

INTRO TO QUANTUM COMPUTING

Week 13 Lab

QUANTUM MECHANICS - 3

<insert name>

<insert date>

PROGRAM FOR TODAY

- Canvas attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
 - This is lab number:
 - Passcode:
- On a scale of 1-5, how much do you agree/disagree with the following statement: I'm glad we took the time to learn the math concepts inherent to quantum computing.
- In general, do you like math?
- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 – Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

QUESTIONS FROM PAST WEEK

What if we tried to measure the spin along x and z at the same time?

LEARNING OBJECTIVES FOR LAB 13

- Understanding the 'quantum' in 'quantum computing'
 - Waves on a string
 - Modes and eigenfunctions
 - Modes in the quantum world
- Using the postulates of quantum mechanics
 - Stern-Gerlach experiment revisited
 - Applying the postulates to the SG experiment
- "Quantum Mechanics" on a guitar string*

*Optional content

MOTIVATION

Why are we learning quantum mechanics?

- Qubits are made of objects that show wave- and particle-like behavior!
- Quantum mechanics is the mathematical description of objects that show such behavior
- Qubits and quantum gates are governed by the rules of quantum mechanics

DO YOU PLAY A MUSICAL INSTRUMENT?

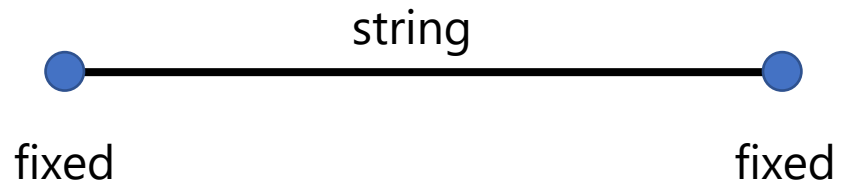


YOU'VE STARTED YOUR QUANTUM CAREER!



"If I were not a physicist, I would probably be a musician. I often think in music. ... I cannot tell if I would have done any creative work of importance in music, but I do know that I get most joy in life out of my **violin**."

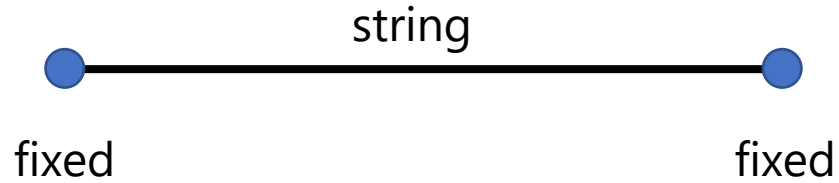
LET'S THINK ABOUT A STRING



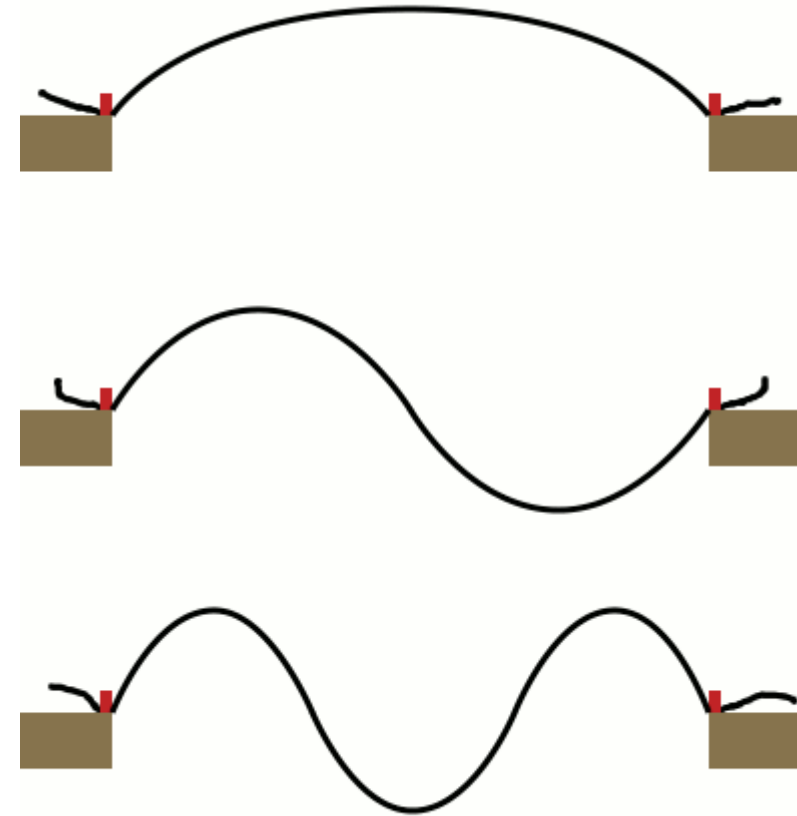
How does this string look when it vibrates?



ONLY SOME SHAPES ALLOWED

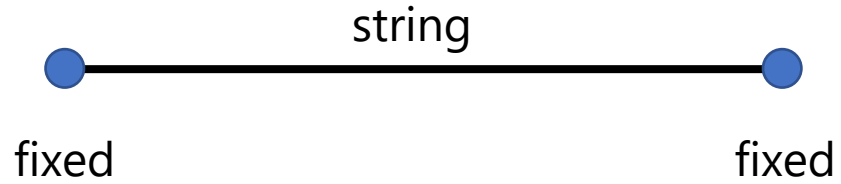


- The string takes on **specific shapes** when it feely vibrates
- These shapes are called **modes**



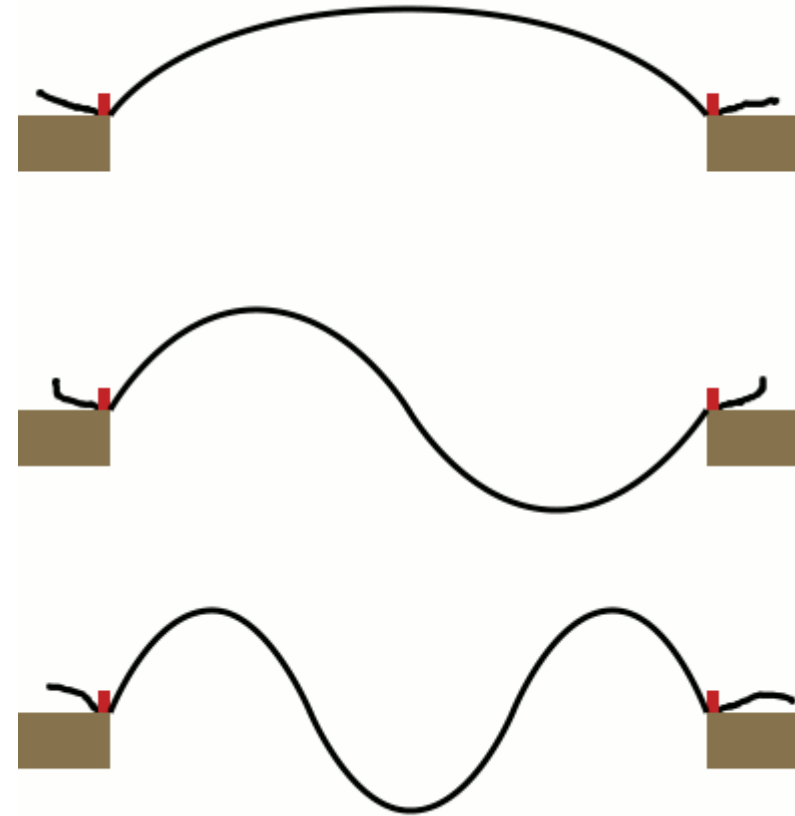
ONLY SOME SHAPES ALLOWED

Modes of guitar strings



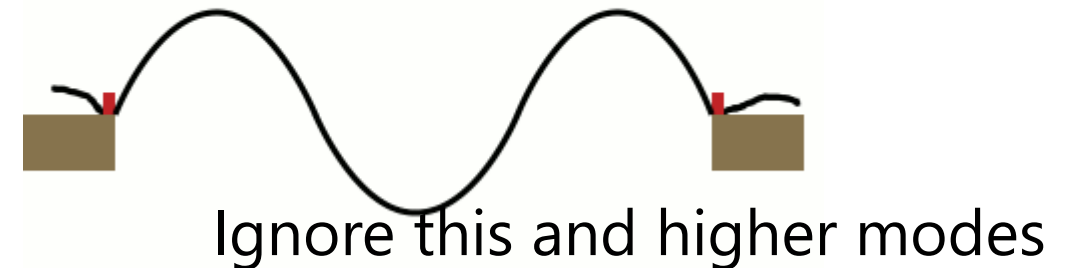
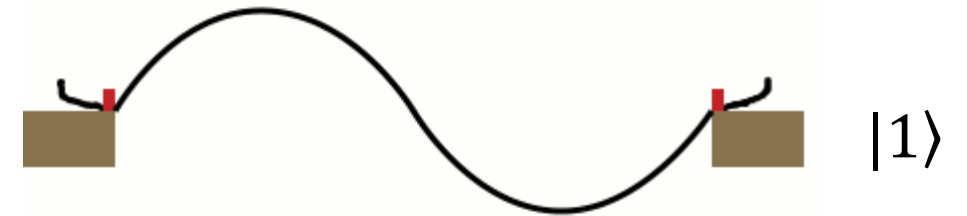
WHY ONLY THESE MODES?

- Modes are a result of **confinement**
- The string has to be fixed at two ends
- These modes are eigenfunctions!



