INTRO TO QUANTUM COMPUTING
LECTURE #11

QUANTUM MECHANICS 1

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ANNOUNCEMENTS

QUBIT
X QUBIT

The <Coding School/>

Happy Birthday Kiera!!

QUANTUM MECHANICS LECTURE SERIES

Lecture 1 - Principles of Quantum Mechanics

What is quantum and how do things behave on quantum length scales?

Lecture 2 - Quantum Two-level Systems and Measurement

Lecture 3 - Postulates of Quantum Mechanics

OBJECTIVES

1. Understand why we need to know the basics of quantum mechanics for quantum computing
2. Understand pre-quantum thinking about particles and waves
3. Understand how particles behave as waves and waves behave as particles
4. Understand how the path of an electron can be represented mathematically

WHAT DOES "QUANTUM" MEAN?

Quantum

Classical

WHAT DOES QUANTUM MECHANICS DO?

Quantum Mechanics: Describes how objects behave at a small scale.

- Describes physics at the microscopic level
- Seemingly incompatible with the types of observations made in our everyday lives
- Leads to counter-intuitive effects
- **Used for describing the behavior of an atom**

The path of an electron around an atom is described by quantum mechanics

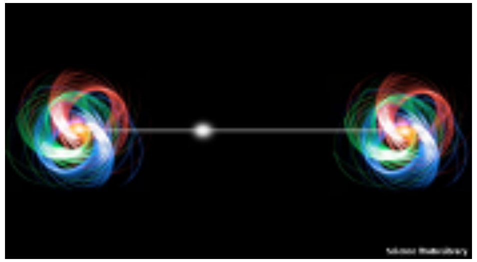
QUANTUM MECHANICS VS QUANTUM COMPUTING

Quantum Computing: Uses quantum phenomena to perform computation.

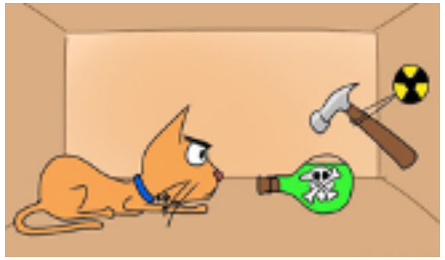
- Quantum mechanics is a tool used by quantum computers to solve certain computational problems that normal computers cannot

These three weird quantum properties enable the design of quantum algorithms which can compute in ways classical computers cannot, making quantum computers more powerful for solving certain types of problems.

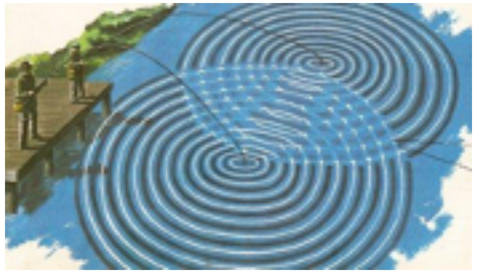
ENTANGLEMENT



SUPERPOSITION



QUANTUM INTERFERENCE

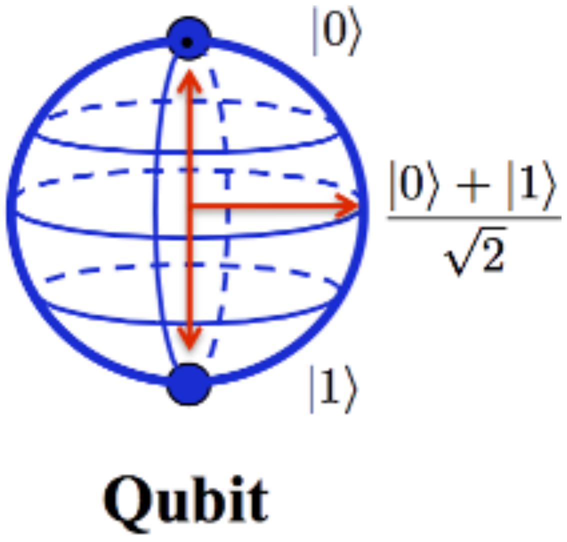


Quantum Computing is an application of Quantum Mechanics.

WHY QUANTUM MECHANICS?

We don't need to know semiconductor physics to learn how to code…

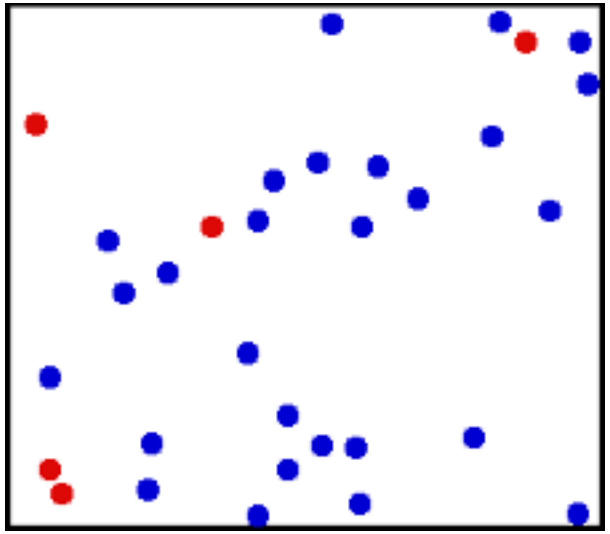
So why do we need to learn quantum mechanics to do quantum computing?

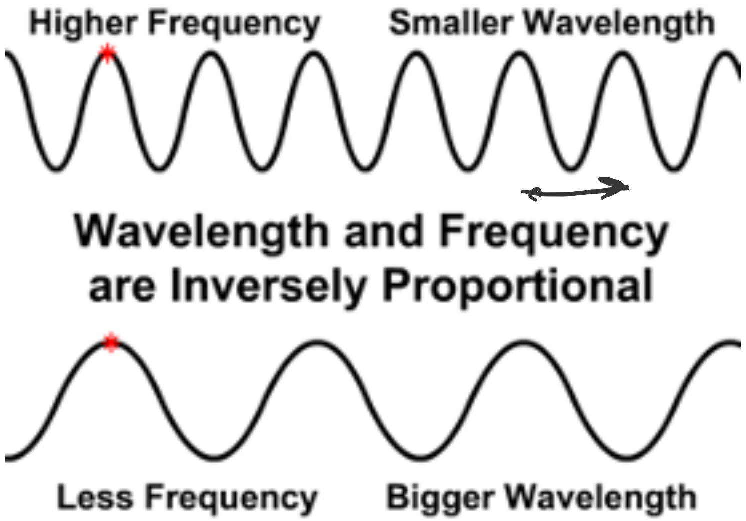


Because qubits are physical systems, and quantum mechanics describes the behavior of these systems!

