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Domáca úloha číslo 06 - komplexné čísla

1) a) $(2-3i)(4+5i)-3i^7+4 = 8+10i-12i-15i^2+3i+4 = 27+i$

b) $2+(2-3i)^3+(1-4i)(-i+5) = 2+(8-3\cdot4\cdot(3i)+3\cdot2(9i^2)-27i^3)+(-i+5+4i^2-20i) = 2+8-36i-54+$

$+27i+1-21i = -43-30i$

c) $8-(2-\sqrt{3}i)(\sqrt{3}i+2)+5i^2 = 8-(2\sqrt{3}i+4-3i^2-2\sqrt{3}i)-5 = -4$

~~d) $(-2-\sqrt{3}i)(2+\sqrt{5}i)^2+5i^7+2i-\frac{4}{i+1} = \ldots$~~

d) $(2-\sqrt{3}i)(2+\sqrt{3}i)^2+5i^7+2i-\frac{4}{i+1} = (2-\sqrt{3}i)(4+4\sqrt{3}i+3i^2)-3i-\frac{4}{i+1} = (2-\sqrt{3}i)(1+4\sqrt{3}i)+\frac{3-3i-4}{i+1} =$

$= (2+8\sqrt{3}i-\sqrt{3}i-4\cdot3i^2)+\frac{-1-3i}{i+1}\cdot\frac{(i-1)}{(i-1)} =$

$= (14+7\sqrt{3}i)+\frac{+2i-4}{-2} = (14+7\sqrt{3}i)+(i+2) = 16+7\sqrt{3}i+i = 16+i(7\sqrt{3}+1)$

~~e) $\frac{4+7i}{2i-1}+\frac{2+i}{i-5} = \ldots = \frac{-269+725i}{130}$~~

e) $\frac{4+7i}{2i-1}+\frac{2+i}{i-5} = \frac{4+7i}{2i-1}\cdot\frac{(2i+1)}{(2i+1)}+\frac{2+i}{i-5}\cdot\frac{(i+5)}{(i+5)} = \frac{8i+4+14i^2+7i}{-5}+\frac{2i+10+i^2+5i}{-26} = \frac{-10+15i}{-5}+\frac{9+7i}{-26} = +2-3i+\frac{9+7i}{-26} =$

$= +2-3i-\frac{9}{26}-\frac{7i}{26} = \frac{43}{26}-\frac{85i}{26}$

~~f) $(i-18):(2i-1)^2+5i^7+2i^{-4} = \frac{i-18}{(2i-1)^2}-5i+\frac{1}{2} = \ldots$~~

f) $(i-18):(2i-1)^2+5i^7+2i^{-4} = \frac{i-18}{-3-4i}\cdot\frac{(-3+4i)}{(-3+4i)}-5i+2 = \frac{-3i-4+54-72i}{25}-5i+2 = \frac{-75i+50}{25}-5i+2 =$

$= -3i+2-5i+2 = 4-8i$

Naskenované pomocou CamScanner

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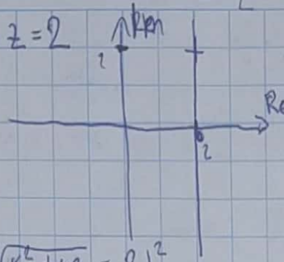
$\mathbb{C} = \{a+bi \mid a \in \mathbb{R}, b \in \mathbb{R}\}$

2) a) $\operatorname{Re} z = 2$ $\operatorname{Re}(z) = 2$ ~~$\operatorname{Re} \neq 0$~~ $z - 2 = 0$



