MÁRIA MATÚŠKOVÁ

Domáca úloha číslo 05 - goniometrické funkcie, úpravy gon. v., gon. rovnice a nerov.

1) a) $45° \to \frac{45° \cdot \pi}{180°} = \frac{9\pi}{36} = \frac{3\pi}{12} = \frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}\right)$

$\alpha = \frac{\alpha \pi}{180°}$

$\sin(45°) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad tg(45°) = tg\frac{\pi}{4} = 1$

$\cos(45°) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad cotg(45°) = cotg\frac{\pi}{4} = 1$

c) $60° \to \frac{60°\pi}{180°} = \frac{\pi}{3}$

$\sin(60°) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad tg(60°) = tg\frac{\pi}{3} = \sqrt{3}$

$\cos(60°) = \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad cotg(60°) = cotg\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

b) $30° \to \frac{30°\pi}{180°} = \frac{\pi}{6}$

$\sin(30°) = \sin\frac{\pi}{6} = \frac{1}{2}$ $\quad tg(30°) = tg\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\cos(30°) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\quad cotg(30°) = cotg\frac{\pi}{6} = \sqrt{3}$

d) $90° \to \frac{90°\pi}{180°} = \frac{\pi}{2}$

$\sin(90°) = \sin\frac{\pi}{2} = 1$ $\quad tg(90°) = tg\frac{\pi}{2} = \emptyset$

$\cos(90°) = \cos\frac{\pi}{2} = 0$ $\quad cotg(90°) = cotg\frac{\pi}{2} = 0$

e) $120° \to \frac{120°\pi}{180°} = \frac{2}{3}\pi$

$\sin(120°) = \sin\frac{2}{3}\pi = \sin\left(\pi - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$; $\quad tg\left(\pi - \frac{\pi}{3}\right) = -\sqrt{3}$

$\cos(120°) = \cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2}$ $\quad cotg\left(\pi - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

f) $150° \to \frac{150°\pi}{180°} = \frac{5\pi}{6}$

$\sin\frac{5}{6}\pi = \sin\left(\pi - \frac{\pi}{6}\right) = \frac{1}{2}$ $\quad tg\left(\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

$\cos\frac{5}{6}\pi = \cos\left(\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\quad cotg\left(\pi - \frac{\pi}{6}\right) = -\sqrt{3}$

g) $720° \to \frac{720°\pi}{180°} = 4\pi$

$\sin 4\pi = 0$ $\quad tg\, 4\pi = 0$

$\cos 4\pi = 1$ $\quad cotg\, 4\pi = \emptyset$

h) $225° \to \frac{225°\pi}{180°} = \frac{5}{4}\pi$

$\sin\frac{5}{4}\pi = \sin\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad tg\left(\pi + \frac{\pi}{4}\right) = 1$

$\cos\frac{5}{4}\pi = \cos\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\quad cotg\left(\pi + \frac{\pi}{4}\right) = 1$

i) $\frac{\pi}{3} \to \frac{\frac{\pi}{3} \cdot 180°}{\pi} = \frac{\frac{180\pi}{3}}{\pi} = 60°$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad tg\frac{\pi}{3} = \sqrt{3}$

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\quad cotg\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

j) $\frac{2}{3}\pi \to \frac{\frac{2}{3}\pi \cdot 180°}{\pi} = 120°$

$\sin\frac{2}{3}\pi = \sin\left(\pi - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ $\quad tg\left(\pi - \frac{\pi}{3}\right) = -\sqrt{3}$

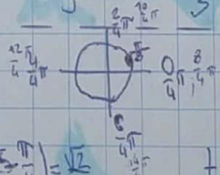
$\cos\frac{2}{3}\pi = -\frac{\sqrt{3}}{2}$ $\quad cotg\left(\pi - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

k) $4\pi = \frac{4\pi \cdot 180°}{\pi} = 720°$

$\sin 4\pi = 0$ $\quad tg\, 4\pi = 0$

$\cos 4\pi = 1$ $\quad cotg\, 4\pi = \emptyset$

l) $\frac{17}{4}\pi = \frac{\frac{17}{4}\pi \cdot 180°}{\pi} = 765°$



$\sin\frac{17}{4}\pi = \sin(765°) = \left(4\pi - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad tg\left(4\pi - \frac{\pi}{4}\right) = 1$

$\cos\left(4\pi - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\quad cotg\left(4\pi - \frac{\pi}{4}\right) = 1$

m) $\frac{24}{3}\pi = \frac{\frac{24}{3}\pi \cdot 180°}{\pi} = 1440°$

$\sin(4 \cdot 360°) = \sin(8\pi) = 0$

$\cos 8\pi = 1$

$tg\, 8\pi = 0$

$cotg\, 8\pi = \emptyset$

n) $\frac{11}{6}\pi = \frac{\frac{11}{6}\pi \cdot 180°}{\pi} = 330°$

$\sin\left(\frac{12}{6}\pi - \frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos\left(2\pi - \frac{\pi}{6}\right) = +\frac{\sqrt{3}}{2}$

$tg\left(2\pi - \frac{\pi}{6}\right) = -\sqrt{3}$

$cotg\left(2\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

o) $\frac{9}{6}\pi = \frac{\frac{9}{6}\pi \cdot 180°}{\pi} = 270°$

$\sin\frac{3}{2}\pi = \sin\left(\pi + \frac{\pi}{2}\right) = -1$

$\cos\left(\pi + \frac{\pi}{2}\right) = 0$

$tg\left(\pi + \frac{\pi}{2}\right) = \emptyset$

$cotg\left(\pi + \frac{\pi}{2}\right) = 0$

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p) $\frac{15}{2}\pi = \frac{\frac{15}{2}\pi \cdot 180°}{\pi} = 1350°$

$\sin\left(36\pi + \frac{3}{2}\pi\right) = \sin\left(7\pi + \frac{\pi}{2}\right) = -1$

$\cos\left(7\pi + \frac{\pi}{2}\right) = 0$

$tg\left(7\pi + \frac{\pi}{2}\right) = \emptyset$

$cotg\left(7\pi + \frac{\pi}{2}\right) = 0$

q) $-\frac{21}{3}\pi = \frac{-\frac{21}{3}\pi \cdot 180°}{\pi} = -1260°$

$\sin(-7\pi) = \sin -(6\pi + \pi) = 0$

$\cos -(6\pi + \pi) = -1$

$tg -(6\pi + \pi) = 0$

$cotg -(6\pi + \pi) = \emptyset$

r) $-\frac{13}{3}\pi = \frac{-\frac{13}{3}\pi \cdot 180°}{\pi} = -780°$

$\sin\left(4\pi - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\cos\left(4\pi - \frac{\pi}{3}\right) = \frac{1}{2}$

$tg\left(4\pi - \frac{\pi}{3}\right) = -\sqrt{3}$

$cotg\left(4\pi - \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$

s) $-\frac{7}{6}\pi = \frac{-\frac{7}{6}\pi \cdot 180°}{\pi} = -210°$

$\sin -\left(\pi + \frac{\pi}{6}\right) = -\frac{1}{2}$

$\cos -\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$tg -\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

$cotg -\left(\pi + \frac{\pi}{6}\right) = -\sqrt{3}$

2) a) $\sin 4\pi = \sin(2 \cdot 2\pi) = 0$ $\qquad \cos 4\pi = \cos(2 \cdot 2\pi) = +1$

b) $\sin 7\pi = \sin(6\pi + \pi) = 0$ $\qquad \cos 7\pi = -1$

c) $\sin(-7\pi) = 0$ $\qquad \cos(-7\pi) = -1$

d) $\sin\left(-\frac{13}{3}\pi\right) = \sin -\left(\frac{12}{3}\pi + \frac{\pi}{3}\right) = \sin -\left(4\pi + \frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ $\qquad \cos -\left(4\pi + \frac{\pi}{3}\right) = +\frac{1}{2}$

