MÁRIA MATÚŠISKOVÁ

Domáca úloha číslo 01 - postupnosti, AP a GP

1) a) $\{3n(2-n)\}_{n=1}^{\infty}$

1. spôsob

$$a_{n+1}-a_n=\left(3(n+1)\left(2-\overset{-n-1}{(n+1)}\right)\right)-\left(3n(2-n)\right)=\left[\overset{3n-3n^2+3-3n}{(3n+3)(1-n)}\right]-\left[\overset{-6n+3n^2}{6n-3n^2}\right]=$$

$$=3-\cancel{3n^2}-6n+\cancel{3n^2}=3-6n \qquad n=2$$

$$a_{n+1}=a_n+(3-6n) \qquad a_{2+1}=a_2+(3-6.2);\ a_3=0+(-9)=-9$$

2. spôsob

$$\frac{a_{n+1}}{a_n}=\frac{(3n+3)(1-n)}{3n(2-n)}=\frac{(3-3n^2)}{3n(2-n)}=\frac{\cancel{3}(1-n^2)}{\cancel{3}n(2-n)}=\frac{1-n^2}{n(2-n)}$$

$$a_{n+1}=a_n\cdot\frac{1-n^2}{n(2-n)} \qquad n=3 \quad a_4=(-9)\cdot\frac{1-3^2}{3(2-3)}=(-9)\cdot\frac{(-8)}{(-3)}=-\frac{72}{3}=-24$$

- $a_1=3.1(2-1)=3.1=3$
- $a_2=3.2(2-2)=6.0=0$
- $a_3=3.3(2-3)=9(-1)=-9$
- $a_4=3.4(2-4)=12(-2)=-24$
- $a_5=3.5(2-5)=15(-3)=-45$

$$a_n=3n(2-n)$$

$$a_{n+1}=3(n+1)\left(2-\overset{-n-1}{(n+1)}\right)=3(n+1)(1-n)$$

$$n=2 \quad a_{2+1}=3(2+1)(1-2)=3.3(-1)=-9$$

b) $\{(3n-1)(3n+1)\}_{n=1}^{\infty}$

$$a_n=(3n-1)(3n+1) \qquad a_{n+1}=\left(3\overset{3n+3-1}{(n+1)}-1\right)\left(3\overset{3n+3+1}{(n+1)}+1\right)=(3n+2)(3n+4)$$

- $a_1=(3.1-1)(3.1+1)=2.4=8$
- $a_2=(3.2-1)(3.2+1)=5.7=35$
- $a_3=(3.3-1)(3.3+1)=8.10=80$
- $a_4=(3.4-1)(3.4+1)=11.13=143$
- $a_5=(3.5-1)(3.5+1)=14.16=224$

1. spôsob

$$a_{n+1}-a_n=\left[\overset{9n^2+12n+6n+8}{(3n+2)(3n+4)}\right]-\left[\overset{9n^2+3n-3n-1}{(3n-1)(3n+1)}\right]=\cancel{9n^2}+18n+8-\cancel{9n^2}+1=$$

$$=18n+9$$

$$a_{n+1}=a_n+(18n+9)$$

$$n=2 \quad a_3=35+(\overset{36}{18.2}+9)=80$$

c) $\left\{\frac{n+1}{n-3}\right\}_{n=4}^{\infty}$

$$a_n=\frac{n+1}{n-3} \qquad a_{n+1}=\frac{n+2}{n-2}$$

- $a_4=\frac{4+1}{4-3}=\frac{5}{1}=5$
- $a_5=\frac{5+1}{5-3}=\frac{6}{2}=3$
- $a_6=\frac{6+1}{6-3}=\frac{7}{3}$
- $a_7=\frac{7+1}{7-3}=\frac{8}{4}=2$
- $a_8=\frac{8+1}{8-3}=\frac{9}{5}$

2. spôsob

$$\frac{a_{n+1}}{a_n}=\frac{n+2}{n-2}\cdot\frac{n-3}{n+1}=\frac{n^2-3n+2n-6}{n^2+n-2n-2}=\frac{n^2-n-6}{n^2-n-2}$$

$$a_{n+1}=a_n\cdot\frac{n^2-n-6}{n^2-n-2} \qquad n=4 \quad a_5=a_4\cdot\frac{4^2-4-6}{4^2-4-2}=5\cdot\frac{6}{10}=5\cdot\frac{3}{5}=\frac{15}{5}=3$$

MÁRIA MATUŠISKOVÁ

$n^3+3n^2+3n+1 \quad -(n^2+2n+1)-n-1 \quad -3n$

d) $\{n^3-n^2-n\}_{n=1}^{\infty}$ $a_n=n^3-n^2-n$ $a_{n+1}=(n+1)^3-(n+1)^2-(n+1)=n^3+2n^2-1$

$a_1=1^3-1^2-1=-1$

1. spôsob

$a_2=2^3-2^2-2=2$

$a_{n+1}-a_n=\cancel{n^3}+\cancel{2n^2}-1-\cancel{n^3}+\cancel{n^2}+n=3n^2+n-1$

$a_3=3^3-3^2-3=15$

$a_{n+1}=a_n+3n^2+n-1$

$a_4=4^3-4^2-4=44$

$n=1 \quad a_2=(-1)+3\cdot1^2+\cancel{1}-\cancel{1}=2$

$a_5=5^3-5^2-5=95$

---

e) $\{1+\frac{1}{n}\}_{n=1}^{\infty}$ $a_n=1+\frac{1}{n}$ $a_{n+1}=1+\frac{1}{n+1}$

$a_1=1+\frac{1}{1}=2$

2. spôsob

$a_2=1+\frac{1}{2}=\frac{3}{2}$

$\frac{a_{n+1}}{a_n}=\frac{1+\frac{1}{n+1}}{1+\frac{1}{n}}=\frac{\frac{n+1+1}{n+1}}{\frac{n+1}{n}}=\frac{n+2}{n+1}\cdot\frac{n}{n+1}=\frac{n(n+2)}{(n+1)^2}$

$a_3=1+\frac{1}{3}=\frac{4}{3}$

$a_{n+1}=a_n\cdot\frac{n(n+2)}{(n+1)^2}$ $n=2$ $a_3=\frac{3}{2}\cdot\frac{2\cdot4}{3^2}=\frac{3}{2}\cdot\frac{8}{9}=\frac{4}{3}$

$a_4=1+\frac{1}{4}=\frac{5}{4}$

$a_5=1+\frac{1}{5}=\frac{6}{5}$

---

f) $\{\sqrt{n^n}\}_{n=1}^{\infty}$ $a_n=\sqrt{n^n}$ $a_{n+1}=\sqrt{(n+1)^{n+1}}$

$a_1=\sqrt{1^1}=1$

1. spôsob

$a_2=\sqrt{2^2}=2$

$a_{n+1}-a_n=(n+1)^{\frac{n+1}{2}}-n^{\frac{n}{2}}$

$a_3=\sqrt{3^3}=\sqrt{27}=3\sqrt{3}$

$a_{n+1}=a_n+(n+1)^{\frac{n+1}{2}}-n^{\frac{n}{2}}$

$a_4=\sqrt{4^4}=16$

$n=1 \quad a_2=\cancel{1}+(1+1)^{\frac{1+1}{2}}-\cancel{1^{\frac{1}{2}}}=2$

$a_5=\sqrt{5^5}=25\sqrt{5}$

---

g) $\{\frac{(n+1)^2}{n-1}\}_{n=1}^{\infty}$ $a_n=\frac{(n+1)^2}{n-1}$ $a_{n+1}=\frac{(n+2)^2}{n}$

$a_1=\frac{(1+1)^2}{1-1}=\frac{4}{0}=0$

1. spôsob

$a_2=\frac{(2+1)^2}{2-1}=\frac{9}{1}=9$

$n^2+4n+4 \quad n^3-n^2+4n^2-4n+4n-4 \quad n^2+2n+1$

$a_{n+1}-a_n=\frac{(n+2)^2}{n}-\frac{(n+1)^2}{n-1}=\frac{(n+2)^2(n-1)-(n+1)^2n}{n(n-1)}=\frac{\cancel{n^3}+3n^2-4-\cancel{n^3}-2n^2-n}{n^2-n}=\frac{n^2-n-4}{n^2-n}$

$a_3=\frac{(3+1)^2}{3-1}=\frac{16}{2}=8$

$a_{n+1}=a_n+\frac{n^2-n-4}{n^2-n}$ $n=2$ $a_3=9+\frac{2^2-2-4}{2^2-2}=9+\frac{-2}{2}=9-1=8$

$a_4=\frac{(4+1)^2}{4-1}=\frac{25}{3}$

$a_5=\frac{(5+1)^2}{5-1}=\frac{36}{4}=9$

MÁRIA HATUŠISKOVÁ

h) $\left\{\frac{n^2-n-6}{n+2}\right\}_{n=1}^{\infty}$ $\quad a_n=\frac{n^2-n-6}{n+2}=n-3$ $\quad a_{n+1}=\frac{(n+1)^2-(n+1)-6}{n+3}=\frac{(n^2+n-6)}{n+3}=\frac{(n-2)(n+3)}{(n+3)}=n-2$

