INTRO TO QUANTUM COMPUTING

Week 13 Lab

QUANTUM MECHANICS - 3

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February 2, 2021

PROGRAM FOR TODAY

- Canvas attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz passcode: 6039

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz

Lab Number: 1 | Quiz Password: 6039

- On a scale of 1-5, how much do you agree/disagree with the following statement: I'm glad we took the time to learn the math concepts inherent to quantum computing.
- In general, do you like math?

- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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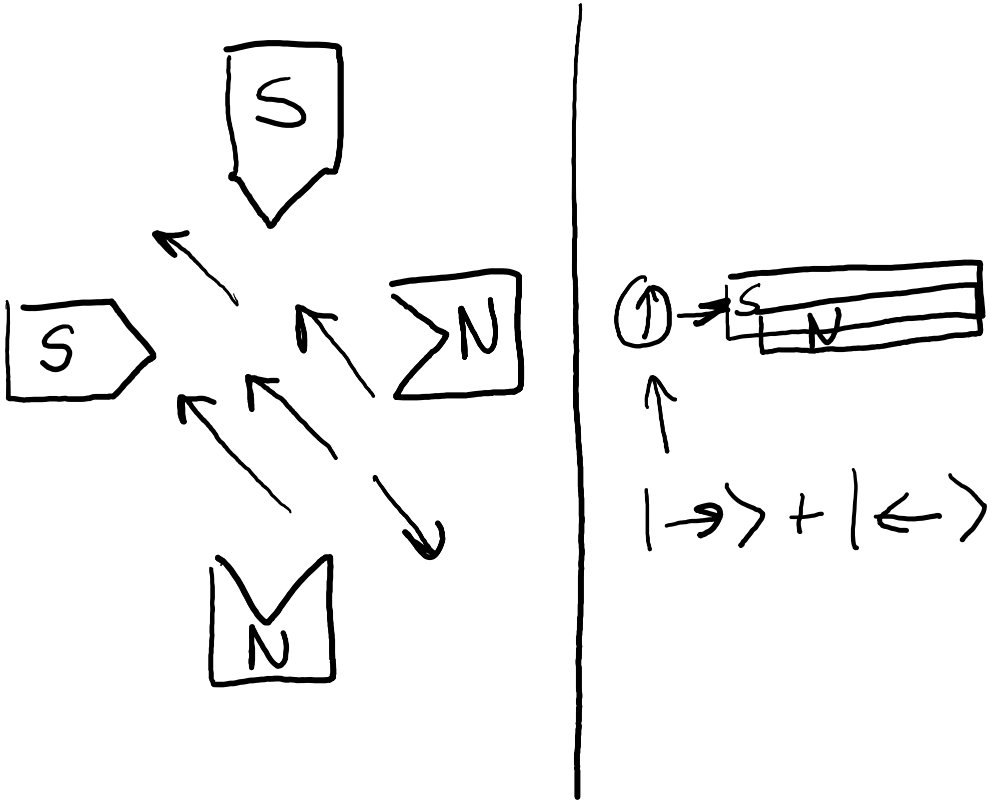
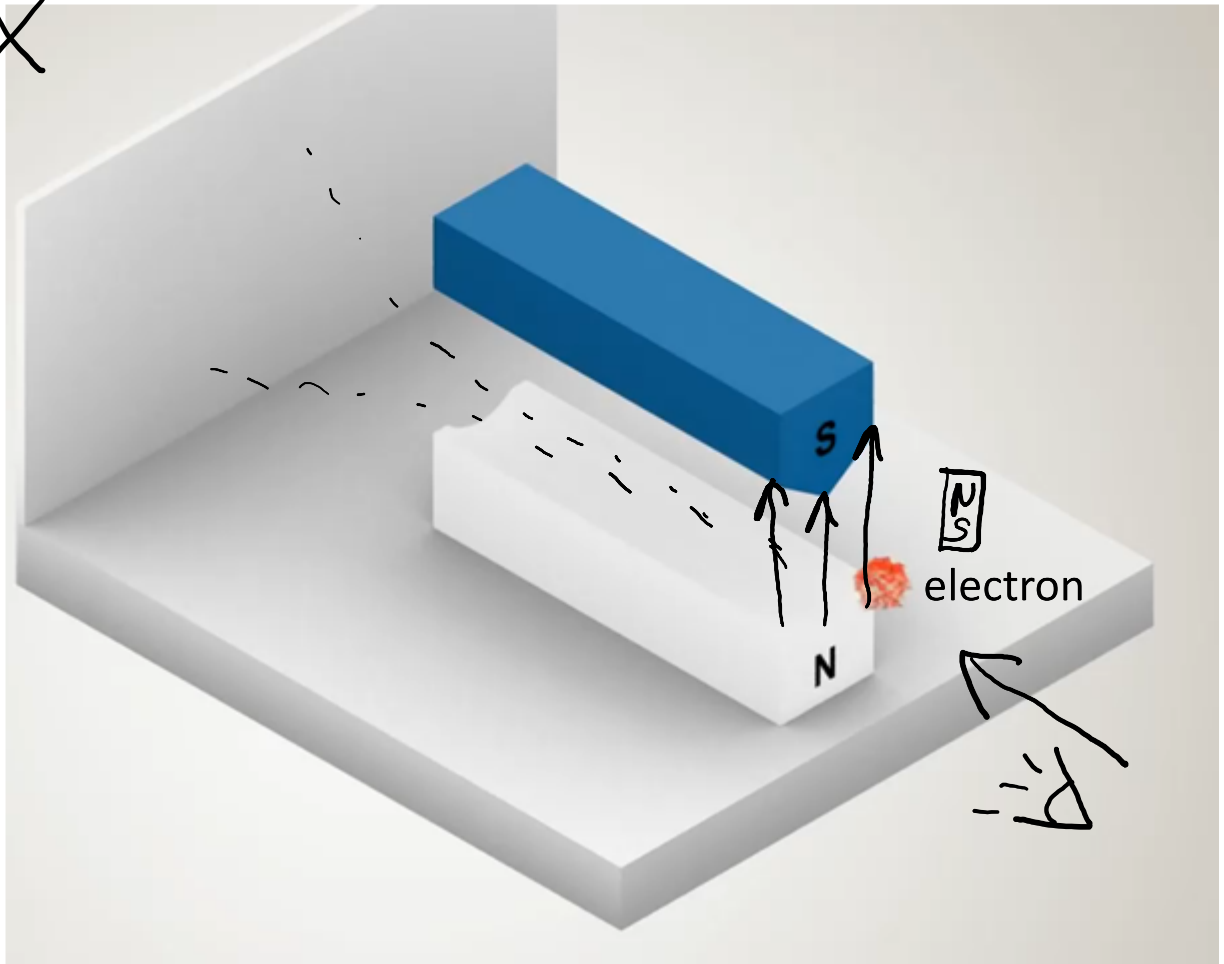
QUESTIONS FROM PAST WEEK

What if we tried to measure the spin along x and z at the same time?

→ NOT measuring Z then X





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LEARNING OBJECTIVES FOR LAB 13

- Understanding the ‘quantum’ in ‘quantum computing’
  - Waves on a string
  - Modes and eigenfunctions
  - Modes in the quantum world
- Using the postulates of quantum mechanics
  - Stern-Gerlach experiment revisited
  - Applying the postulates to the SG experiment
- “Quantum Mechanics” on a guitar string*

*Optional content

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MOTIVATION

Why are we learning quantum mechanics?

- Qubits are made of objects that show wave- and particle-like behavior!

  just a defn

- Quantum mechanics is the mathematical description of objects that show such behavior

  why?

- Qubits and quantum gates are governed by the rules of quantum mechanics

DO YOU PLAY A MUSICAL INSTRUMENT?

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YOU'VE STARTED YOUR QUANTUM CAREER!

"If I were not a physicist, I would probably be a musician. I often think in music. ... I cannot tell if I would have done any creative work of importance in music, but I do know that I get most joy in life out of my **violin**."

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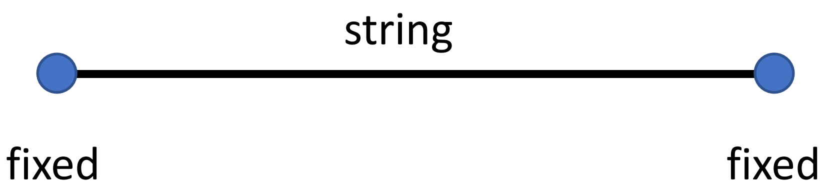
LET'S THINK ABOUT A STRING



How does this string look when it vibrates?

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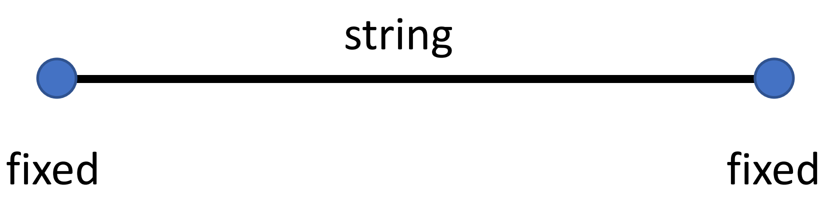
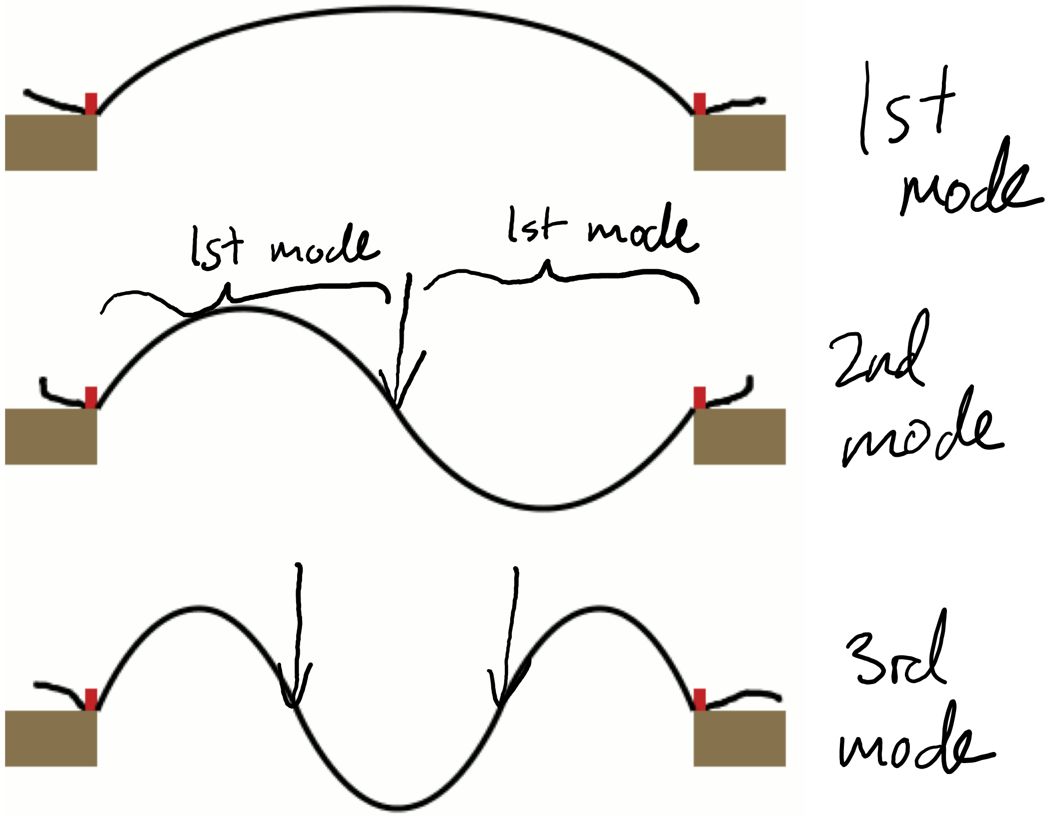
ONLY SOME SHAPES ALLOWED



