

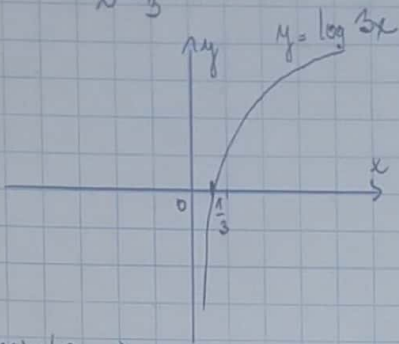
Domáca úloha číslo 04 - logaritmická f., log. rovnice/nerovnice

1) a) $y = \log_3 x$

$\log_3 x = 0$

$3x = 10^0$

$x = \frac{1}{3}$



- $D(f) = (0, \infty)$

- $H(f) = \mathbb{R}$

- parita: $f(-x) = \log_3(-x) = \log_3(-3x)$

$\log_3(-3x) = \log_3(-10^0)$

$x = -\frac{1}{3}$

nie je párna, nie je nepárna

- prostosť: je prostá \Rightarrow $\exists!$ inverzná

- ohraničenosť: nie je ohraničená

- monotónnosť: je rastúca

$x_1 < x_2$
 $3x_1 < 3x_2$

$\log x_1 < \log x_2$

~~\log~~ $f(x_1) < f(x_2)$

- spojitosť: je spojitá

- súradnice priesečníkov s $[0, 0]$

$P_x = [x, 0]$ $0 = \log_3 x$ prípad: $x=0$ $3x=1$ $x=\frac{1}{3}$

$P_y = [0, y]$ $y = \log_3 0$ \emptyset

- periodickosť: nie je periodická

- extrém: nemá

$y = \log_a x$ $a \in \mathbb{R}^+ \setminus \{1\}$

$a > 1$

$0 < a < 1$



b) $y = \log_3(3x-1)$ $\log_3(3x-1) = 0$ $3x-1 = 1$ $3x=2$ $x = \frac{2}{3}$

- $D(f) = (\frac{1}{3}, \infty)$

prípad: $x > \frac{1}{3}$
 $x \in (\frac{1}{3}, \infty)$

- $H(f) = \mathbb{R}$

$-3x = 1$
 $- \log(-3x) = -\frac{1}{3}$

- parita: $f(-x) = \log_3(-3x-1) = -\frac{1}{3}$

nie je ani párna, ani

nepárna

- prostosť: je prostá \Rightarrow $\exists!$ inverzná

- ohraničenosť: nie je ohraničená

- monotónnosť: je rastúca

$x_1 < x_2$

$3x_1 < 3x_2$

$3x_1 - 1 < 3x_2 - 1$

$\log 3x_1 - 1 < \log 3x_2 - 1$

$f(x_1) < f(x_2)$

- spojitosť: je spojitá

- súradnice priesečníkov s $[0, 0]$

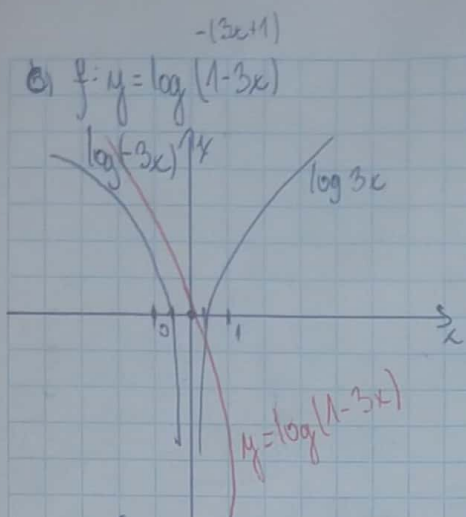
$P_x = [x, 0]$ $x = \frac{2}{3}$

$P_y = [0, y] \emptyset$

- periodickosť: nie je periodická

- extrém: nemá

Matušková



- $D(f) = (-\infty, \frac{1}{3})$
- $H(f) = \mathbb{R}$
- spojitost: je spojitá
- prostota: je prostá
- $\Rightarrow f$: inverzná
- ohraničenost: nie je ohraničená

OP: $1-3x > 0$
 $x < \frac{1}{3}$
 $x=0$
 $y=0$

- párnosť: $f(-x) = \log(1+3x) \neq \frac{2}{3}$
- nie je párna, nie je nepárna

- súradnice s osou $x=0$
 $P_x = [0, 0]$ $0 = \log(1-3x)$ $x=0$
 $P_y = [0, y]$ $y = \log(1-3 \cdot 0)$ $y=0$
 $\log_{10}(1)$ základ = 0

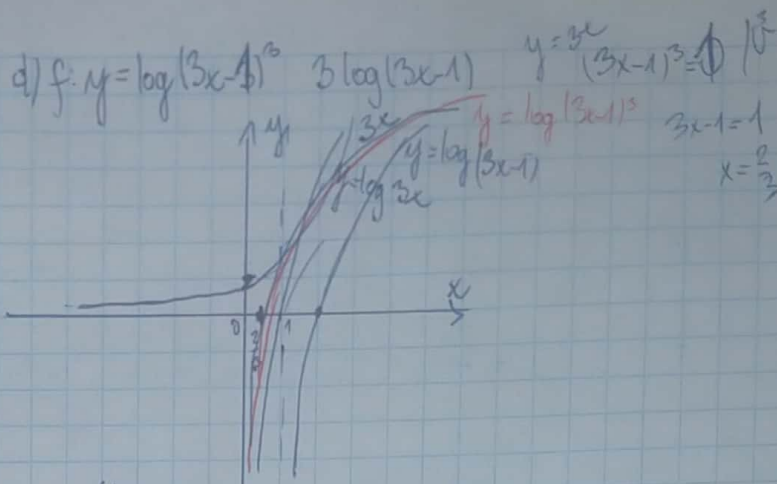
- extrém: nemá

- monotónnosť:

$$x_1 < x_2$$

$$-3x_1 > -3x_2$$

$\log(1-3x_1) > \log(1-3x_2)$
 je klesajúca
 nie je periodická



$$-D(f) = (\frac{1}{3}, \infty)$$

$$-H(f) = \mathbb{R}$$

- spojitost: je spojitá

- prostota: je prostá

$\Rightarrow f$: inverzná

- ohraničenost: nie je ohraničená

- párnosť: $f(-x) = -\log(-3x-1)^3$
 nie je párna, nie je nepárna

- súradnice s osou $[0, 0]$

$$P_x = [x, 0] \quad 0 = \log(3x-1)^3 \quad x = \frac{2}{3}$$

$$P_y = [0, y] \quad y = \log(3 \cdot \frac{2}{3} - 1)^3$$

- extrém: nemá

- monotónnosť: je rastúca

$$x_1 < x_2$$

$$3x_1 < 3x_2$$

$$3x_1 - 1 < 3x_2 - 1$$

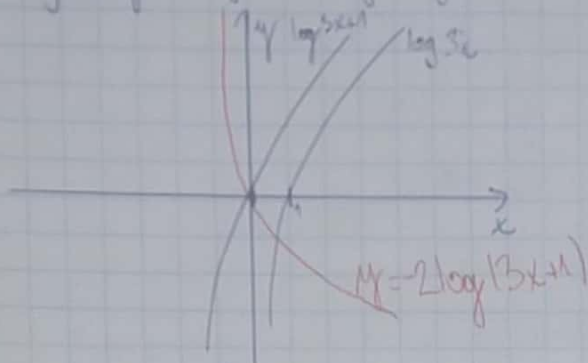
$$\log(3x_1 - 1) < \log(3x_2 - 1)$$

$$3 \log(3x_1 - 1) < 3 \log(3x_2 - 1)$$

- periodickosť: nie je periodická

Matušková

f) $f: y = -2 \log(3x+1) \quad y = \log \frac{1}{(3x+1)^2}$



- $D(f) = \{3x+1 > 0\} \Rightarrow D(f) = (-\frac{1}{3}, \infty)$

- $H(f) = \mathbb{R}$

- spojitost: je spojitá
 $x > -\frac{1}{3}$
 $x \in (-\frac{1}{3}, \infty)$

- prostota: je prostá $\Rightarrow \exists!$ inverzná

- ohraničenost: nie je ohraničená

- párnosť: nie je párna, nie je nepárna

- súradnice s osou (0,0)

