INTRO TO QUANTUM COMPUTING

Week 9 Lab

MATH FOR QUANTUM CONTD.

Sarah Muschinske

12-15-2020

PROGRAM FOR TODAY

- Logistics
- Attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz passcode: 3222

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number: **3**
  - Passcode: **3222**
- If you have NOT attended the Friday Student Assistant Office Hours, why not? Select all that apply.
- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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QUESTIONS FROM LAST WEEK

$$\langle\psi|\psi\rangle = \psi^\dagger \psi$$

$\dagger \equiv *T$ ← transpose

↑ conjugate

$$\psi = ix \quad -iy = \begin{pmatrix} ix \\ -iy \end{pmatrix}$$

$$\psi^\dagger\psi = i(-i)x \cdot x - i(-i)y \cdot y$$

$$= x^2 - y^2$$

$$|\psi\rangle = \left(|0\rangle + e^{i\phi}|1\rangle\right) e^{i\theta}$$

LEARNING OBJECTIVES FOR LAB 5 X9

- Understanding vector spaces and their properties
  - Linear combinations
  - Linear independence
  - Span and Basis vectors
- Demystifying eigenvalues and eigenvectors
  - Applying gates to qubits: Review
  - Eigenvectors for the Z and X matrices
  - Applying gates to qubits using eigenvectors
- TA discussion*

*Optional content

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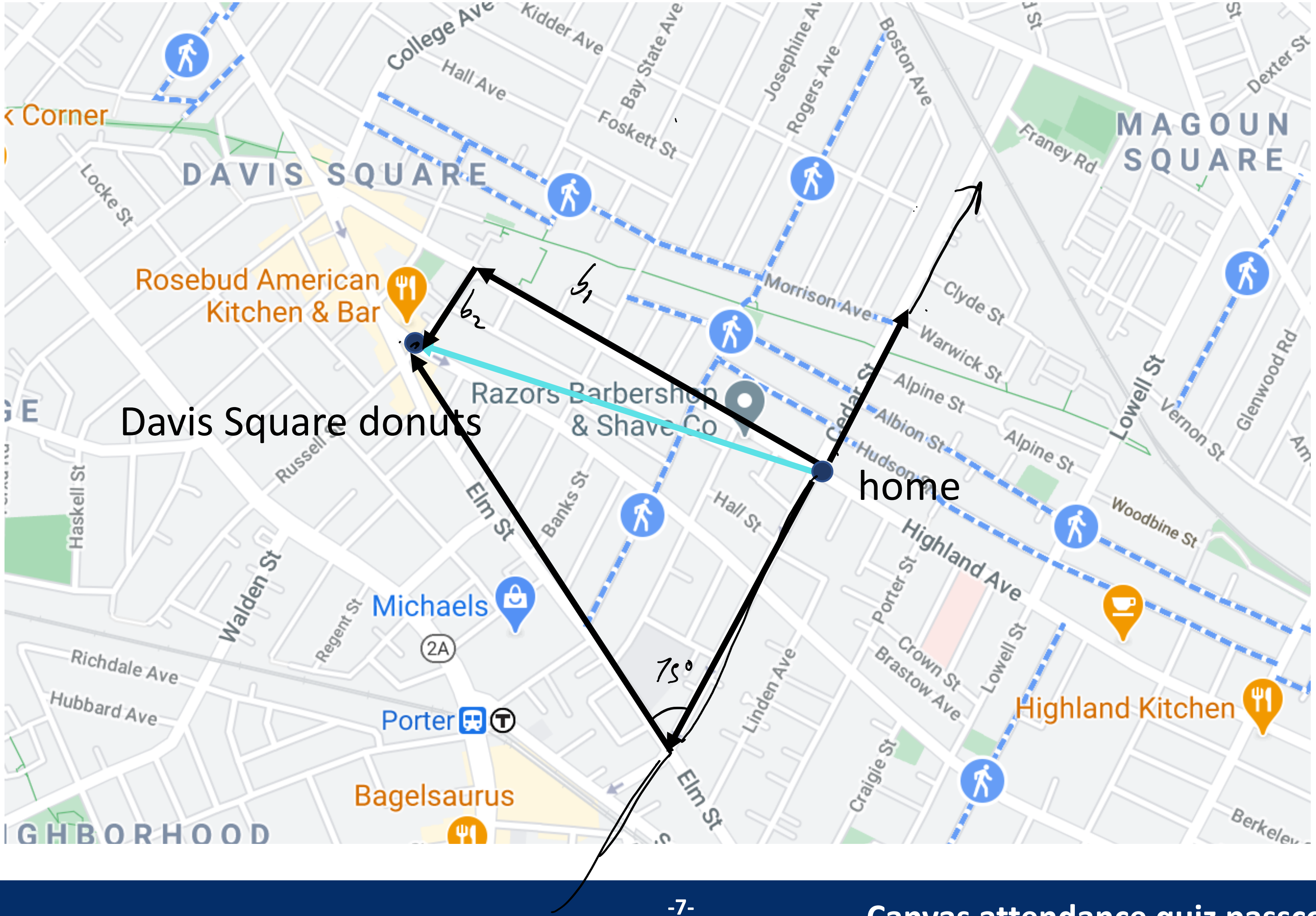
GETTING DONUTS



