlin (2004). As anticipated, the majority party is expected to win, and a higher fraction of majority party supporters vote. Next, if parties adopt identical policies, the expected turnout is zero. Similar to Feddersen and Sandroni (2006), the importance of election represents the weight of citizens' utility based on who wins the election; however, in Feddersen and Sandroni (2006), it is an exogenous parameter, whereas, in our model, it is endogenous and depends on the chosen policies. Comparative statics results show that the more extreme opposition's policy, the higher the importance of the election.

Under the specific assumptions on the population distribution, the closed-form solutions for the cost thresholds are obtained. In addition to the comparative statics results, these distributional assumptions allow us to discuss the policy choices of office-motivated parties. If the parties' incentives are winning the election and hence maximizing the probability of the victory, then both parties will choose the median position, and everyone abstains from voting. This outcome prompts a debate on the paradox of voting. In the case of positive voting costs, a positive voter turnout is feasible only in the existence of differentiated policies and if both parties have motivations beyond winning the election.

The rest of the paper follows accordingly: Section 2 presents the related literature, Section 3 presents the model, provides closed-form solutions, and discusses comparative statics results. Section 4 discusses the results of the office-motivated and policy-motivated parties' choices, and Section 5 concludes. All proofs are in Appendix.

2 Literature Review

The theoretical voter turnout literature originated by Downs (1957) and Riker and Ordeshook (1968). The calculus of voting is defined by the benefit and the cost of voting. Since the benefits of voting are negligible in large elections, and the cost is nonzero, it generated the so-called "paradox of not voting." Ledyard (1984), Palfrey and Rosenthal (1983), Palfrey and Rosenthal (1985), and Myerson and Weber (1993) introduced the pivotal voting calculus models to explain turnout. These models consider the endogenous participation decision by the citizens and incorporate the probability that the citizen is a swing voter. However, these models are heavily criticized since the pivotal probabilities are small in large elections and cannot explain turnout under positive costs without including any further benefits. In the current paper, the chance of any voter being pivotal is zero; however, it does not bear the same criticism since citizens benefit from the social utility and not individual payoffs.

The next body of the literature Morton (1991), Shachar and Nalebuff (1999), and Herrera and Martinelli (2006) make behavioral assumptions on the voters' incentives to participate in the election. In this literature, citizens form community leaders or political blocks and vote in groups. This group mentality allows for incorporating positive voter turnout.

Harsanyi (1977, 1980), Feddersen and Sandroni (2006), Coate and Conlin (2004) developed a theory of voter turnout using a utilitarian approach. The idea is that citizens care about the aggregate payoffs instead of thinking about the individual payoffs, and they follow the strategy of "doing their parts." Coate and Conlin (2004) introduced a new aspect of the theory. Since the elections usually divide society between the supporters and opposers of the policy based on their interests, it is reasonable to assume that citizens are group-utilitarian, i.e., citizens only care about the aggregate payoff of their groups rather than the entire society.

The theory proposed by Feddersen and Sandroni (2006) and Coate and Conlin (2004) is the foundation of the model used in the current paper. If there are two parties named Left and Right, the parties' policies divide the citizens into two groups: Left supporters and Right supporters. We assume that Left-supporting citizens believe that the policy proposed by the Left party is the best for society, and Right-supporting citizens believe that the policy proposed by the Right is the best. Citizens have heterogeneous voting costs, and having a minimum number of voters is socially optimal. Feddersen and Sandroni (2006) proposed a model to overcome this issue by saying an exogenous disagreement between citizens about the policy. In their context, the disagreement refers to the size of the electorate who approves the policy. In this paper, we model disagreement among citizens in a more robust setup: different policy choices give rise to different sizes of Left-supporting and Right-supporting electorates. In addition, this model characterizes the solution under policy differentiation and polarization. A positive share of citizens votes when parties' policies are distinctive, and everyone abstains from voting if parties propose converging policies. Section 4 analysis demonstrates that when the population distribution is uniform, office-motivated candidates choose the median position, and the expected turnout will be zero.

Adams and Merrill III (2003), Adams et al. (2005),Adams et al. (2006), Dreyer and Bauer (2019), study various reasons for citizen's abstention from voting. Their model incorporates the idea that citizens prefer to disengage from politics if they feel unrepresented by their political parties. Similarly, citizens feel that political parties are identical and have incentives to refrain from voting. In the current model's setup, abstention due to indifference rises naturally. Utilitarian citizens with positive voting costs would prefer to minimize the turnout if both parties propose similar policies.

3 Model

Suppose there is a continuum of citizens with ideal policy positions distributed over the $[-1, 1]$ interval with cumulative distribution function $F(\cdot)$ and probability distribution function $f(\cdot)$. Consider an election game where two parties, the Left and the Right, enter the election at no cost with a pre-commitment on the platforms. The Left has an ideal policy position at θ_L, and the Right has an ideal policy position at θ_R. Assume that both parties' proposed policies are fixed at $-1 \leq \theta_L \leq \theta_R \leq 1$. In this section, the parties cannot credibly change the policy positions from their ideal positions. The following sections will discuss the office-motivated.

Every citizen with ideal policy $i \in [-1, 1]$ has a utility function $u_i(\theta) = -(i - \theta)^2$, where θ is the winning policy. The majority rule determines the winner in the election, and each citizen has three actions available: vote for Left, vote for Right, or abstain from voting. The citizen i will support the Left if and only if $u_i(\theta_L) > u_i(\theta_R)$, or when $i \in \Theta_L$, where

$$\Theta_L = \{i \in [-1, 1] : u_i(\theta_L) > u_i(\theta_R)\}. \tag{1}$$

Accordingly, the citizen i will support the Right if and only if $u_i(\theta_R) > u_i(\theta_L)$, or when $i \in \Theta_R$, where

$$\Theta_R = \{i \in [-1, 1] : u_i(\theta_R) > u_i(\theta_L)\}. \tag{2}$$

The citizens with the ideal policy position i such that $u_i(\theta_L) = u_i(\theta_R)$ will be "undetermined," therefore will abstain from voting.[1]

Each citizen has a voting cost, given by an independent uniform distribution over the interval $[0, \overline{c}]$, which is private information. The utilitarian voting models introduced by Feddersen and Sandroni (2006) assume that citizens are "ethical," i.e., citizens follow the rule that, if followed by everyone in their group, will maximize the social utility or the aggregate utility of the group. Analogous to that assumption, citizens receive a payoff of $D > \overline{c}$ for following the "ethical" rule and, therefore, always choose to follow it. The fraction of ethical citizens, e_L and e_R, are independent and uniformly distributed over $[0, 1]$.

A voting rule is a cost-cutoff, in which citizens with a cost higher than the cost cutoff abstain from voting and citizens with a cost below the cutoff vote for their preferred candidate. When choosing the cost cutoffs, Left supporters maximize the social utility of the Left, and Right supporters maximize the social utility of the Right. Denote the cost cutoff point of Left by c_L and the cost cutoff point of Right by c_R. For both parties, the higher the cost threshold, the higher the turnout. However, a higher threshold implies that the social cost will be higher.

If the Left wins the election, then denote the aggregate payoff of the Left supporters by $U_L(\theta_L)$, the aggregate payoff of the Right supporters by $U_R(\theta_L)$, and they will be equal to:

$$U_L(\theta_L) = \int_{\Theta_L} \left[u_i(\theta_L) - E(e_L) \int_0^{c_L} \frac{c_t}{\overline{c}} dt \right] dF(i). \tag{3}$$

$$U_R(\theta_L) = \int_{\Theta_R} \left[u_i(\theta_L) - E(e_R) \int_0^{c_R} \frac{c_t}{\overline{c}} dt \right] dF(i). \tag{4}$$

If the Right wins the election, then denote the aggregate payoff of the Left supporters by $U_L(\theta_R)$, the aggregate payoff of the Right supporters by $U_R(\theta_R)$, and they will be equal to:

$$U_L(\theta_R) = \int_{\Theta_L} \left[u_i(\theta_R) - E(e_L) \int_0^{c_L} \frac{c_t}{\overline{c}} dt \right] dF(i). \tag{5}$$

$$U_R(\theta_R) = \int_{\Theta_R} \left[u_i(\theta_R) - E(e_R) \int_0^{c_R} \frac{c_t}{\overline{c}} dt \right] dF(i). \tag{6}$$

Notice that for both the Left and the Right, the voting cost is independent of whether their party wins or losses the election.

