INTRO TO QUANTUM COMPUTING

LECTURE #12

QUANTUM MECHANICS 2

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Lecture 1 - Principles of Quantum Mechanics

What is quantum and how do things behave on quantum length scales?

Lecture 2 - Quantum Two-Level Systems and Measurement

Objective - What are two-level systems and what can we do with them?

Lecture 3 - Postulates of Quantum Mechanics

Objective - What are the foundational rules of quantum mechanics?

TODAY'S LECTURE

What are two-level systems and what can we do with them?

- ➔ Two-level quantum systems
- ➔ Stern-Gerlach experiment - how do we know electron spin is a two-level system?
- ➔ Quantum measurements - how do we measure a two-level system in different bases?
- ➔ Quantum entanglement - what can we do with two-level systems?

Reminder - Quantum systems have levels

Quantum

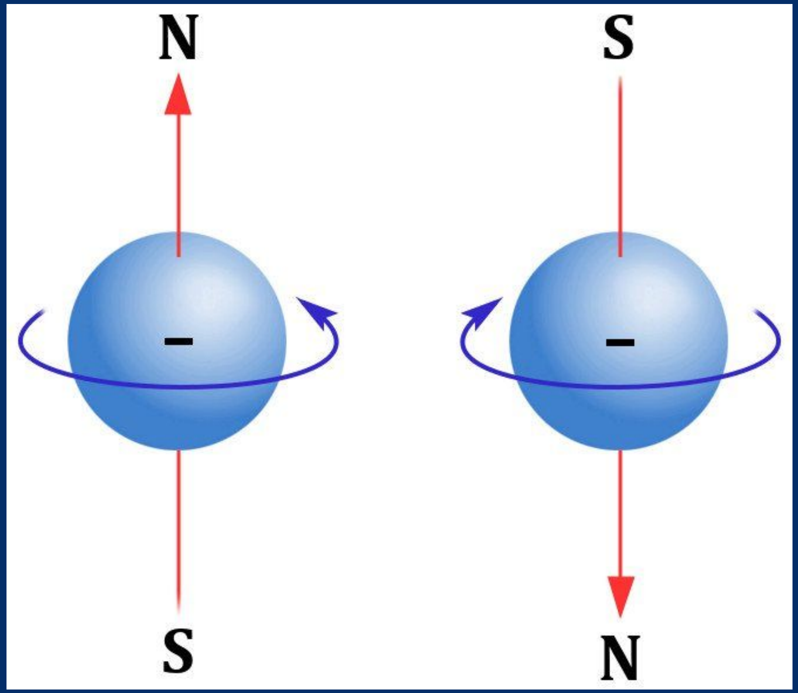
Classical

Two-level systems

A quantum system made out of 2 basis

example:

- Qubits
- Electron spin



Two-level systems

Two-level system:

Qu"bit": $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Three-level system:

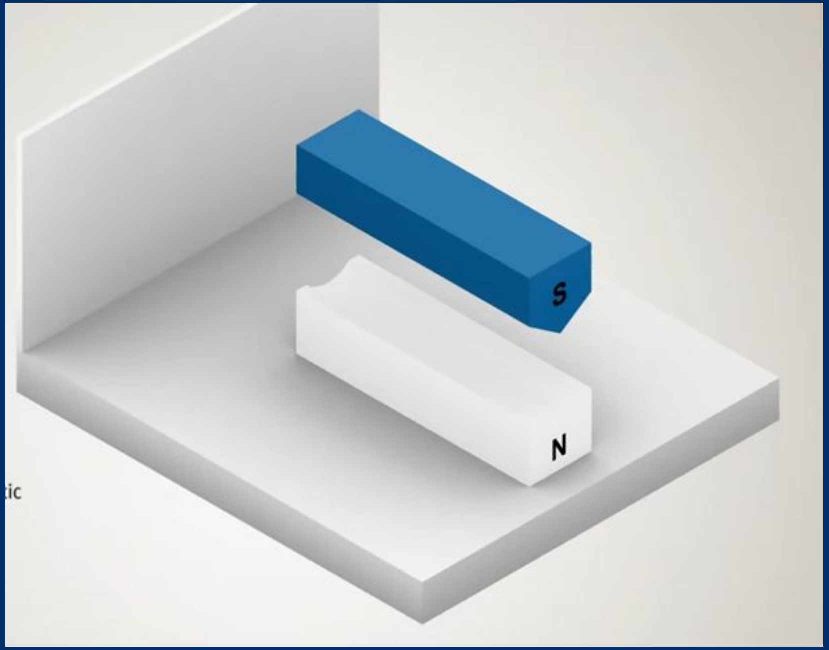
Qu"trit": $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma\,|2\rangle$

Why do we use two level systems for QC?

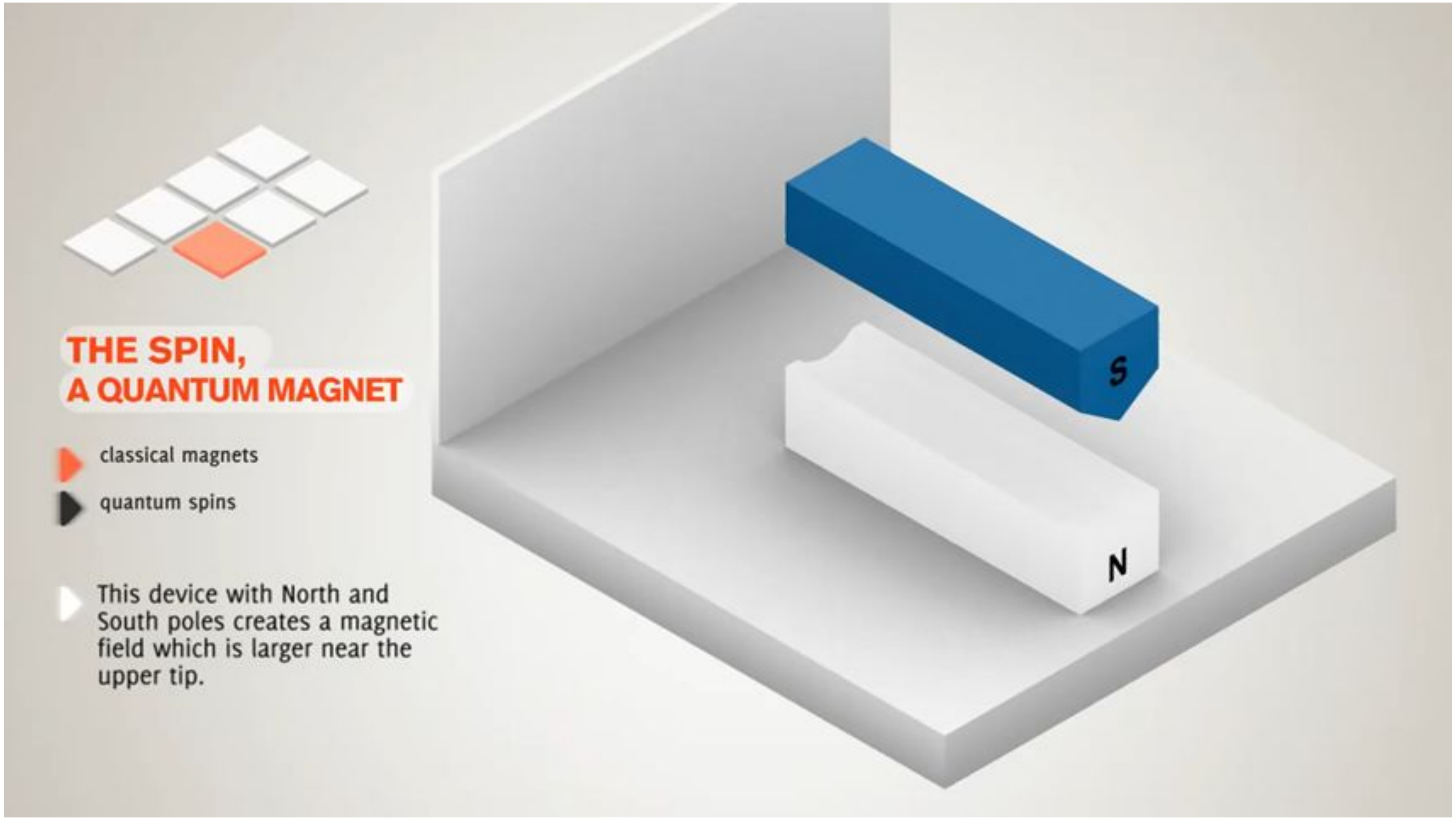
- We can map one level to 0 and another level to 1
- They are easier to control
- Simple linear algebraic properties
- Have been studied for nearly a century

How do we know electron spin is a two-level system?