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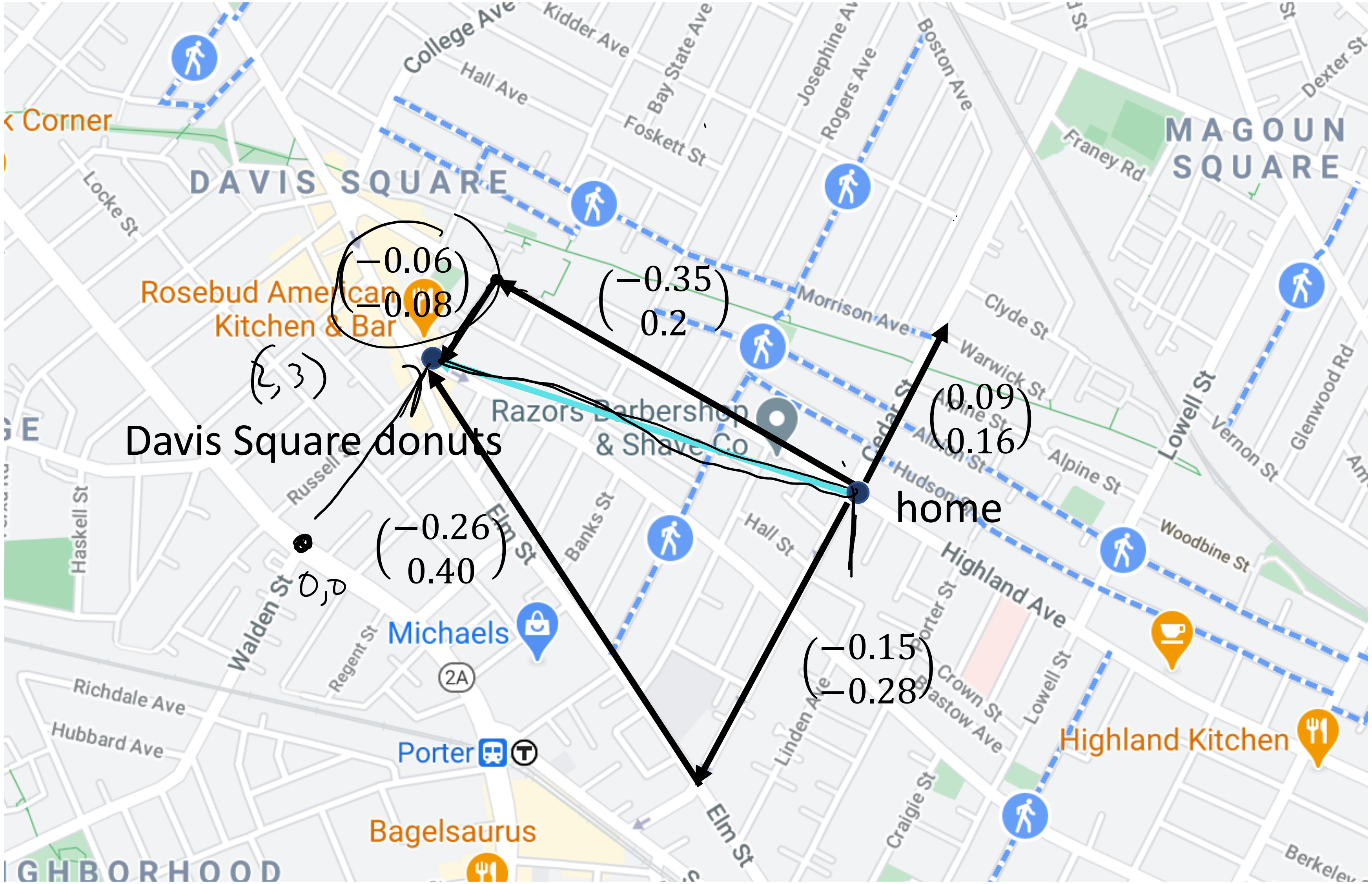
GETTING DONUTS



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LINEAR COMBINATIONS

x, y, z

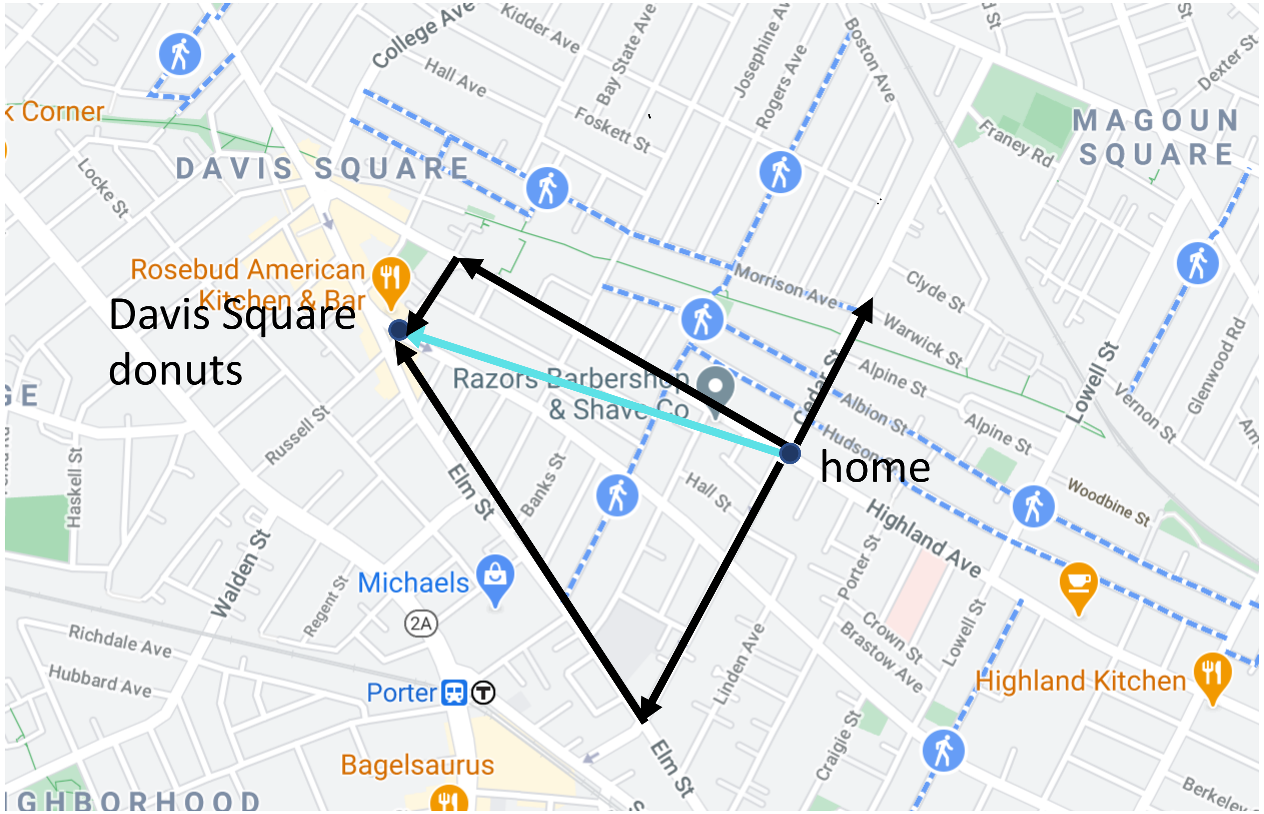
Linear combinations of vectors

$\vec{x}^2 = \vec{x}^T x$

$$y = x + x^2 + x^3 + \cdots$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1^2 \\ x_2^2 \end{pmatrix} \neq \begin{pmatrix} A \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Linear



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EXPRESSING SUPERPOSITION STATES

Superposition state: a **linear combination** of $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{pmatrix} |0\rangle, & |1\rangle \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$$e^{i\pi} = -1$$

$$|\psi_3\rangle = \frac{3i}{\sqrt{10}}|+\rangle - \frac{1}{\sqrt{10}}|-\rangle$$

Can make linear combinations from any vectors!