$a_1=\frac{1^2-1-6}{1+2}=-\frac{6}{3}=-2$

$a_2=2-3=-1$

$a_3=3-3=0$

$a_4=4-3=1$

$a_5=5-3=2$

2. spôsob

$\frac{a_{n+1}}{a_n}=\frac{n-2}{n-3}$

$a_{n+1}=a_n\cdot\frac{n-2}{n-3}$ $\quad n=1$ $\quad a_2=(-2)\cdot\frac{1-2}{1-3}=(-2)\cdot\frac{-1}{-2}=-1$

---

i) $\left\{\binom{n}{2}-\binom{n}{3}\right\}_{n=3}^{\infty}$

$a_n=\frac{n!}{2!(n-2)!}-\frac{n!}{3!(n-3)!}=\frac{n(n-1)(n-2)!}{2!(n-2)!}-\frac{n(n-1)(n-2)(n-3)!}{3\cdot2!(n-3)!}=\frac{n^2-n}{2}-\frac{n^3-3n^2+2n}{6}=$

$=\frac{3n^2-3n-n^3+3n^2-2n}{6}=\frac{-n^3+6n^2-5n}{6}=\frac{(-1)n(n-1)(n-5)}{6}$

$a_1=\frac{(-1)\cdot1(1-1)(1-5)}{6}=0$

$a_2=\frac{(-1)\cdot2(2-1)(2-5)}{6}=1$

$a_3=\frac{(-1)\cdot3(3-1)(3-5)}{6}=\frac{12}{6}=2$

$a_4=\frac{(-1)\cdot4(4-1)(4-6)}{6}=2$

$a_5=\frac{(-1)\cdot5(5-1)(5-5)}{6}=0$

$a_{n+1}=\binom{n+1}{2}-\binom{n+1}{3}=\frac{(n+1)!}{(n-1)!2!}-\frac{(n+1)!}{(n-2)!3!}=\frac{(n+1)n(n-1)!}{(n-1)!2!}-\frac{(n+1)n(n-1)(n-2)!}{(n-2)!3\cdot2!}=$

$=\frac{n^2+n}{2}-\frac{n^3-n}{6}=\frac{3n^2+3n-n^3+n}{6}=\frac{-n^3+3n^2+4n}{6}=\frac{(-1)n(n-4)(n+1)}{6}$

2. spôsob

$\frac{a_{n+1}}{a_n}=\frac{(-1)n(n-1)(n-5)}{6}\cdot\frac{6}{(-1)n(n-4)(n+1)}=\frac{(n-1)(n-5)}{(n-4)(n+1)}$

$a_{n+1}=a_n\cdot\frac{(n-1)(n-5)}{(n-4)(n+1)}$ $\quad n=3$ $\quad a_4=2\cdot\frac{(3-1)(3-5)}{(3-4)(3+1)}=2\cdot\frac{-4}{-4}=2$

---

i) $\left\{\binom{n}{2}-\binom{n}{3}\right\}_{n=3}^{\infty}$ $\quad a_n=\frac{(-1)n(n-1)(n-5)}{6}$

$a_{n+1}=\frac{(-1)n(n-4)(n+1)}{6}$

$a_3=\frac{(-1)\cdot3\cdot(3-1)(3-5)}{6}=$

$=2$

$a_4=\frac{(-1)4(4-1)(4-5)}{6}=2$

$a_5=\frac{(-1)5(5-1)(5-5)}{6}=0$

$a_6=\frac{(-1)6(6-1)(6-5)}{6}=-\frac{30}{6}=-\frac{10}{2}=-5$

$a_7=\frac{(-1)7(7-1)(7-5)}{6}=-\frac{84}{6}=-14$

2. spôsob

$\frac{a_{n+1}}{a_n}=\frac{(-1)n(n-4)(n+1)}{6}\cdot\frac{6}{(-1)n(n-1)(n-5)}=\frac{(n-4)(n+1)}{(n-1)(n-5)}$

$a_{n+1}=a_n\cdot\frac{(n-4)(n+1)}{(n-1)(n-5)}$ $\quad n=3$ $\quad a_4=2\cdot\frac{(3-4)(3+1)}{(3-1)(3-5)}=2\cdot\frac{-4}{-4}=2$

---

j) $\left\{\frac{1+(-1)^n}{2}\right\}_{n=2}^{\infty}$ $\quad a_n=\frac{1+(-1)^n}{2}$ $\quad a_{n+1}=\frac{1+(-1)^{n+1}}{2}$

$a_2=\frac{1+(-1)^2}{2}=1$

1. spôsob

$a_{n+1}-a_n=\frac{1+(-1)^{n+1}}{2}-\frac{1+(-1)^n}{2}=\frac{1+(-1)^n(-1)^1-1-(-1)^n}{2}=\frac{(-1)^n(-1-1)}{2}=\frac{(-1)^n(-2)}{2}=-(-1)^n$

$a_3=\frac{1+(-1)^3}{2}=0$

$a_{n+1}=a_n-(-1)^n$ $\quad n=3$ $\quad a_4=0-(-1)^3=1$

$a_4=\frac{1+(-1)^4}{2}=1$

$a_5=0$

$a_6=1$

MÁRIA MATÚŠISKOVÁ

k) $\left\{3\left(1+\frac{(-1)^n}{2}\right)\right\}_{n=1}^{\infty}$ $\quad a_n = 3\cdot\left(1+\frac{(-1)^n}{2}\right)$ $\quad a_{n+1} = 3\cdot\left(1+\frac{(-1)^{n+1}}{2}\right) =$

$a_1 = 3\left(1+\frac{(-1)^1}{2}\right) = \frac{3}{2}$

$a_2 = 3\left(1+\frac{(-1)^2}{2}\right) = \frac{9}{2}$

$a_3 = 3\left(1+\frac{(-1)^3}{2}\right) = \frac{3}{2}$

$a_4 = \frac{9}{2}$

$a_5 = \frac{3}{2}$

1. spôsob

$$a_{n+1}-a_n = 3\left(1+\frac{(-1)^{n+1}}{2}\right) - 3\left(1+\frac{(-1)^n}{2}\right) = \frac{3(2+(-1)^n(-1)^1) - 3(2+(-1)^n)}{2} =$$

$$= \frac{\cancel{6}+3(-1)^n(-1) - \cancel{6} - 3(-1)^n}{2} = \frac{3((-1)^n(-1)-(-1)^n)}{2} = \frac{3(-1)^n(-1-1)}{2} = \frac{3(-1)^n(-2)}{\cancel{2}} = (-3)(-1)^n$$

$a_{n+1} = a_n + (-3)(-1)^n$ $\quad n=2$ $\quad a_3 = \frac{9}{2} + \frac{(-3)}{1}\cdot\frac{(-1)^2}{1} = \frac{9}{2} - \frac{6}{2} = \frac{3}{2}$

---

l) $\left\{\frac{1}{(n-1)!}\right\}_{n=2}^{\infty}$ $\quad a_n = \frac{1}{(n-1)!}$ $\quad a_{n+1} = \frac{1}{n!}$

2. spôsob

$$\frac{a_{n+1}}{a_n} = \frac{1}{n!}\cdot\frac{(n-1)!}{1} = \frac{1}{n\cancel{(n-1)!}}\cdot\frac{\cancel{(n-1)!}}{1} = \frac{1}{n}$$

$a_{n+1} = a_n \cdot \frac{1}{n}$ $\quad n=4$ $\quad a_5 = \frac{1}{6}\cdot\frac{1}{4} = \frac{1}{24}$

$a_2 = 1$

$a_3 = \frac{1}{2}$

$a_4 = \frac{1}{6}$

$a_5 = \frac{1}{24}$

$a_6 = \frac{1}{120}$

---

m) $\left\{\sqrt{n+1}-\sqrt{n-1}\right\}_{n=1}^{\infty}$ $\quad a_n = \sqrt{n+1}-\sqrt{n-1}$ $\quad a_{n+1} = \sqrt{n+2}-\sqrt{n}$

1. spôsob

$$a_{n+1}-a_n = \left(\sqrt{n+2}-\sqrt{n}\right) - \left(\sqrt{n+1}-\sqrt{n-1}\right) = \sqrt{n+2}-\sqrt{n}-\sqrt{n+1}+\sqrt{n-1}$$

$a_{n+1} = a_n + \sqrt{n+2}-\sqrt{n}-\sqrt{n+1}+\sqrt{n-1}$

$n=3$ $\quad a_4 = \cancel{2}-\cancel{\sqrt{2}} + \sqrt{5}-\sqrt{3}-\cancel{\sqrt{4}}+\cancel{\sqrt{2}} = \sqrt{5}-\sqrt{3}$

$a_1 = \sqrt{2}$

$a_2 = \sqrt{3}-1$

$a_3 = 2-\sqrt{2}$

$a_4 = \sqrt{5}-\sqrt{3}$

$a_5 = \sqrt{6}-2$

---

2) a) $\left\{-\frac{1}{n}\right\}_{n=1}^{\infty}$ $\quad a_n = -\frac{1}{n}$ $\quad a_{n+1} = -\frac{1}{n+1}$

$$\frac{a_{n+1}}{a_n} = \frac{-\frac{1}{n+1}}{-\frac{1}{n}} = \frac{n}{n+1}$$

$a_{n+1} = a_n \cdot \frac{n}{n+1}$

$a_1 = -1$ - dolné oh.