GETTING STARTED - Particles and Waves

Let’s travel back in time, before people discovered quantum mechanics...





wavelength $\downarrow$

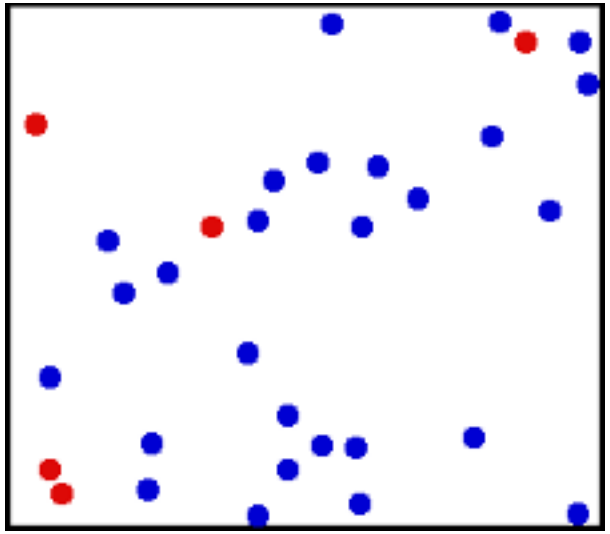
$\lambda \propto 1/f$

$\uparrow$ frequency

We’ll look at how people understood waves and particles.

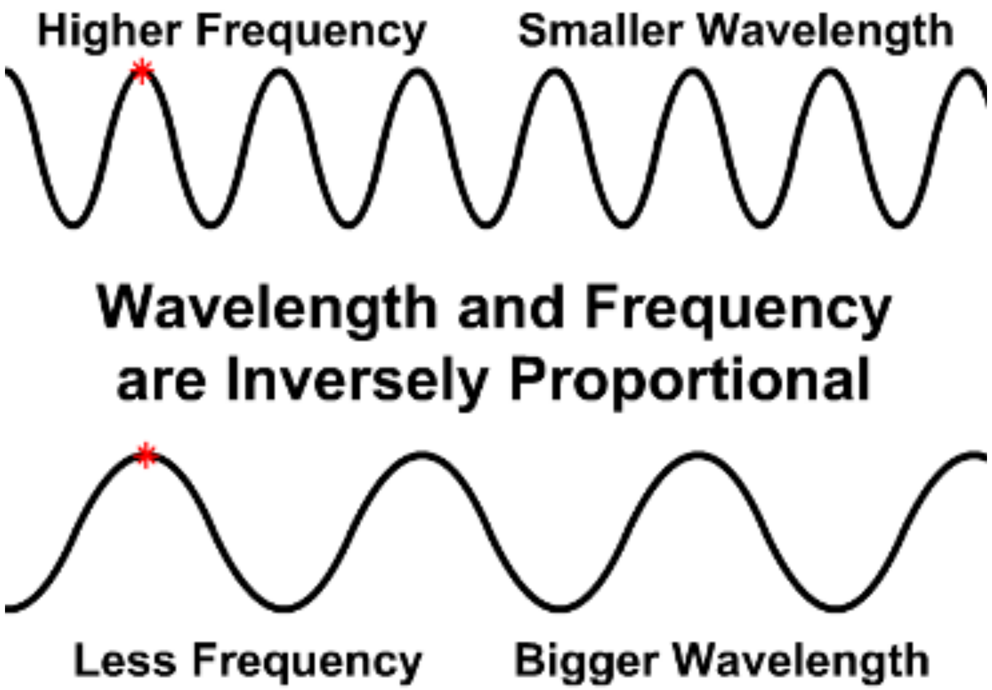
Pre-Quantum Era: Particles

- ➔ Regardless of size, **<u>all</u>** particles have a well defined position: you can point to exactly where they are!
- ➔ Particles are “discrete”
- ➔ When particles collide, they “bounce off” each other



Pre-Quantum Era: Waves

- Unlike particles, waves do not have a well-defined position
- They are "continuous"
- When waves "collide", they will **interfere**

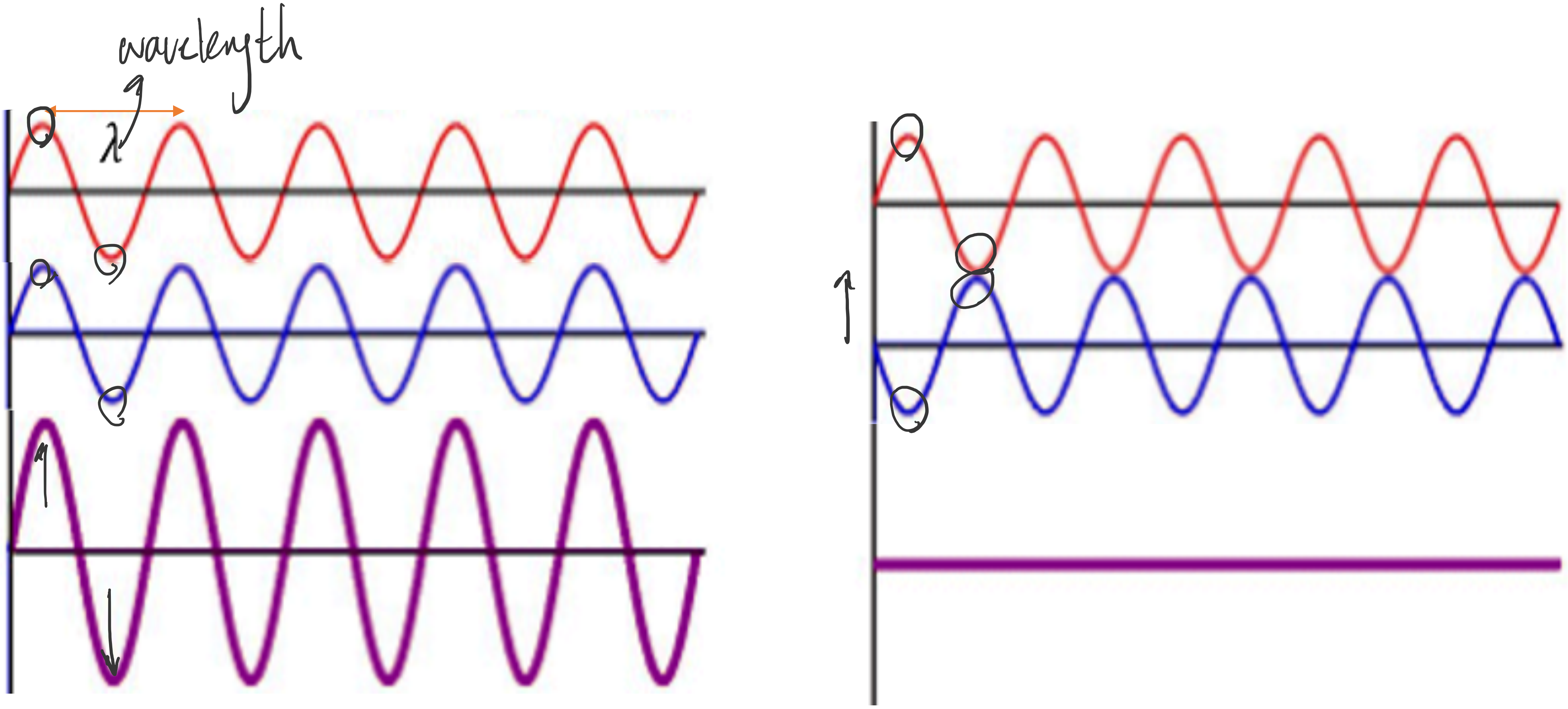


Let's look at what interference means...

