



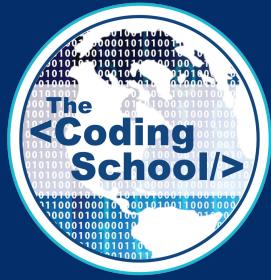
INTRO TO QUANTUM COMPUTING

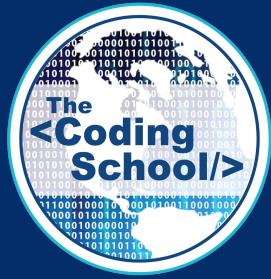
LECTURE #9

MATHEMATICS FOR QUANTUM PT. 3

FRANCISCA VASCONCELOS

12/13/2020





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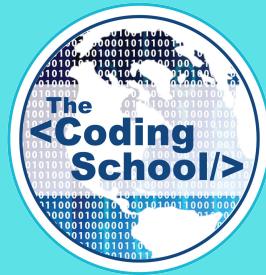
TODAY'S LECTURE

1. Mathematics for Quantum Pt. 3

- a) Vector Spaces
 - a) Properties of Vector Spaces
 - b) Linear Independence
 - c) Span & Bases
 - d) Hilbert Spaces
- b) Eigenvectors & Eigenvalues
 - a) Geometric Interpretation
 - b) Canonical Equation
 - c) Pauli Matrix Eigenvectors/values
 - d) Quantum Measurement

2. Semester 2 Preview! [Get Hyped]

- a) Quantum Mechanics
- b) Quantum Information
- c) Quantum Algorithms
- d) Experimental Quantum Info. Processing
- e) Quantum Applications & Research



MATHEMATICS FOR QUANTUM PT. 3

VECTOR SPACES

VECTOR SPACES

A **vector space** is a collection of vectors which can be added and multiplied, adhering to the following axioms:

$$1. \vec{x} + \vec{y} = \vec{y} + \vec{x}$$

$$2. \vec{x} + (\vec{y} + \vec{z}) = (\vec{x} + \vec{y}) + \vec{z}$$

For all
 $\vec{x}, \vec{y}, \vec{z} \in \mathcal{V}$

3. There is a unique zero vector ($\vec{0}$), such that $\vec{x} + \vec{0} = \vec{x}$

4. For each \vec{x} there is a unique $-\vec{x}$, such that $\vec{x} + (-\vec{x}) = \vec{0}$

(where \mathcal{V} is a vector space)
5. $1\vec{x} = \vec{x}$

$a, b \in \mathbb{C}$
6. $(ab)\vec{x} = a(b\vec{x})$

7. $a(\vec{x} + \vec{y}) = a\vec{x} + b\vec{x}$

8. $(a + b)\vec{x} = a\vec{x} + b\vec{x}$



EXAMPLES

LINEAR INDEPENDENCE

Given a set of vectors $(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k)$ we look at all their possible ***linear combinations*** ($c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k$):

If $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k = \vec{0}$ only when $c_1 = c_2 = \dots = c_k = 0$, then $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k$ are ***linearly independent***.

[Note: orthogonal vectors are linearly independent!!]



If $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k = \vec{0}$ when any c is non-zero, then $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k$ are ***linearly dependent***.

QUANTUM PRACTICE TIME!

State whether the following vectors are linearly dependent or linearly independent.

$$(1) \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -3 \\ -6 \end{pmatrix}$$

$$(2) \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3i \end{pmatrix}$$

$$(3) \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

QUANTUM PRACTICE SOLUTIONS

State whether the following vectors are linearly dependent or linearly independent.

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$$(2) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3i \end{pmatrix}$$

$$(3) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

SPAN

If a vector space \mathcal{V} consists of all linear combinations of $\vec{w}_1, \vec{w}_2, \dots, \vec{w}_k$, then these vectors **span** \mathcal{V} .

In other words, if every vector $\vec{v} \in \mathcal{V}$ can be expressed as $\vec{v} = c_1\vec{w}_1 + c_2\vec{w}_2 + \dots + c_k\vec{w}_k$, then $\vec{w}_1, \vec{w}_2, \dots, \vec{w}_k$ span \mathcal{V} .

BASIS

A **basis** for the vector space \mathcal{V} is the set of vectors having the following two properties at once:

1. The vectors are **linearly independent** (not too many vectors)
2. The vectors **span** \mathcal{V} (not too few vectors)

Thus, every vector in the vector space \mathcal{V} is a **unique** linear combination of the basis vectors.

However, every vector space has **infinitely** many bases...

QUANTUM PRACTICE TIME

State whether the following vectors form a basis for their respective vector space.

(1) $\mathbb{R}^2:$ $\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

(2) $\mathbb{R}^3:$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(3) $\mathbb{R}^2:$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

QUANTUM PRACTICE SOLUTIONS

State whether the following vectors form a basis for their respective vector space.

(1) $\mathbb{R}^2:$ $\begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 7 \end{pmatrix}$

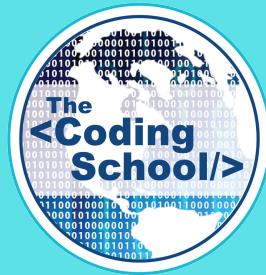
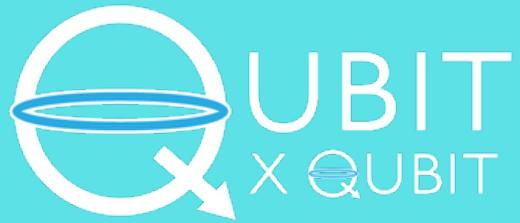
(2) $\mathbb{R}^3:$ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(3) $\mathbb{R}^2:$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

HILBERT SPACES

In quantum computing, you will often hear the term ***Hilbert space***. Although this sounds scary, it is simply a ***vector space equipped with an inner product!***

QUESTIONS?