$a_2 = -\frac{1}{2}$

$a_3 = -\frac{1}{3}$

$a_4 = -\frac{1}{4}$

$a_5 = -\frac{1}{5}$

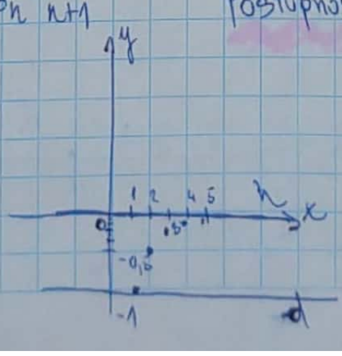
$a_n < 0 \Rightarrow 0$ je horné oh.

$-1 \leq \left\{-\frac{1}{n}\right\}_{n=1}^{\infty} \leq 0$

$d \leq a_n \leq h$

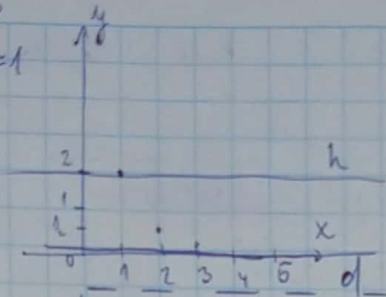
$d=-1 \quad h=0$

Postupnosť $\left\{-\frac{1}{n}\right\}_{n=1}^{\infty}$ je celkovo ohraničená.



MÁRIA MATÚŠIŠKOVÁ

b) $\left\{\frac{2}{n^2}\right\}_{n=1}^{\infty}$



$a_1 = 2$

$a_2 = \frac{1}{2}$

$a_3 = \frac{2}{9}$

$a_4 = \frac{2}{16} = \frac{1}{8}$

$a_5 = \frac{2}{25}$

$h = 2 \; ; \; d = 0$

$0 \leq \left\{\frac{2}{n^2}\right\}_{n=1}^{\infty} \leq 2$

Postupnosť $\left\{\frac{2}{n^2}\right\}_{n=1}^{\infty}$ je celkovo ohraničená.

c) $\left\{\frac{n+2}{n-3}\right\}_{n=4}^{\infty}$ $a_4 = \frac{4+2}{4-3} = \frac{6}{1} = 6$; $a_5 = \frac{5+2}{5-3} = \frac{7}{2} = 3\frac{1}{2}$; $a_6 = \frac{6+2}{6-3} = \frac{8}{3} = 2\frac{2}{3}$; $a_7 = \frac{9}{4} = 2\frac{1}{4}$

$h = 6 \quad d = \{\}$ $\qquad \left\{\frac{n+2}{n-3}\right\}_{n=4}^{\infty} \leq 6$

Postupnosť $\left\{\frac{n+2}{n-3}\right\}_{n=4}^{\infty}$ je ohraničená zhora, celkovo neohraničená.

d) $\left\{1-n^2\right\}_{n=1}^{\infty}$

$a_1 = 1-1^2 = 0 \quad a_2 = 1-2^2 = -3 \quad a_3 = 1-3^2 = -8 \quad a_4 = 1-4^2 = -15 \quad a_5 = -24$

$h = 0 \quad d = \emptyset$ $\qquad \left\{1-n^2\right\}_{n=1}^{\infty} \leq 0$ Postupnosť $\left\{1-n^2\right\}_{n=1}^{\infty}$ je ohraničená zhora, celk. neoh..

e) $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$ $a_1 = \frac{2}{1} = 2$; $a_2 = \frac{3}{2}$; $a_3 = \frac{4}{3}$; $a_4 = \frac{5}{4}$; $a_5 = \frac{6}{5}$

$d = 1 \quad h = 2$ Postupnosť $\left\{\frac{n+1}{n}\right\}_{n=1}^{\infty}$ je

$1 < \left\{\frac{n+1}{n}\right\}_{n=1}^{\infty} \leq 2$ celkovo ohraničená.

f) $\left\{n^2-4n+3\right\}_{n=1}^{\infty}$ $a_1 = 1-4+3 = 0$; $a_2 = 4-8+3 = -1$; $a_3 = 9-12+3 = 0$; $a_4 = 16-16+3 = 3$; $a_5 = 25-20+3 = 8$

$d = -1 \quad h = \emptyset$ $\qquad -1 \leq \left\{n^2-4n+3\right\}_{n=1}^{\infty}$ Postupnosť $\left\{n^2-4n+3\right\}_{n=1}^{\infty}$ je ohran. zdola, celk. neohran..

g) $\left\{\frac{n+1}{n^2-1}\right\}_{n=2}^{\infty}$ $a_n = \frac{n+1}{(n-1)(n+1)} = \frac{1}{n-1}$ $\qquad a_2 = \frac{1}{1} = 1$; $a_3 = \frac{1}{2}$; $a_4 = \frac{1}{3}$; $a_5 = \frac{1}{4}$; ...

$d = 0 \; ; \; h = 1$ $\qquad 0 \leq \left\{\frac{n+1}{n^2-1}\right\} \leq 1$ Postupnosť $\left\{\frac{n+1}{n^2-1}\right\}_{n=1}^{\infty}$ je celkovo ohraničená.

h) $\left\{4(1-(-1)^n\right\}_{n=1}^{\infty}$ $a_1 = 4(1-(-1)^1) = 8$; $a_2 = 4(1-(-1)^2) = 0$; $a_3 = 4(1-(-1)^3) = 8$; $a_4 = 0$

$d = 0 \; ; \; h = 8$ $\qquad 0 \leq \left\{4(1-(-1)^n)\right\}_{n=1}^{\infty} \leq 8$ Postupnosť $\left\{4(1-(-1)^n)\right\}_{n=1}^{\infty}$ je celkovo ohraničená.

i) $\left\{1+\frac{1}{n}\right\}_{n=1}^{\infty}$ $a_1 = 1+\frac{1}{1} = 2$; $a_2 = 1+\frac{1}{2} = \frac{3}{2}$; $a_3 = 1+\frac{1}{3} = \frac{4}{3}$; $a_4 = \frac{5}{4}$; $a_{1000} = \frac{1001}{1000}$

$d = 1 \; ; \; h = 2$ $\qquad 1 < \left\{1+\frac{1}{n}\right\}_{n=1}^{\infty} \leq 2$ Postupnosť $\left\{1+\frac{1}{n}\right\}_{n=1}^{\infty}$ je celkovo ohraničená.

j) $\left\{\frac{1+(-1)^n}{2}\right\}_{n=2}^{\infty}$ $a_2 = \frac{1+(-1)^2}{2} = 1$; $a_3 = 0$; $a_4 = 1$; $a_5 = 0$; $h = 1$; $d = 0$

$0 \leq \left\{\frac{1+(-1)^n}{2}\right\}_{n=2}^{\infty} \leq 1$ Postupnosť $\left\{\frac{1+(-1)^n}{2}\right\}_{n=2}^{\infty}$ je celkovo ohraničená.

k) $\left\{\frac{(n+1)^2}{(n-1)}\right\}_{n=1}^{\infty}$ $a_1 = \frac{4}{0} = 0$; $a_2 = \frac{9}{1} = 9$; $a_3 = \frac{16}{2} = 8$; $a_4 = \frac{25}{3}$; $h = \emptyset$; $d = 0$

$0 \leq \left\{\frac{(n+1)^2}{(n-1)}\right\}_{n=1}^{\infty}$ Postupnosť je ohraničená zdola, celkovo neohraničená.

l) $\left\{3\left(1+\frac{(-1)^n}{2}\right)\right\}_{n=1}^{\infty}$ $a_1 = 3\left(1+\frac{(-1)^1}{2}\right) = \frac{3}{2}$; $a_2 = 3\left(1+\frac{(-1)^2}{2}\right) = \frac{9}{2}$; $a_3 = 3\left(1+\frac{(-1)^3}{2}\right) = \frac{3}{2}$, $h = \frac{9}{2}$; $d = \frac{3}{2}$

$\frac{3}{2} \leq \left\{3\left(1+\frac{(-1)^n}{2}\right)\right\}_{n=1}^{\infty} \leq \frac{9}{2}$ Postupnosť je celkovo ohraničená.

Naskenované pomocou CamScanner

MÁRIA MATUŠIŠKOVÁ

3) a) $a_{n+1} - a_n = \overset{+}{5}$; $\forall n \in N$ $\quad a_{n+1} - a_n > 0$ $\quad$ Postupnosť $\{a_{n+1} - a_n = 5$ je rastúca.

$a_{n+1} = a_n + 5$ $\quad a_{n+1} > a_n$

b) $a_n - a_{n+1} = 2n$; $\forall n \in N$ $\quad$ Postupnosť je klesajúca.

$-a_{n+1} = 2n - a_n$ $\quad a_{n+1} - a_n = -2n$

$a_{n+1} - a_n < 0$

$a_{n+1} = -2n + a_n$

c) $\frac{a_{n+1}}{a_n} = 3$; $\forall n \in N$ $\quad \frac{a_{n+1}}{a_n} > 0$ $\quad$ Postupnosť je rastúca.

d) $\frac{a_n}{a_{n+1}} = \frac{1}{2}$; $\forall n \in N$ $\quad \frac{a_n}{a_{n+1}} > 0$ $\quad$ Postupnosť je rastúca.

