INTRO TO QUANTUM COMPUTING

Week 4 Lab

MATRICES AND LINEAR ALGEBRA

<insert TA name>

<insert date>

PROGRAM FOR TODAY

- Announcement
- Attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz passcode:

ANNOUNCEMENT

Student Assistant Virtual Office Hours

- **Every Friday**, from **8am-8pm EST (UTC-5)**
- Student Assistants are available to review lab and lecture materials, walk through homework problems, or answer any other content-related questions you might have at the end of each week
- You can find a link to office hours under the “Course Materials” module on Canvas.

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number :
  - Passcode:
- The magnitude of the vector (1 0) is:
  - 1
  - 0
  - ½
  - -1
- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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QUESTIONS FROM LAST WEEK

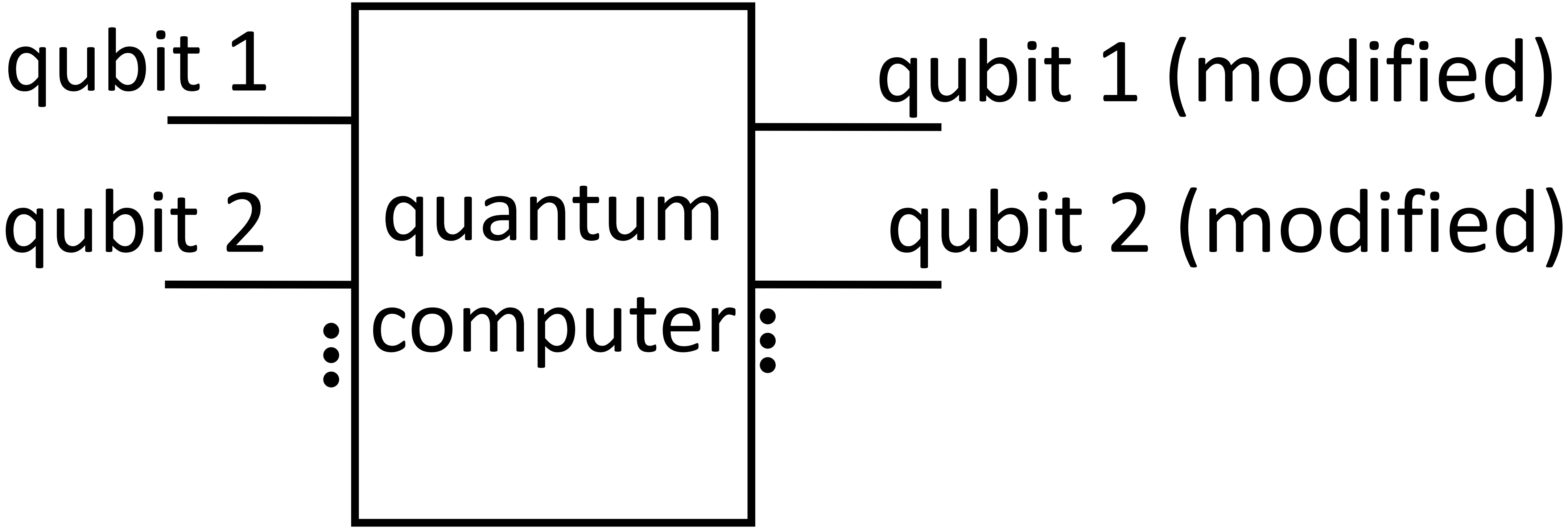
Questions about content from last week?

LEARNING OBJECTIVES FOR LAB 4

- Learning how to compare vectors
  - Normalization
  - Unit vectors
- Getting comfortable with vector inner products
- Relating matrices and vectors to quantum computing
  - Matrix notation
  - Multiplying matrices with vectors
  - Multiplying matrices*
  - Inverting matrices*

*Optional content

SCHEMATIC OF A QUANTUM COMPUTER



REPRESENTING QUBITS

classical bit

either 0 or 1

qubit

Superposition of 0 and 1

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VECTOR NOTATION

scalar

$$v = 2$$

vector

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

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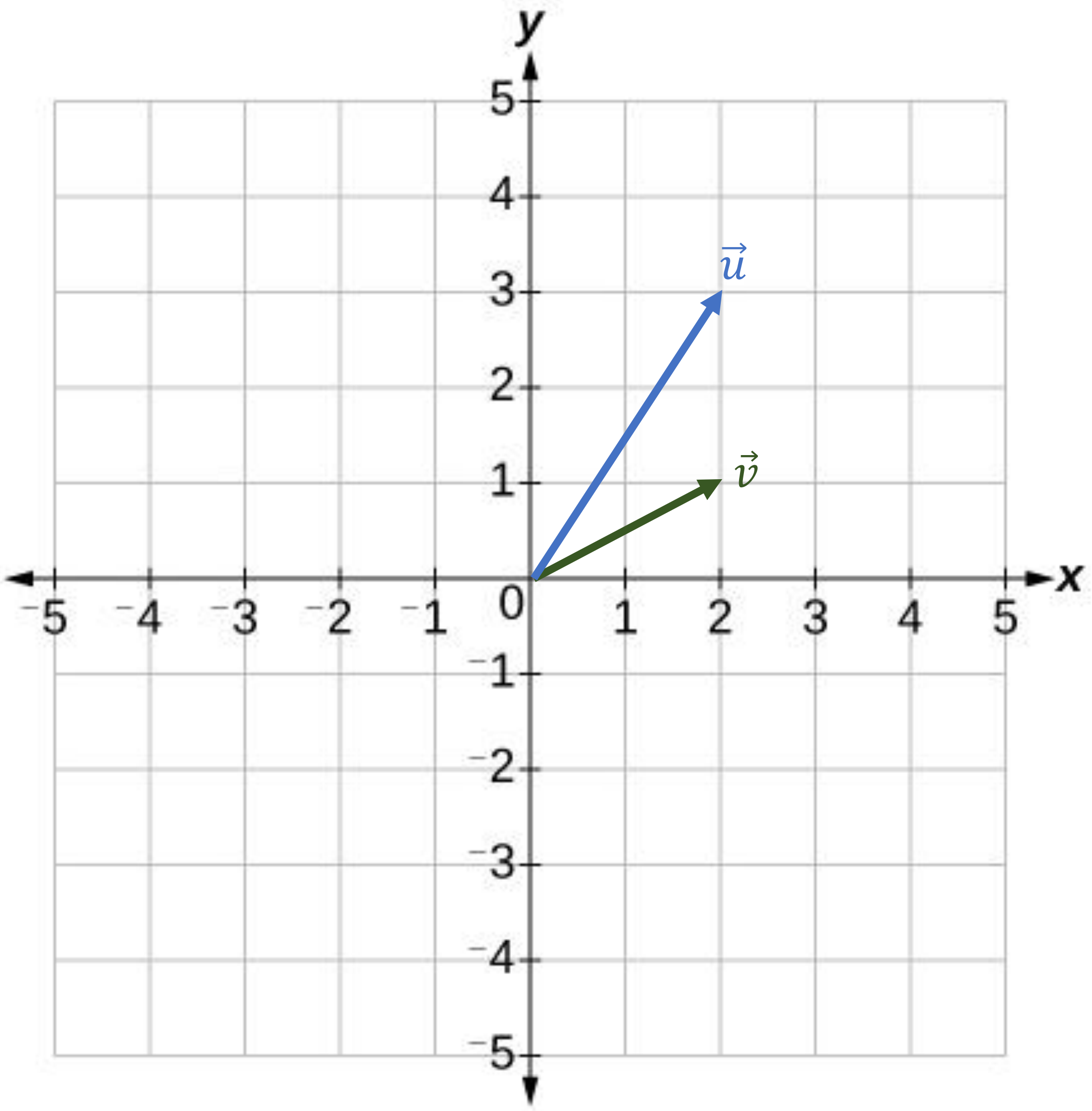
COMPARING VECTORS

Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$



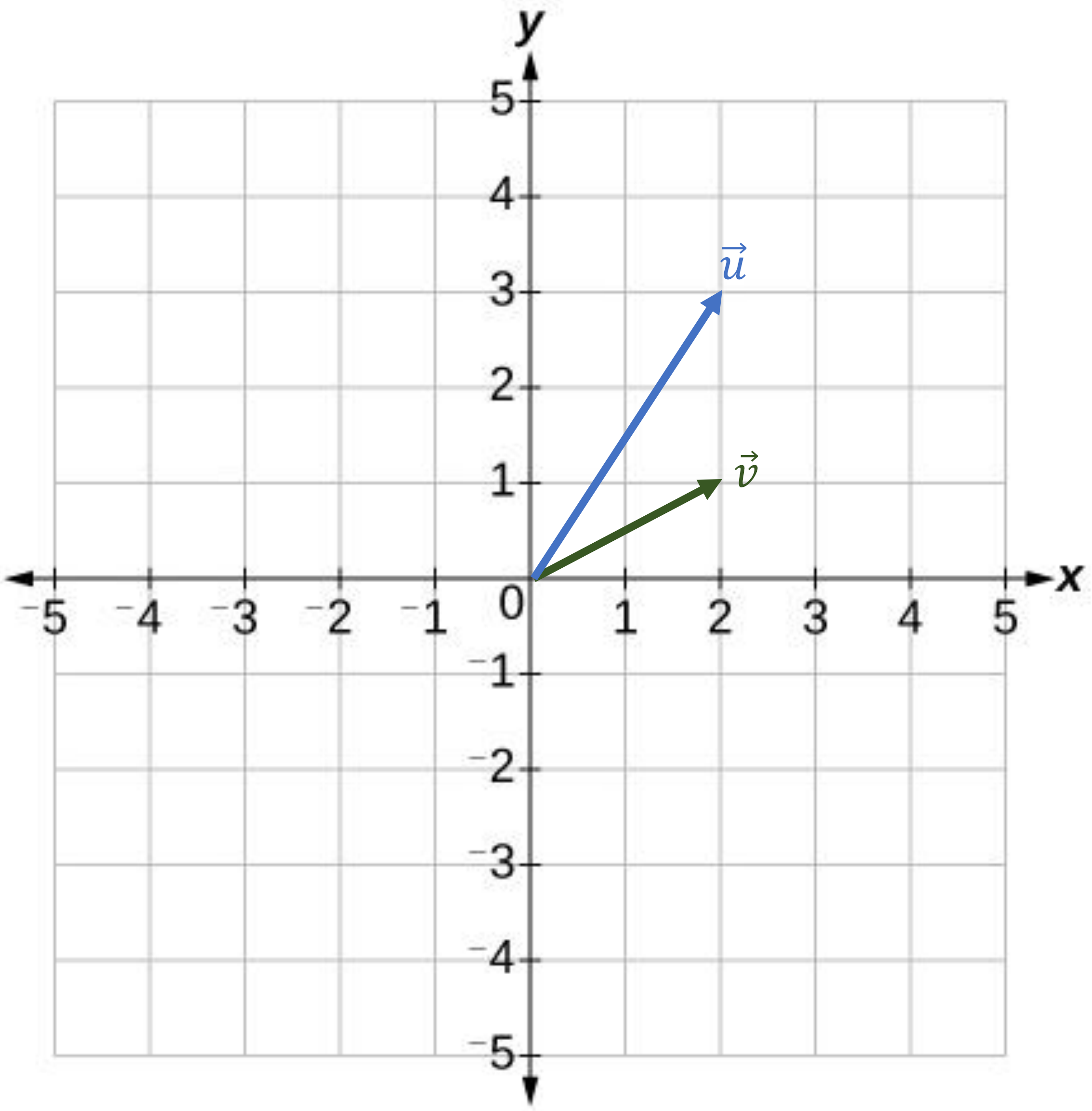
COMPARING VECTORS

Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Idea 1: We can find the lengths of the two vectors

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$



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COMPARING VECTORS

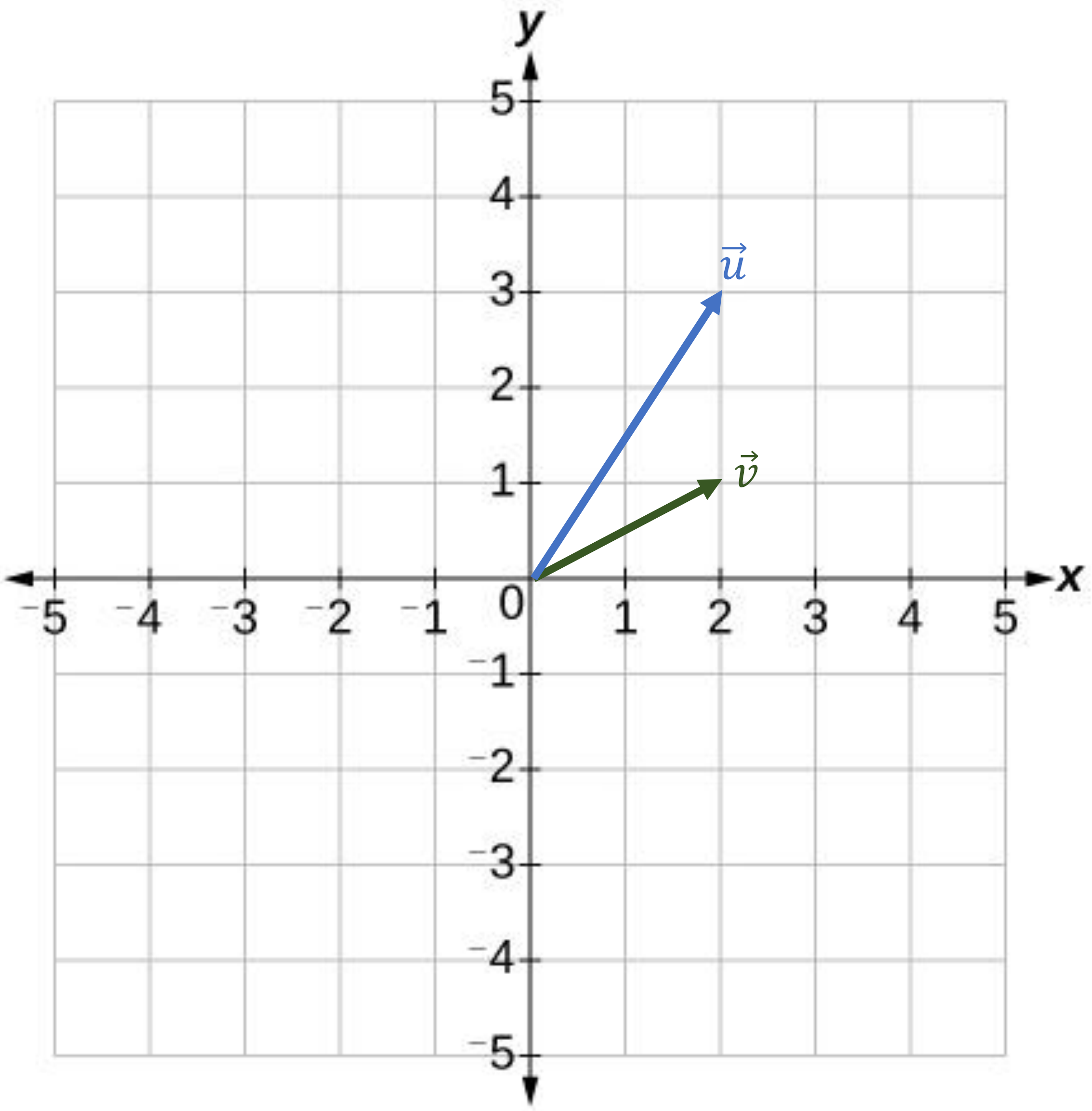
Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Idea 2: We can now compare the directions of the two vectors

- Divide out the lengths of both vectors to form two new **unit vectors**, $\hat{u}$ and $\hat{v}$:

$$\hat{u} = \frac{\vec{u}}{||\vec{u}||}, \hat{v} = \frac{\vec{v}}{||\vec{v}||}$$



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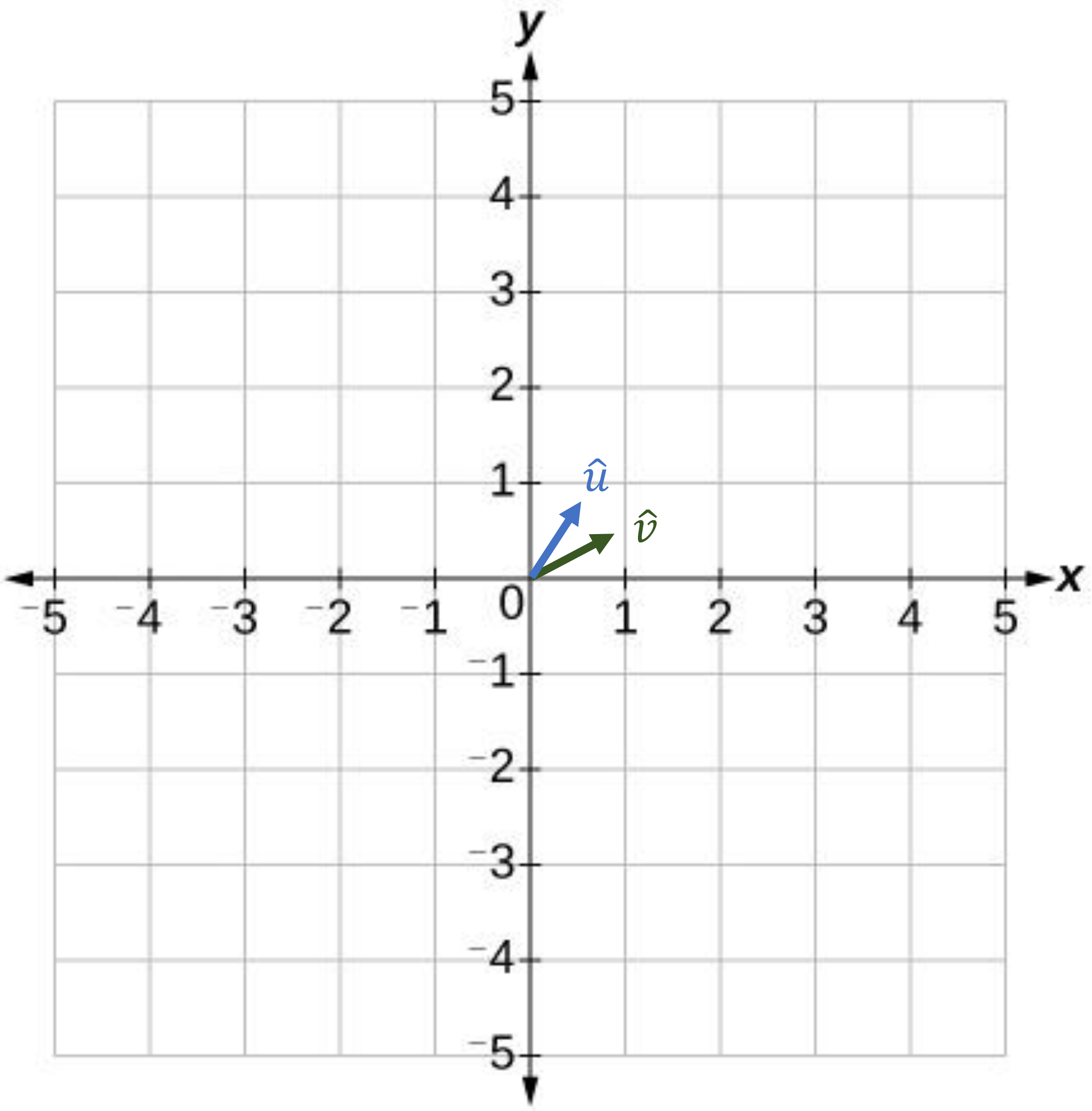

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COMPARING VECTORS

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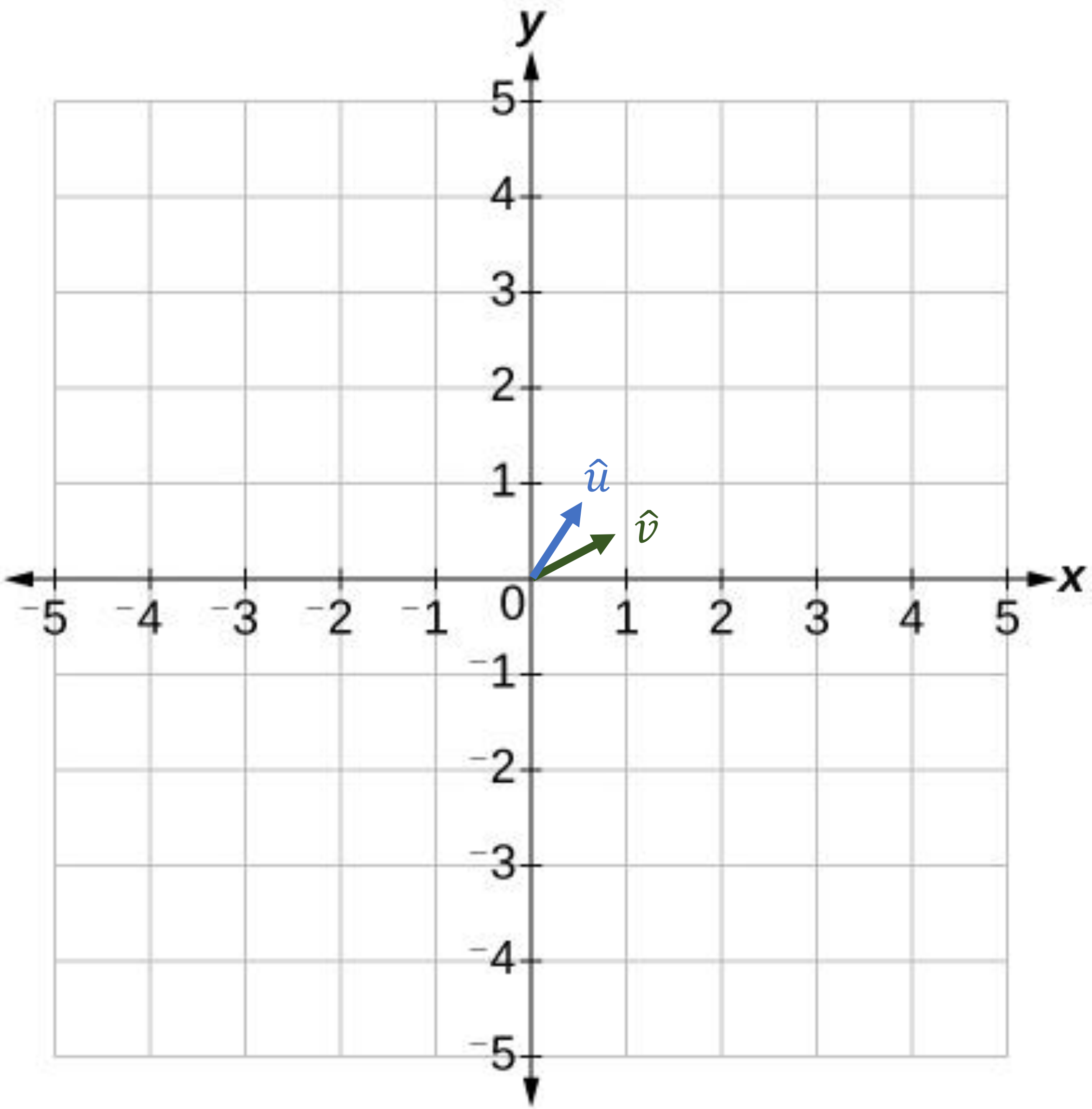
Normalization