To simplify the aggregate payoffs above, denote the share of the Left and Right supporters

[1] If the population cumulative distribution function $F(\cdot)$ does not have a mass point at $u_i(\theta_L) = u_i(\theta_R)$, then the fraction of "undetermined" citizens will be zero.

by $s_L =: s_L(\theta_L, \theta_R; u(\cdot), F(\cdot))$ and $s_R = s_R(\theta_L, \theta_R; u(\cdot), F(\cdot))$.[2] The shares are equal to

$$s_L = \int_{\Theta_L} 1 dF(i), \text{ and } s_R = \int_{\Theta_R} 1 dF(i). \tag{7}$$

The Left party wins the election if there are more Left votes than Right votes. This happens if

$$s_L e_L \frac{c_L}{\overline{c}} \geq s_R e_R \frac{c_R}{\overline{c}} \iff \frac{e_R}{e_L} \leq \frac{s_L c_L}{s_R c_R}. \tag{8}$$

Hence, the Left wins the election with a probability

$$p(c_L, c_R) = G\left(\frac{s_L c_L}{s_R c_R}\right), \tag{9}$$

where G is the cumulative distribution of $\frac{e_R}{e_L}$, with $g(\cdot)$ corresponding probability distribution function, and they are equal to

$$G(x) = \begin{cases} \frac{x}{2}, & \text{if } x \leq 1; \\ 1 - \frac{1}{2x}, & \text{if } x \geq 1. \end{cases} \text{ and } g(x) = \begin{cases} \frac{1}{2}, & \text{if } x \leq 1; \\ \frac{1}{2x^2}, & \text{if } x \geq 1. \end{cases} \tag{10}$$

Following Coate and Conlin (2004), we assume that both the Left and Right supporters are group-rule utilitarians; therefore, their incentive is to maximize the aggregate utility of their own group. The aggregate expected utility of the Left supporting citizens will be

$$U_L(\theta_L) p(c_L, c_R) + U_L(\theta_R) \left(1 - p(c_L, c_R)\right). \tag{11}$$

The aggregate expected utility of the Right supporters will be

$$U_R(\theta_L) p(c_L, c_R) + U_R(\theta_R) \left(1 - p(c_L, c_R)\right). \tag{12}$$

Citizens choose the cost thresholds similar to Feddersen and Sandroni (2006) and Coate and Conlin (2004). Ethical citizens vote by blocks, and their primary objective is to maximize the aggregate payoff of their group.

Definition 1. ***Equilibrium***

For every policy pair (θ_L, θ_R), the Left supporters choose $c_L^*(\theta_L, \theta_R)$ to maximize (11) *subject to $c_L \in [0, \overline{c}]$ constraint. Likewise, the Right supporters choose $c_R^*(\theta_L, \theta_R)$ to maximize* (12) *subject to $c_R \in [0, \overline{c}]$ constraint.*

[2] For the continuous $F(\cdot)$, the shares are calculated in the Appendix A.1.

Assuming all functions are differentiable, if there is an interior solution for citizens' problem, then first-order conditions should define it. Simplifying the Equation (11), it can be rewritten as:

$$
\begin{aligned}
& U_L(\theta_L)p(c_L, c_R) + U_L(\theta_R)\left(1 - p(c_L, c_R)\right) \\
=& \left[\int_{\Theta_L} u_i(\theta_L)dF(i)\right] p(c_L, c_R) + \left[\int_{\Theta_L} u_i(\theta_R)dF(i)\right] \left(1 - p(c_L, c_R)\right) - s_L \frac{c_L^2}{4\bar{c}} \\
=& \left[\int_{\Theta_L} \left(u_i(\theta_L) - u_i(\theta_R)\right) dF(i)\right] p(c_L, c_R) + \int_{\Theta_L} u_i(\theta_R)dF(i) - s_L \frac{c_L^2}{4\bar{c}}. \qquad (13)
\end{aligned}
$$

(14)

Denote by $\phi_L = \phi_L(\theta_L, \theta_R) = \int_{\Theta_L} \left(u_i(\theta_L) - u_i(\theta_R)\right) dF(i)$. It can be interpreted as the importance of winning an election for the Left. Using this notation, Equation (13) can be rewritten as

$$
\phi_L p(c_L, c_R) + \int_{\Theta_L} u_i(\theta_R)dF(i) - s_L \frac{c_L^2}{4\bar{c}}. \qquad (15)
$$

In like manner, denote by $\phi_R = \phi_R(\theta_L, \theta_R) = \int_{\Theta_R} \left(u_i(\theta_R) - u_i(\theta_L)\right) dF(i)$. Notice that $\phi_R(\theta_L, \theta_R) = \phi_L(-\theta_R, -\theta_L)$.

Rewrite the Equation (12) as following:

$$
\begin{aligned}
& U_R(\theta_L)p(c_L, c_R) + U_R(\theta_R)\left(1 - p(c_L, c_R)\right) \\
=& \left[\int_{\Theta_R} u_i(\theta_L)dF(i)\right] p(c_L, c_R) + \left[\int_{\Theta_R} u_i(\theta_R)dF(i)\right] \left(1 - p(c_L, c_R)\right) - s_R \frac{c_R^2}{4\bar{c}} \\
=& \left[\int_{\Theta_R} \left(u_i(\theta_R) - u_i(\theta_L)\right) dF(i)\right] \left(1 - p(c_L, c_R)\right) + \int_{\Theta_R} u_i(\theta_L)dF(i) - s_R \frac{c_R^2}{4\bar{c}} \\
=& \phi_R \left(1 - p(c_L, c_R)\right) + \int_{\Theta_R} u_i(\theta_L)dF(i) - s_R \frac{c_R^2}{4\bar{c}}. \qquad (16)
\end{aligned}
$$

The first order condition of maximizing Equation (15) with respect to c_L is

$$
\phi_L g\left(\frac{s_L}{s_R}\frac{c_L}{c_R}\right)\frac{s_L}{s_R}\frac{1}{c_R} - s_L \frac{c_L}{2\bar{c}} \begin{cases} = 0, & \text{if } c_L \in (0, \bar{c}), \\ \geq 0, & \text{if } c_L \in \{0, \bar{c}\}. \end{cases} \qquad (17)
$$

The first order condition of maximizing Equation (16) with respect to c_R is

$$\phi_R g\left(\frac{s_L}{s_R}\frac{c_L}{c_R}\right)\frac{s_L}{s_R}\frac{c_L}{c_R^2} - s_R\frac{c_R}{2\overline{c}}\begin{cases} = 0, & \text{if } c_R \in (0, \overline{c}), \\ \geq 0, & \text{if } c_R \in \{0, \overline{c}\}. \end{cases} \tag{18}$$

For every policy pair (θ_L, θ_R), the fraction of citizens $T_L := s_L e_L \frac{c_L^*}{\overline{c}}$ will vote for the Left, and the fraction of citizens $T_R := s_R e_R \frac{c_R^*}{\overline{c}}$ will vote for the Right. e_L and e_R are uniformly distributed over $[0, 1]$, thereupon the total expected turnout will be

$$\text{Total Turnout} = E\left[T_L + T_R\right] = E\left[s_L e_L \frac{c_L^*}{\overline{c}} + s_R e_R \frac{c_R^*}{\overline{c}}\right] = \frac{1}{2}s_L\frac{c_L^*}{\overline{c}} + \frac{1}{2}s_R\frac{c_R^*}{\overline{c}}. \tag{19}$$