~~$|z| = 2$~~ ~~$z = \frac{2}{Re}$~~

~~$\sqrt{x^2+y^2} = 2 \,|^2$~~

~~$x^2+y^2 = 4$~~

~~$x = \sqrt{2}, y$~~

$z = \sqrt{x^2+y^2} = 2 \,|^2$

$x^2+y^2 = 4$ $[0,0]$

~~b) $\operatorname{Im} z < 2$~~

3) a) $z = 1 - \sqrt{3}i$

$|z| = \sqrt{1^2 + (-\sqrt{3}i)^2} = \sqrt{1+3} = 2$ $\sin\varphi = \frac{b}{|z|} = -\frac{\sqrt{3}}{2}$ $\cos\varphi = \frac{a}{|z|} = \frac{1}{2}$ $2\pi - \frac{\pi}{3} = \frac{5}{3}\pi + 2k\pi$

$z = 2\left(\cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi\right)$

~~b) $z = 1$~~ $\pi = -0{,}732$ ~~$\cos\varphi = 0{,}536$~~

~~b) $z = 1 - \sqrt{3}$ $|z| = \sqrt{(1-\sqrt{3})^2 + 0^2} = \sqrt{2}$~~ b) $z = 1 - \sqrt{3} = 1 - \sqrt{3}i^0$ ~~$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$~~ $\cos\varphi = \frac{-0{,}732}{0{,}536} = -1{,}36$

~~c) $z = \sin 2 - x$~~

$\cos\varphi = \frac{-0{,}732}{0{,}732} = -1$ ~~$\sin\varphi = -\frac{\sqrt{3}}{2}$ $\cos\varphi = \frac{1}{2}$~~ ~~$z = 2\left(\cos\frac{5}{3}\pi + i \sin \frac{5}{3}\pi\right)$~~ $|z| = (-0{,}732)^2 = 0{,}536 = 0{,}732$

$\sin\varphi = 0$ $z = 0{,}732\,(\cos \pi + i \sin \pi)$

c) $z = \sin 2 - \pi = -2{,}2323$ $|z| = \sqrt{4{,}983} = 2{,}2323$ $\sin\varphi = 0$ $\cos\varphi = -1$

$z = 2{,}2323\,(\cos \pi + i \sin \pi)$

d) $z = \sin(2-\pi) = \sin 2 \cdot \cos\pi - \cos 2 \sin \pi = \sin 2 \cdot (-1) - \cos 2 \cdot 0 = -\sin 2 \doteq -0{,}909$

$z = 0{,}909$ $\cos\varphi = -1$ $\sin\varphi = 0$ $z = 0{,}909\,(\cos \pi + i \sin \pi)$

e) $z = 7i$ $|z| = \sqrt{49} = 7$ $\cos\varphi = 0$ $\sin\varphi = 1$ $z = 7\left(\cos\frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

f) $z = 8\sqrt{3}i - 8$ $|z| = \sqrt{192+64} = \sqrt{256}$ ~~$= 8\sqrt{2}$~~ 16 ~~$\cos\varphi = \frac{8\sqrt{3}}{8\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$~~ ~~$\sin\varphi = \frac{8}{8\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$~~

$z = 16\left(\cos \frac{11}{6}\pi + i \sin \frac{11}{6}\pi\right)$ $\cos\varphi = \frac{8\sqrt{3}}{16} = \frac{\sqrt{3}}{2}$ $\sin\varphi = -\frac{8}{16} = -\frac{1}{2}$ $\frac{11}{6}\pi$

g) $z = \frac{\sqrt{3}}{5} + \frac{i}{5}$ $|z| = \sqrt{\left(\frac{\sqrt{3}}{5}\right)^2 + \left(\frac{1}{5}\right)^2} = \sqrt{\frac{3}{25} + \frac{1}{25}} = \frac{2}{5}$ $\cos\varphi = \frac{\frac{\sqrt{3}}{5}}{\frac{2}{5}} = \frac{\sqrt{3}}{2}$ $\frac{\pi}{6}$ $\sin\varphi = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$ $\frac{\pi}{6}$

$z = \frac{2}{5}\left(\cos\frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

h) $z = 4 - 4i$ $|z| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ $\cos\varphi = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\sin\varphi = -\frac{4}{4\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\frac{\pi}{4}$

