INTRO TO QUANTUM COMPUTING

LECTURE #5

# INTRO TO PROBABILITY & MATHEMATICS FOR QUANTUM PT. 1

FRANCISCA VASCONCELOS

11/15/2020

# TODAY'S LECTURE

1. Intro to Probability
   a) Why Probability?
   b) A Crash Course on Set Notation
   c) The Probabilistic Model
   d) Fundamental Axioms of Probability
   e) Properties of Probability Laws
   f) Discrete Random Variables
   g) Probability Mass Functions
   h) Expectation & Variance
   i) Law of Large Numbers

2. Mathematics for Quantum Pt. 1
   a) Dirac Notation!

# INTRO TO PROBABILITY

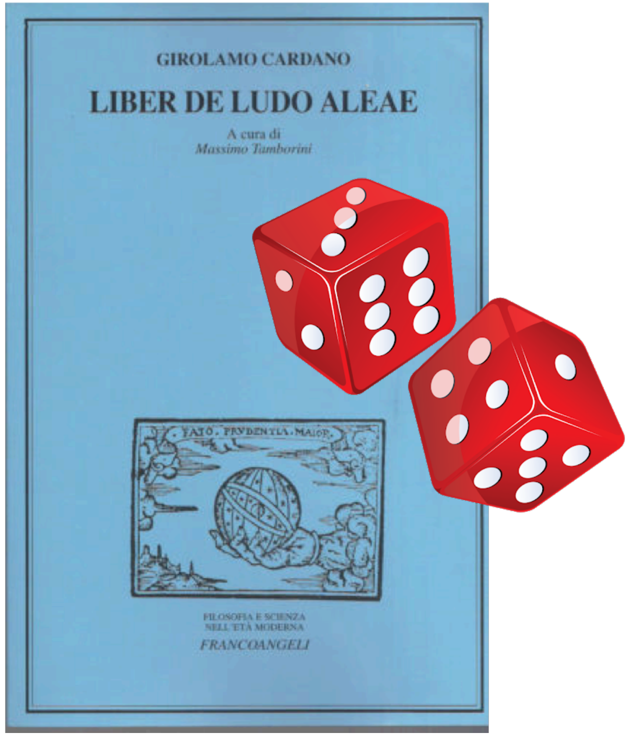
# WHY PROBABILITY?

**Probability is a branch of mathematics that describes the *"likelihood"* of events occurring.**

In fact, the origins of probability were in gambling!


**Girolamo Cardan**, an Italian Renaissance polymath, with interests in physics, mathematics, biology, chemistry, astrology, astronomy, philosophy, writing, and ***gambling*** (he was known to be short of money…).

In 1564 he wrote the book *"Book on Games of Chance"* (published 1663), which was the first systematic treatment of probability. He used the game of dice tossing to explain basic probability (as well as describe effective cheating methods!).

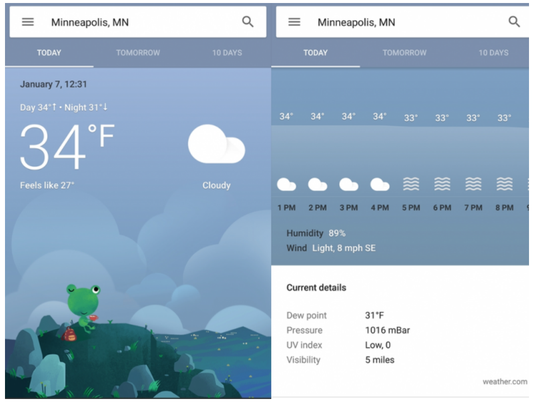


# WHY PROBABILITY?

Probability is used to make predictions in a variety of areas…

**Gambling & Cards**

How likely am I to draw a 2 of spades?

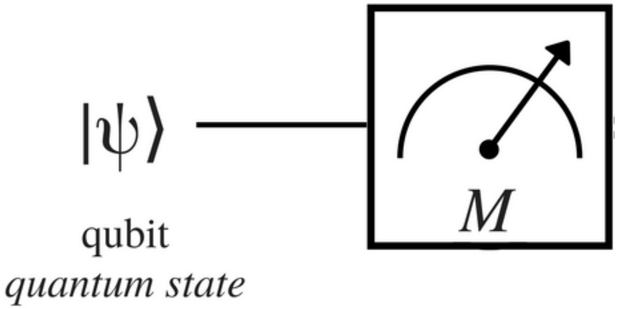



**Weather Forecasting**

What is the likelihood of rain today?

**Sports Predictions**

What are the odds that Portugal wins today's match?

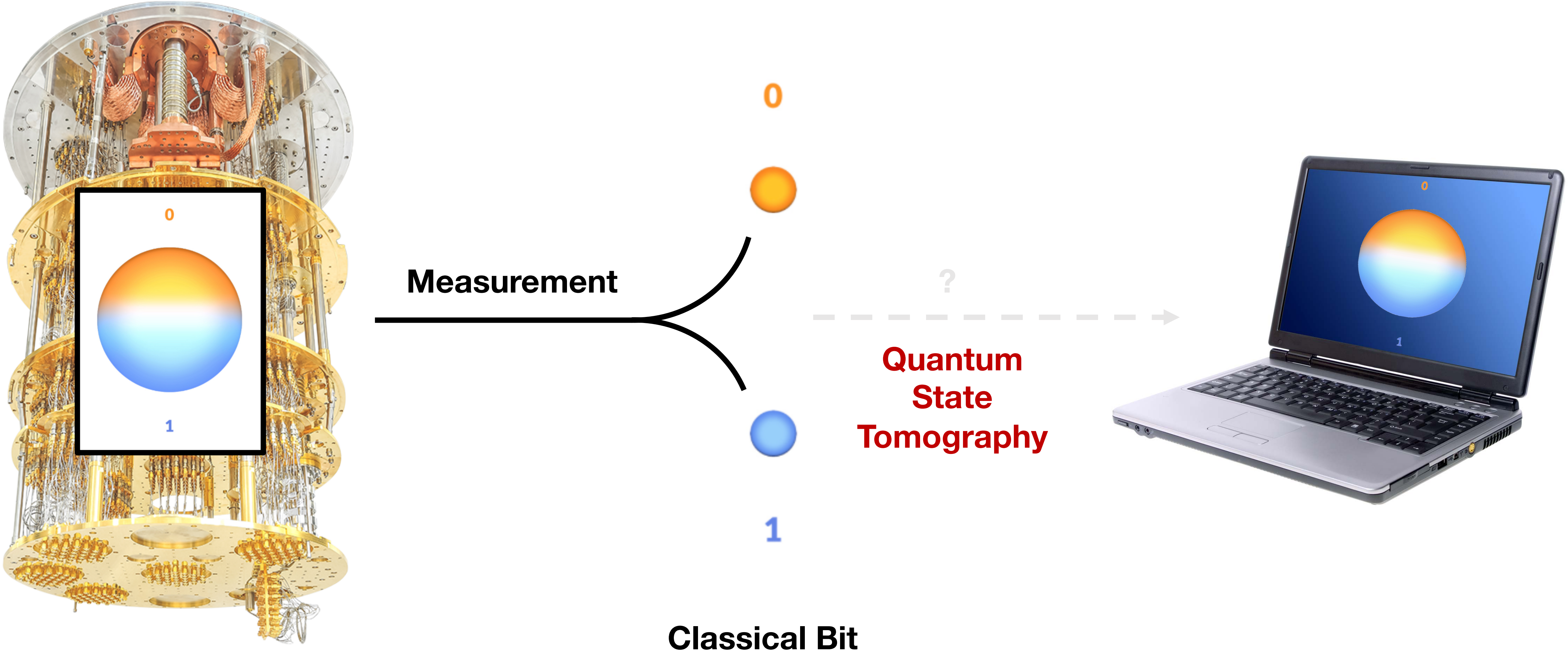



**Quantum Physics!**

What is the probability that this quantum state will collapse to the ground (0) state when I measure it?

**Quantum measurement is inherently probabilistic!!!**

# WHY PROBABILITY?




Quantum State

Classical Bit

# A CRASH COURSE ON SET NOTATION

A **set** is a collection of distinct objects, which can be considered an object in its own right.

The arrangement order of objects in a set does not matter.

The elements of a set can be anything from numbers to people to colors!

$$\boldsymbol{A} = \{1, 7, 8, 25\}$$

$$\boldsymbol{B} = \{Bob, Alice, Joe, Tommy\}$$

$$\boldsymbol{C} = \{Red, Blue, Green\} = \{Green, Blue, Red\} = \{Blue, Red, Green\}$$

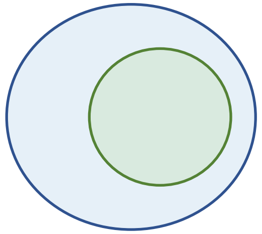
# A CRASH COURSE ON SET NOTATION

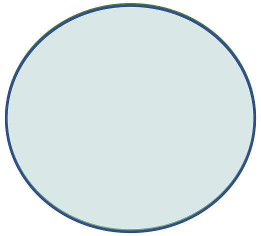
Comparing sets:

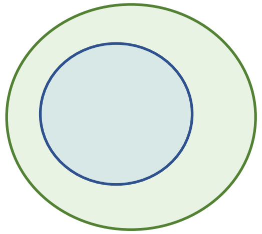
$1 < 5$

$<$ $\leq$ $\geq$ $>$

$A \subset B$ $A \subseteq B$ $A = B$ $A \supseteq B$ $A \supset B$







Set $A$ is a ***subset*** of set $B$.

$B = \{1, 2, 3, 4, 5\}$

$A = \{1, 2\}$ $D = \{1, 5\}$

$C = \{3\}$

Set $A$ is ***equal*** to set $B$.

$A = \{\text{Red}, \text{Orange}, \text{Yellow}\}$

$B = \{\text{Orange}, \text{Yellow}, \text{Red}\}$

Set $A$ is a ***superset*** of set $B$.

$A = \{Q, R, S, T, V\}$

$B = \{Q, S, T\}$

$C = \{R, S\}$

# QUANTUM PRACTICE TIME!

State the relation (⊂, =, ⊃, or None) between the following sets.

1. $\boldsymbol{A} = \{1, 2, 3\}$ and $\boldsymbol{B} = \{1, 2, 3, 4, 5\}$

2. $\boldsymbol{A} = \{Red, Orange, Yellow\}$ and $\boldsymbol{B} = \{Yellow\}$

3. $\boldsymbol{A} = \{NYC, LA, SD\}$ and $\boldsymbol{B} = \{SD, LA, NYC\}$

4. $\boldsymbol{A} = \{X, Y, Q, R\}$ and $\boldsymbol{B} = \{R, Q, X, S\}$

# QUANTUM PRACTICE SOLUTIONS

State the relation (⊂, =, ⊃, or None) between the following sets.