$810° = 2 \cdot 360° + 90°$

e) $\sin\left(-\frac{27}{6}\pi\right) = \sin -\left(4\pi + \frac{\pi}{2}\right) = -1$ $\qquad \cos\left(4\pi + \frac{\pi}{2}\right) = 0$

f) $\sin\left(-\frac{11}{4}\pi\right) = \sin(-495°) =$ $\qquad \cos -\left(3\pi - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$= \sin -\left(2\pi + \frac{\pi}{2} + \frac{\pi}{4}\right) =$

$= \sin -\left(2\pi + \pi - \frac{\pi}{4}\right) =$

$= \sin -\left(3\pi - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

g) $\sin\left(\frac{13}{4}\pi\right) = \sin(585°) =$ $\qquad \cos\left(3\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$= \sin\left(2\pi + \pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

h) $\sin\left(\frac{7}{6}\pi\right) = \sin\left(\pi + \frac{\pi}{6}\right) = -\frac{1}{2}$ $\qquad \cos\left(\pi + \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

i) $\sin\left(\frac{13}{4}\pi\right) = \sin\left(3\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\qquad \cos\left(3\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

j) $\sin\left(\frac{7}{6}\pi\right) = -\frac{1}{2}$ $\qquad \cos\left(\frac{7}{6}\pi\right) = -\frac{\sqrt{3}}{2}$

k) $\sin\left(\frac{14}{3}\pi\right) = \sin\left(2\pi + \frac{\pi}{2} + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\qquad \cos\left(\cancel{2\pi + \frac{\pi}{2}} + \frac{\pi}{6}\right) = \cancel{-\frac{\sqrt{3}}{2}}$ $\quad \frac{15}{6}\pi - \frac{\pi}{6}\Big) = -\frac{1}{2}$

l) $\sin\left(\frac{5}{4}\pi\right) = \sin\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ $\qquad \cos\left(\pi + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

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3) a) $150°$

$\sin 150° = \sin(90° + 60°) = \frac{1}{2}$

$\cos(180° - 30°) = -\frac{\sqrt{3}}{2}$

$tg(180° - 30°) = -\frac{\sqrt{3}}{3}$

$cotg(180° - 30°) = -\sqrt{3}$

b) $300°$

$\sin(360° - 60°) = -\frac{\sqrt{3}}{2}$

$\cos(360° - 60°) = +\frac{1}{2}$

$tg(360° - 60°) = 0$

$cotg(360° - 60°) = \emptyset$

c) $-225°$

$\sin -(180 + 45°) = \frac{\sqrt{2}}{2}$

$\cos -(180° + 45°) = -\frac{\sqrt{2}}{2}$

$tg -(180° + 45°) = +1$

$cotg -(180° + 45°) = 1$

d) $-945°$

$\sin -(720° + 180° + 45°) = \frac{\sqrt{2}}{2}$

$\cos -(3\pi + \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$tg -(3\pi + \frac{\pi}{4}) = -1$

$cotg -(3\pi + \frac{\pi}{4}) = -1$

e) $60°$

$\sin(60°) = \frac{\sqrt{3}}{2}$

$\cos(60°) = \frac{1}{2}$

$tg(60°) = \sqrt{3}$

$cotg(60°) = \frac{\sqrt{3}}{3}$

f) $45°$

$\sin(45°) = \frac{\sqrt{2}}{2}$

$\cos(45°) = \frac{\sqrt{2}}{2}$

$tg(45°) = 1$

$cotg(45°) = 1$

m) $600°$

$\sin(360° + 180° + 60°) = -\frac{\sqrt{3}}{2}$

$\cos(360° + 180° + 60°) = -\frac{1}{2}$

$tg(360° + 180° + 60°) = \sqrt{3}$

$tg(360° + 180° + 60°) = \frac{\sqrt{3}}{3}$

g) $270°$

$\sin 270° = -1$

$\cos 270° = 0$

$tg\, 270° = \emptyset$

$cotg\, 270° = 0$

h) $210°$

$\sin 210° = \sin(180° + 30°) = -\frac{1}{2}$

$\cos(180° + 30°) = -\frac{\sqrt{3}}{2}$

$tg(180° + 30°) = \frac{\sqrt{3}}{3}$

$cotg(180° + 30°) = \sqrt{3}$

i) $45°$

$\sin 45° = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$

$tg\, 45° = 1$

$cotg\, 45° = 1$

j) $0°$

$\sin 0° = 0$

$\cos 0° = 1$

$tg\, 0° = 0$

$cotg\, 0° = \emptyset$

k) $-210°$

$\sin(-210°) = \frac{1}{2}$

$\cos(-210°) = -\frac{\sqrt{3}}{2}$

$tg(-210°) = -\frac{\sqrt{3}}{3}$

$cotg(-210°) = -\sqrt{3}$

l) $750°$

$\sin(2 \cdot 360° + 30°) = +\frac{1}{2}$

$\cos(750°) = \frac{\sqrt{3}}{2}$

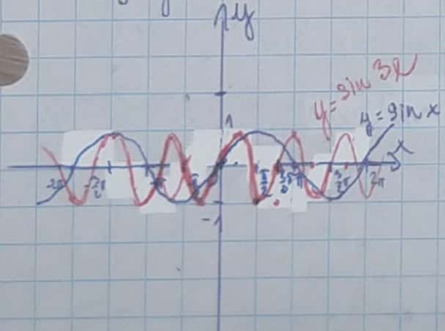
$tg(30°) = \frac{\sqrt{3}}{3}$

$cotg(30°) = \sqrt{3}$

4) a) $f: y = \sin 3x$

$y = a \pm b\, tg(cx + d) + x$

$p = \frac{2\pi}{|c|} = \frac{2\pi}{3}$



Vlastnosti: $D(f) = R$

$H(f) = \langle -1, 1 \rangle$

spojitosť: je spojitá

prostosť: nie je prostá

$\Rightarrow \nexists$ inverzná

ohraničenosť: je ohraničená

$d = -1$

$h = 1$

párnosť: nepárna

súradnice s osou x, y

$P_x = [x, 0]$ $0 = \sin 3x$

$3x = 0 + 2k\pi$

$3x = \pi + 2k\pi$

$P_y [0, y]$ $y = \sin 3 \cdot 0 = 0$

extrémy: min: $\frac{\pi}{2} + \frac{2k\pi}{3}$; $k \in Z$

max: $\frac{\pi}{6} + \frac{2k\pi}{3}$; $k \in Z$

monotónnosť: nie je monotónna

periodickosť: je periodická s periodou $\frac{2}{3}\pi$

jednoznačnosť: nie je jednoznačná

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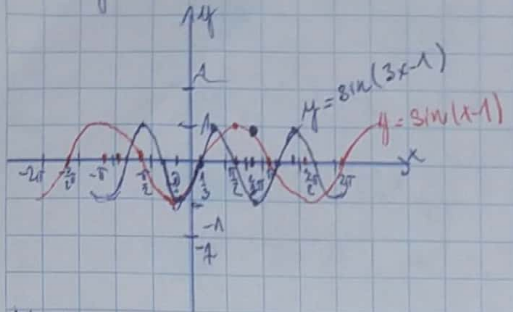
b) $f: y = \sin(3x-1)$ $x = -\frac{d}{c} = -\frac{1}{3}$