- The string takes on **specific shapes** when it feely vibrates
- These shapes are called **modes**



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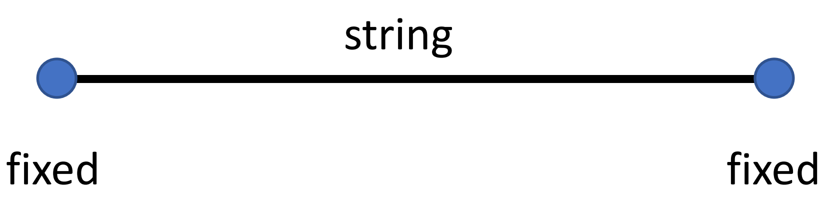

Why?

Modes of guitar strings

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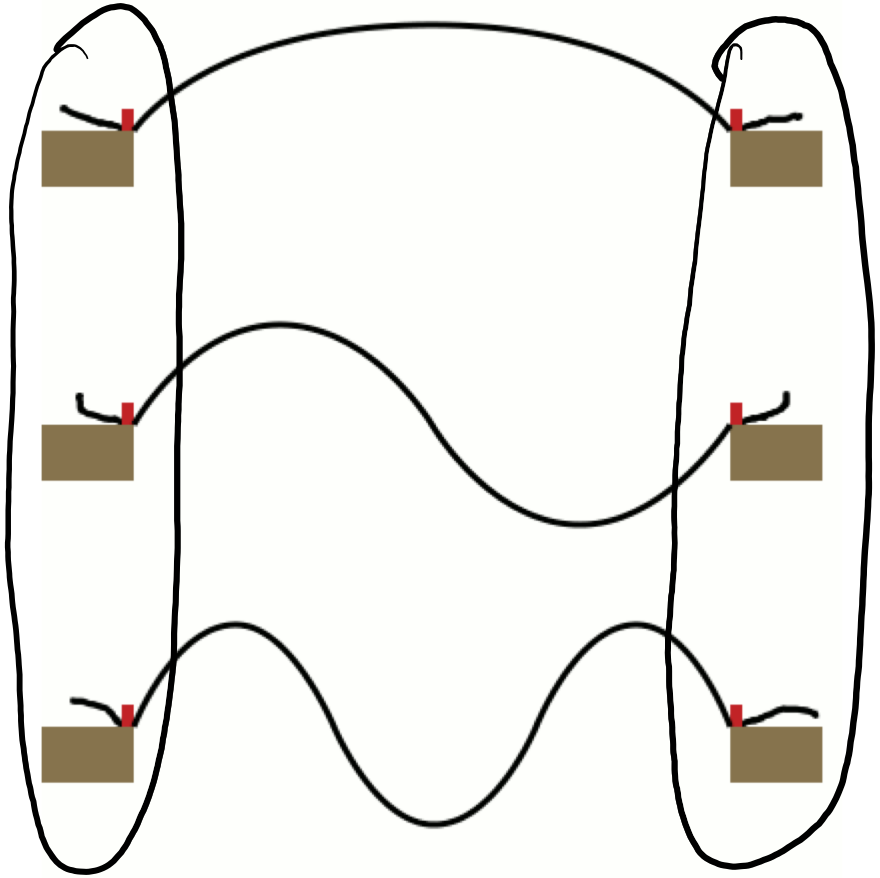
WHY ONLY THESE MODES?

- Modes are a result of **confinement**
- The string has to be fixed at two ends
- These modes are eigenfunctions!



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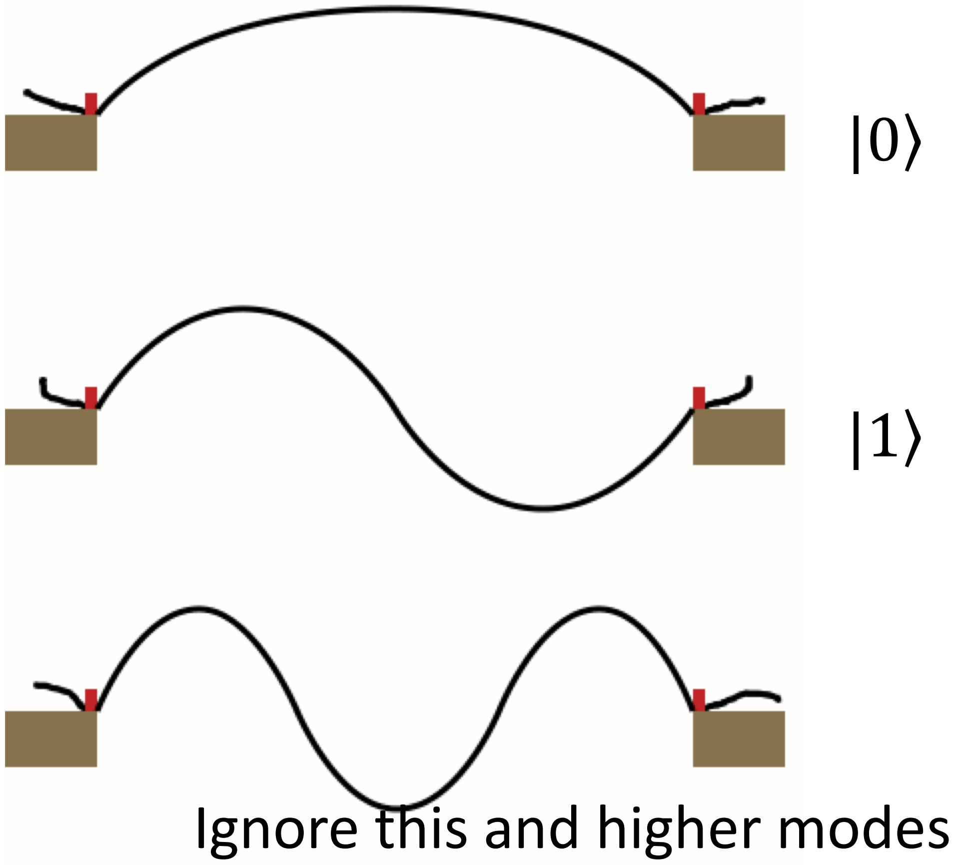
ENTER THE NANOWORLD

- At small scales, **everything** (electrons/atoms/ions…) can behave as a wave ← double slit
- When they are confined, they show quantization, just like a string!
- We can use the modes as qubit states



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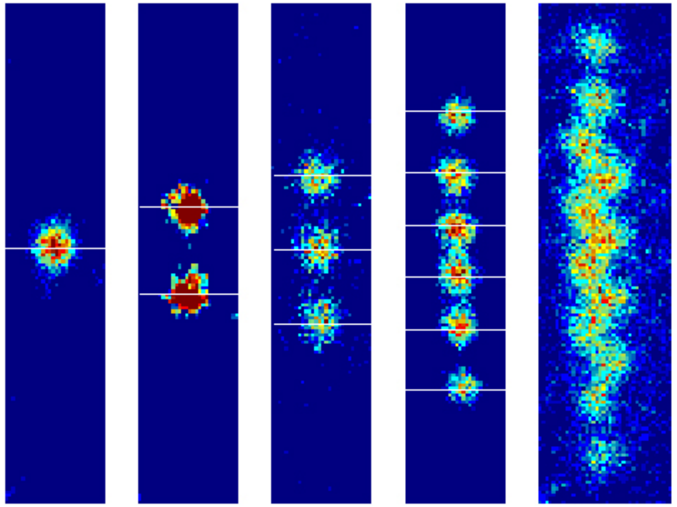
QUANTIZED SYSTEMS IN THE NANOWORLD

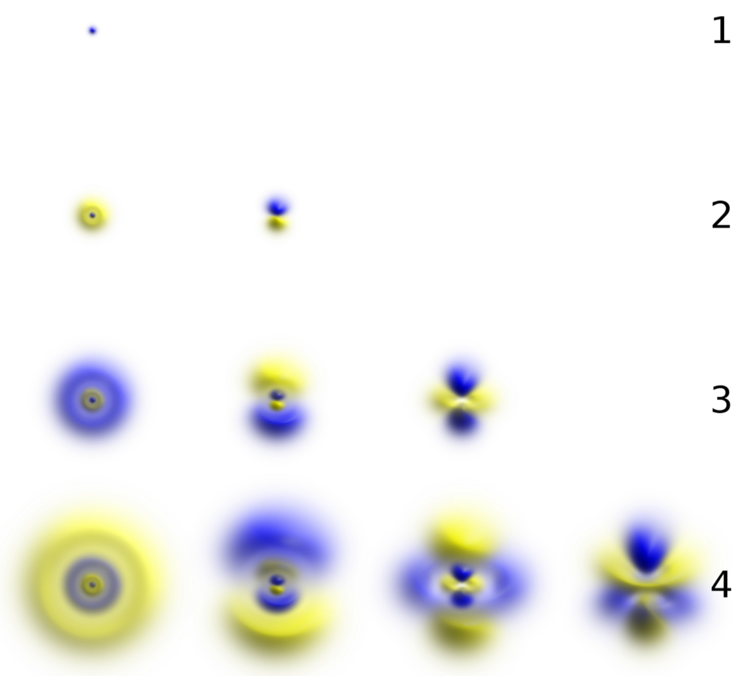
- **Examples:** electrons in atoms, trapped ions, diamond nitrogen vacancy centers $\Big\}$ qubits

- Challenges:
  - Keeping the qubit state 'clean'
  - Minimizing impact of higher modes
  - Reducing noise from the environment
  - Ensuring reliable transfer between modes
  - …

Trapped ions



Atomic orbitals



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QUESTIONS?

Questions on content so far?