1. $\boldsymbol{A} = \{1, 2, 3\}$ and $\boldsymbol{B} = \{1, 2, 3, 4, 5\}$ $\boldsymbol{A \subset B}$

2. $\boldsymbol{A} = \{Red, Orange, Yellow\}$ and $\boldsymbol{B} = \{Yellow\}$ $\boldsymbol{A \supset B}$

3. $\boldsymbol{A} = \{NYC, LA, SD\}$ and $\boldsymbol{B} = \{SD, LA, NYC\}$ $\boldsymbol{A = B}$

4. $\boldsymbol{A} = \{X, Y, Q, R\}$ and $\boldsymbol{B} = \{R, Q, X, S\}$ ***None***

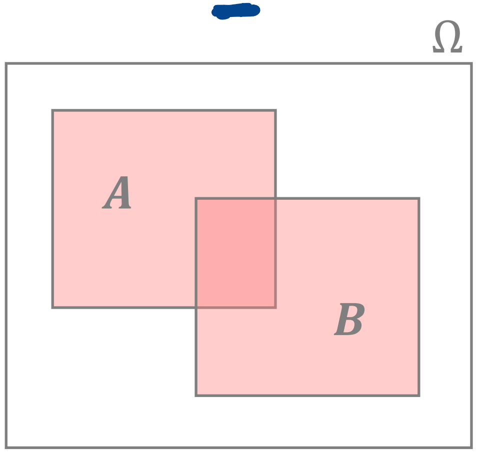
# A CRASH COURSE ON SET NOTATION

Operations on sets:

**UNION (OR)**

nion

$A \cup B$



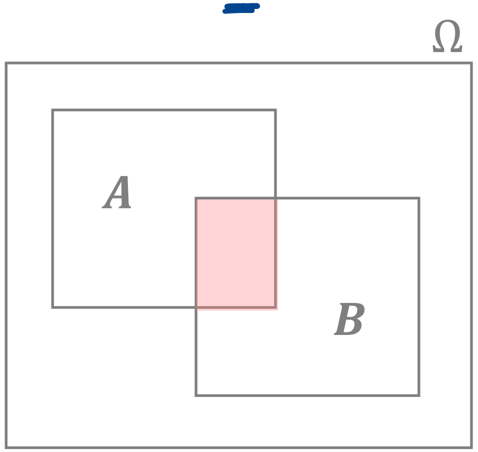

$A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$C = A \cup B = \{1, 2, 3, 4, 5\}$

**INTERSECT (AND)**

$A \cap B$



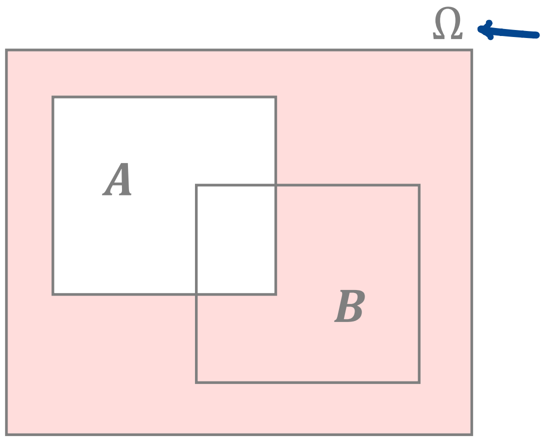

$A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

$D = A \cap B = \{3\}$

**COMPLEMENT (SUBTRACT)**

$A^c$




$\rightarrow \Omega = \{1, 2, 3, 4, 5\}$

$\rightarrow A = \{1, 2, 3\}$

$A^c = \{4, 5\}$

$A \cup A^c = \Omega$

# QUANTUM PRACTICE TIME!

Perform the following set operations.

1. $\boldsymbol{A} = \{1, 2, 3\}$ , $\boldsymbol{B} = \{1, 2, 3, 4, 5\}$, $\boldsymbol{A} \cup \boldsymbol{B}$?

2. $\boldsymbol{A} = \{Red, Orange, Yellow\}$ , $\boldsymbol{B} = \{Yellow\}$, $\boldsymbol{A} \cap \boldsymbol{B}$?

3. $\boldsymbol{A} = \{A, B, C\}$ , $\boldsymbol{\Omega} = \{A, B, C, D, E, F\}$, $\boldsymbol{A}^c$ ?

4. $\boldsymbol{A} = \{1, 2, 3, 4\}$ , $\boldsymbol{B} = \{3, 4, 5, 6\}$, $\boldsymbol{C} = \{1, 5, 8, 9\}$, $(\boldsymbol{A} \cap \boldsymbol{B}) \cup \boldsymbol{C}$ ?

# QUANTUM PRACTICE SOLUTIONS

Perform the following set operations.

1. $\boldsymbol{A} = \{1, 2, 3\}$ , $\boldsymbol{B} = \{1, 2, 3, 4, 5\}$, $\boldsymbol{A} \cup \boldsymbol{B}$? $\boldsymbol{A} \cup \boldsymbol{B} = \{1,2,3,4,5\} = \boldsymbol{B}$

2. $\boldsymbol{A} = \{Red, Orange, Yellow\}$ , $\boldsymbol{B} = \{Yellow\}$, $\boldsymbol{A} \cap \boldsymbol{B}$? $\boldsymbol{A} \cap \boldsymbol{B} = \{Yellow\} = \boldsymbol{B}$

3. $\boldsymbol{A} = \{A, B, C\}$ , $\boldsymbol{\Omega} = \{A, B, C, D, E, F\}$, $\boldsymbol{A}^c$ ? $\boldsymbol{A}^C = \{D, E, F\}$

4. $\boldsymbol{A} = \{1, 2, 3, 4\}$ , $\boldsymbol{B} = \{3, 4, 5, 6\}$, $\boldsymbol{C} = \{1, 5, 8, 9\}$, $(\boldsymbol{A} \cap \boldsymbol{B}) \cup \boldsymbol{C}$ ?

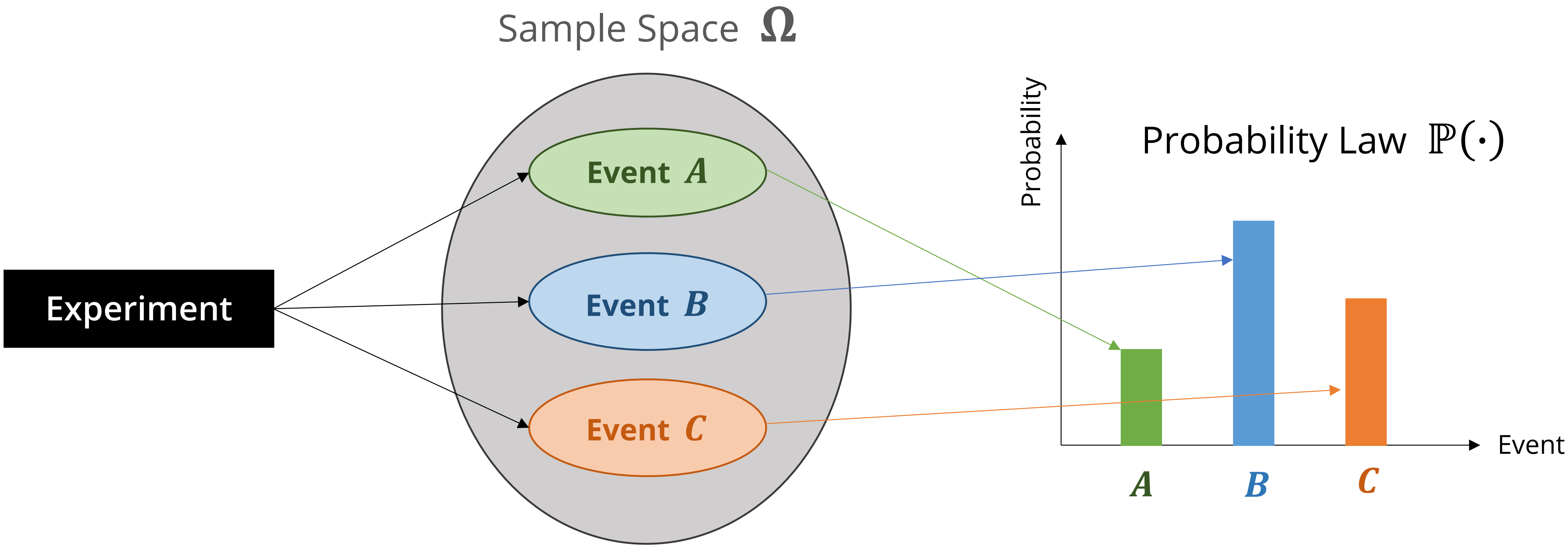
$\rightarrow (\boldsymbol{A} \cup \boldsymbol{B}) \cap \boldsymbol{C} = \{1,2,3,4,5,6\} \cap \{1, 5,8,9\} = \{1, 5\}$

$(A \cap B) \cup C = \{3,4\} \cup \{1,5,8,9\} = \{1,3,4,5,8,9\}$

# THE PROBABILISTIC MODEL

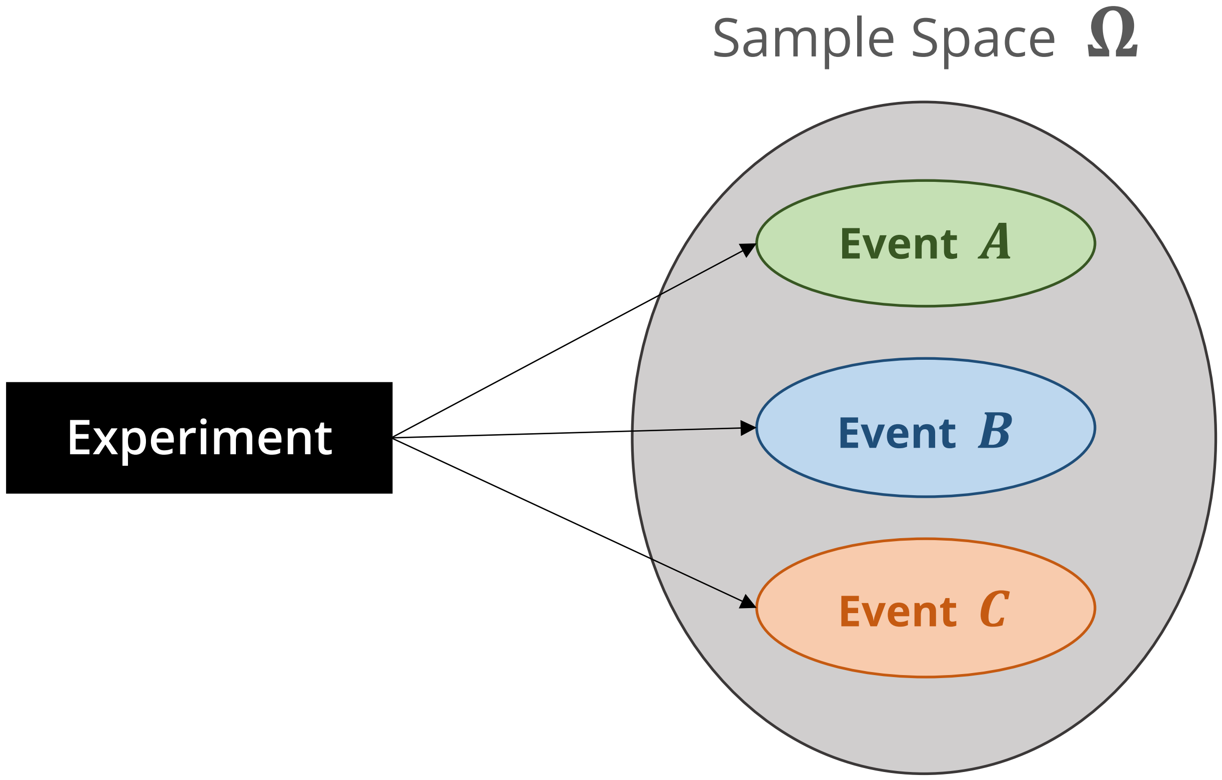
A **probabilistic model** is a way to mathematically describe an unknown situation.

There are two key components: (1) **sample space** and (2) **probability law**.



# THE PROBABILISTIC MODEL – SAMPLE SPACE

The **sample space (Ω)** is the set of all possible outcomes in an experiment.




$$\Omega = \{A, B, C\}$$

(we denote all the events in the space using set notation)

Let's look at some examples….

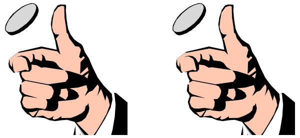
# THE PROBABILISTIC MODEL – SAMPLE SPACE

What is the sample space of a coin toss?


$$\Omega = \{\text{Heads}, \text{Tails}\}$$

# THE PROBABILISTIC MODEL – SAMPLE SPACE

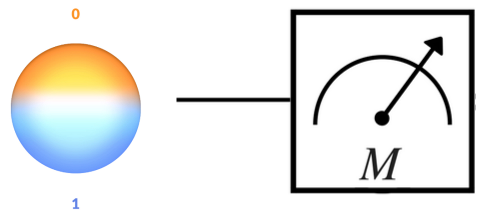
What if we toss two coins?