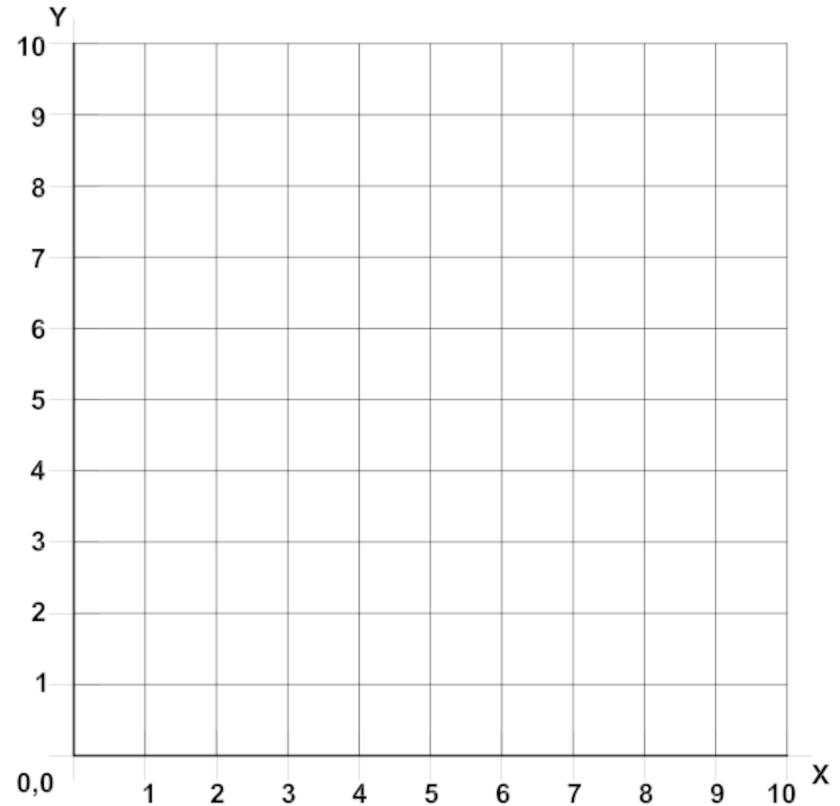
MATHEMATICS FOR QUANTUM PT. 3

EIGENVECTORS & EIGENVALUES

WHY EIGEN?

Typically, multiplying a matrix and vector results in a new vector, which is ***both scaled and rotated***.

Ex) $\begin{pmatrix} -1 & 10 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 8 \\ 1 \end{pmatrix}$



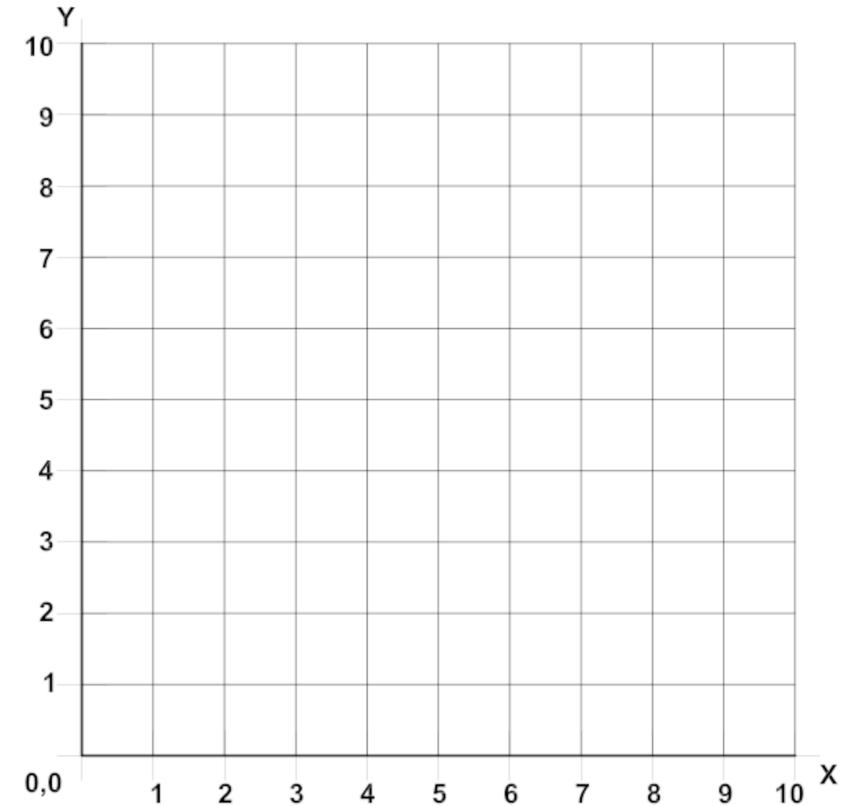
WHY EIGEN?

However, if a matrix is multiplied by its *eigenvector*, the resultant vector is **only scaled** (no rotation!).

Ex)

$$\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 6 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$



WHY EIGEN?

Now, why does this matter?

Our matrix-vector multiplication simplifies to a scalar-vector multiplication!

$$A\vec{v} = \lambda\vec{v}$$

In German, “eigen” means *proper*, *characteristic*, or *own*.

QUANTUM PRACTICE TIME!

State whether the following vectors are eigenvectors of the corresponding matrices. If so, what is the eigenvalue?

(1)

$$A: \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}, \vec{v}: \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(2)

$$A: \begin{pmatrix} 3 & 1 \\ 4 & -3 \end{pmatrix}, \vec{v}: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

QUANTUM PRACTICE SOLUTIONS

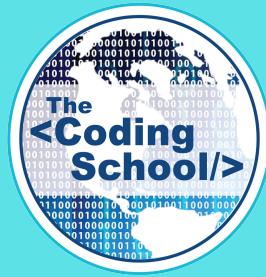
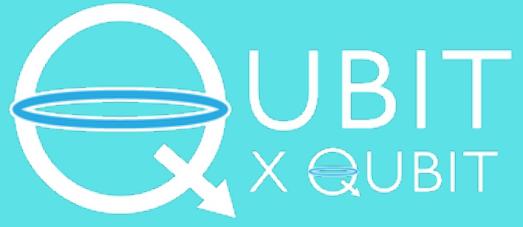
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(2)

$$A: \begin{pmatrix} 3 & 1 \\ 4 & -3 \end{pmatrix}, \vec{v}: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



BREAK TIME!

SOLVING FOR EIGENVECTORS & EIGENVALUES

$$A\vec{v} = \lambda\vec{v}$$

SOLVING FOR EIGENVECTORS & EIGENVALUES

Let's work through an example. Find the eigenvalues of $A = \begin{pmatrix} -6 & 3 \\ 4 & 5 \end{pmatrix}$.

SOLVING FOR EIGENVECTORS & EIGENVALUES

Now find the eigenvectors of $A = \begin{pmatrix} -6 & 3 \\ 4 & 5 \end{pmatrix}$.

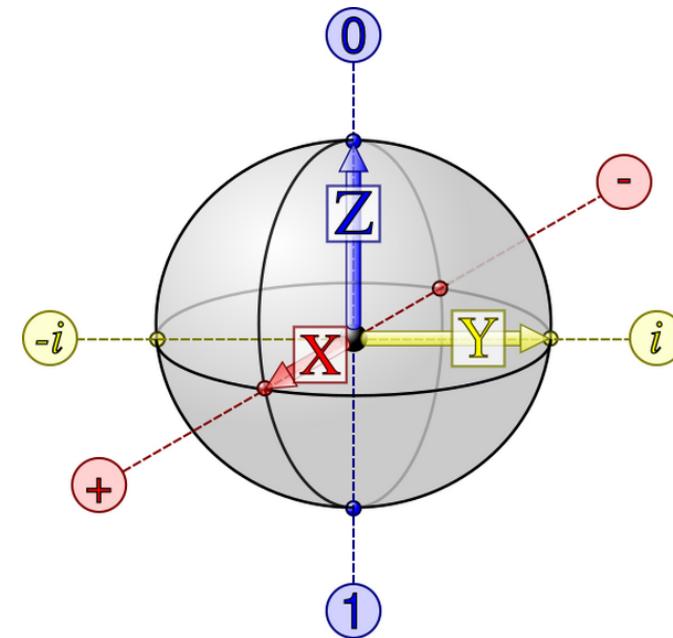
WHY EIGEN FOR QUANTUM?