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THE POSTULATES OF QM

- A quantum state is represented by its wavefunction $|\psi\rangle$: **Wavefunction postulate**

→ matrix

- Classical observables → Operators in QM : **Operators postulate**

↳ anything you can measure

- The only possible results of measuring an observable are the eigenvalues of its operator: **Observation postulate**

- Probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$: **Probability postulate**

$$P_0 = |\langle 0|\psi\rangle|^2$$

- After measurement, $|\psi\rangle$ is equal to the eigenvector corresponding to the observed eigenvalue: **Collapse postulate**

- To evolve $|\psi\rangle$ in time, apply an operator to it: **Evolution postulate**

multiplying by a matrix

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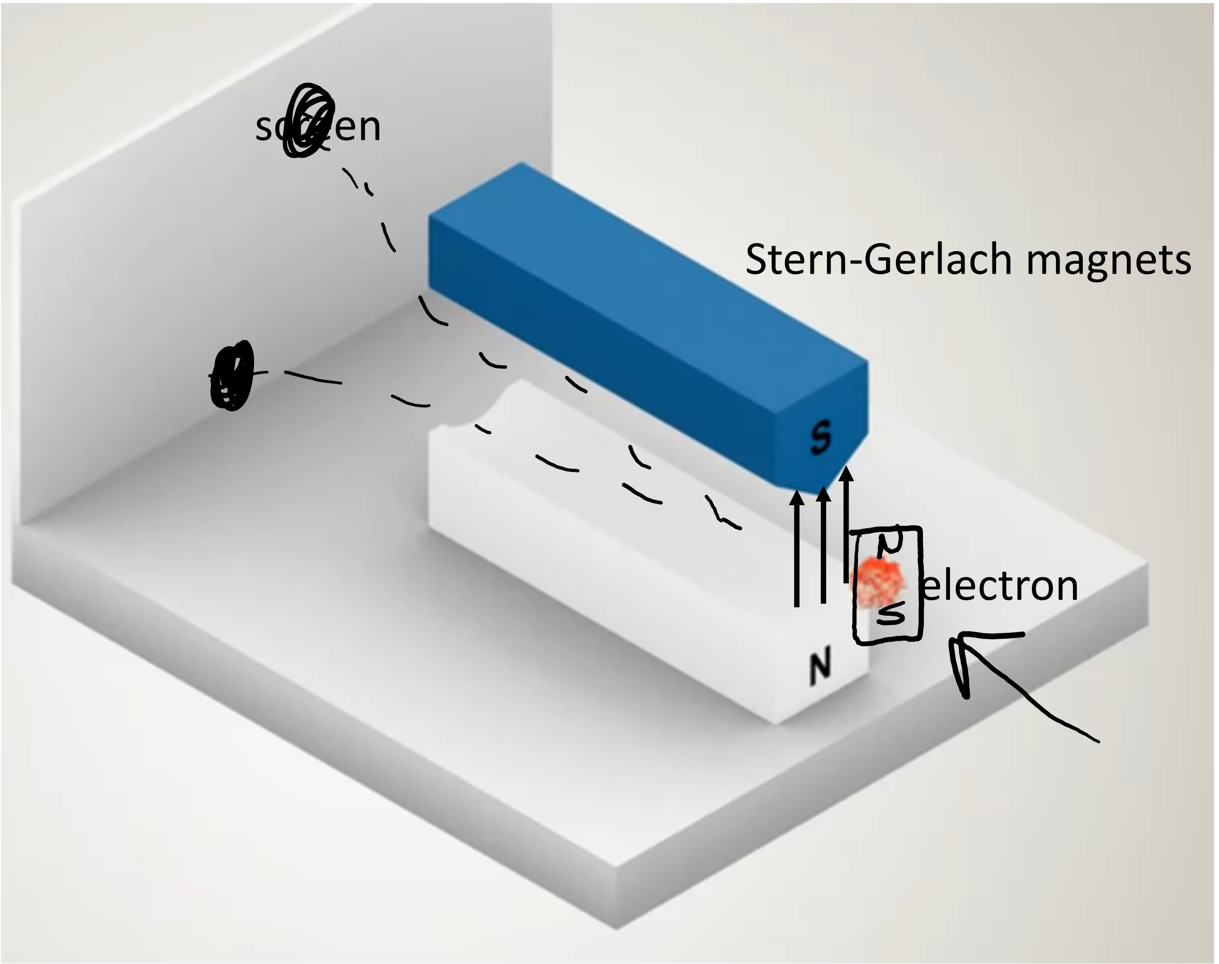
THE STERN GERLACH EXPERIMENT

- Since spin corresponds to a small magnetic field, we can use another magnet to measure it!

- **When measured**, spin always has two directions – one along the direction of the magnetic field and one opposite to it



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MEASURING SPIN - THEORY

- Classical observables → Operators in QM : **Operators postulate**
- In the Stern-Gerlach experiment, the observable is **spin,** and we can measure it along different directions

Observable ← thing you measure	**Operator** ← matrix	**Matrix**
Spin along z-axis	Z or σ_Z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Spin along x-axis	X or σ_X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Spin along y-axis	Y or σ_Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

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MEASURING SPIN - THEORY

The only possible results of measuring an observable are the eigenvalues of its operator: **Observation postulate**

Operator Matrix	**Eigenvalues and eigenvectors**
$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **and** $-1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **and** $-1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ **and** $-1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Check: $Z\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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MEASURING SPIN - THEORY

Operator Matrix **Eigenvalues and eigenvectors**

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \longrightarrow 1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \textbf{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$|0\rangle$ $|1\rangle$

Problem 1: What are the possible outcomes of measuring spin along the z-axis?

Hint: Think about the Observation postulate

MEASURING SPIN - THEORY

Operator Matrix

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenvalues and eigenvectors

$$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

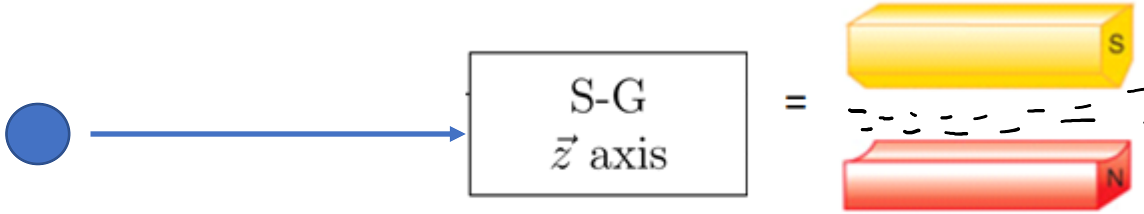
eigenvalue

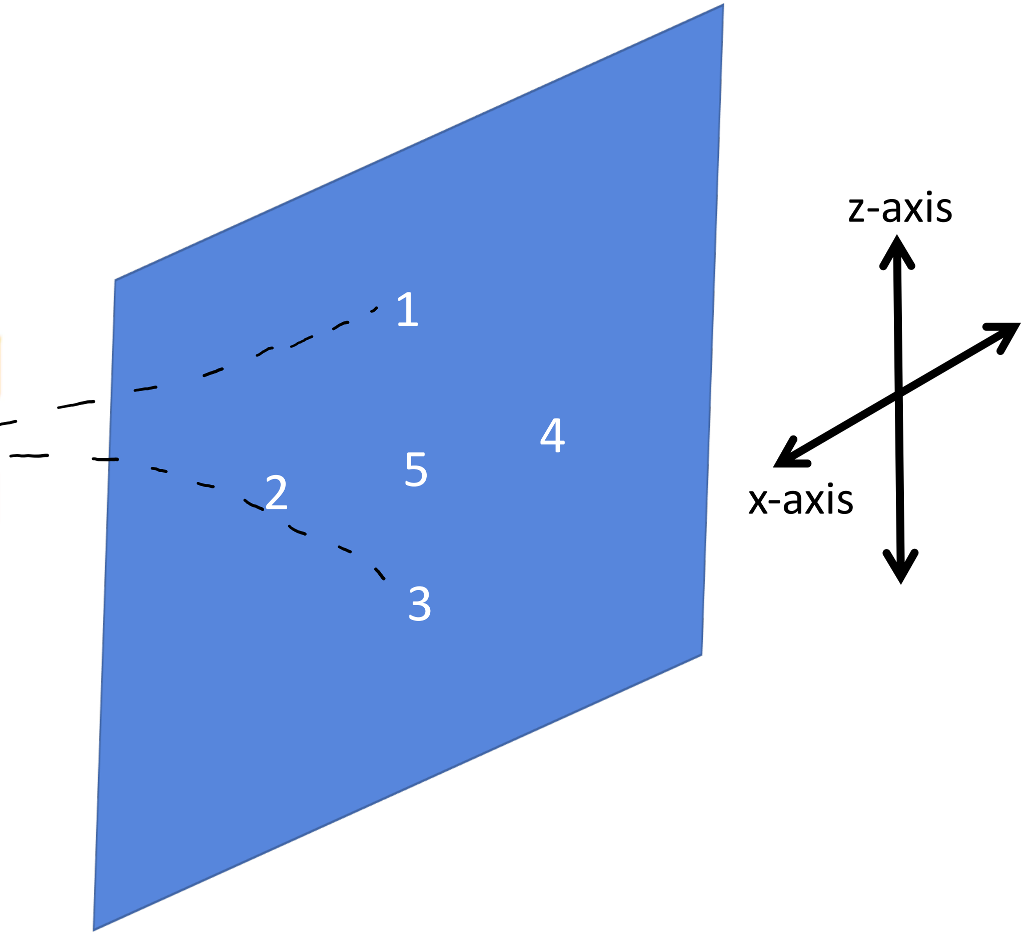
Problem 2: The result of a measurement of spin along the z-axis is -1. What is the wavefunction of the electron immediately after the measurement?

Hint: Think about the collapse postulate

MEASURING SPIN - EXAMPLE

Where does the electron go?





electron state:

$0.71|0\rangle + 0.71|1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

(spin combination of up and down along z)

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MEASURING SPIN - EXAMPLE

$$|\psi\rangle = 0.71|0\rangle + 0.71|1\rangle$$

Operator Matrix

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Eigenvalues and eigenvectors

$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **and** $-1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Problem 3: What is the probability of the result of measuring the spin of the electron in state $|\psi\rangle$ being +1? What is the probability of -1?

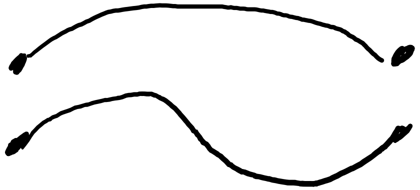
measuring a quantity

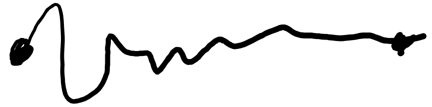
Hint: Think about the Probability postulate - probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$

$$|\langle 0|\psi\rangle|^2 = 0.5$$

KEY TAKEAWAYS

- When a system that shows wave-like behavior is confined, the wave modes get quantized





- At the scale of electrons/atoms/ions, everything shows wave-like behavior. Therefore, everything shows quantization when it is confined!