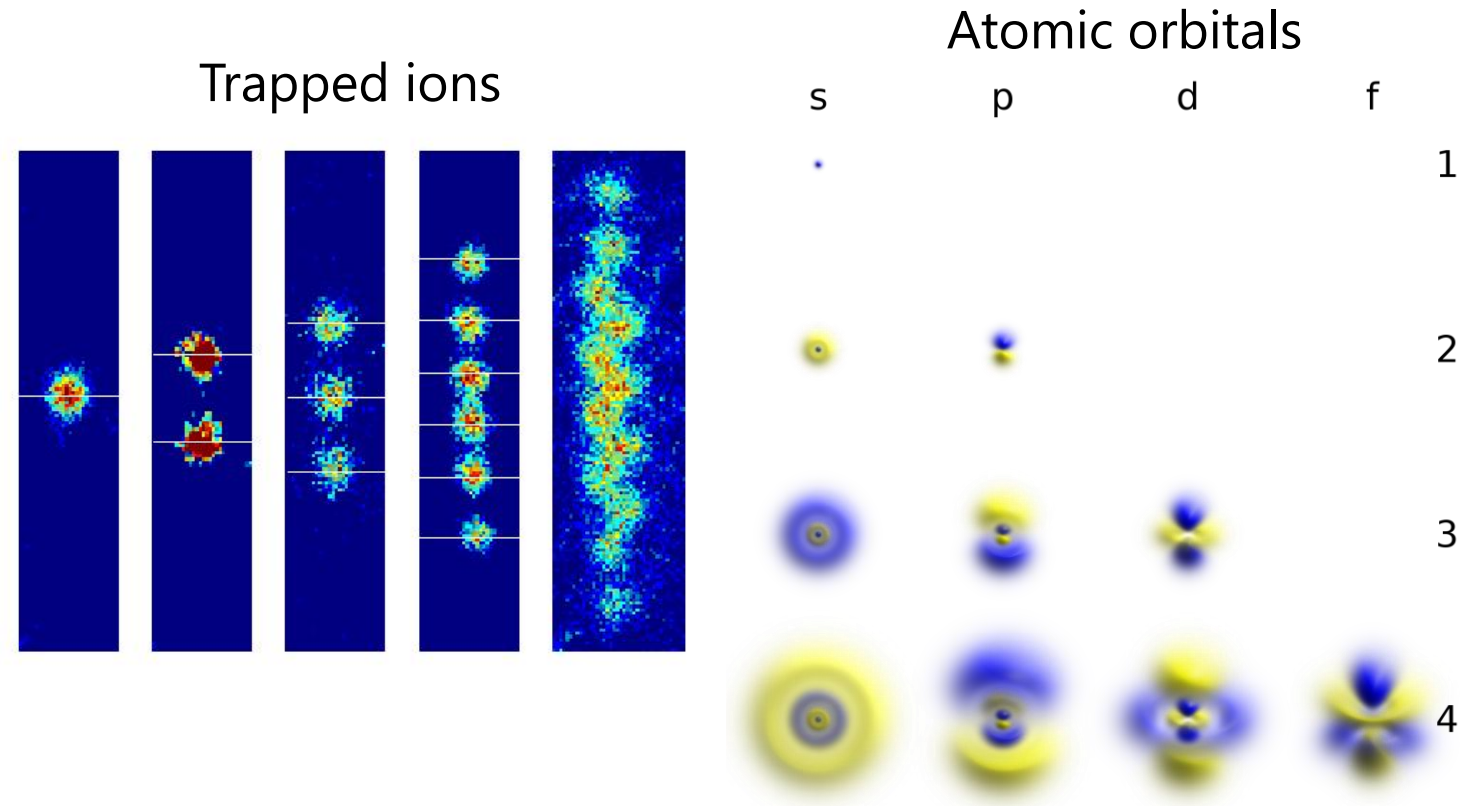
ENTER THE NANOWORLD

- At small scales, **everything** (electrons/atoms/ions...) can behave as a wave
- When they are confined, they show quantization, just like a string!
- We can use the modes as qubit states



QUANTIZED SYSTEMS IN THE NANOWORLD

- **Examples:** electrons in atoms, trapped ions, diamond nitrogen vacancy centers
- **Challenges:**
 - Keeping the qubit state 'clean'
 - Minimizing impact of higher modes
 - Reducing noise from the environment
 - Ensuring reliable transfer between modes
 - ...



QUESTIONS?

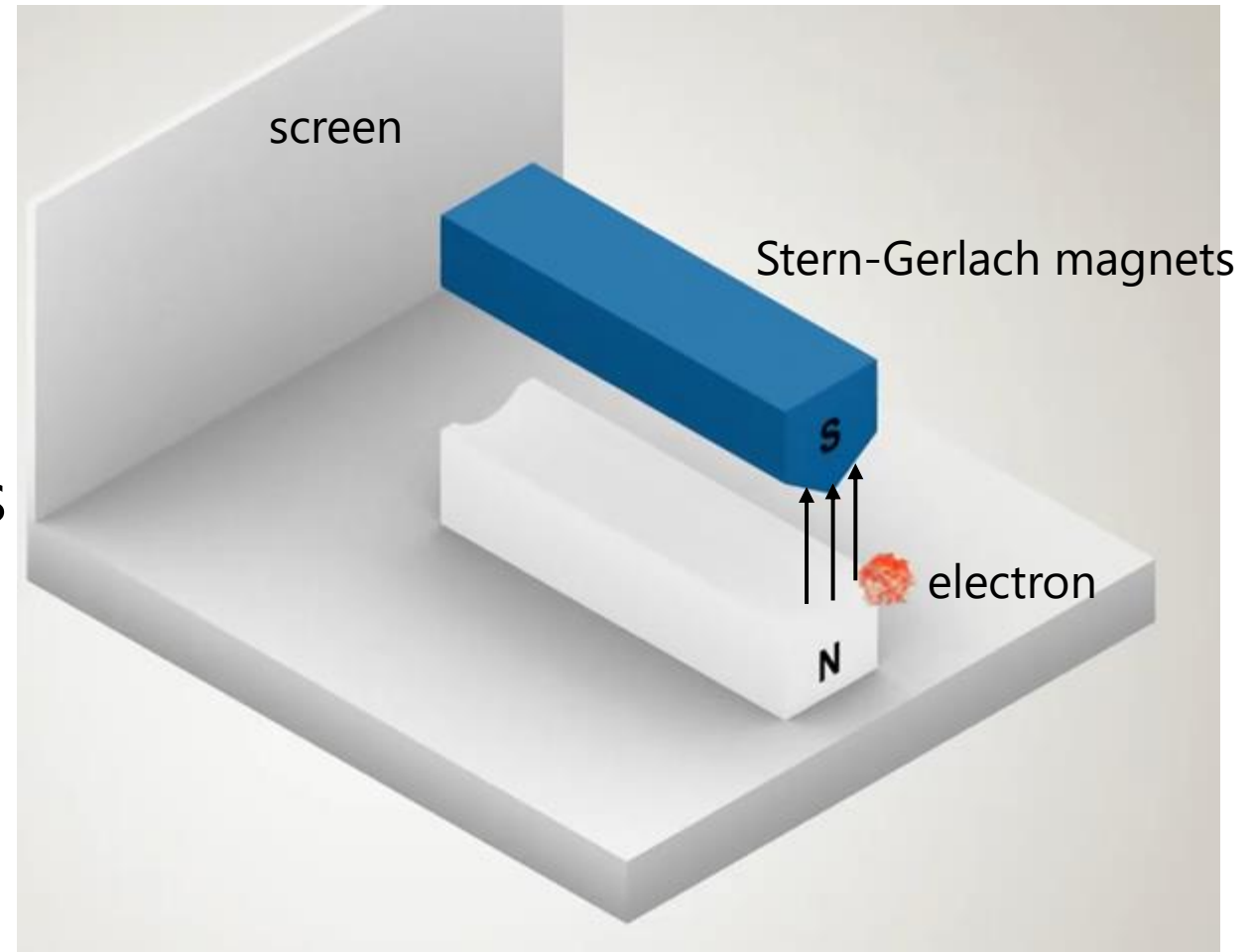
Questions on content so far?

THE POSTULATES OF QM

- A quantum state is represented by its wavefunction $|\psi\rangle$: **Wavefunction postulate**
- Classical observables \rightarrow Operators in QM : **Operators postulate**
- The only possible results of measuring an observable are the eigenvalues of its operator: **Observation postulate**
- Probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$: **Probability postulate**
- After measurement, $|\psi\rangle$ is equal to the eigenvector corresponding to the observed eigenvalue: **Collapse postulate**
- To evolve $|\psi\rangle$ in time, apply an operator to it: **Evolution postulate**

THE STERN GERLACH EXPERIMENT

- Since spin corresponds to a small magnetic field, we can use another magnet to measure it!
- **When measured**, spin always has two directions – one along the direction of the magnetic field and one opposite to it



MEASURING SPIN - THEORY

- Classical observables \rightarrow Operators in QM : **Operators postulate**
- In the Stern-Gerlach experiment, the observable is **spin**, and we can measure it along different directions

Observable	Operator	Matrix
Spin along z-axis	Z or σ_z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Spin along x-axis	X or σ_x	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Spin along y-axis	Y or σ_y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

MEASURING SPIN - THEORY

The only possible results of measuring an observable are the eigenvalues of its operator: **Observation postulate**

Operator Matrix

Eigenvalues and eigenvectors

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$1: \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } -1: \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$1: \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \text{ and } -1: \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\text{Check: } Z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

MEASURING SPIN - THEORY

Operator Matrix

Eigenvalues and eigenvectors

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Problem 1: What are the possible outcomes of measuring spin along the z-axis?