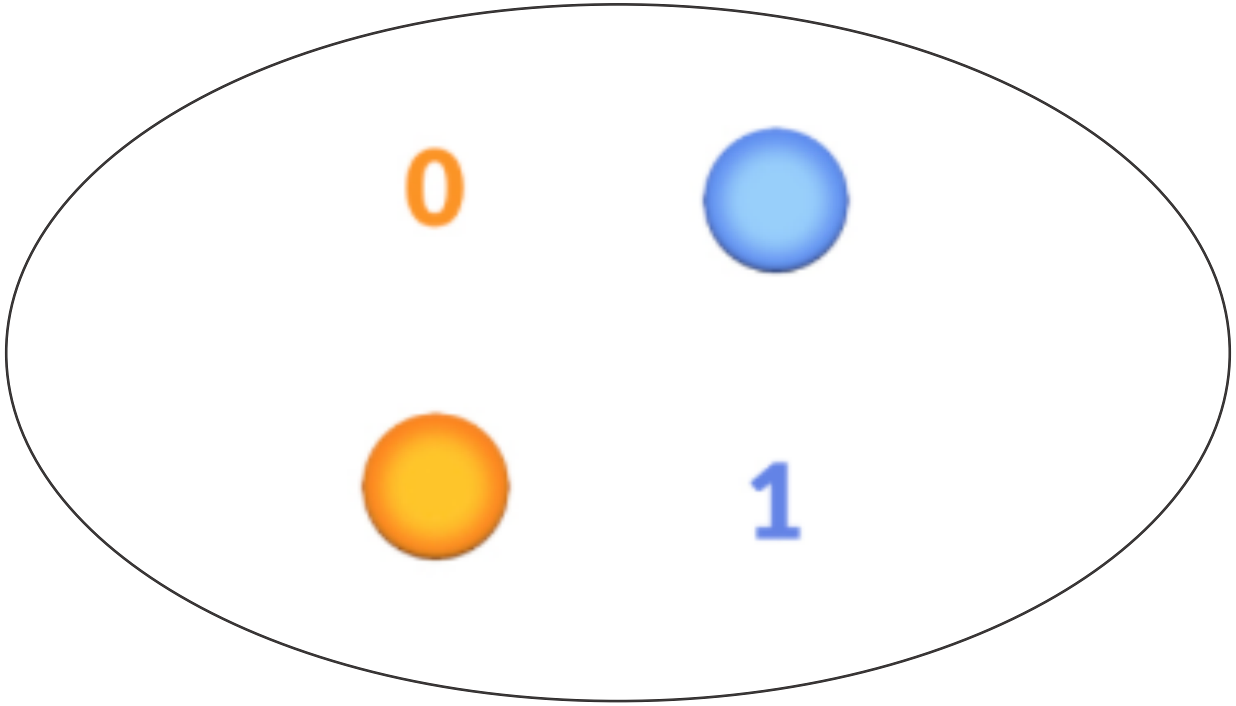



$$\Omega = \{HH, HT, TH, TT\}$$

# THE PROBABILISTIC MODEL – SAMPLE SPACE

What are the different states your qubit will collapse to upon measurement?



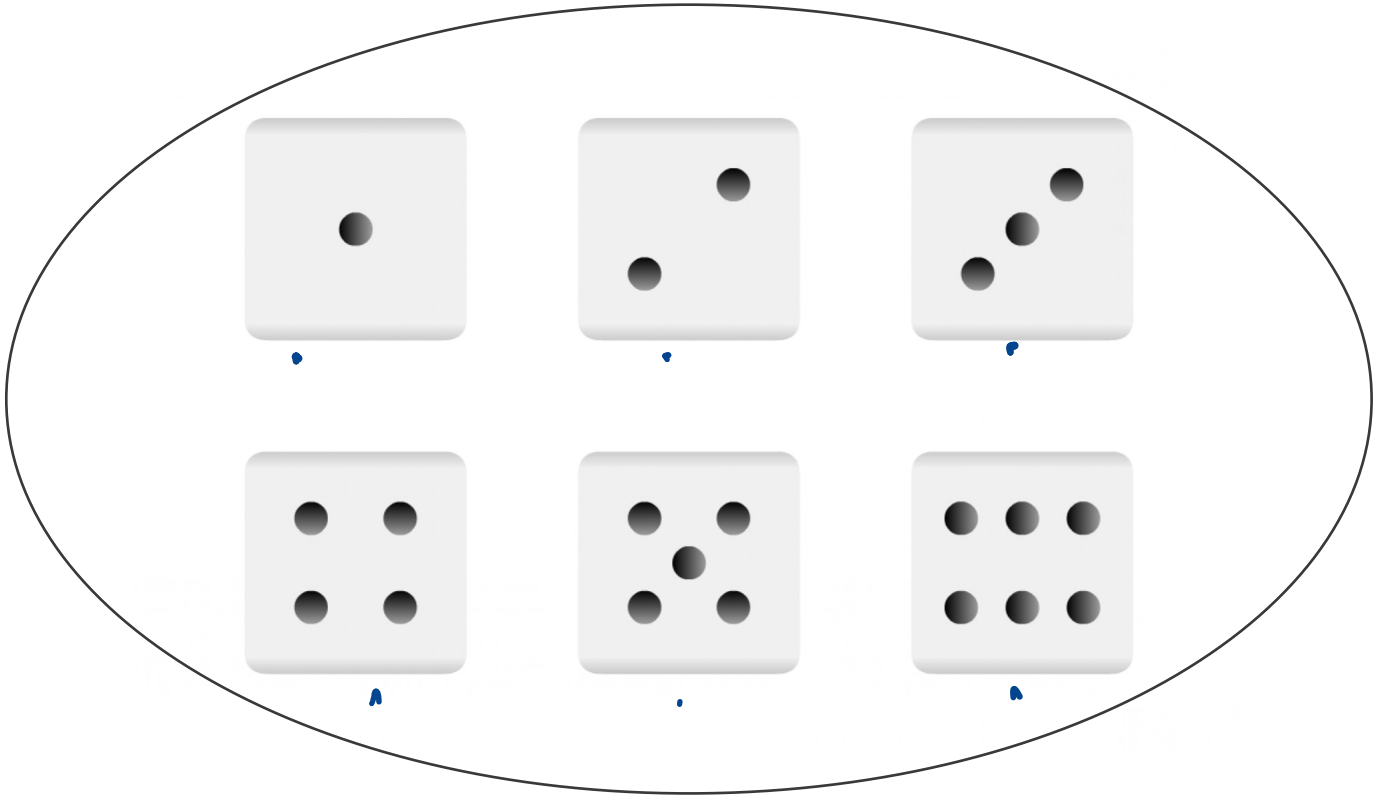

$$\Omega = \{|0\rangle, |1\rangle\}$$

# QUANTUM PRACTICE TIME!

What is the sample space of a 6-sided die toss?

# QUANTUM PRACTICE SOLUTION

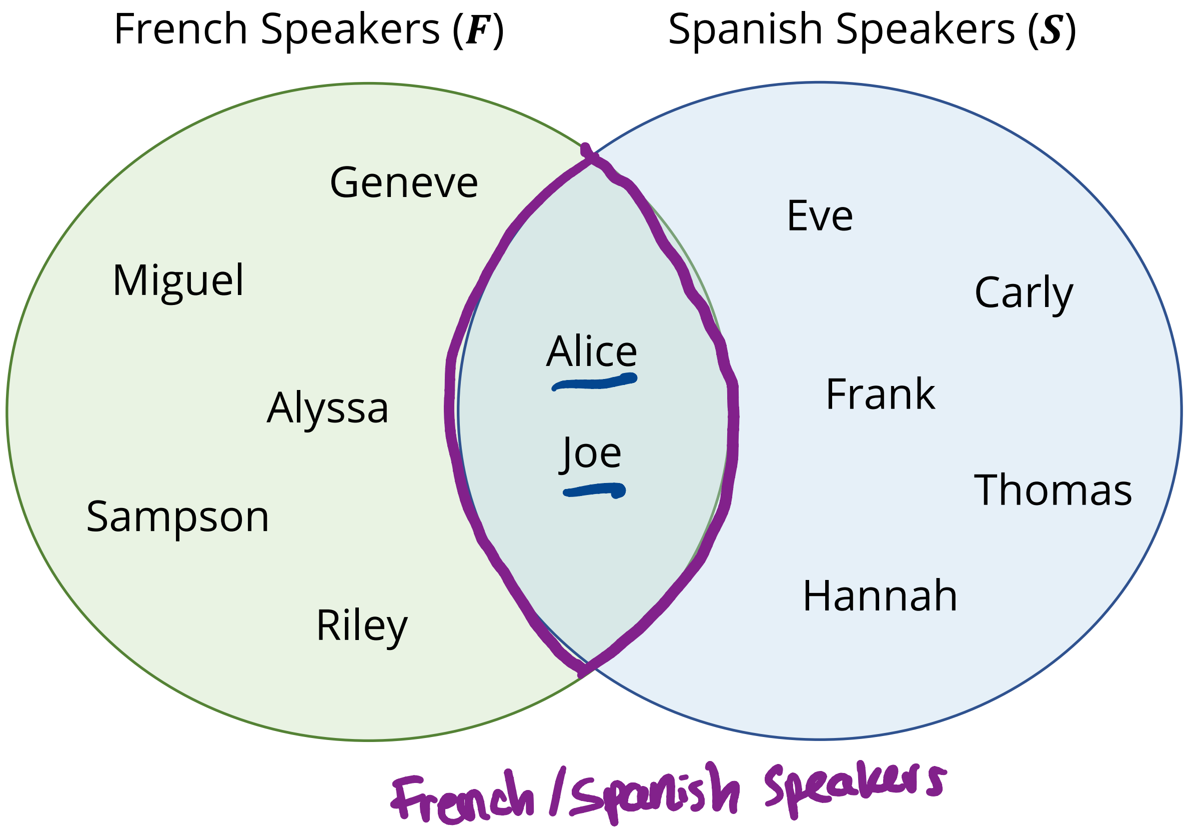
What is the sample space of a 6-sided die toss?



$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

# THE PROBABILISTIC MODEL – SAMPLE SPACE

Elements of the sample space (Ω) must be ***distinct*** and ***mutually exclusive*** (two events can't occur simultaneously!) so that when an experiment is carried out there is a ***unique*** outcome.

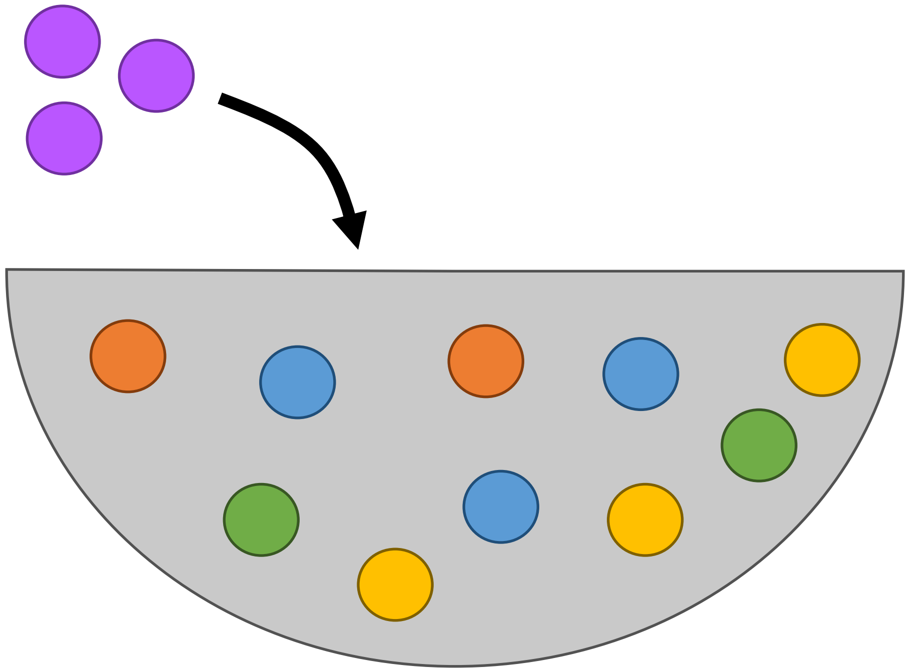


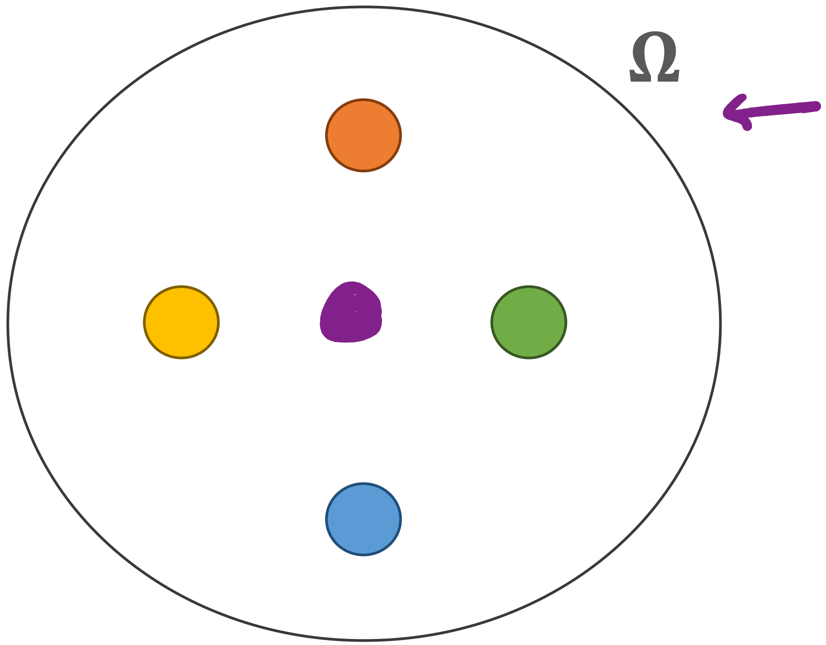
# THE PROBABILISTIC MODEL – SAMPLE SPACE

The sample space (Ω) must be ***collectively exhaustive***, such that no matter what happens in an experiment, we always obtain an outcome in the sample space.