Vlastnosti:

$D(f) = R$

$H(f) = \langle -1, 1 \rangle$

spojitosť: je spojitá

prostosť: nie je prostá $\Rightarrow \nexists!$ inverzná

ohraničenosť: je ohraničená $d=-1$, $h=1$

párnosť: nie je nepárna, ani párna

súradnice s osou $[0,0]$ P_x: $0 = \sin 3x - 1$, $x = 0$; $y = \sin -1$, $y = -\sin 1$, $-0{,}84$

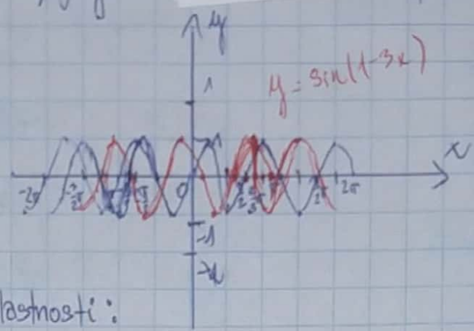
extrémy: max $\frac{\pi}{6} + \frac{1}{3} + \frac{2k\pi}{3}$ min $-\frac{\pi}{3}\pi + \frac{1}{3} + \frac{2k\pi}{3}$, $k \in Z$

monotónnosť: nie je monotónna

jednoznačnosť: nie je jednoznačná

periodickosť: je periodická s periódou $\frac{3k\pi}{3}$

c) $f: y = \sin(1-3x)$ $-(3x-1)$ $p = \frac{2\pi}{|-3|} = \frac{2}{3}\pi$ $x = -\frac{1}{3}$ $-\frac{1}{3} \cdot 0 + \frac{1}{3}$



Vlastnosti:

$D(f) = R$

$H(f) = \langle -1, 1 \rangle$

spojitosť: je spojitá

prostosť: nie je prostá $\Rightarrow \nexists!$ inverzná

ohraničenosť: je ohraničená, $d=-1$; $h=1$

párnosť: ani párna, ani nepárna

súradnice s osou $[0,0]$ $P_x = 0$

$P_y = 0{,}84$ $y = \sin 1$

extrémy: max: $-\frac{\pi}{6} + \frac{1}{3} + \frac{2k\pi}{3}$
min: $-\frac{2}{3}\pi + \frac{1}{3} + \frac{2k\pi}{3}$ $k \in Z$

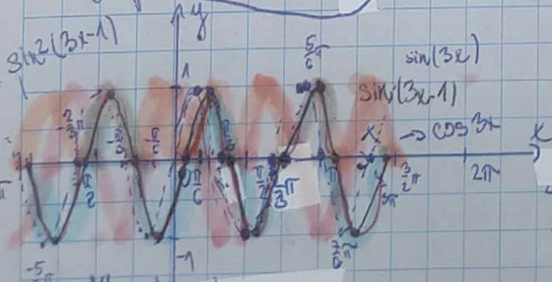
monotónnosť: nie je monotónna

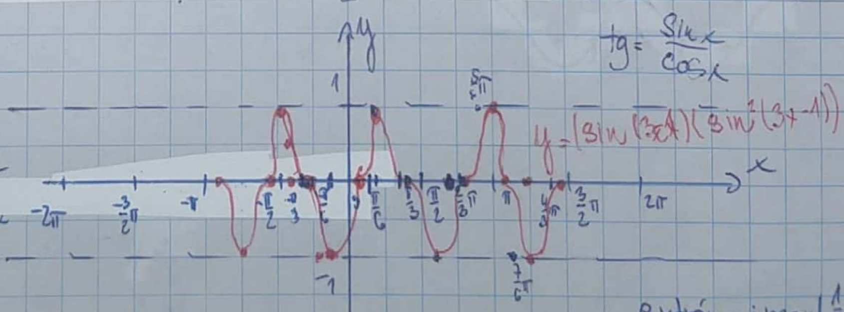
jednoznačnosť: nie je jednoznačná

periodickosť: je periodická s periódou $\frac{2k\pi}{3}$

d) $f: y = \sin(3x-1)^3 = (\sin(3x-1))(\sin^2(3x-1)) = (\sin(3x-1))\left(\frac{1-\cos(3x-1)}{2}\right) = (\sin(3x-1))\left(\frac{1}{2}\right)(1-\cos(3x-1))$

$tg = \frac{\sin x}{\cos x}$





Vlastnosti:

$D(f) = R$

$H(f) = \langle -1, 1 \rangle$

spojitosť: je spojitá

prostosť: nie je prostá $\Rightarrow \nexists!$ inverzná

ohraničenosť: je ohraničená $d=-1$, $h=1$

párnosť: ani párna, ani nepárna

súradnice s osou $[0,0]$ $P_x = 0$

$P_y = -0{,}84$

extrémy: max $\left(\frac{1}{3} + \frac{\pi}{6} + \frac{2k\pi}{3}\right)$ $k \in Z$
min $\left(\frac{\pi}{2} + \frac{1}{3} + \frac{2k\pi}{3}\right)$

monotónnosť: nie je monotónna

periodickosť: je periodická s periódou $\frac{2k\pi}{3}$

$\frac{2}{3}\pi - \frac{\pi}{2} = \frac{4\pi}{6} - \frac{3}{6}\pi = \frac{\pi}{6}$

$\frac{\pi}{2} - \frac{\pi}{6} = \frac{3}{6}\pi - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

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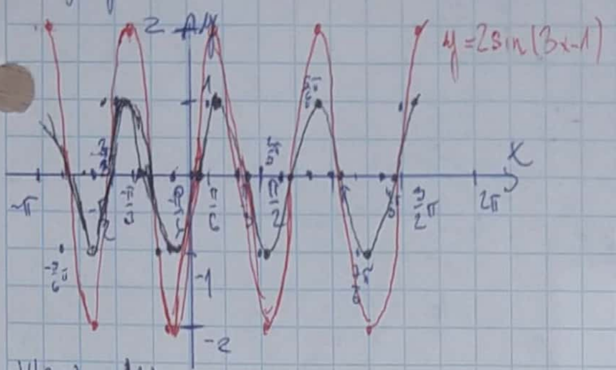
e) $f: y = 2\sin(3x-1)$



Vlastnosti:

$D(f) = R$

$H(f) = \langle -2, 2 \rangle$

Spojitosť: je spojitá

prostosť: nie je prostá

$\Rightarrow \nexists!$ inverzná

ohraničnosť: je ohraničená

$d = -2, h = 2$

párnosť: ani párna, ani nepárna

súradnice s osou [0,0] $P_x = 0$

$P_y = -0{,}84$

extrémy: max $\frac{\pi}{6} + \frac{1}{3} + \frac{2k\pi}{3}$

min $\frac{2}{3}\pi + \frac{1}{3} + \frac{2k\pi}{3}$

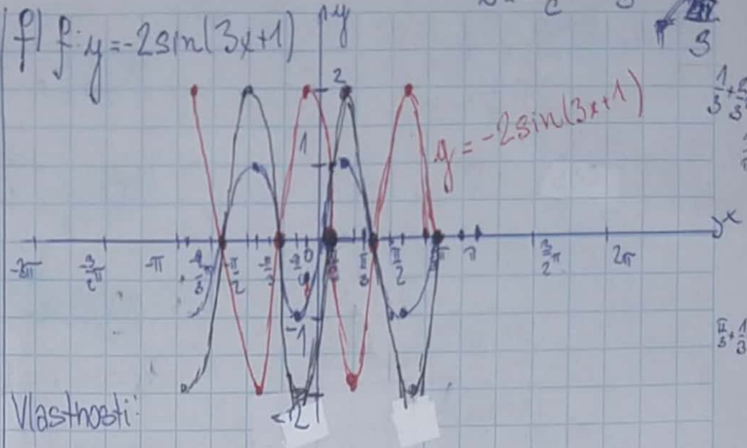
monotónnosť: nie je monotonná

jednoznačnosť: nie je jednoznačnosť

periodickosť: je periodická s periódou $\frac{2k\pi}{3}$

$a \pm b\sin(cx+d)$ $x = -\frac{d}{c} = -\frac{1}{3}$

f) $f: y = -2\sin(3x+1)$



Vlastnosti:

$D(f) = R$

$H(f) = \langle -2, 2 \rangle$

Spojitosť: je spojitá

prostosť: nie je prostá $\Rightarrow \nexists!$ inverzná

ohraničenosť: je ohraničená $d = -2, h = 2$

parita: ani párna, ani nepárna

súradnice s osou [0,0] $P_x = 0$, $P_y = -2\sin 1 = -1{,}683$

extrémy: max.: $\frac{\pi}{2} + \frac{1}{3} + \frac{2k\pi}{3}$

min.: $-\frac{\pi}{6} + \frac{1}{3} + \frac{2k\pi}{3}$

monotónnosť: nie je monotónna

periodickosť: je periodická s periódou $\frac{2k\pi}{3}$

jednoznačnosť: nie je jednoznačná

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IV

5) a) $\cos \varphi = \frac{12}{13}$ pre $\frac{3}{2}\pi < \varphi < 2\pi$

$\cos(x) = \sin(x + \frac{\pi}{2})$

$\left(\frac{12}{13}\right)^2 + \sin(x)^2 = 1$

$\frac{144}{169} + (\sin x)^2 = 1$

169
144
25

$(\sin x)^2 = \frac{25}{169} \quad | \sqrt{\ }$

$|\sin x| = \frac{5}{13}$

$\begin{array}{c|c} + & + \\ \hline - & - \end{array}$

$\sin x = -\frac{5}{13}$

c) $\sin \varphi = 0{,}8$ | pre $\frac{\pi}{2} < \varphi < \pi$ II

$(\cos x)^2 + (\sin x)^2 = 1$

$(\cos x)^2 + \overset{0{,}64}{(0{,}8)^2} = 1$

$\cos^2 x = 0{,}36$

$|\cos x| = 0{,}6$

$\begin{array}{c|c} - & + \\ \hline - & + \end{array}$

$\cos x = -0{,}6$

$tg x = \frac{\sin x}{\cos x} = -\frac{0{,}8}{0{,}6} = -\frac{8}{6} = -\frac{4}{3}$

$cotg\, x = \frac{\cos x}{\sin x} = -\frac{0{,}6}{0{,}8} = -\frac{3}{4}$

e) $tg\, \varphi = -\sqrt{5}$

$cotg\, x = \frac{1}{-\sqrt{5}}$

$\frac{\sin x}{\cos x} = -\sqrt{5}$

$(\cos x)^2 + (\sin x)^2 = 1$

$\cos^2 x + \overset{5}{(-\sqrt{5}\cos x)^2} = 1$

$6\cos^2 x = 1$

$|\cos x| = \frac{1}{\sqrt{6}}$

$\cos x = -\frac{\sqrt{6}}{6}$

$\sin x = (-\sqrt{5}) \cdot \left(-\frac{\sqrt{6}}{6}\right)$

$\sin x = \frac{\sqrt{30}}{6}$

$tg\, x = \frac{\sin x}{\cos x} = \frac{-\frac{5}{13}}{\frac{12}{13}} = -\frac{5}{12}$

$cotg\, x = \frac{\cos x}{\sin x} = \frac{\frac{12}{13}}{-\frac{5}{13}} = -\frac{12}{5}$

b) $tg\, \varphi = \sqrt{2}$ pre $\pi < \varphi < \frac{3}{2}\pi$ III

$tg \cdot cotg\, x = 1$

$cotg\, x = \frac{1}{tg\, x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$tg\, x = \frac{\sin x}{\cos x} = \frac{\sqrt{2}}{1}$

$\frac{\sin x}{\cos x} = \frac{\sqrt{2}}{1}$ ale $\cos x \neq 0$ $\frac{\pi}{2}$; $\frac{3\pi}{2}$ $+2k\pi$ $k \in Z$

$(\cos x)^2 + (\sin x)^2 = 1$

$\sin x = \sqrt{2} \cdot \cos x$

$(\cos x)^2 + (\sqrt{2}\cos x)^2 = 1$

$3\cos^2 x = 1$

$\cos^2 x = \frac{1}{3}$

$\begin{array}{c|c} - & + \\ \hline - & + \end{array}$

$|\cos x| = \frac{\sqrt{3}}{3}$

$\cos x = -\frac{\sqrt{3}}{3}$

$\sin x = \sqrt{2} \cdot \left(-\frac{\sqrt{3}}{3}\right)$

$\sin x = -\frac{\sqrt{6}}{3}$

d) $\cos \varphi = -\frac{3}{5}$

$\left(-\frac{3}{5}\right)^2 + (\sin x)^2 = 1$

$\frac{9}{25} + \sin^2 x = 1$ $\frac{16}{25}$

$|\sin x| = \frac{4}{5}$

$\begin{array}{c|c} - & + \\ \hline & \end{array}$

$\sin x = \frac{4}{5}$

$tg\, x = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}$

$cotg\, x = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$

f) $\cot \varphi = -\frac{8}{15}$ pre $1500° < \varphi < 1620°$

1440 $4 \cdot 360$ $\frac{\pi}{60}$ $4 \cdot 360° / 180\pi$

$\frac{64}{225}$ $\left(-\frac{8}{15}\right)^2 + (\sin x)^2 = 1$

$(\sin x)^2 = \frac{161}{225}$

$\sin x = \frac{\sqrt{161}}{5}$

$tg\, x = \frac{\frac{\sqrt{161}}{5}}{-\frac{8}{5}} = -\frac{\sqrt{161}}{8}$

$cotg\, x = \frac{-\frac{8}{5}}{\frac{\sqrt{161}}{5}} = -\frac{8}{\sqrt{161}}$

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$6\pi + \pi$ $\quad \frac{13}{2} - 6\pi = \frac{\pi}{2}$

g) $tg\,\varphi = 5$ pre $100° < \varphi < 180°$

$cotg\,x = \frac{1}{5}$

$\frac{\sin x}{\cos x} = 5$

$(\cos x)^2 + (\sin x)^2 = 1$

$(\cos x)^2 + (5 \cdot \cos x)^2 = 1$

$6\cos^2 x = 1$

$\cos^2 x = \frac{1}{6}$

$\cos x = -\frac{\sqrt{6}}{6}$

$\sin x = 5 \cdot \left(-\frac{\sqrt{6}}{6}\right) = -\frac{5\sqrt{6}}{6}$

h) $tg\,\varphi = \frac{7}{8}$ pre $-7\pi < \varphi < -\frac{13}{2}\pi$

$cotg\,x = \frac{1}{\frac{7}{8}} = \frac{8}{7}$

$\frac{\sin x}{\cos x} = \frac{7}{8}$

$(\cos x)^2 + (\sin x)^2 = 1$

$\cos^2 x + \frac{49}{64}\cos^2 x = 1$

$\frac{113}{64}\cos^2 x = 1$

$\cos^2 x = \frac{64}{113}$

$\cos x = -\frac{8}{\sqrt{113}}$

$\sin x = \frac{7}{8} \cdot \left(-\frac{8}{\sqrt{113}}\right) = -\frac{7}{\sqrt{113}}$