In quantum computing, we are particularly concerned with the **eigenvectors** of the **Pauli matrices**, which are the quantum states along our **measurement axes** (X, Y, and Z).

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Let's see what the eigenvalues of the Pauli matrices are...

PAULI EIGENVECTORS & EIGENVALUES

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|\psi_{x+}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|\psi_{x-}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

PAULI EIGENVECTORS & EIGENVALUES

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$|\psi_{y+}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|\psi_{y-}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

PAULI EIGENVECTORS & EIGENVALUES

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\psi_{z+}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|\psi_{z-}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

POWER OF THE EIGENVECTORS

The eigenvectors of each Pauli matrix form a basis for \mathbb{C}^2 !!!



POWER OF THE EIGENVECTORS

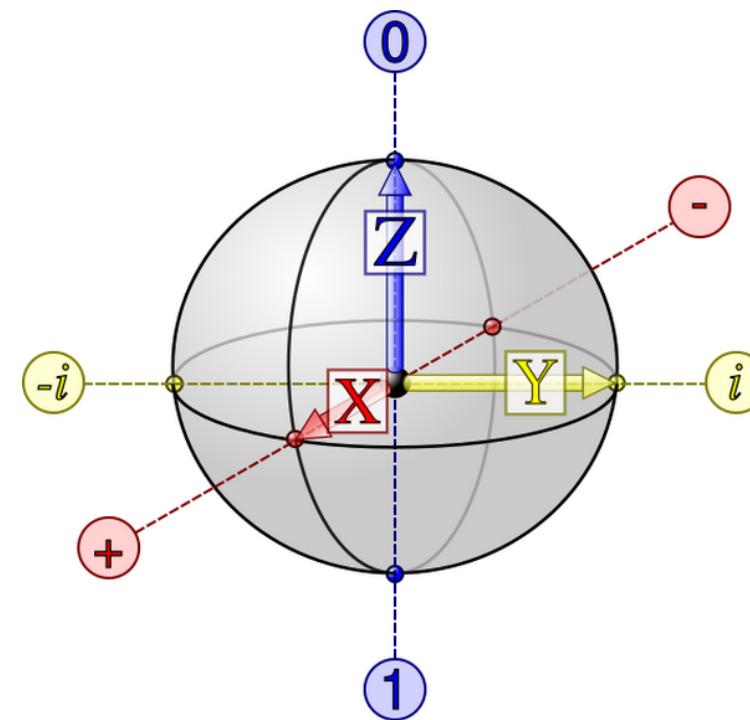
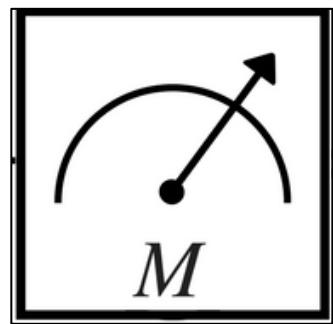
This means that any vector in \mathbb{C}^2 can be expressed as a linear combination of the eigenvectors of a Pauli matrix !!!

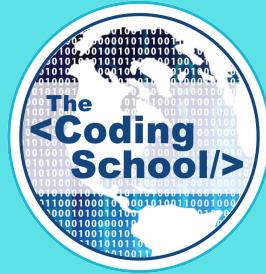
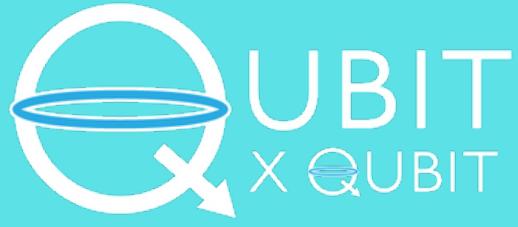
POWER OF THE EIGENVECTORS

This means that any vector in \mathbb{C}^2 can easily be multiplied by a Pauli matrix !!!

POWER OF THE EIGENVECTORS

Which is important for calculating probabilities of measurement along the different axes of the Bloch sphere...



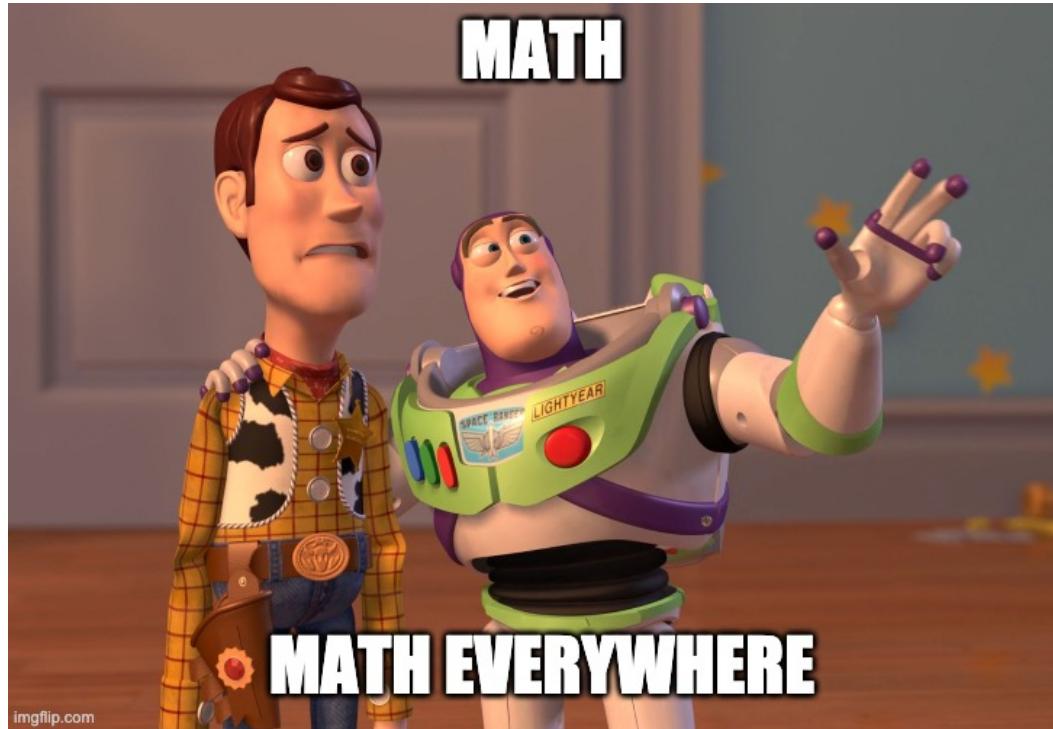


SECOND SEMESTER

QUANTUM HYPEEEEEEE

THIS SEMESTER

We've been building the foundations....



