

## Question 1

1) Probability of a Positive Result for a Sick Person

From the table:  $P(+|D) = 0.009$

So, the probability that the test will return a positive result for a sick person is **0.009** or **0.9%**.

2) Probability of Having the Disease Given a Positive Test Result.

$P(D|+)$ ?

We calculate this using Bayes Theorem

$$P(D|+) = \frac{P(+|D) \cdot P(D)}{P(+)}$$

Where,

$$P(+|D) = 0.009$$

$$P(D) = P(+|D) + P(-|D) \\ = 0.009 + 0.001 = 0.01$$

$$P(+) = P(+|D) \cdot P(D) + P(+|\neg D) \cdot P(\neg D) \\ = (0.009 \cdot 0.01) + (0.099 \cdot 0.99) \\ = 0.00009 + 0.09801 \\ = 0.0981$$

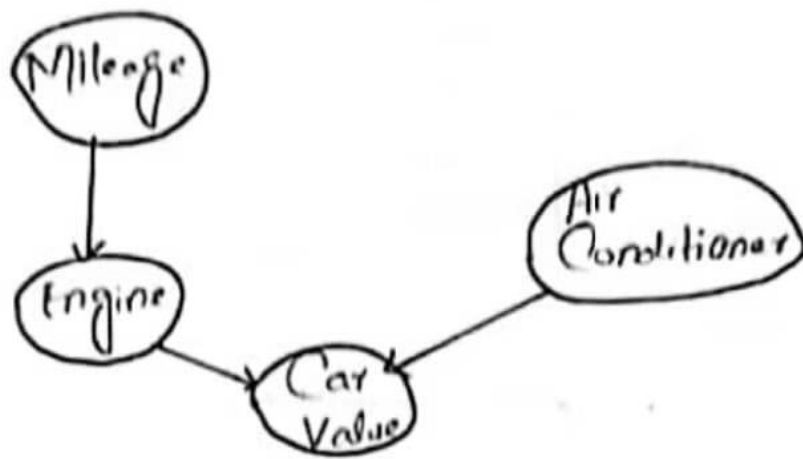
Now, we can calculate

$P(D|+)$ ,

$$P(D|+) = \frac{0.009 \cdot 0.01}{0.0981} = \frac{0.00009}{0.0981}$$

$$= 0.000917$$

So, the probability that you have the disease given a positive test result is **0.0917%** or **0.09%**.



### 1) Mileage

$$P(\text{mileage} = \text{Hi}) = 20/40 = 0.5$$

$$P(\text{mileage} = \text{Lo}) = 20/40 = 0.5$$

Mileage	Probability
Hi	0.5
Low	0.5

### 2) Air Conditioner

$$\text{Working} = 3 + 4 + 1 + 5 + 9 + 0 + 1 + 2 = 25$$

$$\text{Broken} = 1 + 2 + 0 + 4 + 5 + 1 + 0 + 2 = 15$$

$$P(\text{Working}) = 25/40 = 0.625$$

$$P(\text{Broken}) = 15/40 = 0.375$$

Air Cond-	Pro
Working	0.625
Broken	0.375

### 3) Engine

$$P(\text{Hi, Good}) = 3 + 1 + 4 + 2 = 10 = 10/20 = 0.5$$

$$P(\text{Hi, Bad}) = 1 + 0 + 5 + 4 = 10 = 10/20 = 0.5$$

$$P(\text{Lo, Good}) = 9 + 5 + 0 + 1 = 15 = 15/20 = 0.75$$

$$P(\text{Lo, Bad}) = 2 + 2 + 1 + 0 = 5 = 5/20 = 0.25$$

Mileage	Engine	$P(\text{Engine})$
Hi	Good	0.5
Hi	Bad	0.5
Lo	Good	0.75
Lo	Bad	0.25

#### 4) Car value

(1) Engine = Good, Air-Conditioner = working.

$$\text{Car-value} = \text{Hi} = 3 + 9 = 12$$

$$\text{Car-value} = \text{Lo} = 4 + 0 = 4$$

(2) Engine = Good, Air-conditioner = Broken

$$\text{Car-value} = \text{Hi} = 1 + 5 = 6$$

$$\text{Car-value} = \text{Lo} = 2 + 1 = 3$$

(3) Engine = Bad, Air-Conditioner = working

$$\text{Car-value} = \text{Hi} = 1 + 1 = 2$$

$$\text{Car-value} = \text{Lo} = 5 + 2 = 7$$

(4) Engine = Bad, Air-Conditioner = Broken

$$\text{Car-value} = \text{Hi} = 0 + 0 = 0$$

$$\text{Car-value} = \text{Lo} = 4 + 2 = 6$$

Engine	Air-Conditioner	Car-value	$P(\text{Car-value})$
Good	working	Hi	$12/16 = 0.75$
Good	working	Lo	$4/16 = 0.25$
Good	Broken	Hi	$6/9 = 0.667$
Good	Broken	Lo	$3/9 = 0.333$
Bad	working	Hi	$2/9 = 0.222$
Bad	working	Lo	$7/9 = 0.776$
Bad	Broken	Hi	$0/6 = 0$
Bad	Broken	Lo	$6/6 = 1.0$

↔

## Question no 3

Training Set =  $\{(1, 1, 1, +), (1, 0, 1, -), (0, 1, 0, -), (0, 0, 1, -)\}$

•  ~~$h_1$  if  $a$~~

for  $h_1$ :

$$a_1 = 1$$

probability  $h_1 = P(+ | h_1) = \frac{4}{5}$

$$, P(- | h_1) = \frac{1}{5}$$

$$h_2 : P(+ | h_2) = \frac{4}{5}$$

$$, P(- | h_2) = \frac{1}{5}$$

$$h_3: P(+|h_3) = \frac{1}{3}, P(-|h_3) = \frac{2}{3}$$

Using Naive Bayes

$$P(+) = P(h_1) \cdot P(+|h_1) \times P(h_2) \times P(+|h_2) \\ \times P(h_3) \times P(+|h_3)$$

$$= \frac{1}{3} \times \frac{4}{5} \times \frac{1}{3} \times \frac{4}{5} \times \frac{1}{3} \times \frac{1}{3}$$

$$= 0.00790$$

$$P(-) = P(h_1) \times P(-|h_1) \times P(h_2) \times P(-|h_2) \\ \times P(h_3) \times P(-|h_3)$$

$$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{5} \times \frac{1}{3} \times \frac{2}{3}$$

$$= 0.000987$$

As  $P(+) > P(-)$   
So, Naive Bayes Classifier classify

$x = (1, 1, 0)$  as +.