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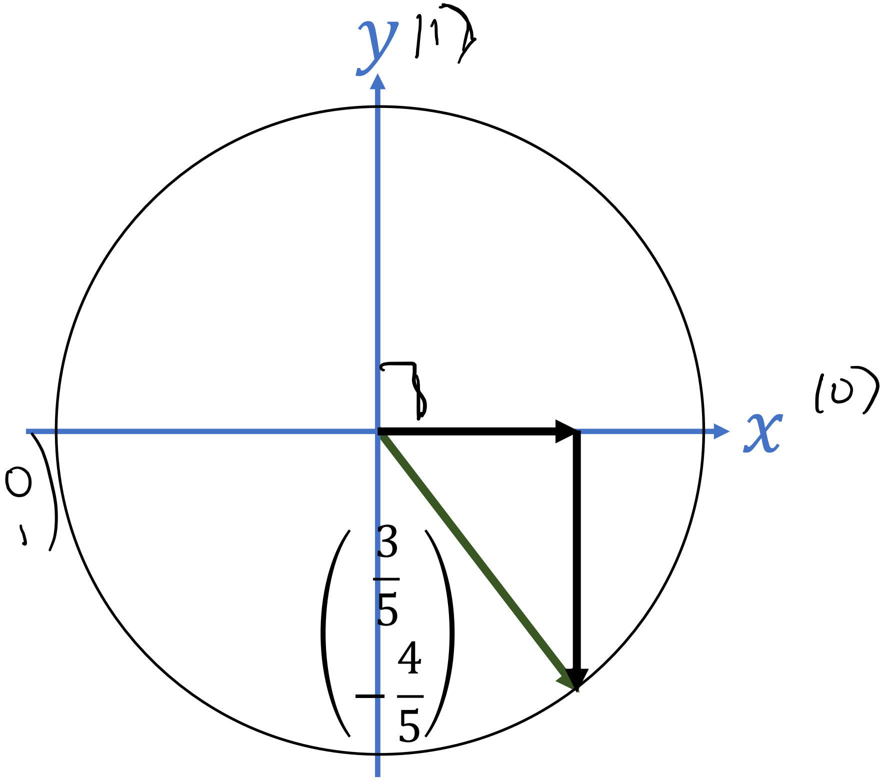
WRITING STATES AS LINEAR COMBINATIONS

Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\frac{3}{5}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(-\frac{4}{5}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



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WRITING STATES AS LINEAR COMBINATIONS

Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Rearrange

$$\frac{1}{\sqrt{2}}\alpha + \frac{4}{5} = \frac{1}{\sqrt{2}}\beta$$

$$= -\frac{1}{10} + \frac{4}{5} = \frac{1}{\sqrt{2}}\beta$$

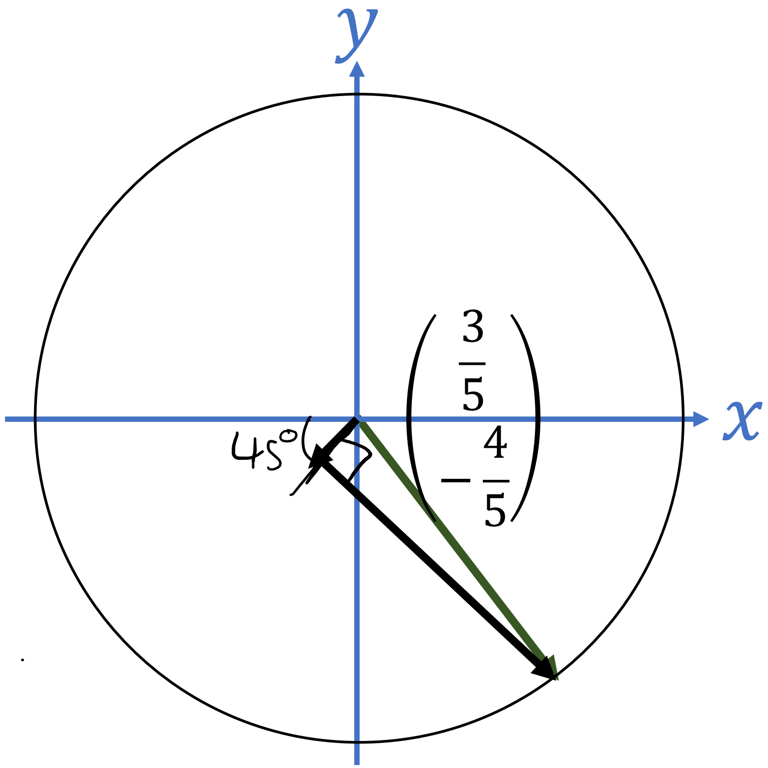
$$\Rightarrow \boxed{\beta = \frac{7}{10}\sqrt{2}}$$

$$\frac{1}{\sqrt{2}}\alpha + \frac{1}{\sqrt{2}}\beta = \frac{3}{5}$$

$$+$$

$$\frac{1}{\sqrt{2}}\alpha - \frac{1}{\sqrt{2}}\beta = -\frac{4}{5}$$

$$= \frac{2}{\sqrt{2}}\alpha = -\frac{1}{5} \Rightarrow \alpha = -\frac{1}{5\sqrt{2}}$$



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WRITING STATES AS LINEAR COMBINATIONS

Can we do this with any two vectors?

Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $\underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{and } \frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix}}$

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LINEAR INDEPENDENCE

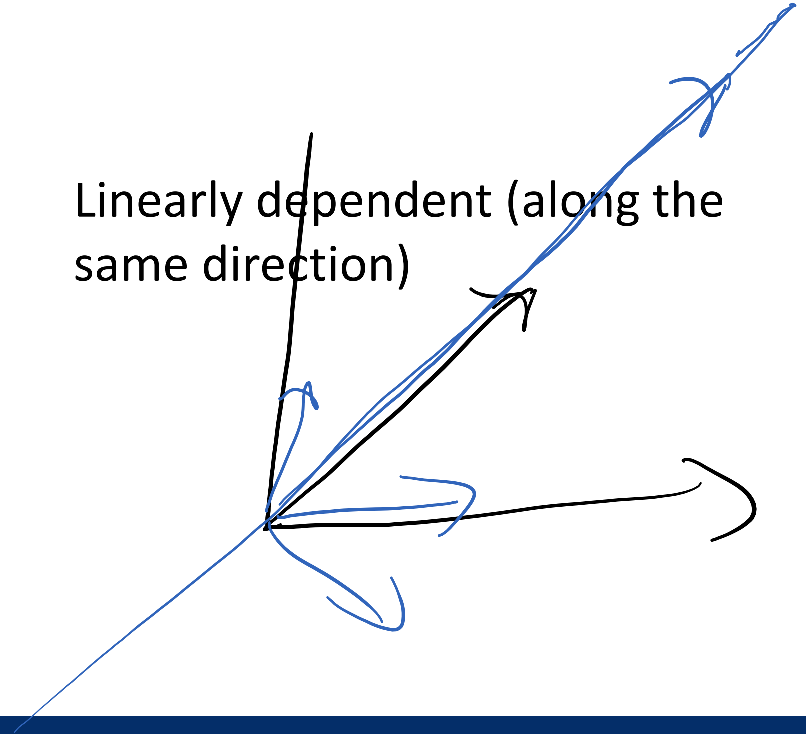
- We need the two (or more) vectors to be in **different** directions!

