INTRO TO QUANTUM COMPUTING

LECTURE #8

MATHEMATICS FOR QUANTUM

Amir Karamlou

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TODAY'S LECTURE

a) Dirac notation

b) Inner product

c) Quantum operations

- Hermitian operations
- Unitary operations

Dirac Notation

Or bra-ket notation allows us to abstract away parts of the complicated underlying math for quantum mechanics!

Bit to Qubit

Bit	Qubit
0	$\|0\rangle$
1	$\|1\rangle$

$|\ \rangle$ is a ket and it indicates that we're talking about a quantum state.

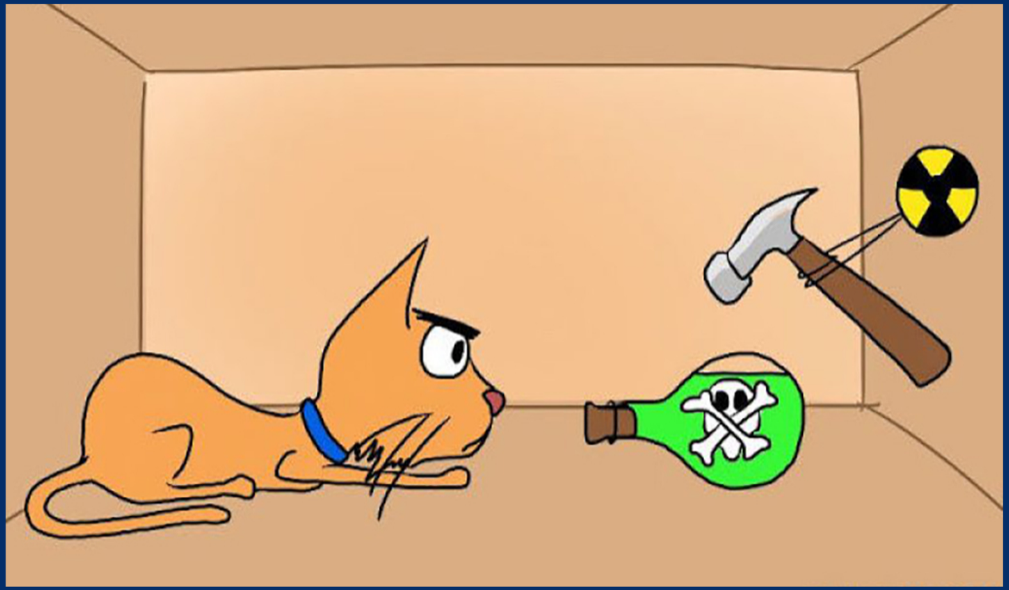
example: QxQ $\mapsto$ $|\text{ QxQ }\rangle$

Ket

Ket (| ⟩): can be represented with a ***column vector***!

Quantum Superposition

Quantum object can be in two states at once!



Superposition

Superposition: a qubit can be $|0\rangle$ and $|1\rangle$ at the same time!

This is how we show it: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Practice

Bra

Bra (⟨ |): can be represented with a ***row vector***!

the bra is the complex-conjugate of the ket!

Practice

Inner Product

We use the inner product to find the overlap between two quantum states

braket (bra+ket): $\langle \psi | \varphi \rangle$

Inner Product

- $\langle 0 | 0 \rangle$:
- $\langle 1 | 0 \rangle$:
- $\langle 0 | 1 \rangle$:
- $\langle 1 | 1 \rangle$:

