INTRO TO QUANTUM COMPUTING

Week 8
Lab

MATHEMATICS FOR QUANTUM

<insert TA name>
<insert date>

PROGRAM FOR TODAY

- Attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number :
  - Passcode:

- How useful is your lab section?
- In general, how is the pacing of the lectures?

- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

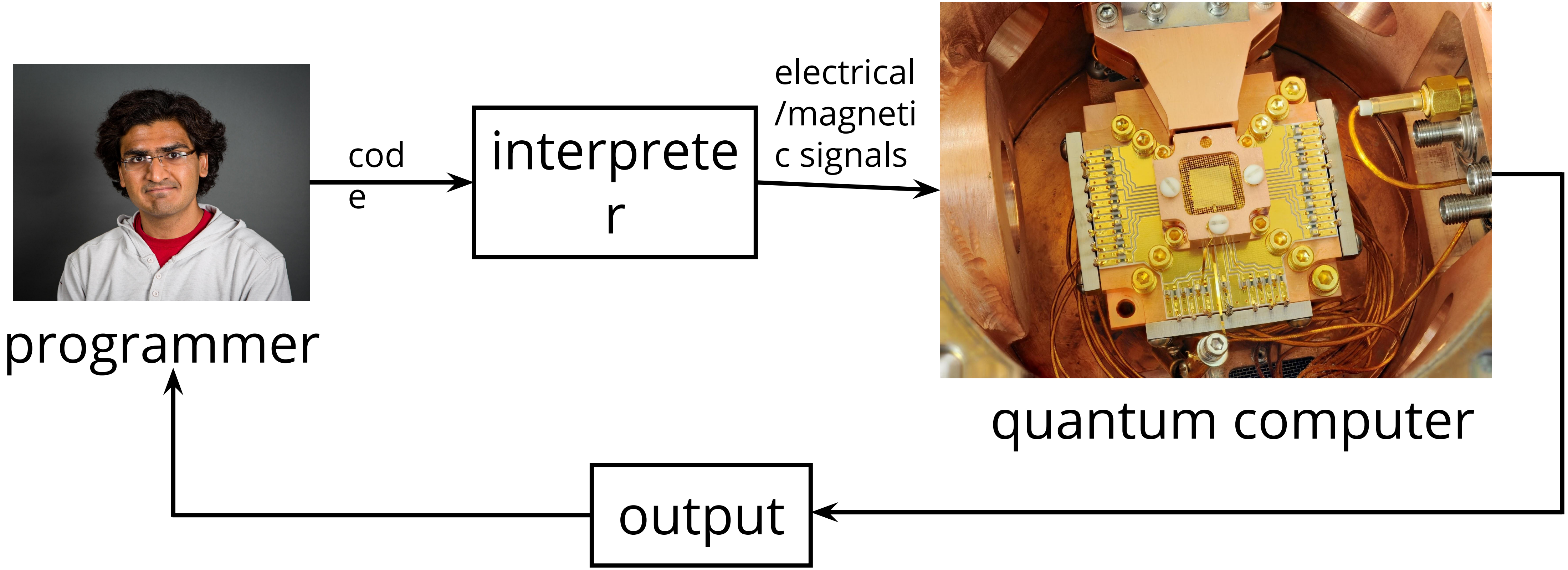
On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

Canvas attendance quiz

QUESTIONS FROM PAST WEEK

Why are we learning Python? Does a quantum computer understand Python?