* e) $\frac{a_n}{a_{n+1}} = (-1)^n$; $\forall n \in N$ $\quad$ Postupnosť je oscilujúca.

* f) $\frac{a_{n+1}}{a_n} = (-1)^n \frac{1}{n}$; $\forall n \in N$ $\quad$ Postupnosť je oscilujúca.

g) $\left\{\frac{3n+1}{2}\right\}_{n=1}^{\infty}$ $\quad a_n = \frac{3n+1}{2}$ $\quad a_{n+1} = \frac{3(n+1)+1}{2} = \frac{3n+4}{2}$ $\quad a_1 = \frac{3+1}{2} = 2$; $a_2 = \frac{7}{2}$; $a_3 = \frac{10}{2} = 5$; $a_4 = \frac{13}{2}$; $a_5 = 8$; ...

$a_{n+1} - a_n = \frac{3n+4}{2} - \frac{3n+1}{2} = \frac{\cancel{3n}+4-\cancel{3n}-1}{2} = \frac{3}{2}$ $\quad a_{n+1} - a_n > 0$

$\frac{3n+1}{2} < \frac{3n+4}{2}$

Postupnosť je rastúca.

$3n+1 < 3n+4$

$1 < 4$ ČBTD

h) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ $\quad a_n = \frac{n}{n+1}$ $\quad a_{n+1} = \frac{n+1}{n+2}$ $\quad a_1 = \frac{1}{2}$; $a_2 = \frac{2}{3}$; $a_3 = \frac{3}{4}$; $a_4 = \frac{4}{5}$; $a_5 = \frac{5}{6}$

$a_{n+1} - a_n = \frac{n+1}{n+2} - \frac{n}{n+1} = \frac{\cancel{n^2}+2n+1-\cancel{n^2}-2n}{(n+2)(n+1)} = \frac{1}{(n+2)(n+1)}$ $\quad +$

$\frac{1}{(n+2)(n+1)} > 0$ $\quad$ P: $n+2 \neq 0$ $\quad n+1 \neq 0$

$n \neq -2$ $\quad n \neq -1$ $\quad \cancel{1<0}$

$(n+2)(n+1) < 0$ $\quad (n+2)(n+1) > 0$

$1 > 0$ $\quad \cancel{n<-2} \vee \cancel{n<-1}$ $\quad n > -2 \vee n > -1$

$\frac{n}{n+1} < \frac{n+1}{n+2}$ $\quad$ Postupnosť je rastúca.

$\frac{n}{n+1} - \frac{n+1}{n+2} < 0$

$\frac{n^2+2n-n^2-2n-1}{(n+1)(n+2)} < 0$

$\frac{-1}{(n+1)(n+2)} < 0$ $\quad -1 < 0$

MÁRIA MATUŠISKOVÁ

i) $\left\{\frac{n^2-1}{n^2+1}\right\}_{n=1}^{\infty}$ $a_n = \frac{n^2-1}{n^2+1}$; $a_{n+1} = \frac{(n+1)^2-1}{(n+1)^2+1} = \frac{n^2+2n+\cancel{1}-\cancel{1}}{n^2+2n+1+1} = \frac{n^2+2n}{n^2+2n+2}$ $a_1 = \frac{0}{2} = 0$; $a_2 = \frac{3}{5}$; $a_3 = \frac{8}{10}$; $a_4 = \frac{15}{17}$; $a_5 = \frac{24}{26}$

$\frac{n^2-1}{n^2+1} \ ? \ \frac{n^2+2n}{n^2+2n+2}$?: $n^2+2n+2 \neq 0$ $\emptyset$ $x^2+1 \neq 0$

$n_{1|2} = \frac{-2\pm\sqrt{4-8}}{2} = \frac{-2\pm\sqrt{-4}}{2}$ $n \notin R$ $x^2 \neq -1$ $x \notin R$

$(n^2-1)(n^2+2n+2) \ ? \ (n^2+2n)(n^2+1)$

$n^4+2n^3+2n^2-n^2-2n-2 \ ? \ n^4+n^2+2n^3+2n$

$\cancel{n^4}+\cancel{2n^3}+\cancel{n^2}-2n-2 \ ? \ \cancel{n^4}+\cancel{2n^3}+\cancel{n^2}+2n$ $\frac{a_{n+1}}{a_n} =$

$-4n \ ? \ 2$

$n > -\frac{1}{2}$

Postupnosť je rastúca.

---

j) $\left\{\frac{n+4}{n-3}\right\}_{n=4}^{\infty}$ $a_n = \frac{n+4}{n-3}$; $a_{n+1} = \frac{n+5}{n-2}$ $a_4 = \frac{8}{1} = 8$; $a_5 = \frac{9}{2} = 4\frac{1}{2}$; $a_6 = \frac{10}{3} = 3\frac{1}{3}$; $a_7 = \frac{11}{4} = 2$

~~Postupnosť je rastúca.~~

$a_{n+1} - a_n = \frac{n+5}{n-2} - \frac{n+4}{n-3} = \frac{(n+5)(n-3)-(n+4)(n-2)}{(n-2)(n-3)} = \frac{n^2+5n-3n-15-(n^2-2n+4n-8)}{(n-2)(n-3)} = \frac{n^2+2n-15-n^2-2n+8}{(n-2)(n-3)} =$

$= \frac{-7}{(n-2)(n-3)}$ $\ominus$ / $\oplus$ $\} \ominus$ Postupnosť je klesajúca.

---

k) $\left\{\frac{n^2}{2-4n}\right\}_{n=1}^{\infty}$ $a_n = \frac{n^2}{2-4n}$; $a_{n+1} = \frac{(n+1)^2}{2-4(n+1)}$ $a_1 = \frac{1}{-2}$; $a_2 = \frac{4}{-6} = -\frac{2}{3}$; $a_3 = \frac{9}{-10}$; $a_4 = \frac{16}{-14} = -\frac{8}{7}$

?. $a_{n+1} - a_n = \frac{(n+1)^2}{2-4(n+1)} - \frac{n^2}{2-4n} = \frac{n^2+2n+1}{2-(4n+4)} - \frac{n^2}{2-4n} = \frac{n^2+2n+1}{-4n-2} - \frac{n^2}{2-4n} = \frac{n^2+2n+1}{(-1)2(2n+1)} - \frac{n^2}{2(1-2n)} =$

$= \cancel{\frac{(n^2+2n+1)(1-2n)-(-1)(2n+1)n^2}{(-1)2(1-2n)(2n+1)}} = \cancel{\frac{-2n^3-3n^2+1+2n^3+n^2}{(-1)2(1-2n)(2n+1)}} = \frac{2n^2-1}{2(2n-1)(2n+1)}$

$16+12-1=27$
$2\cdot 8+3\cdot 4-1$
$2\cdot 8+4=20$

$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(-1)2(2n+1)} \cdot \frac{(-1)2(2n-1)}{n^2} = \frac{2n^3-n^2+4n^2-2n+2n-1}{2n^3+n^2} = \frac{2n^3+3n^2-1}{2n^3+n^2}$

$\cancel{2n^3}+3n^2-1 > \cancel{2n^3}+n^2$ Postupnosť je rastúca.

$2n^2 > 1$

$n^2 > \frac{1}{2}$ $n > \frac{\sqrt{2}}{2}$

$n > \frac{1}{\sqrt{2}}$

---

l) $\left\{\left(1+\frac{1}{n}\right)^2\right\}_{n=1}^{\infty}$ $a_n = \left(1+\frac{1}{n}\right)^2 = 1 + \frac{2}{n} + \frac{1}{n^2} = \frac{(n+1)^2}{n^2}$ $a_{n+1} = \left(1+\frac{1}{n+1}\right)^2 = 1 + \frac{2}{n+1} + \frac{1}{n^2+2n+1} = \frac{(n+2)^2}{(n+1)^2}$ $a_1 = 1$; $a_2 = \frac{9}{4}$; ...