Inner product of superpositions

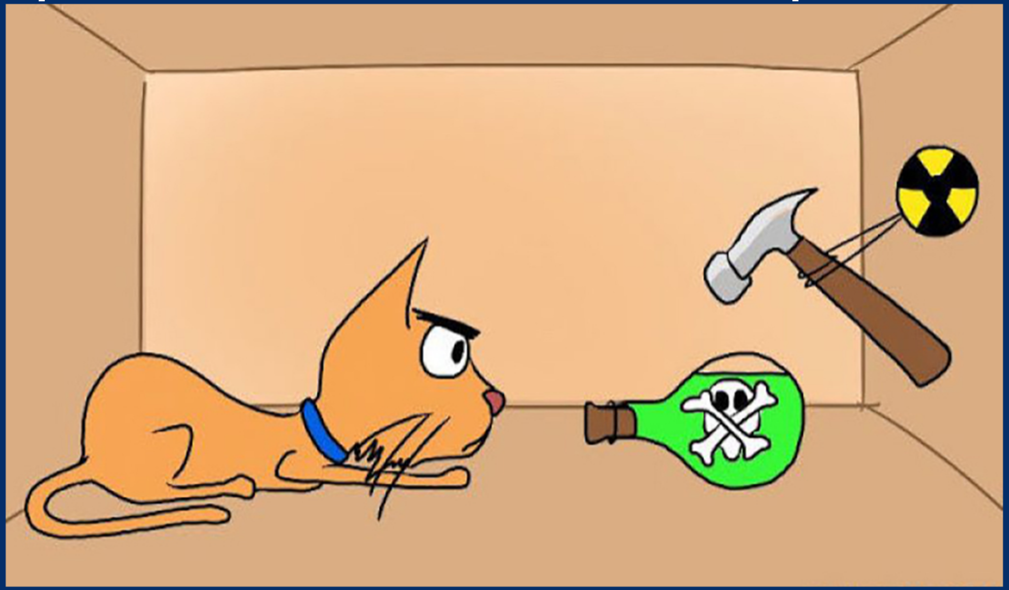
Two definitions

Two states $|\psi\rangle$ and $|\varphi\rangle$ are ***"orthogonal"*** if: $\langle\psi|\varphi\rangle=0$ (orthogonal=perpendicular)

State $|\psi\rangle$ is ***"normal"*** if: $\langle\psi|\psi\rangle=1$

Measurement

collapses the quantum state of the qubit to either 0 or 1



Measurement

Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

measurement: collapses the quantum state of the qubit $|\psi\rangle$ to either $|0\rangle$ or $|1\rangle$

probability of measuring $|0\rangle$: $|\alpha|^2$

probability of measuring $|1\rangle$: $|\beta|^2$

10 MIN BREAK!

Quantum states:

Inputs and outputs of the quantum computer

Quantum operations:

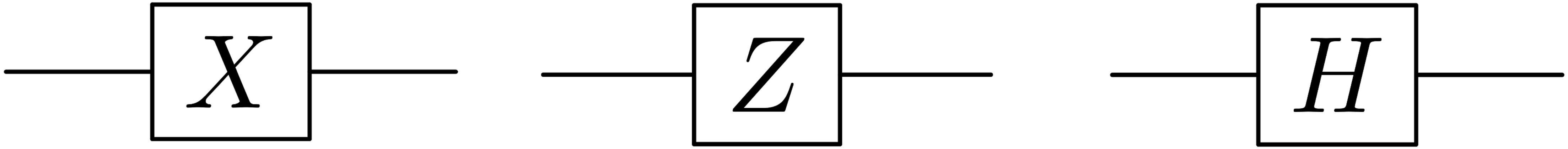
Perform computation in a quantum computer

Quantum Operation

Transforms a quantum state to another

We can represent them as ***matrices***

example: quantum gates!



Important Quantum Operators

Pauli operators:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ Pauli-X operator (X)

$$\sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$$ Pauli-Y operator (Y)

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$ Pauli-Z operator (Z)

Practice

Pauli operators:

Quantum Operation

Quantum operation properties:

1. Linearity

Quantum Operation

Quantum operation properties:

2. Can be composited

Quantum Operation

Quantum operation properties:

3. Order matters

Conjugate Transpose

Conjugate Transpose:

$$\vec{v}^{\dagger} = (\vec{v}^{T})^{*} = (\vec{v}^{*})^{T}$$

Hermitian Operators

All observable operators are Hermitian!!

example:

- Position
- Momentum
- Energy

Hermitian Operators

All observable operators are Hermitian

Hermitian: $A=A^\dagger$ operator is equal to its own conjugate transpose

example:

Reversibility

Given the output of a gate, we can determine what the inputs are.

- **Reversible gate:** preserves all the information
- **Non-reversible gate:** loses some information

Unitary Operators

All reversible quantum operations are unitary!

example:

- Time evolution

Unitary Operators

All reversible operations are unitary → all quantum gates are unitary

Unitary: $A.A^{\dagger}=A^{\dagger}.A= I$

example:

ANNOUNCEMENTS