**EXAMPLE**

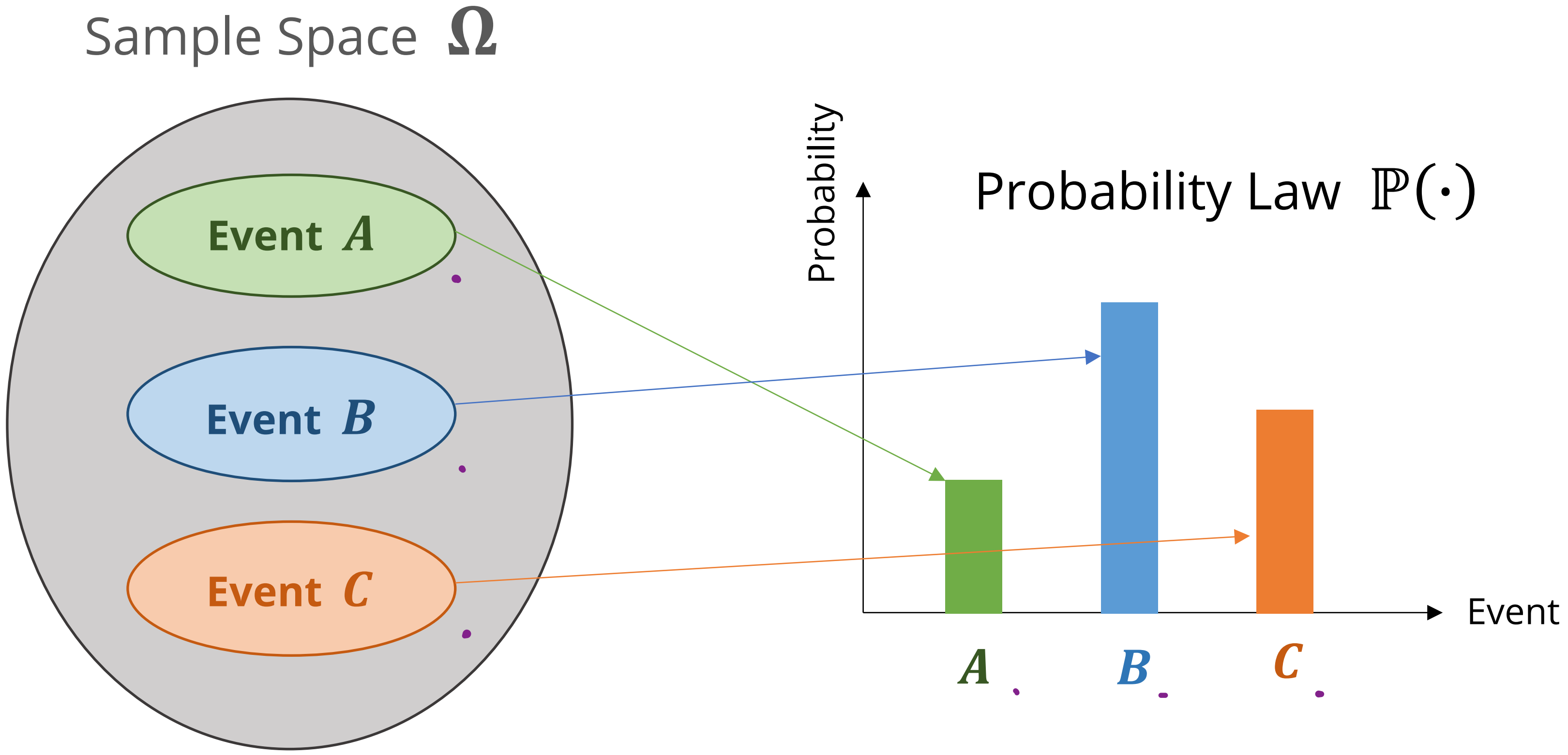
What if we add purple marbles to our bowl?





# THE PROBABILISTIC MODEL – PROBABILITY LAW

A **probability law** assigns to each event, $\boldsymbol{Y}$, a non-negative number, $\mathbb{P}(\boldsymbol{Y})$, that encodes our knowledge/belief about the collective likelihood of elements of $\boldsymbol{Y}$.



# THE PROBABILISTIC MODEL – PROBABILITY LAW

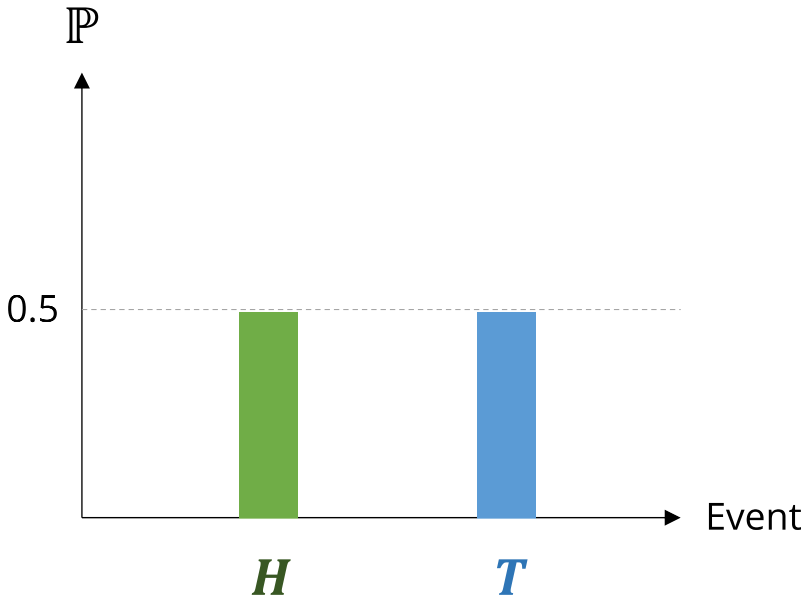
What is the probability law for a *fair* coin toss?

$\Omega = \{H, T\}$

Probability Law

$$\mathbb{P}(H) = \frac{1}{2}$$

$$\mathbb{P}(T) = \frac{1}{2}$$




*We will need to learn some rules about probability to create these generally…*

# THE FUNDAMENTAL AXIOMS OF PROBABILITY

Every probability law must satisfy the following axioms:

1. **NONNEGATIVITY** : The probability of every event in the sample space must be greater than or equal to zero.

$$\mathbb{P}(\boldsymbol{A}) \geq 0, \text{ for every event } \boldsymbol{A}$$

2. **ADDITIVITY** : If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two ***disjoint*** events, then the probability of their union is the sum of their individual probabilities.

$$\mathbb{P}(\boldsymbol{A} \cup \boldsymbol{B}) = \mathbb{P}(\boldsymbol{A}) + \mathbb{P}(\boldsymbol{B})$$

3. **NORMALIZATION** : The probability of the entire sample space ($\Omega$) is equal to 1.

$$\mathbb{P}(\boldsymbol{\Omega}) = 1$$

# THE FUNDAMENTAL AXIOMS OF PROBABILITY

**NONNEGATIVITY** : The probability of every event in the sample space must be greater than or equal to zero.

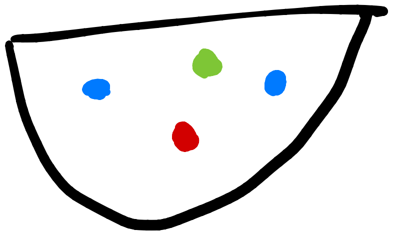
$$\boxed{\mathbb{P}(\boldsymbol{A}) \geq 0, \text{ for every event } \boldsymbol{A}}$$

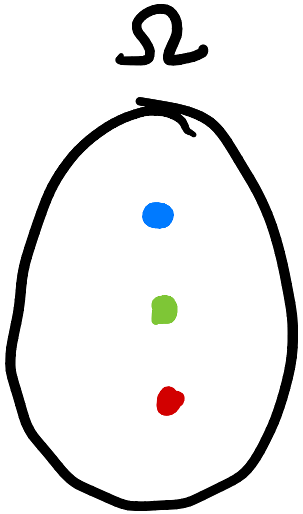
THERE IS NO NEGATIVE PROBABILITY!

# THE FUNDAMENTAL AXIOMS OF PROBABILITY

**ADDITIVITY** : If $A$ and $B$ are two ***disjoint*** events, then the probability of their union is the sum of their individual probabilities.

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$$






$$\mathbb{P}(B) = \frac{2}{4}$$

$$\mathbb{P}(G) = \frac{1}{4}$$

$$\mathbb{P}(R) = \frac{1}{4}$$

$$\mathbb{P}(B \cup G) = \mathbb{P}(B) + \mathbb{P}(G)$$

$$= \frac{2}{4} + \frac{1}{4} = \frac{3}{4} \rightarrow 75\%$$

# THE FUNDAMENTAL AXIOMS OF PROBABILITY

**NORMALIZATION** : The probability of the entire sample space ($\Omega$) is equal to 1.

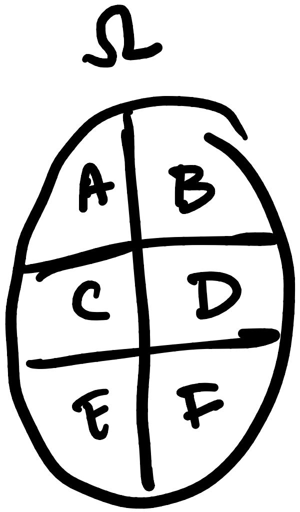
$$\mathbb{P}(\Omega) = 1$$

1 – 100%

½ – 50%

¼ – 25%

0 – 0%




$$\mathbb{P}(\Omega) = 1$$

$$\boxed{0 \leq \mathbb{P}(A) \leq 1}$$

$$\mathbb{P}(A) \geq 0$$

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$$

$$\mathbb{P}(\Omega) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) + \mathbb{P}(D) + \mathbb{P}(E) + \mathbb{P}(F)$$
$$= 1$$

# THE FUNDAMENTAL AXIOMS OF PROBABILITY

There are some further important properties of probability laws that we can derive from the fundamental axioms.

**PROBABILITY OF THE EMPTY SET** :

$$\phi = \{\ \} \qquad \underline{1} = \mathbb{P}(\Omega) = \mathbb{P}(\Omega \cup \phi) = \underline{\mathbb{P}(\Omega)} + \mathbb{P}(\phi) = 1 + \mathbb{P}(\phi)$$

$$\Rightarrow \mathbb{P}(\phi) = 0$$

**COMPLEMENT RULE** :

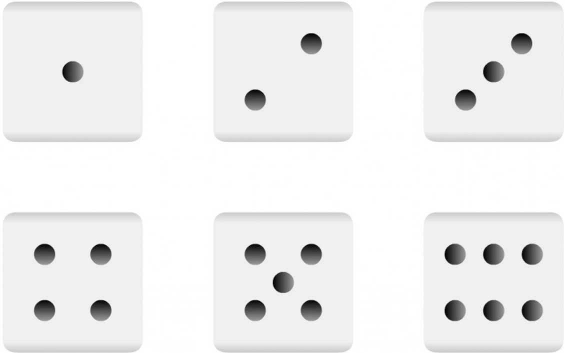
$$1 = \mathbb{P}(\Omega) = \mathbb{P}(A \cup A^c) = \mathbb{P}(A) + \mathbb{P}(A^c)$$


$$\Rightarrow \boxed{\mathbb{P}(A^c) = 1 - \mathbb{P}(A)}$$

# IMPORTANT PROPERTIES OF PROBABILITY LAWS

Every event $\boldsymbol{A}_i$ in our sample space $\boldsymbol{\Omega}$, has probability $0 \leq \mathbb{P}(\boldsymbol{A}_i) \leq 1$ such that $\sum_i \mathbb{P}(\boldsymbol{A}_i) = 1$ .