~~$a_{n+1} - a_n = (1 + \frac{2}{n+1} + \frac{1}{n^2+2n+1}$~~ $\frac{a_{n+1}}{a_n} = \frac{(n+2)^2}{(n+1)^2} \cdot \frac{n^2}{(n+1)^2} = \frac{n^4+4n^3+4n^2}{n^4+4n^3+6n^2+4n+1}$

Postupnosť je klesajúca

$\cancel{n^4}+\cancel{4n^3}+4n^2 < \cancel{n^4}+\cancel{4n^3}+6n^2+4n+1$

$-2n^2-4n-1 < 0$ $\frac{-2+\sqrt{2}}{2} \not< 0$ v $\frac{-2-\sqrt{2}}{2} \not< 0$

$+2n^2+4n+1 > 0$

$n_{1|2} = \frac{-4\pm\sqrt{16-8}}{4} = \frac{-4\pm 2\sqrt{2}}{4} = \frac{\cancel{2}(-2\pm\sqrt{2})}{\cancel{4}} = \frac{-2\pm\sqrt{2}}{2}$

MÁRIA MATÚŠISKOVÁ

m) $\left\{\frac{1-(-1)^n}{4}\right\}_{n=1}^{\infty}$ $a_n=\frac{1-(-1)^n}{4}$ $a_{n+1}=\frac{1-(-1)^{n+1}}{4}$ $a_1=\frac{1-(-1)^1}{4}=\frac{1}{2}; a_2=0; a_3=\frac{1}{2}$

$$a_{n+1}-a_n=\frac{1-(-1)^n\cdot(-1)}{4}-\frac{1-(-1)^n}{4}=\frac{\cancel{1}-(-1)^n(-1)-\cancel{1}+(-1)^n}{4}=\frac{(-1)^n(-1+1)}{4}=0$$

Postupnosť je oscilujúca, konštantná.

n) $\left\{\frac{1+(-1)^n}{n}\right\}_{n=1}^{\infty}$ $a_n=\frac{1+(-1)^n}{n}$ $a_{n+1}=\frac{1+(-1)^{n+1}}{n+1}$ $a_1=\frac{1+(-1)^1}{1}=0$ $a_2=\frac{1+(-1)^2}{2}=1$ $a_3=0; a_4=\frac{1}{2}$

$$a_{n-1}-a_n=\frac{1+(-1)^n(-1)}{n+1}-\frac{1+(-1)^n}{n}=\frac{n+n(-1)^n(-1)-[n+n(-1)^n+1+(-1)^n]}{(n+1)n}=\frac{\cancel{n}+n(-1)^n(-1)-\cancel{n}-n(-1)^n-1-(-1)^n}{(n+1)n}=$$

$$=\frac{(-1)^n(-2n-1)-1}{n(n+1)}=\frac{(-1)^{n+1}(2n+1)-1}{n(n+1)} \text{ ?} \quad \text{osilujúca} \quad ?$$

+

o) $\left\{(-1)^n\frac{1}{n}\right\}_{n=1}^{\infty}$ $a_n=(-1)^n\frac{1}{n}=\frac{(-1)^n}{n}$ $a_{n+1}=(-1)^{n+1}\frac{1}{n+1}=-\frac{(-1)^n}{n+1}$ $a_1=(-1)^1\frac{1}{1}=-1; a_2=(-1)^2\frac{1}{2}=\frac{1}{2}$

$$a_{n+1}-a_n=-\frac{(-1)^n}{n+1}-\frac{(-1)^n}{n}=\frac{(-1)^n(-1)n-[(-1)^n n+(-1)^n]}{n(n+1)}=\frac{(-1)^n(-1)n-(-1)^n n-(-1)^n}{n(n+1)}=$$

$$=\frac{-2n(-1)^n-(-1)^n}{n(n+1)}=\frac{(-1)^n(-2n-1)}{(n+1)n}=\frac{(-1)^{n+1}(+2n+1)}{n(n+1)} \text{ ?}$$

+

p) $\left\{(-1)^n\frac{2}{n^2}\right\}_{n=1}^{\infty}$ $a_n=(-1)^n\frac{2}{n^2}$ $a_{n+1}=(-1)^{n+1}\frac{2}{(n+1)^2}$ $a_1=(-1)^1\frac{2}{1}=-2; a_2=(-1)^2\frac{2}{2^2}=\frac{1}{2};$

$$a_{n+1}-a_n=(-1)^{n+1}\frac{2}{(n+1)^2}-(-1)^n\frac{2}{n^2}=\frac{2(-1)^{n+1}}{(n+1)^2}-\frac{2(-1)^n}{n^2}=\frac{(-1)^{n+1}2(2n^2+2n+1)}{n(n+1)^2}$$ $a_3=(-1)^3\frac{2}{3^2}=-\frac{2}{9}$

$$\frac{a_{n+1}}{a_n}=\frac{\cancel{2}\cancel{(-1)^n}(-1)}{(n+1)^2}\cdot\frac{n^2}{\cancel{2(-1)^n}}=\frac{-n^2}{(n+1)^2}$$ Postupnosť je klesajúca. oscilujúca ?!?

4) a) $\{9-6n\}_{n=1}^{\infty}$ $a_1=9-6\cdot1=3$

$a_{n+1}=a_n+d$ $s_5=\frac{5}{2}(3+(3+4(-6)))=$ $3-24$

$=-\frac{90}{2}=-45$

$a_{n+1}-a_n=d$

$[9-6(n+1)]-[9-6n]=d$ $-6n-6$

$-\cancel{6n}+3-9+\cancel{6n}=d$

$d=-6$ Postupnosť je AP.

b) $\{4-n^2\}_{n=1}^{\infty}$ $a_1=4-1^2=3$

$-n^2-2n-1$ n^2+2n+1

$[4-(n+1)^2]-(4-n^2)=d$

$-\cancel{n^2}-2n+3-4+\cancel{n^2}=d$

$-2n-1=d$

Postupnosť nie je AP.

c) $\{7n+2\}_{n=1}^{\infty}$

$[7(n+1)+2]-(7n+2)=d$

$[7n+7+2]-[7n+2]=d$ 9 $-7n-2$

$d=7$

Postupnosť je AP.

$a_1=7\cdot1+2=9$ $s_5=\frac{5}{2}(9+(9+4\cdot7))=\frac{230}{2}=115$ 46 37 28

d) $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ Postupnosť nie je AP.

$\frac{n+1}{n+2}-\frac{n}{n+1}=d$

$\frac{n^2+2n+1-n^2-2n}{(n+2)(n+1)}=d$

$\frac{1}{(n+2)(n+1)}=d$

e) $\{\log_2 2^n\}_{n=1}^{\infty}$ $\log_a(a^x)=x\log_a(a)$

$\log_2 2^{(n+1)}-\log_2 2^n=d$ $a_1=\log_2 2^1=\log_2 2=1$

$(n+1)\log_2(2)-n\log_2(2)=d$ $a_2=2\log_2 2=2$

rovnaký základ a argument

$(n+1)-n=d$ $s_5=\frac{5}{2}(1+(1+4\cdot1))=$ 5

$d=1$ Postupnosť je AP $=\frac{5\cdot6}{2}=15$

MÁRIA MATUŠISKOVÁ

5) a) $\{a_n\}_{n=1}^{\infty}$

a) $a_2 = 7 \wedge a_3 = 8{,}5$

$a_3 = a_2 + (3-2)d$

$8{,}5 = 7 + 1.d$

$1{,}5 = d$

$a_2 = a_1 + (n-1)d$

$7 = a_1 + 1{,}5$

$a_1 = 5{,}5$

b) $a_4 = -5 \wedge a_6 = 15$

$15 = -5 + (6-4)d$

$10 = 2d$

$d = 5$

$a_4 = a_1 + (n-1)d$

$-5 = a_1 + 3.5$ (15)

$a_1 = -20$

c) $a_1 = 3 \wedge a_3 = -12$

$-12 = 3 + (3-1)d$

$-12 = 3 + 2d$

$-15 = 2d$

$d = -\frac{15}{2}$

$a_2 = 3 + \left(-\frac{15}{2}\right) = -\frac{9}{2}$

$-12 = a_1 + 2 \cdot \left(-\frac{15}{2}\right)$

$a_1 = 3$

d) $a_3 = 2 \cdot a_4 \wedge a_2 = -a_8$

$\frac{a_3}{a_4} = 2 \wedge a_2 + a_8 = 0$

$a_2 + a_8 = 0$

$a_3 = 2 \cdot a_4$

$(a_1 + d) + (a_1 + 7d) = 0$

$(a_1 + 2d) = 2(a_1 + 3d)$ ($2a_1 + 6d$)

$2a_1 + 8d = 0$

$-a_1 - 4d = 0$

$a_1 + 4d = 0$

$-a_1 - 4d = 0$

$d = 0$

$\emptyset$

e) $a_2 - a_1 = 6 \wedge a_{20} - a_{18} = 15$

$(a_1 + d) - a_1 = 6$

$(a_1 + 19d) - (a_1 + 17d) = 15$

$d = 6$

$a_{20} = d = \frac{15}{2}$

?