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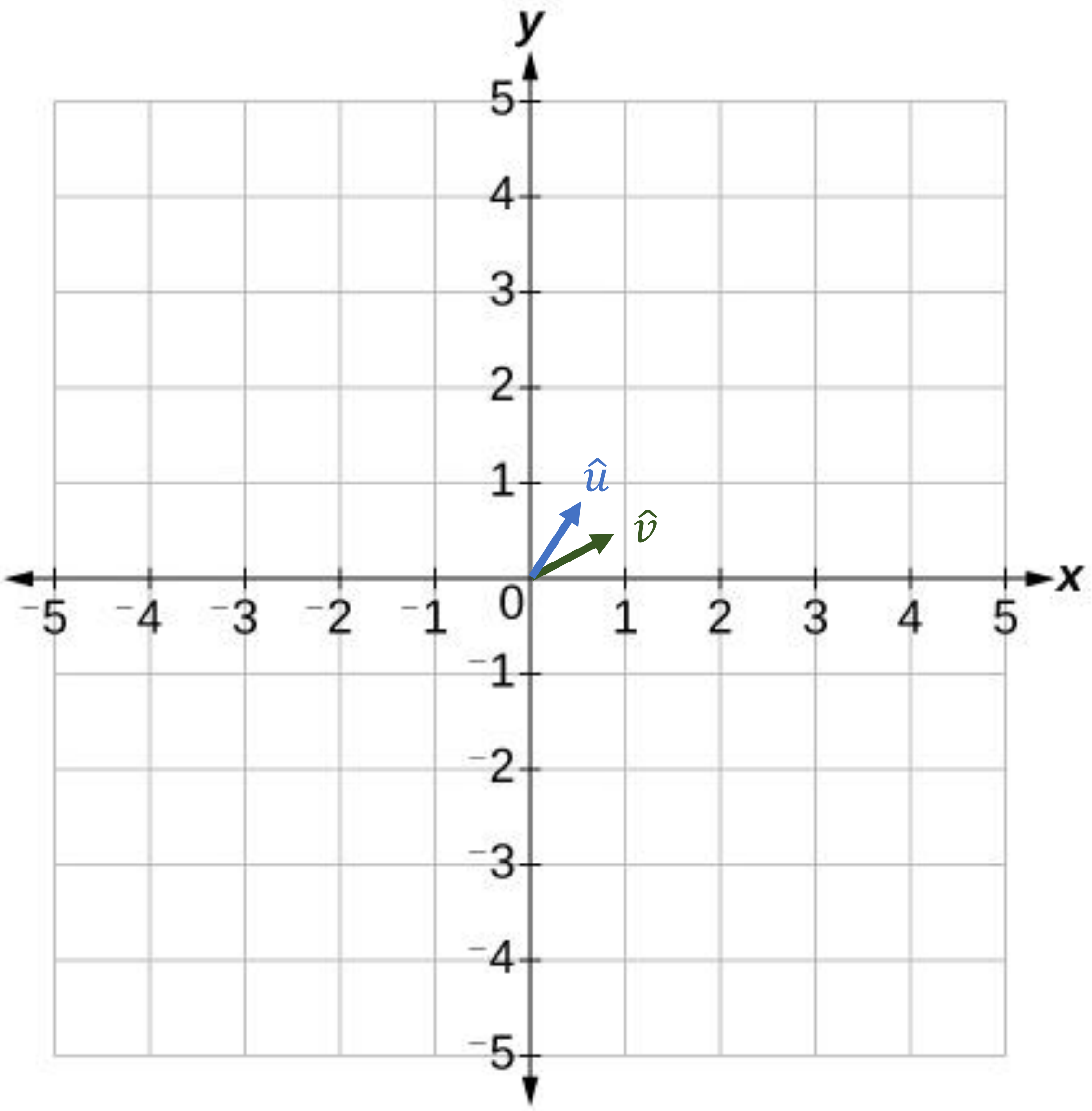
COMPARING VECTORS

Problem: Compare the vectors

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\hat{u} = \frac{\vec{u}}{||\vec{u}||}$, $\hat{v} = \frac{\vec{v}}{||\vec{v}||}$ $\quad \angle \vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$



QUESTIONS

Questions about content so far?

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INNER PRODUCT OF TWO VECTORS

$$\langle \vec{v}, \vec{w} \rangle = \vec{v}^{\dagger}\vec{w} = v_1^* w_1 + \cdots + v_n^* w_n = \sum_{i=1}^{n} v_i^* w_i$$

where
$\vec{v}, \vec{w} \in \mathbb{C}^n$

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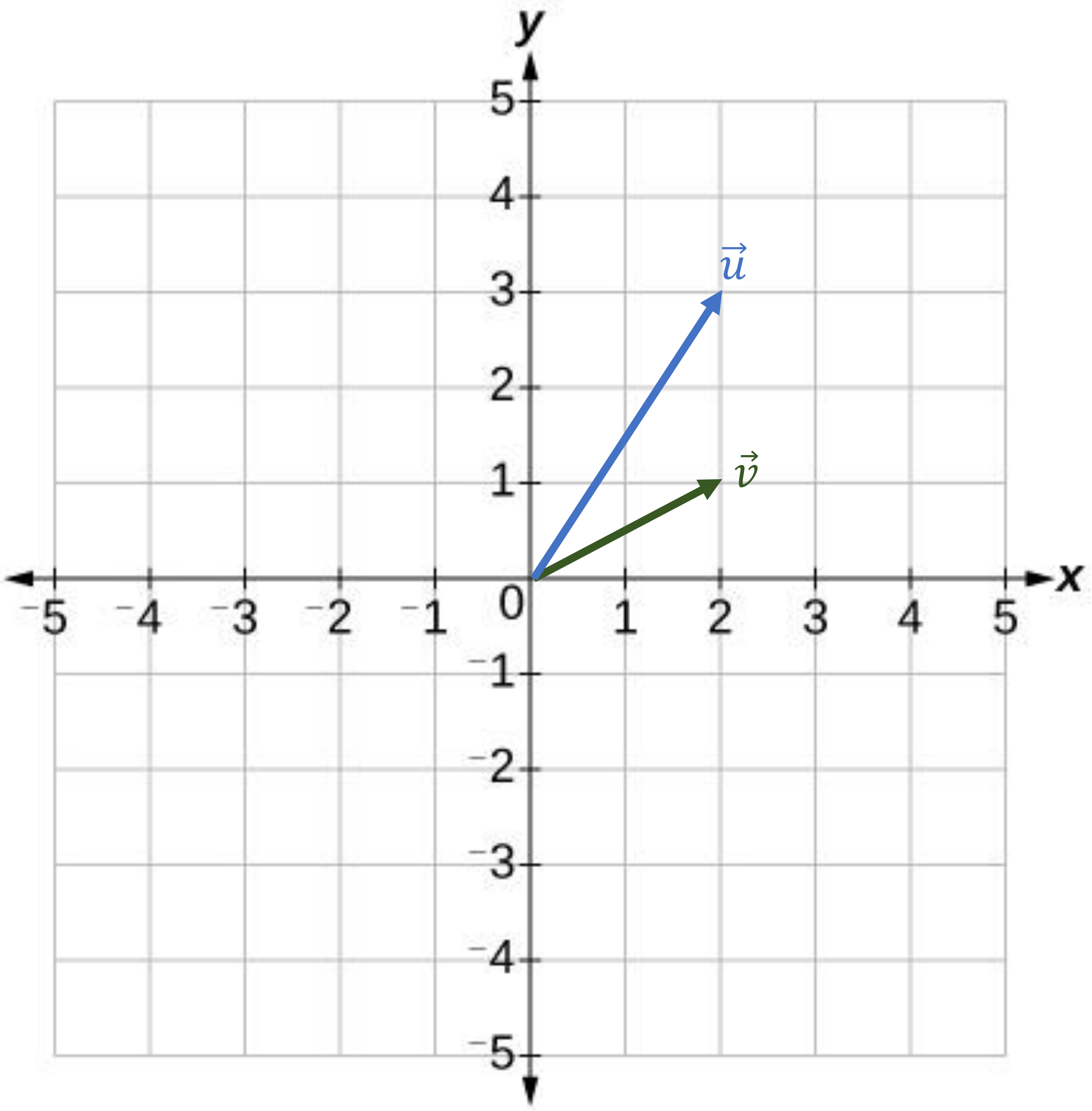
INNER PRODUCT OF TWO VECTORS

Find the inner product of

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$$\langle \vec{u}, \vec{v} \rangle = u_1^* v_1 + u_2^* v_2$$