Interference

Interfering wave

Resultant wave

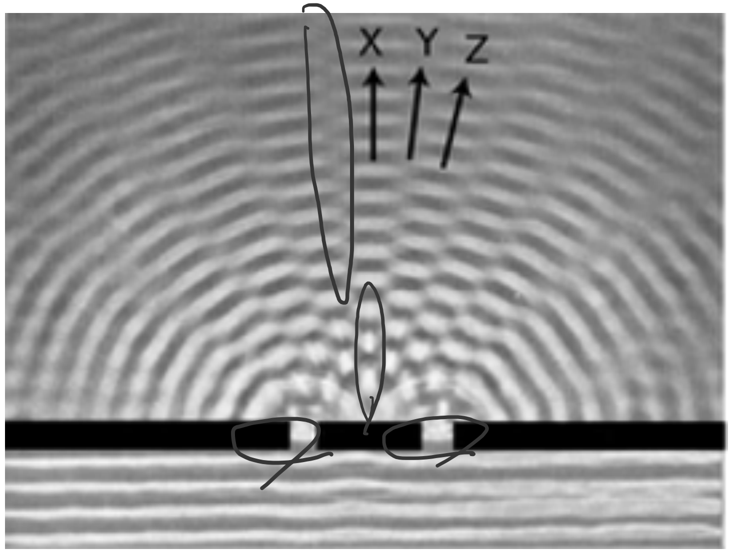


Constructive Interference

Destructive Interference

Interference example

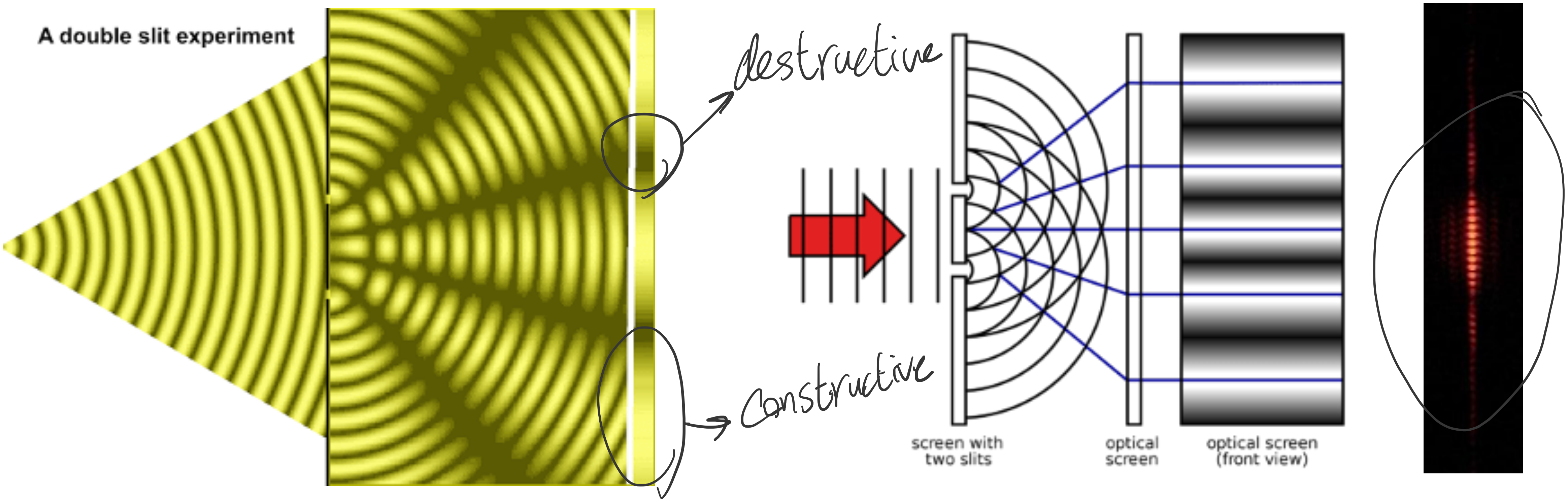
An interference pattern forms when water is passed through two slits.



Is light made out of waves or particles?

Light as a wave - the double slit experiment

How do we know light is a wave?

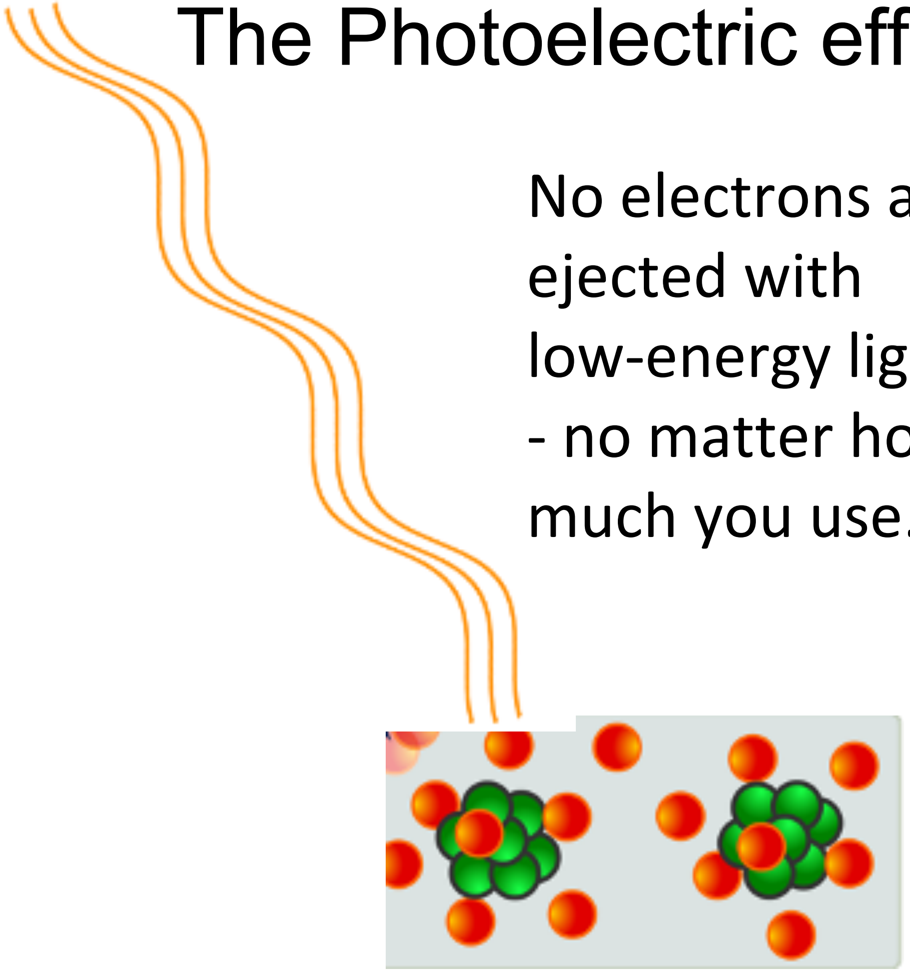


When we pass it through two slits, it forms an interference pattern, just like the water waves.