The Stern-Gerlach experiment

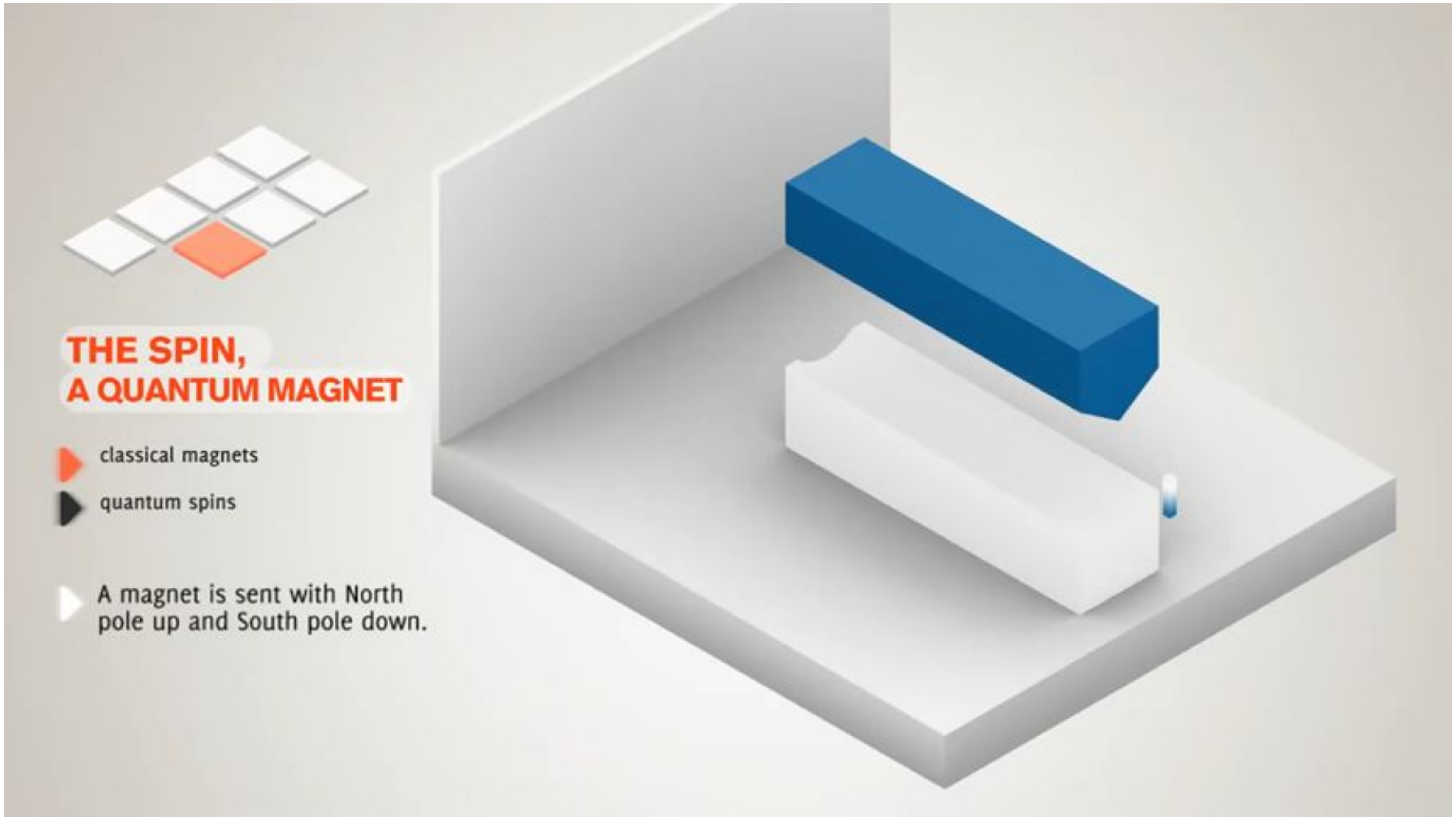


The Stern-Gerlach Experiment



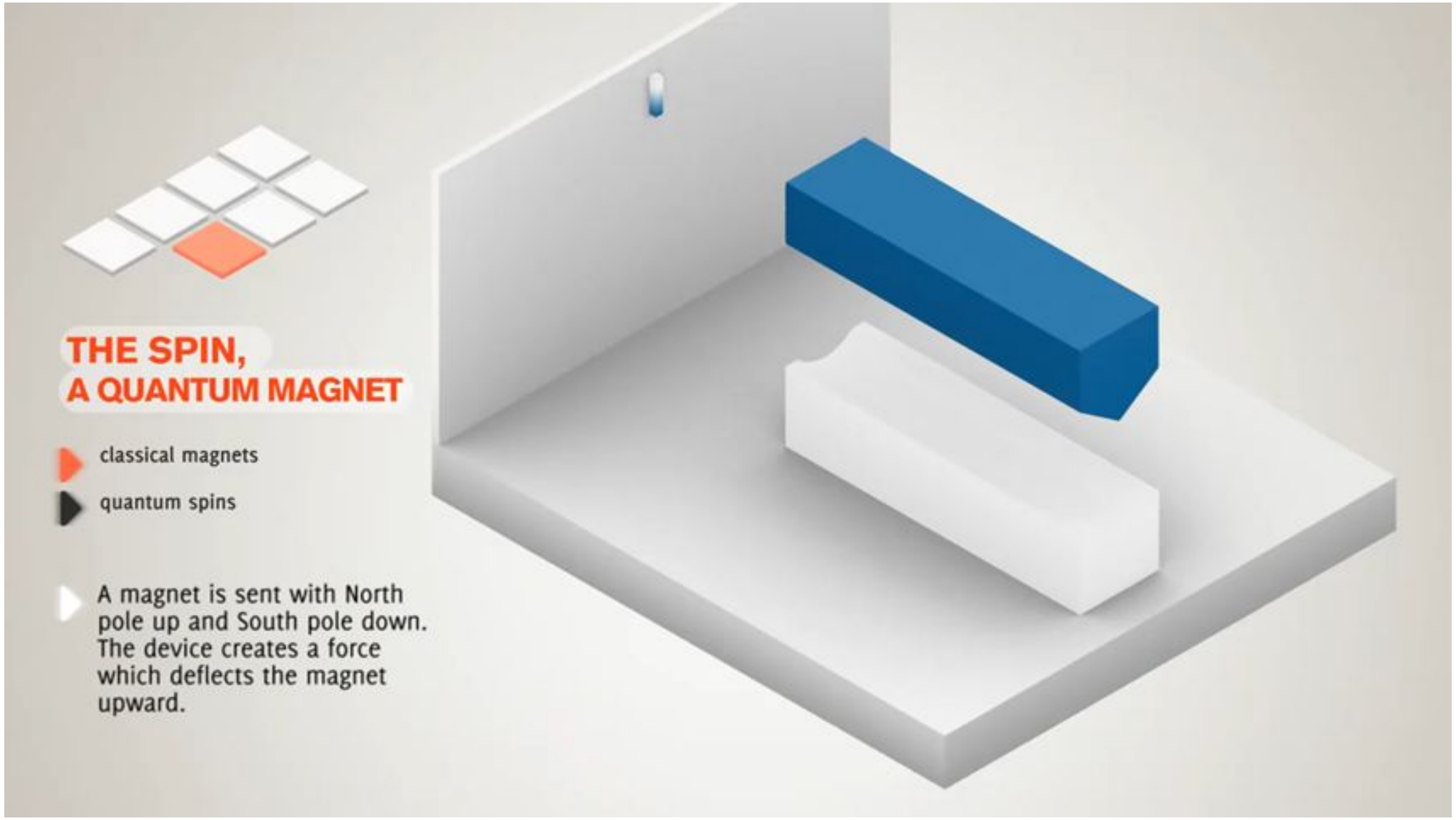