The Left wins the election with probability $p(c_L^*, c_R^*)$, which is equal to

$$PV_L = p(c_L^*, c_R^*) = G\left(\frac{s_L c_L^*}{s_R c_R^*}\right). \tag{20}$$

The Right wins the election with probability $1 - p(c_L^*, c_R^*)$, and since $1 - G(x) = G(\frac{1}{x})$, it can be rewritten as

$$PV_R = 1 - p(c_L^*, c_R^*) = 1 - G\left(\frac{s_L c_L^*}{s_R c_R^*}\right) = G\left(\frac{s_R c_R^*}{s_L c_L^*}\right). \tag{21}$$

Since $G(x)$ is a cumulative distribution function, it implies that it is an increasing function in x. Consequently, the probability of the victory for the Left is increasing with respect to $\frac{s_L c_L^*}{s_R c_R^*}$, and the probability of the victory for the Right is increasing with respect to $\frac{s_R c_R^*}{s_L c_L^*}$. The following subsection summarizes the comparative statics results.

3.1 Comparative Statics Results

The importance of the election $\phi_L(\theta_L, \theta_R)$ and $\phi_R(\theta_L, \theta_R)$, are the subject of examination. While it has been characterized as an exogenous variable in a previous study (Feddersen and Sandroni (2006)), this current model considers it an endogenous variable that relies on both the policy pair and the distribution of the population. The following Proposition outlines the essential comparative statics findings related to these variables.

Proposition 1. *The functions $\phi_L(\theta_L, \theta_R)$ and $\phi_R(\theta_L, \theta_R)$, representing the importance of the election for the Left and the Right, are positive and equal to zero if and only if $\theta_L = \theta_R$. For continuous population distribution $F(\cdot)$, if $\frac{\theta_L+\theta_R}{2} \geq E[F]$, then $\phi_L \geq \phi_R$. In addition, if $F(\cdot)$ is continuously differentiable, then $\phi_L(\theta_L, \theta_R)$ is increasing function in θ_R and $\phi_R(\theta_L, \theta_R)$ is* *a decreasing function in* θ_L.

Proposition 1 implies that if the opposition is more extreme, it is more important to win the election. Denote by m the population median, i.e., $F(m) = \frac{1}{2}$, then the following is true:

$$s_L \geq s_R \iff F(\frac{\theta_L + \theta_R}{2}) \geq \frac{1}{2} \iff \frac{\theta_L + \theta_R}{2} \geq m. \tag{22}$$

To get an intuition about Proposition 1, assume that $F(\cdot)$ is such that $E[F] = m = 0$. In this case, we see that $\frac{\theta_L+\theta_R}{2}$ is strictly positive if $|\theta_R| > |\theta_L|$, which implies the Right is more extreme than the Left. Then the Left will be the majority party, and it will be more important to win the election for the Left than for the Right. This result is very intuitive, considering citizens are utilitarian. The party with minority support is socially inferior to the majority party. In this case, the citizens weigh policy preferences and utilitarian considerations in the equilibrium.

When $F(\cdot)$ is the uniform distribution over $[-1, 1]$, then we can explicitly solve for ϕ_L and ϕ_R. In this case, $\phi_L(\theta_L, \theta_R) = 2s_L^2(\theta_R - \theta_L)$ and $\phi_R(\theta_L, \theta_R) = 2s_R^2(\theta_R - \theta_L)$. Notice that $\phi_R(\theta_L, \theta_R) = \phi_L(-\theta_R, \theta_L)$. Figure 1 depicts the values of $\phi_L(\theta_L, \theta_R)$ and $\phi_R(\theta_L, \theta_R)$ for different combinations of values of θ_L and θ_R. The top graph plots $\phi_L(\theta_L, \theta_R)$ where $\theta_R \in \{0, 0.25, 0.5, 0.75, 1\}$ and $\theta_L \in [-1, \theta_R]$. The bottom graph plots $\phi_r(\theta_L, \theta_R)$ where $\theta_L \in \{-1, -0.75, -0.5, -0.25, 0\}$ and $\theta_R \in [-1, \theta_L]$. It is clear from Figure 1 that $\phi_L(\theta_L, \theta_R)$ is increasing function of θ_R, and $\phi_R(\theta_L, \theta_R)$ is an increasing function of θ_L. Also, $\phi_L(\theta_L, \theta_R)$ and $\phi_R(\theta_L, \theta_R)$ are positive and only equal to zero when $\theta_L = \theta_R$.

Next, the derivation of the closed form solutions for $(c_L^*(\theta_L, \theta_R), c_R^*(\theta_L, \theta_R))$ are characterized under the assumption that $F(\cdot)$ is the uniform distribution over $[-1, 1]$ and can be found in Appendix A.3. Table 1 characterizes the values in closed form solution for different policy pairs under the assumption that $\theta_L \leq \theta_R$.

Under a general $F(\cdot)$ assumption, if the Left is the majority ($\frac{\theta_L+\theta_R}{2} \geq E[F] = m$), we cannot compare c_L^* and c_R^* unless $\phi_L/s_L \geq \phi_R/s_R$. Under the uniform distribution, when Left is the majority, the inequality $\phi_L/s_L \geq \phi_R/s_R$ always holds; hence we can say that $c_L^* \leq c_R^*$. The next proposition summarizes these results for general continuous population distribution.

Proposition 2. *If* $F(\cdot)$ *is continuous,* $\frac{\theta_L+\theta_R}{2} \geq E[F] = m$, *and* $\phi_L/s_L \geq \phi_R/s_R$[3], *then the Left is the majority* ($s_L \geq s_R$)*; whence, the expected fraction of the Left supporters who vote* ($\frac{c_L^*}{2\bar{c}}$) *is greater than or equal to the expected fraction of the Right supporters who vote* ($\frac{c_R^*}{2\bar{c}}$).

[3]Although equilibrium cost thresholds $(c_L^*(\theta_L, \theta_R), c_R^*(\theta_L, \theta_R))$ are determined when $F(\cdot)$ is the uniform distribution over $[-1, 1]$, the comparative statics results can be performed under more general distributions. For the proof, refer to Appendix.