- The postulates of quantum mechanics are the rules that quantum systems are observed to follow

REAL WORLD e^- N S ← postulates → MATH $X, Y, Z, |\psi\rangle, |\langle 0|\psi\rangle|^2$

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FURTHER READING AND RESOURCES

- https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/infwell1d/infwell1d.html - Simulation of the quantum version of a guitar string – the particle in a box

- http://web.mit.edu/8.05/handouts/jaffe1.pdf - Lecture notes from Prof. R. L. Jaffe at MIT, discussing the QM postulates along with linear algebra

- https://www.youtube.com/watch?v=uK2eFv7ne_Q – Lecture 1 of a series of lectures by Prof. R. Shankar on QM

- https://youtu.be/Sut0Aegyoes – Brief video on transmission electron microscopy

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QUESTIONS?

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Questions on content so far?

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POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

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OPTIONAL CONTENT

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WAVEFUNCTION POSTULATE

A quantum state is represented by its wavefunction $|\psi\rangle$

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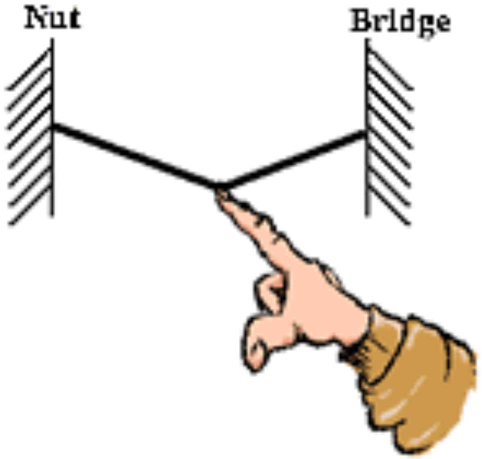
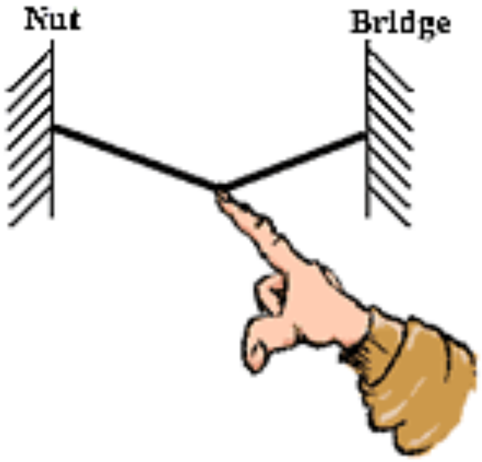
LET'S PLUCK THESE STRINGS!

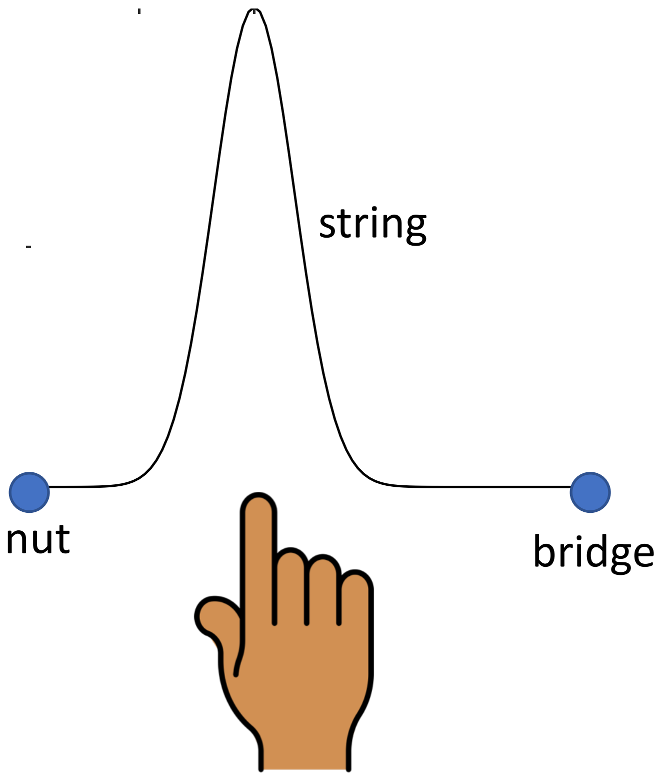


- When we pluck a guitar string, we create a ripple in it

THE GUITAR "WAVEFUNCTION"

- The shape of the string describes everything about it





- The string shape is its "wavefunction"

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DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS

The ripple can be "reconstructed" by adding up the modes

Sum

linear combo

actual shape of string =

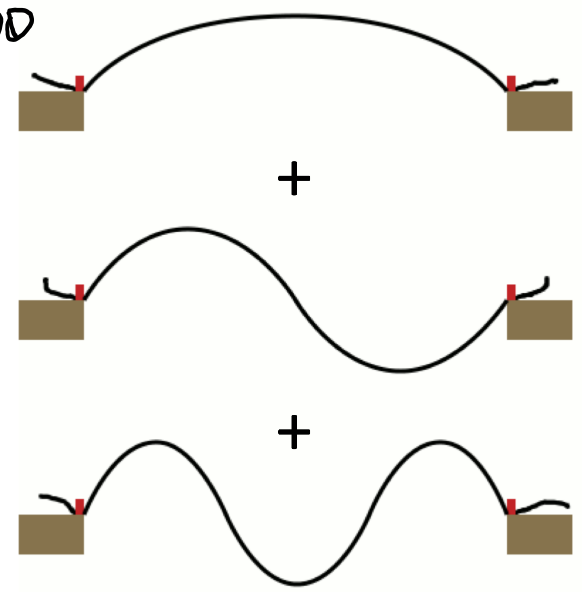
a *

+

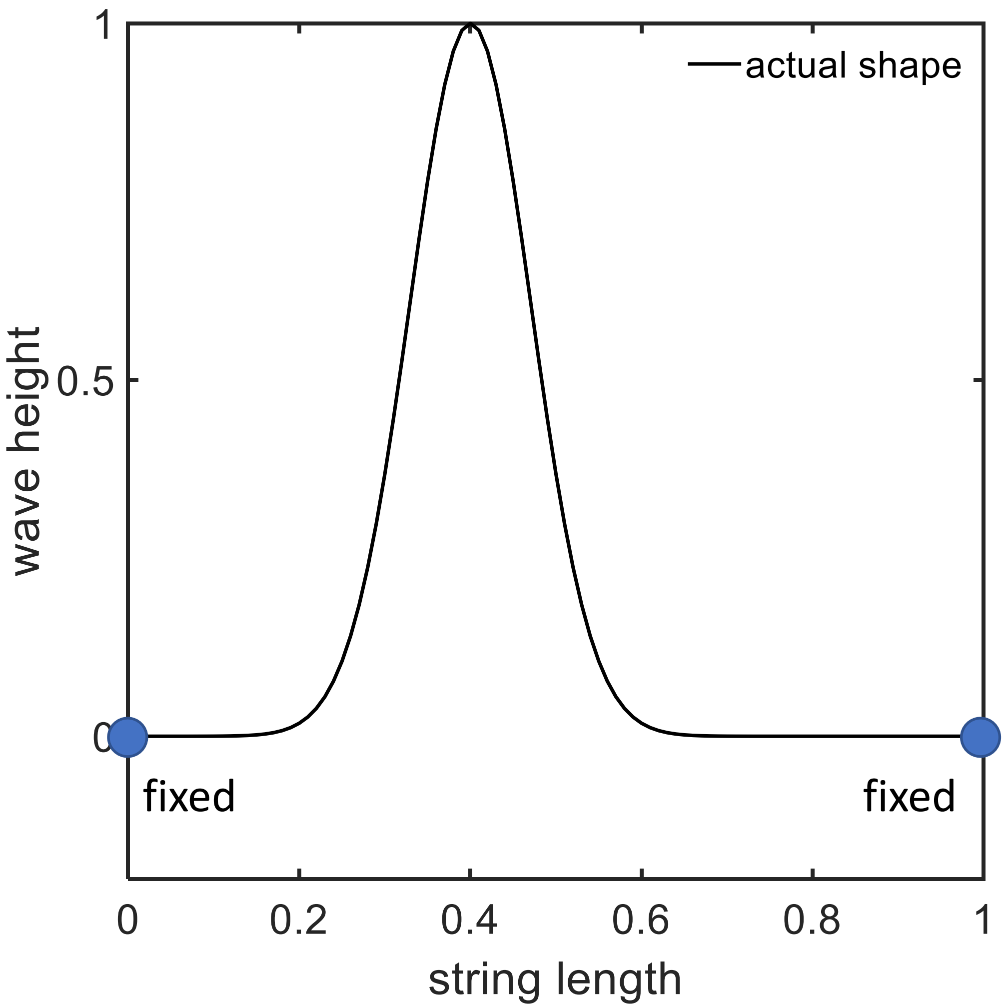
b *

+

c *



+ ....



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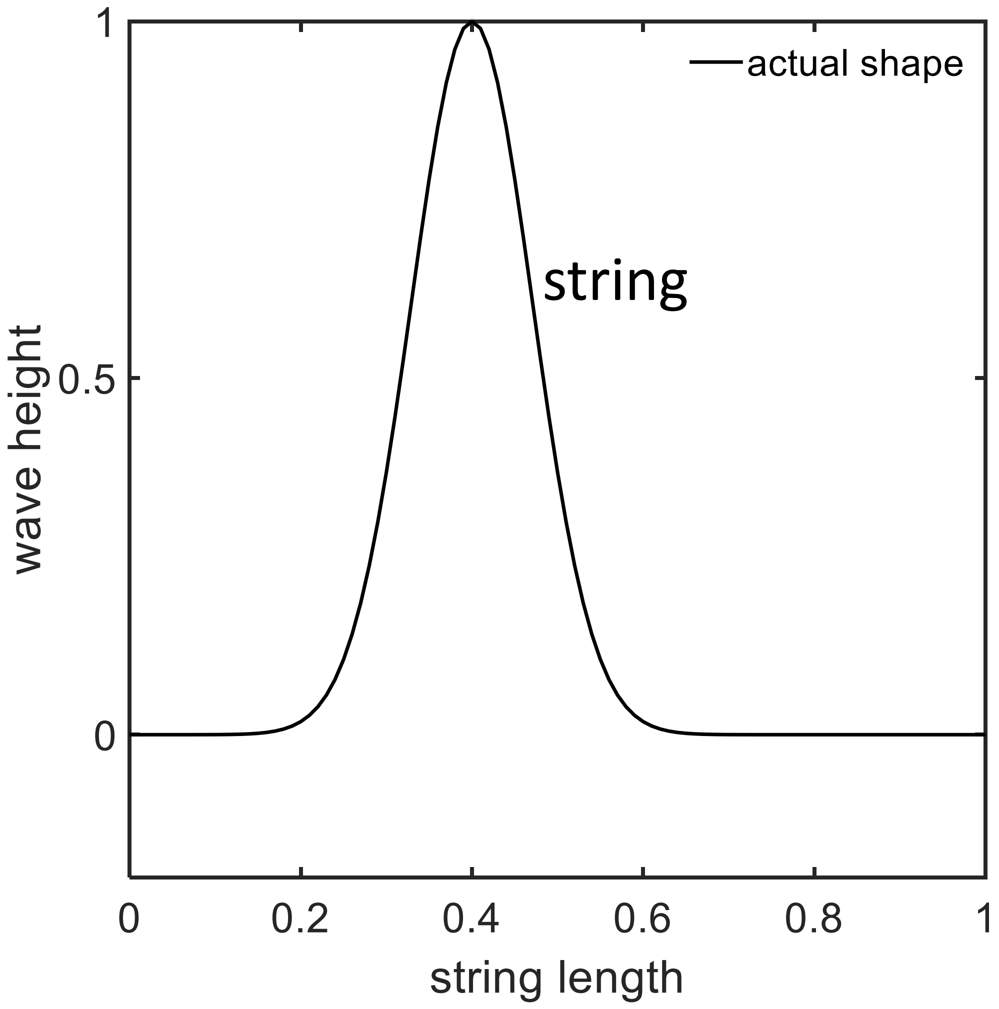
DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