The Stern-Gerlach Experiment

The Stern-Gerlach Experiment

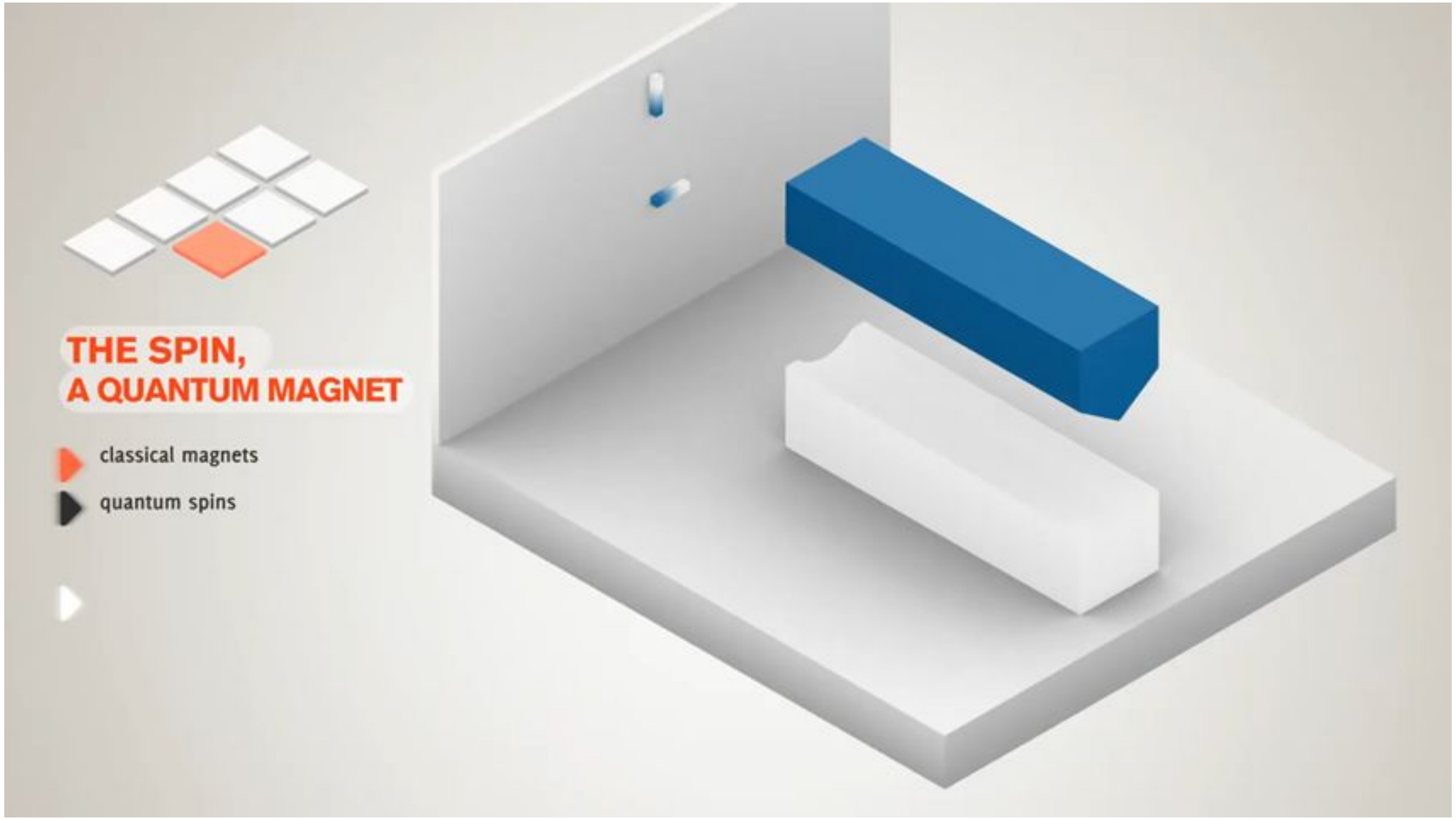


Let's start by putting a regular magnet through the device.