f) $a_4 + a_5 = 4$

$a_4 \cdot a_5 = -5$

$(a_1 + 3d) + (a_1 + 4d) = 4$

$(a_1 + 3d) = \frac{-5}{(a_1 + 4d))}$

$2a_1 + 7d = 4 \Rightarrow a_1 = \frac{4-7d}{2}$

ak $d = 6$: $= \frac{4-7 \cdot 6}{2} = -\frac{38}{2} = -19$

$\left(\frac{4-7d}{2} + 3d\right)\left(\frac{4-7d}{2} + 4d\right) = -5$

$\frac{4-7d}{2} + \frac{6d}{2} = \frac{4-d}{2}$

$\frac{4-7d}{2} + \frac{8d}{2} = \frac{4+d}{2}$

$\frac{16-d^2}{4} = -5$

$16 - d^2 = -20$

$-d^2 = -36$

$d = \pm 6$

ak $= d = -6$

$a_1 = \frac{4-7(-6)}{2} = \frac{46}{2} = 23$

MÁRIA MATUŠISKOVÁ

6) $S_3 = 60$; $a_1 \cdot a_2 \cdot a_3 = 7500$; $d = ?$, $a_1 = ?$

$S_3 = 60$

$60 = \frac{n}{2}(a_1 + (a_1 + (n-1)d)$

$60 = \frac{3}{2}(a_1 + (a_1 + 2d))$

$60 = \frac{3(2a_1 + 2d)}{2}$

$120 = 3(2a_1 + 2d)$

$40 = 2a_1 + 2d$

$20 = a_1 + d$

$-a_1 = d - 20$

$a_1 = 20 - d$

$a_1 \cdot (a_1 + d) \cdot (a_1 + 2d) = 7500$

$(20-d)[(20-d)+d][(20-d)+2d] = 7500$

$(20-d) \cdot 20 \cdot (20+d) = 7500$

$(400 - d^2)20 = 7500$

$400 - d^2 = 375$

$-d^2 = -25$

$d = \pm 5$

ak $d = 5$ — $a_1 = 20 - 5 = 15$

ak $d = -5$ — $a_1 = 20 + 5 = 25$

---

2 ~~*~~ 7) a) $b_n = 6 + a_n$

---

8)

a) $n = 12$; $a_1 = 7$; $d = 0,5$

$S_{12} = \frac{12}{2}(7 + (7 + 11 \cdot 0,5)) = \frac{12 \cdot 19,5}{2} = \frac{234}{2} = 117$

b) $n = 25$; $b_1 = 70$; $d = -5$

$S_{25} = \frac{25}{2}(70 + (70 + 24(-5))) = \frac{25 \cdot 20}{2} = \frac{500}{2} = 250$

c) $n = 100$; $c_1 = -15$; $d = 0,1$

$S_{100} = \frac{100}{2}(-15 + (-15 + 99 \cdot 0,1)) = \frac{100(-20,1)}{2} = \frac{-2010}{2} = -1005$

d) $n = 20$; $a_1 = -7$; $d = 2$

$S_{20} = \frac{20}{2}(-7 + (-7 + 19 \cdot 2)) = \frac{20 \cdot 24}{2} = \frac{480}{2} = 240$

---

8) Koľko členov postupnosti $\{a_n\}_{n=1}^{\infty}$ musíme sčítať, aby súčet bol 252?

$252 = \frac{n}{2}(a_1 + a_n)$

a) $252 = \frac{n}{2}(7 + (7 + (n-1) \cdot 0,5))$

$252 = \frac{n(13,5 + 0,5n)}{2}$

$504 = 13,5n + 0,5n^2$

$0 = 0,5n^2 + 13,5n - 504$

$n_{1/2} = \frac{-13,5 \pm \sqrt{182,25 + 4 \cdot 504 \cdot 0,5}}{2 \cdot 0,5} = \frac{-13,5 \pm 34,5}{1} = \begin{cases} -48 \\ 21 \end{cases}$

$S_{21} = \frac{21}{2}(7 + (7 + 20 \cdot 0,5)) = \frac{504}{2} = 252$

b) Koľko členov postupnosti $\{b_n\}_{n=1}^{\infty}$ môžeme sčítať, aby ich súčet bol kladný?

$S_{28} = \frac{28}{2}(70 + (70 + 27(-5))) = \frac{140}{2} = 70$

$S_{29} = \frac{29}{2}(70 + (70 + 28(-5))) = 0$

Naskenované pomocou CamScanner

$\overset{5}{1,3,5,7,9}$ $\cdot 9 = 45$

MÁRIA MATUŠISKOVÁ

11 − 99 $a_1 = 11$ $d = 2$

9) a) $S_{45} = \frac{45}{2}(11 + 11 + 44 \cdot 2)) = \frac{45 \cdot 110}{2} = \frac{4950}{2} = 2475$

b) 0, 2, 4, 6, 8 — ⑤ ~~100÷998~~ 102 − 996

S $a_1 = 102$

$d = 6$

10)	a_1	d	n	a_n	s_n
a)	2	0,5	33	18	330
b)	0	0,75	11	5	27,5
c)	3	−0,5	13	−3	0
d)	10	10	14	140	1050

a) $a_1 = 2$, $d = ?$, $n = ?$, $S_n = 330$, $a_n = 18$

$330 = \frac{n}{2}(2 + (2 + (n-1)d))$

$330 = \frac{n}{2}(2 + 18)$

$660 = 20n$

$n = 33$

$a_n = a_1 + (n-1)d$

$18 = 2 + 32d$

$16 = 32d$

$d = \frac{16}{32} = \frac{4}{8} = \frac{1}{2}$

b) $a_1 = 0$, $d = ?$, $n = 11$, $s_n = ?$, $a_n = 5$

$S_{11} = \frac{11}{2}(0 + 5)$

$S_{11} = \frac{55}{2} = 27,5$

$5 = 2 + 4 \cdot d$

$3 = 4d$ $d = \frac{3}{4} = 0,75$

c) $a_1 = 3$, $d = -0,5$, $n = ?$, $a_n = ?$, $S_n = 0$

$0 = \frac{n}{2}(3 + (3 + (n-1)(-0,5)))$ $3,5 - 0,5n + 0,5$

$0 = \frac{6,5n - 0,5n^2}{2}$

$0 = n(6,5 - 0,5n)$ $n = 0$; $n = 13$

$-0,5n = -6,5$

$n = 13$

$a_{13} = 3 + 12(-0,5) = -3$

$S_{30} = \frac{30}{2}(3 + (-3))$

$S_{30} = 0$

d) $a_1 = ?$, $d = ?$, $n = 14$, $a_n = 140$, $s_n = 1050$

$1050 = \frac{14}{2}(a_1 + 140)$

$2100 = 14a_1 + 1960$

$140 = 14a_1$

$a_1 = 10$

$140 = 10 + 13d$

$130 = 13d$

$d = 10$

11) a) $\{6^n\}_{n=1}^{\infty}$ $a_1 = 6^1 = 6$

$a_{n+1} = a_n \cdot q \Rightarrow q = \frac{a_{n+1}}{a_n}$ $q = \frac{6^{n+1}}{6^n} = \frac{\cancel{6^n} \cdot 6}{\cancel{6^n}} = 6$

Postupnosť $\{6^n\}_{n=1}^{\infty}$ je GP.

b) $\{n^2\}_{n=1}^{\infty}$ $q = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2}$ $a_1 =$ Postupnosť nie je GP.

c) $\left\{\left(-\frac{2}{3}\right)^n\right\}_{n=1}^{\infty}$

$q = \frac{\left(-\frac{2}{3}\right)^{n+1}}{\left(-\frac{2}{3}\right)^n} = \frac{\cancel{\left(-\frac{2}{3}\right)^n}\left(-\frac{2}{3}\right)}{\cancel{\left(-\frac{2}{3}\right)^n}} = -\frac{2}{3}$

Postupnosť $\left\{\left(-\frac{2}{3}\right)^n\right\}_{n=1}^{\infty}$ je GP.

$a_1 = \left(-\frac{2}{3}\right)^1 = -\frac{2}{3}$

d) $\left\{\frac{n+2}{n}\right\}_{n=1}^{\infty}$ $q = \frac{\frac{n+3}{n+1}}{\frac{n+2}{n}} = \frac{n+3}{n+1} \cdot \frac{n}{n+2} = \frac{n^2 + 3n}{n^2 + 3n + 2}$

Postupnosť $\left\{\frac{n+2}{n}\right\}_{n=1}^{\infty}$ nie je GP.

e) $\{(\sqrt{2})^{n+2}\}_{n=1}^{\infty}$ $q = \frac{(\sqrt{2})^{n+3}}{(\sqrt{2})^{n+2}} = \frac{\cancel{(\sqrt{2})^n} \cdot (\sqrt{2})^3}{\cancel{(\sqrt{2})^n} \cdot (\sqrt{2})^2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$a_1 = (\sqrt{2})^{1+2} = 2\sqrt{2}$ Postupnosť $\{(\sqrt{2})^{n+2}\}_{n=1}^{\infty}$ je GP.