$z = 4\sqrt{2}\left(\cos\frac{7}{4}\pi + i \sin \frac{7}{4}\pi\right)$

i) $z = -8\sqrt{3} + 8i$ $|z| = \sqrt{192+64} = \sqrt{256} = 16$ $\sin\varphi = \frac{8}{16} = \frac{1}{2}$ $\cos\varphi = -\frac{8\sqrt{3}}{16} = -\frac{\sqrt{3}}{2}$ $\pi + \frac{\pi}{6} = \frac{7}{6}\pi$

$z = 16\left(\cos\frac{7}{6}\pi + i \sin\frac{7}{6}\pi\right)$

j) $z = -3 - 3i\sqrt{3} = -3 - 3\sqrt{3}i$ $|z| = \sqrt{9+27} = \sqrt{36} = 6$ $\sin\varphi = -\frac{3\sqrt{3}}{6} = -\frac{\sqrt{3}}{2}$ $\cos\varphi = -\frac{3}{6} = -\frac{1}{2}$ $\frac{\pi}{3}$

$z = 6\left(\cos\frac{4}{3}\pi + i \sin \frac{4}{3}\pi\right)$

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4)

a) $z = (\sqrt{3}+1)^{33}$

$\text{Re } z = (\sqrt{3}+1)^{33}$

b) $z = (1-i)^{16}(i-\sqrt{3})^6$

$z^{16} = (\sqrt{2})^{16}\left(\cos 16 \cdot \frac{7}{4}\pi + i \sin 16 \frac{7}{4}\pi\right) = 256(\cos 28\pi + i \sin 28\pi) =$

$= 256(\cos 0 + i \sin 0) = 256(1 + i \cdot 0) = 256$

$z^6 = 2^6\left(\cos 6\frac{5}{3}\pi + i \sin 6\frac{5}{3}\pi\right) = 64(\cos 10\pi + i \sin 10\pi)$

$= -64$

$256 \cdot (-64) = -16384$

$\text{Re } z = -16384$

c) $z = \left(\frac{i-1}{1+\sqrt{3}i}\right)^{24} =$

$= \frac{i-1}{1+\sqrt{3}i} \cdot \frac{(1-\sqrt{3}i)}{(1-\sqrt{3}i)} = \frac{i+\sqrt{3}-1+\sqrt{3}i}{1+\sqrt{3}} =$

$= \frac{1+\sqrt{3}-1+\sqrt{3}i}{1+\sqrt{3}} = 1 + \frac{1-\sqrt{3}i}{1+\sqrt{3}}$

$= \frac{i+\sqrt{3}-1+\sqrt{3}i}{1+\sqrt{3}} = \frac{i(1+\sqrt{3})+\sqrt{3}-1}{1+\sqrt{3}} = i + \frac{\sqrt{3}-1}{1+\sqrt{3}} = i + \frac{(-1)(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = i + \frac{(-1)(1-\sqrt{3}-\sqrt{3}+3)}{1-3} = i + \frac{(-1)(4-2\sqrt{3})}{-2} =$

$= i + \frac{2(2-\sqrt{3})}{2} = (i + (2-\sqrt{3}))^{24}$ Img Re

$|z| = \sqrt{(2-\sqrt{3})^2+1} =$

d) $z = \left(\frac{1+i}{1-i}\right)^{19} \cdot i^{23} = (-i)^{19} \cdot i^{23} = (-i) \cdot (-i) = -1$ - Re

$\frac{1+i}{i-1} \cdot \frac{(i+1)}{(i+1)} = \frac{1^2+2i+i^2}{i^2-1^2} = \frac{2i}{-2} = -i$

e) $z = (-1+\sqrt{3}i)^{15}$ $\quad |z| = \sqrt{(-1)^2+(\sqrt{3})^2} = 2$ $\quad \sin\varphi = \frac{\sqrt{3}}{2}$ $\quad \cos\varphi = -\frac{1}{2}$ $\quad \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$z^{15} = 2^{15}\left(\cos 15 \cdot \frac{2}{3}\pi + i \sin 15\frac{2}{3}\pi\right) = 2^{15}(\cos 10\pi + i \sin 10\pi) = +32768$ Re