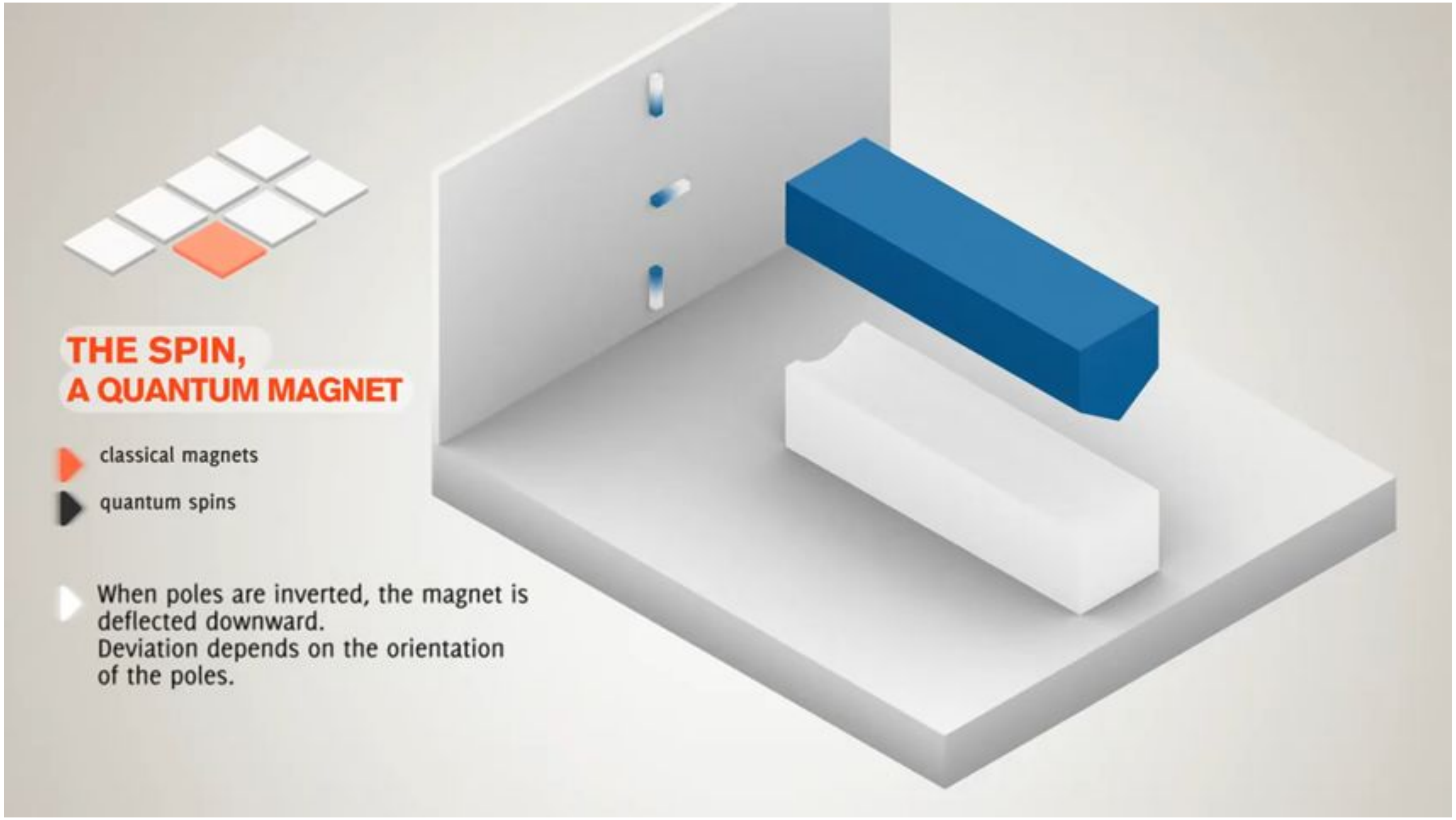
The Stern-Gerlach Experiment



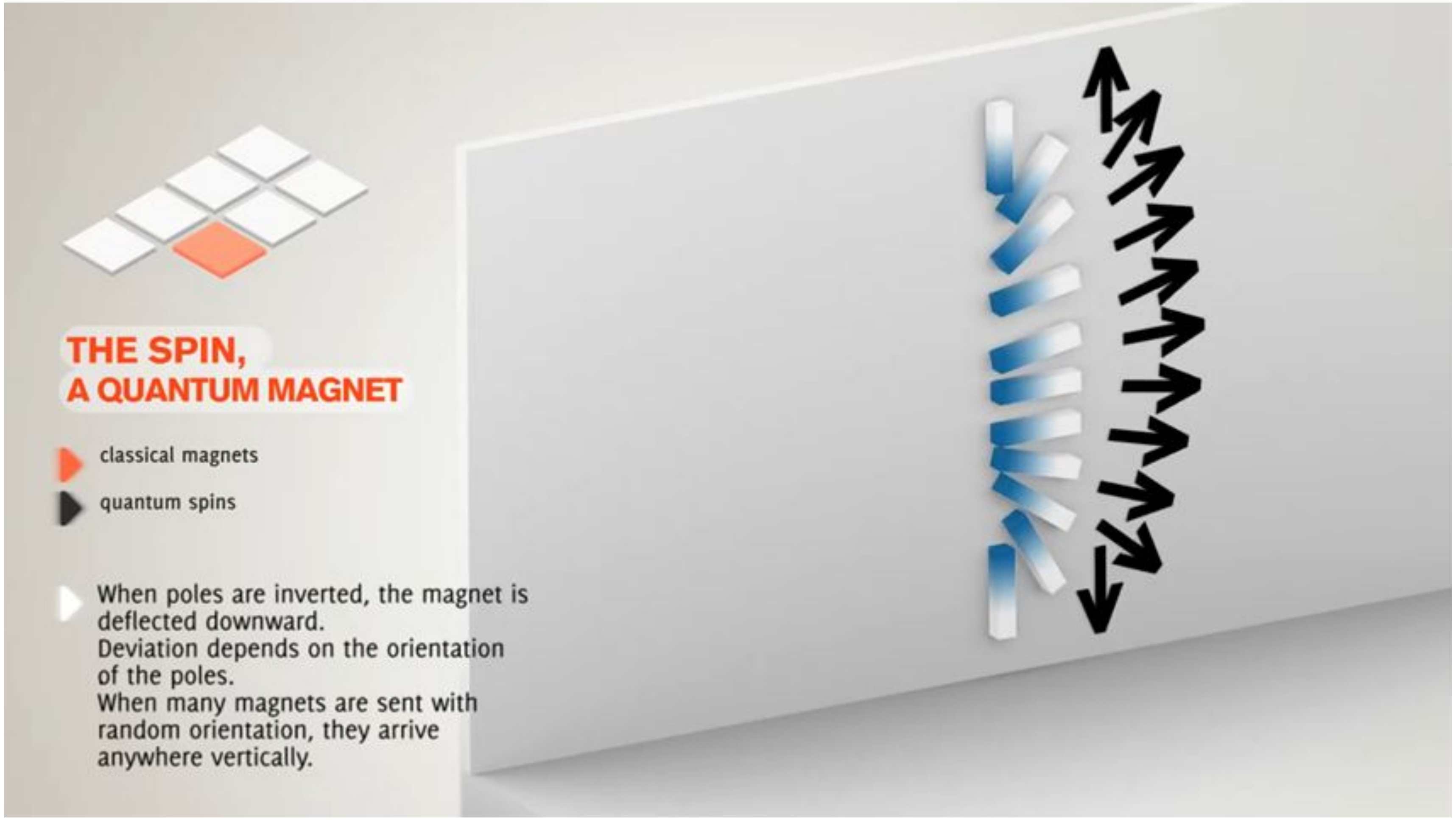
The Stern-Gerlach Experiment



The Stern-Gerlach Experiment

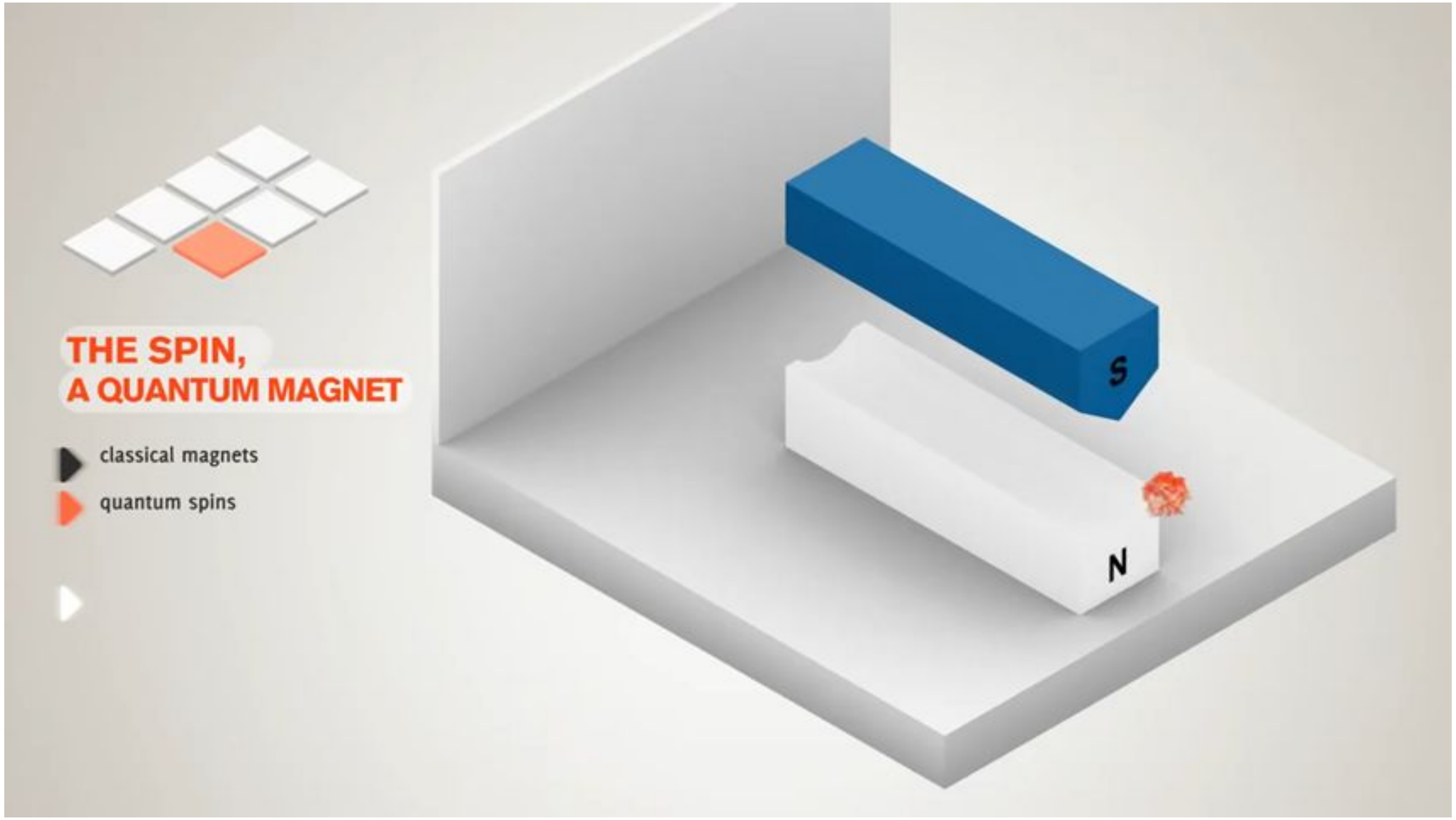


The Stern-Gerlach Experiment



