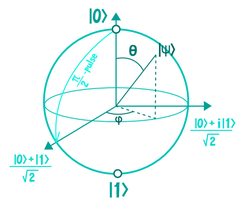
VECTORS AND INTRO TO MATRICES

QUBIT X QUBIT

INTRO TO MATRICES

WHAT DO MATRICES MEAN FOR Q.COMP?



As we saw, we represent q-states as vectors

Our goal is to manipulate these states

q.algorithms

q.gates → Matrices

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

MATRIX

We can think of a matrix as a collection of row vectors or a collection of a column vectors

$$X=\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

Geometrically, they are transformations that allow us to both rotate and scale vectors

MATRIX NOTATION AND SHAPE

An (n x m) matrix is written as

$$X=\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

Matrix Shape: (#rows x #cols)

SOLVING LINEAR SYSTEMS OF EQUATIONS

The identity matrix

It's defined as

$$I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

$a \cdot 1 = a$

$AI = IA = A$

$\vec{v}^{\dagger} I = \vec{v}^{\dagger}$ $\quad I\vec{v} = \vec{v}$

Matrix inversion

$$XX^{-1} = X^{-1}X = I$$

X^{-1} is thus the inverse of Matrix X

Many matrices do not have inverses

MATRIX ADDITION

General equation (only add matrices of the same shape)

$$A+B=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nm} \end{pmatrix}=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1m}+b_{1m} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2m}+b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nm}+b_{nm} \end{pmatrix}$$

MATRIX-VECTOR MULTIPLICATION

General equation (the vector height must match the matrix width)

$$A\vec{x}=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}=\begin{pmatrix} a_{11}x_1 & a_{12}x_2 & \cdots & a_{1m}x_m \\ a_{21}x_1 & a_{22}x_2 & \cdots & a_{2m}x_m \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}x_1 & a_{n2}x_2 & \cdots & a_{nm}x_m \end{pmatrix}=\begin{pmatrix} \langle\vec{a}_1,\vec{x}\rangle \\ \langle\vec{a}_2,\vec{x}\rangle \\ \vdots \\ \langle\vec{a}_n,\vec{x}\rangle \end{pmatrix}$$

MATRIX TRAPOSE

If $X=\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$, then $X^{T}=\begin{pmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1m} & x_{2m} & \cdots & x_{nm} \end{pmatrix}$

Matrix conjugate trapose

If $X=\begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$, then $X^{\dagger}=\begin{pmatrix} x_{11}^{*} & x_{21}^{*} & \cdots & x_{n1}^{*} \\ x_{12}^{*} & x_{22}^{*} & \cdots & x_{n2}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1m}^{*} & x_{2m}^{*} & \cdots & x_{nm}^{*} \end{pmatrix}$

MATRIX-SCALAR MULTIPLICATION

General equation

$$c\cdot A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}=\begin{pmatrix} c\cdot a_{11} & c\cdot a_{12} & \cdots & c\cdot a_{1m} \\ c\cdot a_{21} & c\cdot a_{22} & \cdots & c\cdot a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c\cdot a_{n1} & c\cdot a_{n2} & \cdots & c\cdot a_{nm} \end{pmatrix}$$

MATRIX-MATRIX MULTIPLICATION

General equation (the 1st matrix width must match the second matrix height)

$$AB=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1k} \\ b_{21} & b_{22} & \cdots & b_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mk} \end{pmatrix}=\begin{pmatrix} \langle\vec{a}_1,\vec{b}_1\rangle & \langle\vec{a}_1,\vec{b}_2\rangle & \cdots & \langle\vec{a}_1,\vec{b}_k\rangle \\ \langle\vec{a}_2,\vec{b}_1\rangle & \langle\vec{a}_2,\vec{b}_2\rangle & \cdots & \langle\vec{a}_2,\vec{b}_k\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\vec{a}_n,\vec{b}_1\rangle & \langle\vec{a}_n,\vec{b}_2\rangle & \cdots & \langle\vec{a}_n,\vec{b}_k\rangle \end{pmatrix}$$

LECTURE 4