12) a) $a_2 = 1,5 \wedge a_5 = 40,5$

$a_5 = a_2 \cdot q^3$

$40,5 = 1,5 \cdot q^3$

$27 = q^3$

$q = 3$

$1,5 = a_1 \cdot 3^1$

$a_1 = 0,5$

MÁRIA MAŤUŠISKOVÁ

b) $a_4 = 5 \wedge a_6 = 125$

$a_6 = a_4 \cdot q^2$

$125 = 5 \cdot q^2$

$q^2 = 25$

$q = \pm 5$

ak: $q = 5$

$5 = a_1 \cdot q^4$

$5 = a_1 \cdot 5^4$

$5 = a_1 \cdot 625$

$a_1 = \frac{1}{125}$

ak: $q = -5$

$5 = a_1 (-5)^4$

$a_1 = \frac{1}{125}$

c) $a_1 = 3 \wedge a_3 = -12$

$-12 = 3 \cdot q^2$

$-4 = q^2$

$q \notin R$

d) $a_3 = 48 \wedge a_7 = -3$

$-3 = 48 \cdot q^4$

$-\frac{3}{48} = q^4$

$q \notin R$

e) $a_1 + a_2 = 4 \wedge a_2 - a_4 = -24$

$a_1 + (a_1 \cdot q) = 4 \Rightarrow a_1(1+q) = 4 \Rightarrow a_1 = \frac{4}{1+q}$

$(a_1 \cdot q) - (a_1 \cdot q^3) = -24$

$\left(\frac{4}{1+q} \cdot q\right) - \left(\frac{4}{1+q} \cdot q^3\right) = -24$

$\frac{4q}{1+q} - \frac{4q^3}{1+q} = -24$

$\frac{4q(1-q^2)}{1+q} = -24$

$\frac{4q(1-q)\cancel{(1+q)}}{\cancel{(1+q)}} = -24$

$-4q^2 + 4q + 24 = 0$

$q^2 - q - 6 = 0$

$q_1 = -2$

$q_2 = 3$

ak: $q = -2$

$a_1 = \frac{4}{1-2} = -4$

ak: $q = 3$

$a_1 = \frac{4}{1+3} = 1$

f) $a_2 \cdot a_3 = 9 \wedge a_2 + a_3 = 10$

$a_2 \cdot a_3 = 9$

$a_2 + a_3 = 10 \Rightarrow a_2 = 10 - a_3$

$(10 - a_3) a_3 = 9$

$-a_3^2 + 10a_3 - 9 = 0$

$a_3^2 - 10a_3 + 9 = 0$

$a_{3_1} = 1 \quad a_{3_2} = 9$

ak: $a_3 = 1$

$a_2 = 10 - 1 = 9$

$1 = 9 \cdot q$

$q = \frac{1}{9}$

$9 = a_1 \cdot \frac{1}{9}$

$a_1 = 1$

ak: $a_3 = 9$

$a_2 = 10 - 9 = 1$

$9 = 1q$

$q = 9$

$1 = a_1 \cdot 9$

$a_1 = \frac{1}{9}$

13) 8 648 2

15) a) $n = 12,\ a_1 = 7,\ q = 2$

$S_{12} = 7 \cdot \frac{2^{12}-1}{2-1} = 7 \cdot \frac{4095}{1} = 28672$

b) $n = 5;\ b_1 = 70;\ q = 5$

$S_5 = 70 \cdot \frac{(-5)^5-1}{5-1} = 70 \cdot \frac{(-3125)-1}{4} = \cancel{70}^{35} \cdot \frac{-3126}{\cancel{4}_2} = -54705$

c) $n = 10$

$c_1 = 1500$

$q = 0{,}2$

$S_{10} = 1500 \cdot \frac{0{,}2^{10}-1}{0{,}2-1} \doteq 1875$

$1785 = 7 \cdot \frac{2^n - 1}{1}$

$1785 = 7 \cdot 2^n - 7$

$1778 = 7 \cdot 2^n$

$256 = 2^n$

$2^8 = 2^n$

$n = 8$

$S_8 = 7 \cdot \frac{2^8-1}{2-1} = 7 \cdot \frac{255}{1} = 1785$

MÁRIA MATUŠISKOVÁ

$35\,470 = 70 \cdot \frac{(-5)^n - 1}{-6}$

$35\,470 = \frac{70(-5)^n - 70}{-6}$

$-212\,820 = 70((-5)^n - \cancel{70})$

$-212\,750 = 70(-5)^n$

$-3039{,}29 = (-5)^n$

$\cancel{\emptyset}$

$2\,000 = 1500 \cdot \frac{(0{,}2)^n - 1}{-0{,}8}$

$-1600 = 1500(0{,}2)^n - 1500$

$-100 = 1500(0{,}2)^n$

$-0{,}0\overline{6} = (0{,}2)^n$

$\left(\frac{1}{5}\right)^n = -0{,}0\overline{6}$

$\cancel{\left(\frac{1}{5}\right)^n = -\frac{3}{50}}$

1G)	a_1	q	n	a_n	s_n
a)	90	$\frac{1}{3}$	5	$1,\overline{1}$	$134,\overline{4}$
b)	2	3	7	1458	2186
c)	-4,5	-3	4	121,5	90
d)	3	2	6	96	189

a) $a_1 = 90$

$q = \frac{1}{3}$

$n = 5$

$a_n = ?$

$s_n = ?$

$s_5 = 90 \cdot \frac{\left(\frac{1}{3}\right)^5 - 1}{\frac{1}{3} - 1} = 90 \cdot \frac{\frac{1}{243} - \frac{243}{243}}{\frac{1}{3} - \frac{3}{3}} = 90 \cdot \frac{\frac{242}{243}}{\frac{2}{3}} =$

$= \cancel{90} \cdot \frac{121}{81} = \cancel{9}10 \cdot \frac{121}{9} = 134,\overline{4}$

$a_5 = 90 \cdot \left(\frac{1}{3}\right)^4 = \frac{90}{81} = \frac{10}{9} = 1,\overline{1}$

b) $a_1 = 2$

$q = 3$

$n = ?$

$a_n = 1458$

$s_n = ?$

$s_n = 2 \cdot \frac{3^n - 1}{3 - 1}$

$1458 = 2 \cdot 3^{n-1}$

$729 = 3^n \cdot 3^{-1}$ $n = 7$

$2187 = 3^n$

$3^7 = 3^n$

$s_7 = 2 \cdot \frac{3^7 - 1}{3 - 1} = \cancel{2} \cdot \frac{2186}{\cancel{2}} = 2186$

c) $a_1 = ?$

$q = -3$

$n = 4$

$a_n = 121{,}5$

$s_n = ?$

$s_4 = a_1 \cdot \frac{(-3)^4 - 1}{-3 - 1} = -4{,}5 \cdot \frac{80}{-4} = (-4{,}5)(-20) = 90$

$121{,}5 = a_1 \cdot (-3)^3$

$121{,}5 = a_1 (-27)$

$-4{,}5 = a_1$

d) $a_1 = ?$

$q = 2$

$n = ?$

$a_n = 96$

$s_n = 189$

$189 = a_1 \cdot \frac{2^n - 1}{2 - 1}$

$189 = n \cdot a_1$

$96 = a_1 \cdot 2^{n-1}$

$a_1 = \frac{96}{2^{n-1}}$

$a_1 = \frac{96}{32} = 3$

$189 = \frac{96}{2^n 2^{-1}} \cdot \frac{2^n - 1}{1}$

$\frac{189 \cdot 2^n}{2} = 96 \cdot 2^n - 96$

$189 \cdot 2^n = 192 \cdot 2^n - 192 \quad | -189 \cdot 2^n + 192$

$3 \cdot 2^n = 192$

$2^n = 64$

$2^n = 2^6$

$n = 6$

MÁRIA MATUŠISKOVÁ

17) $a_1+a_3=\frac{10}{9} \wedge \frac{a_4}{a_2}=9$

$a_1+(a_1\cdot q^2)=\frac{10}{9}$

$(a_1\cdot q^3)=9(a_1\cdot q)$

$9a_1+a_1q^2=10 \Rightarrow a_1=\frac{10}{9+q^2}$

$a_1q^3=9a_1q$

$a_1q^3-9a_1q=0$

$a_1q(q^2-9)=0$

$\frac{10}{9+q^2}\cdot q^3=9\cdot\frac{10}{9+q^2}\cdot q$

$\frac{10q^3}{9+q^2}=\frac{90q}{9+q^2}$

$10q^3=90q$

$q^3=9q$

$q^3-9q=0$

$q(q^2-9)=0$

$q(q-3)(q+3)=0$

$q=0 \vee q=3 \vee q=-3$

ak: $q=0$

$a_1=\frac{10}{9+0^2}=1{,}\overline{1}$

$a_2=1{,}\overline{1}\cdot 0=0$

$a_3=0$

$a_4=0$

ak: $q=3$

$a_1=\frac{10}{9+3^2}=\frac{10}{18}=\frac{5}{9}=0{,}\overline{5}$

$a_2=0{,}\overline{5}\cdot 3=1{,}\overline{6}$

$a_3=0{,}\overline{5}\cdot 3^2=5$

$a_4=0{,}\overline{5}\cdot 3^3=15$

ak: $q=-3$

$a_1=0{,}\overline{5}$

$a_2=0{,}\overline{5}(-3)=-1{,}\overline{6}$

$a_3=0{,}\overline{5}(-3)^2=5$

$a_4=0{,}\overline{5}(-3)^3=-15$

18) $a_1=1;\ q=\sqrt{2};\ a_n=32;\ n=?$

$32=1\cdot(\sqrt{2})^{n-1}$

$32=1\cdot(\sqrt{2})^{-1}(\sqrt{2})^n$

$32=\frac{1}{\sqrt{2}}\cdot(\sqrt{2})^n$

$32\sqrt{2}=(\sqrt{2})^n$

$2^5\sqrt{2}=(\sqrt{2})^n$

$2^{\frac{11}{2}}=2^{\frac{n}{2}}$

$\frac{11}{2}=\frac{n}{2}$

$n=11$

$a_{11}=1(\sqrt{2})^{10}$

$a_{11}=2^5$

$a_{11}=32$

19) $a_1=?$

$q=?$

$a_1+a_4=18 \wedge a_2+a_3=12$

$a_1+(a_1q^3)=18 \Rightarrow a_1=\frac{18}{1+q^3}$

$(a_1\cdot q)+(a_1\cdot q^2)=12$

$\frac{18}{1+q^3}\cdot q+\frac{18}{1+q^3}\cdot q^2=12$

$\frac{18q}{1+q^3}+\frac{18q^2}{1+q^3}=12$

$18q+18q^2=12+12q^3$

$9q+9q^2=6+6q^3$

$3q+3q^2=2+2q^3$

$3q(1+q)=2(1+q^2)$

??