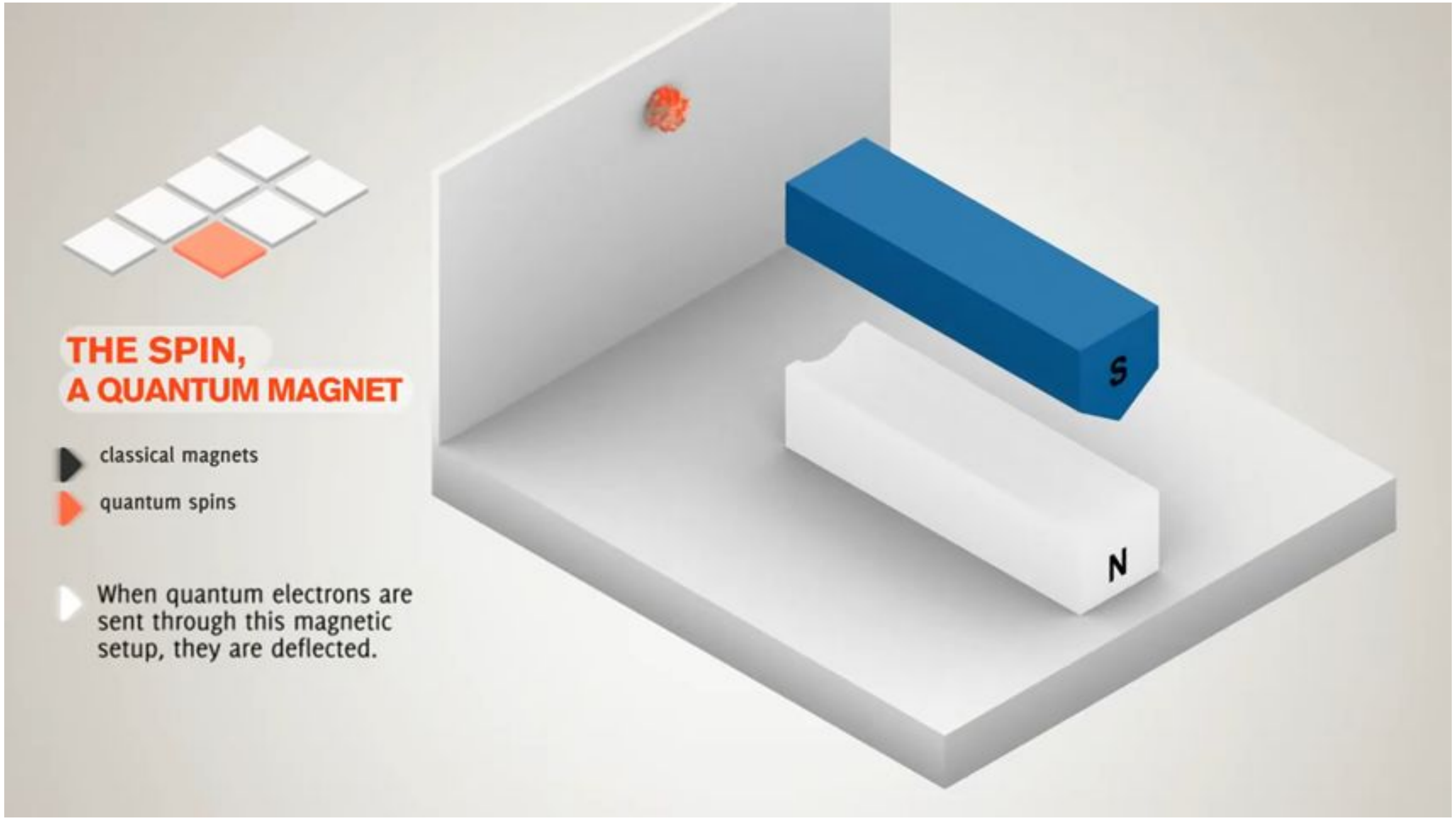
We end up with a **continuum** of positions. The magnets can appear at the top, bottom, or anywhere in between.

The Stern-Gerlach Experiment

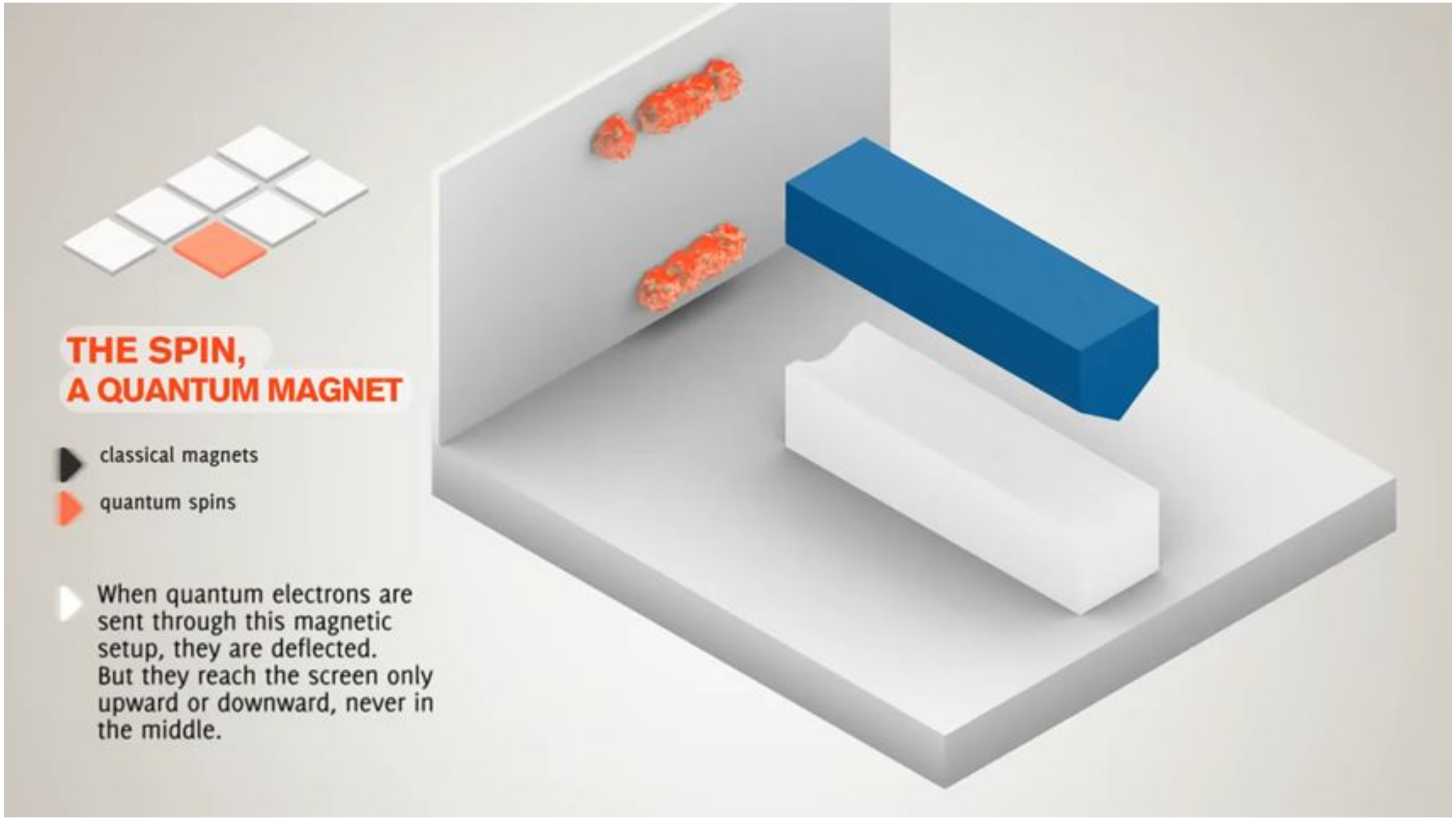


What happens when we use an electron instead of a magnet?