$P(x) = [x, 0] \quad y = -2 \log(3x+1)$
 $1 = \frac{1}{(3x+1)^2}$
 $(3x+1)^2 = 1$

$9x^2 + 6x + 1 = 1$

$3x(3x+2) = 0$

$x = 0 \wedge x = -\frac{2}{3}$

$P_y = [0, y] \quad y = -2 \log(3 \cdot 0 + 1)$
 $y = -2 \cdot 0 = 0$

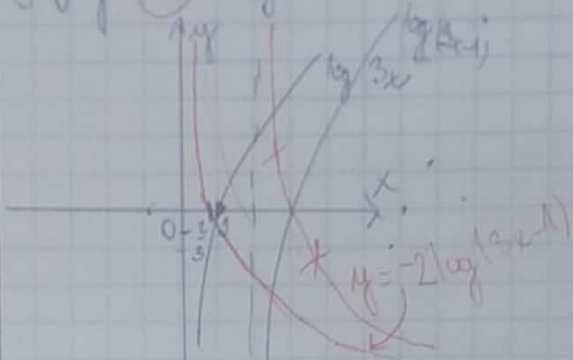
- extrém: nemá

- monotónnosť: je klesajúca

- periodickosť: nie je periodická

$x_1 < x_2 \Rightarrow \frac{1}{(3x_1+1)^2} < \frac{1}{(3x_2+1)^2}$
 $\frac{1}{(3x_1+1)^2} > \log \frac{1}{(3x_1+1)^2}$

g) $f: y = -2 \log \frac{1}{2} \log(3x-1)$



- $D(f) = \{3x-1 > 0\}$

$x > \frac{1}{3}$

$x \in (\frac{1}{3}, \infty)$

- $D(f) = (\frac{1}{3}, \infty)$

- $H(f) = \mathbb{R}$

- prostota: je prostá $\Rightarrow \exists!$ inverzná

- ohraničenost: nie je ohraničená

- spojitost: je spojitá

- párnosť: ani párna, ani nepárna

- súradnice s osou (0,0)

$P_x = \frac{2}{3}$

$P_y = \emptyset$

- extrém: nemá

- monotónnosť: klesajúca

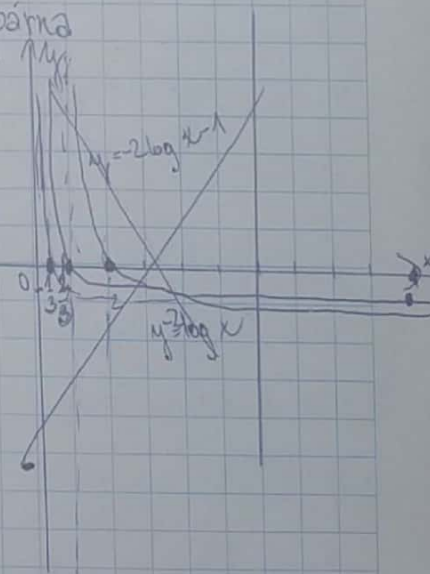
$x_1 < x_2$

$3x_1 - 1 < 3x_2 - 1$

$-2 \log(3x_1 - 1) > -2 \log(3x_2 - 1)$

$f(x_1) > f(x_2)$

- periodickosť: nie je periodická



Madusion

$$y = -\frac{1}{2} \log x \quad \frac{4}{2} = \frac{-\frac{1}{2} \log x}{-\frac{1}{2}} \quad \cdot 2y = \log x \quad y = -\frac{1}{2}$$