Write the state $\underline{\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}}$ as a combination of $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$and $\frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$$\frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow -\frac{4}{\sqrt{8}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Linearly dependent (along the same direction)



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LINEAR INDEPENDENCE

- **Linearly independent** vectors – Vectors that cannot be written as combinations of each other

$$\alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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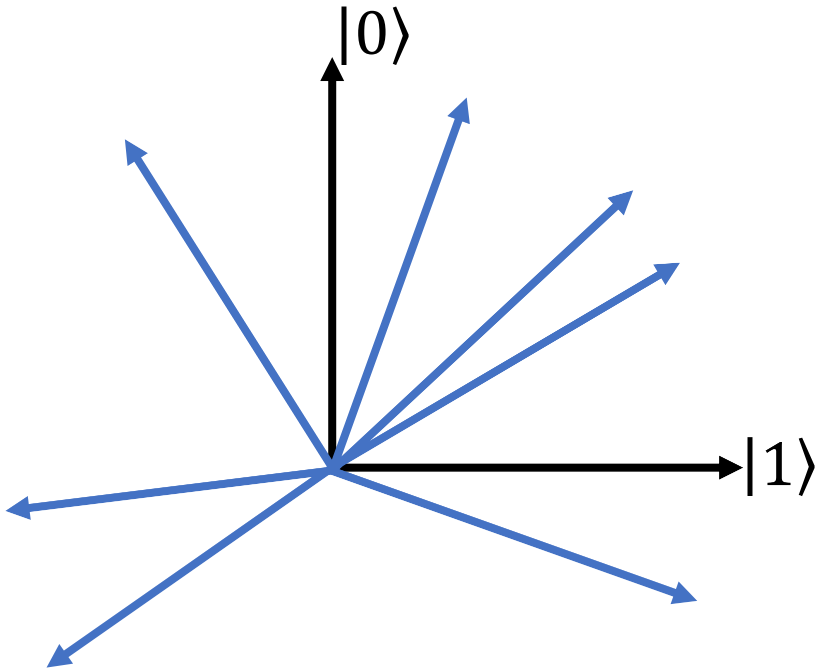
SPAN

- Span: All the vectors that can be written as combinations of the given vector set

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



....and infinitely more

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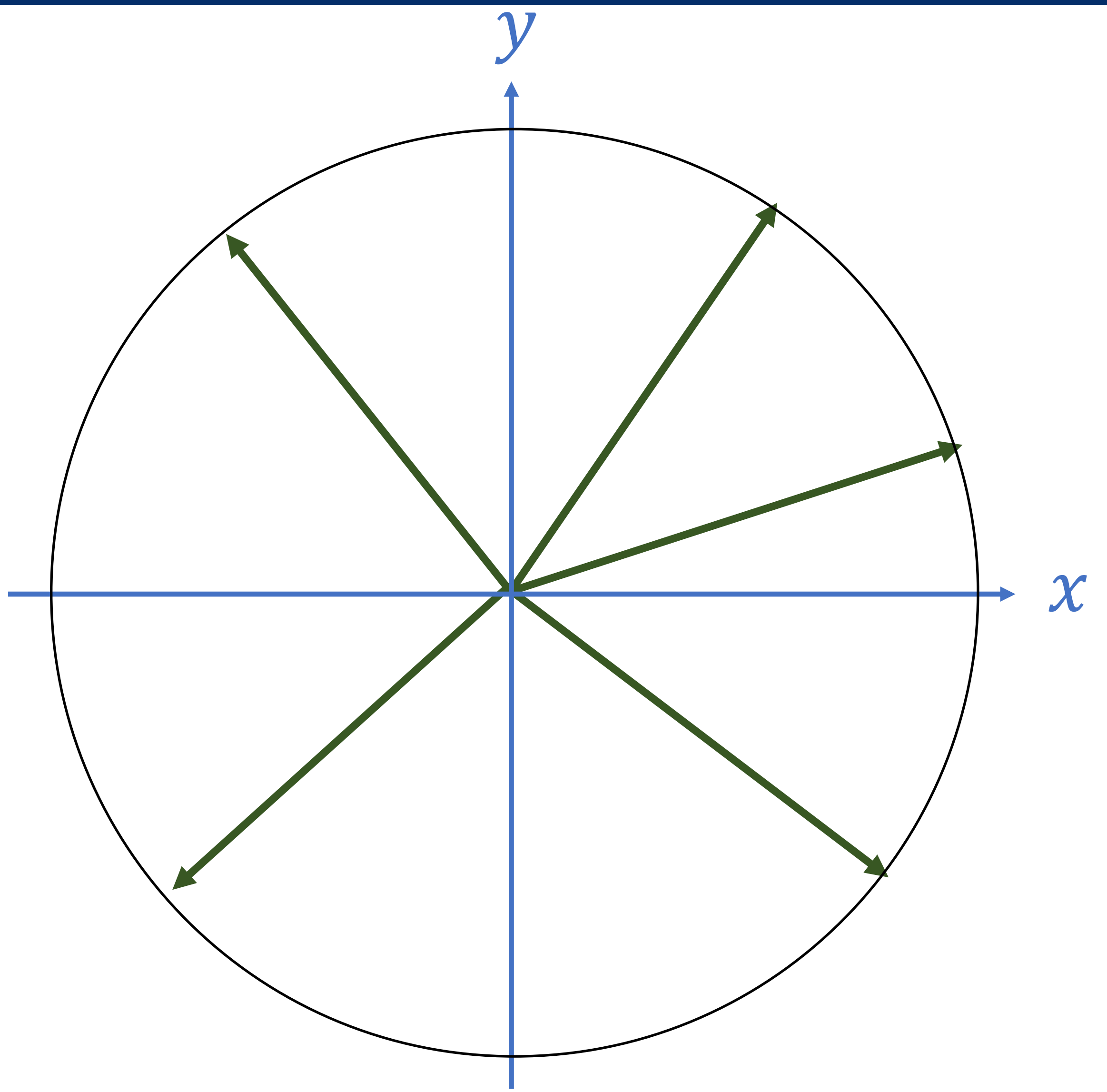
VECTOR SPACE

- **Vector space:** A collection of vectors (states)

- **Examples**
  - All 2-D vectors that line on the unit circle
  - All vectors on the surface of the Bloch sphere