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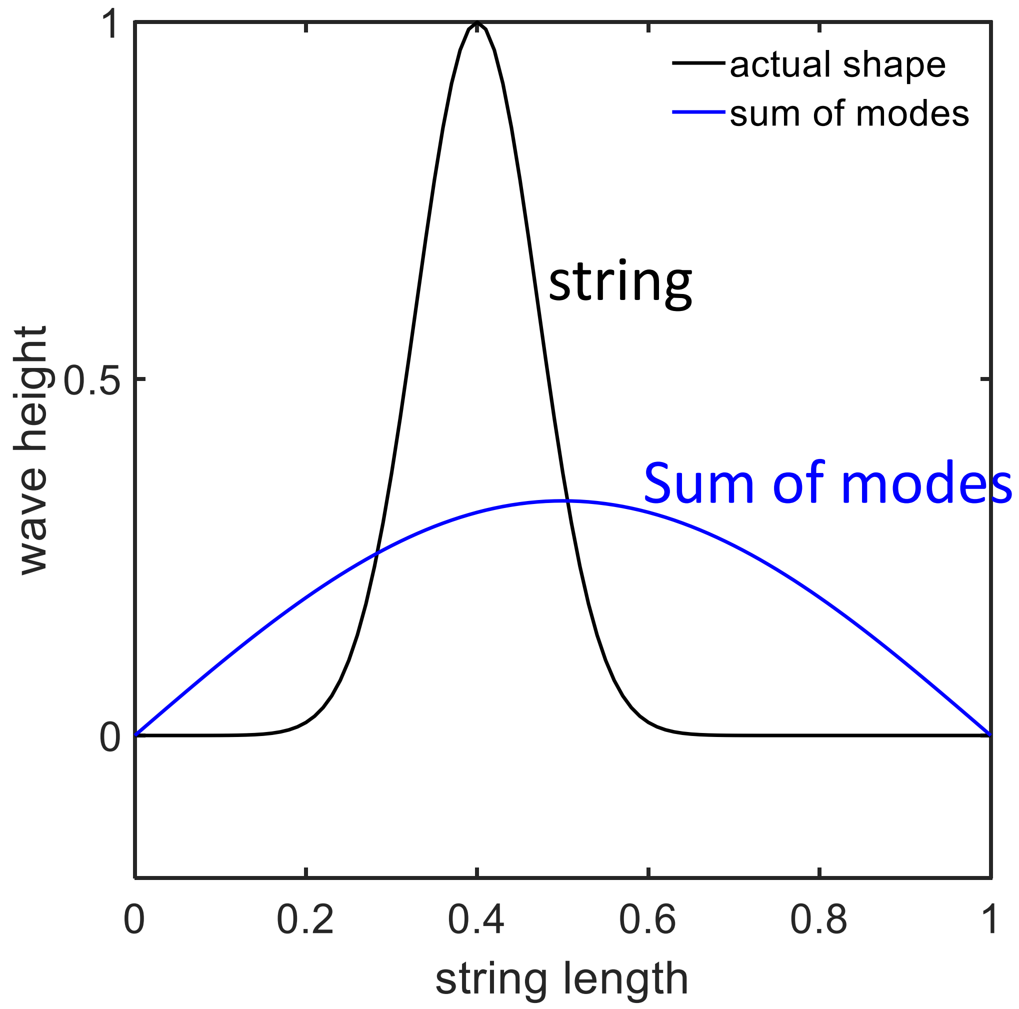
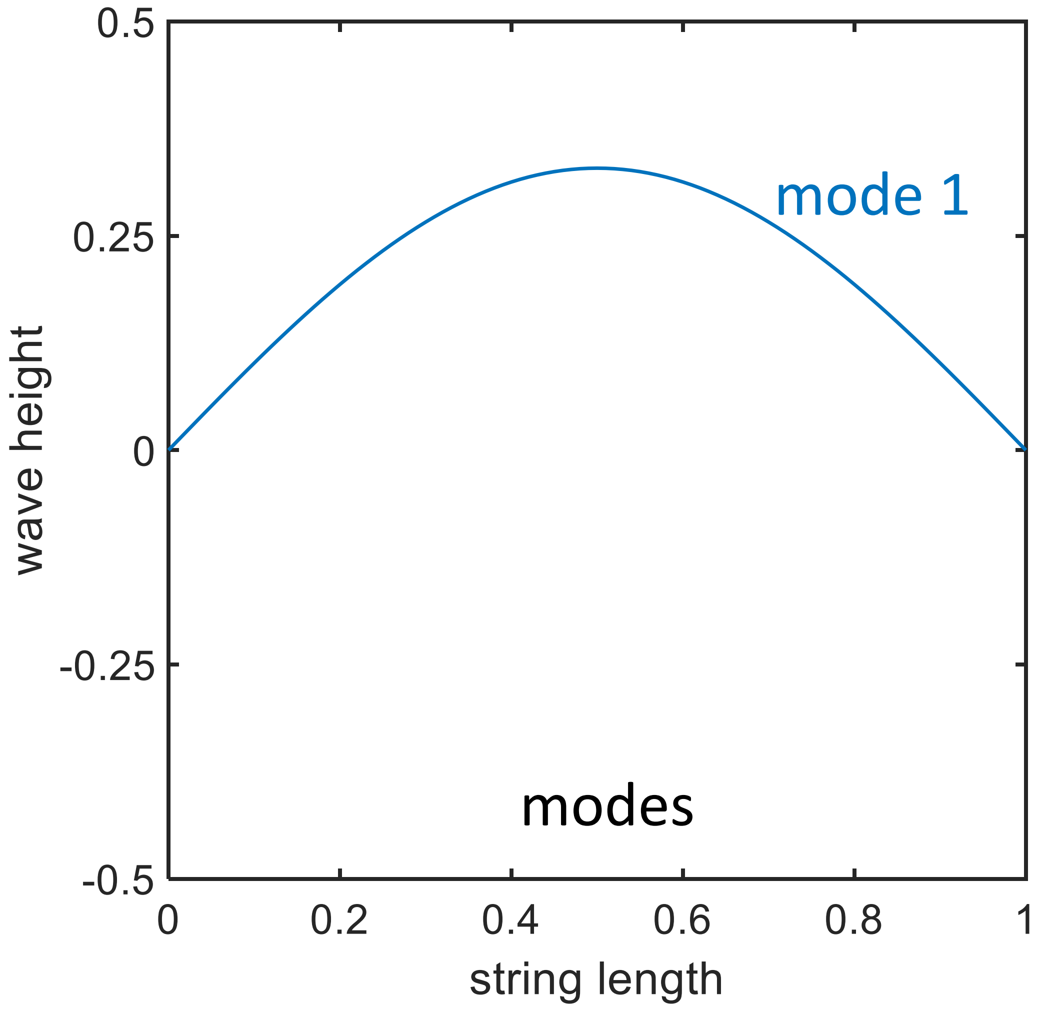
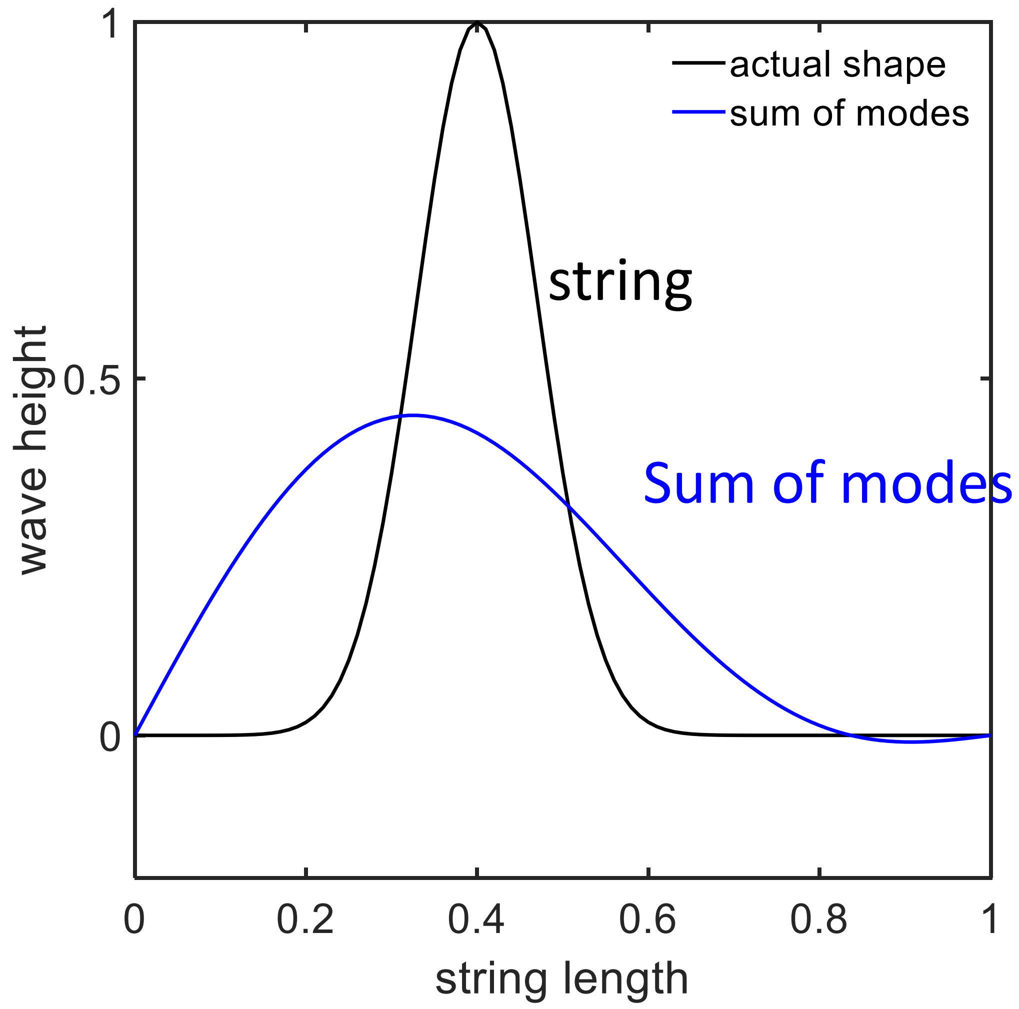
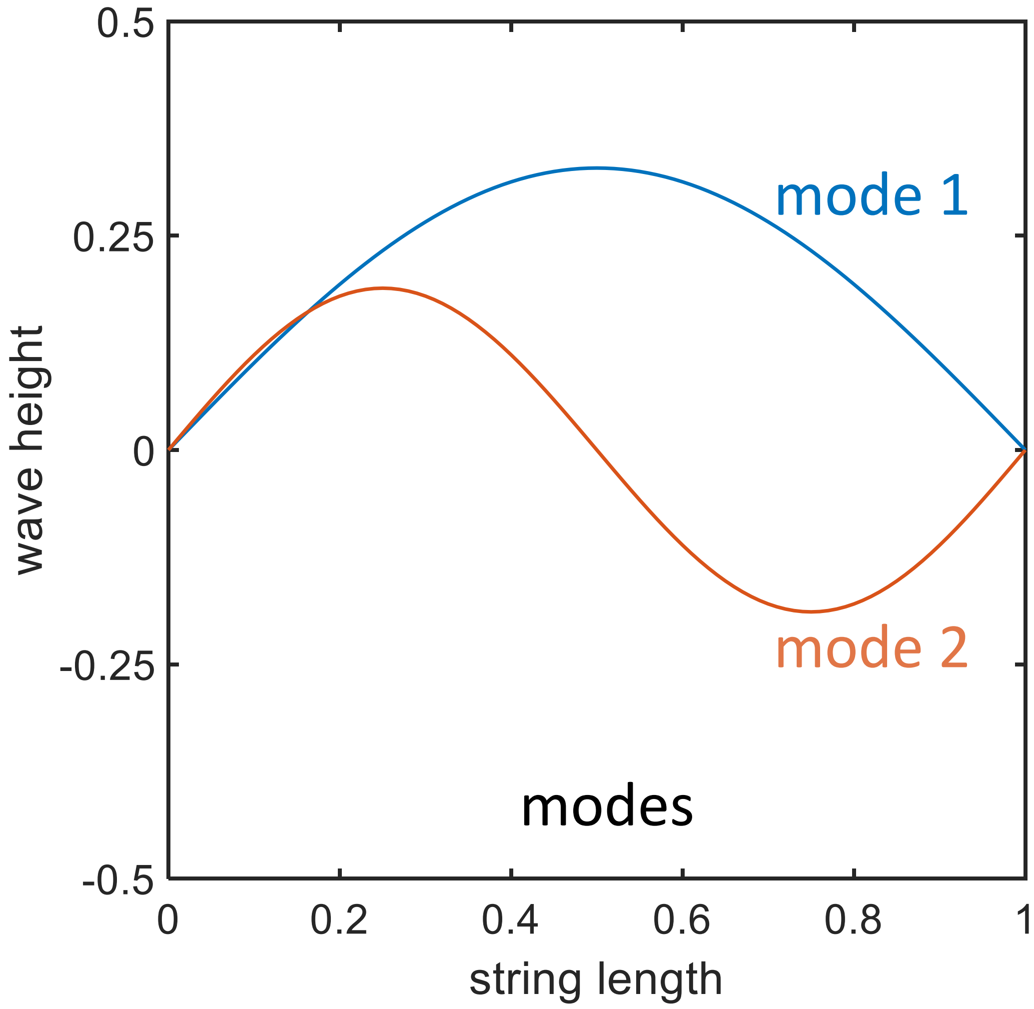