7) ~~a) $\sin x = \frac{3}{5}$, $\cos x = \frac{4}{5}$~~ a) $\sin x \cdot cotg\,x + \cos x =$

$= \sin x \cdot \frac{\cos x}{\sin x} + \cos x = \frac{\sin x \cdot \cos x}{\sin x} + \cos x = 2\cos x$

b) $\frac{cotg\,x + cotg\,y}{tg\,x + tg\,y} = \frac{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}}{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}} = \cancel{\frac{2\cos x}{\sin x} \cdot \frac{\cos x}{2\sin x} = \frac{\cos^2 x}{\sin^2 x} = cotg\,x}$

$= \frac{\cos x \cdot \sin y + \cos y \cdot \sin x}{\sin x \cdot \sin y} \cdot \frac{\cos x \cdot \cos y}{\sin x \cdot \cos y + \sin y \cdot \cos x} = \frac{\sin(x+y)}{\sin x \cdot \sin y} \cdot \frac{\cos x \cdot \cos y}{\sin(x+y)} = cotg\,x \cdot cotg\,y$

c) $\frac{\sin^2 x}{1 - \cos x} = \frac{1 - \cos^2 x}{1 - \cos x} = 1 - \cos x$

d) $\frac{\sin^2 x - 1}{\cos^2 x - 1} = \frac{(-1) \cdot (1 - \sin^2 x)}{(-1) \cdot (1 - \cos^2 x)} = \frac{\cos^2 x}{\sin^2 x} = cotg^2 x$

e) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = \sin^2 x + 2\sin x \cos x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x =$

$= 2\sin^2 x + 2\cos^2 x = 2$

6) a) $\sin x = \frac{3}{5}$, $\cos x = \frac{4}{5}$

$tg\,x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4} = 0{,}75$

b) $\sin x = \frac{2}{3}$, $\cos x = \frac{4}{5}$

$tg\,x = \frac{\sin x}{\cos x} = \frac{\frac{2}{3}}{\frac{4}{5}} = \frac{5}{6} = 0{,}8\overline{3}$

~~c) $\sin x = \frac{2}{3}$, $\cos x =$~~

c) $\sin x = \frac{2}{3}$, $tg\,x = \frac{1}{3}$

$\frac{1}{3} = \frac{\frac{2}{3}}{\cos x}$

$\frac{1}{3} \cdot \cos x = \frac{2}{3}$

$\cos x = \frac{\frac{2}{3}}{\frac{1}{3}} = 2$

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8) a) $\frac{\sin^2\alpha}{1+\cos\alpha} = \frac{1-\cos^2\alpha}{1+\cos\alpha} \cdot \cancel{\frac{1+\cos}{1+\cos\alpha}} = \frac{(1+\cos\alpha)(1-\cos\alpha)}{(1+\cos\alpha)} = 1-\cos\alpha$

$\frac{\sin}{\cos}$

b) $\sin^2\alpha + \cos^2\alpha + tg^2\alpha = 1 + tg^2\alpha =$

$\frac{\cos+\sin}{\cos}$

c) $1-\cos^2\beta + \sin^2\beta = 2\sin^2\beta$

d) $1-\sin^2\alpha + cotg^2\alpha \cdot \sin^2\alpha = \cos^2\alpha + \frac{\cos^2\alpha}{\cancel{\sin^2\alpha}} \cdot \cancel{\sin^2\alpha} = 2\cos^2\alpha$

---

10) a) $\frac{5+\sin x}{1-\sin x} = 3$ OR: $1-\sin x \neq 0$ $x \neq \frac{\pi}{2} + 2k\pi$

$x \in R$

$\sin x \neq 1$

$5+\sin x = 3 - 3\sin x$

$K = \left\{\frac{7}{6}\pi + 2k\pi; \frac{11}{6}\pi + 2k\pi, k \in Z\right\}$

$4\sin x = -2$

III. $x_1 = \pi + \frac{\pi}{6} = \frac{7}{6}\pi + 2k\pi$

$\sin x = -\frac{1}{2}$ $\frac{\pi}{6}$

IV. $x_2 = 2\pi - \frac{\pi}{6} = \frac{11}{6}\pi + 2k\pi$

b) $4\cos^2 x + 4\cos x - 3 = 0$

1) $t = \frac{1}{2}$

2) $t = -\frac{3}{2}$

SUB.: $\cos x = t$

$\cos x = \frac{1}{2}$ $\frac{\pi}{3}$

$\cos x = -\frac{3}{2} = -1\frac{1}{2}$ X

$4t^2 + 4t - 3 = 0$

I. $\frac{\pi}{3} + 2k\pi$

$t_{1/2} = \frac{-4 \pm \sqrt{16+48}}{8} = \frac{-4 \pm \sqrt{64}}{8} = \begin{cases} \frac{1}{2} \\ -\frac{3}{2} \end{cases}$

IV. $x_2 = 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi + 2k\pi$

$K = \left\{\frac{\pi}{3} + 2k\pi; \frac{5}{3}\pi + 2k\pi, k \in Z\right\}$

c) $\sin^2 x - \cos^2 x + \sin x = 0$

1) $t = \frac{1}{2}$

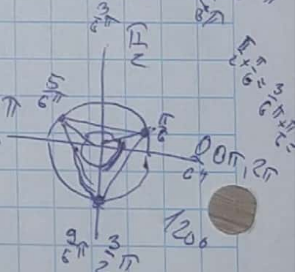
2) $t = -1$

$= 1 + \sin^2 x$

$\cancel{\sin^2 x} - (1 - \sin^2 x) + \sin x = 0$

$\sin x = \frac{1}{2}$ $\frac{\pi}{6}$

$\sin x = -1$ $\frac{3}{2}\pi + 2k\pi$



$2\sin^2 x + \sin x - 1 = 0$

I. $x_1 = \frac{\pi}{6} + 2k\pi$

~~III. $\pi + \frac{2}{3}\pi = \frac{5}{3}\pi + 2k\pi$~~

SUB.: $\sin x = t$

II. $x_2 = \pi - \frac{\pi}{6} = \frac{5}{6}\pi + 2k\pi$

~~IV. $2\pi - \frac{2}{3}\pi =$~~

$2t^2 + t - 1 = 0$

$K = \left\{\frac{\pi}{6} + 2k\pi; \frac{5}{6}\pi + 2k\pi; \frac{3}{2}\pi + 2k\pi, k \in Z\right\} = \left\{\frac{\pi}{6} + \frac{2k\pi}{3}\right\}$

$t_{1/2} = \frac{-1 \pm \sqrt{1+8}}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$

