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# HOMEWORK 3

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## VECTORS AND COMPLEX NUMBERS

1. Calculate the resulting vector of the following operation:

$$\vec{a} = 2 * \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

a)  $\begin{pmatrix} -2 \\ 8 \end{pmatrix}$

b)  $\begin{pmatrix} -7 \\ -5 \end{pmatrix}$

c)  $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

d)  $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$

2. Calculate the resulting vector of the following operation:

$$\vec{b} = 4 * \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 * \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

a)  $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

b)  $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

c)  $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

d)  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$

3. Calculate the magnitude  $r$  and direction  $\theta$  of the following vector. The direction of the vector should be written as an angle in **radians**. You may use a calculator.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

- a)  $r = 4, \theta = -0.93$
  - b)  $r = 5, \theta = 2.21$
  - c)  $r = 5, \theta = 5.35$
  - d)  $r = 7, \theta = 2.21$
4. Calculate the magnitude  $r$  and direction  $\theta$  of the following vector. The direction of the vector should be written as an angle in **radians**. You may use a calculator.

$$\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}$$

- a)  $r = 2, \theta = 4.1$
- b)  $r = 1, \theta = 0.96$
- c)  $r = 1, \theta = -2.18$
- d)  $r = \sqrt{3}, \theta = 2.25$

5. A river that flows north to south, has two ferry terminals directly across the shore from each other. The river flows at 8 m/s with respect to the shore. A boat travels across the river along a trajectory which goes directly between the terminals (east to west) with a net velocity of 15 m/s. Fig. (3.1) shows a sketch of this. What is the actual velocity of the boat? Give your answer as a vector with magnitude (in m/s) and direction (angle  $\theta$  in radians).

(Hint: the boat's net velocity is a combination of its actual velocity and the velocity of the water in the river)

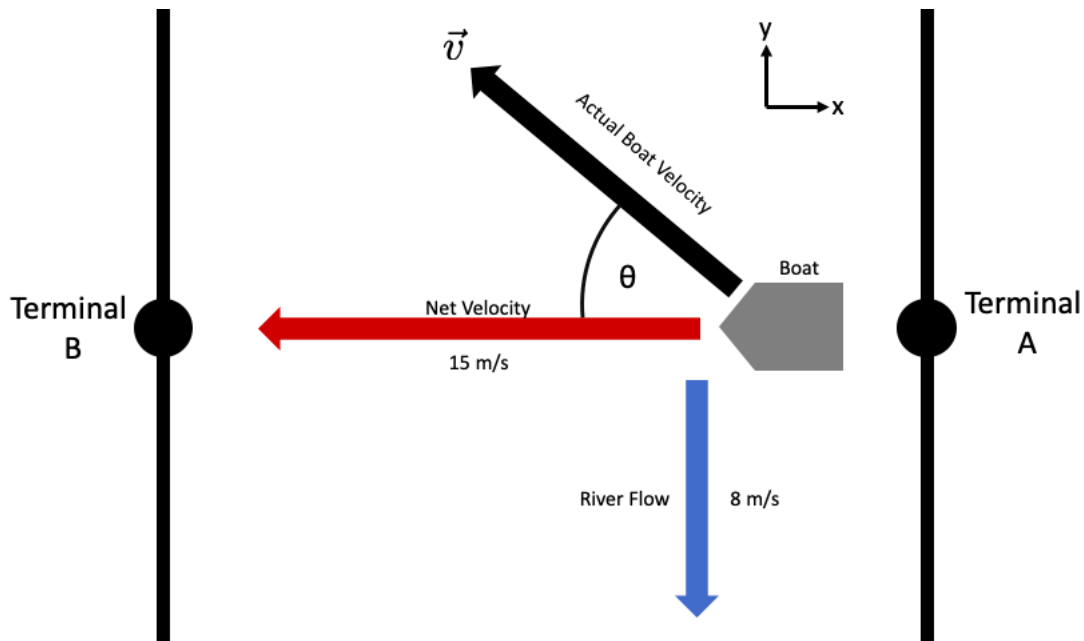


Figure 3.1: Sketch of a boat traveling between ferry terminals.