With just the first mode, the match is not very good

DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS





As we increase the number of modes, the match improves!

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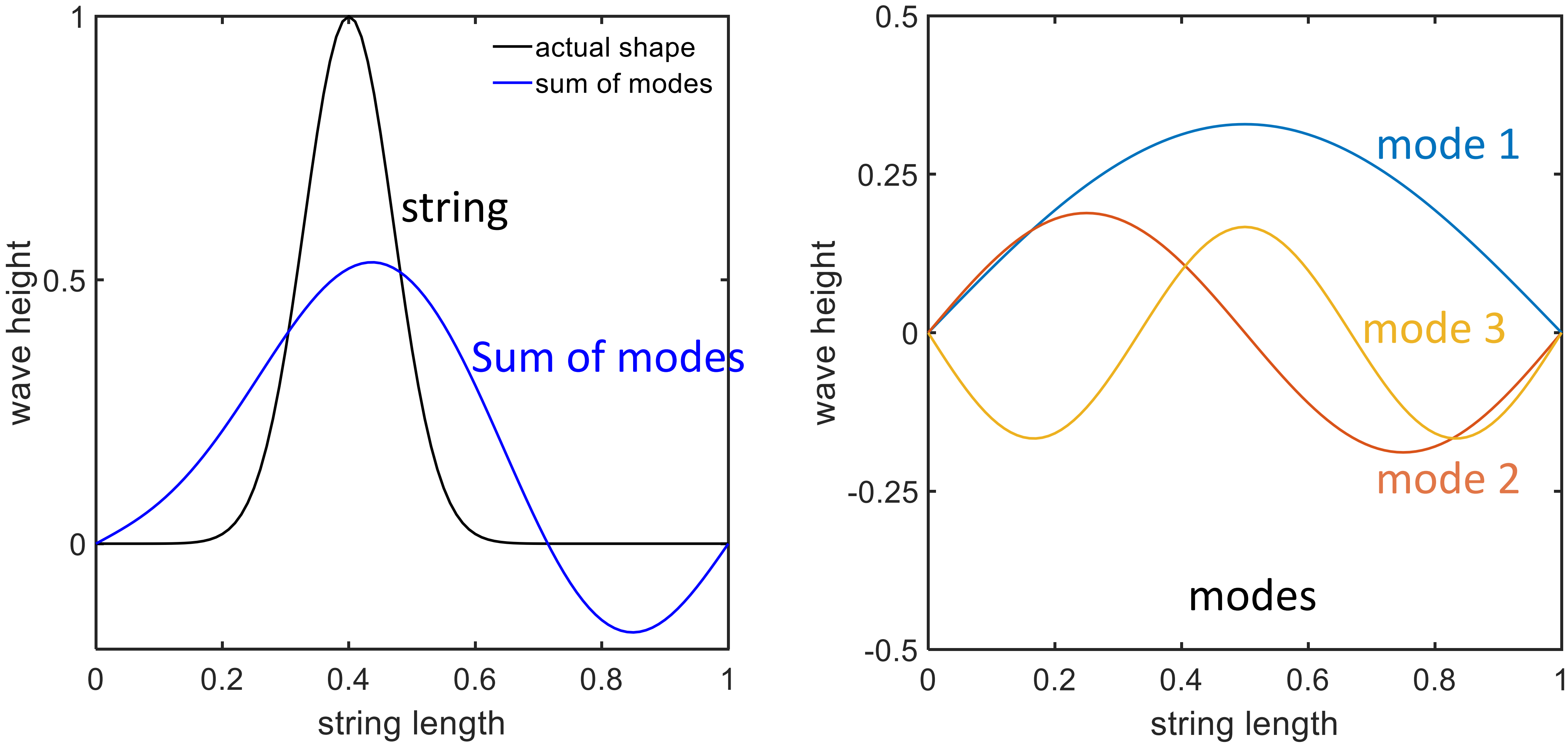
DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



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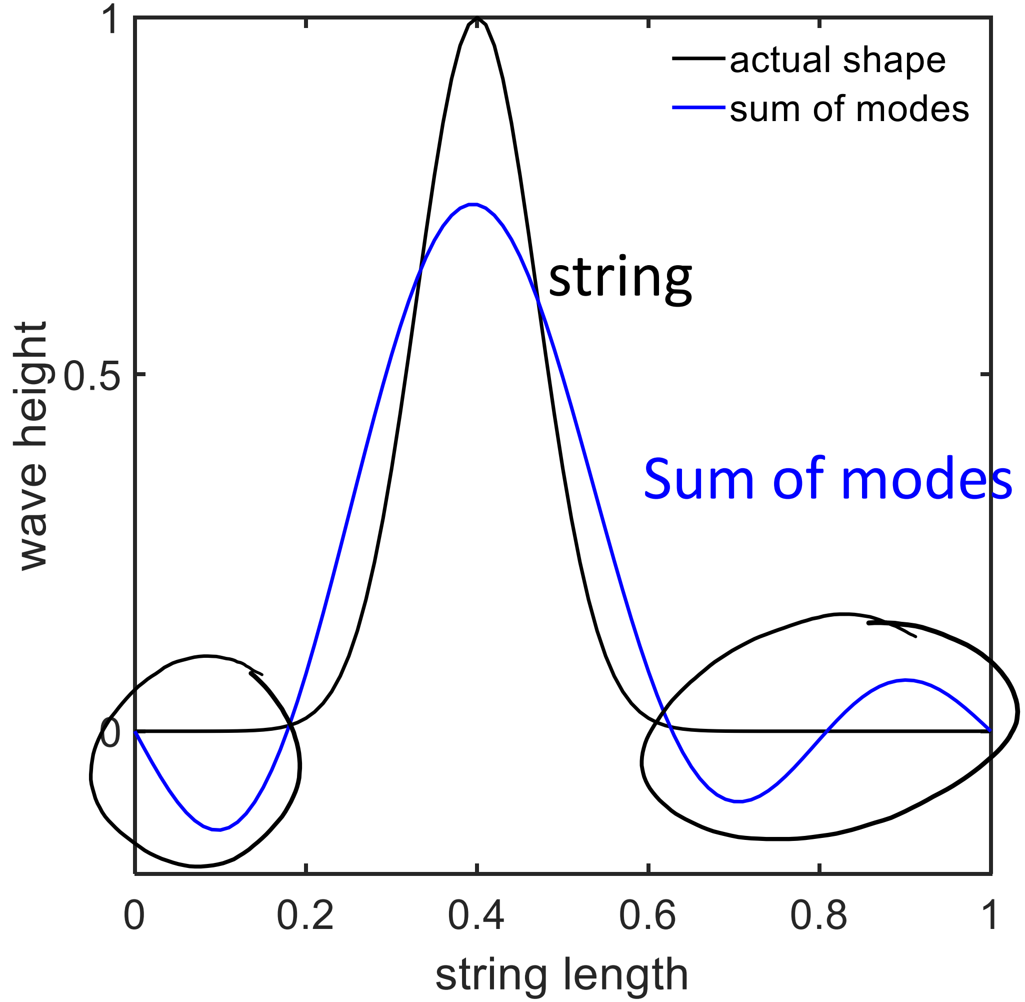