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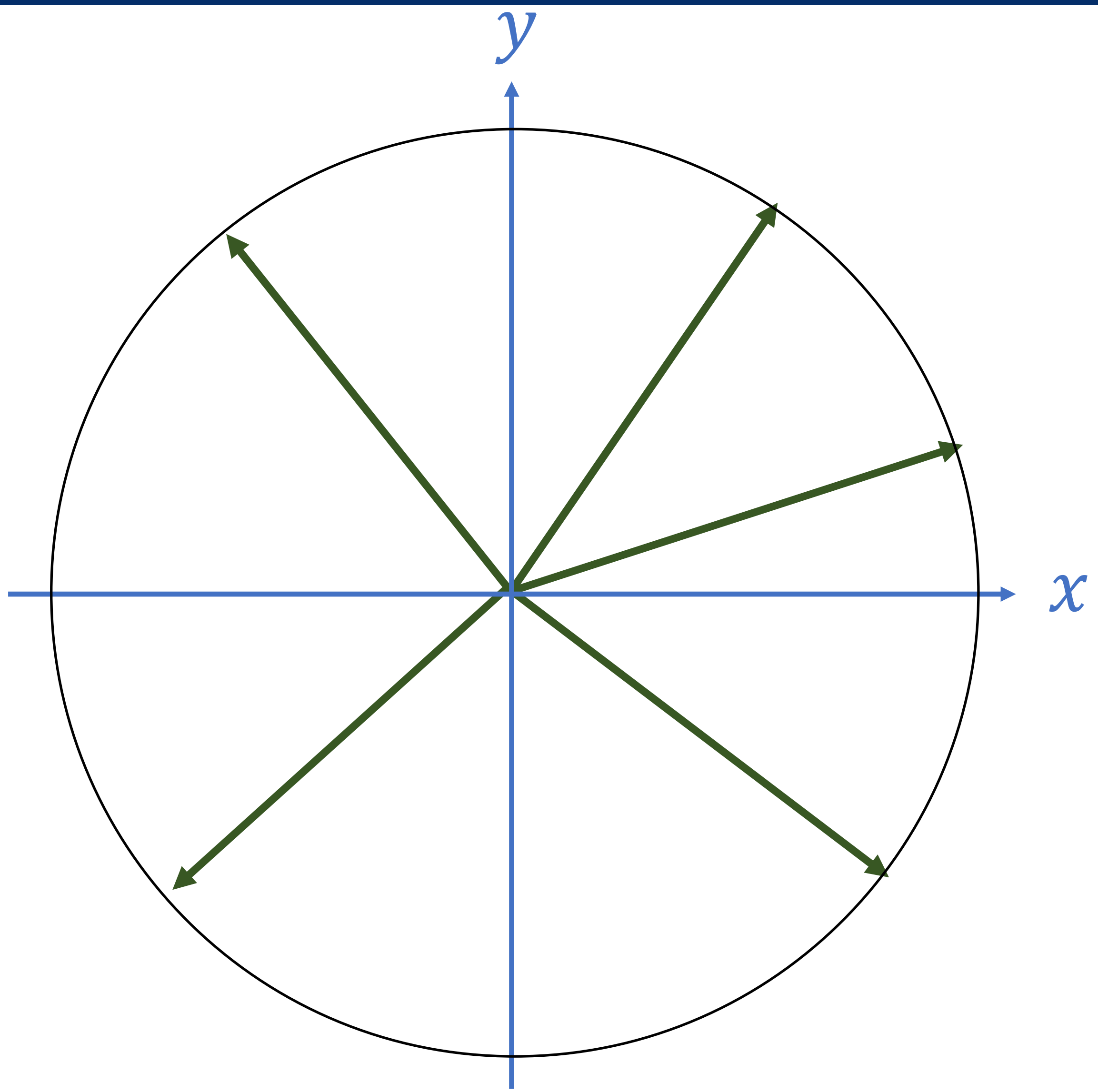
BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- Yes!

- These two vectors form a "basis" for this vector space – they are called **basis vectors**



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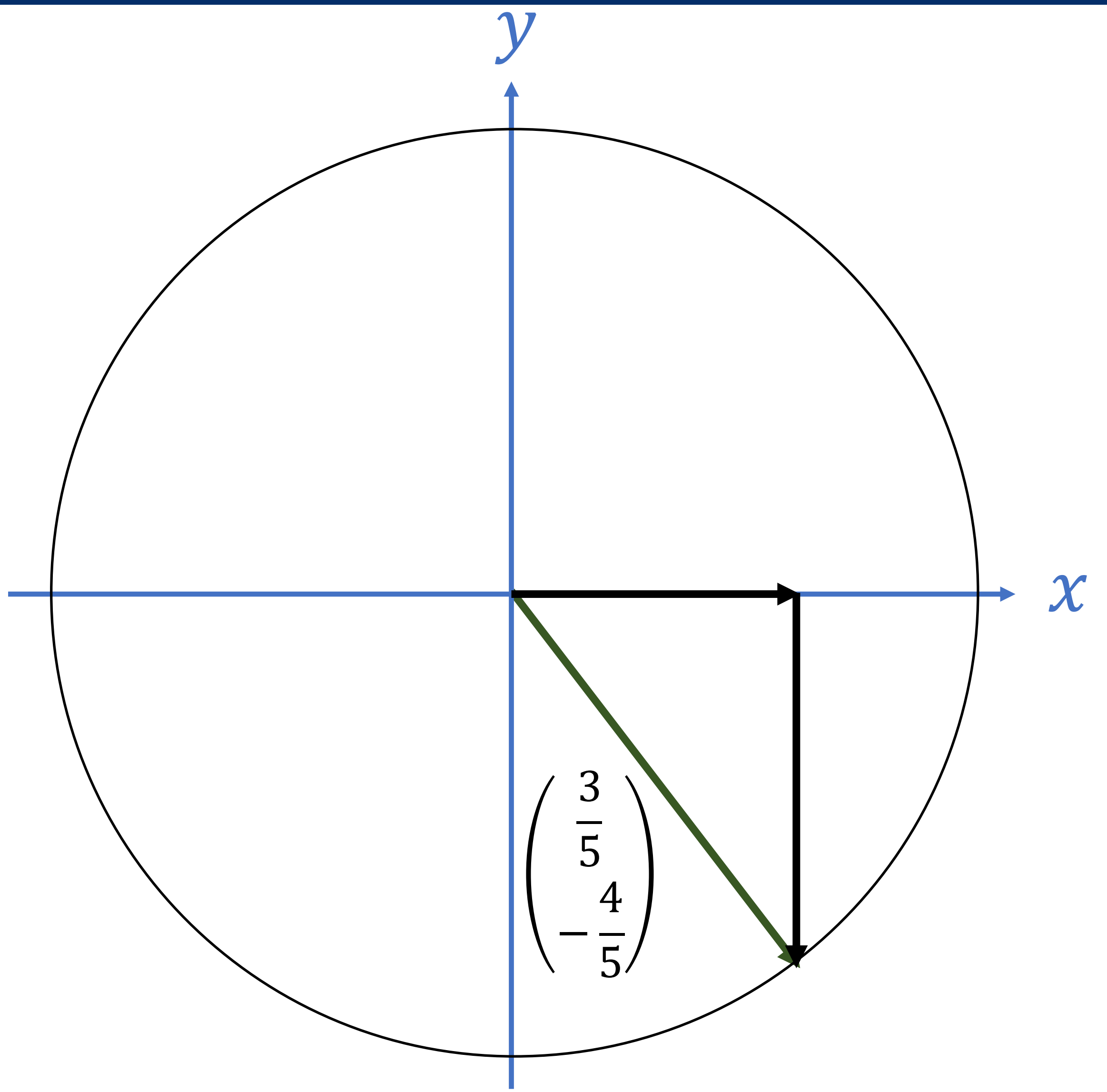
BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- Example: $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = \frac{3}{5}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{4}{5}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



