INTRO TO QUANTUM COMPUTING

LECTURE #24

Near-term Quantum Algorithms

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05/02/2021

ANNOUNCEMENTS

THE QUANTUM ALGOS LANDSCAPE

Deutsch-Jozsa

First theoretical demonstration of quantum advantage!

Uses an Oracle

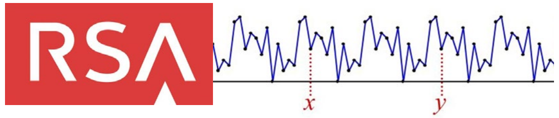
$$O(1) \ll O(2^n)$$

Exponential Quantum Speedup!

QUANTUM ADVANTAGE

Shor's Algorithm

Super-polynomial speedup for factoring using the QFT!



Cracking RSA Requires Factoring & Period Finding

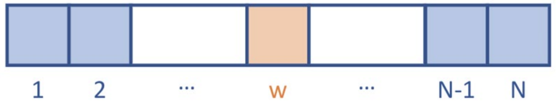
The Quantum Fourier Transform Encodes Frequency in Phase

$$O(\log(n)^3) \ll O(n^{1.9})$$

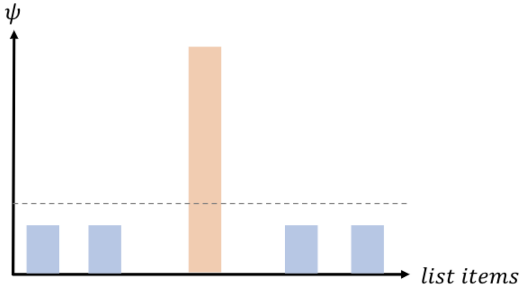
Super-Polynomial Quantum Speedup!

Grover Search

Quadratic speedup for search using amplitude amplification!



Unstructured Search



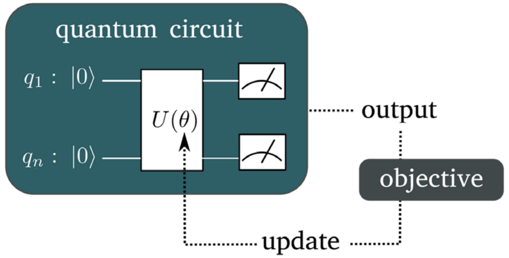
Leverages Amplitude Amplification

$$O(\sqrt{n}) \ll O(n)$$

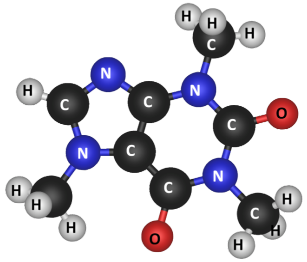
Quadratic Quantum Speedup!

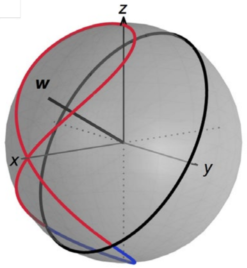
Near-Term Algos

Applications of noisy, small available quantum devices!



Hybrid Quantum-Classical Algos





Using Quantum to Solve Important Problems!

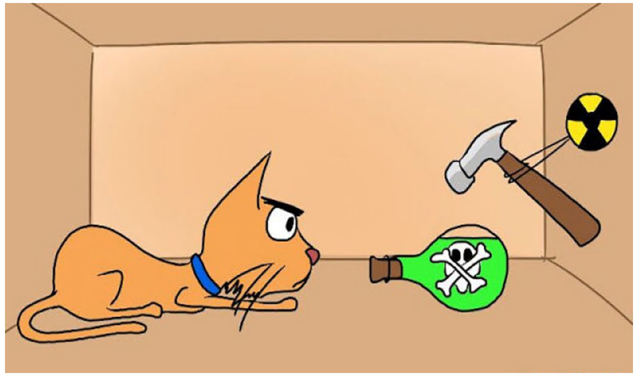
How to program a quantum computer today

- The error in present day quantum computers get out of hand for large quantum circuits
- Largest number factored using Shor's algorithm: 35
- We can't run any of the algorithms discussed without error correction for a meaningful instance
- Near-term algorithms take these restrictions into account when programming a quantum computer

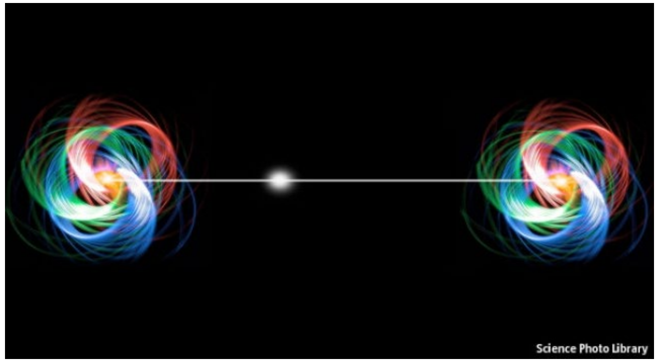
What do we need for running algorithms

- ➔ Many qubits
- ➔ Either with no noise (error), or error-corrected
- ➔ And:

SUPERPOSITION



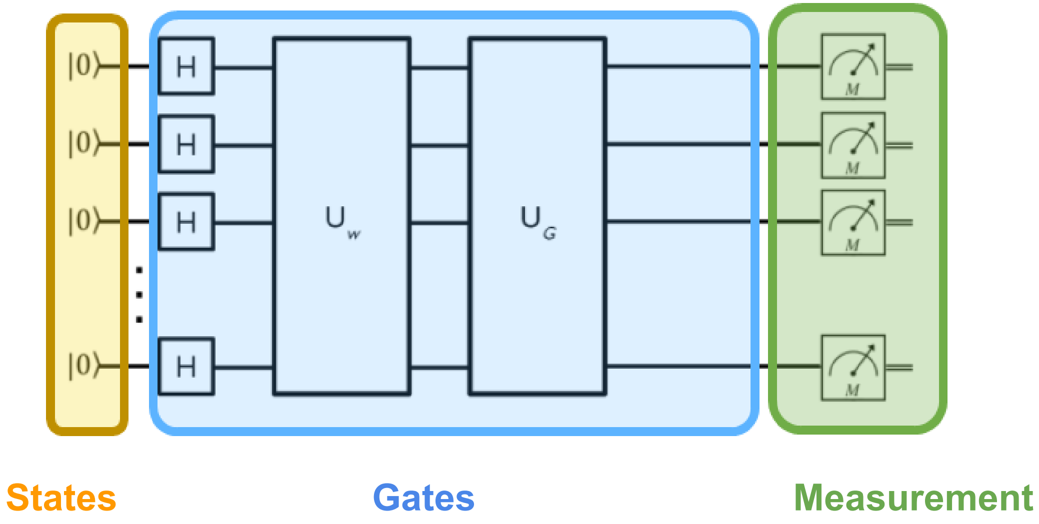
ENTANGLEMENT



QUANTUM INTERFERENCE

Properties of a Quantum Computer

- Enough* qubits to run algorithms
- A way to fix errors if they occur
- A set of gates that are "universal"
- Capability to readout the state at the end of the algorithm



Motivation

What we want for quantum computing:

- No errors (no noise, error correction, perfect gates)
- No limitations on the number of qubits
- No limitations on the number of gates (circuit depth)

What we have:

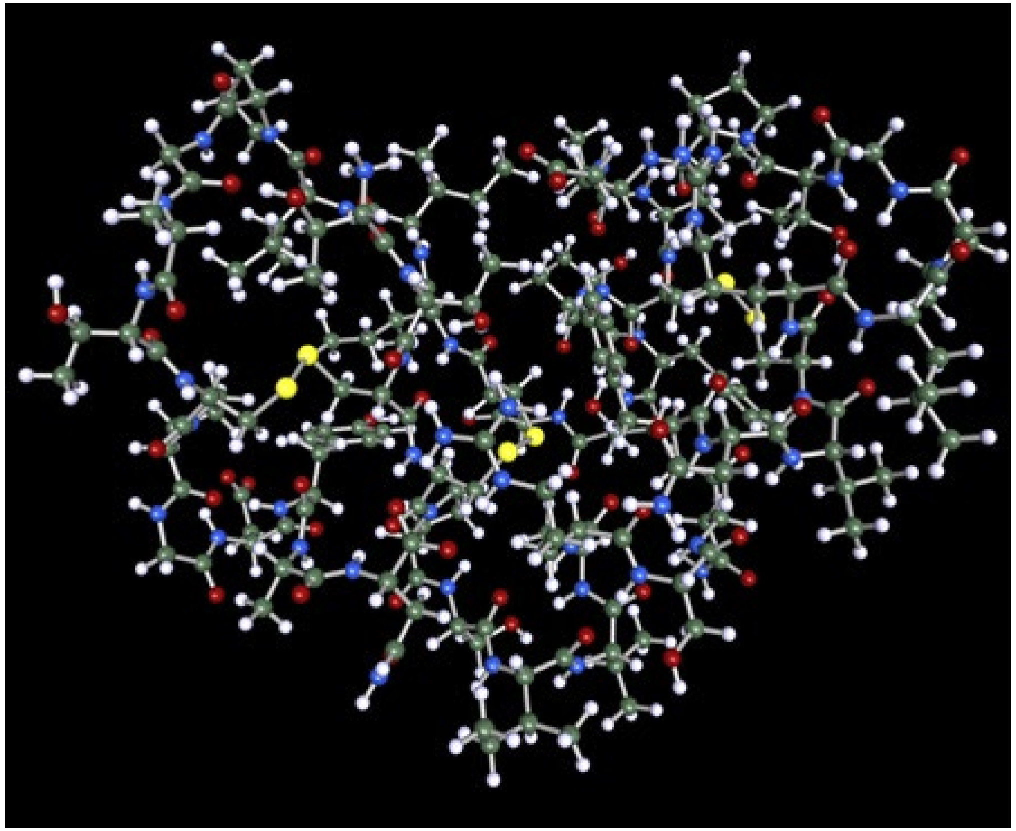
- Limited number of qubits (50-200)
- No error correction -> limited circuit depth
- Known as "noisy intermediate-scale quantum (**NISQ**)" or simply "Near-term quantum" devices

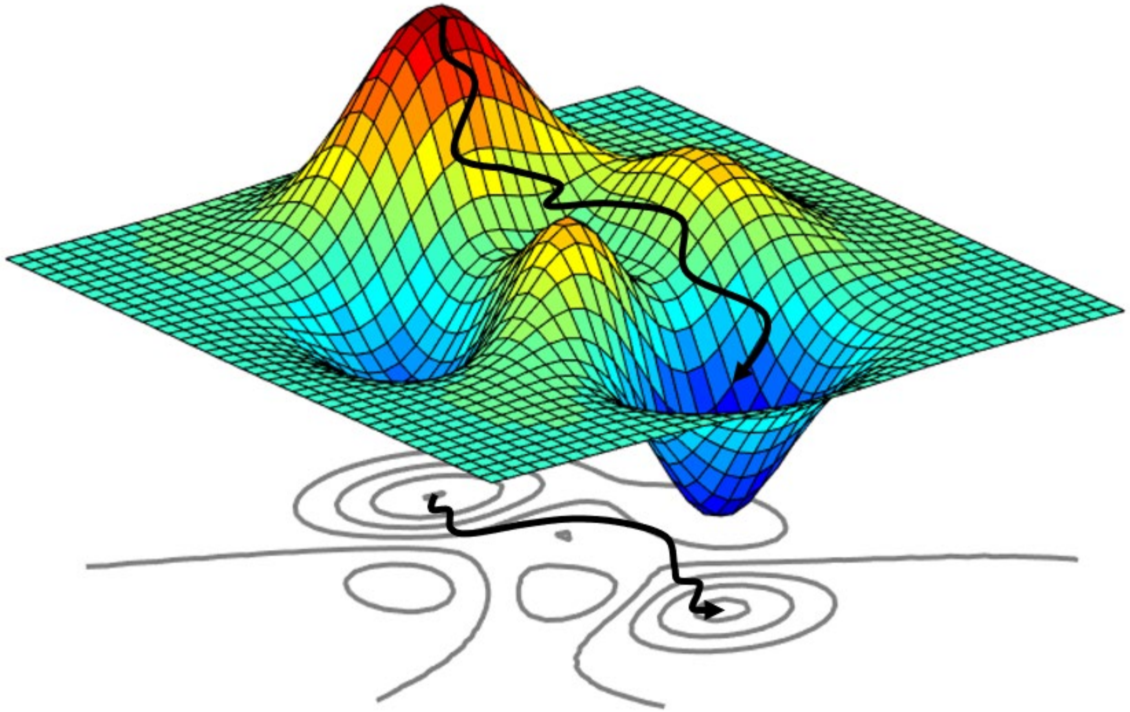
State of the art quantum processor

Today's state of the art quantum computers:

- Superconducting quantum processors
- Number of qubits: 50-100
- T1, T2 ~ 50-100 microseconds
- two-qubit gate fidelity: 99.4%
- No demonstration of ***useful*** quantum advantage

What can we do with near-term computers?





Find the most stable configuration of a molecule

Use cases: medicine, farming, material design

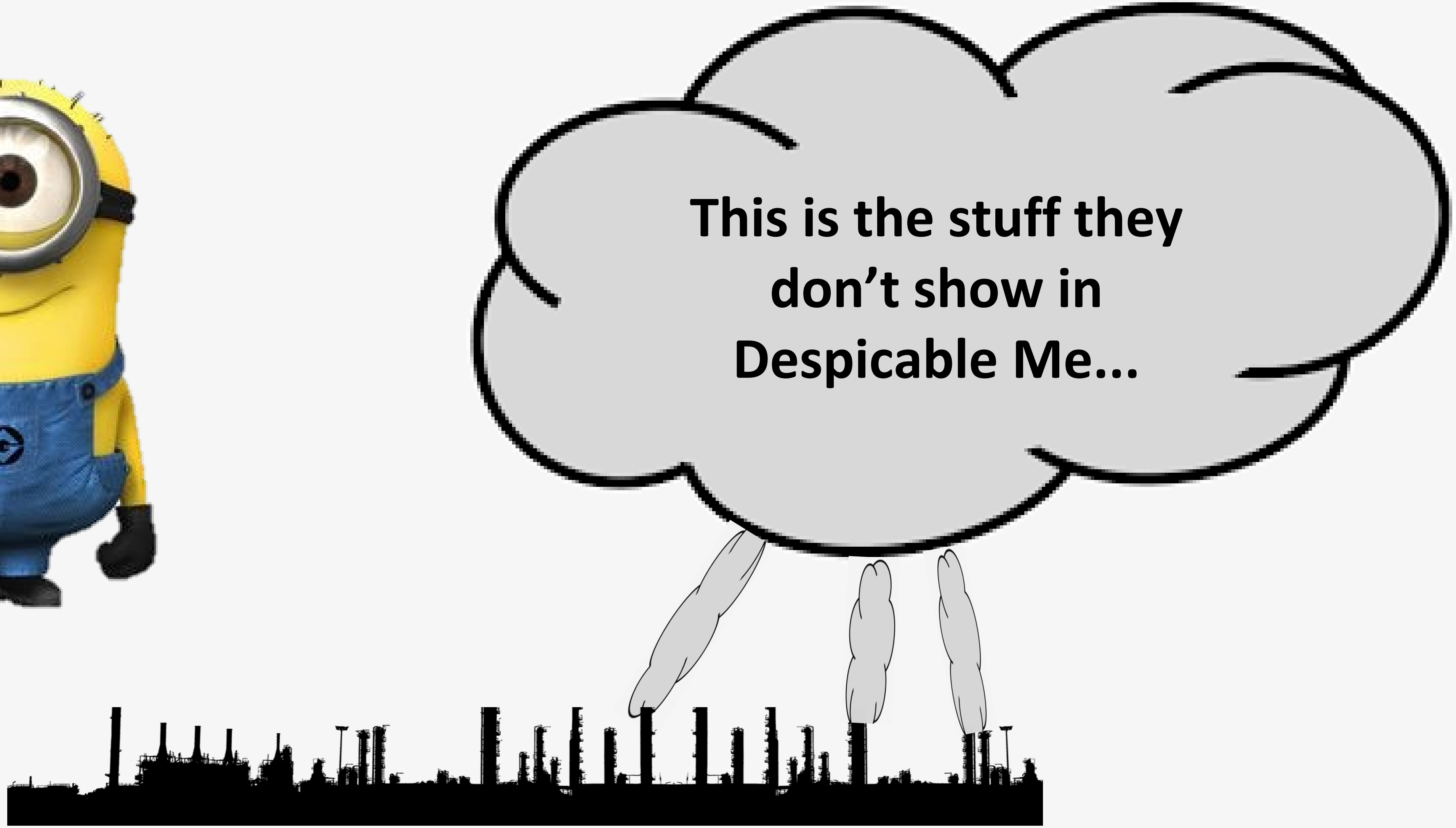
Better optimizing complex problems

Use cases: finance, scheduling, machine learning

Quantum-Classical hybrid algorithm

<u>Idea</u>: Let’s combine the small quantum computers that we have with the classical processing power that’s available to gain an advantage!