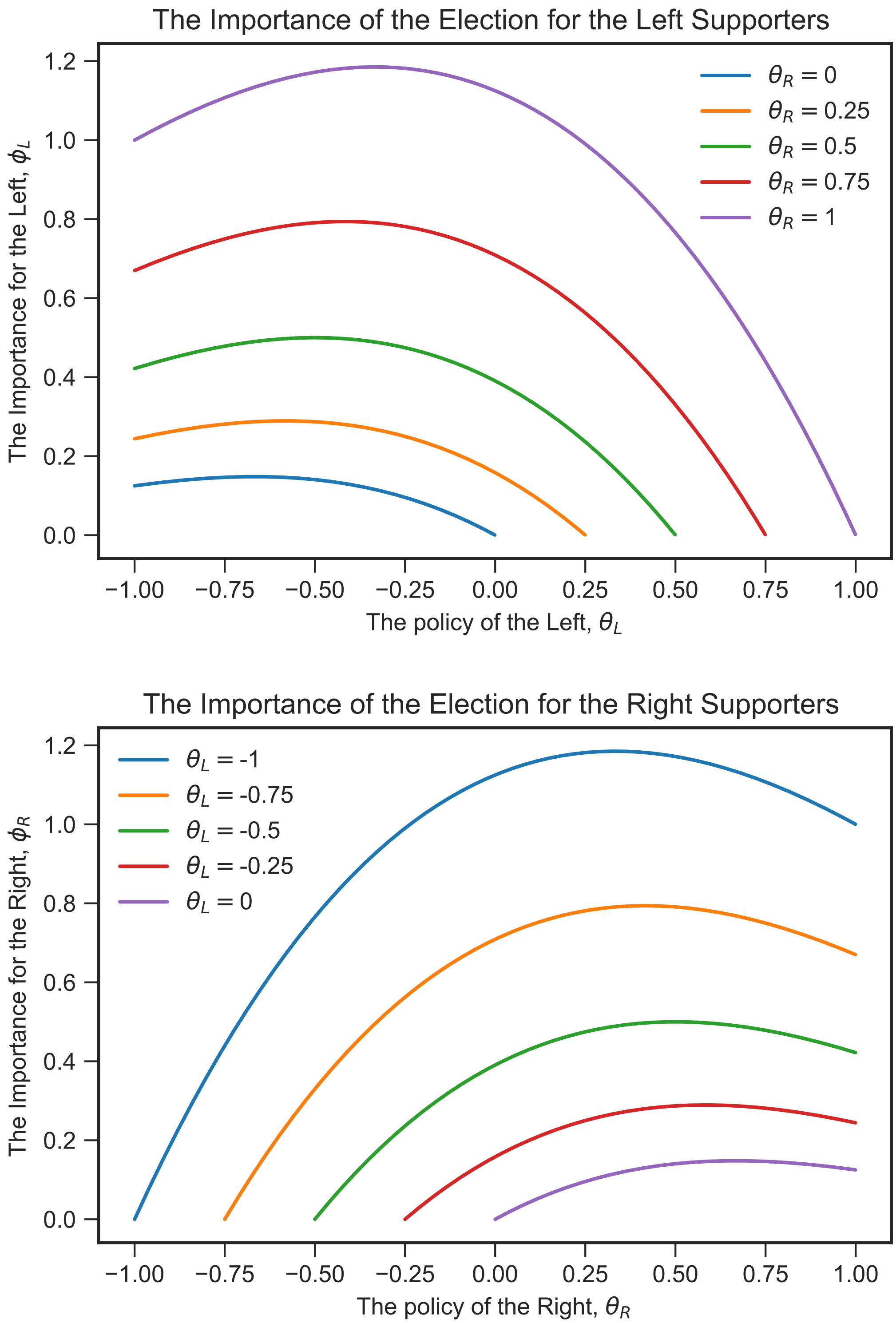

Figure 1: The Importance of Election for the Left and Right Supporters Under Uniform Distribution and $\bar{c} = 1$

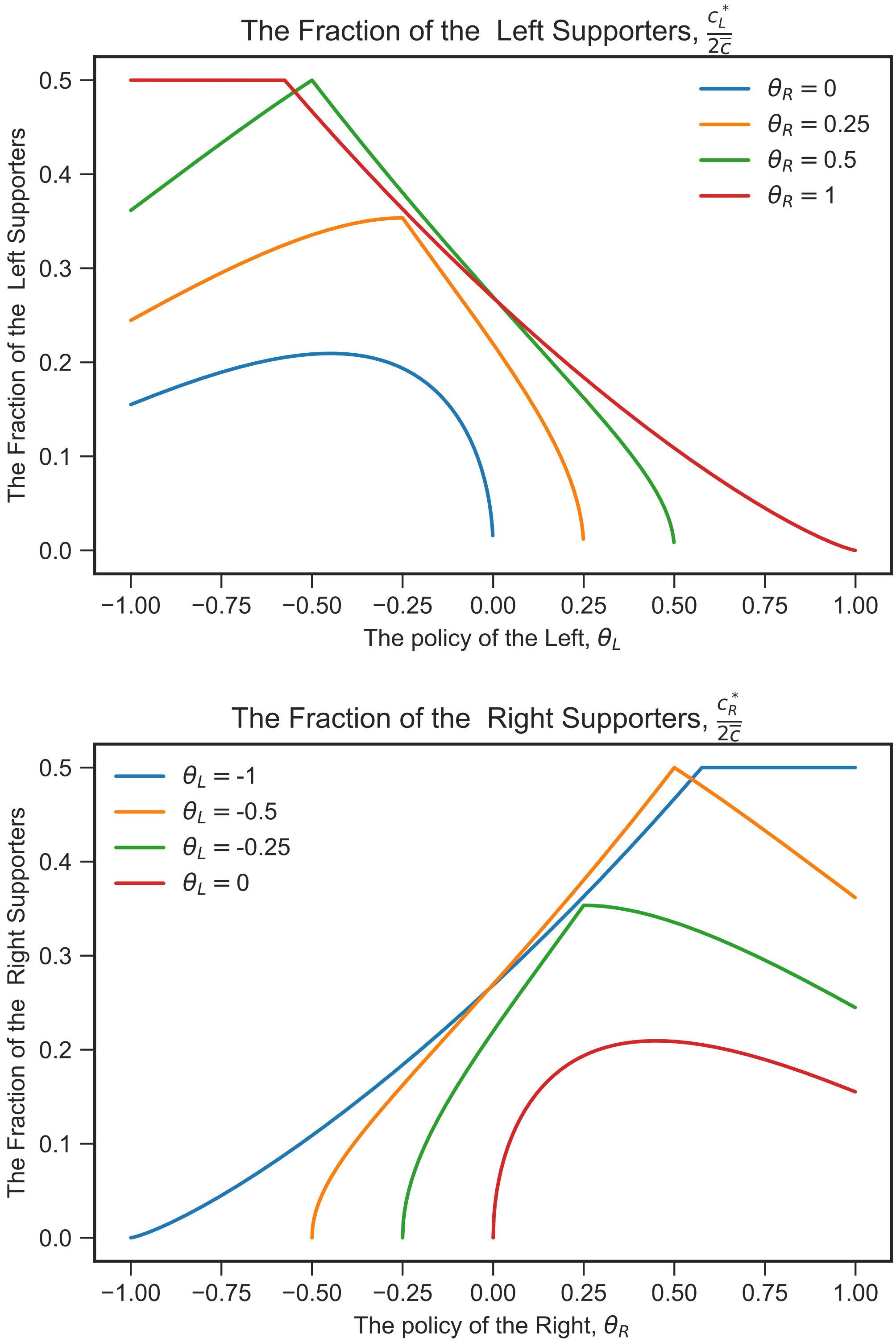

Figure 2: The Expected Fraction of Votes for the Left and Right Under Uniform Population Distribution and $\bar{c} = 1$

Table 1: $(c_L^*(\theta_L,\theta_R), c_R^*(\theta_L,\theta_R))$ Profiles in Closed Form

Case 1:	$\boldsymbol{\theta_L = \theta_R}$
$c_L^* = c_R^* = 0$	$\forall\ \overline{c},$
Case 2:	$\boldsymbol{\theta_L \neq \theta_R}$ and $\boldsymbol{\frac{\theta_L+\theta_R}{2} = E[F]}$
$c_L^* = c_R^* = 0$	if $\phi_L = 0,$
$c_L^* = c_R^* = 2\sqrt{\overline{c}}\phi_L$	if $4\phi_L^2 < \overline{c},$
$c_L^* = c_R^* = \overline{c}$	if $4\phi_L^2 \geq \overline{c}.$
Case 3:	$\boldsymbol{\theta_L \neq \theta_R}$ and $\boldsymbol{\frac{\theta_L+\theta_R}{2} < E[F]}$
$c_L^* = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L^3}{\phi_R s_R s_L}}, c_R^* = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L\phi_R s_L}{(s_R)^3}},$	if $\overline{c} > \sqrt{\frac{\phi_L\phi_R s_L}{(s_R)^3}},$
$c_L^* = \frac{\phi_L}{s_R}, c_R^* = \overline{c},$	if $\frac{\phi_L}{s_R} < \overline{c} \leq \sqrt{\frac{\phi_L\phi_R s_L}{(s_R)^3}},$
$c_L^* = \overline{c}, c_R^* = \overline{c},$	if $\frac{\phi_L}{s_R} \geq \overline{c}.$
Case 4:	$\boldsymbol{\theta_L \neq \theta_R}$ and $\boldsymbol{\frac{\theta_L+\theta_R}{2} > E[F]}$
$c_L^* = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_R\phi_L s_R}{s_L^3}}, c_R^* = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_R^3}{\phi_L s_R s_L}},$	if $\overline{c} > \sqrt{\frac{\phi_L\phi_R s_R}{s_L^3}},$
$c_L^* = \overline{c}, c_R^* = \frac{\phi_R}{s_L},$	if $\frac{\phi_R}{s_L} < \overline{c} \leq \sqrt{\frac{\phi_L\phi_R s_R}{s_L^3}},$
$c_L^* = \overline{c}, c_R^* = \overline{c},$	if $\frac{\phi_R}{s_L} \geq \overline{c}.$

Notes: This table only shows the solutions when $\theta_L \leq \theta_R$ and $F(\cdot)$ is the uniform distribution over $[-1, 1]$.