MÁRIA MATUŠISKOVÁ

20) $b_1 = ?$ $q = ?$

$b_1 + b_2 + b_3 = 31$

$b_1 + b_3 = 26$

$b_1 + (b_1 \cdot q) + b_3 = 31$ $\Rightarrow \frac{5}{q} + \frac{5}{q} \cdot q + b$

$b_1 + b_3 = 26 \quad | \cdot(-1)$

$b_1 + (b_1 q) + b_3 = 31$

$-b_1 - b_3 = -26$

$b_1 q = 5$

$b_1 = \frac{5}{q}$

$b_1 + (b_1 \cdot q) + (b_1 \cdot q^2) = 31$

$b_1 + (b_1 \cdot q^2) = 26 \quad | \cdot(-1)$

$\cancel{b_1} + b_1 q + \cancel{b_1 q^2} = 31$

$-\cancel{b_1} - \cancel{b_1 q^2} = -26$

$b_1 q = 5$

$q = \frac{5}{b_1}$

$b_1 + b_1 \frac{5}{b_1} + b_1 \left(\frac{5}{b_1}\right)^2 = 31$

$b_1 + 5 + \frac{25}{b_1} = 31$

$b_1^2 + 5b_1 - 5 = 0$

$b_{1/2} = \frac{-5 \pm 6{,}7}{2} = \begin{cases} 0{,}85 \\ -5{,}85 \end{cases}$

$q_1 = 5{,}88$

$q_2 = -0{,}854$

---

21) $b_1 = ?$ $q = ?$

$b_1 + b_2 + b_3 = 195$

$b_3 - b_1 = 120$

$\cancel{b_1} + b_1 q + b_1 q^2 = 195$

$b_1 q^2 - \cancel{b_1} = 120$

$b_1 q + 2b_1 q^2 = 215$

$b_1 + b_1 q + b_1 q^2 = 195$

$b_1(q^2 - 1) = 120 \rightarrow b_1 = \frac{120}{q^2 - 1}$

$\frac{120}{(0{,}435)^2 - 1}$ $0{,}189225$ $= \frac{120}{-0{,}81} = -148{,}15$

$\frac{120}{(1{,}435)^2 - 1} = \frac{120}{1{,}06} = 113{,}2$

$\frac{120}{q^2-1} + \frac{120}{q^2-1} \cdot q + \frac{120}{q^2-1} \cdot q^2 = 195$

$120 + 120q + 120q^2 = 195$

$24 + 24q + 24q^2 = 39$

$24q^2 + 24q - 15 = 0$

$8q^2 + 8q - 5 = 0$

$q_{1/2} = \frac{-8 \pm \sqrt{64 + 160}}{16} = \frac{-8 \pm 14{,}97}{16} = \begin{cases} 0{,}435 \\ 1{,}435 \end{cases}$

---

22) $b_1 = ?$ $b_3 = 18$

$q = ?$ $s_3 = 26$

ak: $q = 2{,}63$

$b_1 = \frac{18}{(2{,}63)^2} = \frac{18}{6{,}9169} = 2{,}6$

ak: $q = -0{,}38$

$b_1 = \frac{18}{(-0{,}38)^2} = \frac{18}{0{,}1444} = 124{,}65$

$26 = b_1 \cdot \frac{q^3 - 1}{q - 1}$

$18 = b_1 \cdot q^2 \Rightarrow b_1 = \frac{18}{q^2}$

$26 = \frac{18}{q^2} \cdot \frac{q^3 - 1}{q - 1}$

$26 = \frac{18(q-1)(q^2+q+1)}{q^2(q-1)}$

$26 = \frac{18(q^2+q+1)}{q^2}$

$26q^2 = 18(q^2 + q + 1)$

$26q^2 = 18q^2 + 18q + 18$

$8q^2 - 18q - 18 = 0$

$4q^2 - 9q - 9 = 0$

$q_{1/2} = \frac{9 \pm \sqrt{81 + 64}}{8} = \frac{9 \pm 12{,}04}{8} = \begin{cases} 2{,}63 \\ -0{,}38 \end{cases}$

MÁRIA MATUŠISKOVÁ

23) $b_1 = ?$ $\quad b_2 - b_1 = 18$

$q = ?$ $\quad b_4 - b_3 = 162$

$b_1 \cdot q - b_1 = 18 \Rightarrow b_1(q-1) = 18 \Leftrightarrow b_1 = \frac{18}{q-1}$

$b_1 \cdot q^3 - b_1 \cdot q^2 = 162$

$\frac{18}{q-1} \cdot q^3 - \frac{18}{q-1} \cdot q^2 = 162$

$18q^3 - 18q^2 = 162(q-1)$

$q^3 - q^2 = 9(q-1)$

$q^2(\cancel{q-1}) = 9(\cancel{q-1})$

$q = \pm 3$

ak: $q = 3$

$b_1 = \frac{18}{2} = 9$

ak: $q = -3$

$b_1 = \frac{18}{-4} = -\frac{9}{2} = -4{,}5$

24) $s_5 = ?$

$a_2 = 4$

$a_4 = 13$

$4 = a_1 \cdot \frac{q^2-1}{q-1}$

$13 = a_1 \cdot \frac{q^4-1}{q-1}$

$4 = a_1 \cdot \frac{(\cancel{q-1})(q+1)}{(\cancel{q-1})}$

$13 = a_1 \cdot \frac{(\cancel{q-1})(q+1)(q^2+1)}{(\cancel{q-1})}$

$4 = a_1 \cdot (q+1) \Rightarrow a_1 = \frac{4}{q+1}$

$13 = a_1 \cdot (q+1)(q^2+1)$

$13 = \frac{4}{\cancel{q+1}} (\cancel{q+1})(q^2+1)$

$13 = 4q^2 + 4$

$9 = 4q^2$

$q^2 = \frac{9}{4}$

$q = \pm\frac{3}{2}$

ak: $q = \frac{3}{2}$

$a_1 = \frac{4}{\frac{3}{2} + \frac{1}{1}} = \frac{4}{\frac{5}{2}} = \frac{8}{5} = 1{,}6$

ak: $q = -\frac{3}{2}$

$a_1 = \frac{4}{-\frac{3}{2} + \frac{1}{1}} = \frac{4}{-\frac{1}{2}} = -8$

25) $\frac{1+2+2^2+\dots+2^{11}}{1+2+2^2+\dots+2^5} = ?$

$1+2+2^2+\dots+2^{11} = \{2^{n-1}\}_{n=1}^{11}$

$1+2+2^2+\dots+2^5 = \{2^{n-1}\}_{n=1}^{5}$

$s_{11} = 1 \cdot \frac{2^{11}-1}{2-1} = \frac{2047}{1} = 2047$

$2 = 1 \cdot q$

$q = 2$

$s_5 = 1 \cdot \frac{2^5-1}{2-1} = 32$

$2 = 1 \cdot q$

$q = 2$

$= \frac{2047}{32} \doteq 63{,}97$

MÁRIA MATUŠISKOVÁ

26) $100^2 - 99^2 + 98^2 - 97^2 + \dots + 2^2 - 1^2 = ?$

$\cancel{\{(100-(n-1))^2\}_{n=1}^{100}}$ $\{(-1)^n(100-n)^2\}_{n=1}^{99}$

$S_{99} = 431000$

$S_{99} = 100^2 \cdot \frac{0{,}98^{98}-1}{0{,}98-1}$ $-0{,}862$ $-0{,}02$

$a_2 = 10000 \cdot 0{,}98$

$a_2 = 9800$

$9604 = 10000 \cdot q^2$

$0{,}9604 = q^2$

$q = 0{,}98$

27)

$a_4 + a_5 = 6$

$a_7 \cdot a_8 = 5$

$a_1 \cdot q^3 + a_1 \cdot q^4 = 6$

$a_1 \cdot q^6 \cdot a_1 \cdot q^7 = 5$

$a_1(q^3+q^4) = 6 = a_1 = \frac{6}{q^3+q^4}$

$\frac{6}{q^3+q^4} \cdot q^6 \cdot \frac{6}{q^3+q^4} \cdot q^7 = 5$

$\frac{6}{q^3(1+q)} \cdot q^6 \cdot \frac{6}{q^3(1+q)} \cdot q^7 = 5$

$\frac{6q^3}{(1+q)} \cdot \frac{6q^4}{(1+q)} = 5$

$36q^7 = 5 + 5q$

$a_4 + a_4 \cdot q = 6 \Rightarrow a_4(1+q) \Rightarrow 6 \Rightarrow a_4 = \frac{6}{1+q}$

$a_4 \cdot q^3 \cdot a_4 \cdot q^4 = 5$

$\frac{6}{1+q} \cdot q^3 \cdot \frac{6}{1+q} q^4 = 5$

$36q^7 = 5 + 5q$

?

Naskenované pomocou CamScanner