The Stern-Gerlach Experiment

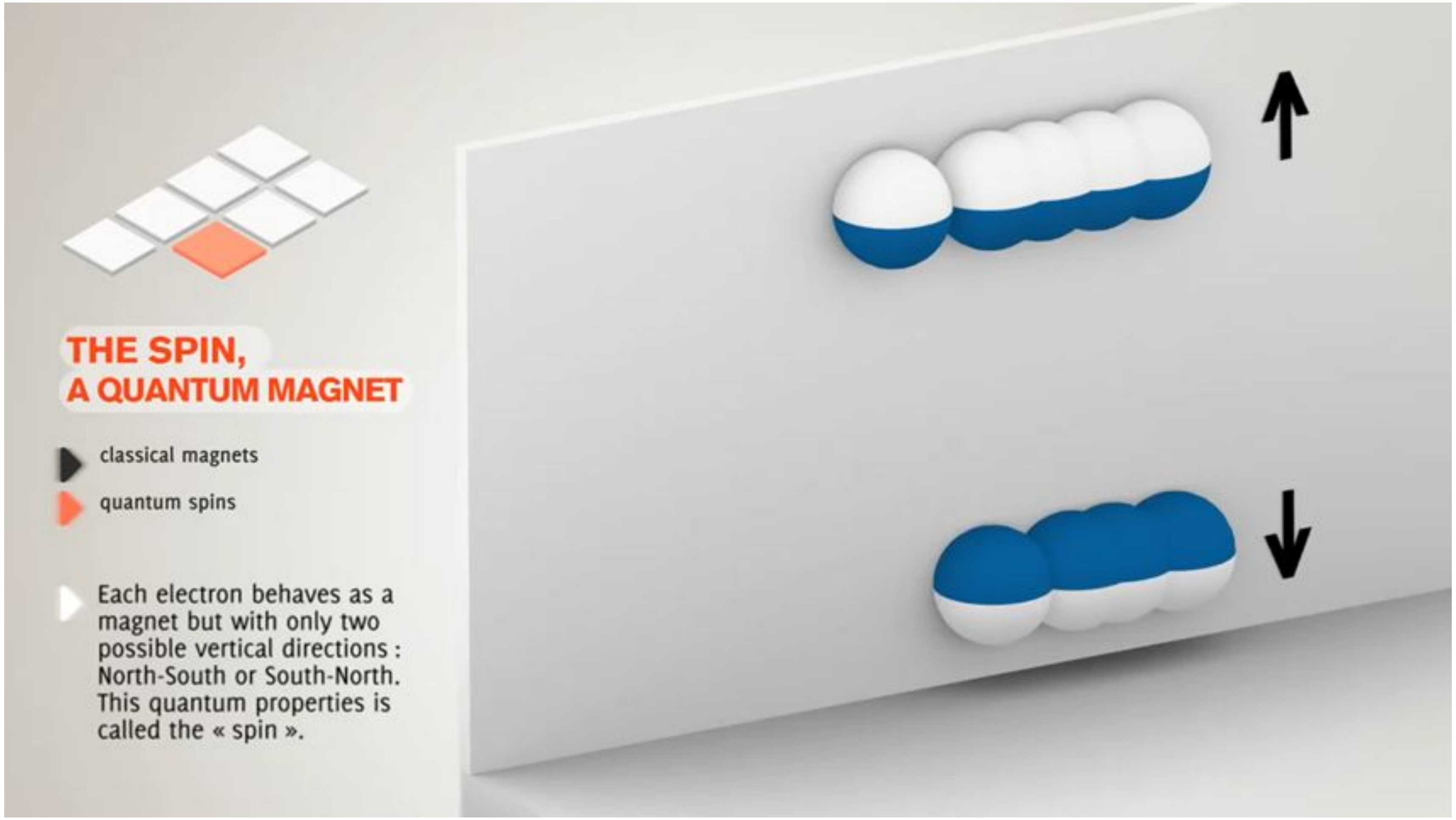


The Stern-Gerlach Experiment



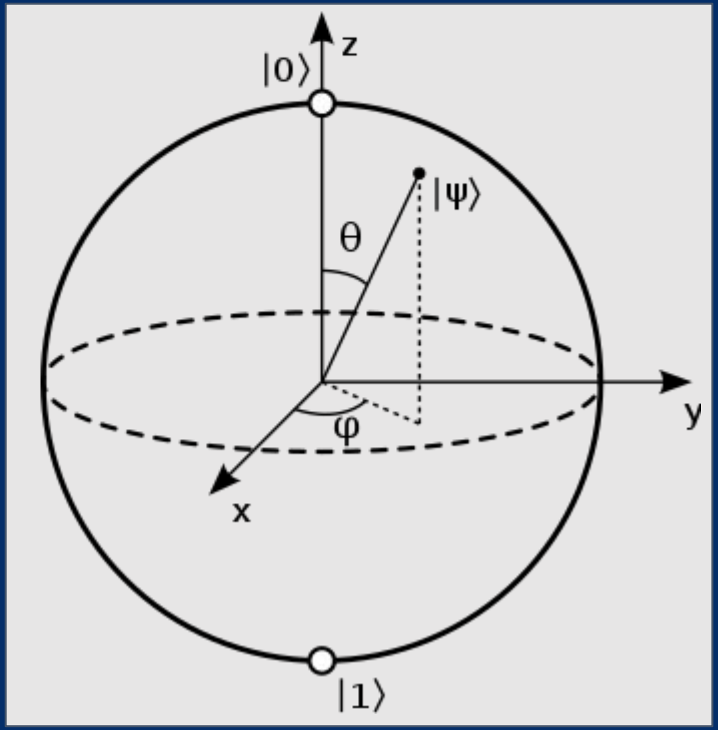
When we used magnets, they could show a continuum of deflections, but electrons can only be deflected to two different levels!

The Stern-Gerlach Experiment



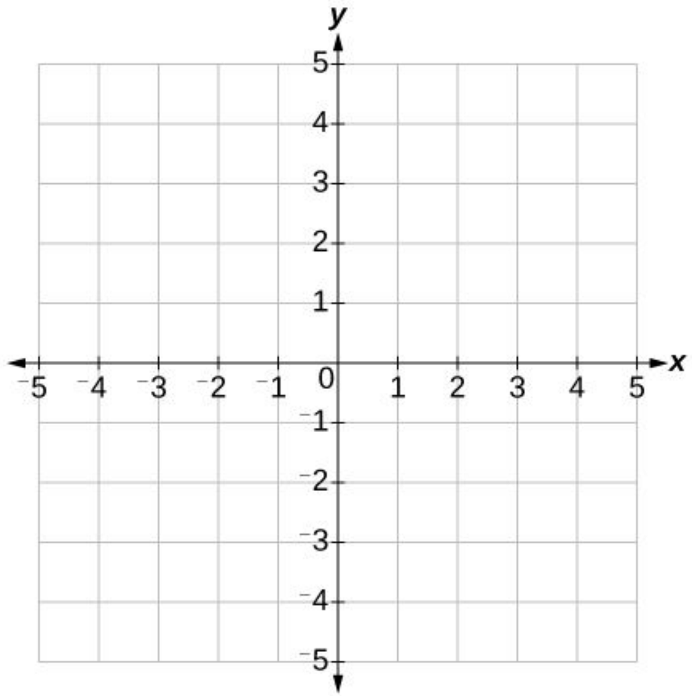
This means electrons can be either “spin up” or “spin down” when they are measured, and nowhere in between- they are two-level systems!