$$10^{-2} = 10$$

$$10^{-2} y = 1$$

$$-2y = 0$$

$$P_y = [0, y] =$$

$$y = 2 - \frac{1}{2} \log(2-3x)$$

$$y = 2 - \frac{1}{2} \log 2$$

$$y = \frac{3}{2} \approx 1,5$$

- extrém: nemá

- monotónnosť: je rastúca

- pre $x_1 < x_2$

$$-3x_1 > -3x_2$$

$$2-3x_1 > 2-3x_2$$

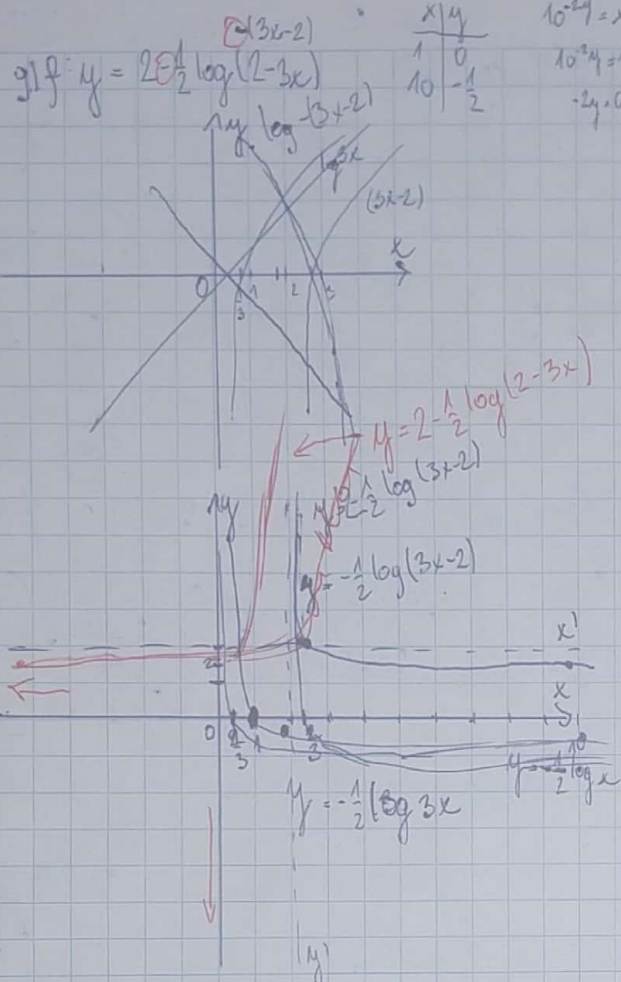
$$\log 2-3x_1 > \log 2-3x_2$$

$$-\frac{1}{2} \log 2-3x_1 < -\frac{1}{2} \log 2-3x_2$$

$$2 - \frac{1}{2} \log(2-3x_1) < 2 - \frac{1}{2} \log(2-3x_2)$$

$$f(x_1) < f(x_2)$$

- periodičnosť: nie je periodická



$$-D(f) = 1) 2-3x > 0$$

$$x < \frac{2}{3}$$

$$x \in (-\infty, \frac{2}{3})$$

$$-H(f) = \mathbb{R}$$

- spojitosť: je spojitá

- prostosť: je prostá \Rightarrow inverzná

- ohranič: je ohraničená

- párnosť: nie je párna ani nepárna

- vrcholice s osou $[0,0]$ OR: $x < \frac{2}{3}$

$$P_x = [x, 0] \quad 0 = 2 - \frac{1}{2} \log(2-3x)$$

$$-2 = -\frac{1}{2} \log(2-3x)$$

$$+4 = \log(2-3x)$$

$$10^4 = 2-3x$$

$$10000 = 2-3x$$

$$-3x = 10000-2$$

$$-3x = 9998$$

$$x = \frac{9998}{-3}$$

$$\log 2x = \log 1$$

$$2x=1$$

$$x=\frac{1}{2}$$

$$\frac{y}{2} = \frac{1}{2} \log x$$

$$2y = \log x$$

$$\frac{x}{10} = \frac{y}{10}$$

$$\log_2(x) = \frac{\ln(x)}{\ln(2)}$$

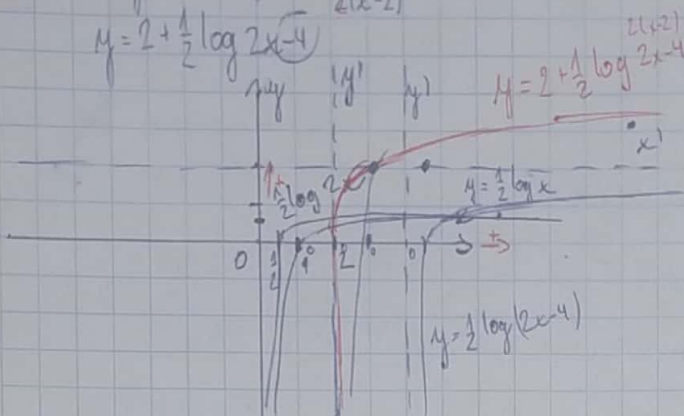
MÁRIA HATUŠKOVÁ

$$h) f: y = 2 + \log_2(2x-4)$$

$$y = 2 + \frac{1}{2} \log_2(2x-4)$$

$$2(x-2)$$

$$y = 2 + \frac{1}{2} \log_2(2x-4)$$



$$-D(f) = 2x-4 > 0 \quad 2x-4 \geq 0$$

$$2x > 4$$

$$x \in (2, \infty)$$

$$x > 2$$

$$P_y = [0, y]$$

-monotónnosť:

$$x_1 < x_2$$

$$2(x_1-2) < 2(x_2-2)$$

$$\frac{1}{2} \log_2 2(x_1-2) < \frac{1}{2} \log_2 2(x_2-2)$$

$$2 + \frac{1}{2} \log_2 2(x_1-2) < 2 + \frac{1}{2} \log_2 2(x_2-2)$$

$$f(x_1) < f(x_2)$$

-spojitosť: je spojitá

-parita: ani párna, ani

nepárna

-prostotať: je prostá $\Rightarrow \exists$ inverzná

-extrémy:

-periodicitať: nie je periodická

-súradnice s osou (0,0)

nema

$$P_x = [x, 0] \quad 0 = 2 + \frac{1}{2} \log_2(2x-4)$$

-ohraničenosť: nie je

ohraničená

$$0 = 2 + \frac{1}{2} \log_2 2 - \frac{1}{2} \log_2(x-2)$$

$$\text{OR: } x > 2$$

$$-4 = \log_2 2 + \log_2(x-2)$$

$$-4 - \log_2(x-2) = \log_2 2$$

$$\log_2(10)^{-4} = \log_2(x-2) = \log_2 2$$

$$\log_2 \frac{(10)^{-4}}{x-2} = \log_2 2$$

$$\frac{1}{(10)^4(x-2)} = 2$$

$$10000x - 20000 = 2$$

$$40000 = 20000x$$

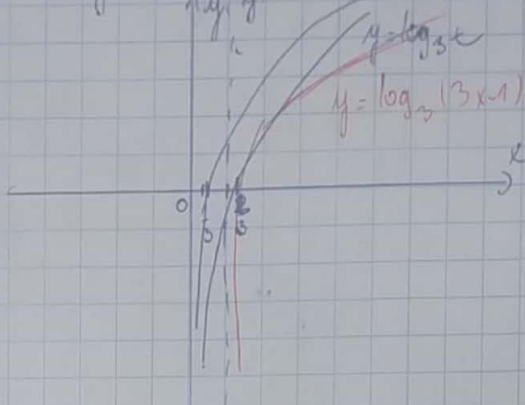
$$x = \frac{40000}{20000} = 2,0$$

$$i) f: y = \log_3(3x-1)$$

$$y = \log_3 3x$$

$$y = \log_3 3x$$

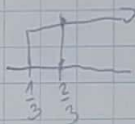
$$y = \log_3(3x-1)$$



$$-D(f) = 3x-1 > 0$$

$$x > \frac{1}{3}$$

$$x \in (\frac{1}{3}, \infty)$$



$$-H(f) = R$$

-súradnice s osou (0,0)

$$P_x = [x, 0]$$

$$P_y = [0, y]$$

$$0 = \log_3(3x-1)$$

$$y = \log_3(1)$$

$$1 = 3x-1$$

$$2 = 3x$$

$$x = \frac{2}{3}$$

-periodicitať: nie je periodická

-spojitosť: je spojitá

-parita: ani párna, ani nepárna

-prostotať: je prostá $\Rightarrow \exists$ inverzná

-extrémy: nema

-ohraničenosť: nie je ohraničená

-monotónnosť: rastúca

$$x_1 < x_2$$

$$3x_1-1 < 3x_2-1$$

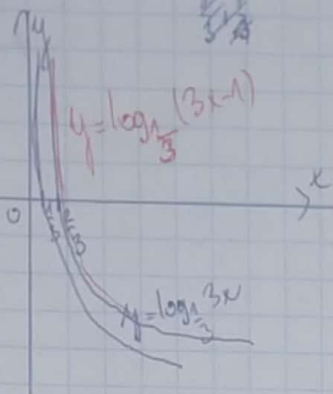
$$\log_3(3x_1-1) < \log_3(3x_2-1)$$

$$f(x_1) < f(x_2)$$

$$0 < \theta < 1$$

*dopositat (k) - (r)

j) $f: y = \log_{\frac{1}{3}}(3x-1)$



-periodickosť: nie je periodická

-extrémy: nemá

$\log_a b \Leftrightarrow a^b = k$ $a \in \mathbb{R}^+ \setminus \{1\}$ $x \in \mathbb{R}^+$ $b \in \mathbb{R}$

2) $x \in \mathbb{R}$

a) $\log_3 \log_4 \log_5 x = 0$

$\log_3 \log_4 \log_5 x \geq \log_3 1$

$\log_4 \log_5 x = 1$

$\log_5 x = 4^1$

$x = 5^4$

$k = \sqrt[5]{625}$

$x = 625$

OR:

$x > 0$

$x \leq 0$

$x \in (0, \infty)$ $x \in \mathbb{R}$

$\log_5 x \leq 0$

$x \leq 1$

$\log_4 (\log_5 x) \leq 0$

$\log_5 x \leq 1$

$x \leq 5$

$x \in (-\infty, 5]$

$x \in (-\infty, 2]$

$x \in (2, \infty)$

b) $\log_4 \log_3 \log_2 x = \frac{1}{2}$

$\log_4 \log_3 \log_2 x = 4^{\frac{1}{2}}$

$\log_3 \log_2 x = 2$

$\log_2 x = 9$

$x = 512$

$k = \sqrt[5]{512}$

OR:

$x \leq 0$

$\log_2 x \leq 0$

$x \leq 1$

$\log_3 (\log_2 x) \leq 0$

$\log_2 x \leq 1$

$x \leq 2$

c) $\log_{\frac{1}{2}} \log_3 (1+20 \log_2 x) = -2$

$\log_3 (1+20 \log_2 x) = \left(\frac{1}{2}\right)^{-2}$

$\log_3 (1+20 \log_2 x) = 4$

$1+20 \log_2 x = 81$

$20 \log_2 x = 80$

$\log_2 x = 4$

$x = 16$

$k = \sqrt[5]{16}$

OR: $x \leq 0$

$\log_3 (1+20 \log_2 x) \leq 0$

$20 \log_2 x \leq -1$

$\log_2 x \leq -\frac{1}{20}$

$x \leq 2^{(-\frac{1}{20})}$

$\log_3 (1+20 \log_2 x) \leq -2$

$1+20 \log_2 x \leq \frac{1}{4}$

$20 \log_2 x \leq 0$

$x \leq 1$

$x \in (-\infty, 1]$

$D(f) = 3x-1 > 0$

$x > \frac{1}{3}$

- $D(f) = (\frac{1}{3}, \infty)$

- $H(f) = \mathbb{R}$

-spojitosť: je spojitá

-parita: ani párna, ani nepárna

-prostosť: nie prostá \Rightarrow

\exists inverzná

-ohraničenosť:

nie je ohraničená

-monotónnosť: klesajúca

$x_1 < x_2$

$3x_1-1 < 3x_2-1$

$\log_{\frac{1}{3}}(3x_1-1) > \log_{\frac{1}{3}}(3x_2-1)$

$f(x_1) > f(x_2)$

-súradnice s osou $[0,0]$

$P_x = [0,0]$ $0 = \log_{\frac{1}{3}}(3x-1)$

$1 = 3x-1$

$x = \frac{2}{3}$

$P_y = [0,4]$

$$d) \log_2 [14 + 2\log_7 (1 + 2\log_{\frac{1}{2}} x)] = 4$$

$$[14 + 2\log_7 (1 + 2\log_{\frac{1}{2}} x)] = 16$$

$$2\log_7 (1 + 2\log_{\frac{1}{2}} x) = 2$$

$$\log_7 (1 + 2\log_{\frac{1}{2}} x) = 1$$

$$1 + 2\log_{\frac{1}{2}} x = 7$$

$$2\log_{\frac{1}{2}} x = 6$$

$$\log_{\frac{1}{2}} x = 3$$

$$x = \frac{1}{8}$$

$$L = \left\{ \frac{1}{8} \right\}$$

$$\text{OR: } (x \leq 0)$$

$$2) 1 + 2\log_{\frac{1}{2}} x \leq 0$$

$$2\log_{\frac{1}{2}} x \leq -1$$

$$\log_{\frac{1}{2}} x \leq -\frac{1}{2}$$

$$x \leq \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$(x \leq \sqrt{2})$$

$$3) 14 + 2\log_7 (1 + 2\log_{\frac{1}{2}} x) \leq 0$$

$$2\log_7 (1 + 2\log_{\frac{1}{2}} x) \leq -14$$

$$\log_7 (1 + 2\log_{\frac{1}{2}} x) \leq -7$$

$$1 + 2\log_{\frac{1}{2}} x \leq \frac{1}{7^7}$$

$$2\log_{\frac{1}{2}} x \leq \frac{1}{7^7} - 1$$

$$\log_{\frac{1}{2}} x \leq \frac{1}{2 \cdot 7^7} - \frac{1}{2}$$

$$x \leq \left(\frac{1}{2}\right)^{\frac{1}{2 \cdot 7^7} - \frac{1}{2}}$$

$$x \in (-\infty, 0) \cup \left(\left(\frac{1}{2}\right)^{\frac{1}{2 \cdot 7^7} - \frac{1}{2}}, \infty\right)$$

$$e) \log_3 \{ 3\log_2 [1 + \log_3 (1 - 2\log_3 x)] \} = \frac{1}{2}$$

$$3\log_2 [1 + \log_3 (1 - 2\log_3 x)] = 3$$

$$1 + \log_3 (1 - 2\log_3 x) = 2$$

$$1 - 2\log_3 x = 1$$

$$\log_3 x = 0$$

$$x = 1$$

$$L = \{1\}$$

$$\text{OR: } (x \leq 0)$$

$$2) 1 - 2\log_3 x \leq 0$$

$$-2\log_3 x \leq -1 \quad x > 0$$

$$\log_3 x \geq \frac{1}{2}$$

$$x \geq \sqrt{3}$$

$$4) 3\log_2 [1 + \log_3 (1 - 2\log_3 x)] \leq 0$$

$$1 + \log_3 (1 - 2\log_3 x) \leq 0$$

$$1 - 2\log_3 x \leq -1$$

$$x \geq 1$$

$$3) 1 + \log_3 (1 - 2\log_3 x) \leq 0$$

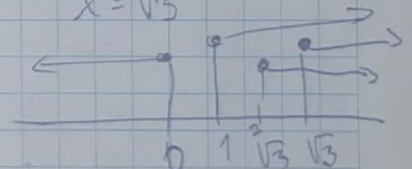
$$\log_3 (1 - 2\log_3 x) \leq -1$$

$$1 - 2\log_3 x \leq \frac{1}{3}$$

$$-2\log_3 x \leq -\frac{2}{3}$$

$$\log_3 x \geq \frac{1}{3}$$

$$x \geq \sqrt[3]{3}$$



$$x \in (-\infty, 0) \cup (1, \infty)$$

$$x \in (1, \infty)$$

$$3) a) \log_2 \frac{1}{8} - 3\log_5 0,2 + \log_3 27 + \log_4 1 = \log_3 x$$

$$\log_2 (2^{-3}) - 3\log_5 (5^{-1}) + \log_3 3^3 + \log_4 1 = \log_3 x$$

$$\log_2 (2^{-3}) - 3\log_5 (5^{-1}) + \log_3 3^3 + 0 = \log_3 x$$

$$-3 - 3 \cdot (-1) + 3 = \log_3 x$$

$$3 = \log_3 x$$

$$3^3 = x$$

$$x = 27$$

$$\log_2 2^x = x$$

$$\text{OR: } x \leq 0$$

$$\frac{2}{10} = \frac{1}{5} = 5^{-1}$$

$$(5^{-1})^3$$

$$8$$

b) $-\log_4 a + \frac{1}{2} \log_4 b^2 - 3 \log_4 2 = \log_4 x$
 $\sim \log_4 a^{-1} + \log_4 b^{\frac{1}{2}} - \log_4 2^3 = \log_4 x$
 $a^{-1} + b^{\frac{1}{2}} - 2^3 = x$
 $\frac{1}{a} + \sqrt{b} - 8 = x$
 $\frac{1}{a} + \sqrt{b} = 8 + x$
 $\frac{1 + a\sqrt{b} - 8a}{a} = x$
 $\frac{a(\sqrt{b}-8)+1}{a} = x$

dopáčiť: $(f) - (g)$
 OR: $a > 0; b > 0; x > 0$

$\frac{\log_4 b^{\frac{1}{2}}}{\log_4 a^{-1}} = \log_4 2^3 = \log_4 x$
 $\frac{\log_4 b^{\frac{1}{2}}}{\log_4 a^{-1}} = \log_4 x \cdot \log_4 2^3$
 $\frac{\log_4 b^{\frac{1}{2}}}{\log_4 a^{-1}} = \log_4 (8x)$
 $\log_4 \frac{b^{\frac{1}{2}}}{a^{-1}} = \log_4 (8x)$
 $\frac{b^{\frac{1}{2}}}{a} = 8x$
 $x = \frac{b^{\frac{1}{2}}}{8a}$

