HOMEWORK 5

DISCRETE PROBABILITY

Questions 1-5 involve the use of a fair coin ($\frac{1}{2}$ probability of flipping heads or tails) and a fair die ($\frac{1}{6}$ of rolling each number).

Questions 3-5 involve using both the die and the coin

1. Using the coin, what is the probability of **flipping heads twice in a row**?

   a) $\frac{1}{8}$
   b) $\frac{1}{2}$
   c) $\frac{1}{4}$
   d) $\frac{1}{3}$

2. Using the die, what is the probability of **rolling 1 twice in a row**?

   a) $\frac{1}{36}$
   b) $\frac{1}{6}$
   c) $\frac{1}{12}$
   d) $\frac{1}{2}$

3. Using both the coin and the die, what is the probability of **flipping tails <u>and</u> rolling 3?**

   a) $\frac{1}{6}$
   b) $\frac{1}{36}$
   c) $\frac{1}{12}$
   d) $\frac{1}{4}$

4. Using both the coin and the die, what is the probability of **flipping heads and rolling an even number?**

   a) $\frac{1}{4}$
   b) $\frac{1}{12}$
   c) $\frac{1}{6}$
   d) $\frac{1}{36}$

5. Using both the coin and the die, what is the probability of **flipping tails and rolling 1 or 2?**

   a) $\frac{1}{4}$
   b) $\frac{1}{12}$
   c) $\frac{1}{2}$
   d) $\frac{1}{6}$

Questions 6-12 are in reference to the following:

Let X and Y be independent discrete random variables that can be one of the following values: {2,5,6,8}. The probability for X and Y to take each value is different. The following table shows the corresponding probability mass functions for X and Y:

v	P(X=v)	P(Y=v)
2	0.1	0.4
5	0.2	0.15
6	0.2	0.35
8	0.5	0.1

For questions 6-9, write answers as decimals rounded to 1 decimal place.

Write your answers as decimals rounded to 1 decimal place

6. What is the expected value of X?
7. What is the expected value of Y?
8. What is the variance of X?
9. What is the variance of Y?
10. Evaluate the probability: P(X=2 **and** Y=2)
    a) 0.41
    b) 0.01
    c) 0.25
    d) 0.04
11. Evaluate the probability: P(X=5 **and** Y=6)
    a) 0.01
    b) 0.55
    c) 0.07
    d) 0.275
12. Evaluate the probability: P(X=8 **and** Y=5)
    a) 0.325
    b) 0.65
    c) 0.075
    d) 0.04

Question 13-21 are in reference to the following quantum states:

$$|\alpha\rangle = a\begin{pmatrix}1\\0\end{pmatrix} \qquad |\beta\rangle = b\begin{pmatrix}1\\1\end{pmatrix} \qquad |\gamma\rangle = c\begin{pmatrix}5\\3e^{i\frac{\pi}{4}}\end{pmatrix}$$

13. Write the following in vector form:

    $\langle\alpha|$

    a) $a\begin{pmatrix}1 & 0\end{pmatrix}$
    b) $a\begin{pmatrix}i & 0\end{pmatrix}$
    c) $a\begin{pmatrix}0 & 1\end{pmatrix}$
    d) $a\begin{pmatrix}\frac{1}{\sqrt{2}} & 0\end{pmatrix}$

14. Write the following in vector form:

    $\langle\beta|$

    a) $b\begin{pmatrix}1 & -1\end{pmatrix}$
    b) $b\begin{pmatrix}i & 1\end{pmatrix}$
    c) $b\begin{pmatrix}1 & 1\end{pmatrix}$
    d) $b\begin{pmatrix}0 & 1\end{pmatrix}$

15. Write the following in vector form:

    $\langle\gamma|$

    a) $c\begin{pmatrix}3 & 5e^{i\frac{\pi}{4}}\end{pmatrix}$
    b) $c\begin{pmatrix}5 & 3e^{-i\frac{\pi}{4}}\end{pmatrix}$
    c) $c\begin{pmatrix}5 & 3e^{i\frac{\pi}{4}}\end{pmatrix}$
    d) $c\begin{pmatrix}5i & e^{i\frac{\pi}{4}}\end{pmatrix}$

One of the most important properties of a quantum state is that it is normalized. This means that for any quantum state $|\psi\rangle$, the following must be satisfied

$$\langle\psi|\psi\rangle = 1$$

Use this relation to solve questions 16-18.

16. Using $\langle\alpha|\alpha\rangle = 1$, determine the value of a.

    (a) $a = \frac{1}{2}$
    (b) $a = \frac{1}{\sqrt{2}}$
    (c) $a = 1$
    (d) $a = 3$

17. Using $\langle\beta|\beta\rangle = 1$, determine the value of b.

    (a) $b = 1$
    (b) $b = 2$
    (c) $b = \frac{1}{2}$
    (d) $b = \frac{1}{\sqrt{2}}$

18. Using $\langle\gamma|\gamma\rangle = 1$, determine the value of a.

    (a) $c = \frac{1}{\sqrt{34}}$
    (b) $c = \frac{1}{4}$
    (c) $c = \frac{1}{\sqrt{72}}$
    (d) $c = \frac{1}{\sqrt{2}}$

19. Calculate the inner product:

    $\langle\alpha|\gamma\rangle$

    a) $\frac{3}{\sqrt{34}}e^{i\frac{\pi}{4}}$
    b) 2
    c) $\frac{5}{\sqrt{34}}$
    d) 1

20. Calculate the inner product:

    $\langle\gamma|\beta\rangle$

    a) $\frac{1}{\sqrt{68}}(5 + 3e^{-i\frac{\pi}{4}})$
    b) $\frac{1}{\sqrt{68}}(5 - 3e^{i\frac{\pi}{4}})$
    c) $3e^{i\frac{\pi}{4}}$
    d) 5

21. Calculate the inner product:

$\langle\beta|\gamma\rangle$

a) $5i$

b) $\frac{1}{\sqrt{68}}(5+3e^{i\frac{\pi}{4}})$

c) $\frac{1}{\sqrt{68}}(5-i3e^{i\frac{\pi}{4}})$

d) $3e^{\frac{\pi}{4}}$