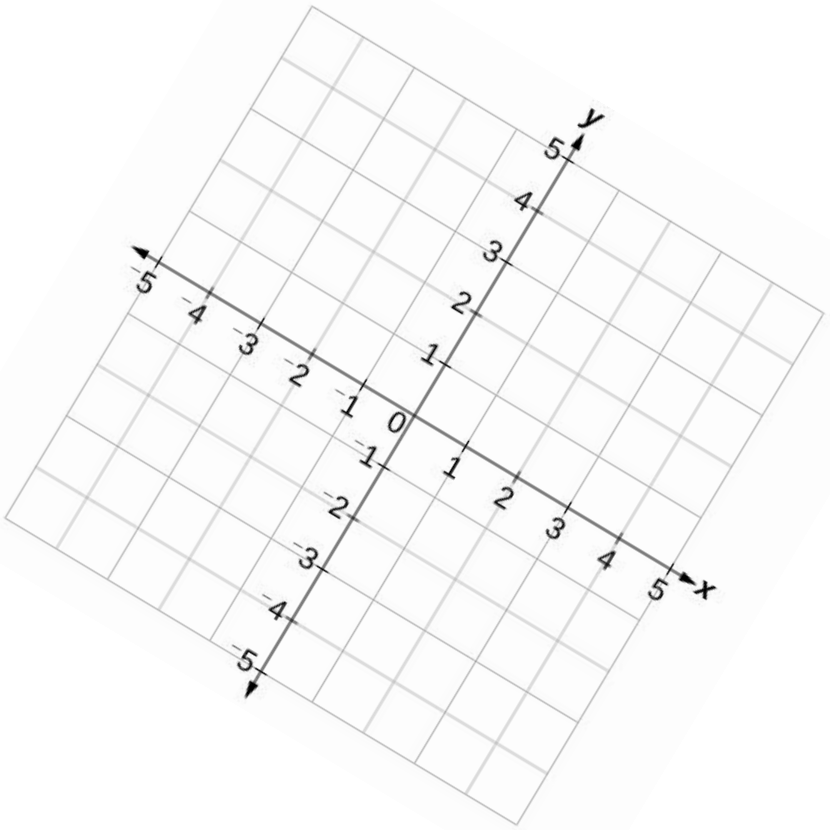
What does measurement of electron spin look like in different bases?



Review - what is a basis?

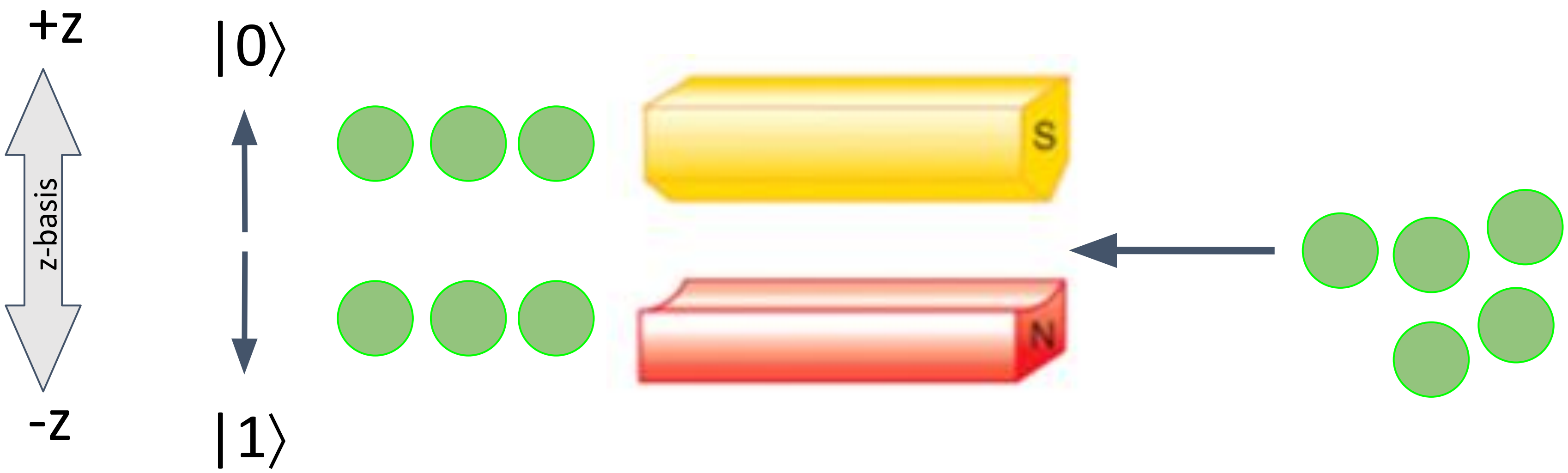
It's the system you are measuring in.





The Stern-Gerlach Experiment

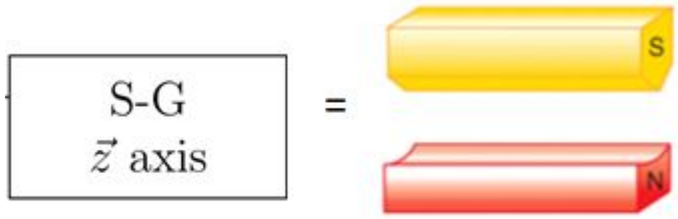
- Measurement in the z-basis:



This is pretty straightforward because the electrons line up along the z direction.

Measurement in z-basis

- Measurement in the z-basis:



Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

measurement: collapses the quantum state of the qubit $|\psi\rangle$ to either $|0\rangle$ or $|1\rangle$

probability of measuring $|0\rangle$: $|\alpha|^2$

probability of measuring $|1\rangle$: $|\beta|^2$

Measurement of a quantum system collapses it to one state

Before measurement: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

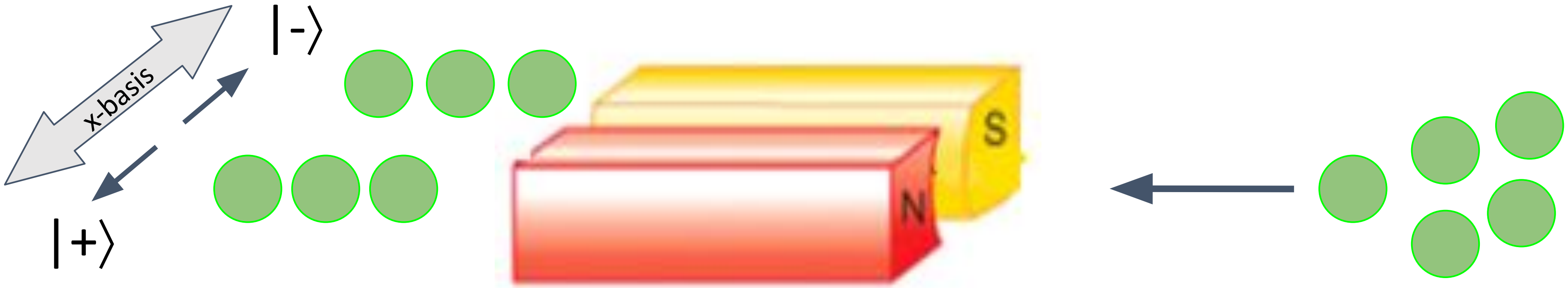
After measurement: $|\psi\rangle = \alpha|0\rangle$

OR

$|\psi\rangle = \beta|1\rangle$

Measurement in other bases

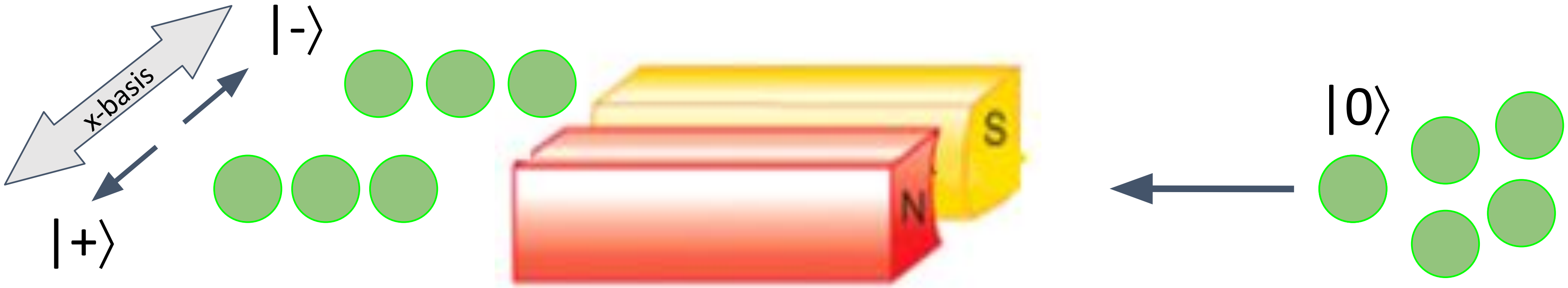
We could have aligned the experiment like this:



Now, the electrons would go to the two different sides instead of up and down.
How do we represent the + and - states?