$10:2=5$

5) a) $z = \sqrt[4]{-1-\sqrt{3}i}$ $\quad |z| = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{1+3} = 2$ $\quad \sin\varphi = \frac{-\sqrt{3}}{2}$ $\quad \cos\varphi = -\frac{1}{2}$ $\quad \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

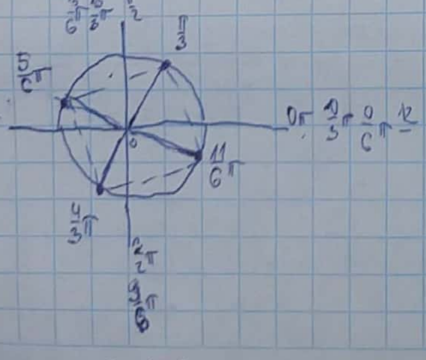
$z = 2\left(\cos\frac{4}{3}\pi + i \sin\frac{4}{3}\pi\right)$

$z_0 = 2\left(\cos\frac{\frac{4}{3}\pi+0\cdot2\pi}{4} + i \sin\frac{\frac{4}{3}\pi+0\cdot2\pi}{4}\right) = 2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$

$z_1 = 2\left(\cos\frac{\frac{4}{3}\pi+1\cdot2\pi}{4} + i \sin\frac{\frac{4}{3}\pi+2\pi}{4}\right) = 2\left(\cos\frac{\frac{10}{3}\pi}{4} + i \sin\frac{5}{6}\pi\right) = 2\left(\cos\frac{5}{6}\pi + i \sin\frac{5}{6}\pi\right)$

$z_2 = 2\left(\cos\frac{\frac{4}{3}\pi+2\cdot2\pi}{4} + i \sin\frac{\frac{4}{3}+\frac{12}{3}\pi}{4}\right) = 2\left(\cos\frac{\frac{16}{3}\pi}{4} + i \sin\frac{4}{3}\pi\right) = 2\left(\cos\frac{4}{3}\pi + i \sin\frac{4}{3}\pi\right)$

$z_3 = 2\left(\cos\frac{\frac{4}{3}\pi+2\cdot3\pi}{4} + i \sin\frac{\frac{4}{3}\pi+\frac{18}{3}\pi}{4}\right) = 2\left(\cos\frac{\frac{22}{3}\pi}{4} + i \sin\frac{11}{6}\pi\right) = 2\left(\cos\frac{11}{6}\pi + i \sin\frac{11}{6}\pi\right)$



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b) $\sqrt[4]{8\sqrt{3}i-8}$ $\quad |z|=\sqrt{(-8)^2+(8\sqrt{3})^2}=\sqrt{64+192}=16$ $\quad \sin\varphi=\frac{8\sqrt{3}}{16}=\frac{\sqrt{3}}{2}$ $\quad \cos\varphi=-\frac{8}{16}=-\frac{1}{2}$

$\pi-\frac{\pi}{3}=\frac{2}{3}\pi$

$z=16\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)$

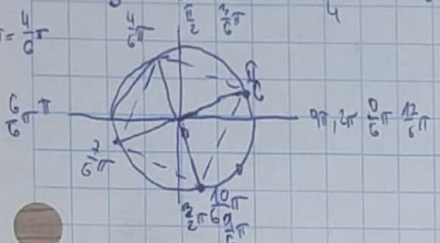
$z_0=16\left(\cos\frac{\frac{2}{3}\pi+2\cdot0\pi}{4}+i\sin\frac{\pi}{6}\right)=16\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

$z_1=16\left(\cos\frac{\frac{2}{3}\pi+2\cdot1\pi}{4}+i\sin\frac{\frac{2}{3}\pi+\frac{6}{3}\pi}{4}\right)=16\left(\cos\frac{2}{3}\pi+i\sin\frac{2}{3}\pi\right)$