$k = \left\{ \frac{b^{\frac{1}{2}}}{8a} \right\}$

c) $-\log_4 a + \frac{1}{2} \log_4 b^2 - 3 \log_4 2 = \log_4 x$
 $10 \log x^2 + 4 \log x^5 + 3 \log x^3 + 2 \log \sqrt{x} = 100$
 $10 \cdot 2 \log x + 4 \cdot 5 \log x + 3 \cdot 3 \log x + 2 \cdot \frac{1}{2} \log x = 100$ OR: $x > 0$
 $20 \log x + 20 \log x + 9 \log x + \log x = 100$
 $50 \log x = 100$
 $\log x = 2$
 $x = 10^2 = 100$

$k = \left\{ \frac{5 \cdot 100^2}{2} \right\}$

d) $\log x^2 + \log \sqrt{x} - \log \frac{1}{x} = \frac{35}{2}$ (OR: $x > 0$)
 $2 \log x + \frac{1}{2} \log x - (\log 1 - \log x) = \frac{35}{2}$
 $2 \log x + \frac{1}{2} \log x + \log x = \frac{35}{2}$
 $2 \log x + \frac{3}{2} \log x = \frac{35}{2}$
 $2 \log x + \log(x^{\frac{3}{2}}) = \frac{35}{2}$
 $2 \log x + \log x^{\frac{3}{2}} = \frac{35}{2}$
 $\log(x^2 \cdot x^{\frac{3}{2}}) = \frac{35}{2}$
 $\frac{7}{2} \log x = \frac{35}{2}$
 $\log x = 5$
 $x = 10^5 = 100000$

e) $\ln 2x + \ln x^2 - \ln^3 \sqrt{x} = 1 + \ln 2 - \ln x^{-\frac{1}{3}}$
 $\ln 2x + 2 \ln x - \frac{1}{3} \ln x = 1 + \ln 2 - \frac{1}{3} \ln x$
 $\ln 2 + \ln x + 2 \ln x - \frac{1}{3} \ln x = 1 + \ln 2 - \frac{1}{3} \ln x$
 $3 \ln x - \frac{1}{3} \ln x = 1 - \frac{1}{3} \ln x$ OR:
 $3 \ln x = 1$ $x > 0$
 $\ln x = \frac{1}{3}$
 $x = e^{\frac{1}{3}}$
 $x = \sqrt[3]{e}$
 $x \approx 1,396$

$k = \left\{ \frac{3 \sqrt[3]{e}}{2} \right\}$

$k = \left\{ \frac{5 \cdot 100000^2}{2} \right\}$

4) $x \in \mathbb{R}$

OR: $2x-2 \geq 0$

$x-5 > 0$

a) $\log \sqrt{2x-2} = \log(x-5)$

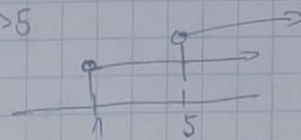
$2x \geq 2$

$x > 5$

$\sqrt{2x-2} = |x-5|^2$

$x \geq 1$

$2x-2 = x^2-10x+25$



$x \in (5, \infty)$

$-x^2+12x-27=0$

$L = \{9\}$

Sk: $\log \sqrt{2 \cdot 5 - 2} = \log(5-5)$

$\log 2 = \log 0$

\emptyset

$\log \sqrt{2 \cdot 9 - 2} = \log(9-5)$

$\log 4 = \log 4 \checkmark$

$x_1 = 3$

$x_2 = 9$

b) $\log \sqrt{2x+3} = \log(x-3)$

OR: $2x+3 \geq 0$

$x-3 > 0$

$\sqrt{2x+3} = |x-3|^2$

$2x \geq -3$

$x > 3$

$2x+3 = x^2-6x+9$

$x \geq -\frac{3}{2}$

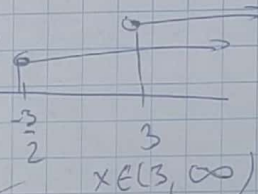
$L = \{4+\sqrt{10}\}$

$-x^2+8x-6=0$

$x^2-8x+6=0$

$x_{1/2} = \frac{8 \pm \sqrt{64-24}}{2} = \frac{8 \pm \sqrt{40}}{2} = \frac{8 \pm 2\sqrt{10}}{2} = 4 \pm \sqrt{10}$

Sk: $\log \sqrt{2(4+\sqrt{10})+3} = \log(4+\sqrt{10}-3)$
 $\log \sqrt{11+2\sqrt{10}} = \log(1+\sqrt{10})$



