Homework review 9

Vector spaces

Independence

Span

Basis

Eigen

Vector Space

- Collection of stuff
- Abstraction
- E.g. the number 10, could represent 10 books, pizzas, universes etc.
- Similarly, vector space- collection of certain vectors

Vector Space- mathsy

- Add vectors
- Multiply a scalar/ Scale vectors

For all

$\vec{x}, \vec{y}, \vec{z} \in \boldsymbol{\mathcal{V}}$

(where $\boldsymbol{\mathcal{V}}$ is a vector space)

$a, b \in \mathbb{C}$

1. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$
2. $\vec{x} + (\vec{y} + \vec{z}) = (\vec{x} + \vec{y}) + \vec{z}$
3. There is a unique zero vector ($\vec{0}$), such that $\vec{x} + \vec{0} = \vec{x}$
4. For each $\vec{x}$ there is a unique $-\vec{x}$, such that $\vec{x} + (-\vec{x}) = \vec{0}$
5. $1\vec{x} = \vec{x}$
6. $(ab)\,\vec{x} = a(b\vec{x})$
7. $a(\vec{x} + \vec{y}) = a\vec{x} + b\vec{x}$
8. $(a + b)\vec{x} = a\vec{x} + b\vec{x}$

Questions

Considera vector space V which contains real 3-component vectors $|a\rangle$, $|b\rangle$ and $|c\rangle$. For **Questions 1-4**, state whether the following would also be contained in V. (Answer True if it is contained in V or False if it is not.)

1. $|w\rangle = 2\,|a\rangle$

2. $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Linear Independence/ Dependence

- Linearly dependent if at least one vector can be expressed in terms of other vectors e.g. {(1,1), (5,5)}
- Alternatively, if the determinant of the given vectors written as column matrices is 0, then they're linearly dependent.
- **Independent if not dependent**
- Mathematically:

If $c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k = \vec{0}$ only when $c_1 = c_2 = \cdots = c_k = 0$, then $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k$ are ***linearly independent***.

Span

- What vectors can you create when you scale your original vectors and add them?
- All vectors that you can create with a ***linear combination*** (i.e. scaling and adding) of the original vectors is the span of those vectors.
- https://youtu.be/k7RM-ot2NWY

Basis of a vector space

- A set
- Contains **linearly independent** vectors
- **Spans** all of that space
- Number of elements in that set = dimension of that space, e.g. 2 basis vectors for $\boldsymbol{R^2}$, 3 basis vectors for $\boldsymbol{R^3}$ etc. (Optimal number)

7. True or False: The span of a set of vectors is a vector space.

Homework 9-11

For **Questions 9-11**, state whether the following vectors are linearly independent.

9. $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ 10. $\left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}3\\6\end{pmatrix}\right\}$ 11. $\left\{\begin{pmatrix}3\\-4\end{pmatrix}, \begin{pmatrix}7\\1\end{pmatrix}\right\}$

Homework 12-14

For **Questions 12-14**, state whether the following sets of vectors span $\mathbb{R}^2$. Answer True if the span of the set is $\mathbb{R}^2$ or False if it is not.

12. $\left\{\begin{pmatrix}1\\3\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}\right\}$

14. $\left\{\begin{pmatrix}5\\5\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right\}$

Eigen stuff

- Practice matrix multiplication (and finding determinants + inverses)
- Eigen means self/ own and we'll see why
- https://youtu.be/PFDu9oVAE-g

Eigenvector of matrix

- An eigenvector is a vector in the direction of *an invariant line* (i.e. the vector's span) through the origin
- Remember: normal matrix vector multiplication- scales and rotates a vector
- Eigenvectors: they're only stretched/diminished by the **eigenvalue** but NOT rotated
- So vector remains on its own span.
- Powerful: changes matrix vector multiplication to scalar vector multiplication, makes things significantly simpler

Eigenvectors mathsy

$$A\vec{v} = \lambda\vec{v}$$

15. $\begin{pmatrix} 1 & 4 \\ 0 & 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

16. $\begin{pmatrix} 7 & 0 \\ -3 & 4 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$