$z_2=16\left(\cos\frac{\frac{2}{3}\pi+2\cdot2\pi}{4}+i\sin\frac{\frac{2}{3}\pi+\frac{12}{3}\pi}{4}\right)=16\left(\cos\frac{7}{6}\pi+i\sin\frac{7}{6}\pi\right)$

$z_3=16\left(\cos\frac{\frac{2}{3}\pi+2\cdot3\pi}{4}+i\sin\frac{\frac{2}{3}\pi+\frac{18}{3}\pi}{4}\right)=16\left(\cos\frac{5}{3}\pi+i\sin\frac{5}{3}\pi\right)$

$\frac{5}{6}\pi=\frac{10}{6}\pi$ $\quad \frac{2}{3}\pi=\frac{4}{6}\pi$



c) $\sqrt[3]{1+\sqrt{3}i}$ $\quad |z|=\sqrt{1+3}=2$ $\quad \sin\varphi=\frac{\sqrt{3}}{2}$ $\quad \cos\varphi=\frac{1}{2}=\frac{\pi}{3}$

$z=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

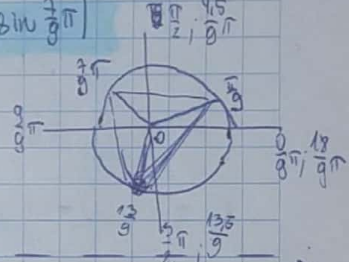
$z_0=2\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)$ $\quad z_1=2\left(\cos\frac{\pi}{9}+\frac{2}{3}\pi+i\sin\frac{\pi}{9}+\frac{2\pi}{3}\right)=$

$=2\left(\cos\frac{7}{9}\pi+i\sin\frac{7}{9}\pi\right)$

$z_2=2\left(\cos\frac{7\pi}{9}+\frac{2}{3}\pi+i\sin\frac{7}{9}\pi+\frac{2}{3}\pi\right)=$

$=2\left(\cos\frac{13}{9}\pi+i\sin\frac{13}{9}\pi\right)$

~~$z_3=2\left(\cos\frac{13}{9}\pi+\frac{2}{3}\pi+i\sin\frac{13}{9}\pi+\frac{2}{3}\pi\right)=2($~~



d) $\sqrt[4]{i}$ $\quad |z|=\sqrt{0^2+1^2}=1$

$\sin\varphi=\frac{1}{1}=1$ $\quad \cos\varphi=0$ $\quad \frac{\pi}{2}$

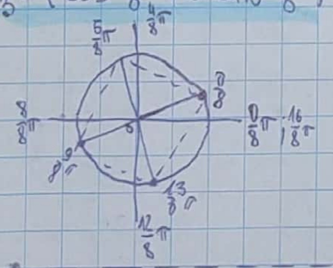
$z=1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$z_0=1\left(\cos\frac{\pi}{8}+i\sin\frac{\pi}{8}\right)$

$z_1=\left(\cos\frac{5}{8}\pi+i\sin\frac{5}{8}\pi\right)$

$z_2=\left(\cos\frac{9}{8}\pi+i\sin\frac{9}{8}\pi\right)$

$z_3=\left(\cos\frac{13}{8}\pi+i\sin\frac{13}{8}\pi\right)$



e) $\sqrt[4]{-1-i}$ $\quad |z|=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$ $\quad \sin\varphi=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$ $\quad \cos\varphi=-\frac{\sqrt{2}}{2}$

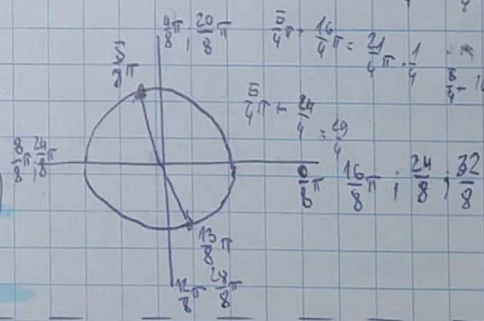
$z=\sqrt{2}\left(\cos\frac{5}{4}\pi+i\sin\frac{5}{4}\pi\right)$