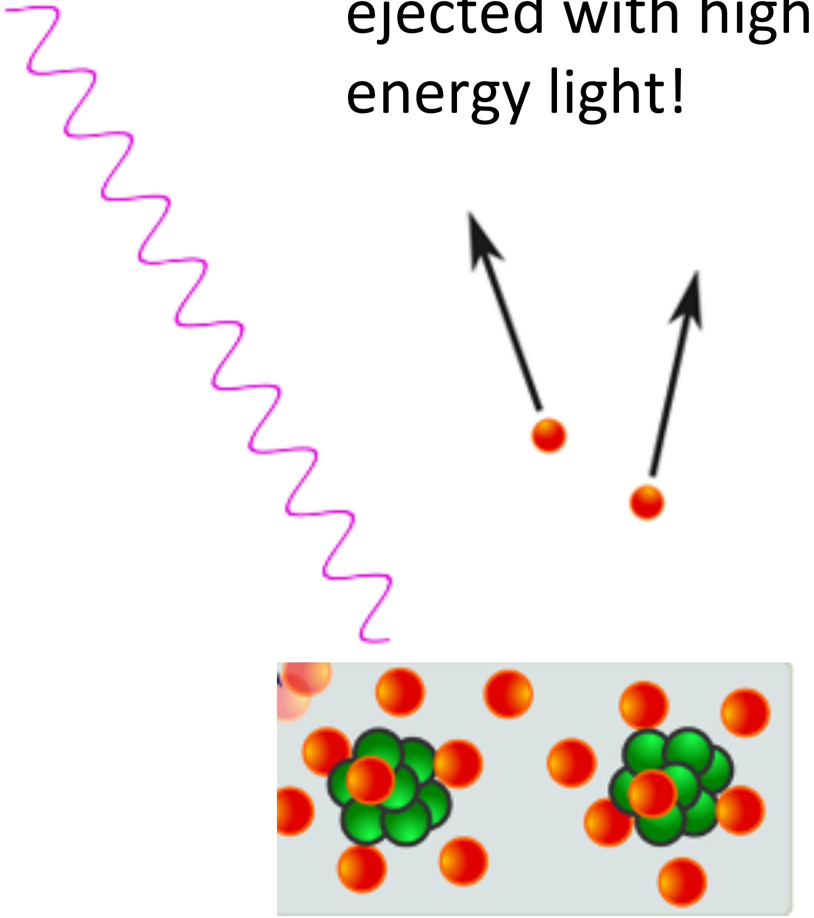
Can light also be a particle?

The Photoelectric effect

No electrons are ejected with low-energy light - no matter how much you use.



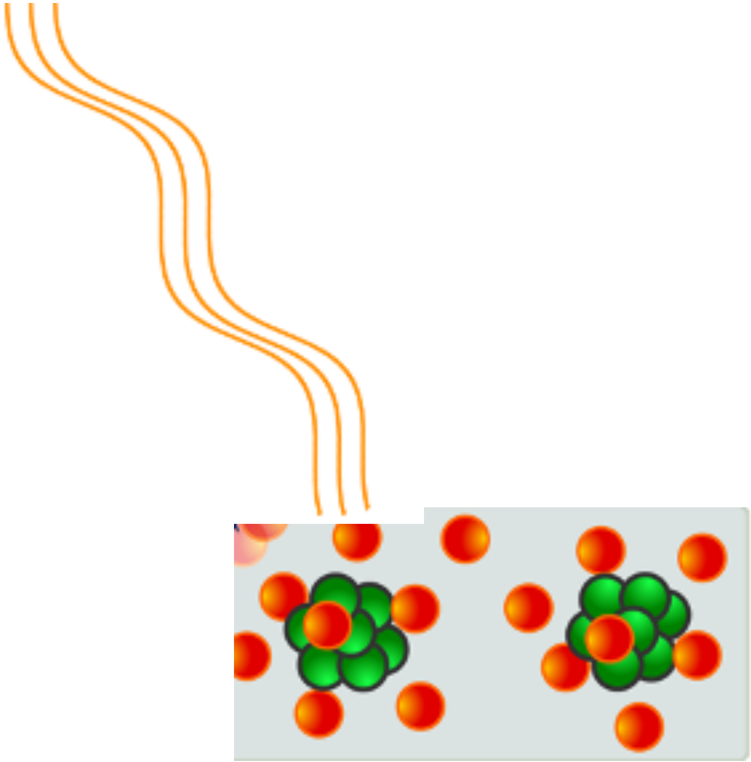
Electrons are ejected with high energy light!

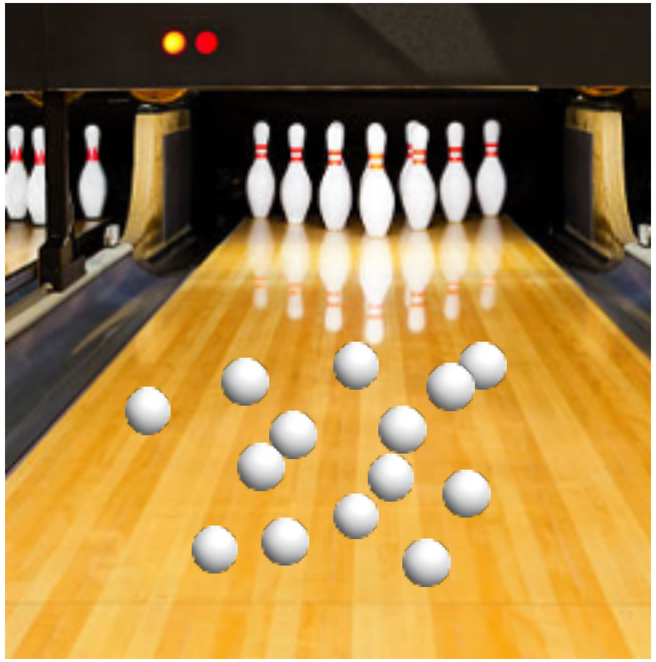


This doesn’t make sense if light is only a wave, because low-energy waves could eventually dislodge the electron.

Can light also be a particle?

