MDS
Probability theory
seminar 1

Maria Gerasimova

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Problem 1. A fair coin is flipped 3 times. Consider two events:
A: the first toss yields heads.
B: there are exactly 2 tails.
Are these events independent?

Problem 2. A fair dice is tossed one time. Let A be the event "number of points is even". Are A and B independent if:

1. $B =\{$number of points $> 2\}$

2. $B =\{$number of points $> 3\}$

3. $B =\{$number of points $> 4\}$

Problem 3. Two fair dice are tossed simultaneously.
$A =\{$the first dice yields 3$\}$
$B =\{$the sum of points is even$\}$
$C =\{$the number of points on the second dice $>$
$>$ the number of points on the first dice$\}$

1. Find $P(A), P(B), P(C)$.

2. Find $P(A|C)$.

3. Are there any pairs of independent events among A, C and $B \cap C$?

Problem 4. Company A goes bankrupt with probability 0.4. Company B independently goes bankrupt with probability 0.2. Find the probability of the following events:

1. Only one company goes bankrupt.

2. At least one company goes bankrupt.

3. Both companies go bankrupt.

4. Company A goes bankrupt if it's known that only one company goes bankrupt.

5. Company A goes bankrupt if it's known that at least one company goes bankrupt.

Problem 5. A businessman plans to open a new shop. If there are competitor shops nearby, then this shop will be profitable with probability 0.3. If there are no such shops nearby, then it will be profitable with probability 0.4. Probability of finding a building without competitor shops nearby is 0.8. Find probability that this shop will be profitable.

Problem 6. If on the day before the exam Alex goes only to the cinema, he passes the exam with probability 0.6. If on the day before the exam Alex goes only to the bar, he passes the exam with probability 0.5. If on the day before the exam he goes both to the cinema and to the bar, then he passes the exam with probability 0.1. If on the day before the exam he stays at home, he passes the exam with probability 0.9. Alex decides to go to the cinema and to the bar independently with probabilities 0.2 and 0.6 respectively.

1. Find the probability that Alex passes the exam.

2. Find the probability that Alex was at the bar and at the cinema yesterday if it's known that he passed the exam.

3. Find the probability that Alex was at the bar yesterday if it's known that he passed the exam.

4. Alex passed the exam. What is more likely - that he was at the bar yesterday or that he was at the cinema yesterday?

Problem 7. We flip a fair coin 10 times. What is the probability that 2 or more heads appear?