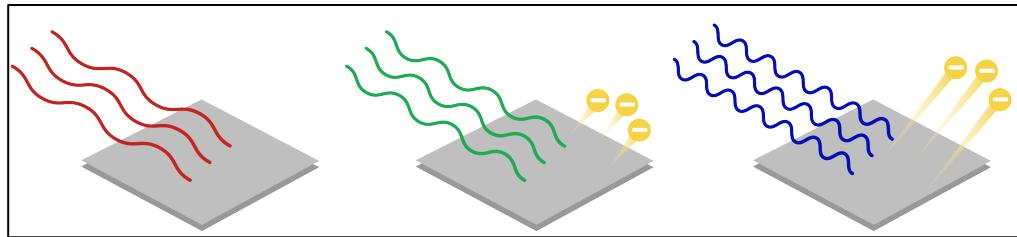
NEXT SEMESTER



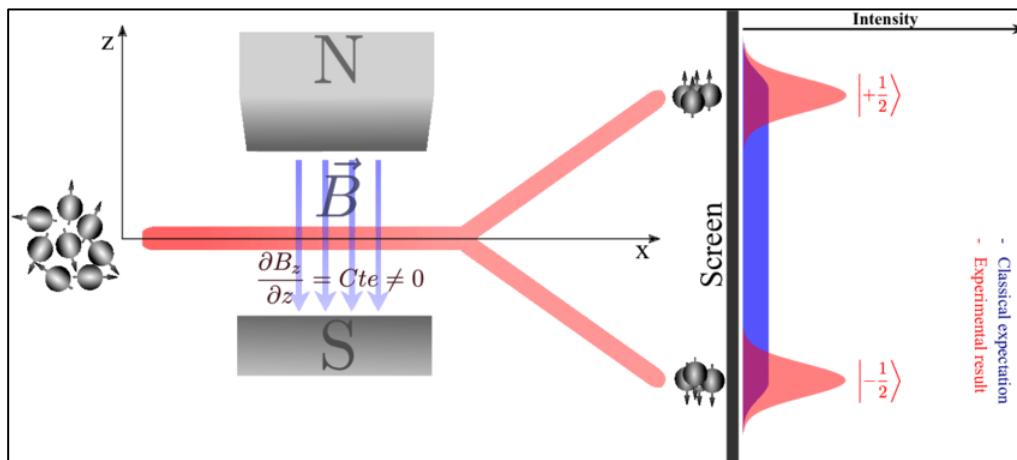
- a) Quantum Mechanics
- b) Quantum Information
- c) Quantum Protocols
- d) Quantum Algorithms
- e) Experimental Quantum Info. Processing
- f) Quantum Applications & Research

QUANTUM MECHANICS

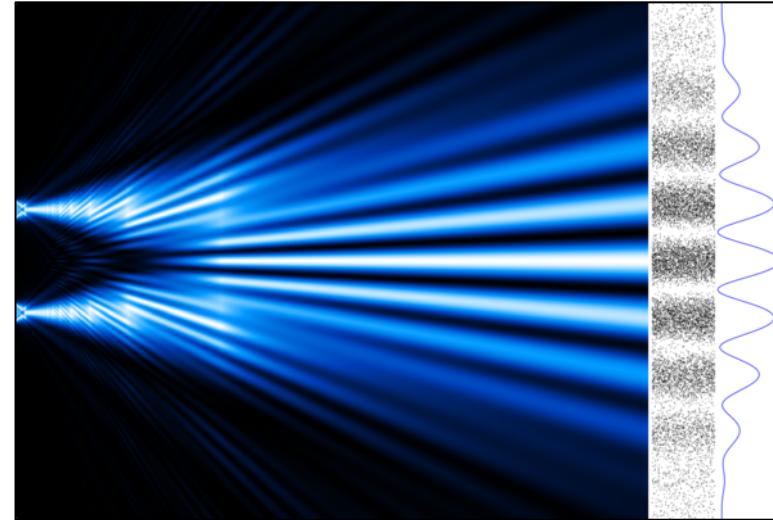
How does physics work at the microscopic scale?