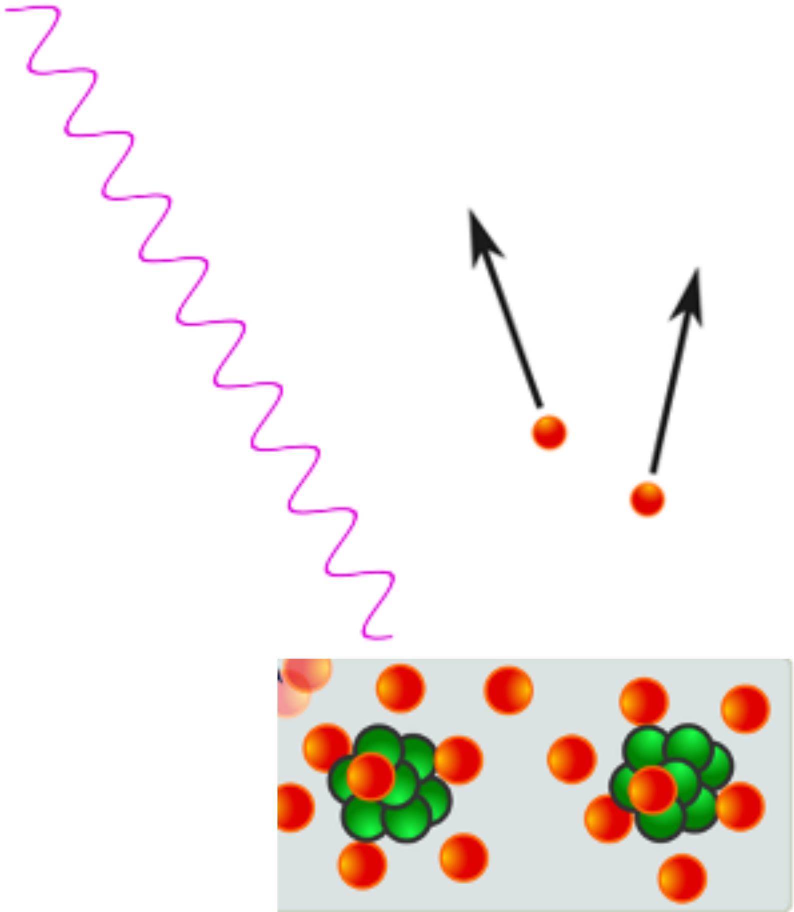
What if light was also made of particles?





Then it would make sense that no matter how many “particles” of light you use, the electron is not ejected, just like if you tried to use a lot of ping-pong balls to knock over bowling pins. It wouldn’t work.

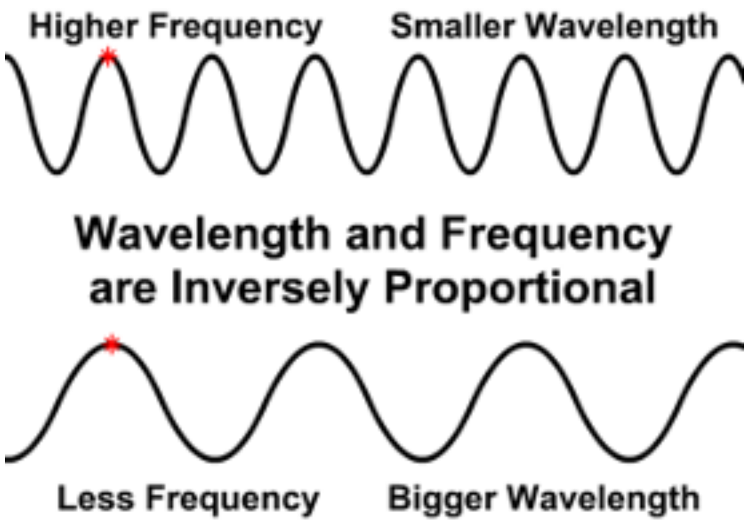
Can light also be a particle?



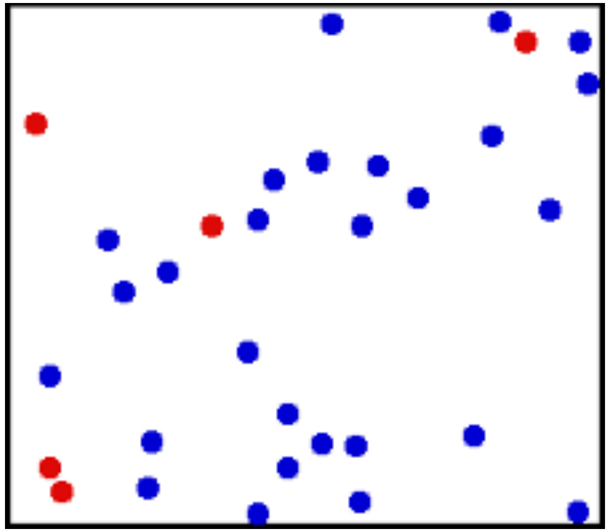
And when you try to use a high-energy "particle" of light to remove the electron, it works! Just like how a bowling ball would work to knock over the bowling pins.

Light is a wave and a particle

Waves



Photons ("particles" of light)



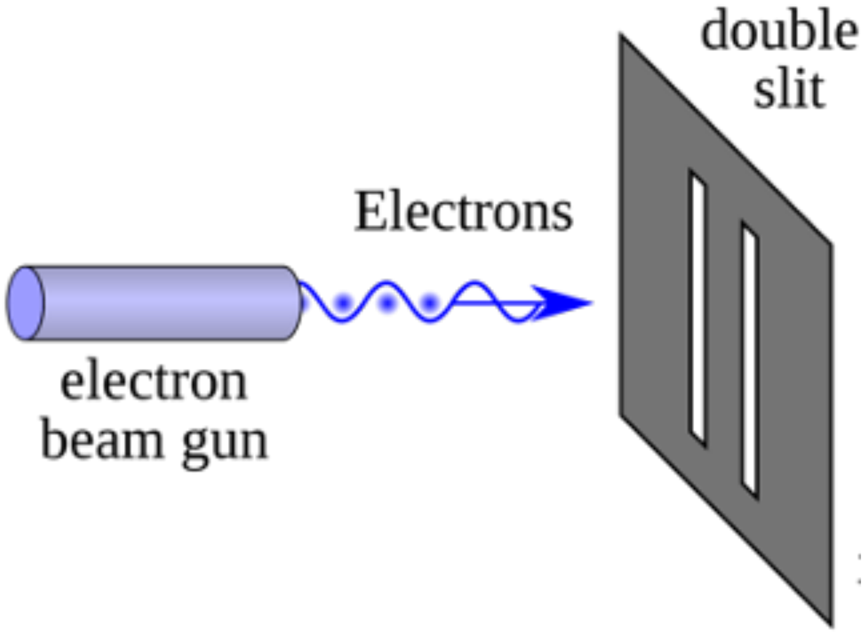
Wave-particle duality

In quantum mechanics all objects can be described as both a ***wave*** and a ***particle***