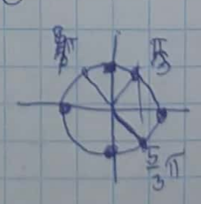
$x \in R$

11) a) $\cos^2 x - \sin^2 x - \frac{1}{4} = 0$

b) ~~$15\sin x + 10\cos x = 12$~~ $\sin^2 x + 3\cos^2 x + \cos x = 1$

$1 - \sin^2 x - 3\sin^2 x - \frac{1}{4} = 0$

$\cancel{1} - \cos^2 x + 3\cos^2 x + \cos x = \cancel{1}$



$-2\sin^2 x = -\frac{3}{4}$

$2\cos^2 x + \cos x + 1 = 0$

$\sin x = \sqrt{\frac{3}{8}}$

SUB: $\cos x = t$ $\cos x(2\cos x + 1) = 0$

$x = 37{,}76° = 37°46'$

~~$2t^2 + t + 1 = 0$~~ $\cos x = 0 \wedge \cos x = \frac{1}{2}$ $\frac{\pi}{3}$

~~$t_{1/2} = -1 \pm 1 - 8$~~ $\frac{3}{2}\pi; \frac{\pi}{2} + 2k\pi \wedge \frac{\pi}{3} + 2k\pi; \frac{4}{3}\pi + 2k\pi$

$K = \left\{\frac{\pi}{3} + 2k\pi; \frac{4}{3}\pi + 2k\pi; \frac{\pi}{2} + 2k\pi\right\}$

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12) a) $\sin x = \cos x - 1$

$\sin x - \cos x = -1 \quad | \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = -\frac{\sqrt{2}}{2}$

$\sin\left(\frac{\pi}{4}\right)\cdot \sin x - \cos\left(\frac{\pi}{4}\right)\cdot\cos x = -\frac{\sqrt{2}}{2}$

$\cos\left(x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

SUB.: $x - \frac{\pi}{4} = t$

$\cos t = -\frac{\sqrt{2}}{2} \approx \frac{\pi}{4}$

II. $\pi - \frac{\pi}{4} = \frac{3}{4}\pi + 2k\pi$

III. $\pi + \frac{\pi}{4} = \frac{5}{4}\pi + 2k\pi$

1) $x - \frac{\pi}{4} = \frac{3\pi}{4} + 2k\pi$

$x = \pi + 2k\pi$

2) $x - \frac{\pi}{4} = \frac{5}{4}\pi + 2k\pi$

$x = \frac{3}{2}\pi + 2k\pi$

$K = \left\{\pi + 2k\pi;\ \frac{3}{2}\pi + 2k\pi,\ k \in Z\right\}$

---

b) $15\sin x + 10\cos x = 12$

$5(3\sin x + 2\cos x) = 12$

$3\sin x + 2\cos x = \frac{12}{5}$

$3\,tg\,x + 2 = \frac{12}{5\cos x}$

$3\,tg\,x = \frac{10\cos x - 12}{5\cos x}$

$3\,tg\,x = \frac{2(5\cos x - 6)}{5\cos x}$

$\frac{12}{5}\cdot\frac{1}{\cos}$ $\quad 2 - \frac{12}{5\cos x}$ $\quad 10\cos x - 12$

~~$5\sin x + 10\sin x + 10\cos x = 12$~~

~~$5\sin x + 10(\sin x + \cos x) = 12$~~

$15\sin x = 12 - 10\cos x \quad |^2$

$225\sin^2 x = 144 - 240\cos x + 100\cos^2 x$

$225\sin^2 x - 144 - 81 + 81 = -240\cos x + 100\cos^2 x$

$225\sin^2 x - 225 + 81 = -240\cos x + 100\cos^2 x$

$-225(1 - \sin^2 x) + 81 = -240\cos x + 100\cos^2 x$

$-225\cos^2 x + 81 = -240\cos x + 100\cos^2 x$

$-335\cos^2 x + 240\cos x + 81 = 0$

SUB.: $\cos x = t$

~~$-335t^2 + 240t + 81 = 0$~~

~~$t_{1,2} = \frac{-240 \pm \sqrt{57600 + 108540}}{-670}$~~

$= \frac{-240 \pm 6\sqrt{4615}}{-670} = \frac{1}{2}$

$-325t^2 + 240t + 81 = 0$

$t_{1,2} = \frac{-240 \pm \sqrt{57600 + 105300}}{650} = \frac{-240 \pm 30\sqrt{181}}{-650} = \frac{24 \pm 3\sqrt{181}}{65} = \frac{3(8 \pm \sqrt{181})}{65}$

1) $\cos x = \frac{24 + 3\sqrt{181}}{65}$

$x_1 \doteq 0{,}988$

2) $\cos x = \frac{24 - 3\sqrt{181}}{65}$

$x_2 \doteq -0{,}252$

~~$K = \left\{\frac{3(8-\sqrt{181})}{65};\ \frac{3+(8+\sqrt{181})}{65}\right\}$~~

$K = \{0{,}988 + 2k\pi;\ -0{,}252 + 2k\pi\}$

---

c) $\sin x + \sqrt{3}\cos x = 1 \quad | \cdot \frac{\sqrt{2}}{2}$

~~$\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}\sqrt{3}}{2}\cos x = 1$~~

$\frac{2}{2}\sin x + \frac{2\sqrt{3}}{2}\cos x = \frac{2}{2}$

$\frac{2(\sin x + \sqrt{3}\cos x)}{2} = 1$

$\frac{2}{1}\left(\frac{\sin x}{2} + \frac{\sqrt{3}\cos x}{2}\right) = 1$

$2\left(\cos\frac{\pi}{3}\cdot\sin x + \sin\frac{\pi}{3}\cdot\cos x\right) = 1$

$2\left(\sin\left(x + \frac{\pi}{3}\right)\right) = 1$

$\sin\left(x + \frac{\pi}{3}\right) = \frac{1}{2}$

SUB.: $t = x + \frac{\pi}{3}$

$\sin t = \frac{1}{2}$

$t_1 = \frac{\pi}{6} + 2k\pi$

$t_2 = \frac{5}{6}\pi + 2k\pi$

1) $t_1 = \frac{\pi}{6} + 2k\pi$

$x_1 = -\frac{\pi}{6} + 2k\pi$

2) $t_2 = \frac{5}{6}\pi + 2k\pi$

$x_2 = \frac{\pi}{2} + 2k\pi$

$K = \left\{-\frac{\pi}{6} + 2k\pi;\ \frac{\pi}{2} + 2k\pi,\ k \in Z\right\}$

$\frac{\pi}{6} - \frac{\pi}{3}$ $\quad \frac{\pi}{6} - \frac{2\pi}{6}$ $\quad -\frac{\pi}{6}$

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d) $12\cos x + 5\sin x = 11{,}2$

$5\sin x = 11{,}2 - 12\cos x \;|^2$

$25\sin^2 x = 125{,}4 - 268{,}8\cos x + 144\cos^2 x$

$25\sin^2 x - 125{,}4 = -268{,}8\cos x + 144\cos^2 x$

$25\sin^2 x - 25 - 100{,}4 = -268{,}8\cos x + 144\cos^2 x$

$-25\cos^2 x - 100{,}4 = -268{,}8\cos x + 144\cos^2 x$

$-169\cos^2 x + 268{,}8\cos x - 100{,}4 = 0$

$t_{1,2} = \frac{-268{,}8 \pm \sqrt{72253{,}44 - 67870{,}4}}{-338}$ $\quad 4383{,}04$

1) t_1

$x_1 \doteq 0{,}991$

2) t_2

$x_2 \doteq 0{,}6$

$K = \{0{,}6 + 2k\pi;\ 0{,}991 + 2k\pi\}$

---

13) a) $x \in R$

$\cos 2x = 2\cos x$

$\cos^2 x - \sin^2 x = 2\cos x$

$\cos^2 x - (1 - \cos^2 x) = 2\cos x$ $\quad -1 + \cos^2 x$

$2\cos^2 x - 2\cos x - 1 = 0$

$t_{1,2} = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm 2\sqrt{3}}{4} = \frac{1 \pm \sqrt{3}}{2}$ $\quad (12)$