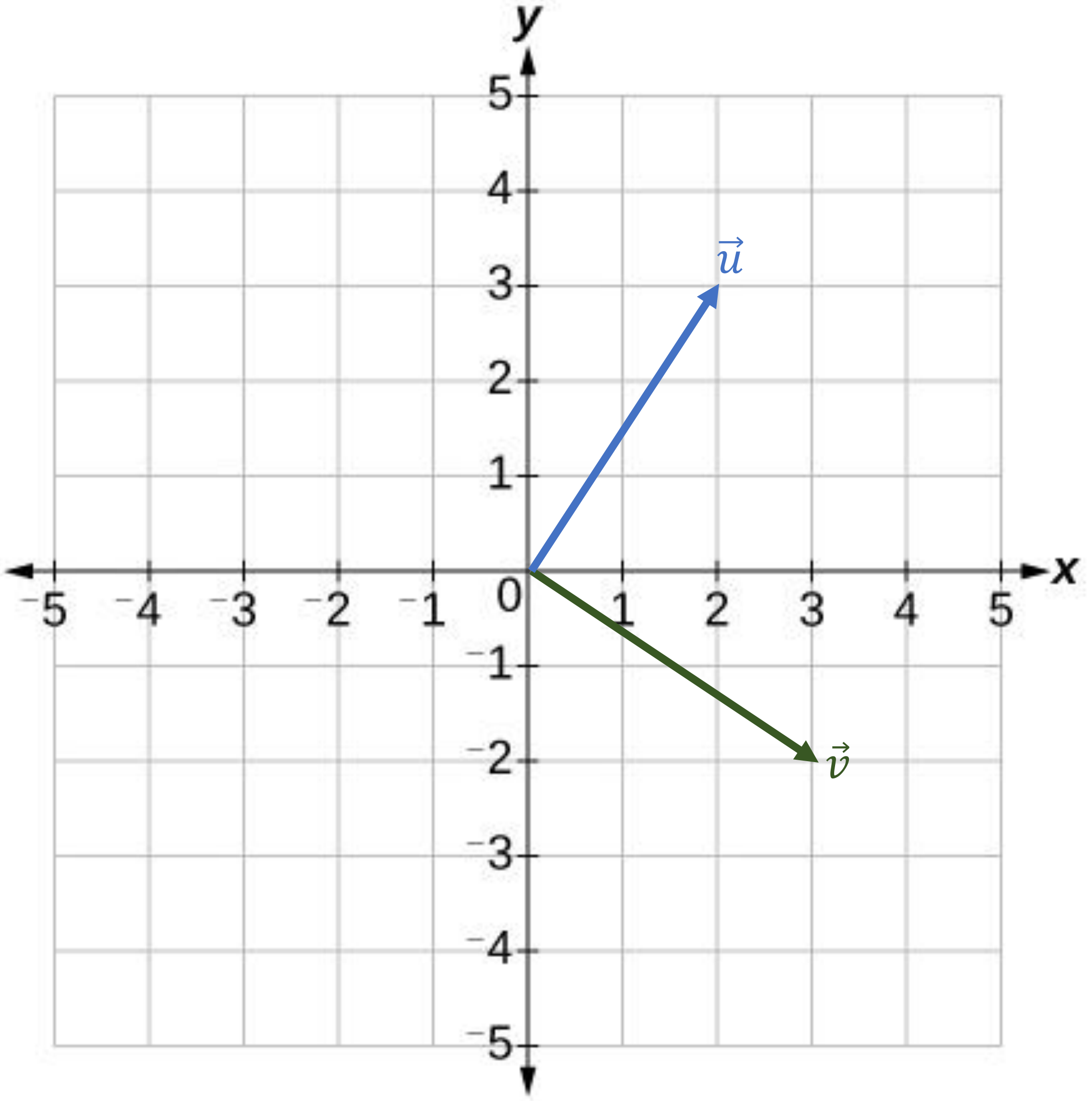


ORTHOGONALITY OF VECTORS

Find the inner product of

$\vec{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$$\langle \vec{u}, \vec{v} \rangle = u_1^* v_1 + u_2^* v_2$$



QUESTIONS

Questions about content so far?

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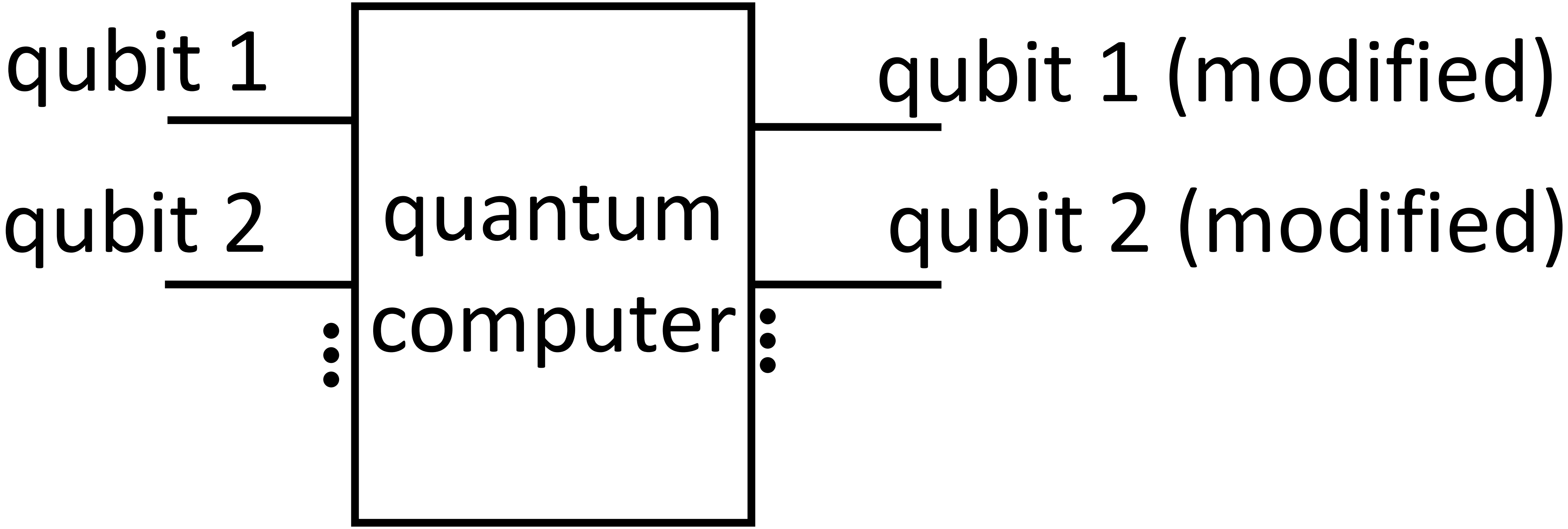
SCHEMATIC OF A QUANTUM COMPUTER