Step into our Quantum-Classical Factory

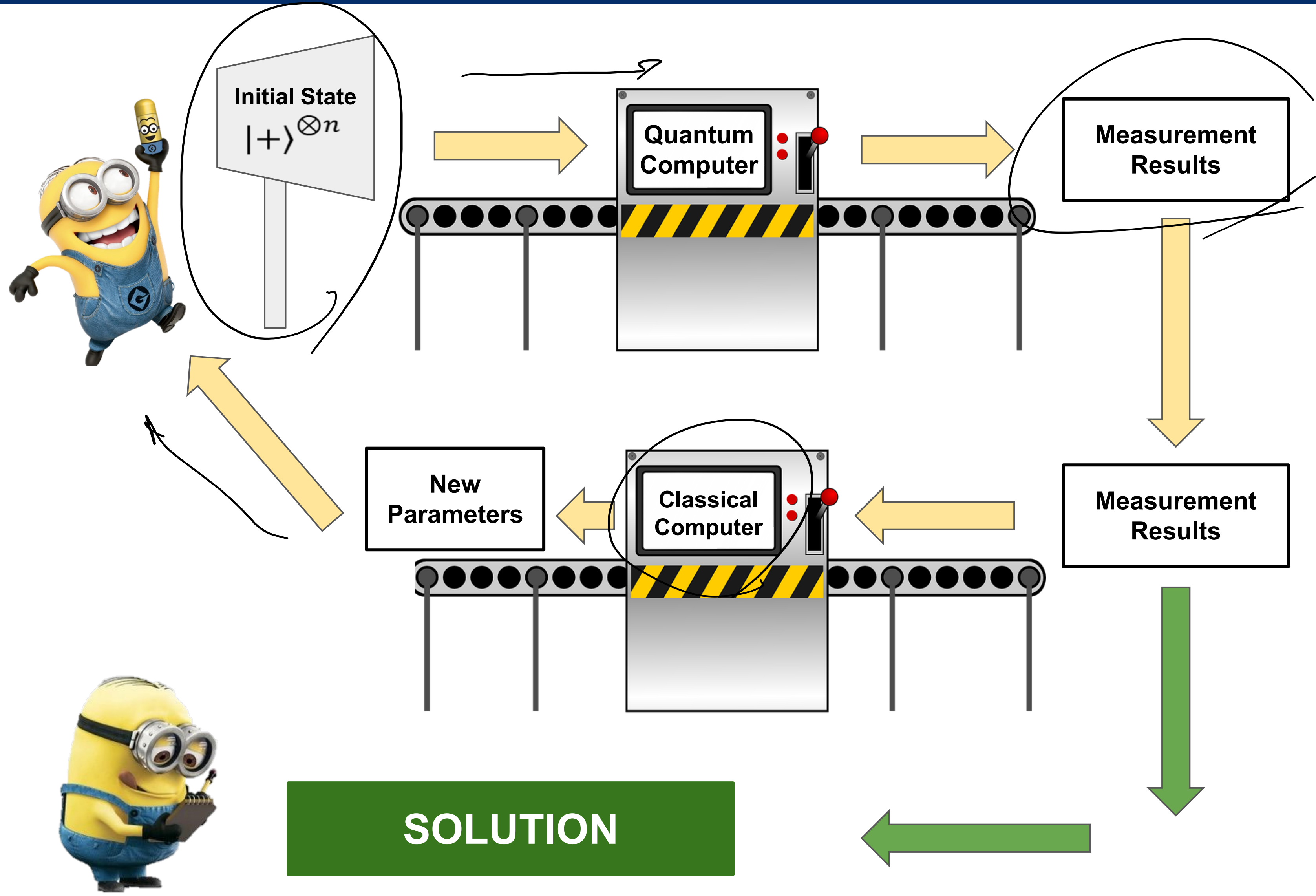


Step into our Quantum-Classical factory

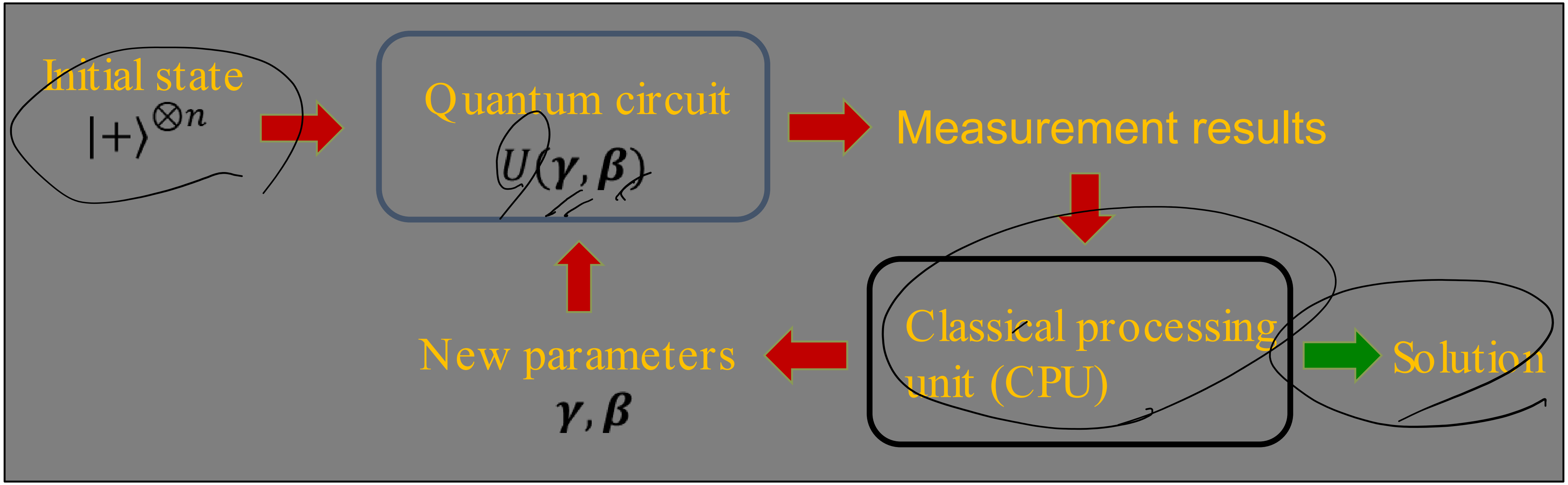
QC: quantum computer
CC: classical computer

- **QC**: run your quantum circuit
- **QC**: send the results of your circuit to CC
- **CC**: figure out how to modify the circuit
- **CC**: send new instructions to QC

Repeat!