Particles as waves

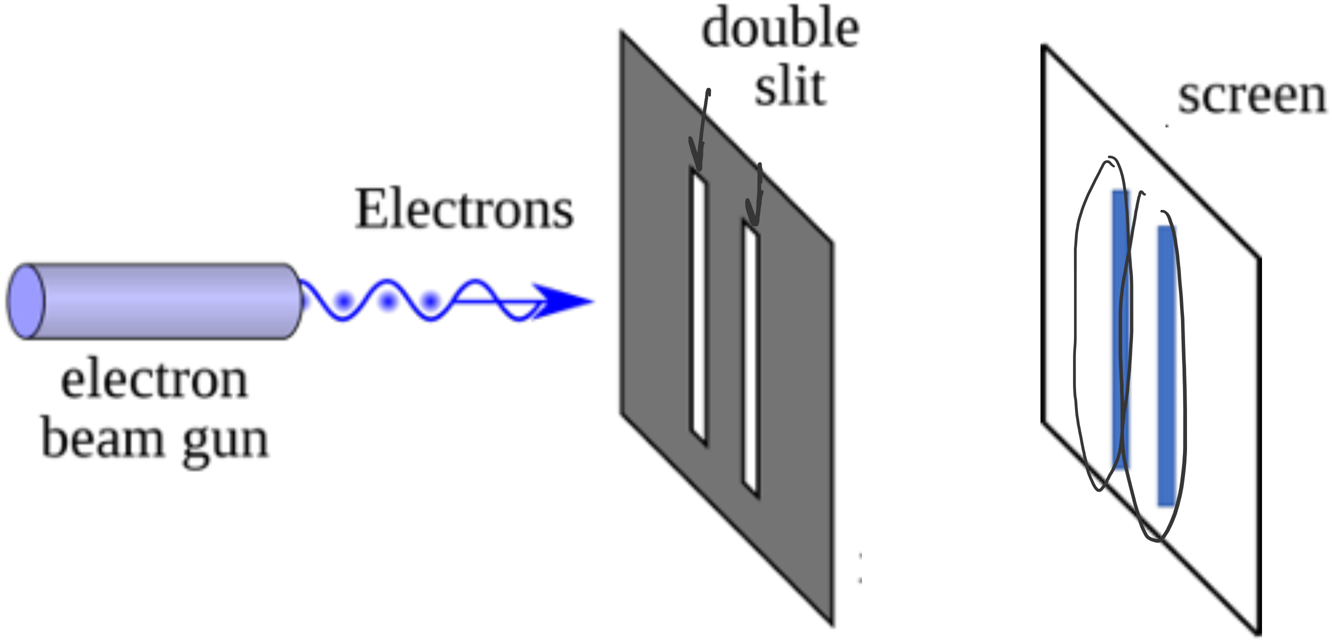
We saw an experiment that showed how waves behave as particles…

But where’s the proof that particles behave as waves?



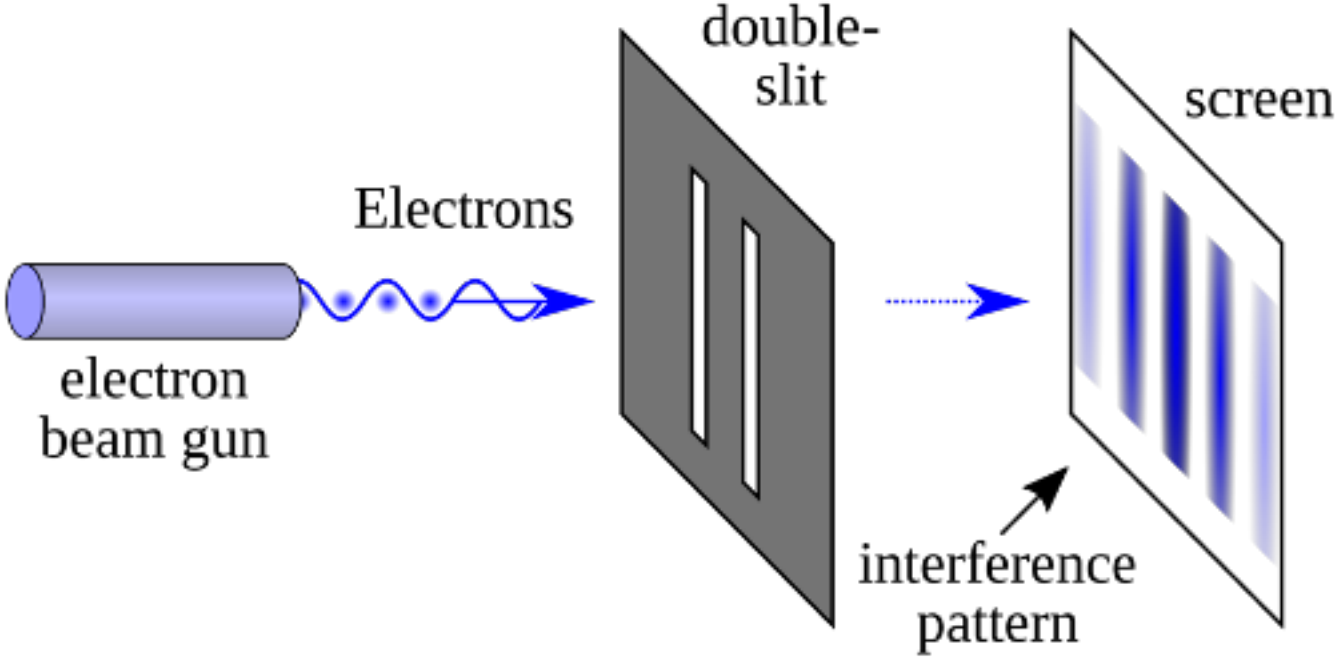
Let’s go back to the double-slit experiment, except with electrons instead of light.

Particles as waves - double slit experiment

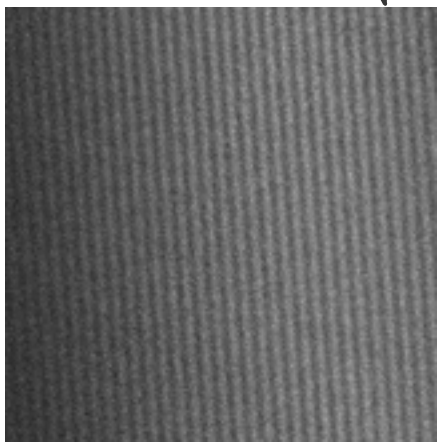


Electrons are particles, right? So we expect to see two lines where the electrons went through the slits...

Particles as waves - double slit experiment



Real actual image of electron interference taken by our very own Akshay!!

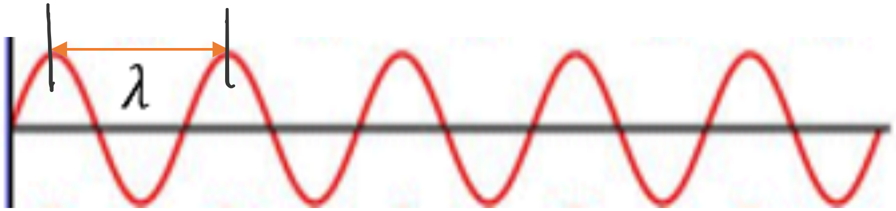


But we actually see an interference pattern!! Just like we saw with the waves!