Hint: Think about the Observation postulate

MEASURING SPIN - THEORY

Operator Matrix

Eigenvalues and eigenvectors

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

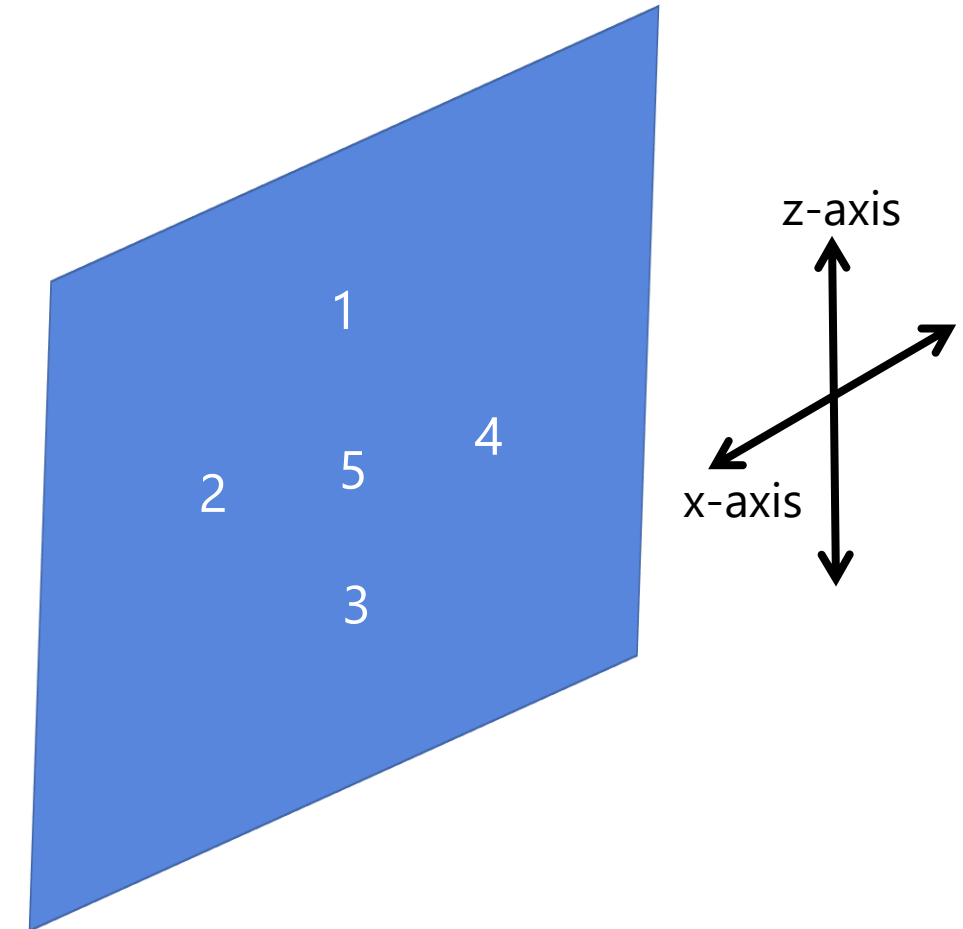
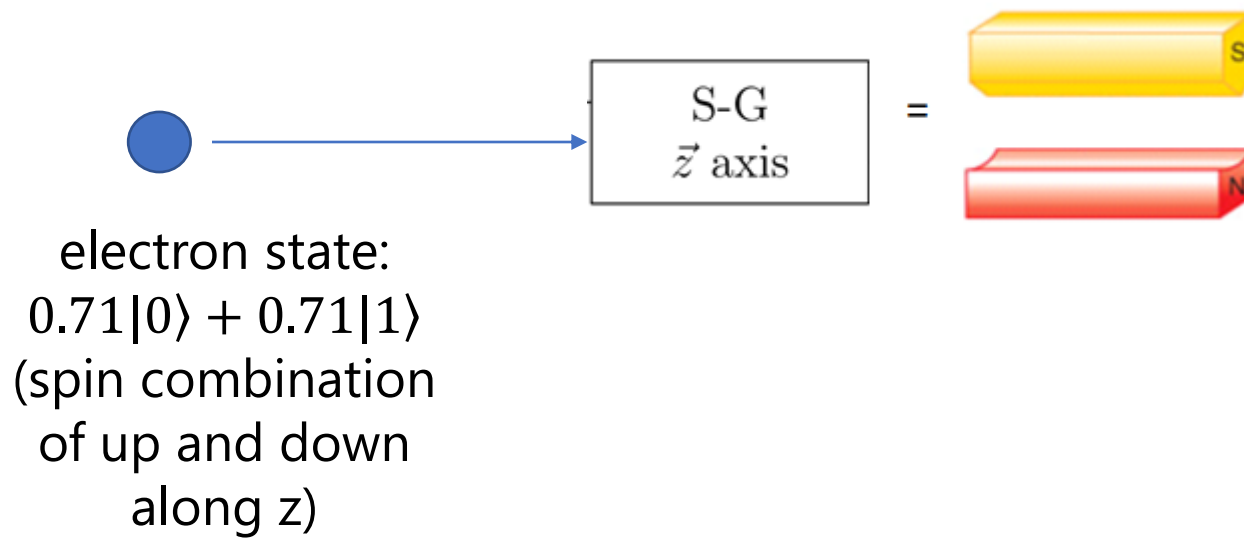
$$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Problem 2: The result of a measurement of spin along the z-axis is -1. What is the wavefunction of the electron immediately after the measurement?

Hint: Think about the collapse postulate

MEASURING SPIN - EXAMPLE

Where does the electron go?



MEASURING SPIN - EXAMPLE

$$|\psi\rangle = 0.71|0\rangle + 0.71|1\rangle$$

Operator Matrix

Eigenvalues and eigenvectors

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } -1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Problem 3: What is the probability of the result of measuring the spin of the electron in state $|\psi\rangle$ being +1? What is the probability of -1?

Hint: Think about the Probability postulate - probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$

KEY TAKEAWAYS

- When a system that shows wave-like behavior is confined, the wave modes get quantized
- At the scale of electrons/atoms/ions, everything shows wave-like behavior. Therefore, everything shows quantization when it is confined!
- The postulates of quantum mechanics are the rules that quantum systems are observed to follow

FURTHER READING AND RESOURCES

- https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/infwell1d/infwell1d.html - Simulation of the quantum version of a guitar string – the particle in a box
- <http://web.mit.edu/8.05/handouts/jaffe1.pdf> - Lecture notes from Prof. R. L. Jaffe at MIT, discussing the QM postulates along with linear algebra
- https://www.youtube.com/watch?v=uK2eFv7ne_Q – Lecture 1 of a series of lectures by Prof. R. Shankar on QM
- <https://youtu.be/Sut0Aegyoaes> – Brief video on transmission electron microscopy

QUESTIONS?

Questions on content so far?

POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

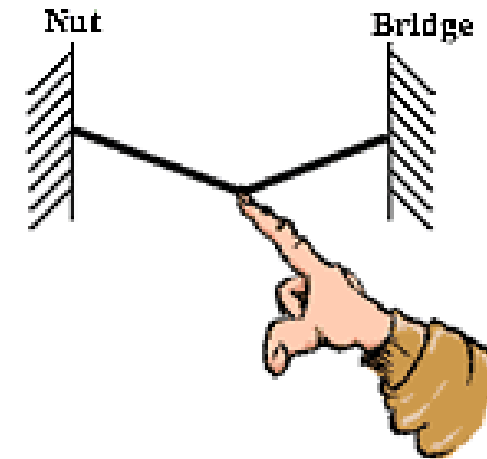
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OPTIONAL CONTENT

WAVEFUNCTION POSTULATE

A quantum state is represented by its wavefunction $|\psi\rangle$

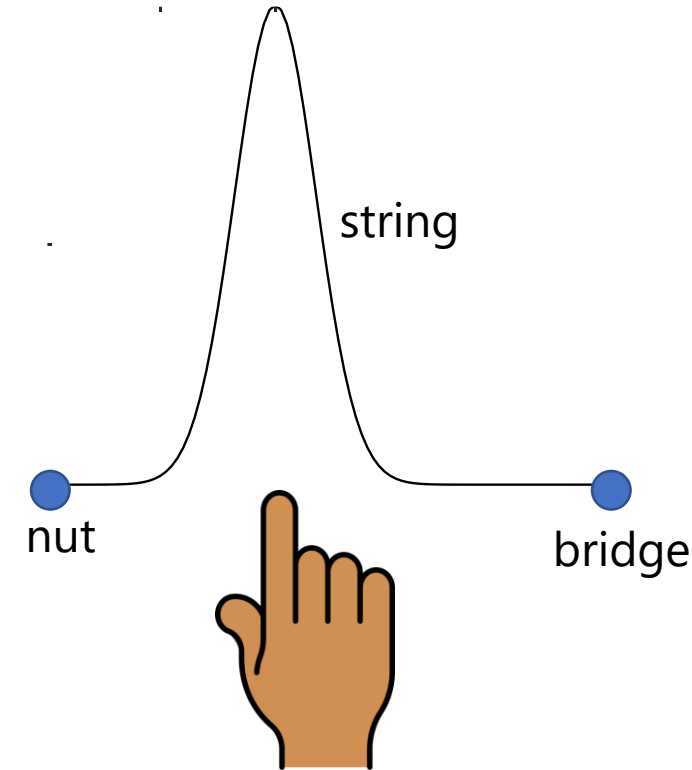
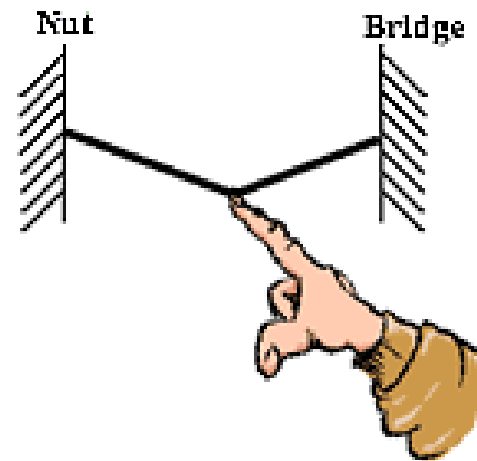
LET'S PLUCK THESE STRINGS!