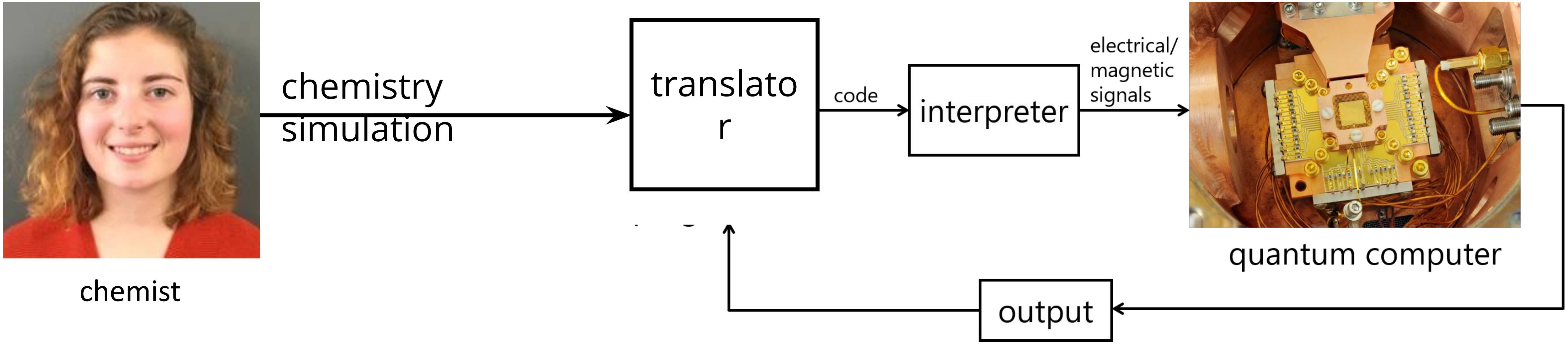


Python is widely used to code, and IBM provides an interpreter for python code to run quantum computers

Canvas attendance quiz

QUESTIONS FROM PAST WEEK

There are more layers in the stack!



More discussion in semester 2 😊

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LEARNING OBJECTIVES FOR LAB 8

- Using bra-ket notation to express qubits and inner products
  - Superposition states: Review
  - Normalization
  - Inner products: Review
- Understanding how to apply quantum gates to qubits: Review
- Develop intuition for measurements of qubit states
  - Measuring superposition qubit states
  - Expressing measurements with bra-ket notation*

*Optional content

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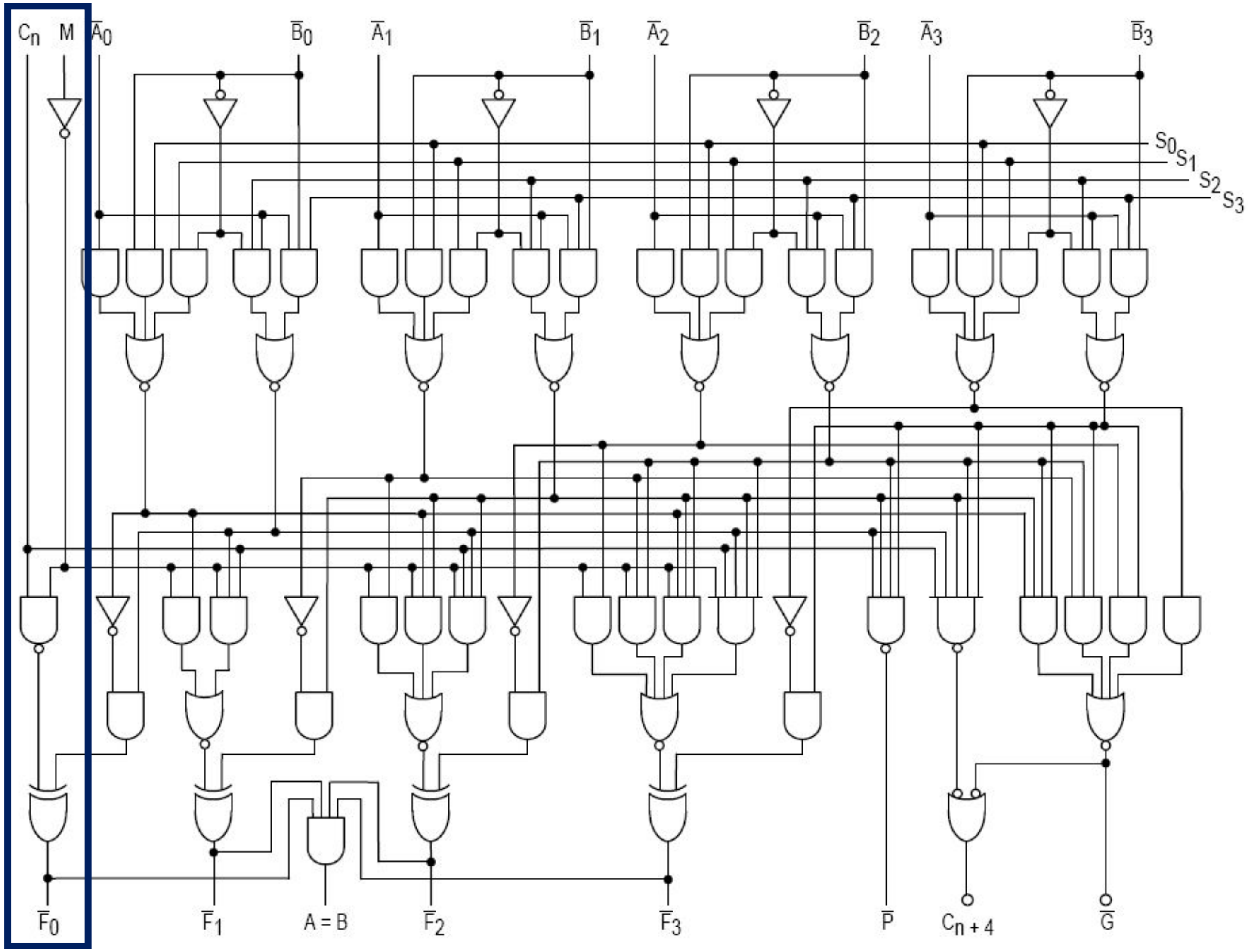
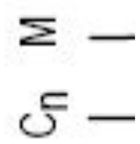
UNDERSTANDING A CLASSICAL COMPUTER