$\Omega$

H
T

$$\mathbb{P}(H) = \frac{1}{2} \qquad \mathbb{P}(T) = \frac{1}{2}$$

$$\mathbb{P}(H) + \mathbb{P}(T) = \frac{1}{2} + \frac{1}{2} = 1$$

$\mathbb{P}(\Omega) = 1$

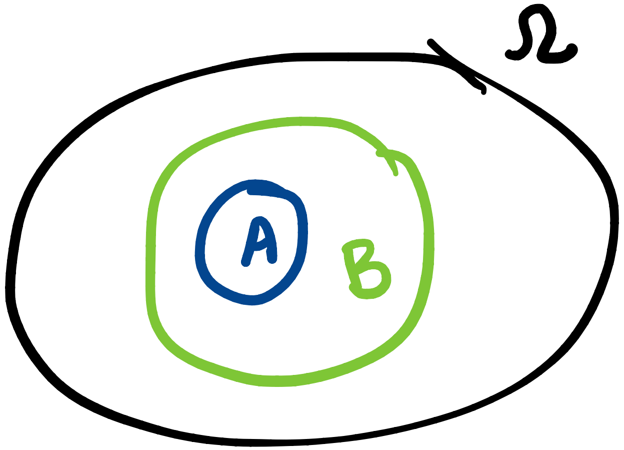
$1 = \mathbb{P}(1) + \mathbb{P}(2) + \mathbb{P}(3) + \mathbb{P}(4) + \mathbb{P}(5) + \mathbb{P}(6)$

$\mathbb{P}(1) = \mathbb{P}(2) = \mathbb{P}(3) \ldots = \mathbb{P}(6)$

$1 = 6 * \mathbb{P}(1) \Rightarrow \mathbb{P}(1) = \frac{1}{6}$

# IMPORTANT PROPERTIES OF PROBABILITY LAWS

If $A \subseteq B$, then $\mathbb{P}(A) \leq \mathbb{P}(B)$.



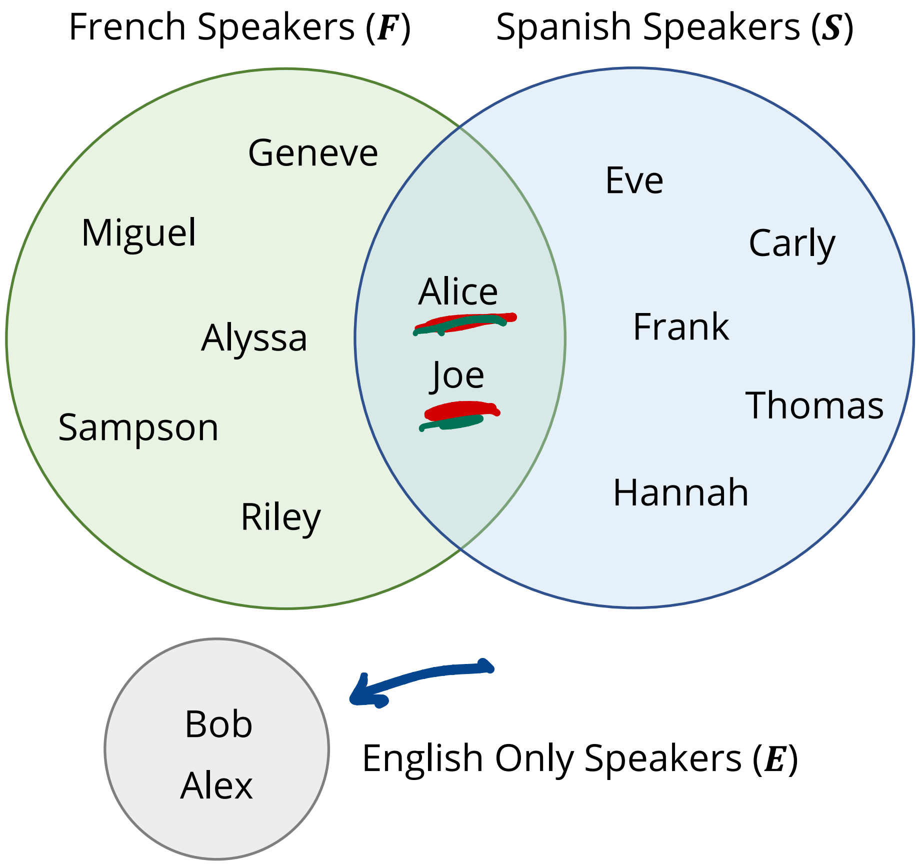

$A \subset B \Rightarrow \mathbb{P}(A) < \mathbb{P}(B)$

$A = B \Rightarrow \mathbb{P}(A) = \mathbb{P}(B)$

Generally, $\mathbb{P}(F \cup S) = \mathbb{P}(F) + \mathbb{P}(S) - \mathbb{P}(F \cap S)$ .

Thus, $\mathbb{P}(F \cup S) \leq \mathbb{P}(F) + \mathbb{P}(S)$ .

F & S are not disjoint




14 students total

12 students speak F or S

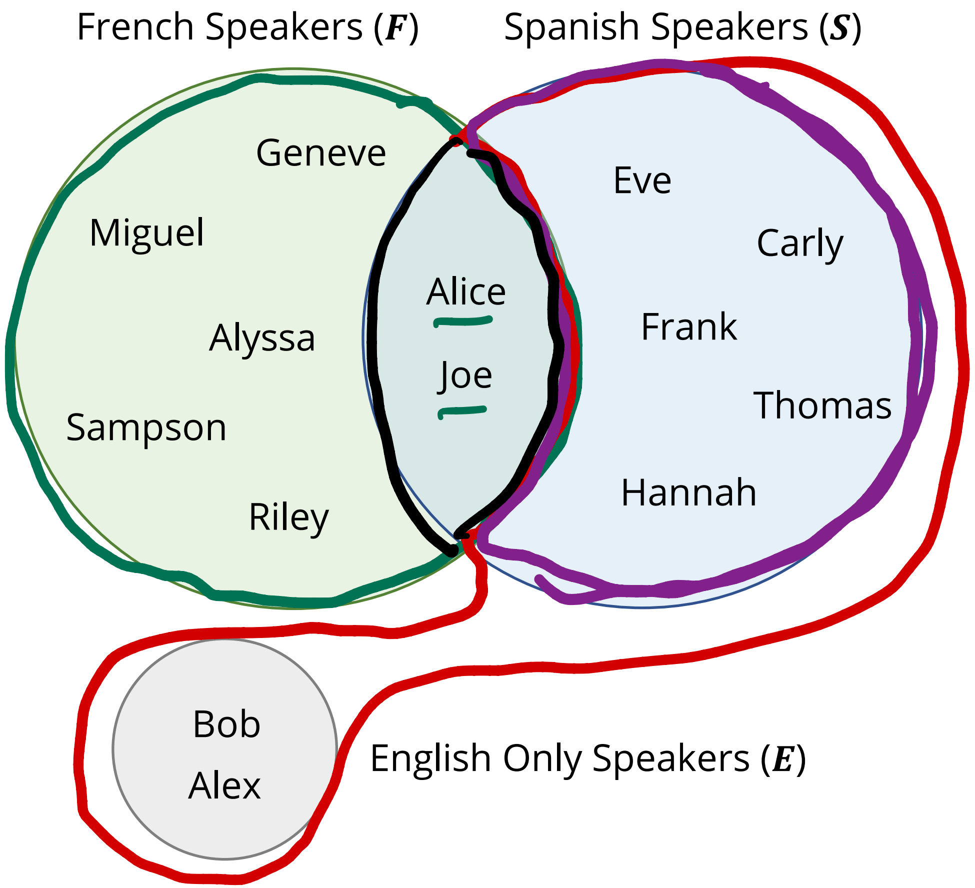
$\frac{12}{14}$

WRONG → $\mathbb{P}(F \cup S) = \frac{7}{14} + \frac{7}{14} = \frac{14}{14} = 1$

$\mathbb{P}(F \cup S) = \frac{7}{14} + \frac{7}{14} - \frac{2}{14} = \frac{14-2}{14} = \frac{12}{14}$

# IMPORTANT PROPERTIES OF PROBABILITY LAWS

$$\mathbb{P}(\boldsymbol{F} \cup \boldsymbol{S}) = \mathbb{P}(\boldsymbol{F}) + \mathbb{P}(\boldsymbol{F}^c \cap \boldsymbol{S})$$




$$\frac{7}{14} + \frac{5}{14} = \frac{12}{14} \checkmark$$

$A = \{1, 2, 3\} = \{3, 2, 1\} = \{3, 1, 2\}$

$B = \{1, 2, 4\}$ $\quad$ $D = A \cup B = \{1, 2, 3, 4\}$

# 10 MIN BREAK!

Until 3:26 PM EST

QUANTUM

1-QUBIT

$\Omega$

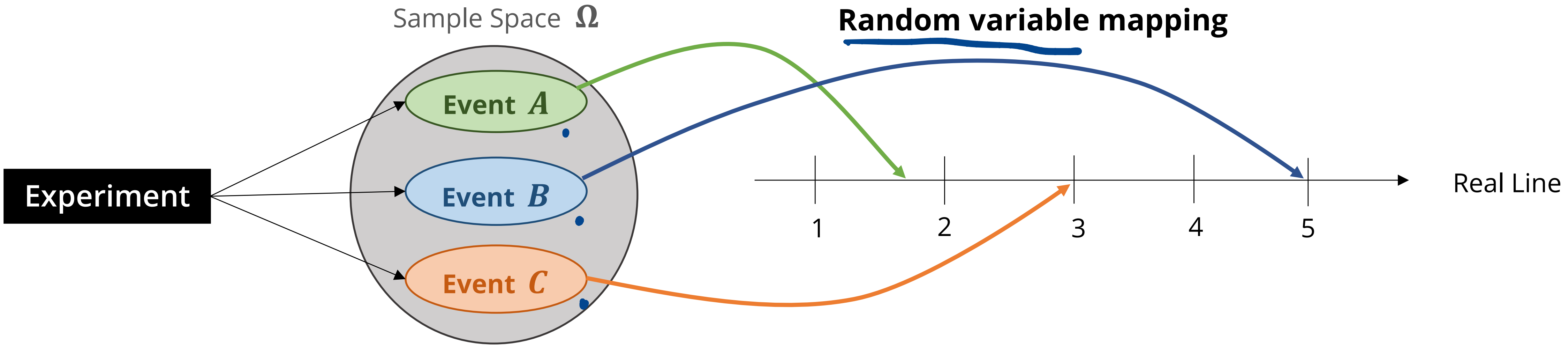
$|\psi\rangle \rightarrow \{|0\rangle, |1\rangle\}$

# DISCRETE RANDOM VARIABLES

Frequently, outcomes of probabilistic models are numerical (i.e. instrument readings, stock prices).

In other cases, the outcomes may not be numerical, but may be associated with numerical values of interest (i.e. in a group of people, we might care about their height).

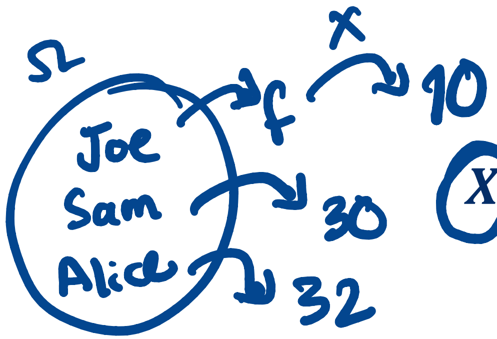
**Random variable (RV)**: a ***function*** that assigns a numerical value to each possible outcome of the experiment.



# DISCRETE RANDOM VARIABLES

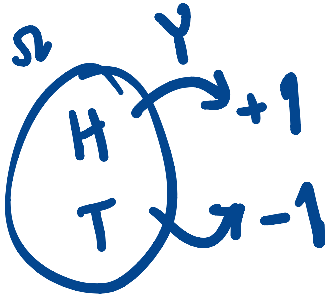
Let's look at some examples...