Proposition 2 resembles Property 1 in Feddersen and Sandroni (2006). In both setups, the majority party is expected to win the election, and the minority is expected to lose. There are a few distinguishable differences between Proposition 2 and Property 1 in Feddersen and Sandroni (2006) that are worth mentioning. Unlike Feddersen and Sandroni (2006), the majority party supporters vote with higher intensity than the minority. Foremost, there is no polarization in Feddersen and Sandroni (2006). The size of the electorate for the Left and the Right is predetermined. If two different policy positions yield to the same size of the electorate for the Left and the Right, then the result will be identical in Feddersen and Sandroni (2006) setup, which is a key distinction to the current one. Next, citizens do not have a preference for alternative policies, which determines both the electorate's share and the voting costs.

The Proposition below conducts comparative statics results for the cost of voting and the importance of the election on the expected voter turnout.

Proposition 3. *Assume conditions of Proposition 2 hold. Both the expected fraction of votes for the Left(Right) and the turnout rates for the Left(Right) increase as the importance of election ϕ_L (ϕ_R) increases. If the overall cost of voting $\bar{c}$ increases, then the turnout rates decrease.*[4]

Proposition 3 conducts the comparative statics results of the turnout and the fraction of the votes from the changes of the model parameters. The results are consistent with the literature. If the election becomes more important, then the benefit of voting increases; hence the turnout increases. On the other hand, if the voting is costlier, the turnout is lower as well.

Proposition 4. *Assume population distribution is uniform $[-1,1]$. Expected turnout for both parties is strictly positive and goes to zero if and only if $\theta_L^* = \theta_R^*$. If $\frac{\theta_L+\theta_R}{2} \geq 0$, then the Left is the majority ($s_L \geq s_R$); Then total expected turnout for the Left ($\frac{1}{2} s_L \frac{c_L^*}{\bar{c}}$) is greater than the total expected turnout for the Right ($\frac{1}{2} s_R \frac{c_R^*}{\bar{c}}$). The expected turnout for the Left is an increasing function of the polarization ($\theta_R - \theta_L$) and the policy of the Right, θ_R.*

The proof of this Proposition follows directly from the Propositions 1, 2, and Proposition 3. The resulting conclusion is a contrast to the findings in Feddersen and Sandroni (2006) since, in their model, the turnout can only be zero if the share of the supporters for the minority goes to zero. Under our assumptions, even if the $\theta_L^* = \theta_R^* = 0$ and the shares for both the Left and the Right are 0.5, the turnout for both parties converges to zero. If $\theta_L = -0.5$, and $\theta_R = 0.5$, and suppose the population is uniformly distributed, then the shares will be equal again; however, the turnout levels will be nonzero. This difference is noteworthy because Feddersen and Sandroni (2006) calls the share of the minority the disagreement level. Here, the disagreement is between policy differences. If both parties propose identical policies, there is no turnout since the social cost will be minimized by everyone abstaining from voting. Under such circumstances, polarization is a better predictor of turnout than the share of the supporters. Proposition 4 also summarizes the relationship between policy polarization and voter turnout. The more differentiated the policies, the higher the total turnout.

Figure 3 depicts the turnout levels for the Left and the Right. The top graph plots the turnout levels for the Left when $\theta_L \in [-1, \theta_R]$ and $\theta_R \in \{0, 0.25, 0.5, 0.75, 1\}$. The bottom graph shows the turnout for the Right when $\theta_R \in [\theta_L, 1]$ and $\theta_L \in \{0, -0.25, -0.5, -0.75, -1\}$.

[4] The proofs of this Proposition are straightforward and left for the reader.

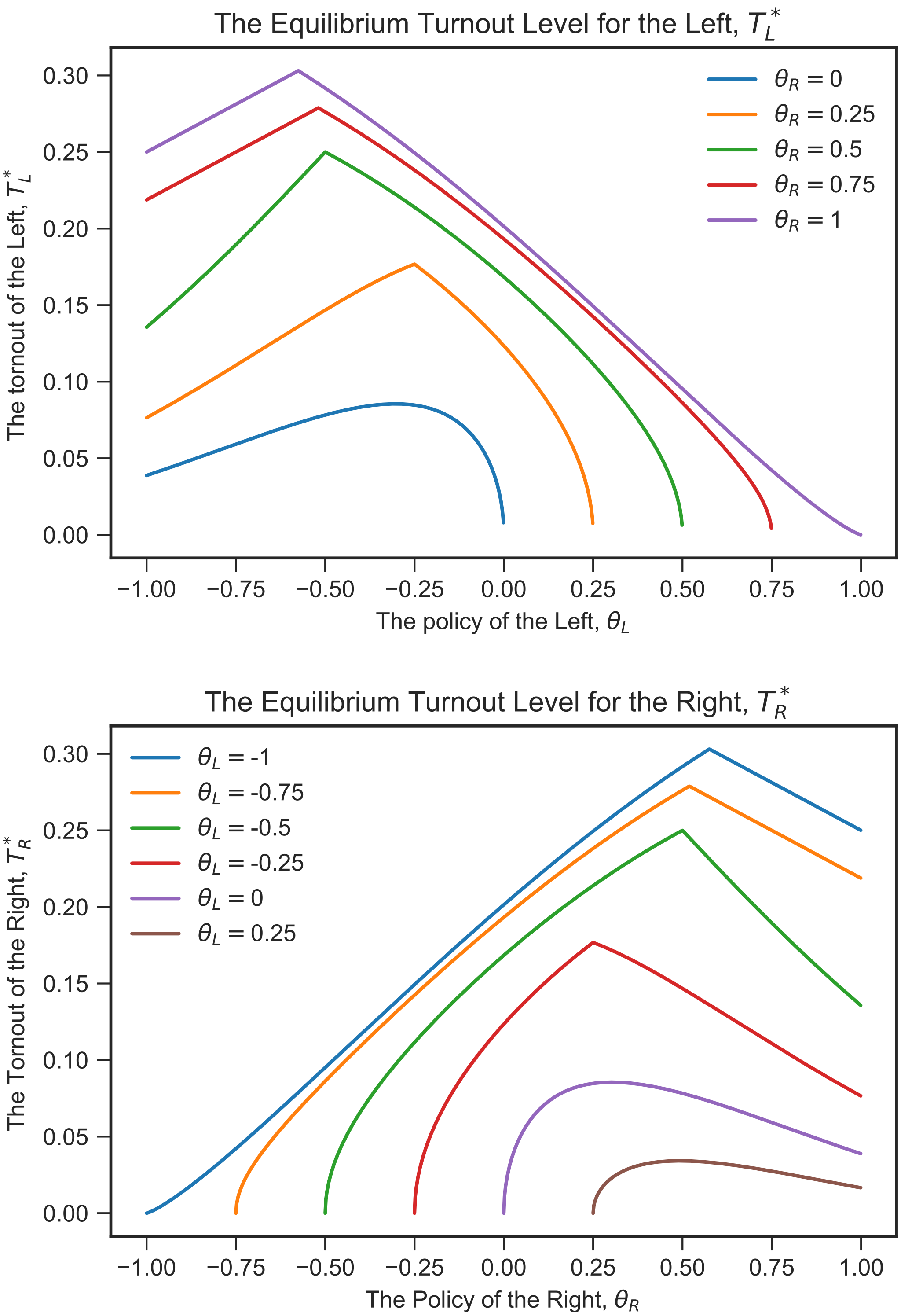

Figure 3: The Equilibrium Turnout Level for the Left and Right Under Uniform Population Distribution and $\bar{c} = 1$

The conclusions of Proposition 4 can be seen clearly depicted in those figures. The expected turnout is positive for both parties and goes to zero if and only if $\theta_L = \theta_R$. The higher θ_R, the higher the level of the turnout for the Left, and it is an increasing function of the polarization $(\theta_R - \theta_L)$.