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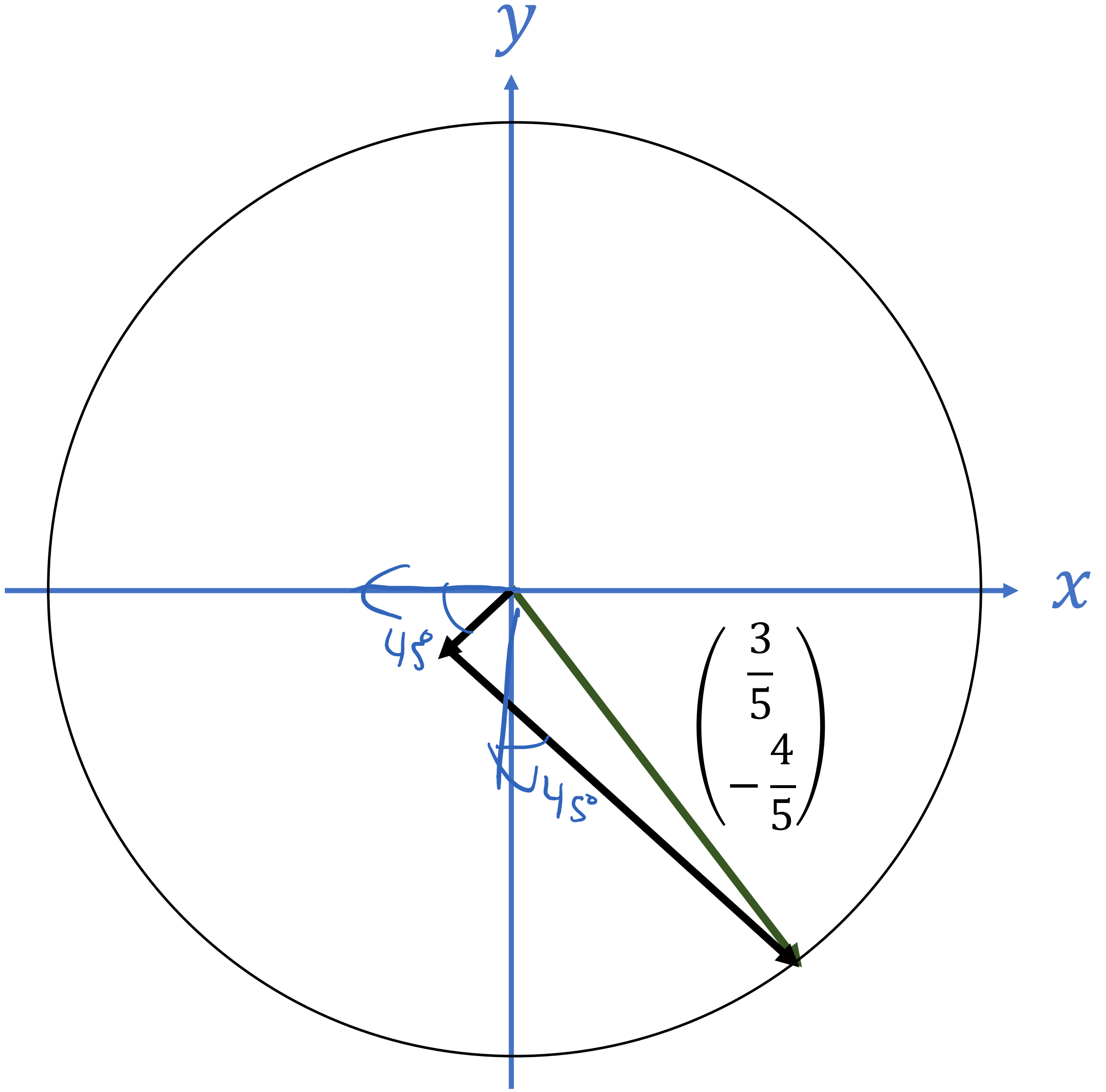
ANOTHER BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- There are other options!
- Example: $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = -\frac{\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{7\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

- There are an infinite number of choices for basis!



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QUESTIONS

Questions on content so far?

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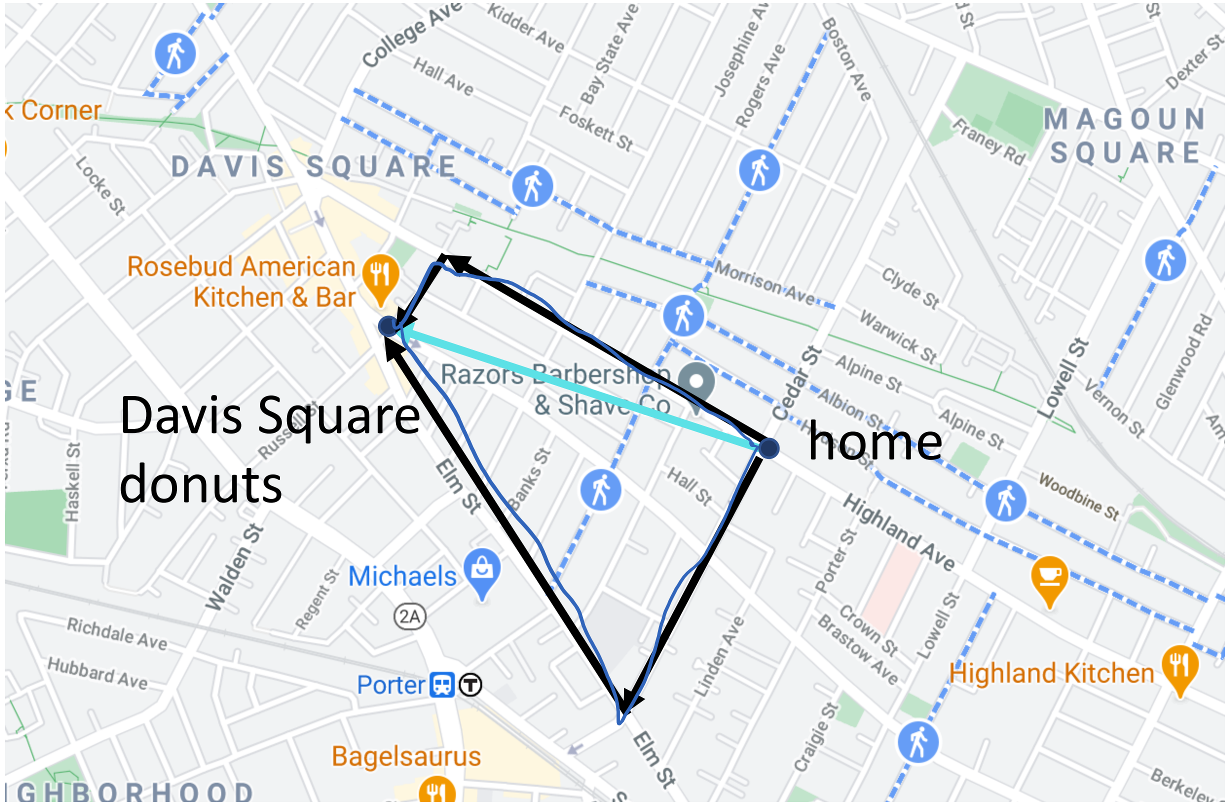
WHY USE DIFFERENT BASES?