$x \in (3, \infty)$

c) $\log_2 \sqrt{x+1} = 3 - \log_2 4^{22}$

OR: $x+1 \geq 0$

Sk: $\log_2 \sqrt{3+1} = 3 - \log_2 4$

$\log_2 \sqrt{x+1} = 3-2$

$x \geq -1$

$\log_2 2 = 3$

$\log_2 \sqrt{x+1} = 1 \quad [\log_2 2 = 1]$

$x \in (-1, \infty)$

$1 = 3-2$

$\sqrt{x+1} = 2^1$

$L = \{3\}$

$1=1 \checkmark$

$x+1=4$

$x=3$

d) $\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1$

$\frac{1}{2} \log_8 (x+30) + \frac{1}{2} \log_8 x = 1$

$x = -32$

OR: $x+30 \geq 0$

$\log_8 (\sqrt{x+30} \cdot \sqrt{x}) = \log_8 8$

$\frac{1}{2} \log_8 x + \log_8 (x+30) = 2$

$x = 2$

$x \geq -30$

$\sqrt{x+30} \cdot \sqrt{x} = 8^{1/2}$

$\log_8 (\sqrt{x+30} \cdot \sqrt{x}) = 2$

$L = \{2, 3\}$

$x \geq 0$

$x \in (0, \infty)$

$(x+30)x = 64$

$x^2+30x-64=0$

$x^2+32x-2x-64=0$

$x^2+30x-64=0$

$x(x+32)-2(x+32)=0$

$x_{1/2} = \frac{-30 \pm \sqrt{900+256}}{2}$

$(x+32)(x-2)=0$

dopocitat: $(x-1)$

5) $x \in \mathbb{R}$

a) $\frac{\log(x^2+13)}{2\log(x+5)} = 1$

$\log(x^2+13) = 2\log(x+5)$

$\log(x^2+13) = \log(x+5)^2$

$\log(x^2+13) = \log(x+5)^2$

$x^2+13 = x^2+10x+25$

$-10x = 12$

$x = -\frac{6}{5} = -1\frac{1}{5}$

OR: $x^2+13 > 0$

$x^2 > -13$

\mathbb{R}

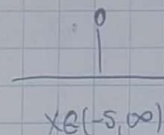
$x+5 \neq 0$

$x \neq -5$

$x \in \mathbb{R}$

$x+5 > 0$

$x > -5$



$K = \left\{ -\frac{6}{5} \right\}$

b) $\frac{\log(2x+13)}{\log(x+5)} = 2$

$\log(2x+13) = 2\log(x+5)$

$\log(2x+13) = \log(x+5)^2$

$2x+13 = x^2+10x+25$

$-x^2-8x-12=0$

$x^2+8x+12=0$

$x^2+6x+2x+12=0$

$x(x+6)+2(x+6)=0$

$(x+2)(x+6)=0$

$x_1 = -2; x_2 = -6$

OR: $2x+13 > 0$

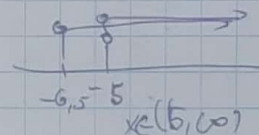
$2x > -13$

$x > -\frac{13}{2} = -6\frac{1}{2}$

$x+5 > 0$

$x > -5$

$x \neq -5$



$K = \{-2, -6\}$

c) $\frac{\log(x^2+3)}{\log(x+3)} = 2$

$\log(x^2+3) = 2\log(x+3)$

$\log(x^2+3) = \log(x+3)^2$

$x^2+3 = x^2+6x+9$

$-6x = 6$

$x = -1$

OR: $x^2+3 \geq 0$

$x^2 \geq -3$

\mathbb{R}

$x+3 \neq 0$

$x \neq -3$

$x+3 > 0$

$x > -3$

$x \in (-3, \infty)$

$K = \{-1\}$

doplniť (x) = (-1)