Proposition 5. *Assume population distribution is uniform* $[-1, 1]$*. The probability of the victory for the Left,* $PV_L = p(c_L^*, c_R^*) = G\left(\frac{s_L c_L^*}{s_R c_R^*}\right)$*, is an increasing function of the policy of the Left,* θ_L *on the interval* $[-1, \theta_R)$*, and the policy of the Right,* θ_R *on the interval* $(\theta_L, 1]$*. The probability of the victory for the Right,* $PV_R = 1 - p(c_L^*, c_R^*) = 1 - G\left(\frac{s_L c_L^*}{s_R c_R^*}\right)$*, is a decreasing function of the policy of the Right,* θ_R*, on the interval* $(\theta_L, 1]$ *and the policy of the Left,* θ_L *on the interval* $[-1, \theta_R)$*. When* $\theta_L = \theta_R$*,* $PV_L = p(c_L^*, c_R^*) = PV_R = \frac{1}{2}$*.*

Figure 4 demonstrates the validity of the Proposition 5 for the Left and Right supporters. As you can see, under the uniform distribution assumption, the winning probability of the Left depends on the policy positions of both the Left and the Right. Both plots show the discontinuity at $\theta_L = \theta_R$ point, where both $PV_L = p(c_L^*, c_R^*)$ and $PV_R = 1 - p(c_L^*, c_R^*)$ drop to 0.5 value. As long as the policies are close but distinguishable, the majority party will have a higher chance of victory. But when policies converge, the turnout goes to zero; therefore, the probability of the victory becomes one-half. This allows us to discuss the implications of this result for the office-motivated candidates whose target is to maximize the chance of victory. Notice that so far, the policy positions were fixed, and parties could not credibly change their policies. Proposition 5 paves the foundation for the discussion of the next section regarding the policy choices of the office-motivated candidates.

4 Office Motivated Parties

Assume both parties, the Left and the Right, must choose a policy from $[-1, 1]$ such that $\theta_L \leq \theta_R$. Both parties understand that only a fraction of their supporters is "ethical." Intuitively, they take into account the voting behavior of the citizens resulting in $(c_L^*(\theta_L, \theta_R), c_R^*(\theta_L, \theta_R))$ and corresponding expected turnout levels.[5]

To summarize, the election game can be considered as a two-stage dynamic game:

1. Parties propose the pair of policies (θ_L, θ_R).

2. Citizens determine the cost thresholds (c_L, c_R) for the Left and the Right supporters, and ethical citizens follow it.

[5]Feddersen and Sandroni (2006) have an identical assumption. However, they assumed that the so-called "level of disagreement" between parties is what we call in this model share of Right and Left.

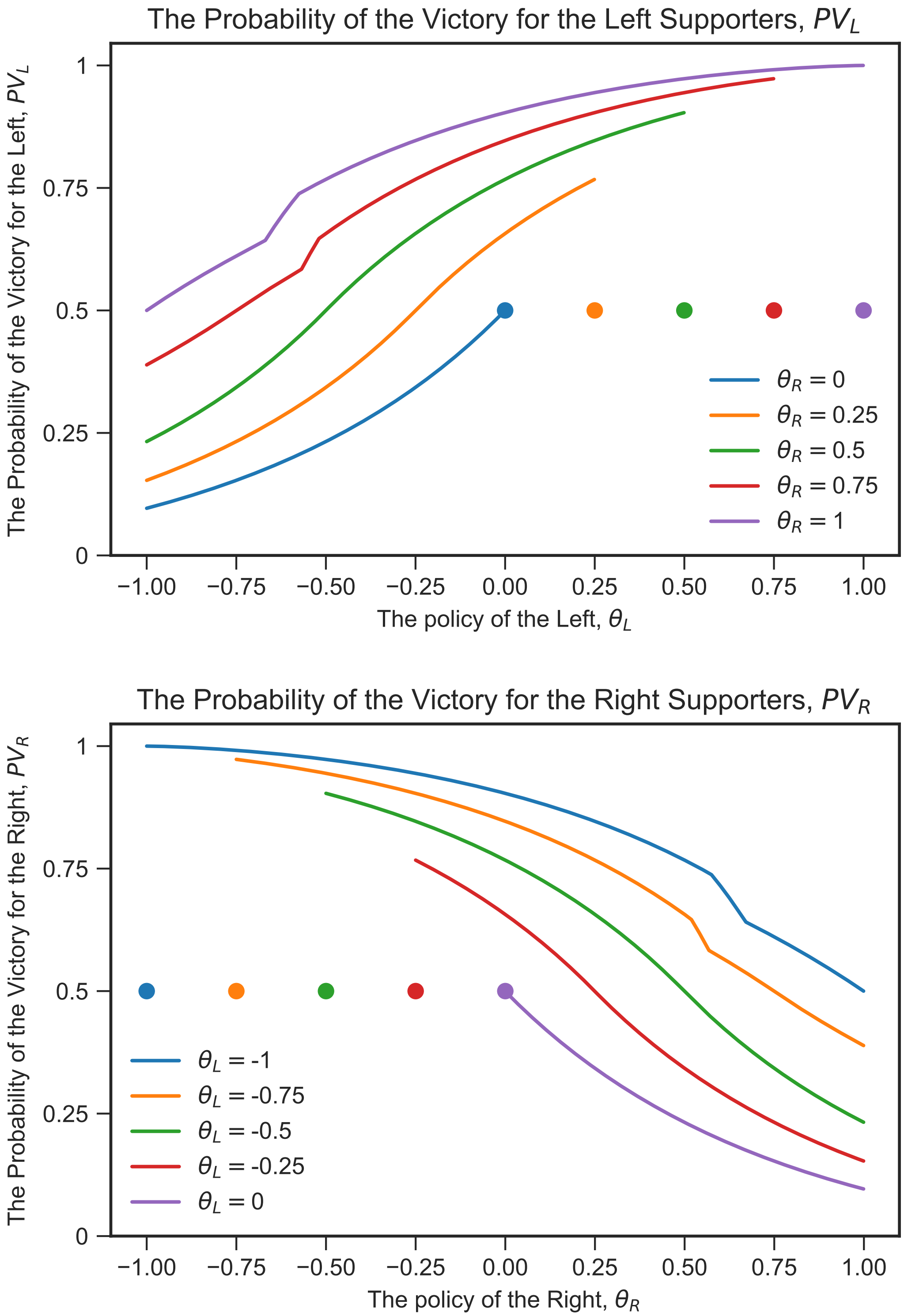

Figure 4: The Probability of the Victory for the Left and the Right Under Uniform Distribution and $\bar{c} = 1$

3. The party with the majority votes wins the election.

Next, we look for a backward induction type of solution in equilibrium. Citizens choose the cost thresholds similar to Harsanyi (1980), Feddersen and Sandroni (2006), and Coate and Conlin (2004). Ethical citizens vote by blocks, and their primary objective is to maximize the aggregate payoff of their group. However, the parties' objectives must be considered as well. For every policy pair, the Left and the Right supporters determine their cost thresholds. Parties taking into account cost thresholds, choose policies to maximize their objectives. We will the settings when parties are office-motivated.