Suppose we are interested in the age of different family members...




$$\boldsymbol{X}(f) = \begin{cases} 10, & \text{if } f = \text{Joe} \\ 30, & \text{if } f = \text{Sam} \\ 32, & \text{if } f = \text{Alice} \end{cases}$$

Now suppose we are betting on a coin flip. You win \$1 if it is heads and lose \$1 if it is tails...

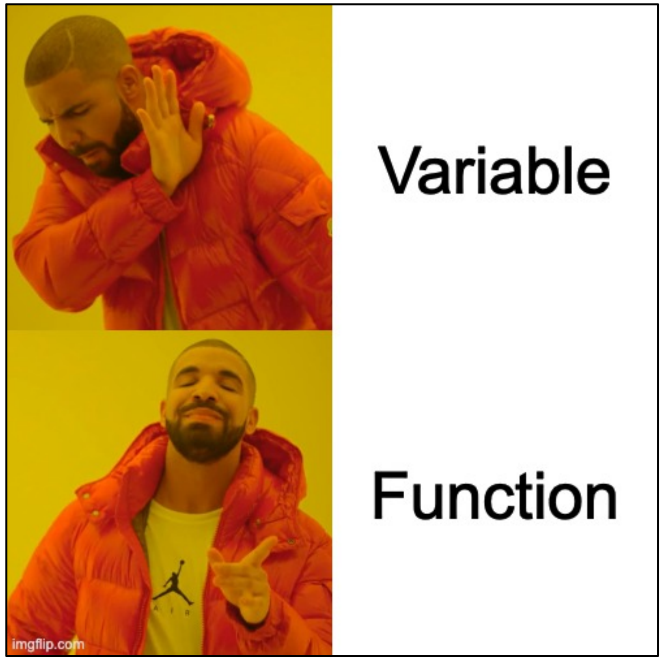



$$\boldsymbol{Y}(c) = \begin{cases} +1, & \text{if } c = \text{heads} \\ -1, & \text{if } c = \text{tails} \end{cases}$$

# DISCRETE RANDOM VARIABLES

Remember that even though it is called a random ***variable***, it is actually a ***function*** which maps elements from the sample space $\boldsymbol{\Omega}$ to $\mathbb{R}$.

$$\boldsymbol{X}\colon\ \boldsymbol{\Omega} \rightarrow \mathbb{R}$$





# PROBABILITY MASS FUNCTIONS

Every discrete random variable has an associated ***probability mass function*** (PMF), which gives the *probability* of each numerical value that the random variable can take.

It is a means of describing the discrete ***probability distribution***!

Going back to our previous example:

Now suppose we are betting on a *fair* coin flip. You win \$1 if it is heads and lose \$1 if it is tails…

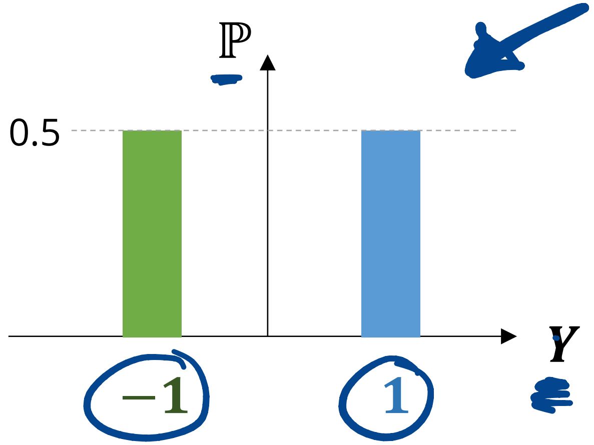
**Random Variable**

$$y = \boldsymbol{Y}(c) = \begin{cases} +1, & \text{if } c = \text{heads} \\ -1, & \text{if } c = \text{tails} \end{cases}$$

**Probability Mass Function**

$$f_{\boldsymbol{Y}}(y) = \begin{cases} 0.5, & \text{if } y = +1 \\ 0.5, & \text{if } y = -1 \end{cases}$$

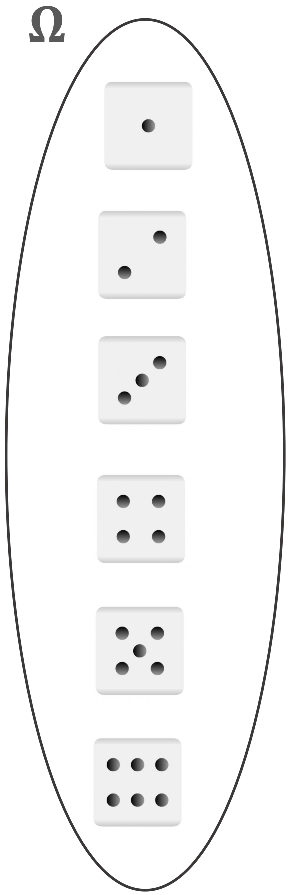
**Probability Distribution**




**Remember, the sum of the elements of the probability mass function must add to 1!**

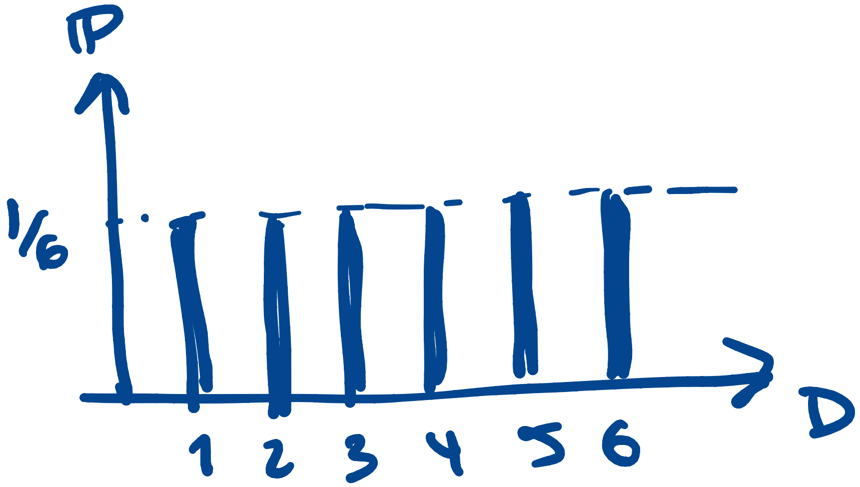
# PROBABILITY MASS FUNCTIONS

Now, let's define the **random variable**, **probability mass function**, and **probability distribution** for the roll of fair 6-sided die.



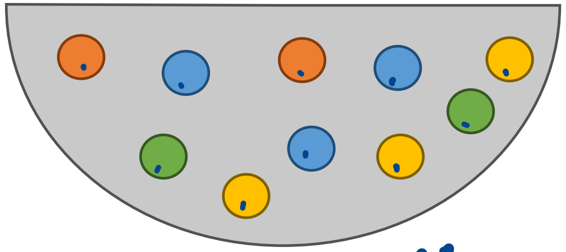

$$D(r) = \begin{cases} 1, & \text{if } r = \boxed{\cdot}\ 1 \\ 2, & \text{if } r = \boxed{\cdot\cdot}\ 2 \\ 3, & \text{if } r = \boxed{\cdot\cdot\cdot}\ 3 \\ 4, & \vdots \\ 5, & \vdots \\ 6, & \vdots \end{cases}$$

$$P_D(d) = \{\ 1/6, \text{ if } d = 1,2,3,4,5,6$$



# PROBABILITY MASS FUNCTIONS

So far we have only considered cases of uniform probability. Let's now consider an example in which the probabilities are not uniform. Define the **sample space**, **random variable**, **probability mass function**, and **probability distribution** for pulling different colored marbles from the bowl.



10

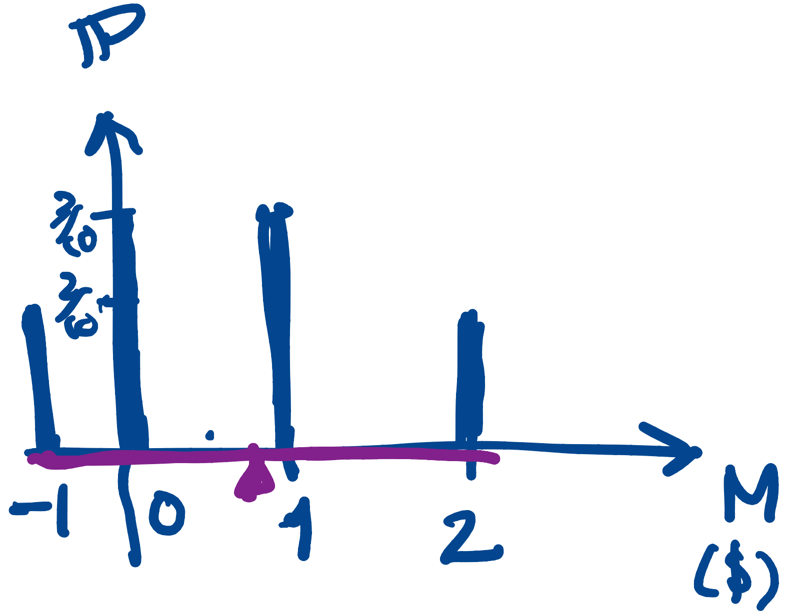
$\Omega$

Blue
Green
Yellow
Orange

$$M(p) = \begin{cases} +2, & p = \text{orange} \\ +1, & p = \text{blue} \\ 0, & p = \text{yellow} \\ -1, & p = \text{green} \end{cases}$$

$$P_M(m) = \begin{cases} 2/10, & m = 2 \\ 3/10, & m = 1 \\ 3/10, & m = 0 \\ 2/10, & m = -1 \end{cases}$$

$$\frac{2}{10} + \frac{3}{10} + \frac{3}{10} + \frac{2}{10} = \frac{10}{10} = 1 \checkmark$$



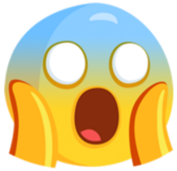
# EXPECTATION

The ***expectation*** is the weighted (in proportion to the probabilities) average of the possible values of a random variable.

It is sometimes referred to as the *mean* of a random variable or *center of gravity* of the PMF.

$$\langle X \rangle = \mathbb{E}[X] = \sum_x x \, \mathbb{P}(X = x)$$

**Look it's a linear combination!!**



$$\text{PMF} = \begin{cases} 1/2, & p = +1 \\ 1/2, & p = -1 \end{cases}$$

$$\langle x \rangle = \tfrac{1}{2}(+1) + \tfrac{1}{2}(-1)$$
$$= \tfrac{1}{2} - \tfrac{1}{2} = 0$$

# QUANTUM PRACTICE TIME!

What the expectation of a 6-sided die toss?

# QUANTUM PRACTICE SOLUTION

What is the expectation of a 6-sided die toss?

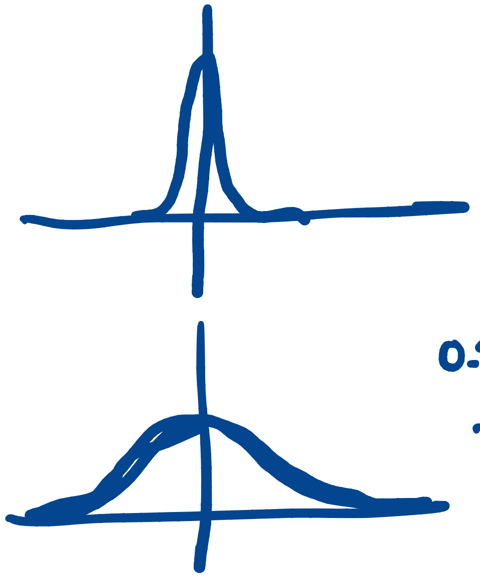
$$\langle \boldsymbol{X} \rangle = \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3) + \frac{1}{6}(4) + \frac{1}{6}(5) + \frac{1}{6}(6)$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

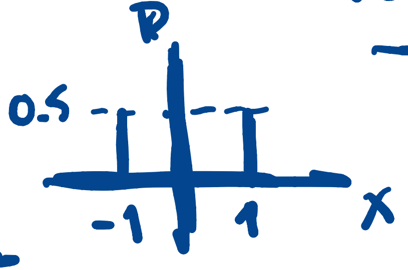
# VARIANCE

The ***variance*** is expected value of the random variable $\boxed{(\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}])^2}$ and provides a measure of the *dispersion* of $\boldsymbol{X}$ about its mean.

$$\mathrm{var}(\boldsymbol{X}) = \mathbb{E}[\,(\boldsymbol{X} - \mathbb{E}[\boldsymbol{X}])^2\,] = \mathbb{E}[\boldsymbol{X}^2] - \mathbb{E}[\boldsymbol{X}]^2 = \sum_x (x - \mathbb{E}[\boldsymbol{X}])^2\, \mathbb{P}(\boldsymbol{X} = x)$$



$\mathbb{E}[x] = 0$




$$\mathrm{var}(x) = \frac{1}{2}(-1)^2 + \frac{1}{2}(1)^2$$

$$= \frac{1}{2}(1) + \frac{1}{2}(1) = \frac{1}{2} + \frac{1}{2} = 1$$

# QUANTUM PRACTICE TIME!

What is the variance of a 6-sided die toss?