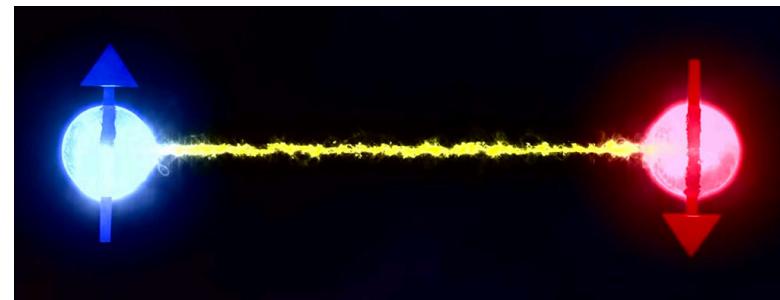
Photoelectric Effect



Stern Gerlach Experiment



Double Slit Experiment

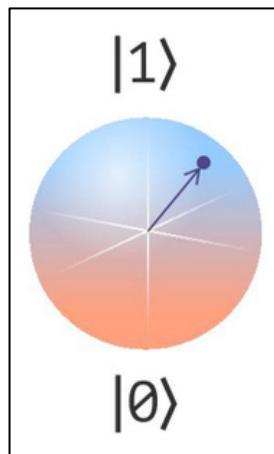


Quantum Entanglement

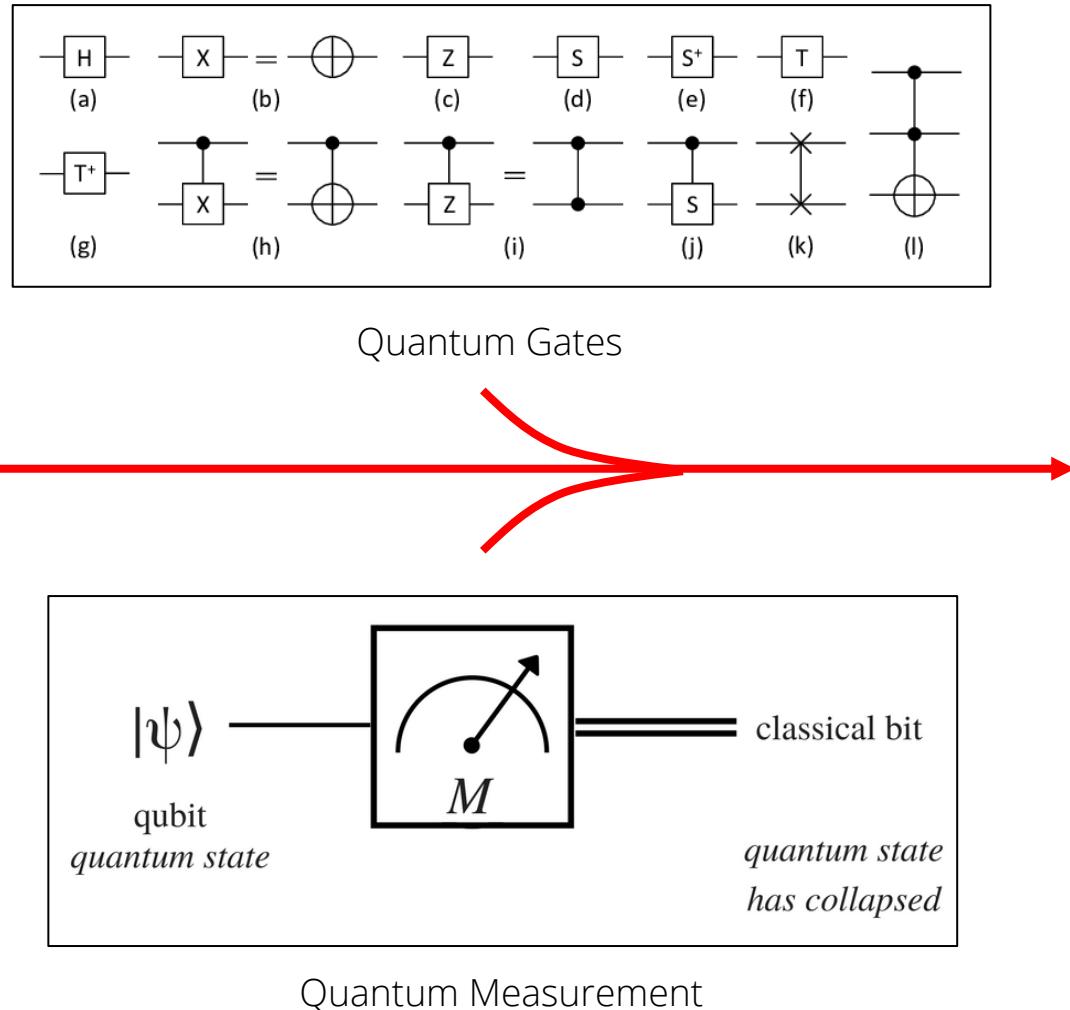
POSTULATES OF QUANTUM MECHANICS

QUANTUM INFORMATION

How can information be stored and manipulated in quantum systems?

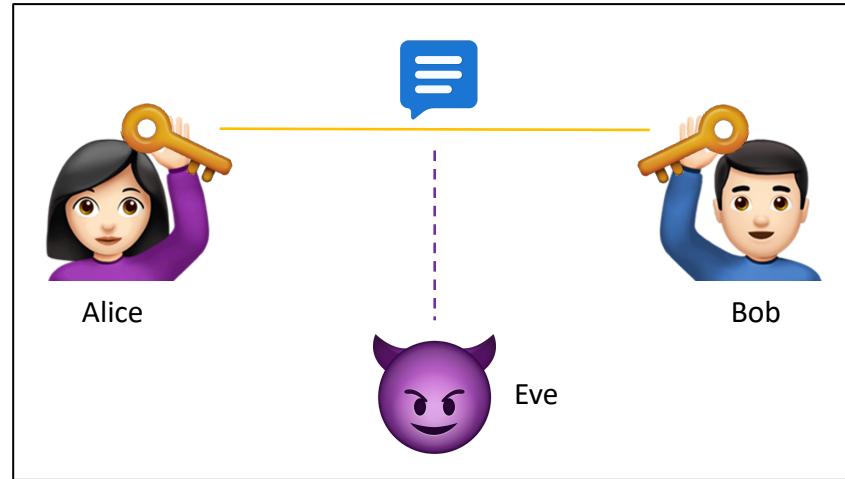


The Qubit &
Bloch Sphere

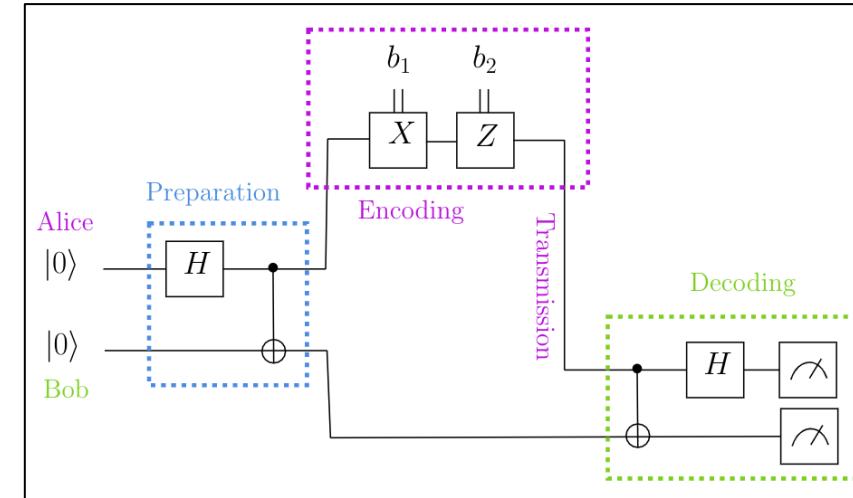


QUANTUM PROTOCOLS

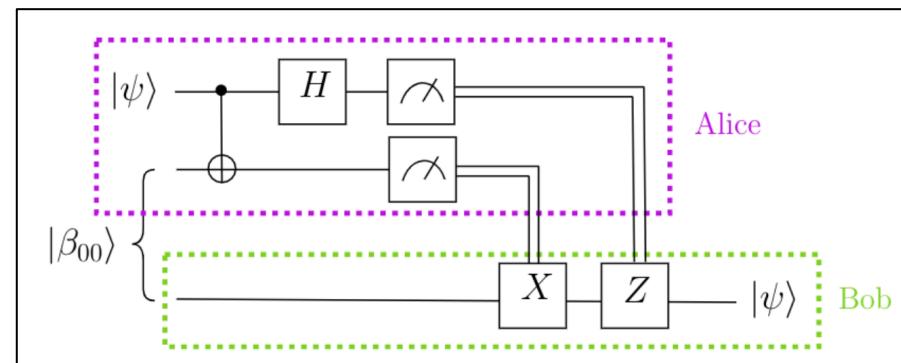
How can quantum systems be used to safely and efficiently transmit information?