Same vector, two bases:

$$\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = \frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} + \frac{7\sqrt{2}}{10}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix}$$



But why?

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APPLYING GATES TO QUBITS: REVIEW

$$|\psi\rangle_{in} \longrightarrow \boxed{Z} \longrightarrow |\psi\rangle_{out}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\psi\rangle_{out} = Z|\psi\rangle_{in}$$

Multiply the matrix of Z with the column vector $|\psi\rangle_{in}$ to get $|\psi\rangle_{out}$

Example: Find $Z\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$

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APPLYING GATES TO QUBITS: REVIEW

Find $Z\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$ and $Z\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

$|0\rangle$

$= 1\begin{pmatrix}1\\0\end{pmatrix}$

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linearly independent

↓

every row gives an eigenvalue & eigenvector

$$Z\begin{pmatrix}1\\0\end{pmatrix} = \underbrace{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}_{\text{2 eigenvalues \& 2 eigenvectors}}\begin{pmatrix}1\\0\end{pmatrix} = 1\cdot\begin{pmatrix}1\\0\end{pmatrix}$$

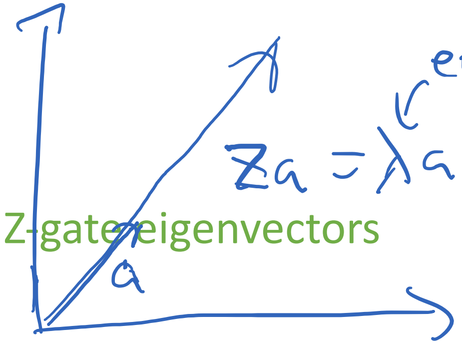
$$= \frac{1}{2}\begin{pmatrix}2\\0\end{pmatrix}$$

eigenvalues

Z-gate eigenvectors

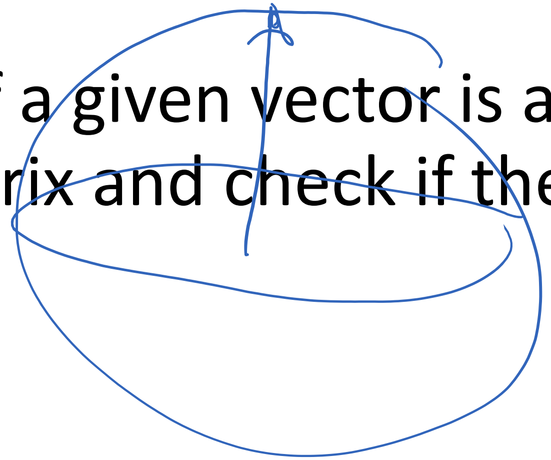
eigenvalue

$Za = \lambda a$



$$Z\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \underbrace{-1}_{\lambda}\cdot\begin{pmatrix}0\\1\end{pmatrix}$$

$|0\rangle$



Takeaway: To find if a given vector is an eigenvector of a matrix, multiply the vector with the matrix and check if the result is a scalar times the same vector

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WHY USE EIGENVECTORS?

Find $Z\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$

$$\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \underbrace{\frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}\begin{pmatrix}0\\1\end{pmatrix}}$$

$$Z\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \frac{3}{5}Z\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}Z\begin{pmatrix}0\\1\end{pmatrix} = \underbrace{\frac{3}{5}(1)\begin{pmatrix}1\\0\end{pmatrix}} - \underbrace{\frac{4}{5}(-1)\begin{pmatrix}0\\1\end{pmatrix}}$$

$$= \frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} + \frac{4}{5}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}$$

Takeaway: Applying a gate to qubit states is really easy, if you can write the vector as a linear combination of the eigenvectors of that gate!

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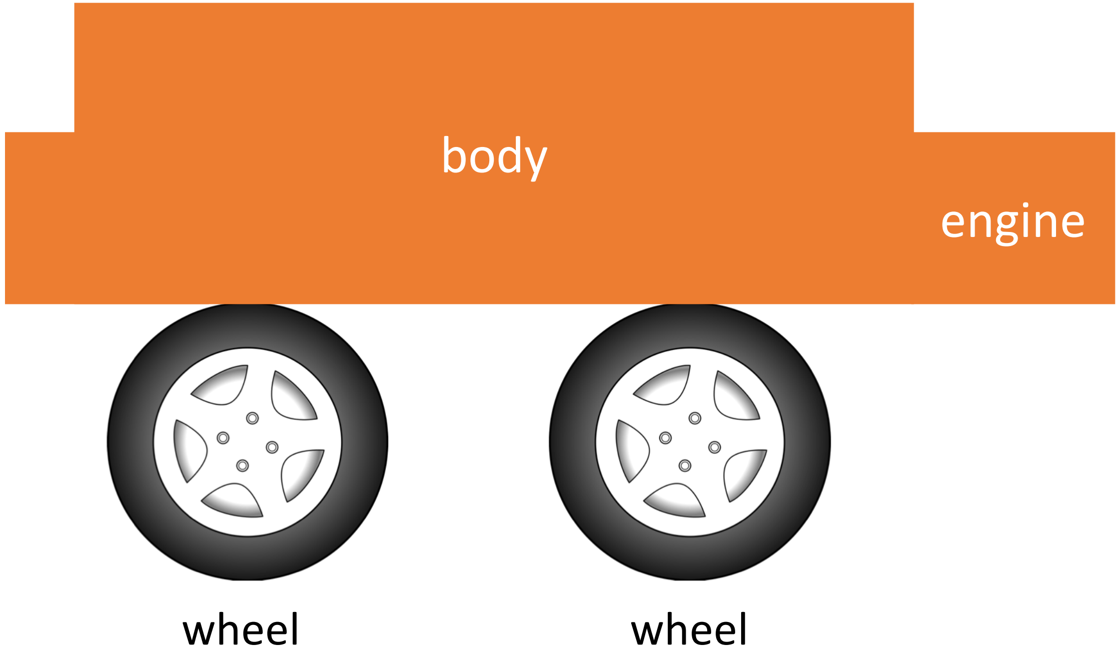
CARS VS EIGENVECTORS