$E[D] = 3.5$

# QUANTUM PRACTICE SOLUTION

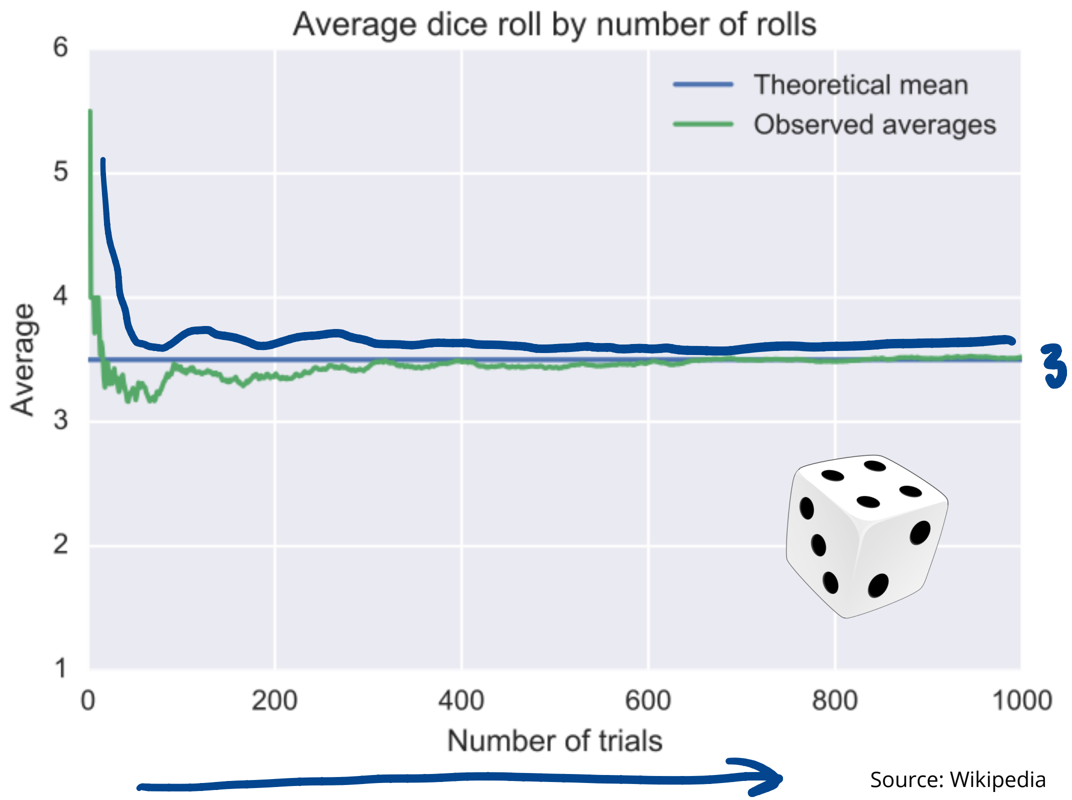
What is the variance of a 6-sided die toss?

$$\mathrm{var}(\boldsymbol{X}) = \mathbb{E}[\boldsymbol{X}^2] - \overset{3.5}{\boxed{\mathbb{E}[\boldsymbol{X}]}}^2 = \frac{1}{6}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) - (3.5)^2$$

$$= \frac{91}{6} - 12.25 = 2.92$$

# LAW OF LARGE NUMBERS

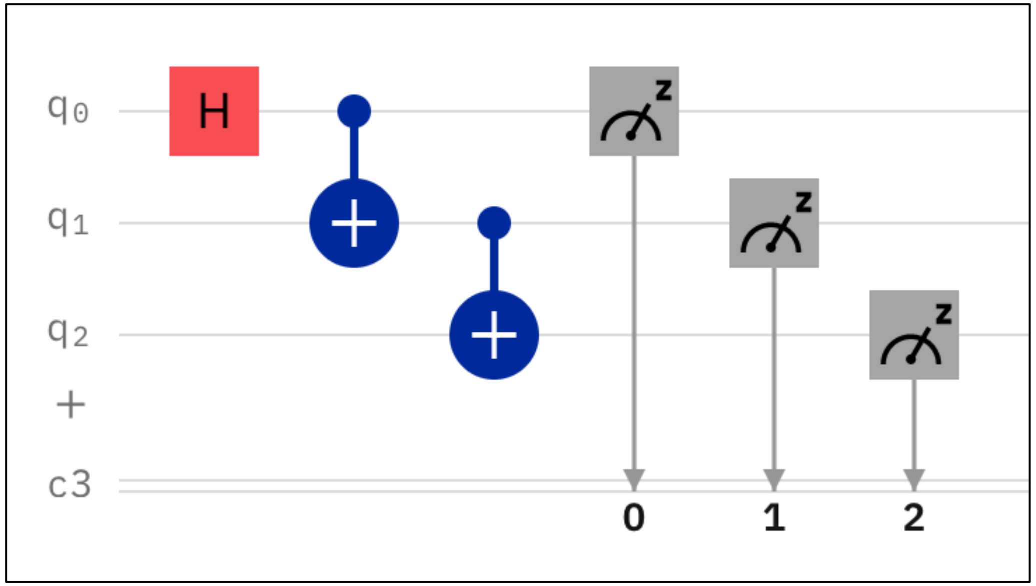
***Law of Large Numbers*** : The average of a large number of trials should be close to the *expected value* and will tend become closer to the expected value as the number of trials increases.



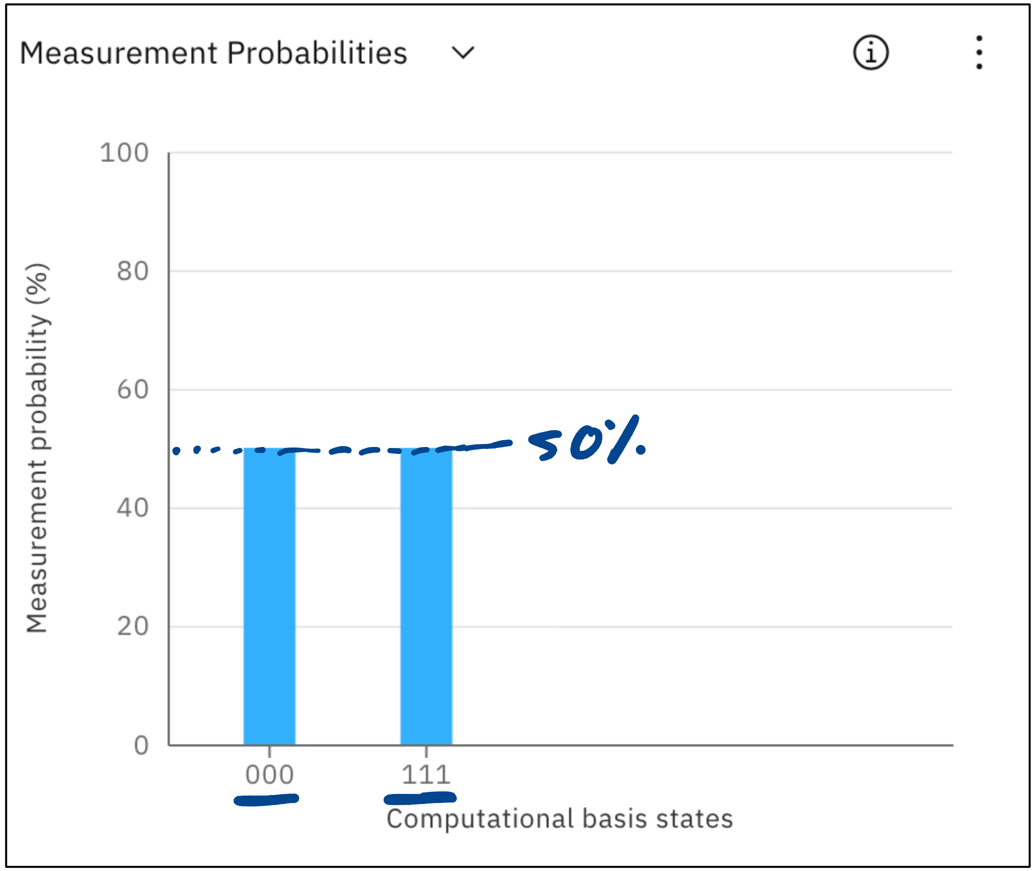

$3.5 = \langle x \rangle$

Source: Wikipedia

Running and measuring the Bell State circuit…

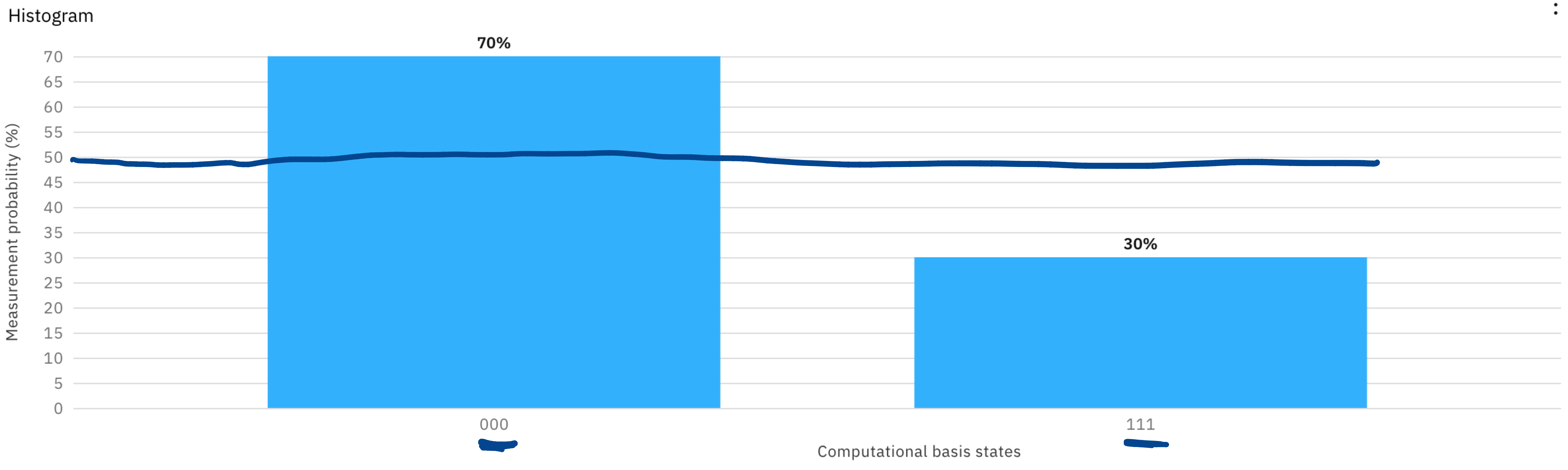



Ideally we expect a 50-50 distribution over the states $|000\rangle$ and $|111\rangle$ .