BB84 Quantum Key Distribution



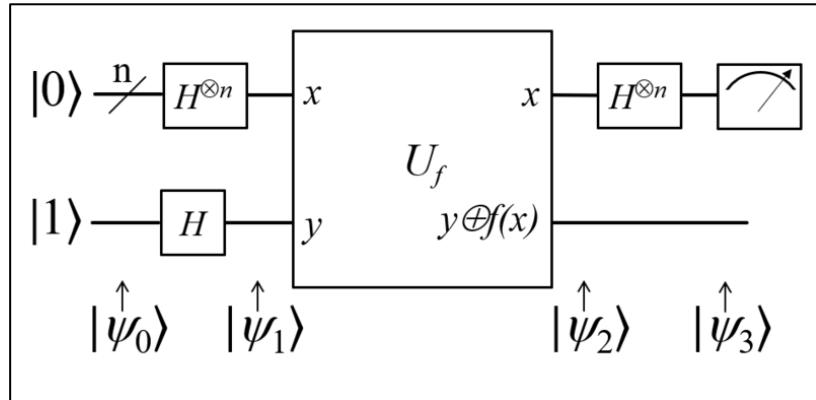
Superdense Coding



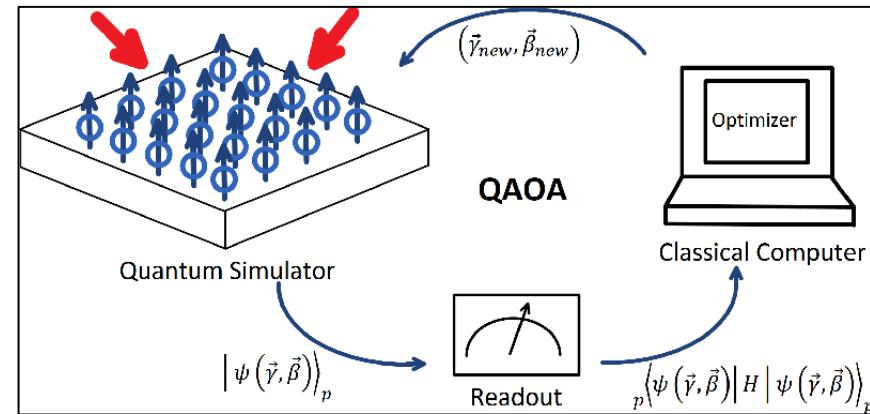
Quantum
Teleportation

QUANTUM ALGORITHMS

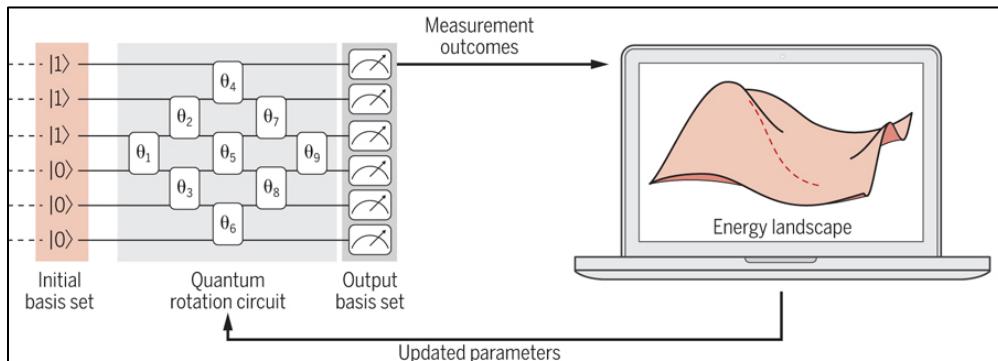
How can quantum systems provide an advantage to classical systems for computation?



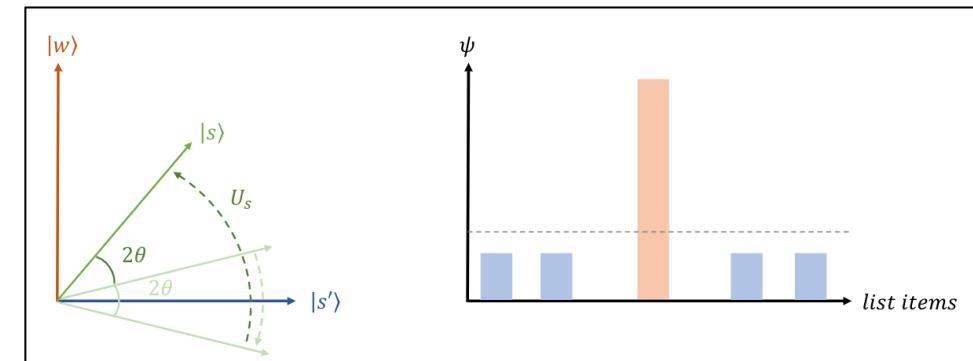
Deutsch-Josza Algorithm



Quantum Approximate Optimization Algorithm



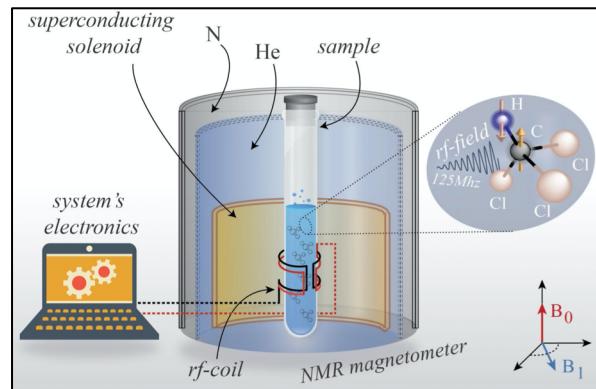
Variational Quantum Eigensolver



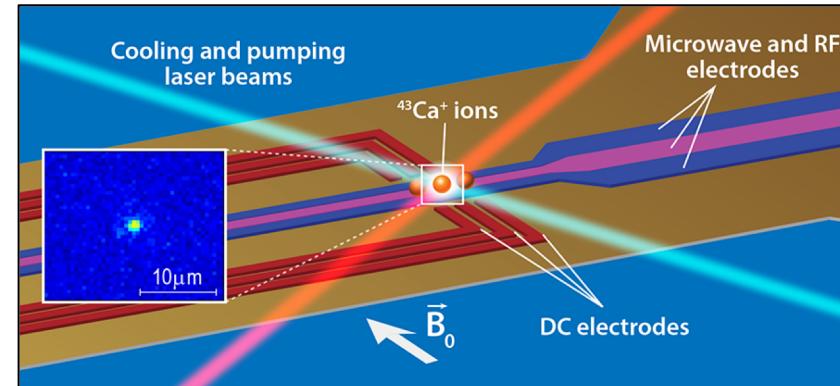
Grover's Algorithm

EXPERIMENTAL QUANTUM INFO. PROCESSING

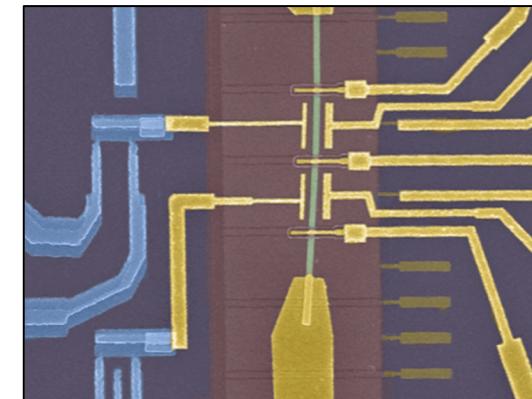
What physical platforms are we trying to build quantum computers with?