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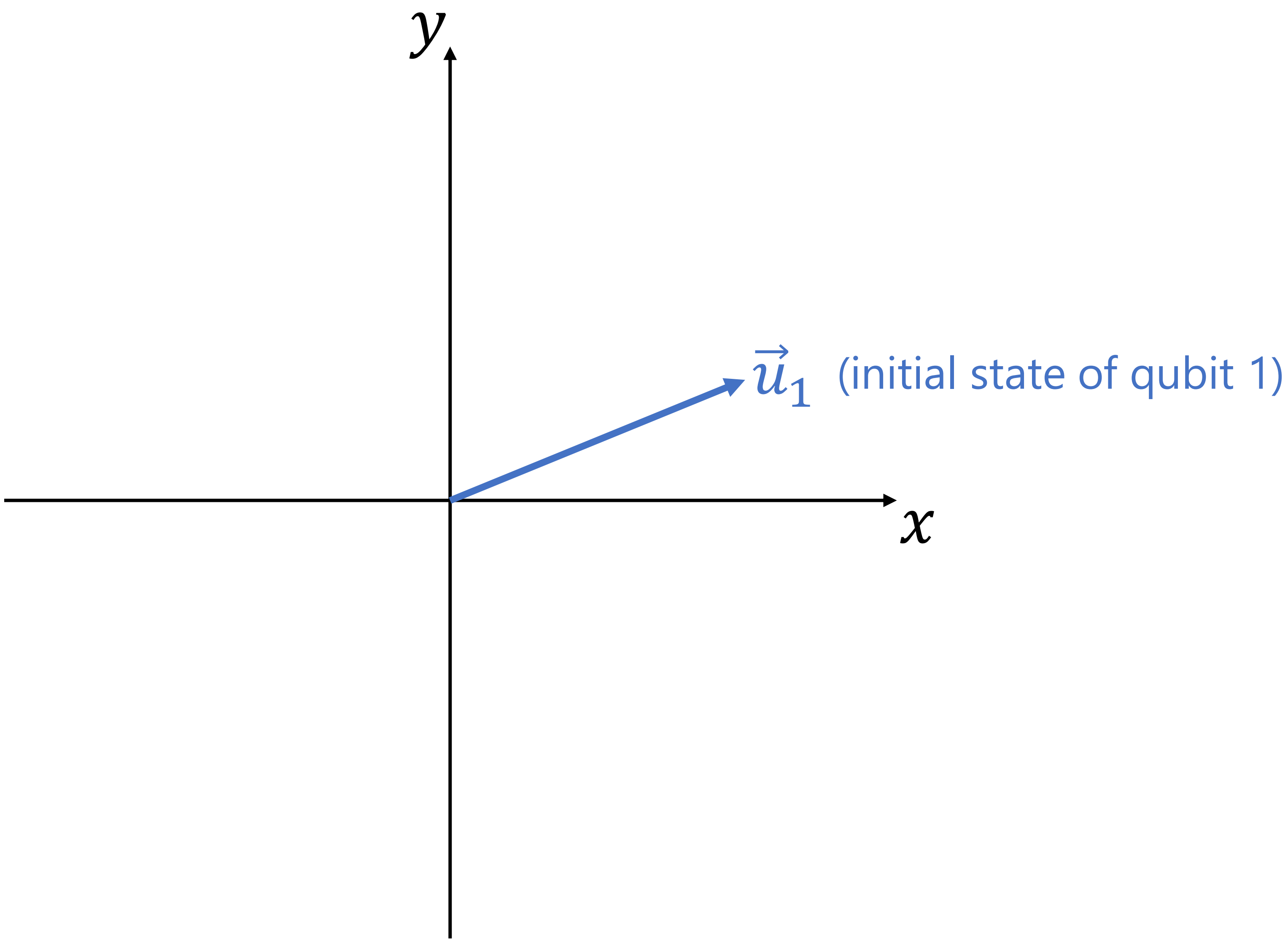
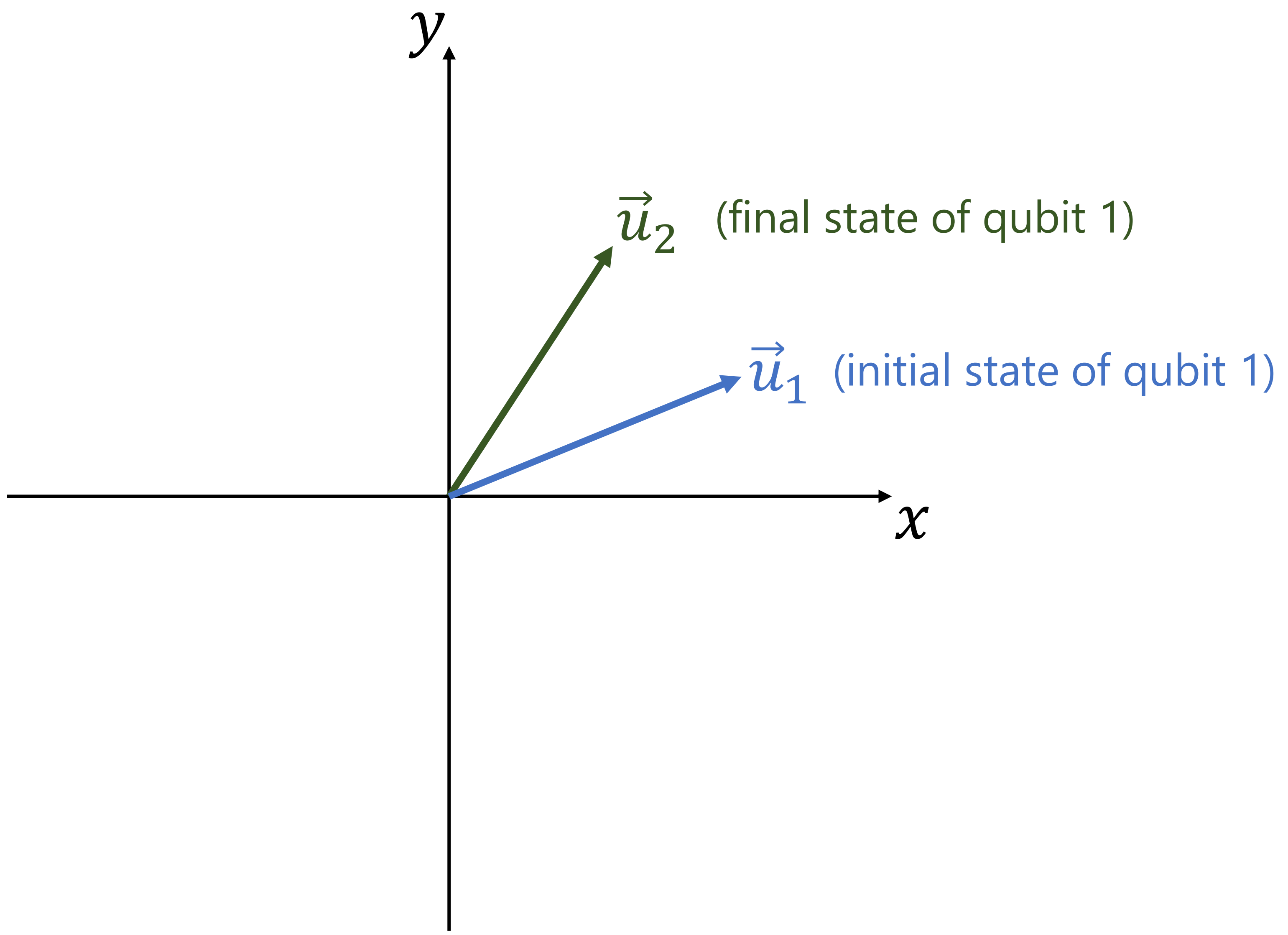
TRANSFORMING VECTORS



Canvas attendance quiz passcode:

TRANSFORMING VECTORS



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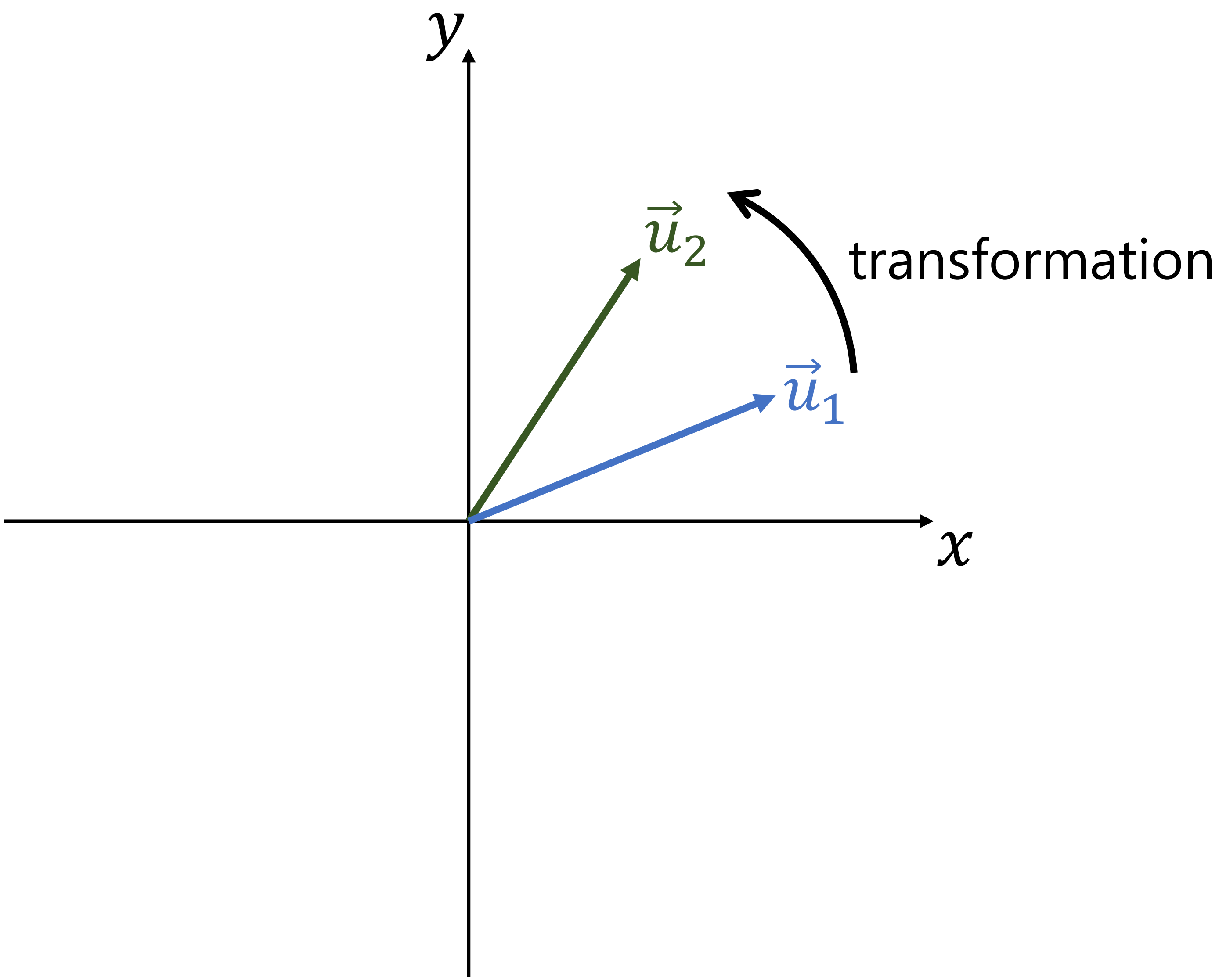