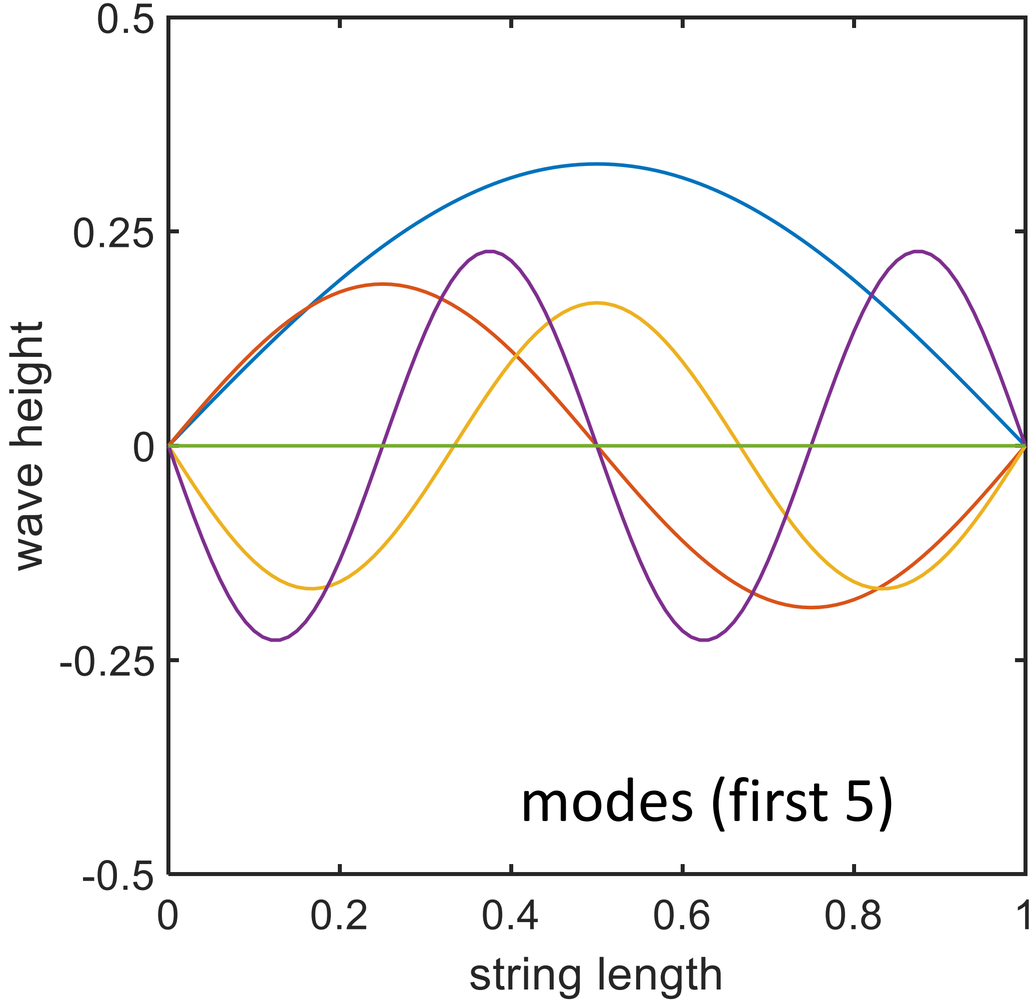

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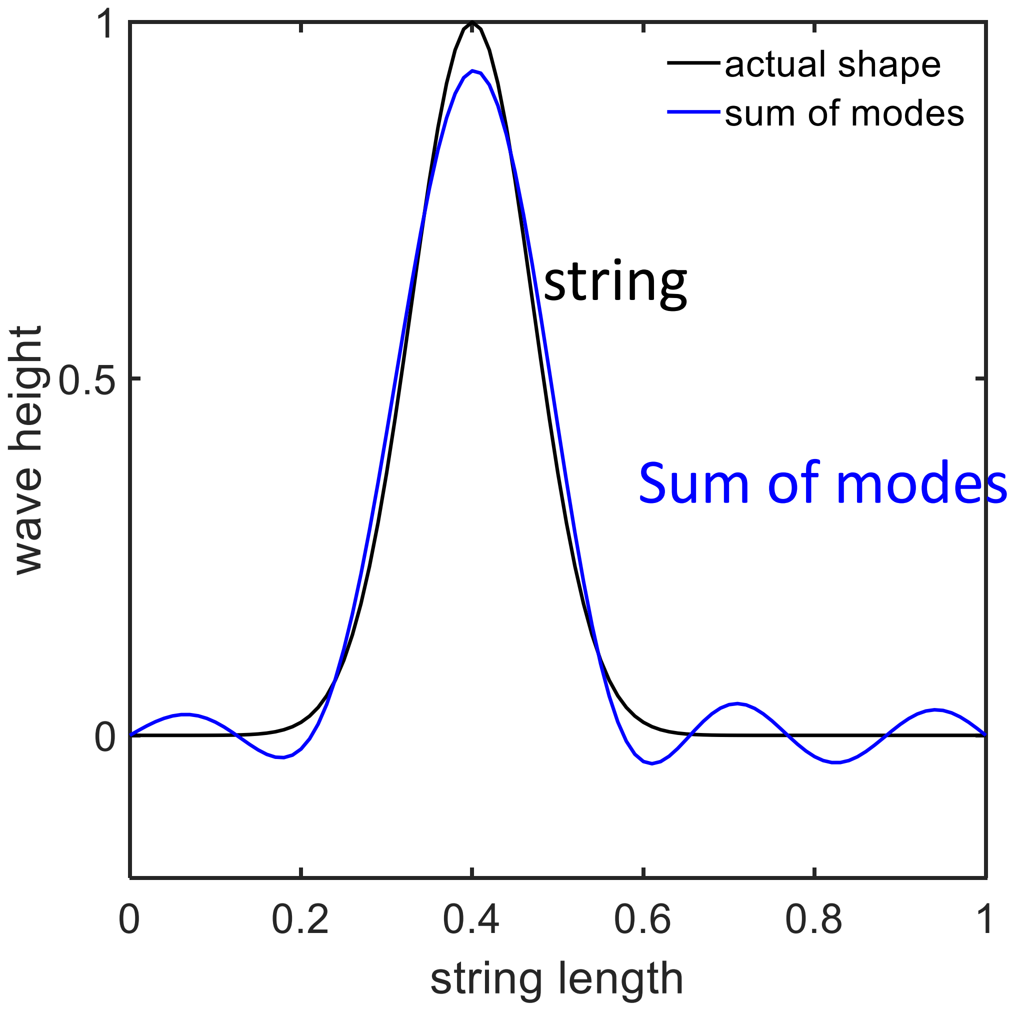

