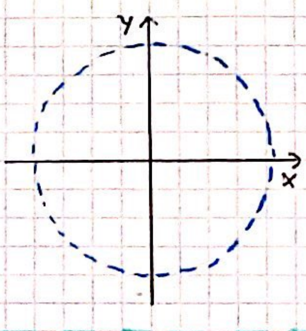
trigonometry
by Maria Delgado

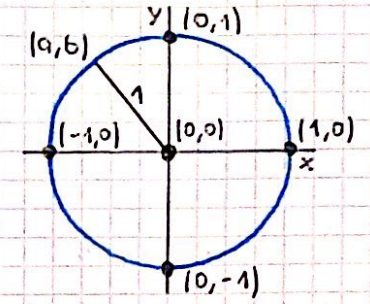
What are sin, cos, tan?

- We are talking about circles in trig



UNIT CIRCLE

1. It's a circle
2. Centered at the origin
3. With radius 1



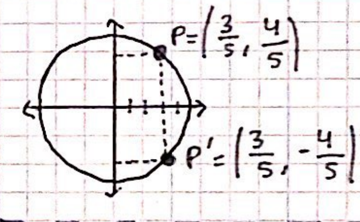
Equation of a circle $x^2+y^2=r^2 \dashrightarrow (a)^2+(b)^2=1$

Practice

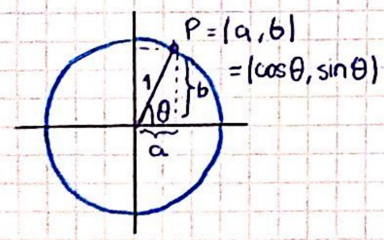
"The point $P=\left(\frac{3}{5}, b\right)$ lies on the unit circle. Solve for b."

Using $x^2+y^2=r^2 \rightarrow \left(\frac{3}{5}\right)^2+y^2=1$

$= \frac{9}{25}+b^2=1 \qquad \therefore b=\pm\frac{4}{5}$



sine and cosine

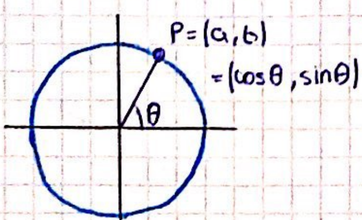


$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{1} = b \qquad \sin\theta = b$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{1} = a \qquad \cos\theta = a$

"We define $\cos\theta$ and $\sin\theta$ as the x and y coordinates of a point on the unit circle

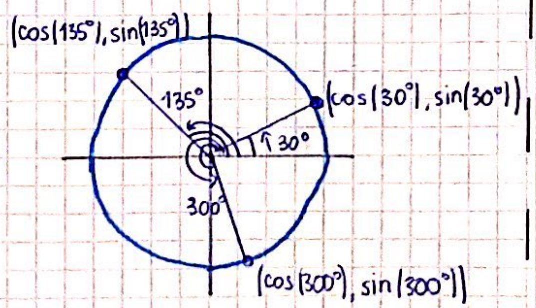
tan and inverse tan



$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{b}{a}$

$\rightarrow \theta = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$\rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

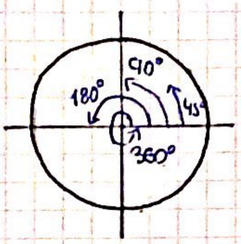


Degrees and Radians

- What are radians?

  - Different ways of expressing grades

$\approx$ radians = rads = (Not need a unit attached to it)



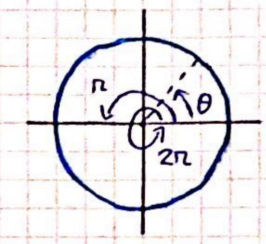
$360° = 2\pi$ rads

$180° = \pi$ rads

$90° = \pi/2$ rads

$45° = \pi/4$ rads

- Why radians?

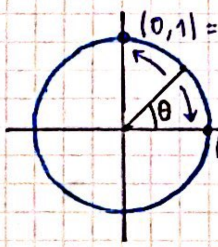


circumference $C = 2\pi r$

$C = 2\pi$, $C_{1/2} = 2\pi/2 = \pi$

$$\boxed{1° = \frac{\pi}{180} \text{ rad}}$$

Special angles

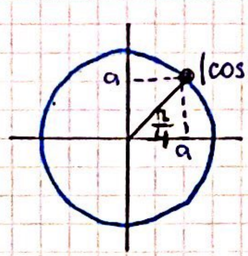


$(0,1) = (\cos(\frac{\pi}{2}), \sin(\frac{\pi}{2})) \to \cos(\frac{\pi}{2}) = 0$

$\quad \llcorner \sin(\frac{\pi}{2}) = 1$

$(1,0) = (\cos 0°, \sin 0°) \to \cos(0) = 1$

$\quad \llcorner \sin(0) = 0$



$(\cos(\frac{\pi}{4}), \sin(\frac{\pi}{4})) = (a,a) \to \cos\frac{\pi}{4} = a = \frac{\sqrt{2}}{2}$

$\quad \llcorner \sin\frac{\pi}{4} = a = \frac{\sqrt{2}}{2}$

$x^2 + y^2 = 1$

$2a^2 = 1 \longrightarrow a = \pm\frac{\sqrt{2}}{2}$

Practice

"The point $Q = (c,d)$ lies on the unit circle, and makes a 45° angle, with x axis. c,d?"

