HOMEWORK 2

HIGH SCHOOL MATH REVIEW

Before moving onto complex topics in quantum computing, it is important that we solidify our foundation in trigonometry and algebra. Having a good understanding of these concepts will be very helpful in getting a handle on the more advanced mathematics to be introduced later in the course.

Problems 1-4 are required, Problem 5 is an optional challenge problem.

Problem 1: Trigonometric Functions

Without using a calculator, evaluate the following expressions involving trigonometric functions:

i) $\sin\left(\frac{\pi}{4}\right)$

ii) $\sin\left(\frac{\pi}{2}\right)$

iii) $\sin\left(\frac{\pi}{3}\right)$

iv) $\sin(0)$

v) $\cos(0)$

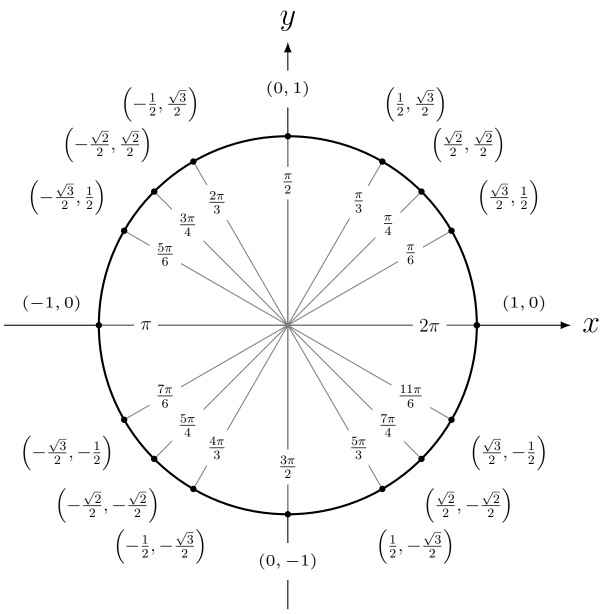
vi) $\cos\left(\frac{\pi}{3}\right)$

vii) $\cos\left(\frac{3\pi}{4}\right)$

viii) $\cos\left(-\frac{\pi}{6}\right)$

ix) $\tan\left(\frac{2\pi}{3}\right)$

x) $\tan\left(\frac{5\pi}{4}\right)$



Problem 2: Coordinate Systems

A solid understanding of Cartesian and polar coordinates, and being able to convert between them will be important moving forward.

a) Convert the following points from Cartesian coordinates to polar coordinates. You may use a calculator for this problem.

i) $(x = 3, y = 4)$

ii) $(x = \sqrt{3}, y = 1)$

iii) $(x = -2, y = 2)$

iv) $(x = 0, y = 1)$

v) $(x = 5, 12)$

b) Convert the following points from polar coordinates to Cartesian coordinates:

vi) $(r = 1, \theta = 0)$

vii) $(r = 3, \theta = \frac{\pi}{4})$

viii) $(r = 3, \theta = \frac{9\pi}{4})$

ix) $(r = 5, \theta = \frac{2\pi}{3})$

x) $(r = 2, \theta = \frac{11\pi}{6})$

Problem 3: Exponents and Factoring

a) Use the rules for exponents to simplify the following expressions:

i) $\frac{x^5 x^2}{x^3}$

ii) $e^2 e^{-4}$

b) Factor and simplify the following expressions:

i) $x^2 + 10x$

ii) $x^2 + 4x + 3$

iii) $x^2 + 8x + 16$

iv) $x^2 - 64$

v) $x^2 - 3x - 28$

vi) $\frac{e^3 x^2 + e^2 x}{ex + 1}$

Problem 4: Summations

Summation notation will feature quite frequently once we begin learning more about probability and quantum superposition, so we would like to get familiar with them.

a) Evaluate the following summations:

i) $\sum_{k=1}^{9} k$

ii) $\sum_{k=1}^{5} k^2$

iii) $\sum_{k=4}^{10} (2k - 1)$

b) Consider the following table:

Value	Weights
2	4
10	6
14	3
15	2
19	1
25	0

i) Compute the arithmetic mean of the values in the table.

ii) Compute the weighted mean of the values in the table.

Problem 5: Optional Challenge Problem

The Orbit Equation

This is an optional problem, and is quite mathematically intensive. We will explore a real world application of coordinate system transformation.

In many fields of physics, we are often interested in describing the motion of objects in space under the influence of gravity. For example, we may want to describe the motion of a planet around a star. This motion is described by the "orbit equation".

a) First take a moment to think about which coordinate system (Cartesian or polar) might be better to describe this type of system. Explain the reasoning behind your choice.

The orbit equation in Cartesian coordinates can be written as:

$$x = [(1 + \frac{y^2}{x^2})^{\frac{1}{2}} + \varepsilon]^{-1} \tag{2.1}$$

This equation is quite cumbersome, it is long and x appears on both sides of the equations so it is not immediately apparent how x and y depend on each other. As a reminder, we can convert between Cartesian and polar coordinates with the equations

$$x = r\cos\theta \qquad y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$$

b) Using the relationships between Cartesian and polar coordinates and trigonometric identities, show that the orbit equation (Eq(2.1)) can be written in polar coordinates as:

$$r = \frac{1}{1 + \varepsilon\cos\theta}$$

Notice how in this form of the equation the coordinate variables r and θ are separated, appearing only on one side of the equals sign. This allows us to much more easily infer how they relate to each other. Also, the presence of the $\cos\theta$ directly hints at periodicity in this equation, which makes sense for orbits. Try to plot the orbit equation, and see how the parameter ε, which is called the "orbit eccentricity", changes the shape of an orbit.