Car #1



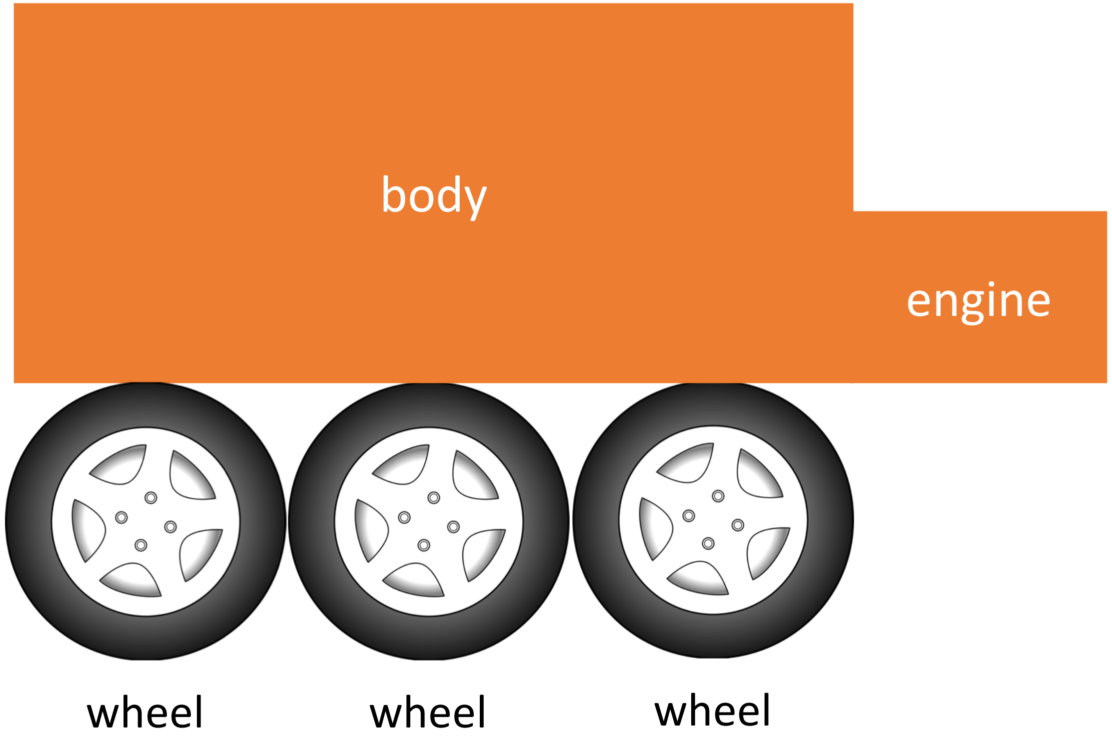
Car #1 = 4 * wheel +

 * body

+  * engine

CARS VS EIGENVECTORS

Car #2



Car #2 = 6 * wheel +

* body

+  * engine

- We know how to work with wheels, engines and bodies, and we combine them in different ways to make different cars
- We know how to work with eigenvectors, and we combine them in different ways to make different qubit states

EIGENVECTORS FOR THE X GATE

Example: Find $X\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

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EIGENVECTORS FOR THE X GATE

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = 1\cdot\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

$|+\rangle$

eigenvalues

X-gate eigenvectors

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = -1\cdot\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix}$$

$|-\rangle$

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USING EIGENVECTORS OF THE X GATE

Find $X\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\underbrace{\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}}$

$$\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}\overset{|+\rangle}{\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}} + \frac{7\sqrt{2}}{10}\overset{|-\rangle}{\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}}$$

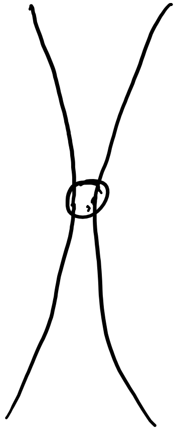
$$X\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}\left(X\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}\right) + \frac{7\sqrt{2}}{10}\left(X\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}\right) = X\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}\underset{\lambda_1}{(1)}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix} + \frac{7\sqrt{2}}{10}\underset{\lambda_2}{(-1)}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{-4}{5}\\\frac{3}{5}\end{pmatrix}$$

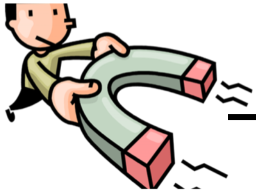
Takeaway: Applying a gate to qubit states is really easy, if you can write the vector as a linear combination of the eigenvectors of that gate!

IMPORTANT TAKEAWAYS

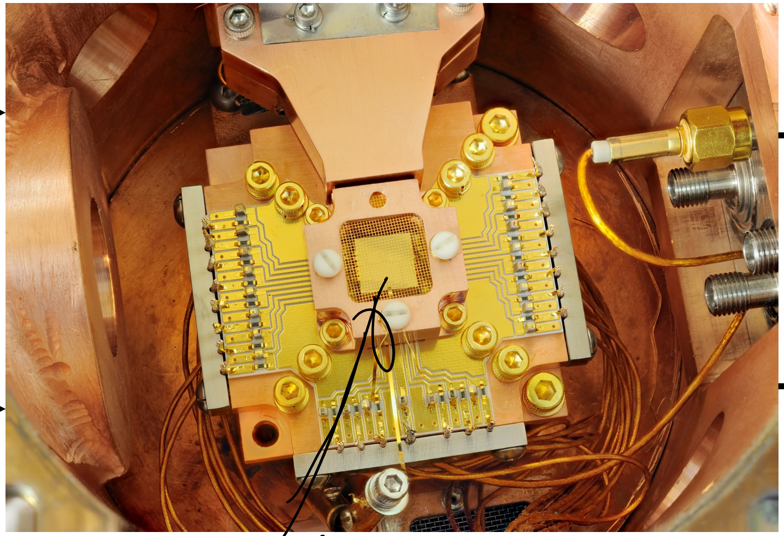
- **Vector space**: A collection of vectors
  - A given qubit state in a vector space can be expressed as a combination of two (or more) states in the space
  - **Linear independence:** These states cannot themselves be written as combinations of one another
  - **Basis**: These states can be combined to express every other state in the vector space
- Applying a gate to its **eigenvector** results in the same vector times a constant (called the **eigenvalue**)
- Applying a gate to any other vector can be broken down into two steps
  - Express the vector as a linear combination of the eigenvectors of the gate
  - Apply the gate to the eigenvectors to find the result

WHY ALL THE MATH?





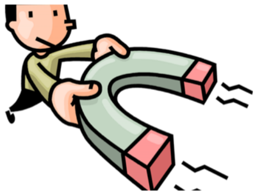
quantum computer



superconductor

$H\Psi = E\Psi$

Hamiltonian



Isaac Chuang
& Michael Nielsen
quantum
textbook

Qutip
Qiskit

- We use currents and magnetic fields to control the quantum computer, and get currents and magnetic fields out of it
- **The math is our attempt at describing the physics in a quantum computer**
- **Why use vectors for qubits?** They can't be described by just a single current or magnetic field number
- **Why use matrices for quantum gates?** Gates change qubits in the same way as matrices change vectors

QUESTIONS?

Questions on content so far?

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POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

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OPTIONAL CONTENT

TA discussion!

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