$45° = \pi/4$

$Q = (\cos(\pi/4), \sin(\pi/4))$

$\therefore c = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\therefore d = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

"The point $R = (g,h)$ lies on the unit circle and makes a 45° angle, with y axis. g,h?"

$45° = \pi/4$

$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

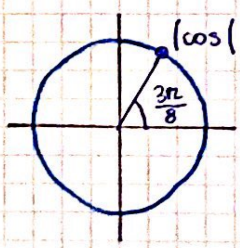
$\therefore h = \frac{\sqrt{2}}{2} = \sin(\frac{3\pi}{4})$

$\therefore g = \frac{-\sqrt{2}}{2} = \cos(\frac{3\pi}{4})$

Useful trig identities

Pythagorean Identity $\to \cos(\theta)^2 + \sin(\theta)^2 = 1$

"You are told that $\cos(\frac{3\pi}{8}) = \frac{1}{2}\sqrt{2-\sqrt{2}}$. What is $\sin(\frac{3\pi}{8})$?"



$(\cos(\frac{3\pi}{8}), \sin(\frac{3\pi}{8}))$

$x^2 + y^2 = 1$

$(\cos(\frac{3\pi}{8})^2) + (\sin(\frac{3\pi}{8})^2) = 1$

$(\frac{1}{2}\sqrt{2-\sqrt{2}})^2 + \sin^2(\frac{3\pi}{8}) = 1$

$\to \frac{1}{2} - \frac{\sqrt{2}}{4} + \sqrt{\frac{1-\cos(\frac{3\pi}{4})}{2}} = 1$

$= \frac{1}{2} - \frac{\sqrt{2}}{4} + \sqrt{\frac{2+\sqrt{2}}{4}} = 1$

$\therefore \sin(\frac{3\pi}{8}) = \frac{\sqrt{2+\sqrt{2}}}{2}$

"Find $\cos(-\pi/4)$, $\sin(-\pi/4)$"

$\cos(\pi/4) = \frac{\sqrt{2}}{2} \to \cos(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\sin(\pi/4) = \frac{\sqrt{2}}{2} \to \sin(-\pi/4) = -\frac{\sqrt{2}}{2}$

Using $\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

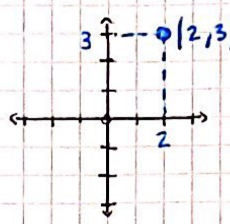
$\cot(-\theta) = -\cot\theta$

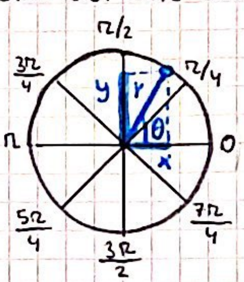
$\sec(-\theta) = \sec\theta$

$\csc(-\theta) = -\csc\theta$

Coordinates

- We need a system to point out distances and directions



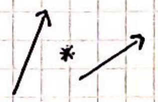


SINELAND-1A POLAR COORDINATES

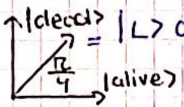
$x = r\cos\theta$

$y = r\sin\theta$

Vectors, QM states and complex numbers



operations between vectors are easier with their compone.

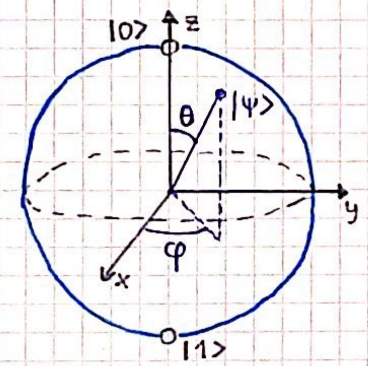


$= |L\rangle \cos\left(\frac{\pi}{4}\right) + |D\rangle \sin\left(\frac{\pi}{4}\right) \to \frac{|L\rangle}{\sqrt{2}} + \frac{|D\rangle}{\sqrt{2}}$

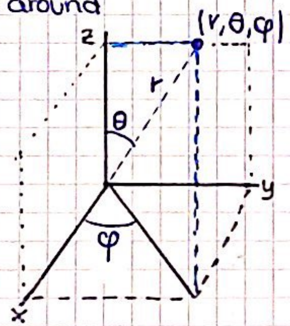
↳ QM states are vectors in Hibert Space

complex numbers have a super useful polar form which makes everything intuitive

The Bloch sphere



Quantum states in quantum computing are often represented on a Bloch spheres and operations in quantum computing amounts to moving the state around



SINELAND-1B SPHERICAL COORDINATES

$z = r\cos\theta$

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

Practice

"$(r,\theta,\phi) = (2, \pi/4, \pi/4)$"

$z = 2\cos(\pi/4) = 2\cdot\frac{1}{\sqrt{2}} = \sqrt{2}$

$x = 2\sin(\pi/4)\cos(\pi/4) = 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 1$

$y = 2\sin(\pi/4)\sin(\pi/4) = 2\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = 1$

Mathematicians use different convention than Physicists and switch θ and φ

WAVES

* Enter Sineland 2

- Quantum Mechanics: Wave Matter Duality!

de Broglie

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Planck's constant $h = 6.626 \cdot 10^{-34}\ m^2\ kg/s$

Why do moving objects around us not appear wavy?

Ex: golf ball (46g, 1m/s)

$\rightarrow \lambda = 10^{-32}\ m$

* $\sin(2\pi + x) = \sin(x)$ ← Trigonometric Functions are periodic

$$\psi(x) = A \sin(kx + \phi)$$

A = amplitude

k = wave-vector

ϕ = pase