d) c) XER

$$\log_{a^y} b = \frac{1}{y} \cdot \log_a b$$

a) $\log_2 x + \log_8 x = 8$

$$\log_2 3 = 8$$

$$\log_2 x + \log_{2^3} x = 8$$

OR: $x > 0$

Sk: $\log_2 64 + \log_8 64 = 8$

$$\log_2 x + \frac{1}{3} \log_2 x = 8$$

$$6 \log_2 2 + \log_8 8 = 8$$

$$\log_2 (x \cdot \sqrt[3]{x}) = 8 \log_2 3$$

$$8 = 8 \checkmark$$

$$x \sqrt[3]{x} = 3^8$$

$$L = \{5643\}$$

$$\frac{4}{3} \log_2 x = 8$$

$$\log_2 x = 6$$

$$x = 2^6 = 64$$

b) $\log_9 x + \log_3 x = 6$

OR: $x > 0$

$$\log_{3^2} x + \log_3 x = 6$$

Sk: $\log_9 81 + \log_3 81 = 6$

$$\frac{1}{2} \log_3 x + \log_3 x = 6$$

$$2 \log_3 9 + \log_3 3 = 6$$

$$\log_3 (x^{\frac{3}{2}}) = 6$$

$$6 = 6 \checkmark$$

$$\frac{3}{2} \log_3 x = 6$$

$$L = \{81\}$$

$$\log_3 x = 4$$

$$x = 81$$

c) $\log_7 2 + \log_{49} x = \log_{\frac{1}{7}} 3$

OR: $x > 0$

$$\log_7 2 + \frac{1}{2} \log_7 x = -\frac{1}{2} \log_7 3$$

Sk: $\log_7 2 + \log_{49} 2 = \log_{\frac{1}{7}} 3$

$$\log_7 (2 \cdot \sqrt{x}) = \log_7 \frac{1}{3}$$

$$2x^{\frac{1}{2}} = \frac{1}{3}$$

$$L = \{\frac{1}{36}\}$$

$$x = \frac{1}{36}$$

$$\frac{1}{36} = \frac{1}{36}$$

$$Sk: \log_7 2 + 2 \log_7 x = -\frac{1}{2} \log_7 3$$

$$\log_7 2 + 2 \log_7 \frac{1}{36} = \log_7 \frac{1}{3}$$

$$\log_7 2 + \log_7 \frac{1}{36} = \log_7 \frac{1}{3}$$

d) $\log_{16} x + \log_4 x + \log_2 x = 7$

OR: $x > 0$

$$4 \log_2 x + 2 \log_2 x + \log_2 x = 7$$

Sk: $\log_{16} 2 + \log_4 2 + \log_2 2 = 7$

$$\log_2 (x^4 \cdot x^2 \cdot x) = 7$$

$$\log_2 16 + \log_2 4 + \log_2 2 = 7$$

$$L = \{2\}$$

$$7 \log_2 x = 7$$

$$4 \cdot 2 + 2 \cdot 1 + 1 = 7$$

$$7 = 7 \checkmark$$

$$\log_2 x = 1$$

$$x = 2$$

7) a) $\log_2 (4 \cdot 3^x - 6) = \log_2 (9^x - 6) = 1$

$$3^x = 3$$

$$x = 1$$

$$3^x = 3 \quad x = 1$$

$$9^x - 6 > 0$$

$$\log_2 \left(\frac{4 \cdot 3^x - 6}{9^x - 6} \right) = 1$$

$$\frac{4 \cdot 3^x - 6}{9^x - 6} = 2$$

$$t = 1$$

$$3^x = 1 \quad 3^0$$

$$9^x > 6$$

$$\frac{4 \cdot 3^x - 6}{9^x - 6} = 2$$

$$4 \cdot 3^x - 6 = 2(9^x - 6)$$

$$t^2 - 2t - 3 = 0$$

$$L = \{1, 3\}$$

OR: $4 \cdot 3^x - 6 > 0$

$$4 \cdot 3^x > 6$$

$$3^x > \frac{3}{2}$$

$$3^x > \frac{3}{2}$$

$$b) \log_7(2^x-1) + \log_7(2^x-7) = 1$$

$$\log_7((2^x-1)(2^x-7)) = 1$$

$$\log_7((2^x-1)(2^x-7)) = \log_7 7$$

$$(2^x-1)(2^x-7) = 7$$

~~$$(2^x-1)(2^x-7) = 7$$~~

$$\text{SUB: } t = 2^x$$

$$(t-1)(t-7) = 7$$

$$t^2 - 7t - t + 7 = 7$$

$$t^2 - 8t = 0$$

$$t(t-8) = 0$$

$$t = 0 \vee t = 8$$

$$c) \log 10 + \frac{1}{3} \log(3^{2x} + 271) = 2$$

~~$$\log 10 + 1 + \frac{1}{3} \log(3^{2x} + 271) = 2$$~~

$$\frac{1}{3} \log(3^{2x} + 271) = 1$$

$$\log(3^{2x} + 271) = 3$$

$$3^{2x} + 271 = 10^3$$

$$3^{2x} = 729$$

$$3^{\frac{1}{2}} = 3^3$$

$$\sqrt{x} = 3$$

$$x = 9$$

$$d) 2 + \log_2(3^{x-2} + 1) = \log_2(9^{x-2} + 7)$$

$$2 + \log_2(3^{x-2} + 1) = \log_2(3^{2x-4} + 7)$$

$$2 + \log_2 4 + \log_2(3^{x-2} + 1) = \log_2(3^{2x-4} + 7)$$

$$\log_2(4 \cdot (3^{x-2} + 1)) = \log_2(3^{2x-4} + 7)$$

$$2 \cdot 3^{x-2} + 4 = 3^{2x-4} + 7$$

$$2 \cdot 3^x \cdot 3^{-2} - 3^x \cdot 3^{-4} = 3$$

~~$$2 \cdot 3^x \cdot 3^{-2} - 3^x \cdot 3^{-4} = 3$$~~

$$\text{SUB: } t = 3^{x-2}$$

$$t = 0 \quad 1 \cdot 2^x = 0$$

$$t = 8 \quad 2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$L = \{3\}$$

$$\text{OR: } 2^x - 1 > 0 \quad 2^x > 7$$

$$2^x > 1$$

$$\emptyset$$

$$x > 0$$

$$x \in (0, \infty)$$

$$\text{sk: } \log_7(2^3-1) + \log_7(2^3-7) = 1$$

$$1 + 0 = 1$$

$$1 = 1 \checkmark$$

$$\text{OR: } 3^{2x} + 271 > 0$$

$$3^{2x} > -271$$

$$3^{2x} > -271$$

$$\mathbb{R}$$

$$L = \{9\}$$

$$4t - t^2 = 3$$

$$-t^2 + 4t - 3 = 0$$

$$t^2 - 4t + 3 = 0$$

$$t_1 = 3$$

$$3^{x-2} = 3$$

$$x-2 = 1$$

$$x = 3$$

$$t_2 = 1$$

$$3^{x-2} = 1$$

$$x-2 = 0 \quad x = 2$$

$$L = \{2, 3\}$$

* dopočítaj: $(e) - (c)$

8. xER

$$t = -1 \quad \log x = -1$$

$$\text{OR: } x > 0 \quad x \neq 0$$

$$a) \log x - \frac{1}{\log x} = 0$$

$$x = 10^{-1} = \frac{1}{10}$$

$$x \in (0, \infty)$$

$$\log x = t, \text{ SUB: } \log x = t \quad t = 1 \quad \log x = 1$$

$$t - \frac{1}{t} = 0 \quad t \neq 0$$

$$x = 10$$

$$\frac{t^2 - 1}{t} = 0$$

$$L = \left\{ \frac{1}{10}, 10 \right\}$$

$$t^2 = 1$$

$$t = \pm 1$$

$$b) \log x + \frac{3}{\log x} = 4$$

$$t = 1 \quad \log x = 1$$

$$\text{OR: } x > 0 \quad x \neq 0$$

$$\text{Sk: } \log 10 + \frac{3}{\log 10} = 4$$

$$\text{SUB: } t = \log x$$

$$x = 10$$

$$x \in (0, \infty)$$

$$t + \frac{3}{t} = 4$$

$$t_2 = 3 \quad \log x = 3$$

$$\log 1000 + \frac{3}{\log 1000} = 4$$

$$\frac{t^2 + 3}{t} = 4$$

$$t \neq 0$$

$$x = 10^3$$

$$L = \left\{ 10, 1000 \right\}$$

$$3 \cdot 1 + \frac{3}{3} = 4$$

$$t^2 - 4t + 3 = 0$$

$$c) \log x + \frac{4}{\log x} = 4$$

$$t = 2 \quad \log x = 2$$

$$\text{OR: } x > 0, x \neq 0$$

$$\text{Sk: } \log 100 + \frac{4}{\log 100} = 4$$

$$\text{SUB: } t = \log x$$

$$t \neq 0$$

$$x = 10^2$$

$$2 + \frac{4}{2} = 4 \checkmark$$

$$t^2 + 4t + 4 = 0$$

$$L = \left\{ 2, 2 \right\}$$

$$d) \log x - \frac{20}{\log x} = 1$$

$$t_1 = -4 \quad \log x = -4$$

$$x = 10^{-4} = \frac{1}{10^4}$$

$$\text{OR: } x > 0, x \neq 0$$

$$\text{SUB: } \log x = t \quad t \neq 0$$

$$t_2 = 5 \quad \log x = 5$$

$$\text{Sk: } -4 \log 10 - \frac{20}{-4 \log 10} = 1$$

$$5 \log 10 - \frac{20}{5 \log 10} = 1$$

$$t^2 - t - 20 = 0 \quad t \neq 0$$

$$x = 10^5$$

$$-4 + \frac{20}{-4} = 1$$

$$5 - 4 = 1$$

$$L = \left\{ \frac{1}{10^4}, 10^5 \right\}$$

$$-4 + 5 = 1$$

$$1 = 1 \checkmark$$

$$e) \frac{1}{1 + \log x} + \frac{5}{3 - \log x} = 3$$

$$\frac{4(2+t)}{(1+t)(3-t)} = 3$$

$$3t^2 - 2t - 1 = 0$$

$$L = \left\{ \frac{\sqrt{10}}{10}, 10 \right\}$$

$$\text{SUB: } \log x = t \quad t \neq -1$$

$$t \neq 3$$

$$4(2+t) = 3(1+t)(3-t)$$

$$t_{1,2} = \frac{2 \pm \sqrt{4+12}}{6} = \frac{2 \pm 4}{6} = \frac{1}{3}, -\frac{1}{2}$$

$$\frac{1}{1+t} + \frac{5}{3-t} = 3$$

$$8+4t = 3(1+t)(3-t)$$

$$t_1 = 1 \quad \log x = 1$$

$$\text{OR: } x > 0$$

$$\frac{3-t+5(1+t)}{(1+t)(3-t)} = 3$$

$$8+4t = 3(1+t)(3-t)$$

$$x = 10$$

$$x \neq 0$$

$$x \in (0, \infty)$$

$$\frac{8+4t}{(1+t)(3-t)} = 3$$

$$t^2 + 2t - 5 = 0$$

$$t = -\frac{1}{3} \quad \log x = -\frac{1}{3}$$

$$8+4t = 3(1+t)(3-t)$$

$$x = 10^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{10}}$$

f) $\log x^3 - \frac{6}{\log x} = 7$

sub.: $\log x = t$

$t^3 - \frac{6}{t} = 7$

$\frac{t^4 - 6}{t} = 7$

$\log x^3 - \frac{6}{\log x} = 7 \quad +2 \log x = 2$

$3 \log x - \frac{6}{\log x} = 7$

sub.: $\log x = t$

$3t - \frac{6}{t} = 7$

$\frac{3t^2 - 6}{t} = 7$

$t \neq 0$

$3t^2 - 6 = 7t$

$3t^2 - 7t - 6 = 0$

$t_{1/2} = \frac{7 \pm \sqrt{49 + 72}}{6} = \frac{19}{6} = 2$
 $\frac{-4}{6} = -\frac{2}{3}$

OR: $x > 0 \quad x \neq 0$

$\log_{10} x = 0$

$x = 1$

$K = \left\{ 10^2, \frac{1}{\sqrt[3]{10^2}} \right\}$

g) $\frac{20}{\log x^2} - \log x^3 = 1$

$t = -2 \log x = 2$

$x = 10^{-2} = \frac{1}{10^2}$

OR: $x > 0$
 $x \neq 0$

$\frac{20}{2 \log x} - 3 \log x = 1$

sub.: $t = \log x$

$t = \frac{5}{3}$

$\log x = \frac{5}{3}$

$x = 10^{\frac{5}{3}} = \sqrt[3]{10^5} = 10 \sqrt[3]{10^2}$

$\frac{20}{2t} - 3t = 1$

$t \neq 0$

$\frac{20 - 6t^2}{2t} = 1$

$20 - 6t^2 = 2t$

$-6t^2 - 2t + 20 = 0$

$6t^2 + 2t - 20 = 0$

$3t^2 + t - 10 = 0$

$t_{1/2} = \frac{-1 \pm \sqrt{1 + 120}}{6} = \frac{-1 \pm 11}{6} = \frac{10}{6} = \frac{5}{3}$
 $\frac{-12}{6} = -2$