$z_0=\sqrt{2}\left(\cos\frac{5}{8}\pi+i\sin\frac{5}{8}\pi\right)$

$z_1=\sqrt{2}\left(\cos\frac{13}{8}\pi+i\sin\frac{13}{8}\pi\right)$

$z_2=\sqrt{2}\left(\cos\frac{21}{8}\pi+i\sin\frac{21}{8}\pi\right)$

$z_3=\sqrt{2}\left(\cos\frac{29}{8}\pi+i\sin\frac{29}{8}\pi\right)$



f) $\sqrt[6]{i}$ $\quad |z|=\sqrt{0+1^2}=1$ $\quad \sin\varphi=\frac{1}{1}=1$ $\quad \cos\varphi=0$

$z=1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

$z_0=\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

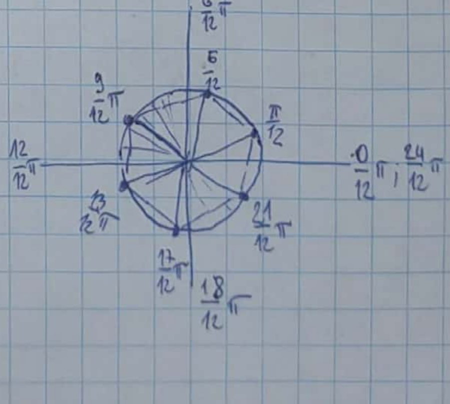
$z_1=\left(\cos\frac{5}{12}\pi+i\sin\frac{5}{12}\pi\right)$

$z_2=\left(\cos\frac{9}{12}\pi+i\sin\frac{9}{12}\pi\right)$

$z_3=\left(\cos\frac{13}{12}\pi+i\sin\frac{13}{12}\pi\right)$

$z_4=\left(\cos\frac{17}{12}\pi+i\sin\frac{17}{12}\pi\right)$

$z_5=\left(\cos\frac{21}{12}\pi+i\sin\frac{21}{12}\pi\right)$



$\frac{17}{2}\pi\cdot\frac{1}{6}$ $\quad \frac{8}{2}+\frac{20}{2}=\frac{21}{2}$

$\cancel{-8}+12i \quad + \cancel{4}+12i$
$-9 \quad -1+24i$

$\to 4+12i+9i^2 \quad 4+12i$

6) a) $z^2-4z+8=0$

$$z_{1/2}=\frac{4\pm\sqrt{16-32}}{2}=\frac{4\pm\sqrt{-16}}{2}=\frac{4\pm4i}{2}=2\pm2i$$

b) $z^2-(2+3i)z-(1+3i)=0$

$$z_{1/2}=\frac{(2+3i)\pm\sqrt{(2+3i)^2-4\cdot(-(1+3i))}}{2}=\frac{2+3i\pm\sqrt{-1}}{2}=$$

$$=\frac{2+3i\pm i}{2}=\frac{\cancel{2+4i}}{\cancel{2}}=\cancel{1+2i}\ \begin{cases}1+2i\\1+i\end{cases}$$

c) $\sqrt[3]{\cancel{4+\sqrt{3}i}}$ $\cancel{|z|=\sqrt{1+3}=2 \quad \sin}$

$x^4+8x^2+16=0 \qquad x^2=-4$

SUB.: $x^2=t \qquad x=\sqrt{-4}$

$t^2+8t+16=0 \qquad x=2i$

$$t_{1/2}=\frac{-8\pm\sqrt{64-64}}{2}=-\frac{8}{2}=-4$$

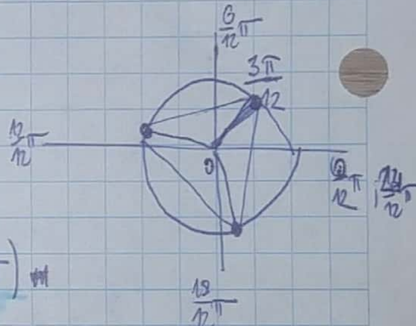
d) $x^4+16=0 \qquad \cancel{x^2=t} \qquad x^4=-2^4$