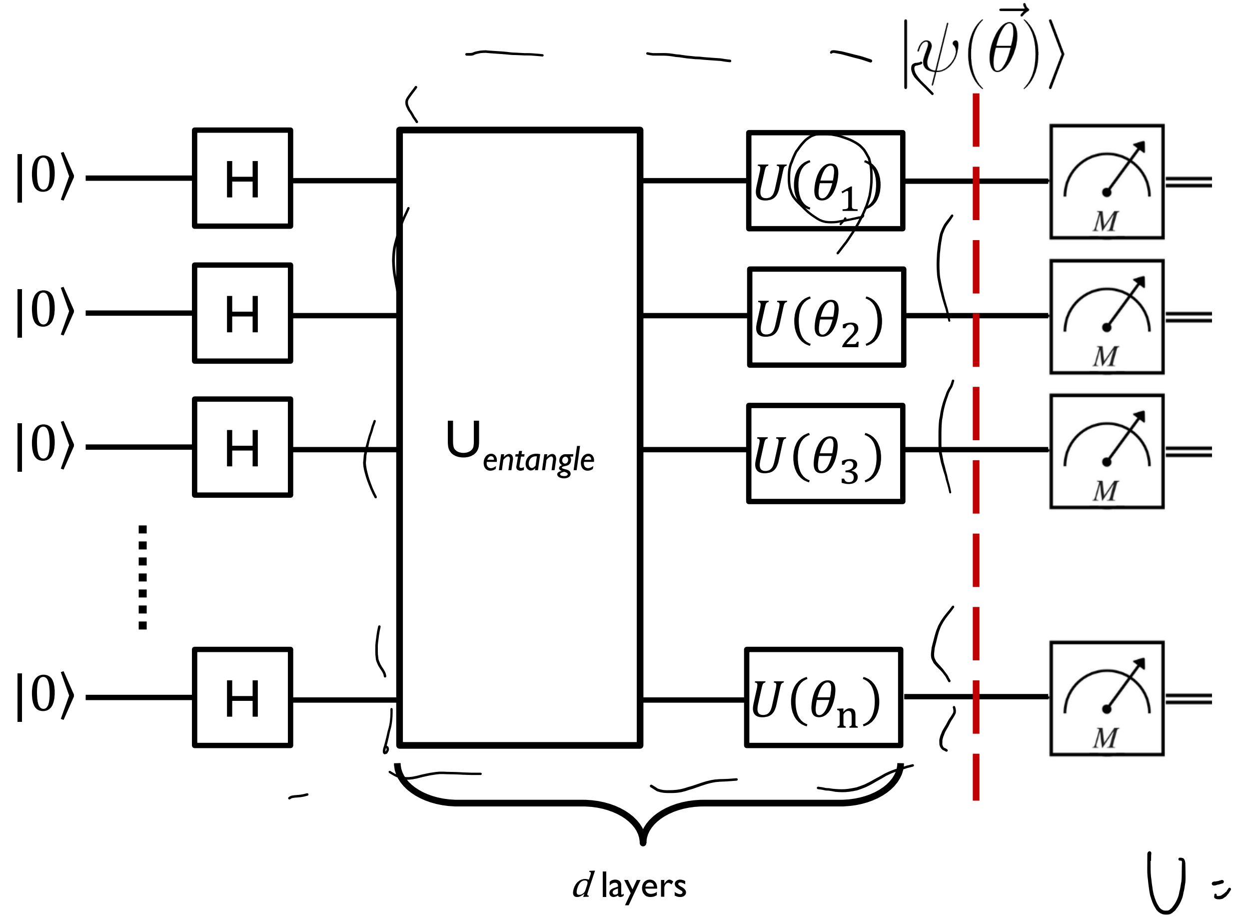


The workflow

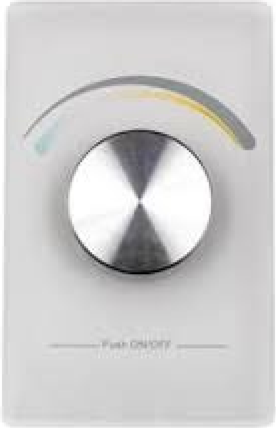


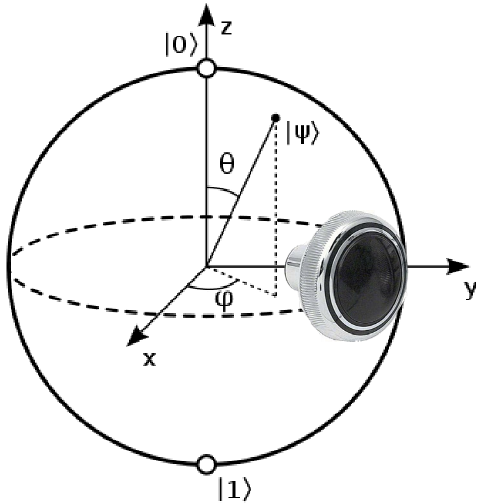
Variational quantum circuits

idea: Let's use "tunable" (variational) quantum circuits



We have noisy qubits and gates, so we try to compensate by adding tunable parameters





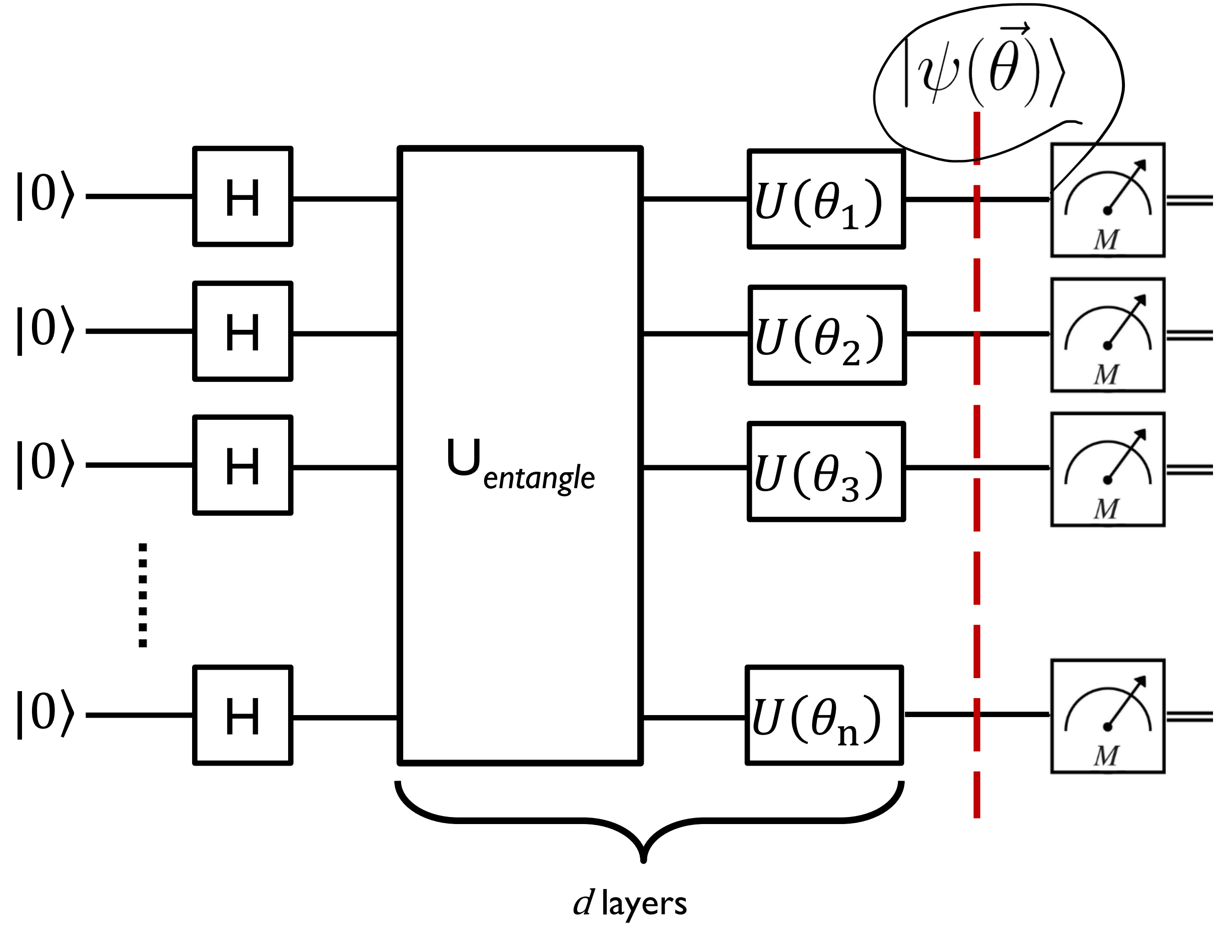
$U = R_x(\theta)$ $\theta = \pi \rightarrow U = X$