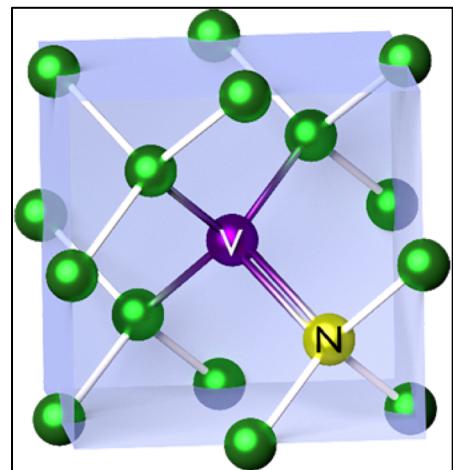
Nuclear Magnetic Resonance



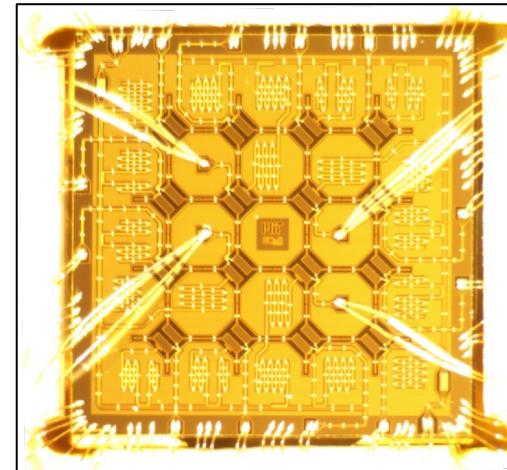
Trapped Ion



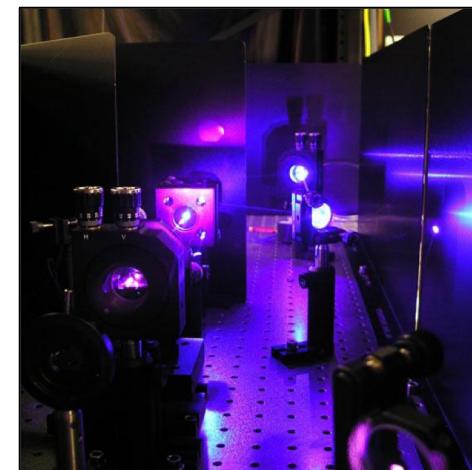
Majorana/Topological



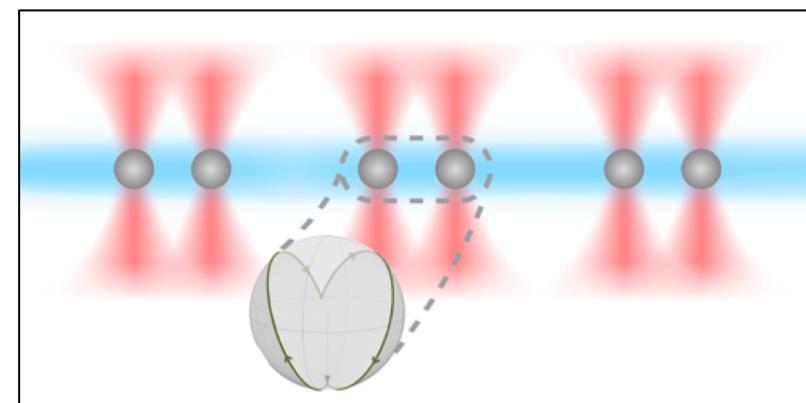
Diamond NV Centers



Superconducting



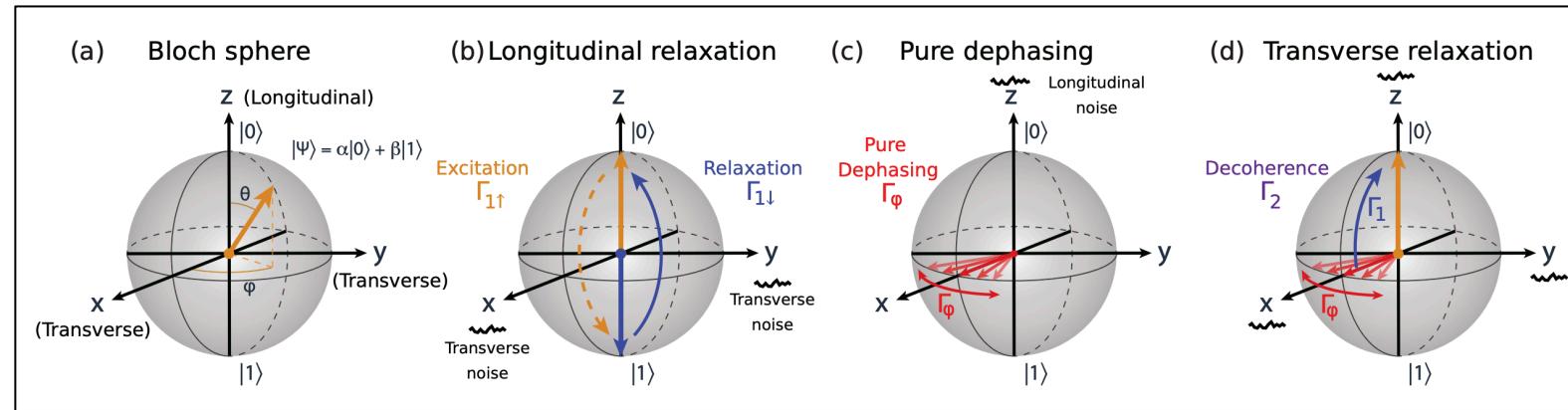
Photonics



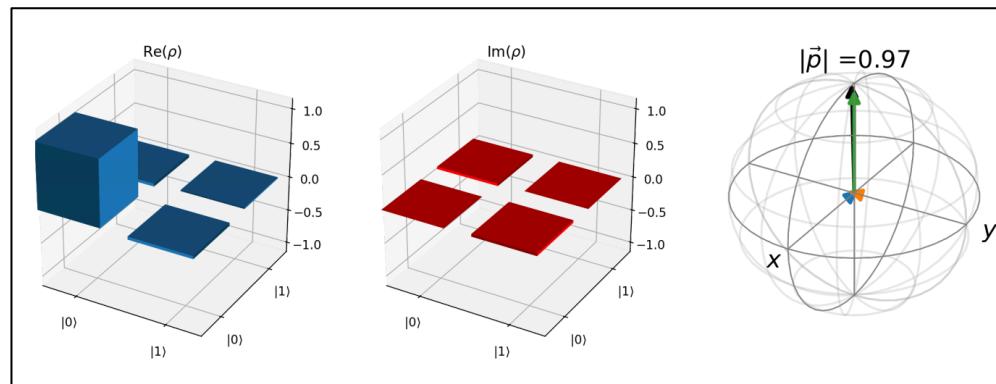
Neutral Atoms

EXPERIMENTAL QUANTUM INFO. PROCESSING

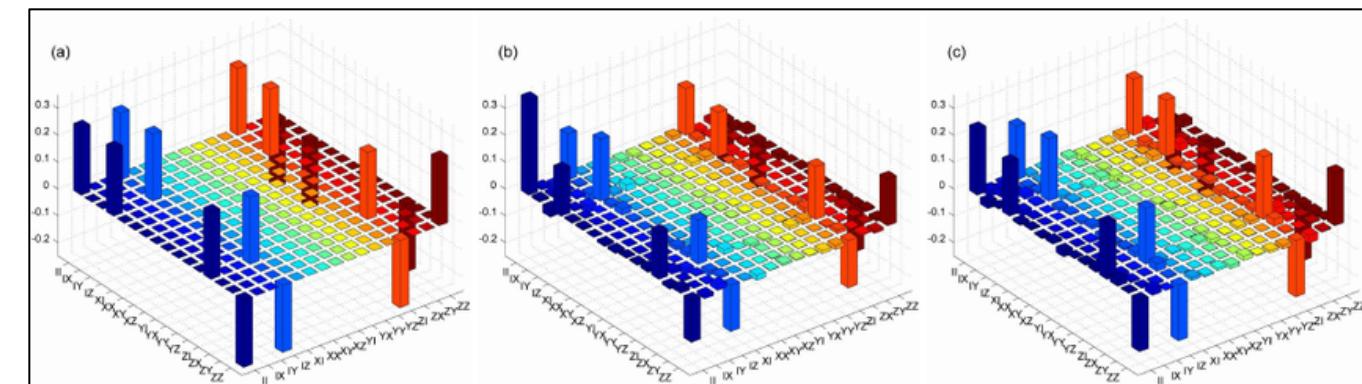
How can we assess the performance of qubits and quantum computing devices?



Measuring Qubit Lifetimes: T1 & T2



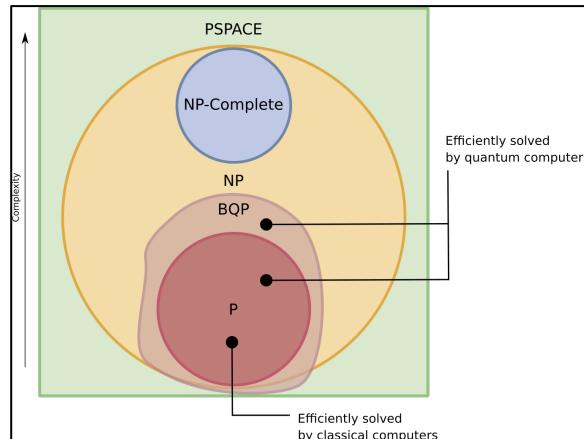
Assessing State Fidelity: Quantum State Tomography