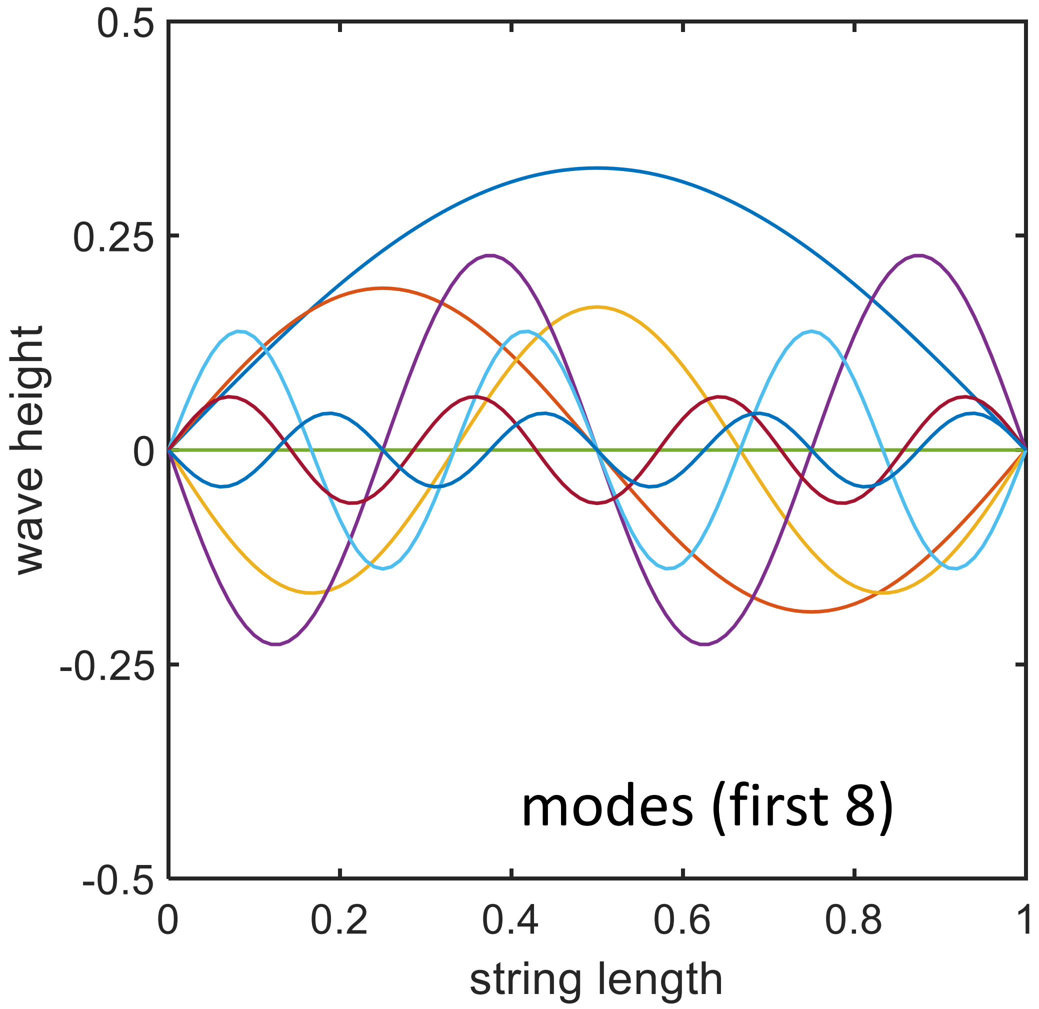

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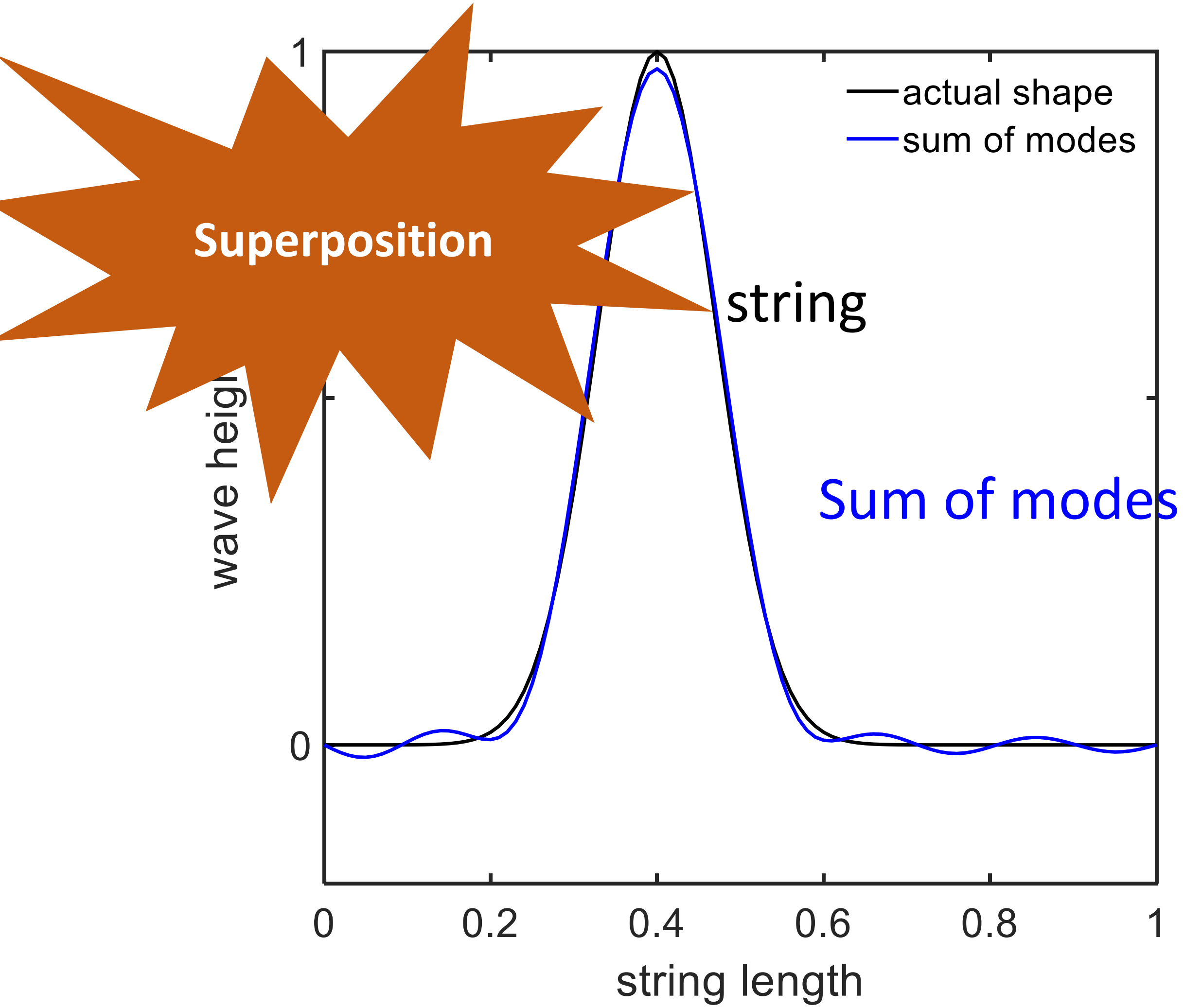
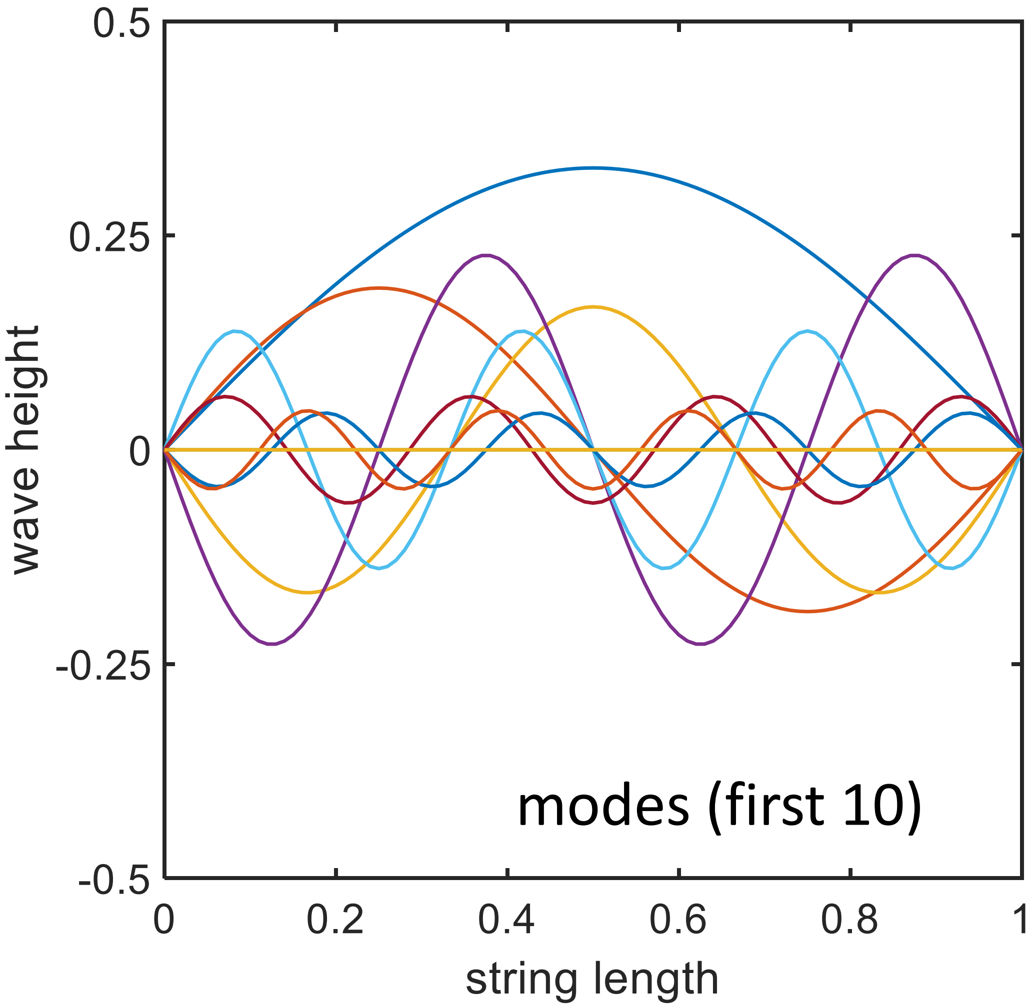
DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS





As we increase the number of modes, the match improves!

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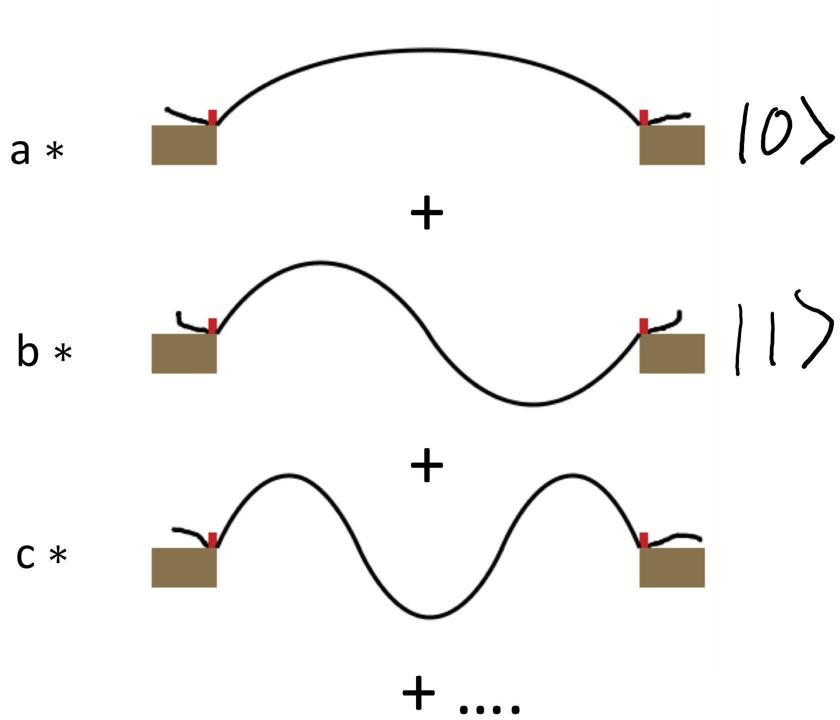
MODE SUPERPOSITION

On a string

In a qubit

actual shape of string =



$$|\psi\rangle = 0.71|0\rangle + 0.71|1\rangle$$

This method of expressing a function as a superposition of modes is known as Fourier analysis

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OBSERVATION POSTULATE

- The only possible results of measuring an observable are the eigenvalues of its operator

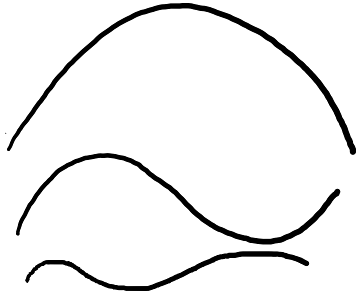
- **On the guitar:** If you try to see the string, you will only see one of the modes, not the ripple

PROBABILITY POSTULATE

- Probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$

$$|\langle 0|\psi\rangle|^2$$

- **On the guitar:** When you see the string, the probability of seeing a mode depends on its contribution to the initial wavefunction (the ripple when you pluck the string)



COLLAPSE POSTULATE

$$\frac{d}{dt} f(t) = a f(t) \qquad f(t) = e^{at}$$

constant

- After measurement, $|\psi\rangle$ is equal to the eigenvector corresponding to the observed eigenvalue

$$\frac{d}{dt}|\psi\rangle = H|\psi\rangle \qquad |\psi(t)\rangle = e^{Ht}|\psi\rangle$$

Hamiltonian

$$U = e^{Ht}$$

- **On the guitar:** After you see the guitar string, it remains in the mode you observed

$$\frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \xrightarrow{Z} |\uparrow\rangle$$