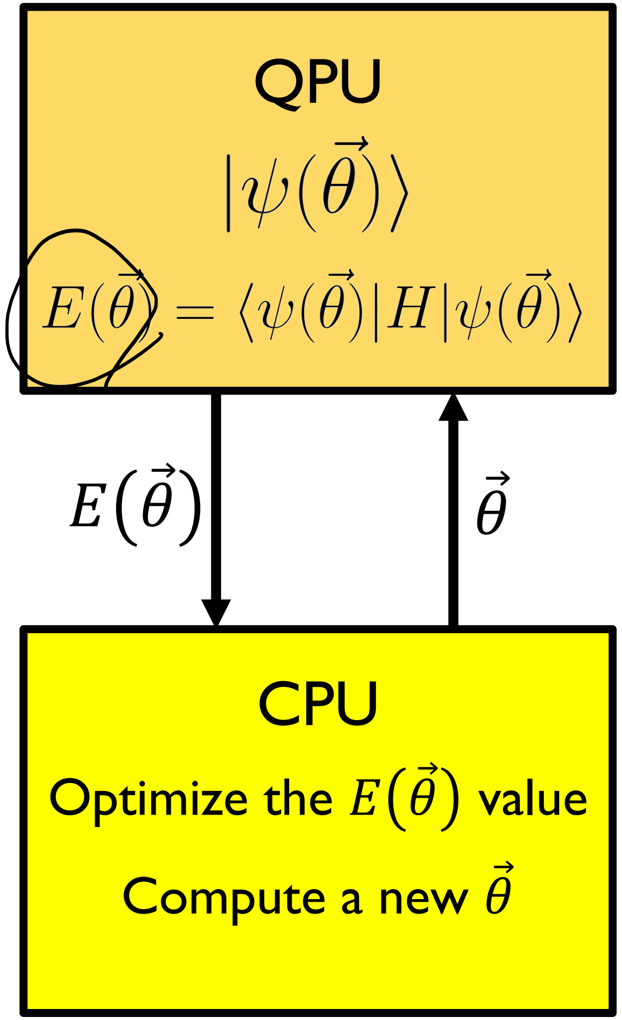
$\theta = \pi/2$ $U = e^{i\pi/4 X}$

Variational quantum circuits

The tunable circuit template is called an *ansatz*

We fill in the template with our choice for the tunable parameters!

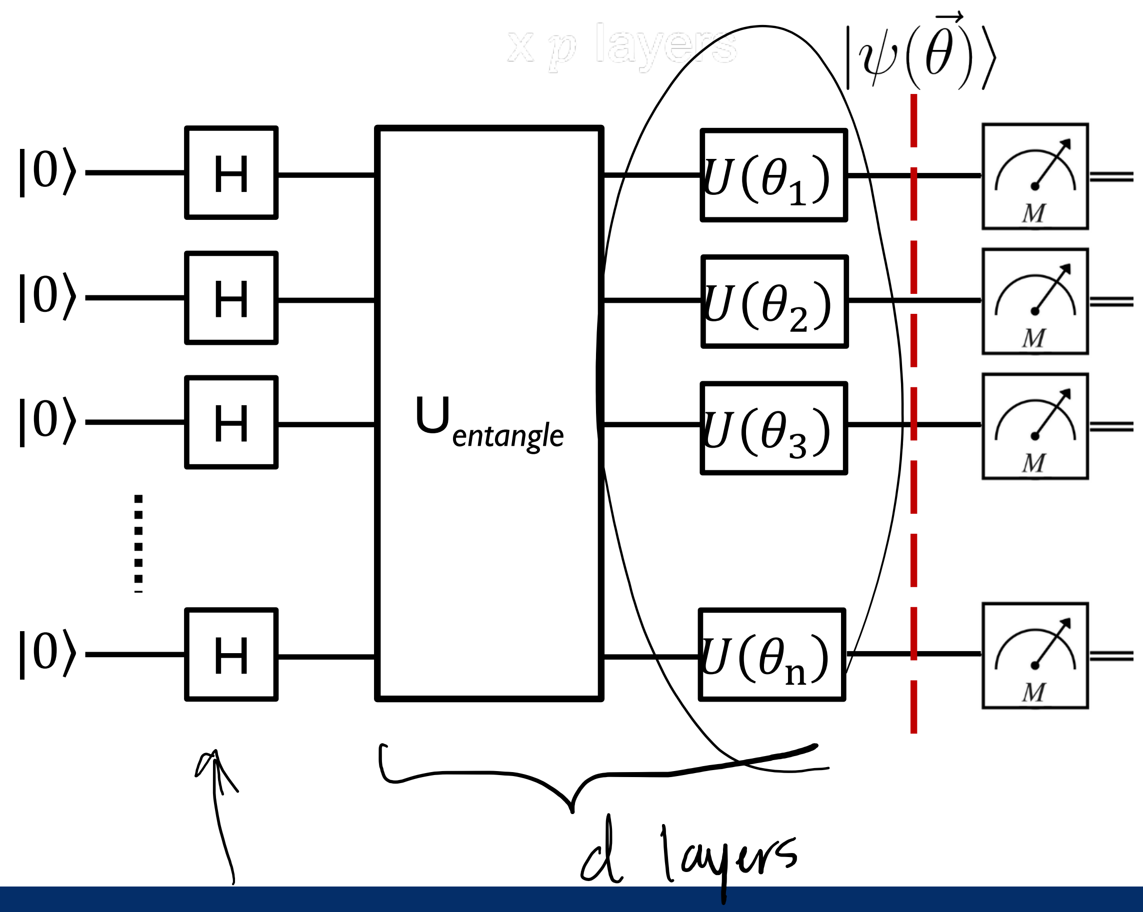




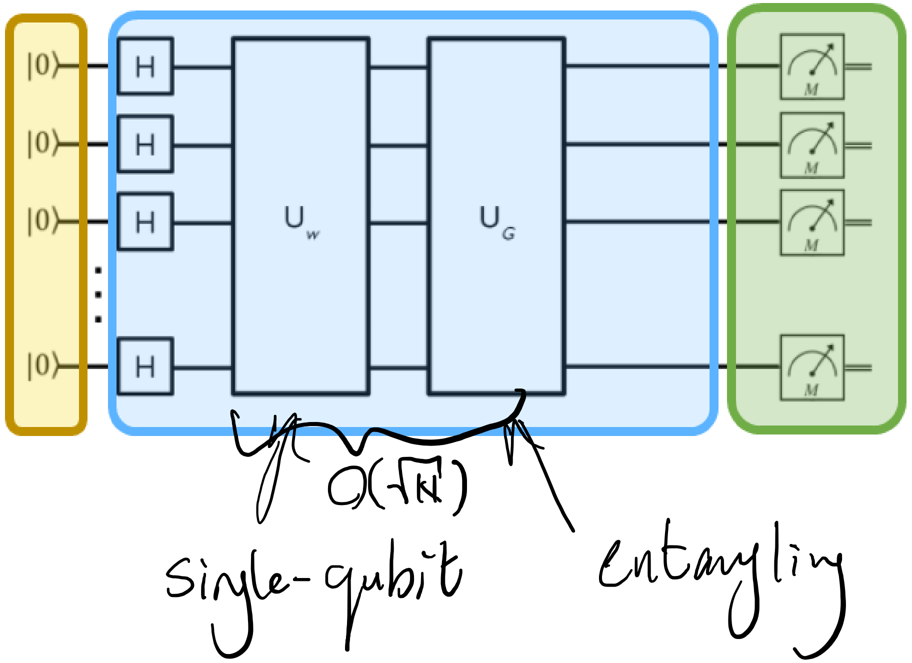
Variational quantum circuits

The circuit for Grover's was pre-fixed

But we can change the parameters of the variational circuits



Similar to Grover's algorithm circuit:



Quantum-Classical hybrid algorithms

Few notes:

- Variational circuits are a heuristic approach, trying to imitate amplitude amplification
- Not every variational circuit is created equal
  - Expressibility : how many different quantum states can we produce with our ansatz
- The art of variational quantum-classical algorithms is coming up with good ansatz

Quantum-Classical hybrid algorithms

Two famous variational algorithms:

- ***V***ariational ***Q***uantum ***E***igensolver (VQE)
  - Used for continuous systems (e.g. chemistry simulations)

- ***Q***uantum ***A***pproximate ***O***ptimization ***A***lgorithms (QAOA)
  - Used for combinatorial optimization

VQE

Recent breakthroughs!