$\cos x = \frac{1+\sqrt{3}}{2}$ $\qquad \cos x = \frac{1-\sqrt{3}}{2}$

$\emptyset$

---

b) $\operatorname{tg} x = \sin 2x$

OR: $\cos x \neq 0$

$x \neq \frac{\pi}{2} + 2k\pi$

$x \neq \frac{3}{2}\pi + 2k\pi$

$x \neq \pi + 2k\pi$

$\frac{\sin x}{\cos x} = 2\sin x \cdot \cos x$

~~$\sin x = 2\sin x \cdot \cos^2 x$~~

~~$\sin x = 2\sin x(1 - \sin^2 x)$~~

~~$\sin x = 2\sin x - 2\sin^3 x$~~

$2 = \cos^2 x$

~~$2 = 1 - \sin^2 x$~~

$1 = \sin^2 x$

$\sin x = \pm 1$

$\sin x = 2\sin x \cdot \cos^2 x$

$0 = \sin x - 2\sin x \cdot \cos^2 x$

$0 = \sin x(1 - 2\cos^2 x)$

$\sin x = 0 \wedge \cos x = \frac{\sqrt{2}}{2}$

$\pi + 2k\pi,\ \boxed{2k\pi}$ $\qquad \boxed{\frac{\pi}{4} + 2k\pi},\ \frac{7}{4}\pi + 2k\pi$

$K = \{2k\pi;\ \frac{\pi}{4} + 2k\pi\}$

$\frac{+1}{2}$

$\sqrt{2}$

c) $\sin 2x = 3\sin^2 x$

$2\sin x \cdot \cos x = 3\sin^2 x$

$2\sin x \cdot \cos x - 3\sin^2 x = 0$

$\sin x(2\cos x - 3\sin x) = 0$

$2\cos x = 3\sin x \quad |:\cos$

$\frac{2}{3} = \frac{\sin x}{\cos x}$

$\frac{2}{3} = \operatorname{tg} x$

$K = \{33{,}69°\}$

$\frac{2\cos x}{3}$ $\quad \sin x$

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$x \in R$

14) a) $\sin^2 x - 2\sin x \cdot \cos x - \cos^2 x = 0$ $\quad -1+ \quad -(1-\sin^2 x)$

~~$\sin^2 x - 2\sin x \cdot \cos x + \sin^2 x = 1$~~

~~$2\sin^2 x - 2\sin x \cos x = 1$~~

$2\sin x(\sin x - \cos x) = 1$

$2\sin x = 1 \wedge \sin x \cos x$

$2\sin x(\sin x - \cos x) = 1$

$2\sin x = 1 \quad \wedge \quad \sin x - \cos x + 1 \quad$ OR: $\cos x \neq 0$, $x \neq \pi + 2k\pi$

$\sin x = \frac{1}{2} \qquad \operatorname{tg} x = 1 + \frac{1}{\cos x}$

$\frac{\pi}{6} + 2k\pi$

$\frac{5\pi}{6} + 2k\pi$

~~b) $6\sin^2 x + 3\sin x \cdot \cos x - 5\cos^2 x = 2$~~

OR: $\cos x \neq \frac{\pi}{2} + 2k\pi, k \in Z$

$\frac{\sin^2 x - 2\sin x \cdot \cos x - \cos^2 x}{\cos^2 x} = 0$

$\operatorname{tg}^2 x - 2\operatorname{tg} x - 1 = 0$

SUB.: $\operatorname{tg} x = t$

$t^2 - 2t - 1 = 0$

$t_{1,2} = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

$\operatorname{tg} x = 1 + \sqrt{2} \qquad \operatorname{tg} x = 1 - \sqrt{2}$

$67{,}5^\circ \qquad -22{,}5^\circ$

$K = \{-22{,}5^\circ + k180^\circ, \ 67{,}5^\circ + k180^\circ\}$

---

b) $6\sin^2 x + 3\sin x \cdot \cos x - 5\cos^2 x = 2 \quad$ OR: $x \neq \frac{\pi}{2} + 2k\pi$

~~$6\operatorname{tg}^2 x + 3\operatorname{tg} x - 5 =$~~

$6\sin^2 x + 3\sin x \cdot \cos x - 5\cos^2 x = 2\sin^2 x + 2\cos^2 x$

$4\sin^2 x + 3\sin x \cdot \cos x - 7\cos^2 x = 0 \;|: \cos^2 x$

$4\operatorname{tg}^2 x + 3\operatorname{tg} x - 7 = 0$

SUB.: $\operatorname{tg} x = t$

$4t^2 + 3t - 7 = 0$

$t_{1,2} = \frac{-3 \pm \sqrt{9 + 112}}{8} = \frac{-3 \pm 11}{8} = \begin{cases} 1 \\ -\frac{7}{4} \end{cases}$

$\operatorname{tg} x = 1 \qquad \operatorname{tg} x = -\frac{7}{4}$

$x = \frac{\pi}{4} + k\pi, k \in Z \qquad -60{,}25^\circ + k180^\circ$

$K = \{\frac{\pi}{4} + k\pi, -60{,}25^\circ + k \cdot 180^\circ, k \in Z\}$

---

c) $\sin^2 x + \frac{3}{2}\cos^2 x = \frac{5}{2}\sin x \cdot \cos x$

$\sin^2 x + \frac{3}{2}\cos^2 x - \frac{5}{2}\sin x \cdot \cos x = 0 \quad$ OR: $x \neq \frac{\pi}{2} + 2k\pi$

$\operatorname{tg}^2 x - \frac{5}{2}\operatorname{tg} x + \frac{3}{2} = 0$

$2\operatorname{tg}^2 x - 5\operatorname{tg} x + 3 = 0$

SUB.: $\operatorname{tg} x = t$

$2t^2 - 5t + 3 = 0$

$t_{1,2} = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4} = \begin{cases} \frac{3}{2} \\ -\frac{1}{2} \end{cases}$

$\operatorname{tg} x = -\frac{1}{2} \qquad \operatorname{tg} x = \frac{3}{2}$

$-26{,}56^\circ \qquad 56{,}3^\circ$

$K = \{-26{,}56 + k \cdot 180^\circ, 56{,}3^\circ + k \cdot 180^\circ, k \in Z\}$

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15) a) $\frac{1+\cos x}{\sin x} = 2\,tg\,\frac{x}{2}$ OP: $\sin x \neq 0$ $\frac{x}{2} \neq \frac{\pi}{2} + k\pi, k \in Z$

$x \neq \pi + 2k\pi$ $x \neq \pi + 2k\pi$

$x \neq 2k\pi$

$\cancel{\frac{1+\cos x}{\sin x} = x}$

$\frac{1}{\sin x} + \cot g\, x = 2\,tg\,\frac{x}{2}$??

b) ~~x sin x =~~ $a \cdot \sin x = b \cdot \cos x$

$\frac{\sin x}{\cos x} = \frac{b}{a}$

$tg\, x = \frac{b}{a}$

c) $a \cdot \sin x = b \cdot tg\, x$

$a \cdot \sin x = b \cdot \frac{\sin x}{\cos x}$ OP: $\cos x \neq 0$

$a \cdot \cancel{\sin x} \cdot \cos x = b \cdot \cancel{\sin x}$ $x \neq \frac{\pi}{2} + 2k\pi$ $x \neq \frac{3\pi}{2} + 2k\pi$

$\cos x = \frac{b}{a}$

d) $\sin(a+x) + \sin(a-x) = c$

$\sin a \cdot \cos x + \cancel{\sin x \cdot \cos a} + \sin a \cdot \cos x - \cancel{\sin x \cdot \cos a} = c$