- a)  $|\vec{v}| = 15, \theta = 0$
- b)  $|\vec{v}| = 8, \theta = 2.06$
- c)  $|\vec{v}| = 17, \theta = 0.49$
- d)  $|\vec{v}| = 23, \theta = 0.98$

6. Add the following complex numbers:

$$(1 + 3i) + (4 + 4i)$$

a)  $5 + 7i$

b)  $5 - i$

c)  $5$

7. Add the following complex numbers:

$$(2 - i) + (-2 + i)$$

a)  $2i$

b)  $4 + 2i$

c)  $0$

8. Add the following complex numbers:

$$(i) + (3)$$

a)  $3(1 + i)$

b)  $1 + 3i$

c)  $3 + i$

9. Multiply the following complex numbers:

$$(5 + 2i) \cdot (5 - 2i)$$

a)  $29$

b)  $25 + 4i$

c)  $21$

d)  $0$

10. Multiply the following complex numbers:

$$(2 - 7i) \cdot (3 - 2i)$$

a)  $6 + 14i$

b)  $-(8 + 25i)$

c)  $20i$

d)  $5 - 9i$

11. What is the complex conjugate of the following complex number?

$$(1 + 4i)$$

- a)  $1 + 4i$
- b)  $-1 - 4i$
- c)  $1 - 4i$
- d)  $-1 + 4i$

12. What is the complex conjugate of the following complex number?

$$(-4 - 2i)$$

- a)  $4 + 2i$
- b)  $-4 + 2i$
- c)  $-4 - 2i$
- d)  $4 - 2i$

13. Find the modulus of the following complex number

$$1 + i$$

- a) 2
- b) 0
- c)  $\frac{1}{\sqrt{2}}$
- d)  $\sqrt{2}$

14. Find the modulus of the following complex number

$$5 - 12i$$

- a) 7
- b) 17
- c) 13
- d)  $\sqrt{17}$

15. Express the following complex number in polar form (express your angles in radians).

$$1 + i$$

- a)  $\frac{\pi}{4}\sqrt{2}$
- b)  $e^{i\frac{\pi}{2}}$
- c)  $e^{\frac{\pi}{4}}$
- d)  $\sqrt{2}e^{i\frac{\pi}{4}}$

16. Express the following complex number in polar form (express your angles in radians).

$$1 - i$$

- a)  $e^{i\frac{\pi}{2}}$
- b)  $\sqrt{2}e^{-i\frac{\pi}{4}}$
- c)  $e^{-\frac{\pi}{4}}$
- d)  $\frac{\pi}{4}\sqrt{2}$

17. Express the following complex number in polar form (express your angles in radians).

$$5i$$

- a)  $e^{5i}$
- b)  $5e^{i\frac{\pi}{2}}$
- c)  $\sqrt{5}e^{i\frac{\pi}{2}}$
- d)  $e^{-i\frac{\pi}{4}}$

18. What is the complex conjugate of the following complex number?

$$e^{i\frac{\pi}{2}}$$

- a)  $-e^{i\frac{\pi}{2}}$
- b)  $e^{-i\frac{\pi}{2}}$
- c)  $-i\frac{\pi}{2}$

19. What is the complex conjugate of the following complex number?

$$5e^{-i\frac{\pi}{6}}$$

- a)  $5e^{i\frac{\pi}{6}}$
- b)  $e^{5i}$
- c)  $\sqrt{5}e^{i\frac{\pi}{6}}$

20. What is the modulus of the following complex number?

$$\alpha = 4e^{i\pi}$$

- a)  $\pi$
- b) 2
- c) 4

21. What is the modulus of the following complex number?

$$\beta = 7e^{i\frac{\pi}{2}}e^{-i\frac{\pi}{3}}$$

a)  $\sqrt{7}$

b) 7

c)  $\frac{\pi}{6}$