ARTICLE

Received 9 Dec 2013 | Accepted 27 May 2014 | Published 23 Jul 2014 DOI: 10.1038/ncomms5213 OPEN

A variational eigenvalue solver on a photonic quantum processor

Alberto Peruzzo[1,*,†], Jarrod McClean[2,*], Peter Shadbolt[1], Man-Hong Yung[2,3], Xiao-Qi Zhou[1], Peter J. Love[4], Alán Aspuru-Guzik[2] & Jeremy L. O'Brien[1]

LETTER

doi:10.1038/nature23879

Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets

Abhinav Kandala[1*], Antonio Mezzacapo[1*], Kristan Temme[1], Maika Takita[1], Markus Brink[1], Jerry M. Chow[1] & Jay M. Gambetta[1]

PHYSICAL REVIEW X **6**, 031007 (2016)

Scalable Quantum Simulation of Molecular Energies

P. J. J. O'Malley,[1,*] R. Babbush,[2,†] I. D. Kivlichan,[3] J. Romero,[3] J. R. McClean,[4] R. Barends,[5] J. Kelly,[5] P. Roushan,[5] A. Tranter,[6,7] N. Ding,[2] B. Campbell,[1] Y. Chen,[5] Z. Chen,[1] B. Chiaro,[1] A. Dunsworth,[1] A. G. Fowler,[5] E. Jeffrey,[5] E. Lucero,[5] A. Megrant,[5] J. Y. Mutus,[5] M. Neeley,[5] C. Neill,[1] C. Quintana,[1] D. Sank,[5] A. Vainsencher,[1] J. Wenner,[1] T. C. White,[5] P. V. Coveney,[7] P. J. Love,[6] H. Neven,[2] A. Aspuru-Guzik,[3] and J. M. Martinis[5,1,‡]

LETTER

https://doi.org/10.1038/s41586-019-1040-7

Error mitigation extends the computational reach of a noisy quantum processor

Abhinav Kandala[1*], Kristan Temme[1], Antonio D. Córcoles[1], Antonio Mezzacapo[1], Jerry M. Chow[1] & Jay M. Gambetta[1]

QUANTUM COMPUTING

Hartree-Fock on a superconducting qubit quantum computer

Google AI Quantum and Collaborators*†

The simulation of fermionic systems is among the most anticipated applications of quantum computing. We performed several quantum simulations of chemistry with up to one dozen qubits, including modeling the isomerization mechanism of diazene. We also demonstrated error-mitigation strategies based on *N*-representability that dramatically improve the effective fidelity of our experiments. Our parameterized ansatz circuits realized the Givens rotation approach to noninteracting fermion evolution, which we variationally optimized to prepare the Hartree-Fock wave function. This ubiquitous algorithmic primitive is classically tractable to simulate yet still generates highly entangled states over the computational basis, which allowed us to assess the performance of our hardware and establish a foundation for scaling up correlated quantum chemistry simulations.

10 MIN BREAK!

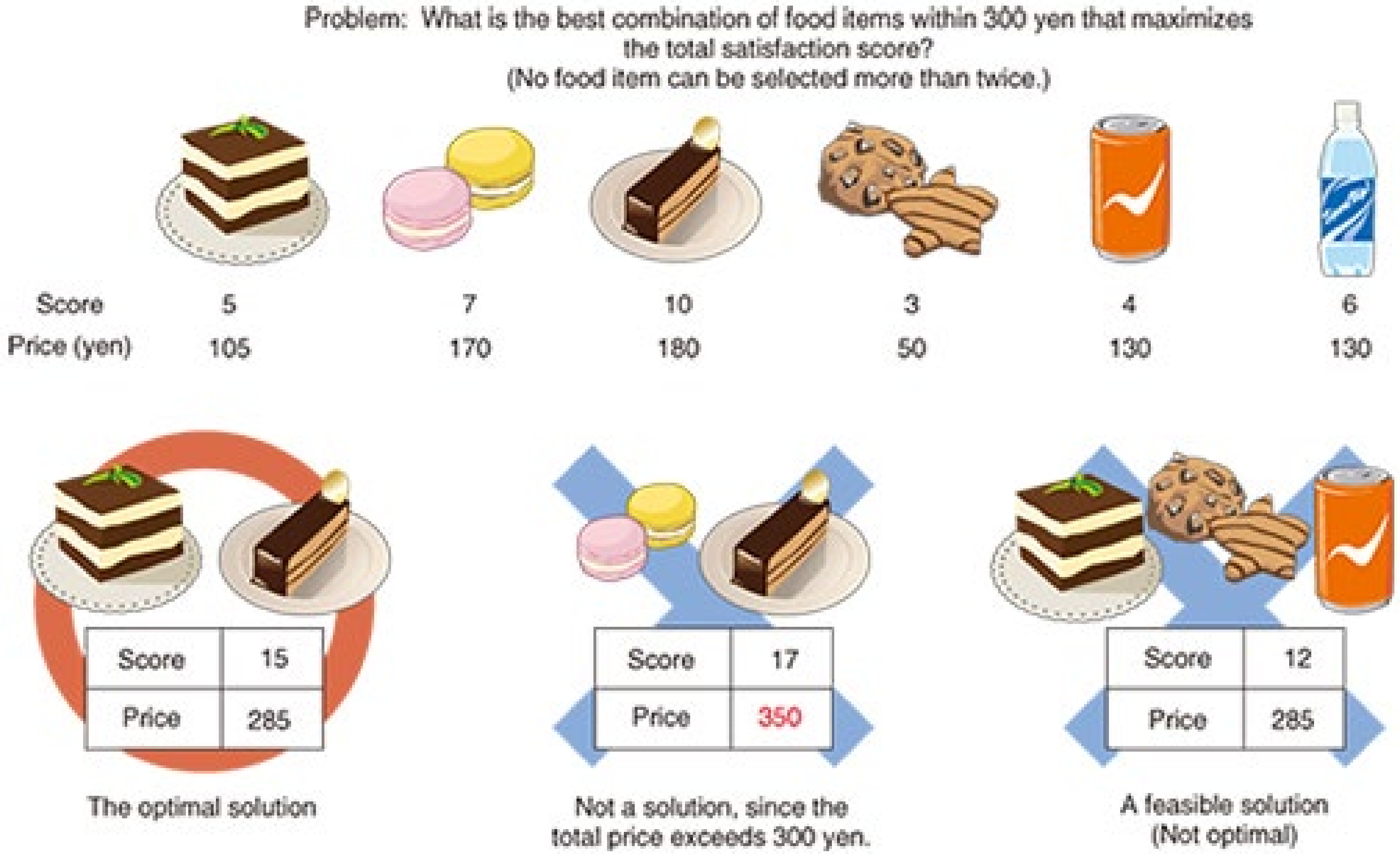
Till 2:46 pm EST

Quantum Approximate Optimization Algorithm (QAOA)

A variational quantum algorithm used for combinatorial optimization problems

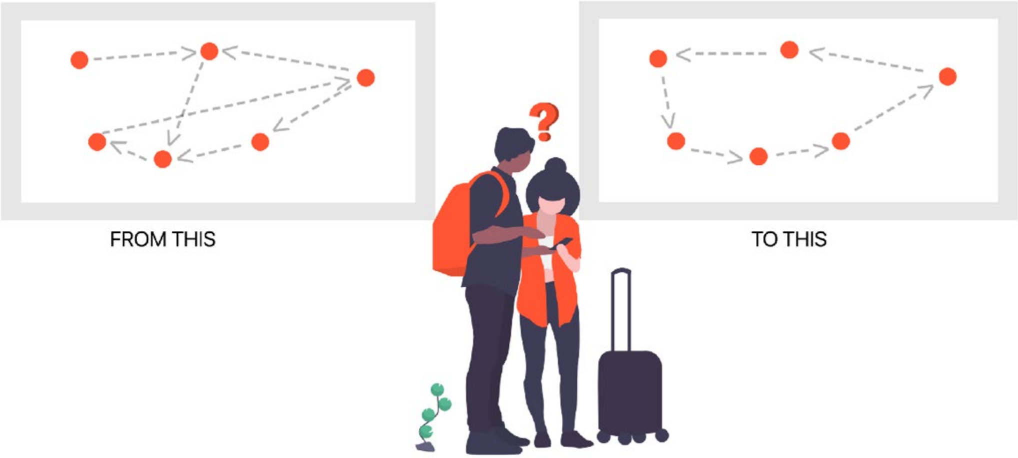
Candy shop problem

What is combinatorial optimization?