$\cancel{\text{SUB.: } x^2=t} \qquad \cancel{\text{SUB}} \qquad x=\sqrt[4]{-2^4}$

$\cancel{t^2+16=0} \qquad x=2i$

7) $\sqrt[3]{z}$ $\quad z=3(\cos 0+i\sin 0)$

a) $\frac{2}{3}\pi$ $\quad z_0=5\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

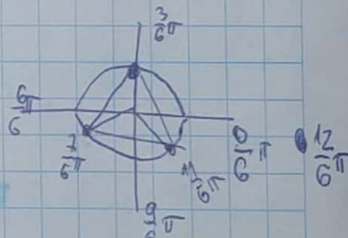


$$z_1=5\left(\cos\frac{\pi}{4}+\frac{2}{3}\pi+i\sin\frac{\pi}{4}+\frac{2}{3}\pi\right)=5\left(\cos\frac{11}{12}\pi+i\sin\frac{11}{12}\pi\right)$$

$\frac{3}{12}\pi$

$$z_2=5\left(\cos\frac{11}{12}\pi+\frac{2}{3}\pi+i\sin\frac{11}{12}\pi+\frac{2}{3}\pi\right)=5\left(\cos\frac{19}{12}\pi+i\sin\frac{19}{12}\pi\right)$$

b) $\frac{2}{3}\pi$ $\quad z_0=-2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

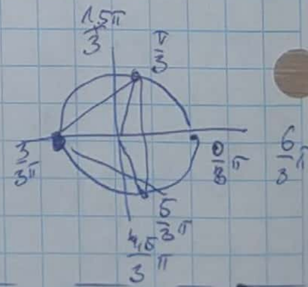


$$z_1=-2\left(\cos\frac{\pi}{2}+\frac{2}{3}\pi+i\sin\frac{\pi}{2}+\frac{2}{3}\pi\right)=-2\left(\cos\frac{7}{6}\pi+i\sin\frac{7}{6}\pi\right)$$

$$z_2=-2\left(\cos\frac{7}{6}\pi+\frac{2}{3}\pi+i\sin\frac{7}{6}\pi+\frac{2}{3}\pi\right)=-2\left(\cos\frac{11}{6}\pi+i\sin\frac{11}{6}\pi\right)$$

$\frac{3}{6}\pi$

c) $z_0=3\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$



$$z_1=3\left(\cos\frac{\pi}{3}+\frac{2}{3}\pi-i\sin\frac{\pi}{3}+\frac{2}{3}\pi\right)=3\left(\cos\pi-i\sin\pi\right)$$

$$z_2=3\left(\cos\frac{3}{3}\pi+\frac{2}{3}\pi-i\sin\frac{3}{3}\pi+\frac{2}{3}\pi\right)=3\left(\cos\frac{5}{3}\pi-i\sin\frac{5}{3}\pi\right)$$

d) $z_0=-6\left(-\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

$$z_1=-6\left(-\cos\frac{\pi}{6}+\frac{2}{3}\pi+i\sin\frac{\pi}{6}+\frac{2}{3}\pi\right)=-6\left(-\cos\frac{5}{6}\pi+i\sin\frac{5}{6}\pi\right)$$

$$z_2=-6\left(-\cos\frac{5}{6}\pi+\frac{2}{3}\pi+i\sin\frac{5}{6}\pi+\frac{2}{3}\pi\right)=-6\left(-\cos\frac{9}{6}\pi+i\sin\frac{9}{6}\pi\right)$$