Measurement in other bases

We could have aligned the experiment like this:



If the incoming electrons were all in the |0⟩ (up) state, then we will observe that half of them will go to |+⟩ and half will go to |-⟩.

Measurement in other bases

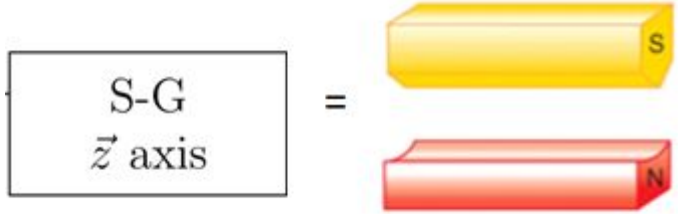
We could have aligned the experiment like this:

If the incoming electrons were all in the |-⟩ (up) state, then we also observe that half of them will go to |+⟩ and half will go to |-⟩.

So we can represent|+⟩ and |-⟩ as combinations of |0⟩ and |1⟩.

The Stern-Gerlach Experiment

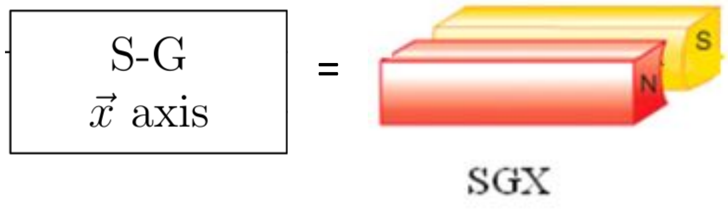
Stern-Gerlach in multiple basis:



SGX

Measurement in x-basis

- Measurement in the x-basis:



Qubit: $|\psi\rangle = |0\rangle$

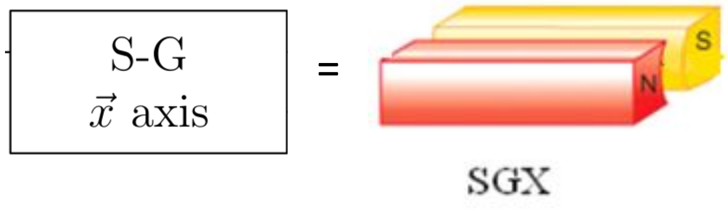
measurement in x basis: collapses the quantum state of the qubit $|\psi\rangle$ to either $|+\rangle$ or $|-\rangle$

$|+\rangle : (|0\rangle+|1\rangle)/\sqrt{2}$

$|-\rangle : (|0\rangle-|1\rangle)/\sqrt{2}$

Measurement in x-basis

- Measurement in the x-basis:



Qubit: $|\psi\rangle = |0\rangle$

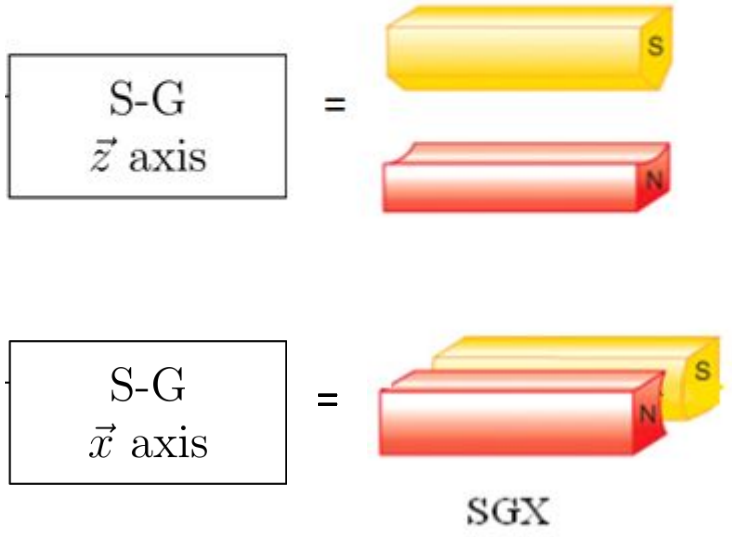
measurement in x basis: collapses the quantum state of the qubit $|\psi\rangle$ to either $|+\rangle$ or $|-\rangle$

probability of measuring $|+\rangle$:

probability of measuring $|-\rangle$:

The Stern-Gerlach Experiment

- Revisiting Stern-Gerlach in multiple basis:



Classical analogy

Let's assume an object has two properties: ***Color*** and ***Shape***

Color: red or blue

Shape: square or circle

So far...

- We know what two-level systems are and why we use them
- We know electron spin is a two level system
- We know how to measure electron spin in different bases

Next...

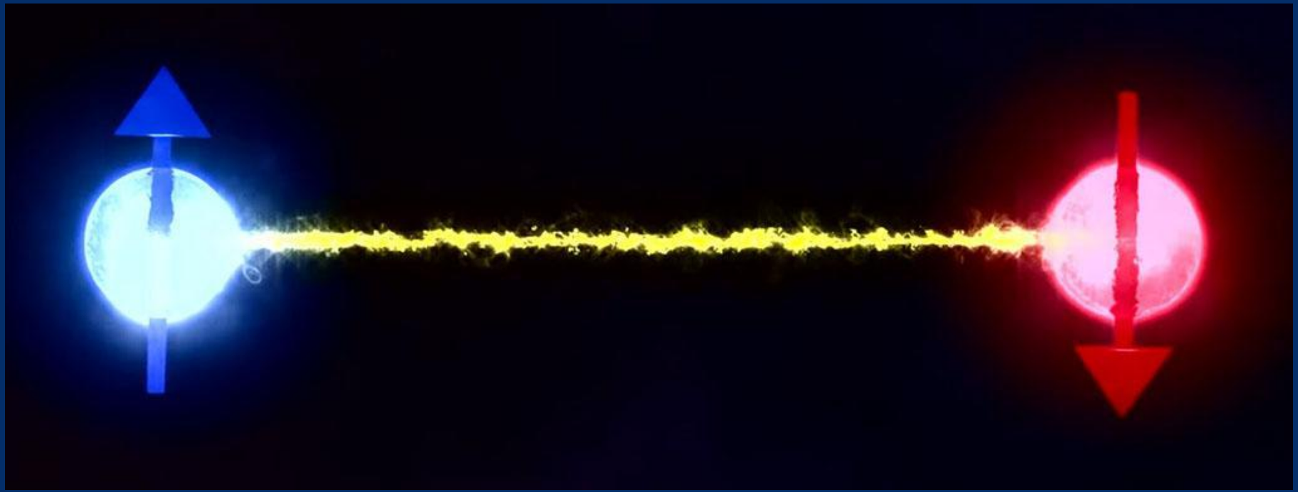
- We will see an interesting property that arises out of two-level systems

10 MIN BREAK!

What can we do with two-level systems?

Quantum Entanglement

Quantum correlation between two (or more) objects where each of their states will depend on the state of the other



Entanglement

Entanglement

Entangled state: $|\psi\rangle = \sqrt{0.5}|01\rangle + \sqrt{0.5}|10\rangle$

what if we only measure qubit A?

- If qubit A is 0 $\rightarrow$ the quantum state of qubit B is immediately set to $|1\rangle$
- If qubit A is 1 $\rightarrow$ the quantum state of qubit B is immediately set to $|0\rangle$

Entanglement

Entanglement is preserved under local operations:

Bell states

There are different ways we can arrive at an entangled state.

$$|\square_{00}\rangle = \sqrt{0.5}|00\rangle + \sqrt{0.5}|11\rangle$$

$$|\square_{01}\rangle = \sqrt{0.5}|01\rangle + \sqrt{0.5}|10\rangle$$

$$|\square_{10}\rangle = \sqrt{0.5}|00\rangle - \sqrt{0.5}|11\rangle$$

$$|\square_{11}\rangle = \sqrt{0.5}|01\rangle - \sqrt{0.5}|10\rangle$$

Applications of Entanglement

- Quantum Teleportation
- Quantum Cryptography
- Superdense Coding
- Quantum speedups

Quantum teleportation

Quantum speedups

ANNOUNCEMENTS