- When we pluck a guitar string, we create a ripple in it

THE GUITAR “WAVEFUNCTION”

- The shape of the string describes everything about it

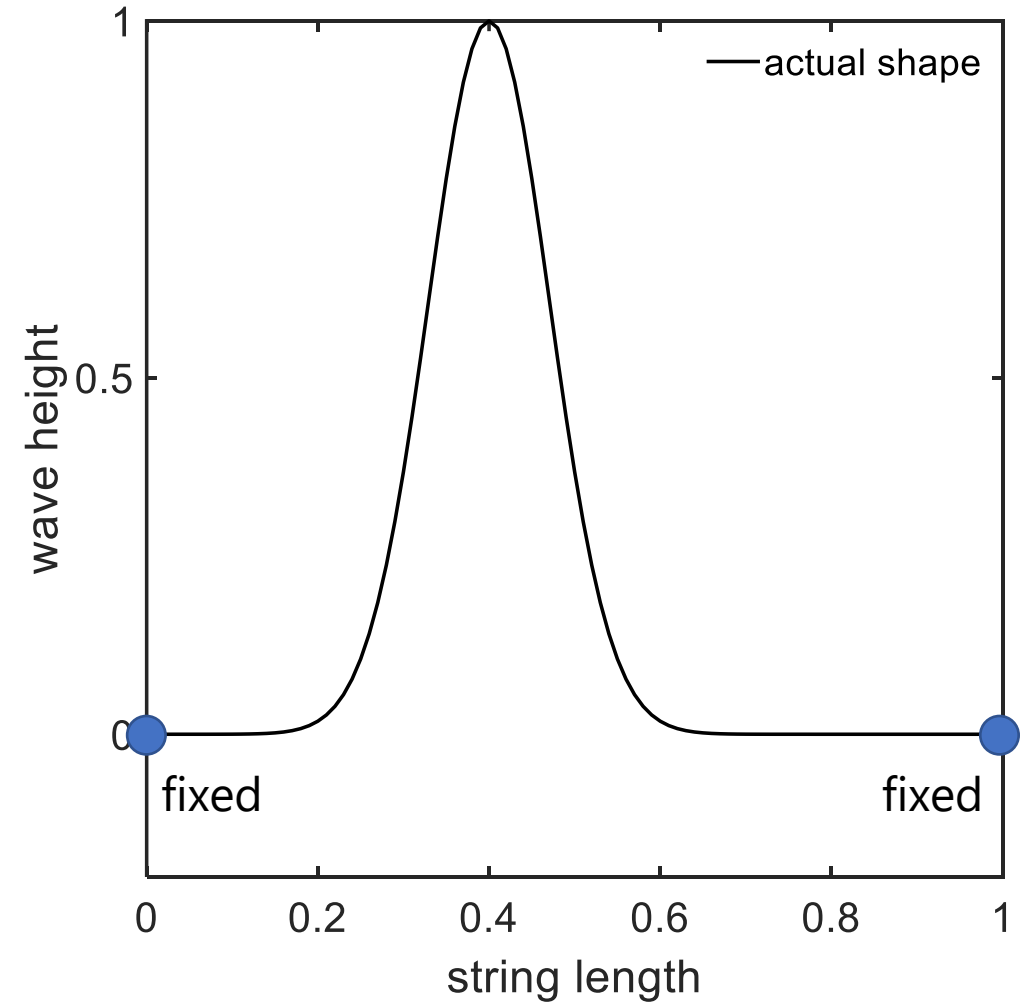
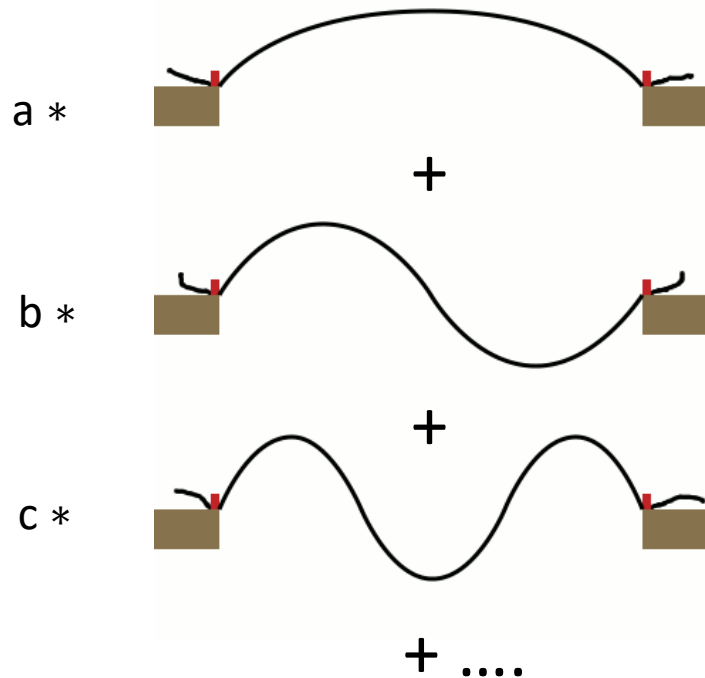


- The string shape is its “wavefunction”

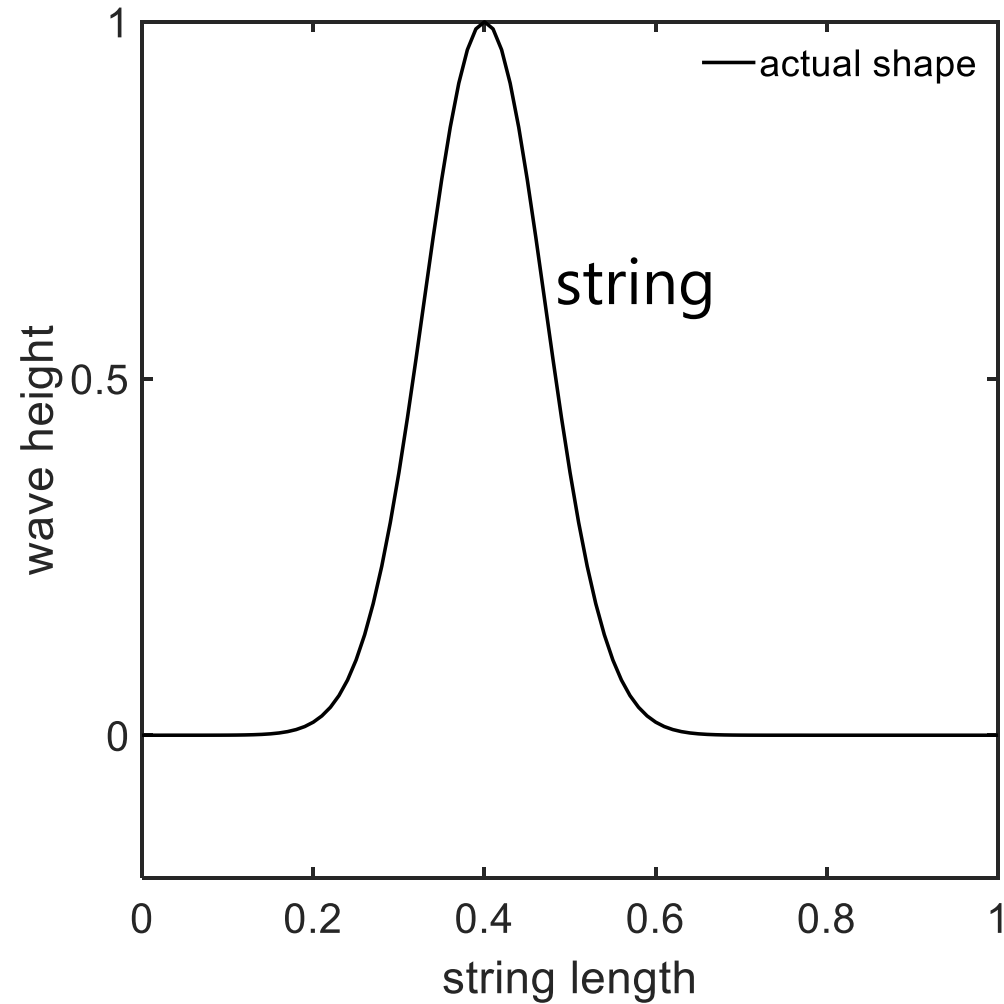
DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS

The ripple can be “reconstructed” by adding up the modes

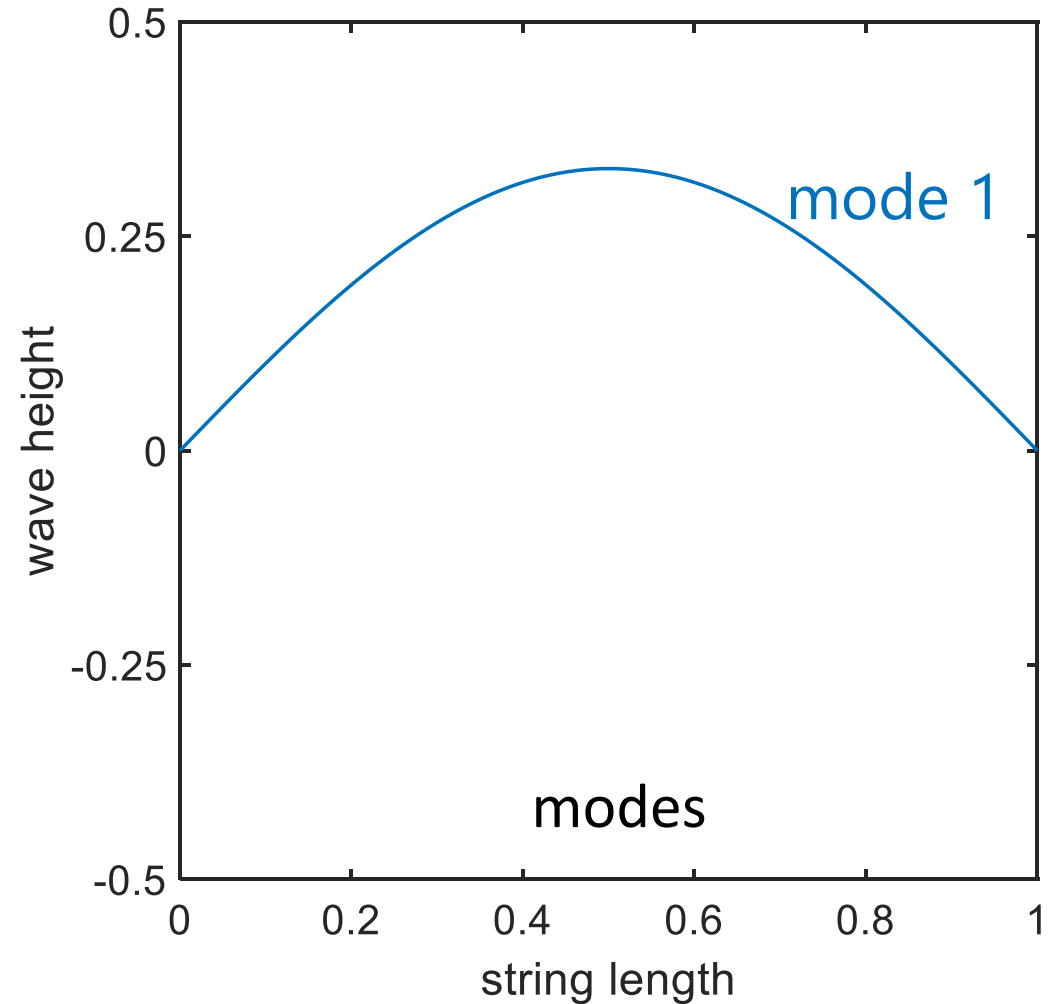
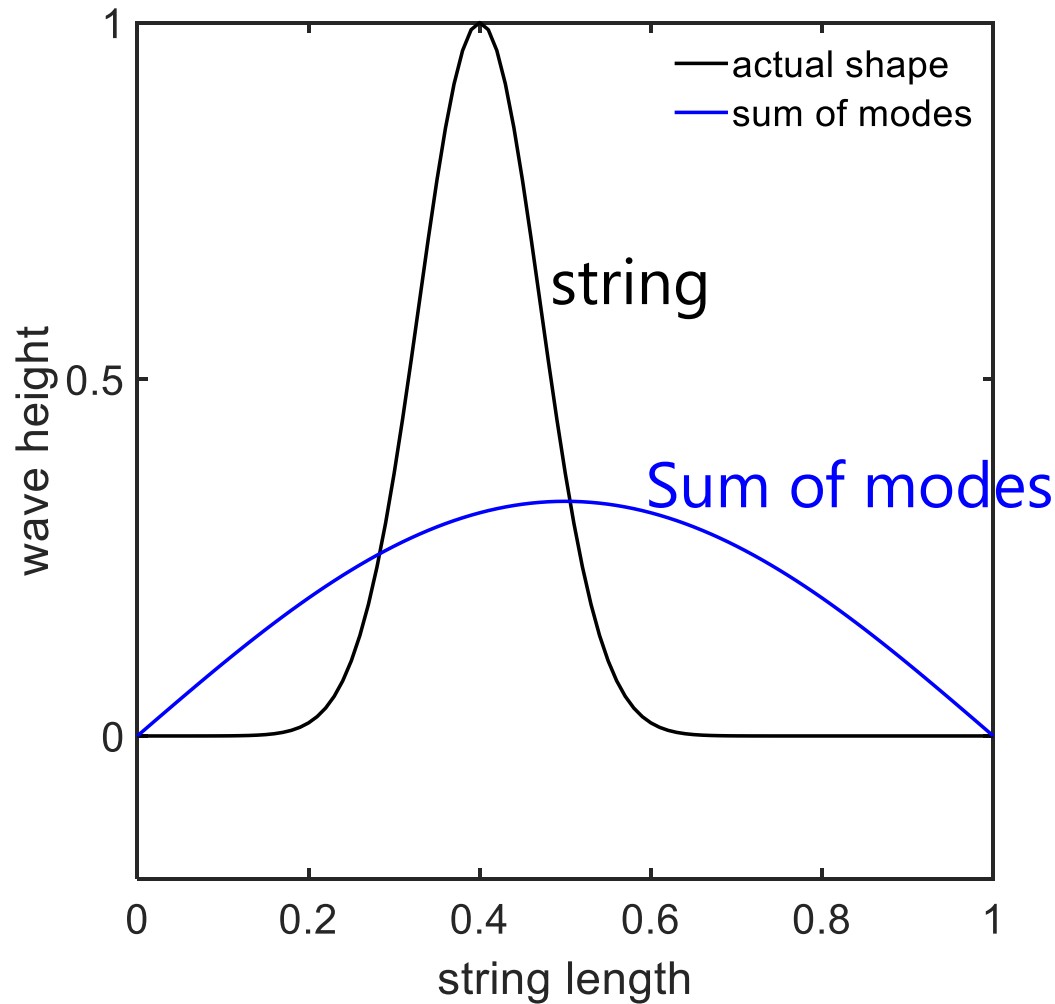
actual
shape of
string =



DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS

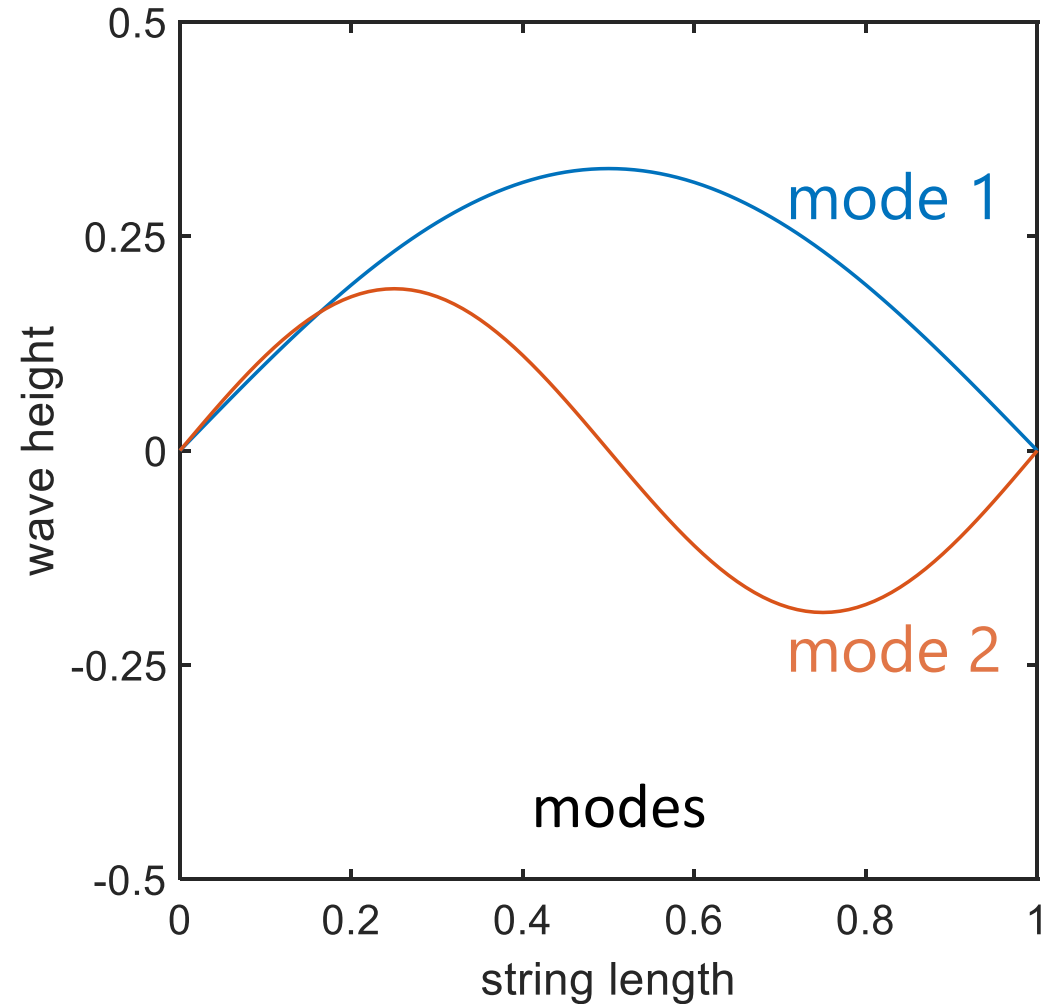
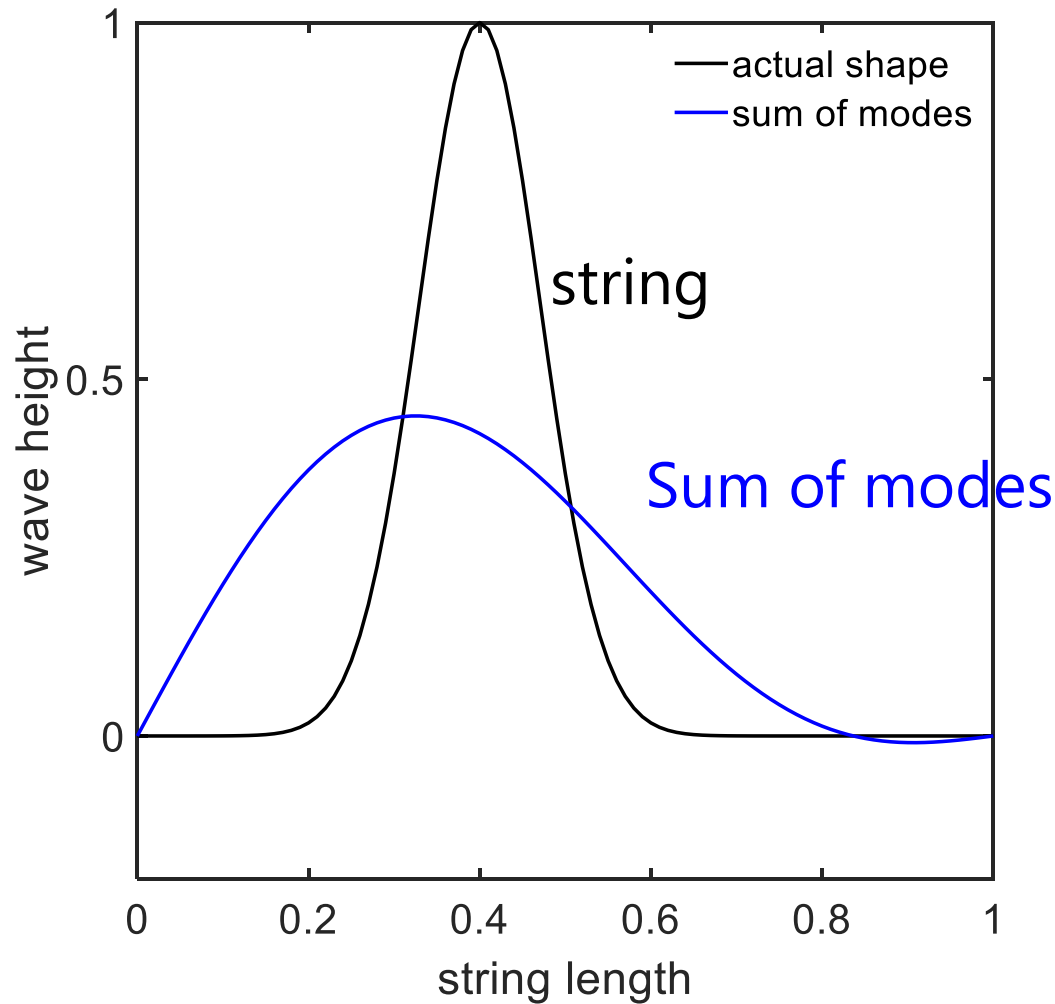


DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



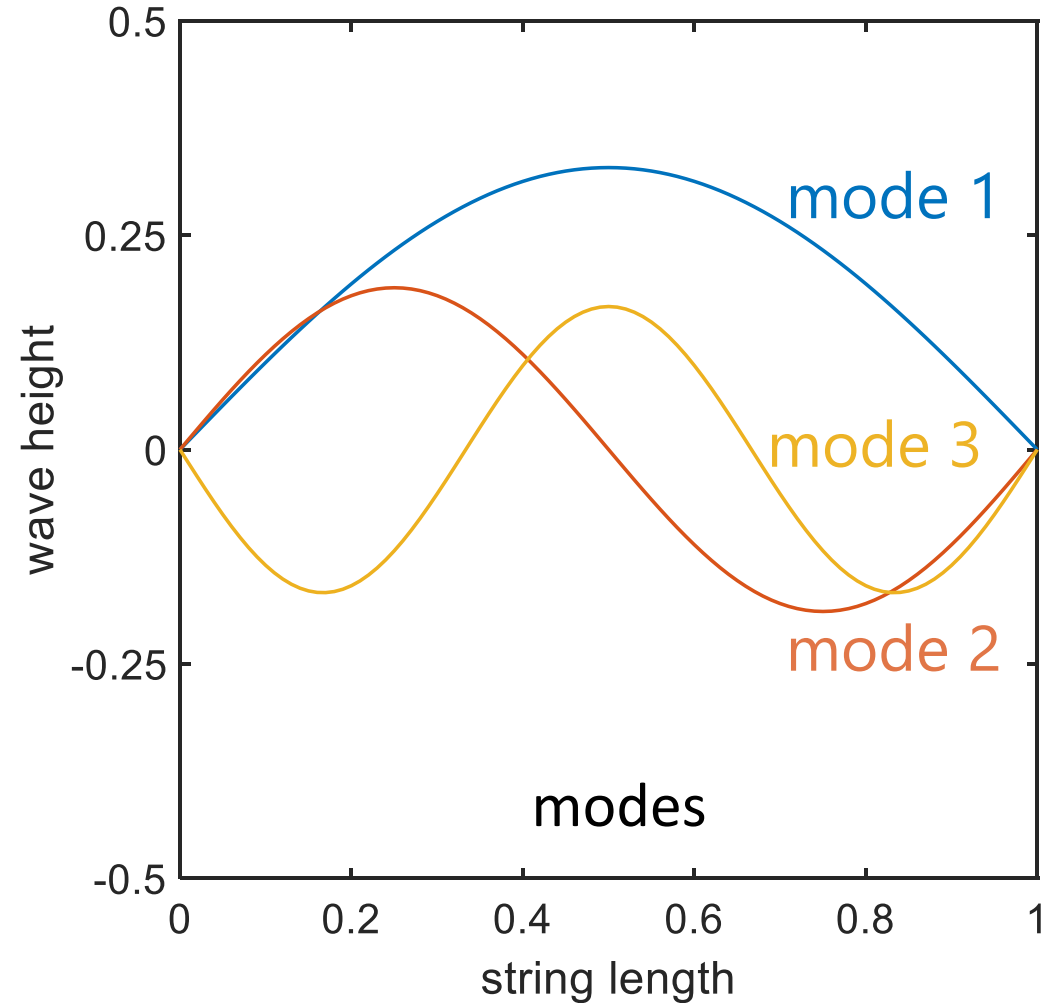
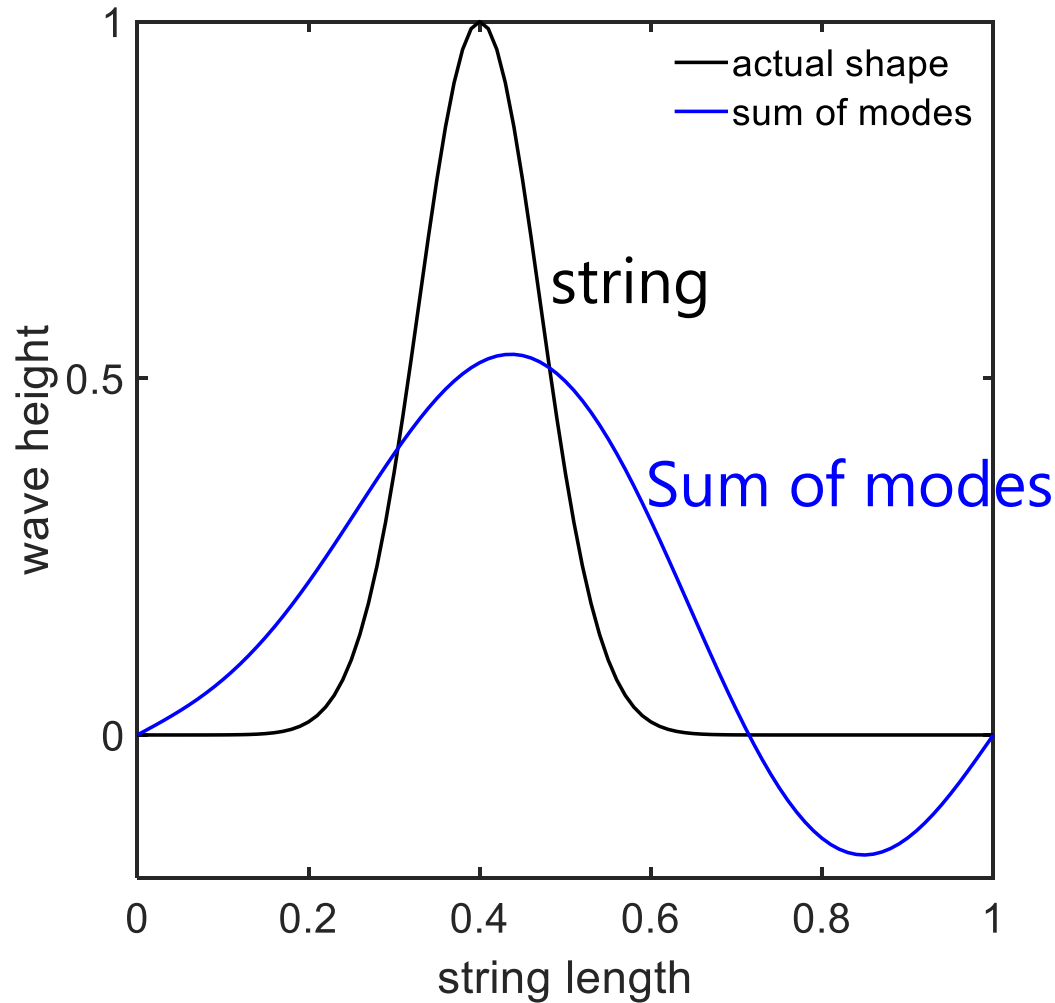
With just the first mode, the match is not very good

DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



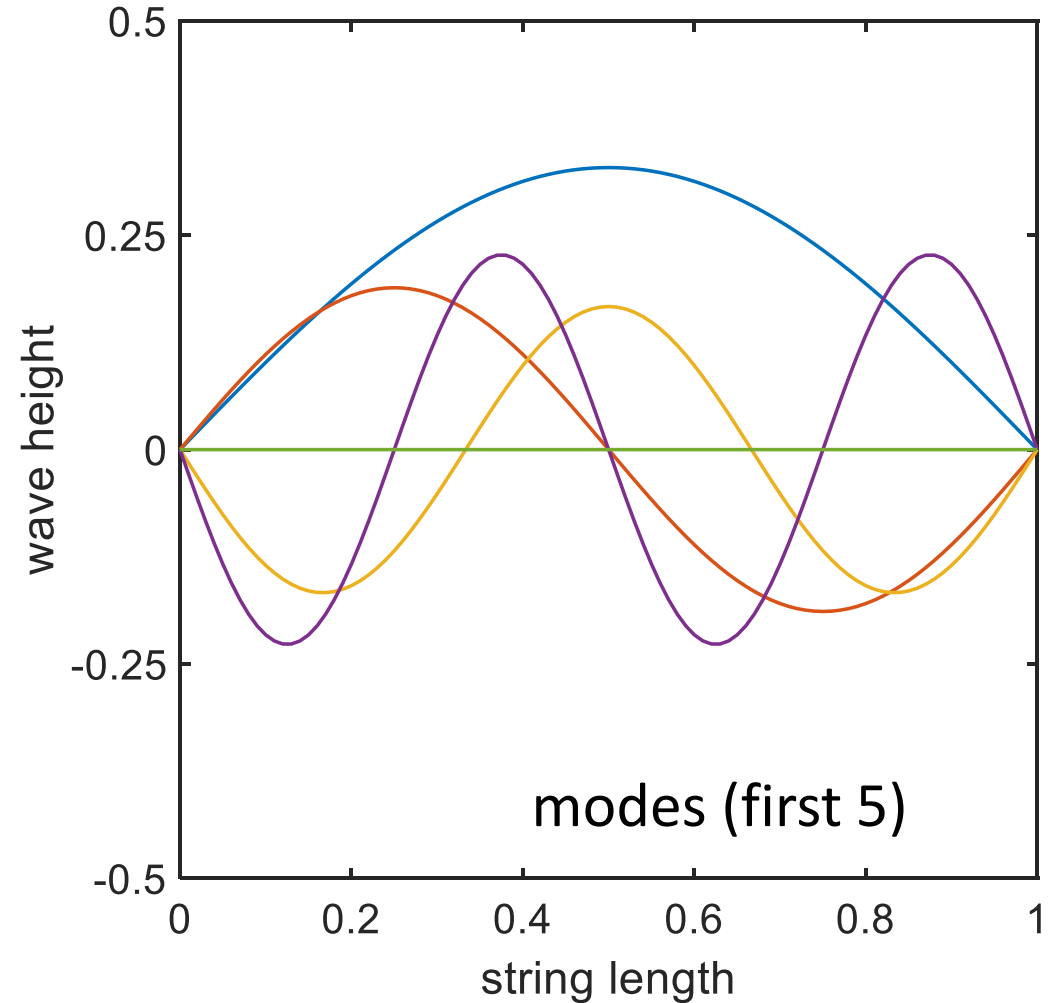
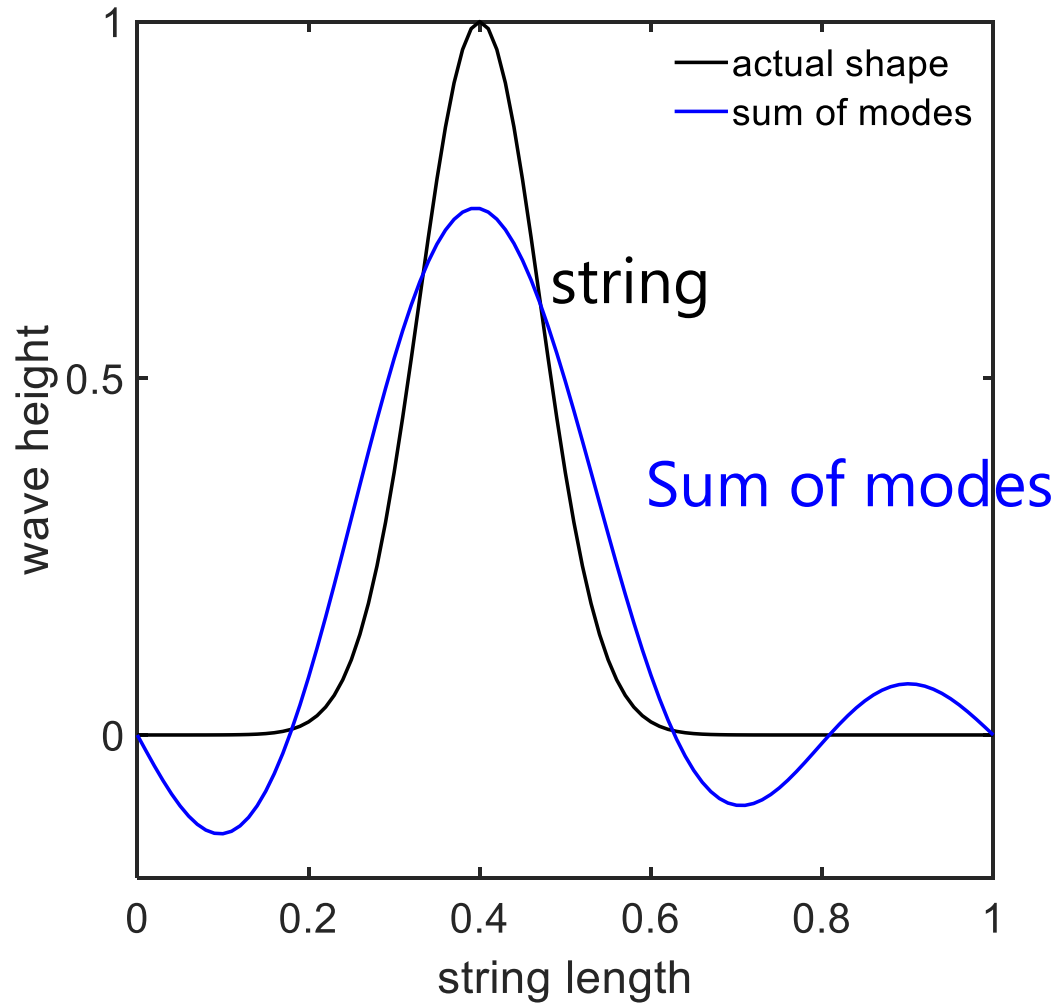
As we increase the number of modes, the match improves!

DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



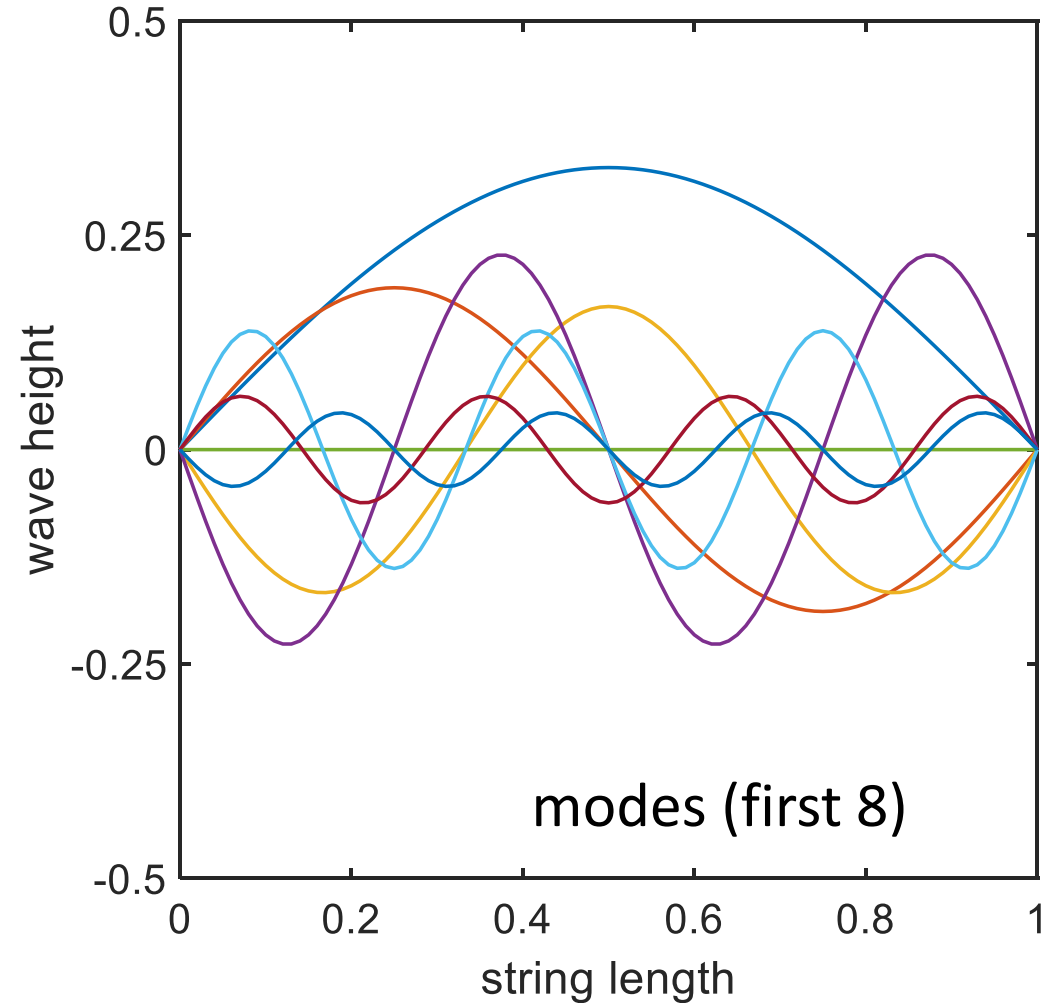
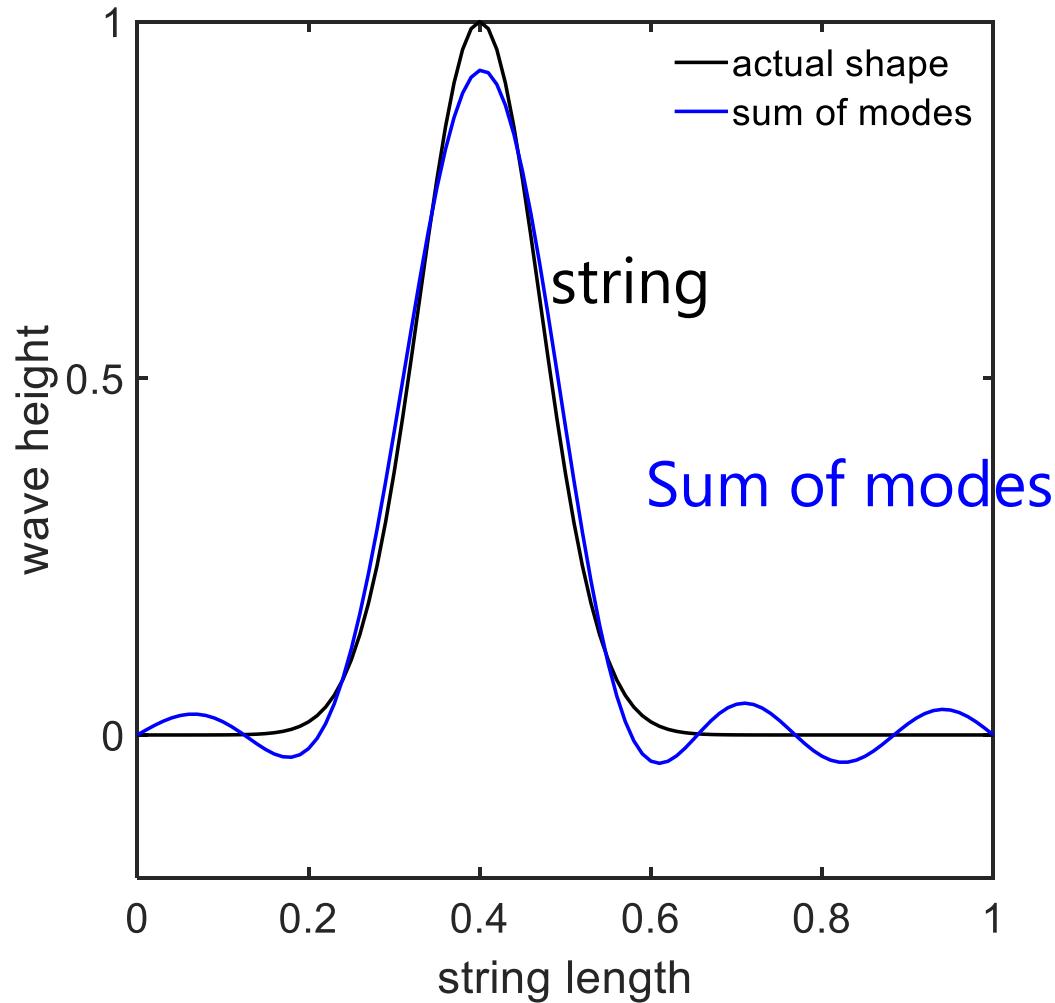
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DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