Therefore, electrons exhibit wave behavior!

De Broglie wavelength

All particles have a wavelength!



$$\lambda = \frac{h}{P}$$

- ➔ λ: wavelength
- ➔ h: Planck's constant ($6.626 \times 10^{-34} m^2\ kg / s$)
- ➔ P: momentum (m*v)

mass velocity

If we're made out of waves why can't Fran walk through walls?

$$\lambda = \frac{h}{m \cdot v}$$

$$\lambda_{Fran} = \frac{6.626 \times 10^{-34} \, (m^2 \cdot kg/s)}{(60 \, kg) \times (5 \, m/s)} \simeq 2.21 \times 10^{-36} \, m$$

lower wavelength — higher wavelength

more particle-like behavior — more wave-like behavior

Wifi wavelength: 0.125 m

X-ray wavelength: 10^{-9} m

Why do we care that particles are also waves?

- Because this means that particles are not discrete and solid objects. They don't have a well-defined position.
- Instead, they are defined by a "**wavefunction**" $\Psi(x,t)$.

$$\Psi(x,t)$$

psi (the wavefunction)

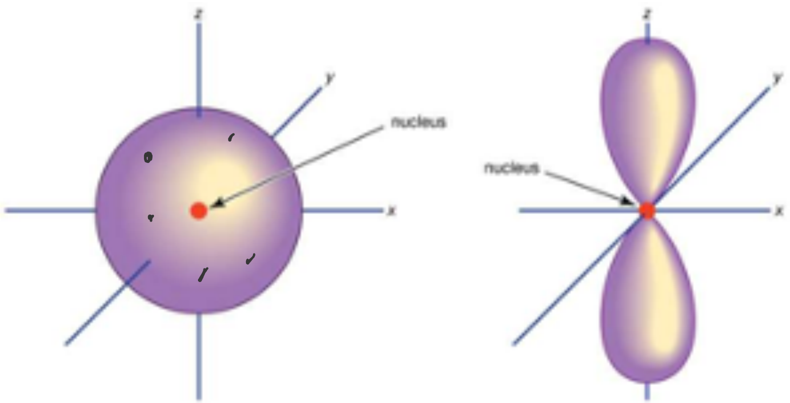
time

position

- But the wavefunction doesn't tell you exactly where the particle is...

The Wavefunction

- Instead, the wavefunction is a “probability cloud.”
- It shows how likely it is for a particle to be in a certain region of space at a given time.



These are pictures of the wavefunction of our favorite particle, the electron. They show that there’s a chance the electron will be somewhere within those regions.

Wave-particle duality in quantum computing

- Qubits behave like waves
- The superposition states of the qubit can interfere with one another
- Using interference we can amplify the probability of the correct answer
- Quantum interference leads to a quantum speed-up

Seach Algorithm
(Amplitude Amplification)

So far...

- We defined “quantum”
- We went over the difference between quantum mechanics and quantum computing
- We looked at experiments that prove that waves act as particles and particles act as waves
- We introduced the wavefunction

Next...

- We’ll mathematically describe the relationship between the wavefunction and the energies of a quantum state.

10 MIN BREAK!

$$\lambda = \frac{h}{m \cdot v} \qquad v\uparrow \quad \lambda\downarrow \qquad \lambda \propto \frac{1}{v}$$

Heisenberg Uncertainty Principle

exactly known velocity $\Delta v = 0$

$$\Delta x \, \Delta p = m \times \Delta x \, \Delta v \geq \frac{h}{4\pi}$$

$h \rightarrow \sim 10^{-34}$

$\Delta x\uparrow \quad \Delta v\downarrow$

$\Delta x\downarrow \quad \Delta v\uparrow$

v_{Fran} to pass through a wall $\sim 10^{-30}$ m/s

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

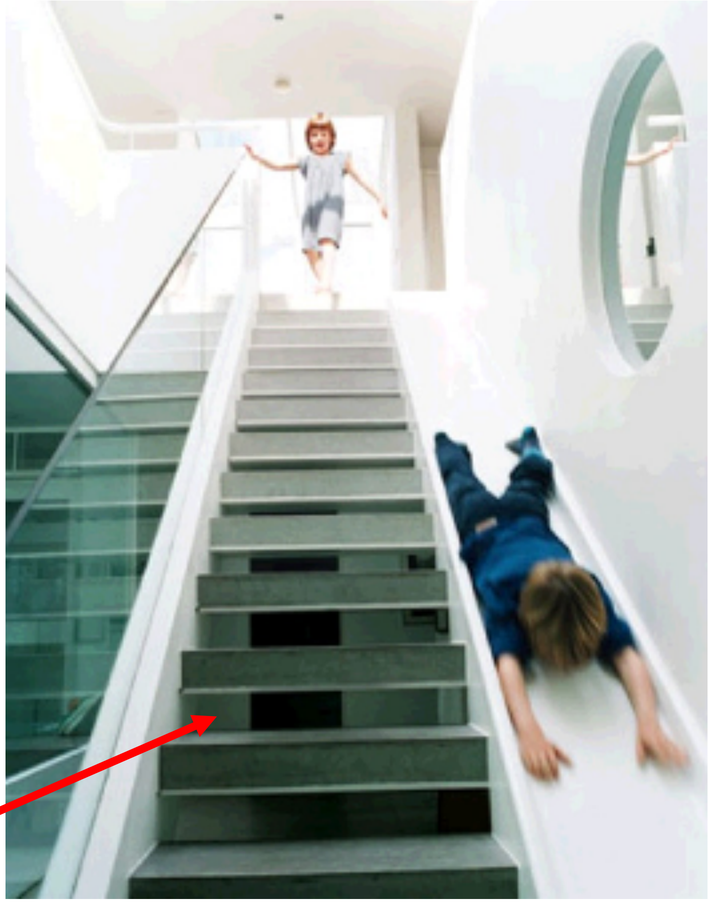
$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$ superposition

$$H|\psi\rangle = \frac{H|0\rangle + H|1\rangle}{\cancel{\sqrt{2}}}$$

$$H|\psi\rangle \rightarrow (|0\rangle + \cancel{|1\rangle})/2 + (|0\rangle - \cancel{|1\rangle})/2 = |0\rangle$$

Mathematically representing quantum states

- We know that the wavefunction describes the probability of finding a particle somewhere in space.
- We can use this information, plus something called the Hamiltonian operator, to find the allowed energy states of that wavefunction.



Quantum - only certain energies are allowed

Hamiltonian

A quantum operator that gives the energy of a quantum state

energy

$$\hat{H}|\psi\rangle = \mathrm{E}|\psi\rangle$$

$\hat{H}$ "^" hat indicates that $\hat{H}$ is a quantum operator

The Hamiltonian

The form of this equation should look familiar...

$$\underline{A}\vec{v} = \lambda\vec{v}$$

Operator

Eigenvector

Eigenvalue

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

Operator

Eigenstate

Eigenvalue

The eigenvalues are the allowed solutions to an eigenvalue equation. So we're using the Hamiltonian operator in an eigenvalue equation to find the **allowed energy states** of the wavefunction.