$K = \left\{ \frac{1}{10^2}, 10 \sqrt[3]{10^2} \right\}$

g) a) $\log_2^2 x + 2 \log_2 x - 3 = 0$

OR: $x > 0$

$\log_2 x^2 + 2 \log_2 x - 3 = 0$

~~\log_2^2~~

sub.: $\log_2 x = t$

~~$K = \left\{ \frac{1}{10^2}, 10^3 \right\}$~~

$t^2 + 2t - 3 = 0$

$t = -3 \log_2 x = -3 \quad x = 2^{-3} = \frac{1}{8}$

$K = \left\{ 2, \frac{1}{8} \right\}$

~~$x = 10^3 = \frac{1}{10^3}$~~

$t = 1 \log_2 x = 1 \quad x = 10 \quad x = 2$

b) $\log^2 x - 3 \log x = \log x^2 - 4$

$\log x^2 - 3 \log x = 2 \log x - 4$

$t = 4 \log x = 4$

OR: $x > 0$
 $x \in (0, \infty)$

~~$\log^2 x - 3 \log x = \log x^2 - 4$~~

sub.: $t = \log x$

$x = 10^4$

~~$\log^2 x + \frac{4}{x}$~~

$t^2 - 3t = 2t - 4$

$t = 1 \log x = 1$

$K = \{ 10, 10^4 \}$

~~$x = 10^3 = \frac{4}{10^3}$~~

$t^2 - 5t + 4 = 0$

$x = 10$

c) $4 \log_9 x (\log_9 x - 1) = 2 + \log_9 x$ OR: $x > 0$ $t = \frac{5 \pm \sqrt{5}}{8}$ $\log_9 x = \frac{5 \pm \sqrt{5}}{8}$
~~SUB: $\log_9 x = t$ $4 \log_9 x^2 - 4 \log_9 x = 2 + \log_9 x$~~
 ~~$4t(t-1) = 2+t$ $4 \log_9 x^2 - 5 \log_9 x - 2 = 0$~~
 ~~$4t^2 - 4t = 2+t$ SUB: $\log_9 x = t$~~
 ~~$4t^2 - 5t - 2 = 0$ $4t^2 - 5t - 2 = 0$~~
 ~~$t_{1/2} = \frac{5 \pm \sqrt{25+40}}{8}$ $t_{1/2} = \frac{5 \pm \sqrt{65}}{8} = \frac{5 \pm \sqrt{5}}{8}$~~
 $k = \left\{ 9^{\frac{5-\sqrt{5}}{8}}, 9^{\frac{5+\sqrt{5}}{8}} \right\}$

10.) $x \in \mathbb{R}$ OR: $x > 0$ $\log_a (b^c) = c \cdot \log_a (|b|)$
a) $x^{\log x} = 10000$ $a^{\log_a x} = x$
 $\log_a (x^{\log x}) = \log 10000$ $\log x = 2$
 $\log (x^{\log x}) = 4 \log 10$ $x = 10^2$
 $\log x (\log |x|) = 4 \log 10$ $\log x = -2$
 $\log x \cdot \log x = 4$ $x = 10^{-2} = \frac{1}{10^2}$
 $\log x^2 = 4$
 $\log x = \pm \sqrt{4}$
 $\log x = \pm 2$

$k = \left\{ 10^2, \frac{1}{10^2} \right\}$

b) $x^{\log 3\sqrt{x}} = 1000$ c) $x^{1-\frac{1}{2}\log x} = 10$ OR: $x > 0$
 $\log 3\sqrt{x} = 1000$ $\log x (x^{1-\frac{1}{2}\log x}) = \log_{10} 10$
 $\log (x^{\log 3\sqrt{x}}) = \log 10^4$ $\log x (1-\frac{1}{2}\log x) (\log |x|) = 1$
 $(\log 3\sqrt{x}) \cdot \log (x^x) = 4 \log 10$ $\log x - \frac{\log x^2}{2} = 1$
 ~~$\log 3 + \log \sqrt{x} \cdot \log x = 4$~~
 ~~$(\log 3 + \log \sqrt{x}) \log x = 4$~~
 ~~$(\log 3\sqrt{x}) \cdot \log x = 4$~~
SUB: $\log x = t$
 $t - \frac{t^2}{2} = 1$
 $\frac{2t - t^2}{2} = 1$
 $2t - t^2 = 2$
 $t^2 - 2t - 2 = 0$
 $k = \emptyset$

11) a) $x^{\log x} = 1000, x > 0$

$t = -1 \quad \log x = -1$
 $x = \frac{1}{10}$

$x > 0$
 $x \in (0, \infty)$

$(\log x \log x) \cdot 2 \log x$

$\log x \log x = 2 \log x + 3 \quad t = 3 \quad \log x = 3$

$\log x^2 = 2 \log x + 3$

$x = 10^3$

Sub: $t = \log x$

$t^2 - 2t + 3 = 0$

$x = \frac{1}{10}, 10^3$

-nerovnice

1)

a) $\frac{\log x + 1}{\log x - 2} < 0$

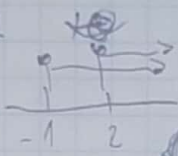
$\log x + 1$

OR: $x > -1 \quad x \neq 2 \quad \log x - 2 \neq 0$

$x > 2$

$\log x \neq 2$

$x \neq 100$



$(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

$(-\infty, 0) \cup (0, 100) \cup (100, \infty)$

$\log(x+1) - \log(x-2) < 0$

$\log(x+1) < \log(x-2)$

$x+1 < x-2$

$x < -2$

$\log x + 1 < 0$

$\log x + 1 > 0$

$\log x - 2 > 0$

$\log x - 2 < 0$

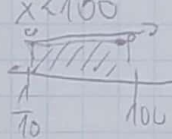
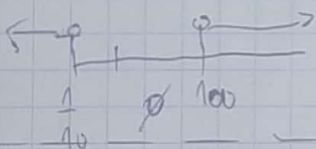
$x < \frac{1}{10}$

$x > \frac{1}{10}$

$x > 100$

$x < 100$

$x \in (\frac{1}{10}, 100)$



Sub: $\log x = t$

$\frac{t+1}{t-2} < 0$

$(-\infty, -1) \cup (2, \infty)$

$t+1$	-	+
$t-2$	-	+
	+	-

b) $\frac{2 - \log x}{\log x} > 1$

Sub: $t = \log x$

c) $\frac{1}{\log x} + \frac{1}{3} \geq 0$ OR: $x > 0$

$2 - \log x - \log x > 1 \quad \frac{2-t}{t} > 1$

$\frac{1}{\log x} + \frac{1}{3} \geq 0$

OR: $x > 0 \quad x > 0 \quad \frac{2-t}{t} - 1 > 0$

$\frac{3 + \log x}{3 \log x} \geq 0$

$\log x \in (0, 1) \quad 2 - \log x > 1 \quad \frac{2-t}{t} > 0$

Sub: $t = \log x$

$\log x > 0 \quad \log x < \frac{1}{2} \quad \frac{2-2t}{2} > 0$

$\frac{3+t}{3t} \geq 0$

$x > 1 \quad x < \sqrt{10} \quad \frac{2(1+t)}{2} > 0$

$(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

$\log x < 1 \quad x < 10 \quad \frac{1+t}{1} > 0$

$3+t$	-	+	+
$3t$	-	+	+
	+	+	+

$x \in (1, 10) \quad x \in (\sqrt{10}, 10) \quad x \in (0, 1)$

$x \in (1, 10) \quad x \in (\sqrt{10}, 10) \quad x \in (0, 1)$

$x = (0, \sqrt{10})$