Definition 2. ***Equilibrium***

Citizens: *For every policy pair* (θ_L, θ_R), *the Left supporters choose* $c_L^*(\theta_L, \theta_R)$ *to maximize* (11) *subject to* $c_L \in [0, \overline{c}]$ *constraint. Likewise, the Right supporters choose* $c_R^*(\theta_L, \theta_R)$ *to maximize* (12) *subject to* $c_R \in [0, \overline{c}]$ *constraint.*

Office Motivated Parties: *Considering the cost thresholds* $(c_L^*(\theta_L, \theta_R), c_R^*(\theta_L, \theta_R))$, *both parties choose policies* (θ_L^*, θ_R^*) *to maximize their probability of victory.*

In this case, the solution to citizens' equilibrium cost threshold levels does not change; hence we can move straight to the parties' policy choices. The following proposition summarizes the results of this case.

Proposition 6. *Assume population distribution is uniform* $[-1, 1]$. *Office-motivated parties choose median policy position* $\theta_L^* = 0$, $\theta_R^* = 0$, *and the expected turnout levels are zero.*

Proposition 6 is a direct follow-up of the Proposition 5. Figure 4 demonstrates that both parties will have higher chances of victory the closer they choose their policy to their opponents. Since office-motivated parties' objectives are to maximize their winning probabilities, both will choose to move closer to the center. This tendency results in both parties choosing the median policy position. Unfortunately, this result implies that the expected turnout rates for both parties will be zero. This contradicts the claims made by Feddersen and Sandroni (2006), claiming that the proposed utilitarian model answers the voting paradox. When the voting costs are nonzero, the office-motivated candidates choose median positions resulting in zero turnout levels; hence, the utilitarian models proposed by Feddersen and Sandroni (2006) did not solve the paradox of voting.

5 Conclusion

As demonstrated from the modeling of this paper, which expanded from the work of Feddersen and Sandroni (2002) and Coate and Conlin (2004), citizens have greater turnouts as polarization increases along the lines of spatial policy competition. Ethical or civil considerations influence the citizens voting decisions and allow them to have potentially positive turnout even with positive voting costs. Parties aim to win the election, so they are bound to choose policies aligned with their positions to maximize the potential winning probabilities or objective functions. If the parties' objectives are purely winning the electron, then the model predicts median policies and no positive turnout.

Future researchers would be urged to follow up on a three-party system with office-motivated and policy-motivated parties. The presence of more than two parties could reveal some interesting scenarios and lead to the existence of "Duverger's Law," where only two major parties receive votes. Would ethical citizens be more inclined to vote for the central option if a third party was too extreme? Another potential avenue to explore outside the scope of this paper would be to review the factors to increase a citizen's civic duty to create a bigger pool of ethical voters.

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A Appendix

A.1 Calculating The Share of The Left Supporters

Every citizen for whom $u_i(\theta_L)$ is greater than $u_i(\theta_R)$ supports the Left party, which means that if:

$$u_i(\theta_L) = -(i-\theta_L)^2 > u_i(\theta_R) = -(i-\theta_R)^2, \tag{A.1.1}$$

then citizen i is Left-leaning. To simplify further, let's do some algebra:

$$-(i-\theta_L)^2 + (i-\theta_R)^2 > 0. \tag{A.1.2}$$

For citizen i if

$$i < \frac{\theta_L+\theta_R}{2} \tag{A.1.3}$$

then citizen i is Left supporter; otherwise, they support the Right.

To calculate the share of Left supporting citizens, notice that every citizen in the interval $[-1, \frac{\theta_L+\theta_R}{2})$ is Left supporter, and every citizen in the interval $(\frac{\theta_L+\theta_R}{2}, 1]$ is Right supporter. Assume $F(\cdot)$ is continuous, therefore

$$s_L = s_L(\theta_L,\theta_R) = \int_{-1}^{\frac{\theta_L+\theta_R}{2}} 1dF(i) = F(\frac{\theta_L+\theta_R}{2}) - F(-1) = F(\frac{\theta_L+\theta_R}{2}). \tag{A.1.4}$$

$$s_R = s_R(\theta_L,\theta_R) = \int_{\frac{\theta_L+\theta_R}{2}}^{1} 1dF(i) = F(1) - F(\frac{\theta_L+\theta_R}{2}) = 1 - F(\frac{\theta_L+\theta_R}{2}). \tag{A.1.5}$$

In case $F(\cdot)$ is uniform, we have

$$s_L = s_L(\theta_L,\theta_R) = \frac{\frac{\theta_L+\theta_R}{2}+1}{2} = \frac{\theta_L+\theta_R+2}{4}, \tag{A.1.6}$$

and

$$s_R = s_R(\theta_L,\theta_R) = \frac{1-\frac{\theta_L+\theta_R}{2}}{2} = \frac{2-\theta_L-\theta_R}{4} = 1 - s_L. \tag{A.1.7}$$

A.2 Proof of Propositions 1

Case 1: Continuous $F(\cdot)$

From the definitions of ϕ_L and ϕ_R:

$$\phi_L(\theta_L,\theta_R) = \int_{\Theta_L} (u_i(\theta_L) - u_i(\theta_R))\, dF(i) = (\theta_R^2-\theta_L^2)s_L - 2(\theta_R-\theta_L)\int_{\Theta_L} idF(i). \tag{A.2.1}$$

$$\phi_R(\theta_L,\theta_R) = \int_{\Theta_R} (u_i(\theta_R) - u_i(\theta_L))\, dF(i) = (\theta_L^2 - \theta_R^2)s_R + 2(\theta_R - \theta_L) \int_{\Theta_R} idF(i) \quad \text{(A.2.2)}$$

From here,

$$\phi_L - \phi_R = (\theta_R^2 - \theta_L^2)(s_L + s_R) - 2(\theta_R - \theta_L) \left(\int_{\Theta_L} idF(i) + \int_{\Theta_R} idF(i) \right). \quad \text{(A.2.3)}$$

Since $F(\cdot)$ is continuous, $\int_{\Theta_L} idF(i) + \int_{\Theta_R} idF(i) = E[F]$.

$$\phi_L - \phi_R = 2(\theta_R - \theta_L) \left[\frac{\theta_L + \theta_R}{2} - E[F] \right]. \quad \text{(A.2.4)}$$

Hence, if $\frac{\theta_L+\theta_R}{2} \geq E[F]$, then $\phi_L \geq \phi_R$.

Finally, if $F(\cdot)$ is continuously differentiable, then using Leibniz integral rule, we see that the derivative of $\phi_L(\theta_L, \theta_R)$ with respect to θ_R is greater than equal to zero, hence $\phi_L(\theta_L, \theta_R)$ is an increasing function of θ_R. The proof for the $\phi_R(\theta_L, \theta_R)$ is symmetrical, since $\phi_R(\theta_L, \theta_R) = \phi_L(-\theta_R, \theta_L)$.