Classical Energy Equation

Total energy = kinetic energy + potential energy

Energy you have based on how fast you're going

Energy you have stored up based on how high up you are (because when you fall you'll have a lot of energy!)

Example of the Energy Equation

$E = KE + V$ pot. as function of position

$= \frac{1}{2} m v^2 + V(x)$

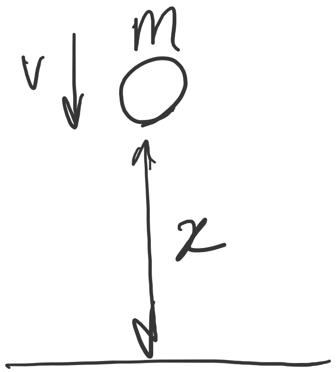
for this example

$\boxed{V(x) = m \cdot g \cdot x}$.

momentum $\quad$ velocity

$p = m \cdot v \rightarrow p^2 = m^2 v^2$

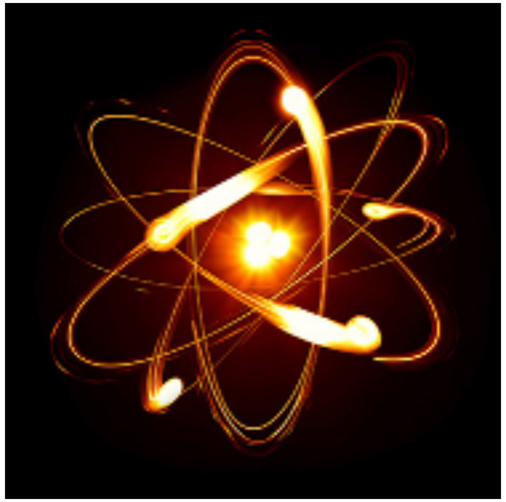
$E = \frac{p^2}{2m} + V(x)$



Hamiltonian form

We need a Hamiltonian that captures the different ways a quantum system could have energy.

Let's take the example of an electron around a nucleus.



Total energy = kinetic energy + potential energy

Energy based on how fast the electron is going

Stored energy (not from gravity this time, but from how far the negative electron is from the positive nucleus).

Energy equation to Hamiltonian

Formal tool: Canonical quantization

$$E = \frac{p^2}{2m} + V(x)$$

$p \longmapsto$ quantum momentum operator $\hat{p}$

↳ if I apply this operator to a quantum state, it tells me the momentum of that quantum state

$x \longmapsto$ quantum position operator $\hat{x}$

↳ if I apply this operator to a quantum state, it tells me the position

$$E \longmapsto \hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = \frac{\hat{p}\,\hat{p}}{2m} + V(\hat{x})$$

the Schrodinger Equation

Describes how the wavefunction changes in time based on its Hamiltonian

the Schrodinger Equation

Describes how the quantum state (state of a qubit) changes in time based on its energy values.

Schrodinger Equation

imaginary number

$i = \sqrt{-1}$

Planck's constant

rate of change

Hamiltonian

$$i \frac{h}{2\pi} \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

$\hbar$

with respect to time

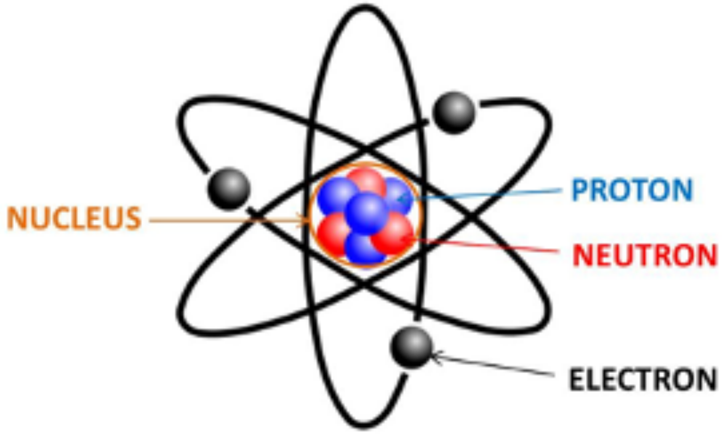
quantum state

Why Schrodinger Equation

- The Schrodinger equation tells us how our quantum hardware behaves
- It tells us how to operate our quantum hardware
- It allows us to engineer and implement quantum gates
- And many more applications outside of quantum computing in physics and chemistry!!

Why do we need quantum mechanics

How does an atom work?



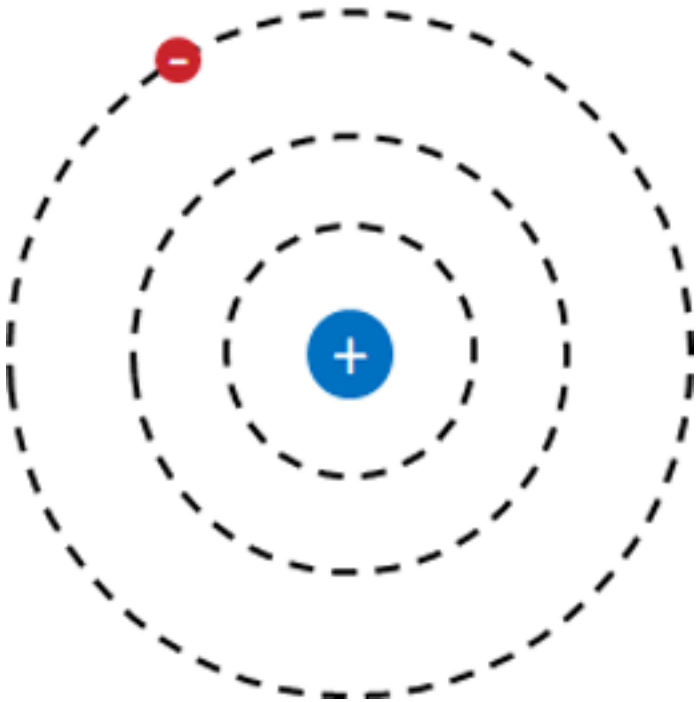
- Atoms are made out of electrons, protons, and neutrons
- Protons and neutrons are in the nucleus of the atom
- Electrons orbit the nucleus at different distances based on their energy

One problem: Accelerating charges (the electron) lose energy. This will cause the electron to eventually collapse into the nucleus

Why do we need quantum mechanics

Bohr's atom:

- Contrary to classical EM, electrons were allowed to orbit in certain discrete stable orbits. Orbits in between are not allowed.
- The stable orbits are those for which the angular momentum of the electron is quantized
- Electrons can only "switch" between orbits of different n by absorbing or emitting a single photon



Solution: Electrons can only stay in certain quantized orbits, so they will not eventually collapse into the nucleus.