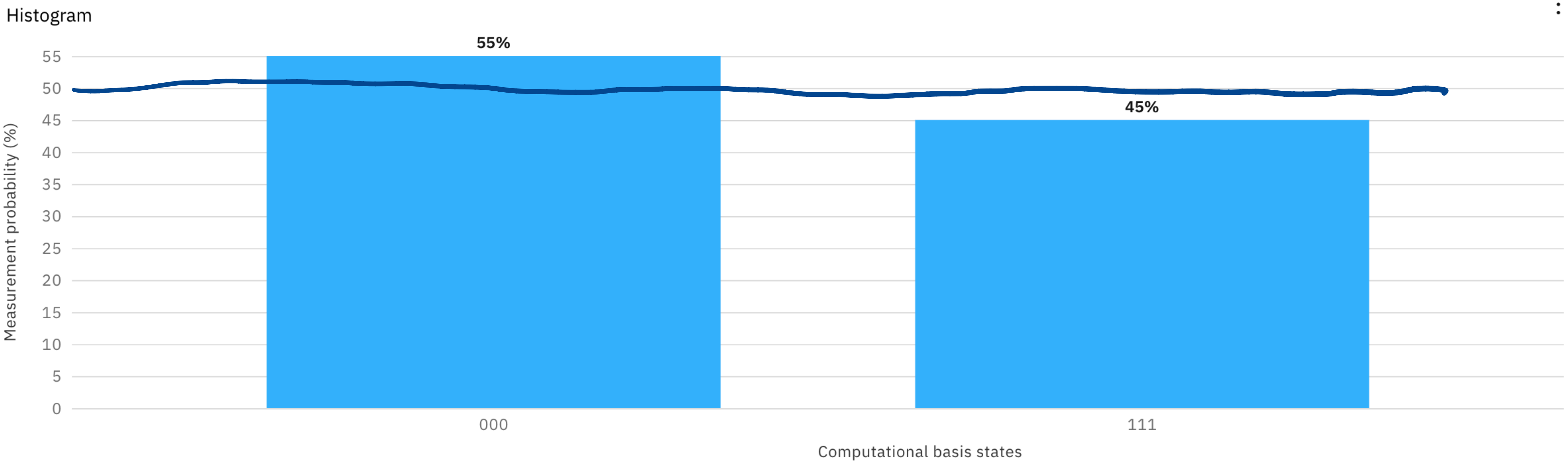
# LAW OF LARGE NUMBERS - QUANTUM

**10 shots** → trials/measurements



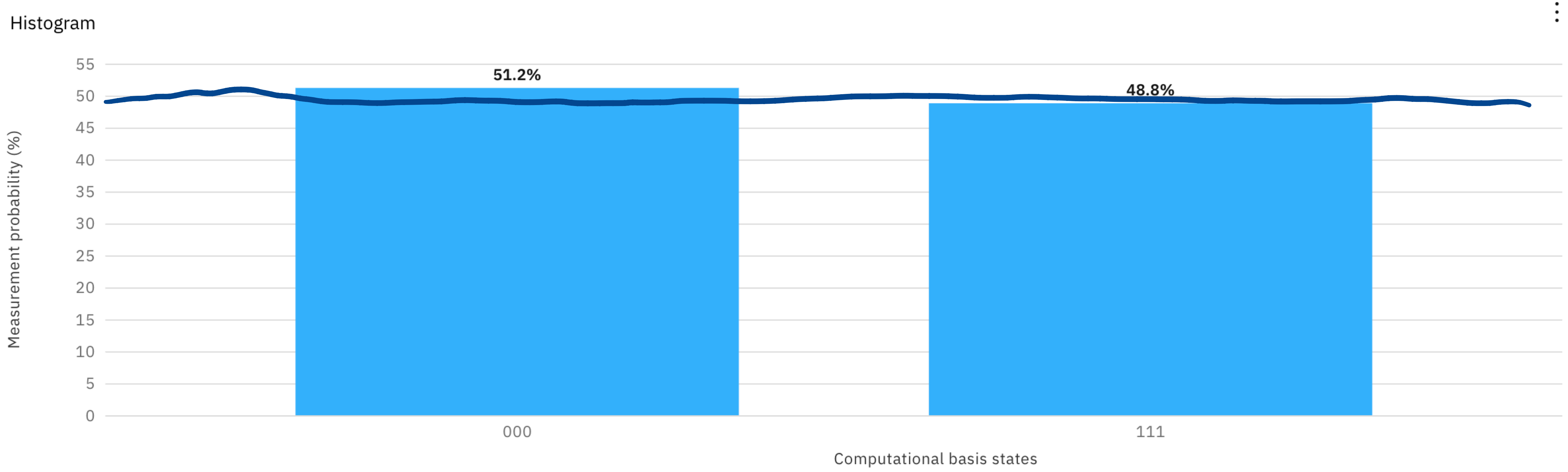
# LAW OF LARGE NUMBERS - QUANTUM

**100 shots**

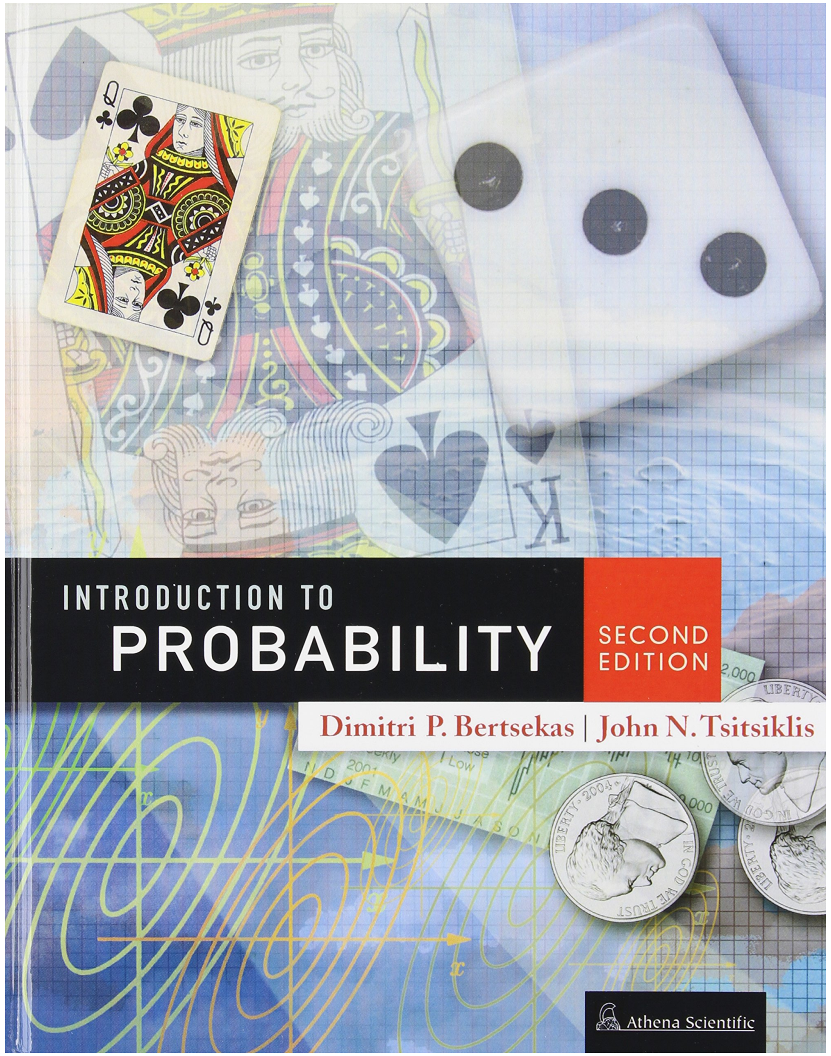


# LAW OF LARGE NUMBERS - QUANTUM

## 1000 shots



# FURTHER PROBABILITY RESOURCES



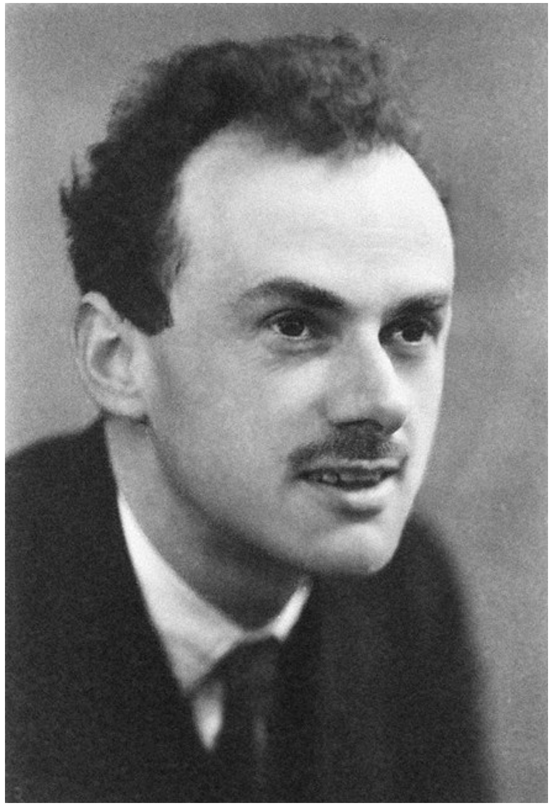

**MIT OCW**

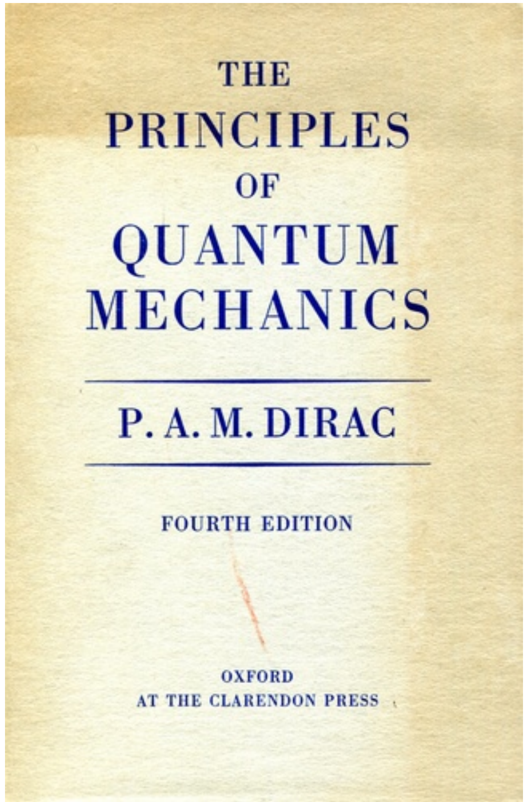
**6.041 or 18.600**

# MATHEMATICS FOR QUANTUM PT. 1

# DIRAC NOTATION - A BRIEF HISTORY

***Paul Dirac*** established the most general theory of quantum mechanics and discovered the relativistic equation for the electron. In 1930 he published "*The Principles of Quantum Mechanics*", which quickly became the standard text in the field. In the third edition of the book, he introduced Dirac or bra-ket notation…





# DIRAC NOTATION – BRAS AND KETS

A ***ket*** is simply a column vector!

$$|v\rangle = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

A ***bra*** is the conjugate transpose of a ket (row vector)!

$$\langle v| = |v\rangle^{\dagger} = \begin{pmatrix} v_1^* & v_2^* & \cdots & v_n^* \end{pmatrix}$$

# DIRAC NOTATION – QUANTUM STATES

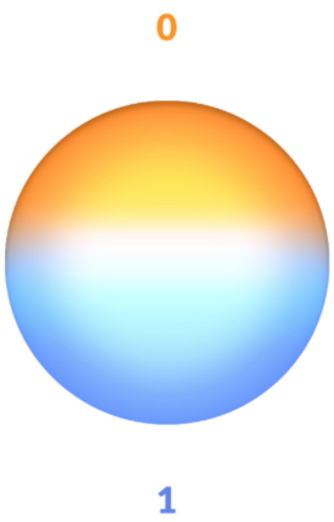
0

**Ground State**  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

**Excited State**  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

1

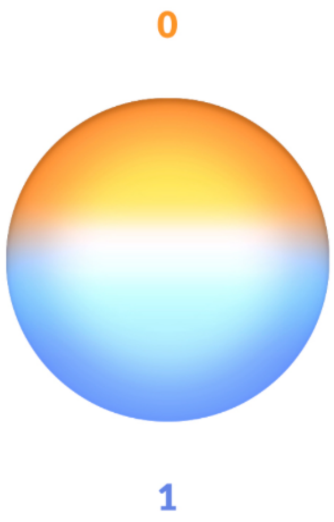
**Superposition State**




$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

# DIRAC NOTATION – QUANTUM STATES

**Superposition State**




$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$\alpha, \beta \in \mathbb{C}$ are **probability amplitudes**, with $|\alpha|^2 + |\beta|^2 = 1$

$\alpha^*\alpha$ $\beta^*\beta$

# DIRAC NOTATION – QUANTUM STATES

How would we represent an equal superposition state?

H – $\frac{1}{2}$

T – $\frac{1}{2}$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Schrodinger's

# ANNOUNCEMENTS

11/22: PYTHON LECTURE**

11/24-11/28: NO LAB

11/29: PYTHON LECTURE**