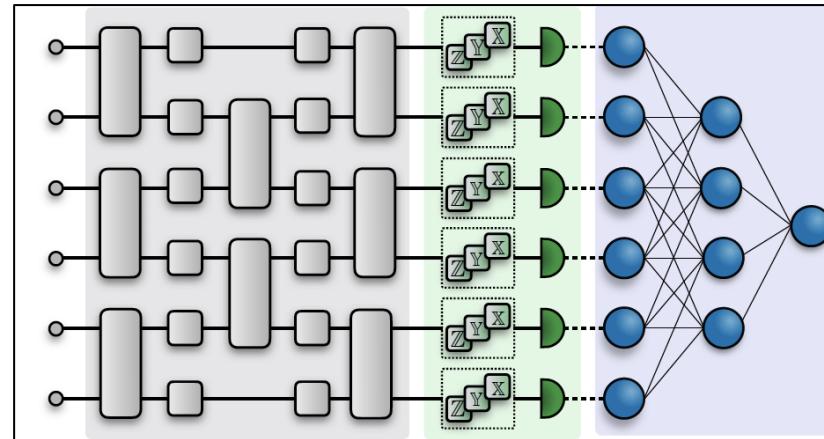
Assessing Gate Fidelity: Quantum Gate Tomography & Randomized Benchmarking

QUANTUM APPLICATIONS & RESEARCH

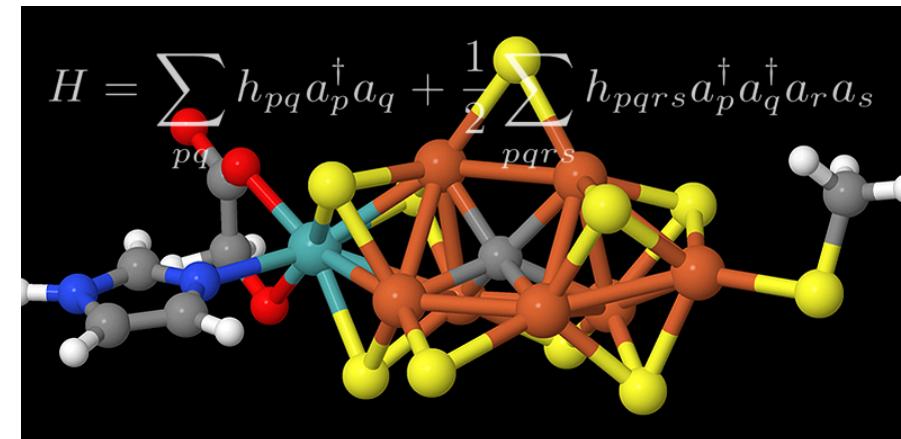
What problems are researchers working on now?



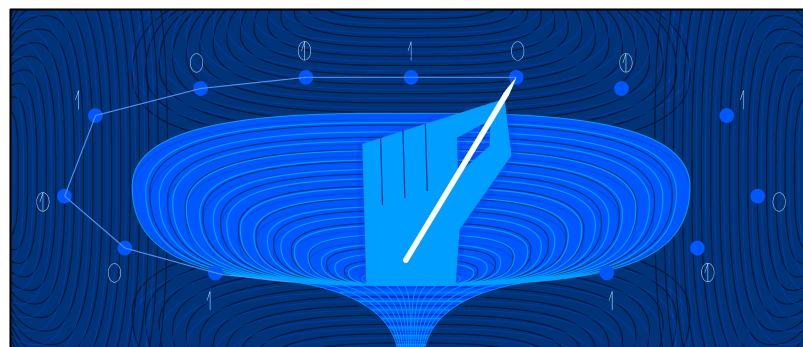
Quantum Complexity Theory



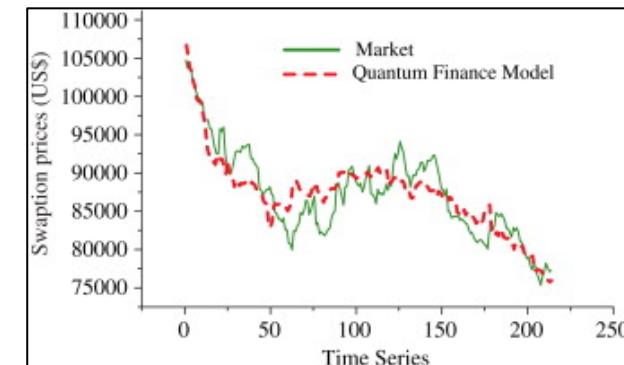
Quantum Machine Learning



Quantum Chemistry

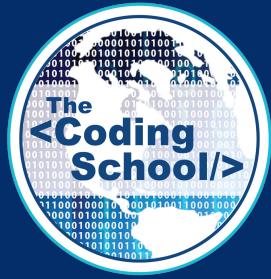


Quantum Gravity



Quantum Finance

and much more...



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Happy and healthy holidays!

To a quantum 2021!

<3 the QxQ course staff