Optimization in industry

Traveling Salesman Problem



Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

Nurse Scheduling Problem

	Day 1			Day 2			...	Day 7			Number of shifts
Shifts	M	E	N	M	E	N	...	M	E	N	
Nurse 1							...				6
Nurse 2							...				7
Nurse 3							...				7
Nurse 4							...				7
....	...	...	...	...	...	...	...	...	...	...	...
Nurse J							...				6
Number of nurses assigned	15	18	13	13	18	13	...	13	14	13	314

How can we find an optimal way to assign nurses to shifts within a set of hard and soft constraints, such as allowing no more than two consecutive shifts, considering nurses who are on vacation, and assigning nurses who work well together?

Combinatorial optimization

Let's consider we have:

- ***n*** bits
- ***m*** clauses (conditions)

Example:

- ***3*** bits: b1, b2, b3
- ***2*** clauses:
  - 3*b1 - 2*b2 + b3
  - b1 + b2 + b3

$3 \times b_1 - 2 \times b_2 + b_3$

if $b_1 = 1$, $b_2 = 0$, $b_3 = 1$

$\rightarrow 3 \times 1 - 2 \times 0 + 1 = 4$

What are some values for b1, b2, and b3 that minimize the value of the clauses?

Combinatorial optimization

Cost function: tells us what the cost for different values of the bits are

We can define the cost function as the sum of our clauses!

Back to our example:

- ***3*** bits: b1, b2, b3
- ***2*** clauses:
  - 3*b1 - 2*b2 + b3
  - b1 + b2 + b3

Cost function: C(b1,b2,b3)=[3*b1 - 2*b2 + b3]+[b1 + b2 + b3]=4*b1 - b2 + 2*b3

Combinatorial optimization

Goal: find values for the bits (b1,b2,b3) that minimize (or maximizes) the value of the cost function!

How can we do it?

for 3 bits:

One idea: Let's try every single combination of 0s and 1s

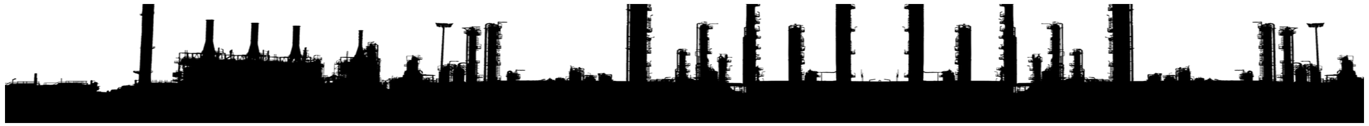
$2^3 = 8$ combos

Problem: The problem grows exponentially with the number of bits :(

QAOA is a heuristic quantum algorithm for tackling this problem!

Back to the Classical-Quantum Factory

The Classical-Quantum Factory just hired a new team of purple minions. The problem? The purple and yellow minions do not speak the same language. They cannot understand each other.



Back to the Classical-Quantum Factory

Once they were given translating devices,
both teams could work together!

Like teams of minions, we can translate between classical bits and qubits.

Translating from bits to qubits

We need to “translate” the problem from classical bits to qubits

Let’s map the value of a bit to an operator for a qubit

Using this mapping we can turn a classical cost function into a quantum Hamiltonian, which a quantum computer understands

Translating from bits to qubits

Let's map the value of a bit to an operator for a qubit

$$b_k \mapsto \frac{1}{2}(1 - Z_k)$$

$Z|0\rangle = (+1)|0\rangle$

$Z|1\rangle = (-1)|1\rangle$

$b = 0 \longrightarrow |0\rangle$ $\quad\quad \frac{1}{2}(1-Z)|0\rangle = (1/2 - 1/2)|0\rangle = 0 \times |0\rangle$

$b = 1 \longmapsto |1\rangle$ $\quad\quad 1/2(1-Z)|1\rangle = 1/2(1+1)|1\rangle = 1 \times |1\rangle$

Translating from bits to qubits

Example: C = b1*b2

$b_1 \mapsto \frac{1}{2}(1 - Z \otimes I)$

$b_2 \mapsto \frac{1}{2}(1 - I \otimes Z)$

$$b_k \mapsto \frac{1}{2}(1 - Z_k)$$

$H = ?$

$b_1 \times b_2 \mapsto \frac{1}{4}(1 - Z \otimes I)(1 - I \otimes Z) = \frac{1}{4}(1 - I \otimes Z - Z \otimes I + Z \otimes Z)$

Z to QB1

$H|11\rangle = \frac{1}{4}(1 - Z_1 - Z_2 + Z_1 Z_2)|11\rangle$

what we expect from C

$= \frac{1}{4}(1 + 1 + 1 + 1)|11\rangle = 1 \times |11\rangle$

$H|10\rangle = \frac{1}{4}(1 - Z_1 - Z_2 + Z_1 Z_2)|10\rangle$

what we expect from C

$= \frac{1}{4}(1 + 1 - 1 + (-1 \times 1))|10\rangle = 0 \, |10\rangle$

Translating from bits to qubits

Example: C = b1 xor b2

$$C = b_1 + b_2 - 2 b_1 \times b_2$$

$$b_1 \mapsto \tfrac{1}{2}(1 - Z_1) \qquad Z_1 = Z \otimes I$$

$$b_2 \mapsto \tfrac{1}{2}(1 - Z_2) \qquad Z_2 = I \otimes Z$$

$$b_k \mapsto \frac{1}{2}(1 - Z_k)$$

$$H = \tfrac{1}{2}(1 - Z_1) + \tfrac{1}{2}(1 - Z_2) - \tfrac{2}{4}(1 - Z_1 - Z_2 + Z_1 Z_2)$$

$$= \tfrac{1}{2} - \cancel{Z_1/2} + \cancel{\tfrac{1}{2}} - \cancel{Z_2/2} - \cancel{\tfrac{1}{2}} + \cancel{Z_1/2} + \cancel{Z_2/2} - Z_1 Z_2/2$$

$$= \tfrac{1}{2}(1 - Z_1 Z_2)$$

$$H|10\rangle = \tfrac{1}{2}(1 - (-1 \times 1))|10\rangle = 1 \times |10\rangle$$

$$H|11\rangle = \tfrac{1}{2}(1 - (-1 \times -1))|11\rangle = 0 \times |11\rangle$$