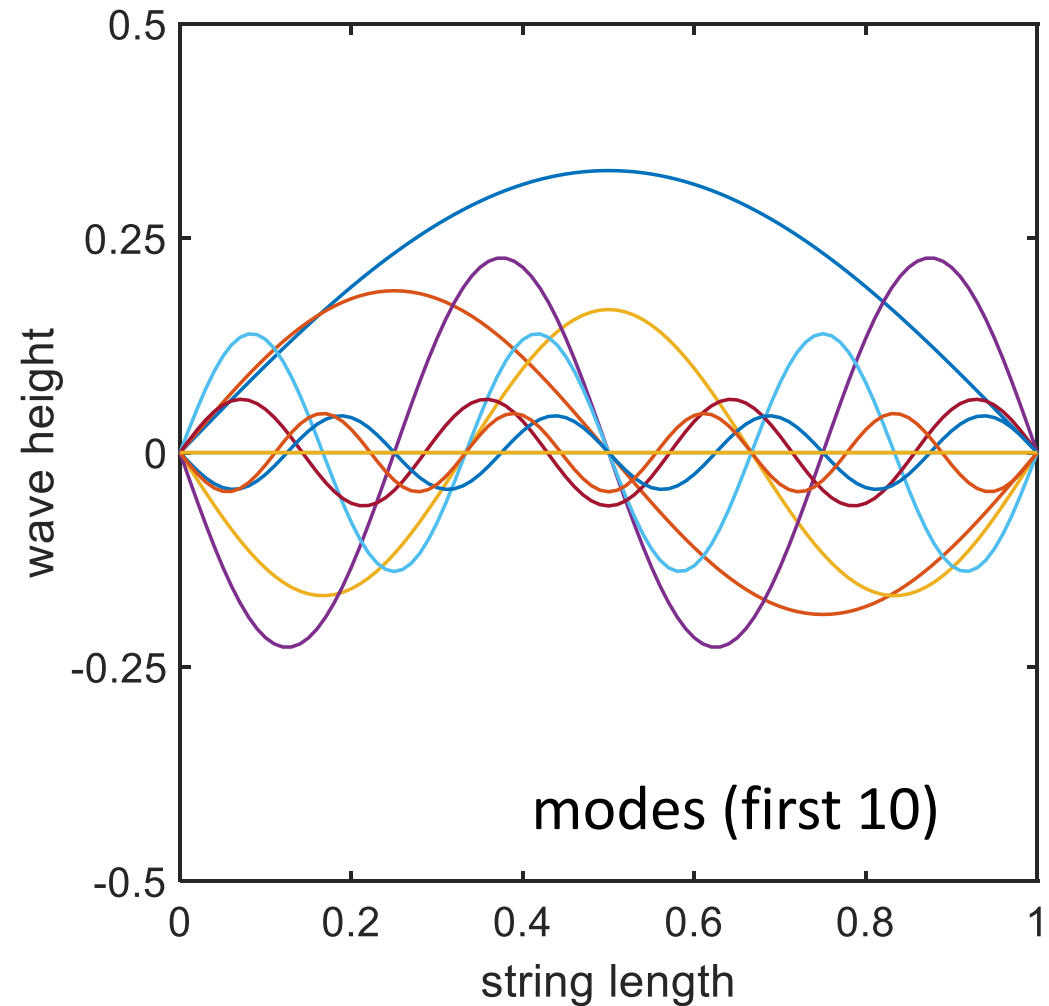
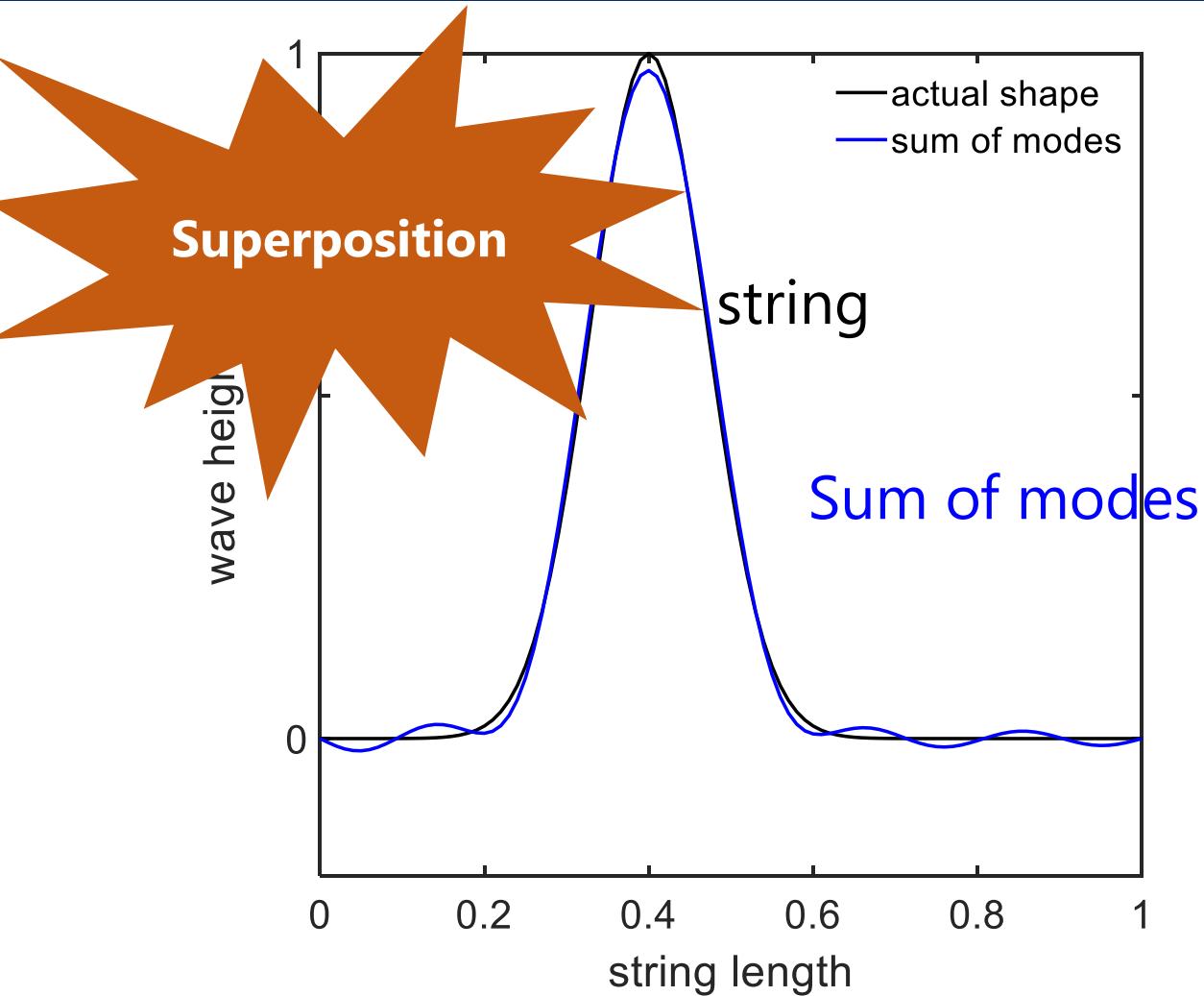
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DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



As we increase the number of modes, the match improves!

DESCRIBING THE RIPPLE WITH EIGENFUNCTIONS



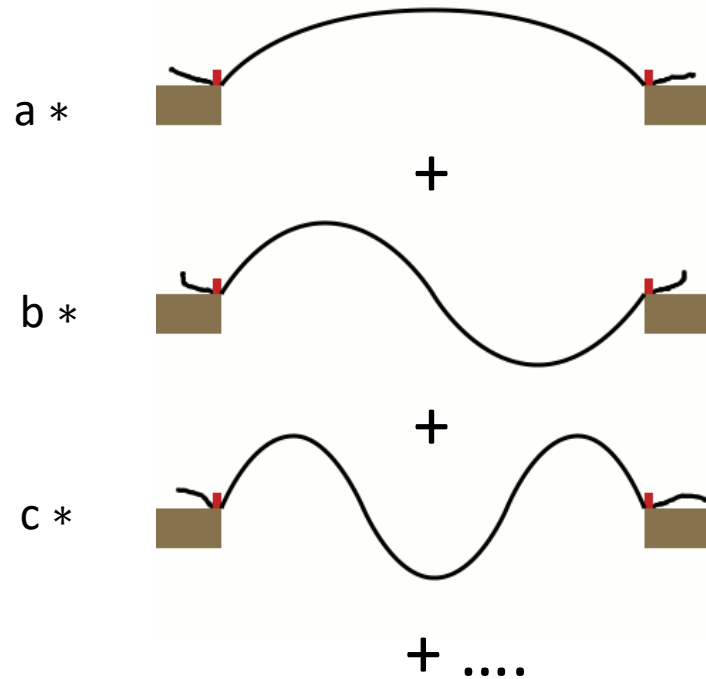
As we increase the number of modes, the match improves!

MODE SUPERPOSITION

On a string

In a qubit

actual
shape of =
string



$$|\psi\rangle = 0.71|0\rangle + 0.71|1\rangle$$

This method of expressing a function as a superposition of modes is known as Fourier analysis

OBSERVATION POSTULATE

- The only possible results of measuring an observable are the eigenvalues of its operator
- **On the guitar:** If you try to see the string, you will only see one of the modes, not the ripple

PROBABILITY POSTULATE

- Probability of observing an eigenvalue = modulus square of inner product of corresponding eigenvector with $|\psi\rangle$
- **On the guitar:** When you see the string, the probability of seeing a mode depends on its contribution to the initial wavefunction (the ripple when you pluck the string)

COLLAPSE POSTULATE

- After measurement, $|\psi\rangle$ is equal to the eigenvector corresponding to the observed eigenvalue
- **On the guitar:** After you see the guitar string, it remains in the mode you observed