Case of Uniform $F(\cdot)$:

Let's calculate the values of ϕ_L and ϕ_R in terms of policies θ_L and θ_R.

$$\begin{aligned} \phi_L =& U_L(\theta_L) - U_L(\theta_R) = \int_{-1}^{\frac{\theta_L+\theta_R}{2}} -(i-\theta_L)^2 di - \left(\int_{-1}^{\frac{\theta_L+\theta_R}{2}} -(i-\theta_R)^2 di \right) \\ =& (\theta_R^2 - \theta_L^2)s_L + 0.5(\theta_R - \theta_L)(1 - \frac{(\theta_R+\theta_L)^2}{4}) = 2s_L^2(\theta_R - \theta_L). \end{aligned} \quad \text{(A.2.5)}$$

Next,

$$\begin{aligned} \phi_R =& U_R(\theta_R) - U_R(\theta_L) = \int_{\frac{\theta_L+\theta_R}{2}}^{1} -(i-\theta_R)^2 di - \left(\int_{\frac{\theta_L+\theta_R}{2}}^{1} -(i-\theta_L)^2 di \right) \\ =& (\theta_L^2 - \theta_R^2)s_R + 0.5(\theta_R - \theta_L)(1 - \frac{(\theta_R+\theta_L)^2}{4}) = 2s_R^2(\theta_R - \theta_L). \end{aligned} \quad \text{(A.2.6)}$$

These derivations complete the proof of the Proposition 1.

A.3 Derivation of Table 1

Case 1: Assume Right is the majority, i.e., $\frac{\theta_L+\theta_R}{2} < E[F] = m$. Let's remember that in this case, $\phi_L < \phi_R$ and $g(x) = \begin{cases} \frac{1}{2}, & \text{if } x \leq 1; \\ \frac{1}{2x^2}, & \text{if } x \geq 1. \end{cases}$

The FOCs for the Left and The Right are:

$$\phi_L g\left(\frac{s_L}{s_R}\frac{c_L}{c_R}\right)\frac{s_L}{s_R}\frac{1}{c_R} - s_L\frac{c_L}{2\overline{c}}\begin{cases} = 0, & \text{if } c_L \in (0,\overline{c}),\\ \geq 0, & \text{if } c_L \in \{0,\overline{c}\}.\end{cases} \tag{A.3.1}$$

$$\phi_R g\left(\frac{s_L}{s_R}\frac{c_L}{c_R}\right)\frac{s_L}{s_R}\frac{c_L}{c_R^2} - s_R\frac{c_R}{2\overline{c}}\begin{cases} = 0, & \text{if } c_R \in (0,\overline{c}),\\ \geq 0, & \text{if } c_R \in \{0,\overline{c}\}.\end{cases} \tag{A.3.2}$$

Let's find an interior solution first. Divide Equation (A.3.1) to the Equation (A.3.2) in case they both are zero, and we get the following:

$$\frac{\phi_L}{\phi_R}\frac{\frac{1}{c_R}}{\frac{c_L}{c_R^2}} = \frac{s_L}{s_R}\frac{c_L}{c_R} \Rightarrow \frac{c_L}{c_R} = \sqrt{\frac{\phi_L}{\phi_R}\frac{s_R}{s_L}}. \tag{A.3.3}$$

In this case

$$\frac{s_L}{s_R}\frac{c_L}{c_R} = \frac{s_L}{s_R}\sqrt{\frac{\phi_L}{\phi_R}\frac{s_R}{s_L}} = \sqrt{\frac{\phi_L}{\phi_R}\frac{s_L}{s_R}} \leq 1 \Rightarrow g\left(\frac{s_L}{s_R}\frac{c_L}{c_R}\right) = \frac{1}{2}. \tag{A.3.4}$$

Therefore,

$$c_L = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L^3}{\phi_R s_R s_L}},\ c_R = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L\phi_R s_L}{(s_R)^3}}. \tag{A.3.5}$$

Notice that in order for this to be an interior solution, we must have $c_L, c_R \in [0,\overline{c}]$. Therefore, the solution can be characterized accordingly:

$$\begin{cases} c_L = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L^3}{\phi_R s_R s_L}}, c_R = \sqrt{\overline{c}}\sqrt[4]{\frac{\phi_L\phi_R s_L}{(s_R)^3}}, & \text{if } \overline{c} > \sqrt{\frac{\phi_L\phi_R s_L}{(s_R)^3}},\\ c_L = \frac{\phi_L}{s_R}, c_R = \overline{c}, & \text{if } \frac{\phi_L}{s_R} < \overline{c} \leq \sqrt{\frac{\phi_L\phi_R s_L}{(s_R)^3}},\\ c_L = \overline{c}, c_R = \overline{c}, & \text{if } \frac{\phi_L}{s_R} \geq \overline{c}.\end{cases} \tag{A.3.6}$$

Notice that it is always the case that $\frac{\phi_L}{s_R} \leq \sqrt{\frac{\phi_L\phi_R s_L}{(s_R)^3}}$ since $\sqrt{\frac{\phi_L}{s_L}} \leq \sqrt{\frac{\phi_R}{s_R}}$.

Case 2: Assume Left is the majority, i.e., $\frac{\theta_L+\theta_R}{2} > E[F] = m$ and in this case, $\phi_L > \phi_R$. From Equation (A.3.3), it follows that

$$\frac{s_L}{1-s_L}\frac{c_L}{c_R} = \sqrt{\frac{\phi_L}{\phi_R}\frac{s_L}{1-s_L}} \geq 1 \Rightarrow f\left(\frac{s_L}{1-s_L}\frac{c_L}{c_R}\right) = \frac{1}{2}\frac{c_R^2}{c_L^2}\frac{(1-s_L)^2}{s_L^2}. \tag{A.3.7}$$

Plugging Equation (A.3.7) into Equation (A.3.1), we solve

$$\begin{cases} c_L = \sqrt{\bar{c}}\sqrt[4]{\frac{\phi_R \phi_L s_R}{s_L^3}}, c_R = \sqrt{\bar{c}}\sqrt[4]{\frac{\phi_R^3}{\phi_L s_R s_L}}, & \text{if } \bar{c} > \sqrt{\frac{\phi_L \phi_R s_R}{s_L^3}}, \\ c_L = \bar{c}, c_R = \frac{\phi_R}{s_L}, & \text{if } \frac{\phi_R}{s_L} < \bar{c} \leq \sqrt{\frac{\phi_L \phi_R s_R}{s_L^3}}, \\ c_L = \bar{c}, c_R = \bar{c}, & \text{if } \frac{\phi_R}{s_L} \geq \bar{c}. \end{cases} \tag{A.3.8}$$

Case 3: Assume Left and Right are tied, i.e., $\frac{\theta_L+\theta_R}{2} = E[F] = m$.
Notice that in this case, $\phi_L = \phi_R$, $s_L = s_R = \frac{1}{2}$ and hence $c_L = c_R$.
Plugging Equation (A.3.7) into Equation (A.3.1), we solve

$$\begin{cases} c_L = c_R = 0, & \text{if } \phi_L = 0, \\ c_L = c_R = 2\sqrt{\bar{c}}\phi_L, & \text{if } 4\phi_L^2 < \bar{c}, \\ c_L = c_R = \bar{c}, & \text{if } 4\phi_L^2 \geq \bar{c}. \end{cases} \tag{A.3.9}$$

A.4 Proof of Proposition 2

Proposition 2 states that if $\frac{\theta_L+\theta_R}{2} \geq E[F] = m$, then the Left is the majority ($s_L \geq s_R$); whence, the expected fraction of the Left supporters who vote ($\frac{c_L^*}{2\bar{c}}$) is greater than or equal to the expected fraction of the Right supporters who vote ($\frac{c_R^*}{2\bar{c}}$). Notice that in the interior equilibria, we have

$$\frac{c_L^*}{c_R^*} = \sqrt{\frac{\phi_L}{\phi_R}\frac{s_R}{s_L}}. \tag{A.4.10}$$

Also, if the Left is the majority, then $\sqrt{\frac{\phi_L}{s_L}} \geq \sqrt{\frac{\phi_R}{s_R}}$. Therefore,

$$\frac{c_L^*}{c_R^*} = \sqrt{\frac{\phi_L}{s_L} \times \frac{s_R}{\phi_R}} \geq 1. \tag{A.4.11}$$

This concludes the proof of Proposition 2.