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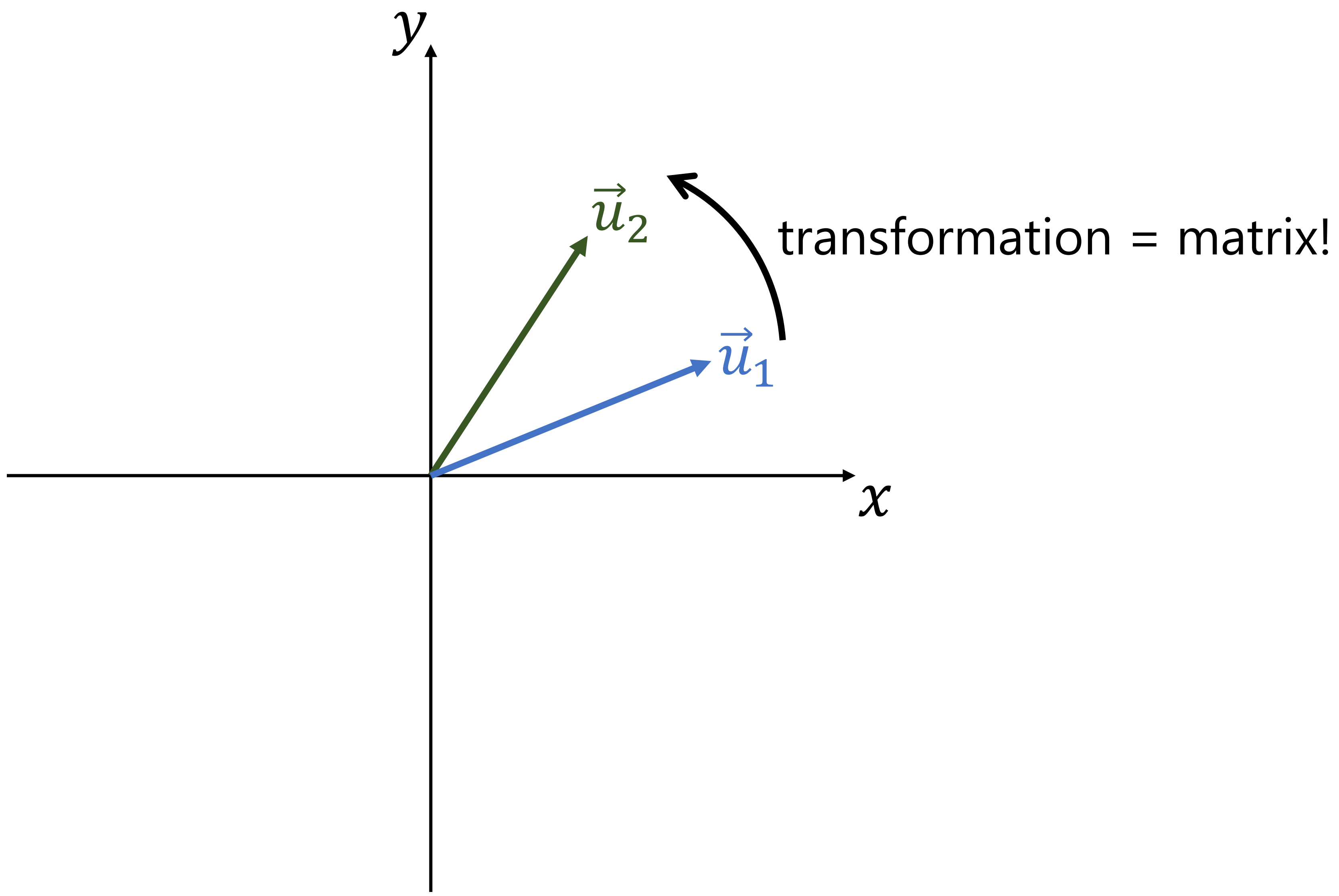

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MATRIX NOTATION

Vector

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Matrix

$$\boldsymbol{S} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

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USING MATRICES TO TRANSFORM VECTORS

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} a_{11} * x_1 + a_{12} * x_2 + \cdots + a_{1m} * x_m \\ a_{21} * x_1 + a_{22} * x_2 + \cdots + a_{2m} * x_m \\ \vdots \\ a_{n1} * x_1 + a_{n2} * x_2 + \cdots + a_{nm} * x_m \end{pmatrix}$$

Note: The vector height must match the matrix width.

$$(n \times m) \times (m \times 1)$$
$$\downarrow$$
$$(n \times 1)$$

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USING MATRICES TO TRANSFORM VECTORS

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} a_{11} * x_1 + a_{12} * x_2 + \cdots + a_{1m} * x_m \\ a_{21} * x_1 + a_{22} * x_2 + \cdots + a_{2m} * x_m \\ \vdots \\ a_{n1} * x_1 + a_{n2} * x_2 + \cdots + a_{nm} * x_m \end{pmatrix}$$

Note: The vector height must match the matrix width.

$$(n \times m) \times (m \times 1)$$

$$\downarrow$$

$$(n \times 1)$$

$$\boldsymbol{A}\vec{x} = \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_{\text{transformation}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{\text{input}} = \underbrace{\begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}}_{\text{output}}$$

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APPLYING MATRICES TO VECTORS

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = ?$$

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

APPLYING MATRICES TO VECTORS

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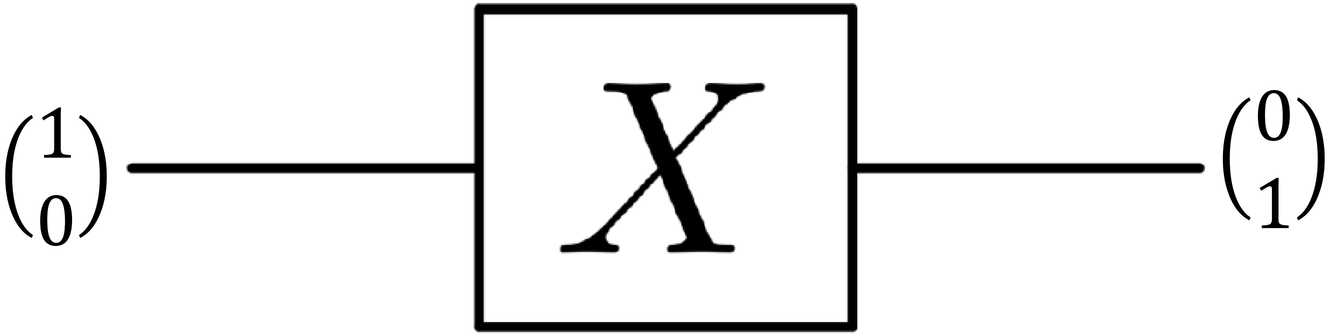
APPLYING MATRICES TO VECTORS

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

input qubit   output qubit



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APPLYING MATRICES TO VECTORS

Multiplying constant with matrix

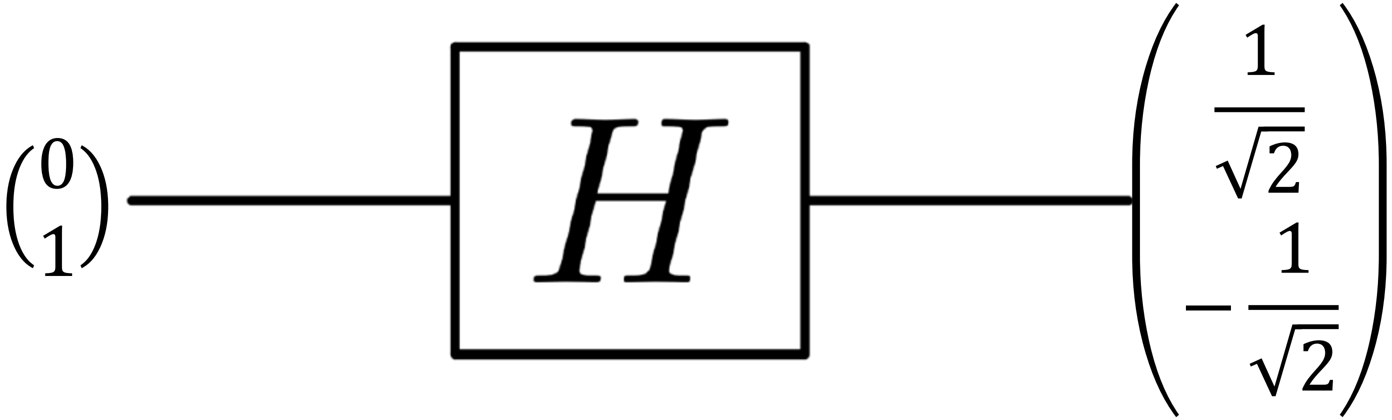
$$c * \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} c * a_{11} & c * a_{12} \\ c * a_{21} & c * a_{22} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = ?$$

$$\boldsymbol{A}\vec{x} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

APPLYING MATRICES TO VECTORS

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$



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IMPORTANT TAKEAWAYS

- Qubit → vector
- Quantum gate → matrix
- Operating a quantum gate on a qubit → multiplying matrix with vector

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Questions about content so far?

POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the conten

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OPTIONAL CONTENT

MATRIX MULTIPLICATION

- Qubit → vector
- Quantum gate → matrix
- Operating a quantum gate on a qubit → multiplying matrix with vector
- Operating multiple gates on a qubit → matrix multiplication and multiplying matrix with vector

MATRIX MULTIPLICATION

$$\boldsymbol{AB} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{m} a_{1i} * b_{i1} & \sum_{i=1}^{m} a_{1i} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{1i} * b_{ik} \\ \sum_{i=1}^{m} a_{2i} * b_{i1} & \sum_{i=1}^{m} a_{2i} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{2i} * b_{ik} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{m} a_{ni} * b_{i1} & \sum_{i=1}^{m} a_{ni} * b_{i2} & \cdots & \sum_{i=1}^{m} a_{ni} * b_{ik} \end{pmatrix}$$

Remember to always check your shapes! : $(n \times m) \times (m \times k) \longrightarrow (n \times k)$

Note: The first matrix width must match the second matrix height!

MATRIX MULTIPLICATION

$$\boldsymbol{AB} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

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MATRIX MULTIPLICATION

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = ?$$

$$\boldsymbol{AB} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

MATRIX MULTIPLICATION

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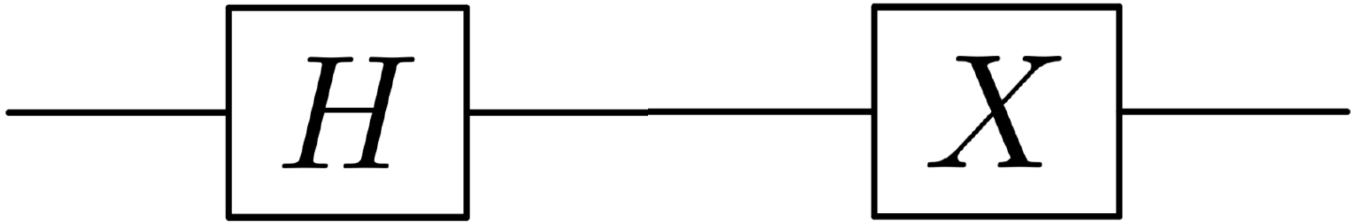
MATRIX MULTIPLICATION



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

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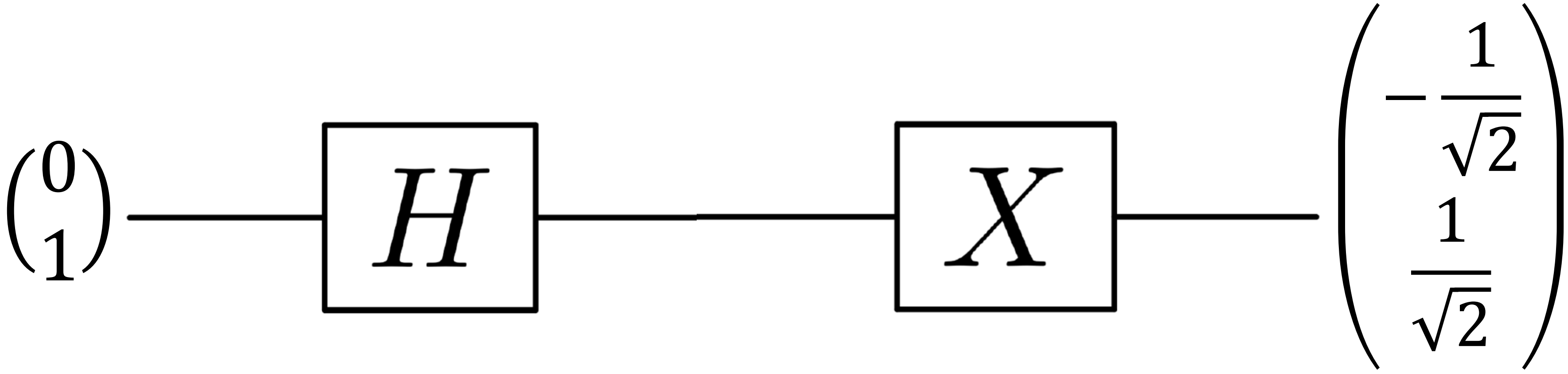
MATRIX MULTIPLICATION



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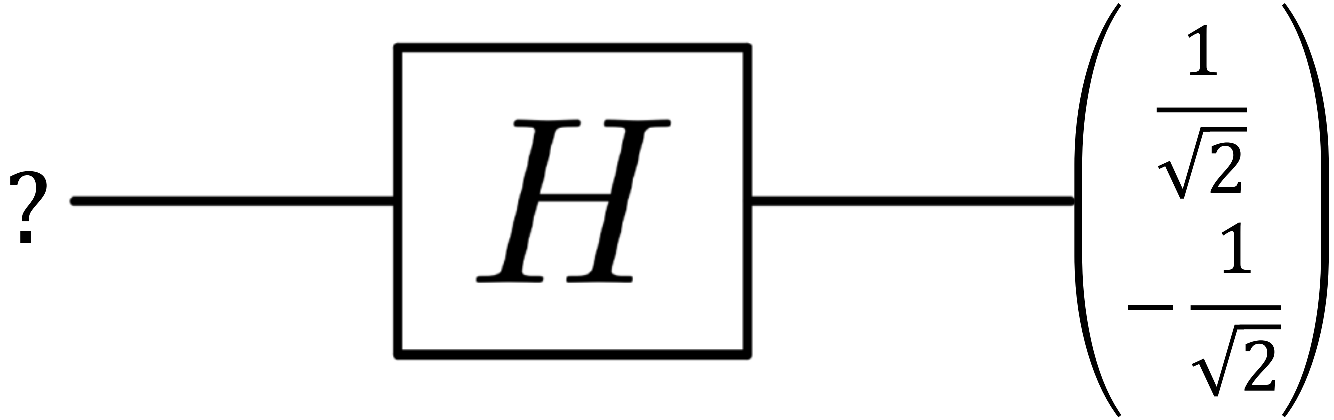
MATRIX INVERSION



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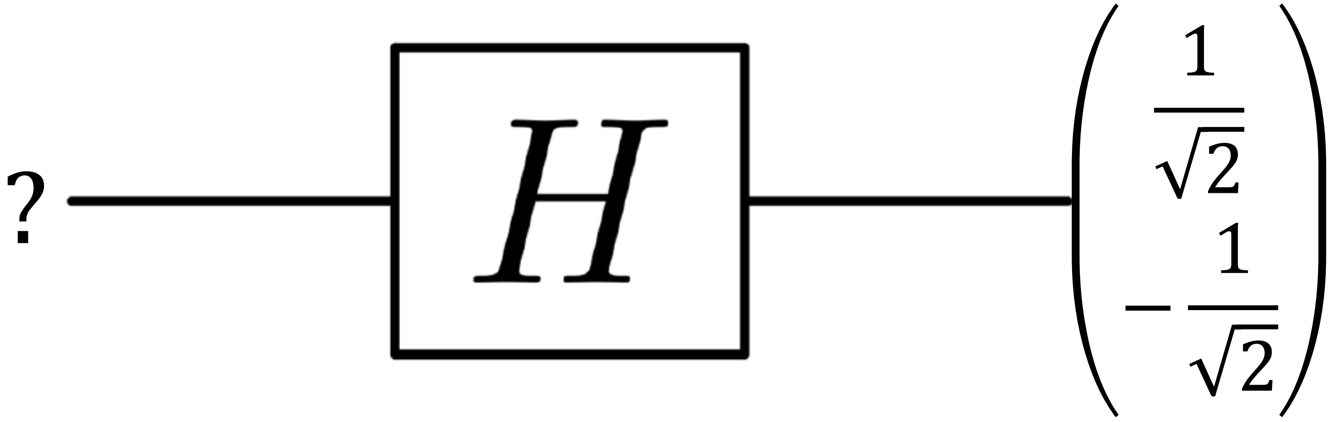
MATRIX INVERSION

$$\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{\boldsymbol{H}}\begin{pmatrix}?\\ ?\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$



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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

$$\vec{?} = \vec{y}/\boldsymbol{H}$$

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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

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MATRIX INVERSION

$$\boldsymbol{H}\vec{?} = \vec{y}$$

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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MATRIX INVERSION

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad ; \boldsymbol{H}^{-1} = ?$$

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MATRIX INVERSION

If $\boldsymbol{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\boldsymbol{X}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\boldsymbol{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \; ; \boldsymbol{H}^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

MATRIX INVERSION

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

$$\begin{pmatrix} ? \\ ? \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

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MATRIX INVERSION

$$\vec{?} = \boldsymbol{H}^{-1}\vec{y}$$

$$\begin{pmatrix} ? \\ ? \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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