$2 \sin a \cdot \cos x = c$

16) $x \in R$

a) $\sin \overset{\sin(2x+x)}{3x} = \sin 2x - \sin x$

$\sin 2x \cdot \cos x + \sin x \cdot \cos 2x = 2 \sin x \cdot \cos x - \sin x$

$2 \sin x \cdot \cos x \cdot \cos x + \sin x (\cos^2 x - \sin^2 x) = 2 \sin x \cdot \cos x - \sin x$

$2 \sin x \cdot \cos^2 x + \sin x \cdot \cos^2 x - \sin x \cdot \sin^2 x = 2 \sin x \cdot \cos x - \sin x$

$3 \sin x \cdot \cos^2 x - \sin x \cdot \sin^2 x - 2 \sin x \cdot \cos x + \sin x = 0$

$\sin x (3\cos^2 x - \sin^2 x - 2\cos x + 1) = 0$

$3(1-\sin^2$

$\sin x (3\cos^2 x - \cancel{1} + \cos^2 x - 2\cos x + \cancel{1}) = 0$

$-(1-\cos^2 x)$

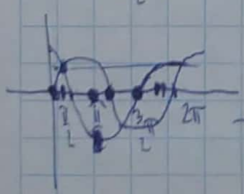
$\sin x (4\cos^2 x - 2\cos x) = 0$

$\sin x (2\cos x (2\cos x - 1)) = 0$

$\sin x = 0 \wedge \cos x = 0 \wedge \cos x = \frac{1}{2}$

$x_1 = \pi + 2k\pi$ $x_1 = \frac{\pi}{2} + 2k\pi$ $x_1 = \frac{\pi}{3} + 2k\pi$

$x_2 = 2k\pi$ $x_2 = \frac{3}{2}\pi + 2k\pi$ $x_2 = \frac{5}{3}\pi + 2k\pi$



$K = \left\{ \frac{k\pi}{2} ; \frac{\pi}{3} + \frac{2k\pi}{3} ; k \in Z \right\}$

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b) $2\sin^2 x + \sin^2 2x = 2$

$2\sin^2 x + (2\sin x \cos x)^2 = 2$

$2\sin^2 x + 4\sin^2 x \cdot \cos^2 x = 2$

~~$2\sin^2 x (1 + 2\cos^2 x) - 2 = 0$~~

$2\sin^2 x + 4\sin^2 x (1 - \sin^2 x) = 2$

$2\sin^2 x + 4\sin^2 x - 4\sin^4 x = 2$

$6\sin^2 x - 4\sin^4 x = 2$

SUB.: $\sin^2 x = t$

$3t - 2t^2 = 1$

$-2t^2 + 3t - 1 = 0$

$2t^2 - 3t + 1 = 0$

$t_{1/2} = \frac{3 \pm \sqrt{9-8}}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$

$\sin^2 x = 1$

$|\sin x| = 1$

$\sin x = 1 \rightarrow x = \frac{\pi}{2} + 2k\pi$

$\sin x = -1 \rightarrow x = \frac{3}{2}\pi + 2k\pi$

$+2k\pi$

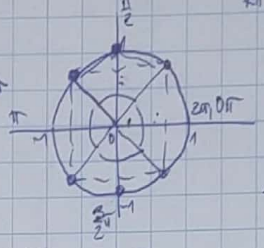
2π

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{\sqrt{2}}{2} \rightarrow \frac{\pi}{4} + 2k\pi ; \frac{3}{4}\pi + 2k\pi$

$\sin x = -\frac{\sqrt{2}}{2} \rightarrow \frac{5}{4}\pi + 2k\pi ; \frac{7}{4}\pi + 2k\pi$

2π



$\varphi = \frac{\pi}{4} + \frac{k\pi}{2}$

$\varphi = \frac{\pi}{2} + 2k\pi$

$K = \left\{ \frac{\pi}{2} + k\pi ; \frac{\pi}{4} + \frac{k\pi}{2} ; k \in Z \right\}$

$\frac{5}{4}\pi + 2k\pi$

$\frac{\pi}{2} - k\pi$

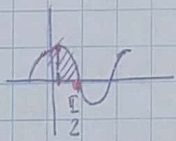
17) a) $\cos 3x > \frac{\sqrt{3}}{2}$

SUB.: $3x = t$

$\cos t > \frac{\sqrt{3}}{2}$, $\frac{\pi}{6}$

$\frac{\pi}{6} + 2k\pi$

$\frac{11}{6}\pi + 2k\pi$



$3x = \frac{\pi}{6} + 2k\pi$ $\quad$ $3x = \frac{11}{6}\pi + 2k\pi$

$x = \frac{\pi}{2} + \frac{2k\pi}{3}$ $\quad$ $x = \frac{11}{2}\pi + \frac{2k\pi}{3}$

$K = \left(-\frac{7}{2}\pi + \frac{2k\pi}{3} ; \frac{\pi}{2} + \frac{2k\pi}{3} \right)$

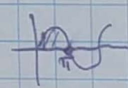
b) $\sin\left(x + \frac{\pi}{3}\right) \geq \frac{1}{2}$

SUB.: $x + \frac{\pi}{3} = t$

$\sin t \geq \frac{1}{2}$, $\frac{\pi}{6}$, $\frac{5}{6}\pi + 2k\pi$, $\frac{\pi}{6} + 2k\pi$

$x + \frac{\pi}{3} = \frac{\pi}{6} + 2k\pi$ $\quad$ $x + \frac{\pi}{3} = \frac{5}{6}\pi + 2k\pi$

$x = -\frac{\pi}{6} + 2k\pi$ $\quad$ $x = \frac{\pi}{2} + 2k\pi$



$\frac{\pi}{6}$ $\frac{2\pi}{6}$

$K = \left\langle -\frac{\pi}{6} + 2k\pi ; \frac{\pi}{2} + 2k\pi \right\rangle$

18) a) $tg^2 x - (1 + \sqrt{3})\, tg\, x + \sqrt{3} < 0$

$\frac{\sin^2 x}{\cos^2 x} - (1+\sqrt{3}) \frac{\sin x}{}$

$\sin x + \sqrt{3} \cos x > 0$

SUB.: $tg\, x = t$

$t^2 - (1 + \sqrt{3})t + \sqrt{3} < 0$

~~$t^2 - t - \sqrt{3}t + \sqrt{3} < 0$~~ $16 - 4\sqrt{3}$

$t_{1/2} = \frac{(1+\sqrt{3}) \pm \sqrt{(1+\sqrt{3})^2 - 4 \cdot \sqrt{3}}}{2} =$

$1+3$

$4 - 2\sqrt{3}$

$1+\sqrt{3} - 3 + \sqrt{3} + 2\sqrt{3}$

b) $2\cos^2 x + 5\cos x + 2 \geq 0$

SUB.: $\cos x = t$

$2t^2 + 5t + 2 \geq 0$

$t_{1/2} = \frac{-5 \pm \sqrt{25 - 16}}{4} = \frac{-5 \pm 3}{4} = \begin{cases} -\frac{1}{2} \\ -2 \end{cases}$

$\cos x = -\frac{1}{2}$, $\frac{\pi}{3}$ $\quad$ $\cos x = -2$, $\emptyset$

$\frac{2}{3}\pi + 2k\pi$

$\frac{4}{3}\pi + 2k\pi$

$K = \left\langle -\frac{2}{3}\pi + 2k\pi ; \frac{2}{3}\pi + 2k\pi \right\rangle$