QAOA circuit

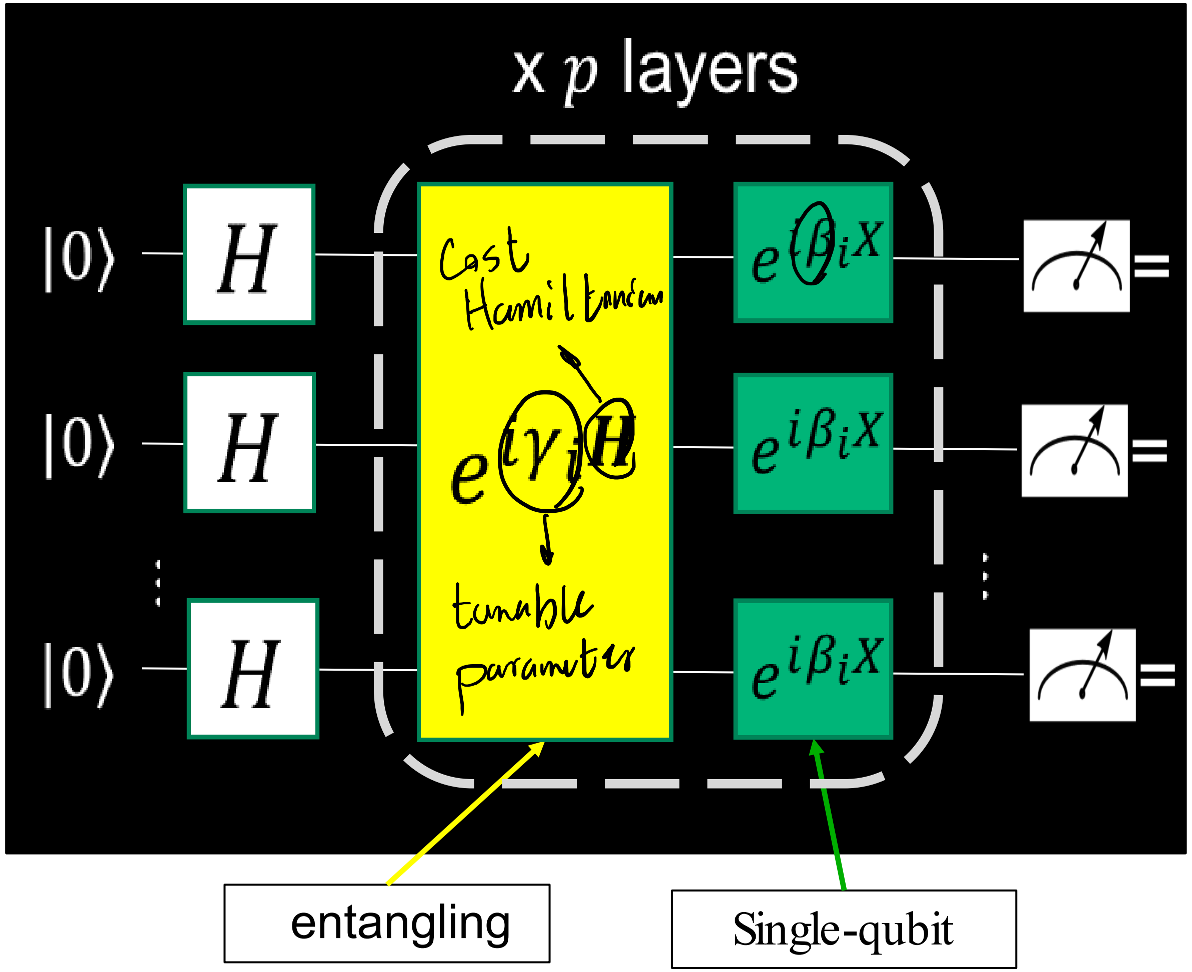
Hermitian

1. Map cost function to a Hamiltonian H

2. Entangling unitary of the template (ansatz)

3. Single-qubit unitary of the template (ansatz)

4. Measurement



QAOA

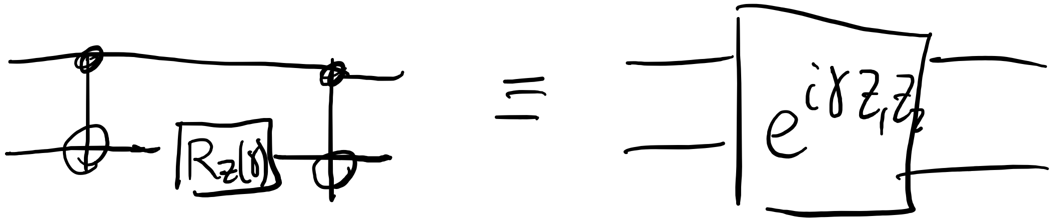
How to make the entangling layer using our gates? CNOTs , $R_z(\gamma)$

$$H = Z_1 + Z_2 + Z_1 Z_2 \qquad U_{ent}(\gamma) = e^{i\gamma H} = e^{i\gamma(Z_1 + Z_2 + Z_1 Z_2)}$$

$$e^{i\gamma H} = e^{i\gamma Z_1} \times e^{i\gamma Z_2} \times e^{i\gamma Z_1 Z_2}$$

$$e^{i\gamma Z_1} = R_z(-2\gamma) \otimes I \quad , \quad e^{i\gamma Z_2} = I \otimes R_z(-2\gamma)$$

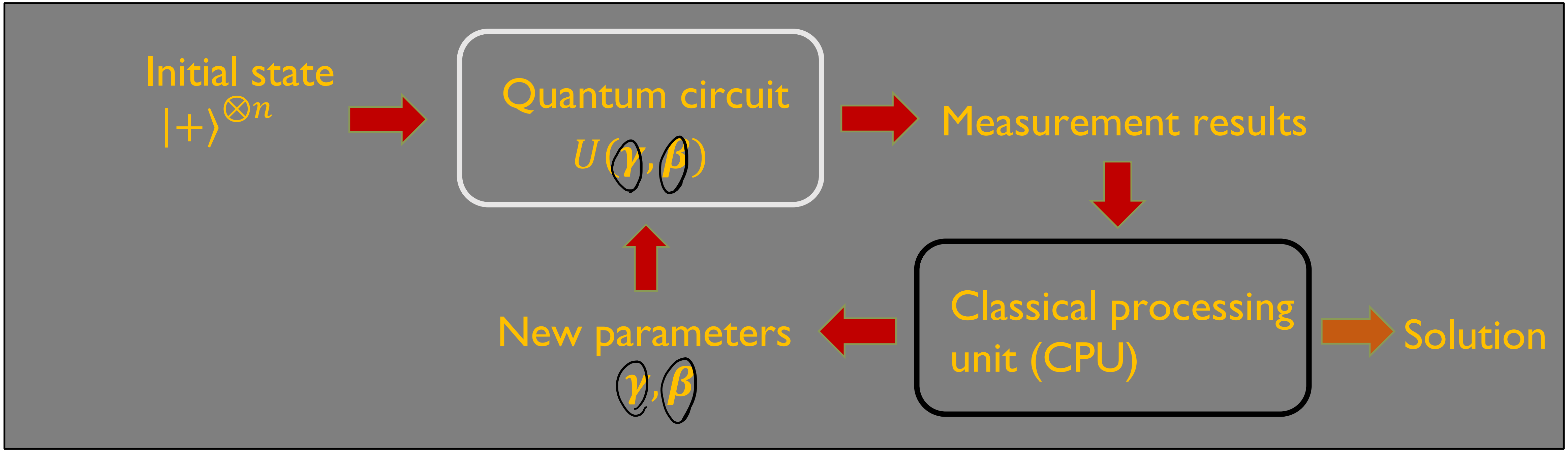
$$e^{i\gamma Z_1 Z_2} =$$



$$R_z(\theta) = e^{-i\theta/2\, Z}$$

QAOA workflow

Find the optimal parameters that minimize the expected value of the cost Hamiltonian:



QAOA

This is an active area of research!!

A Quantum Approximate Optimization Algorithm

Edward Farhi and Jeffrey Goldstone
Center for Theoretical Physics
Massachusetts Institute of Technology
Cambridge, MA 02139

Sam Gutmann

2014

ARTICLES
https://doi.org/10.1038/s41567-020-01105-y
nature physics
Check for updates

Quantum approximate optimization of non-planar graph problems on a planar superconducting processor

Quantum approximate optimization is computationally universal

Seth Lloyd
Departments of Mechanical Engineering and Physics
Massachusetts Institute of Technology
Xanadu

Analyzing the Performance of Variational Quantum Factoring on a Superconducting Quantum Processor

Amir H. Karamlou[1,2,*], William A. Simon[1,*], Amara Katabarwa[1],
Travis L. Scholten[3], Borja Peropadre[1], and Yudong Cao[1,†]
[1] *Zapata Computing, Boston, MA 02110 USA*
[2] *Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA and*
[3] *IBM Quantum, IBM T. J. Watson Research Center, Yorktown Heights, NY 10598*
(Dated: March 11, 2021)

Looking forward

Want to learn more?

- The original QAOA paper: https://arxiv.org/abs/1411.4028
- QAOA in the Qiskit Textbook: https://qiskit.org/textbook/ch-applications/qaoa.html
- First ever VQE demonstration: https://www.nature.com/articles/ncomms5213
- VQE in the Qiskit Textbook: https://qiskit.org/textbook/ch-applications/vqe-molecules.html

Looking forward

What's next?

- Variational circuits can be executed on near-term quantum computers
- Use your knowledge to design and execute your own variational circuit
- Combine your quantum resources with your classical algorithms
- Experiment!

Special thanks to

Our All-Stars!!

Katie

Sarah

Rachel

Kiera

Welcome to your quantum journey!