CLASSICAL COMPUTER -SIMPLIFIED



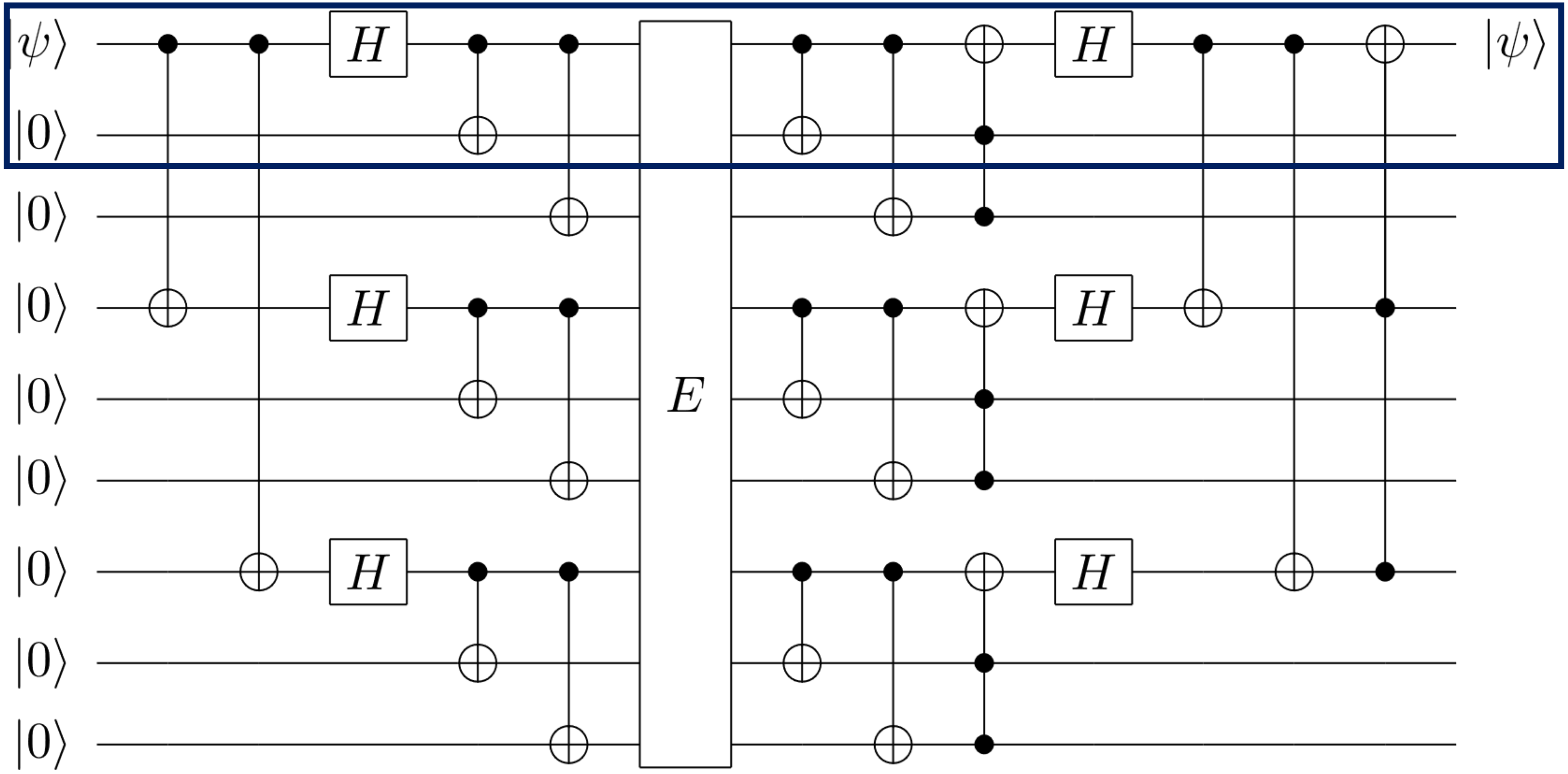
input
s

computer

Using a classical computer □ send input bits □ measure output bits

Canvas attendance quiz

UNDERSTANDING A QUANTUM COMPUTER



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QUANTUM COMPUTER-SIMPLIFIED

$|\psi\rangle$ –

$|0\rangle$ –

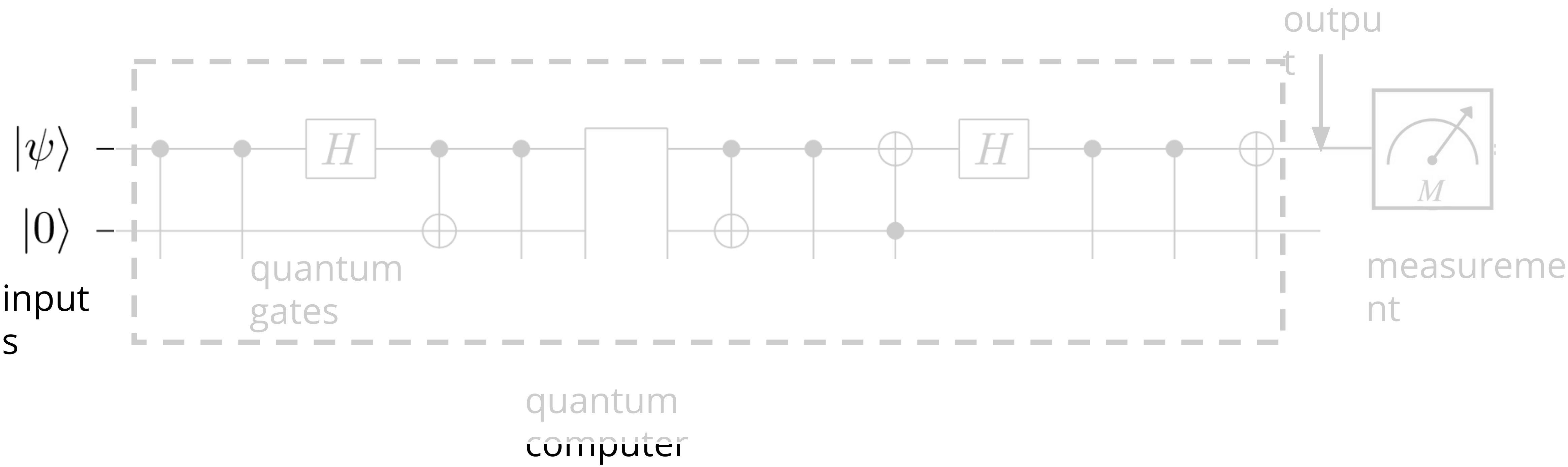
input
s

computer

Using a quantum computer send input qubits measure output qubits

Canvas attendance quiz

EXPRESSING QUBITS



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BRA-KET NOTATION: REVIEW

conjugate transpose

ket: column vector

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

bra: row vector

$$|0\rangle^{\dagger} = \langle 0| = \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$|1\rangle^{\dagger} = \langle 1| = \begin{pmatrix} 0 & 1 \end{pmatrix}$$

INNER PRODUCT NOTATION: REVIEW

inner
product:

$$\langle\psi|\psi\rangle$$

br
a

ke
t

What is $\langle 0|0\rangle$?

$$\langle 0|0\rangle = |0\rangle^{\dagger}|0\rangle$$
$$= (1 \quad 0)\begin{pmatrix}1\\0\end{pmatrix} = 1$$

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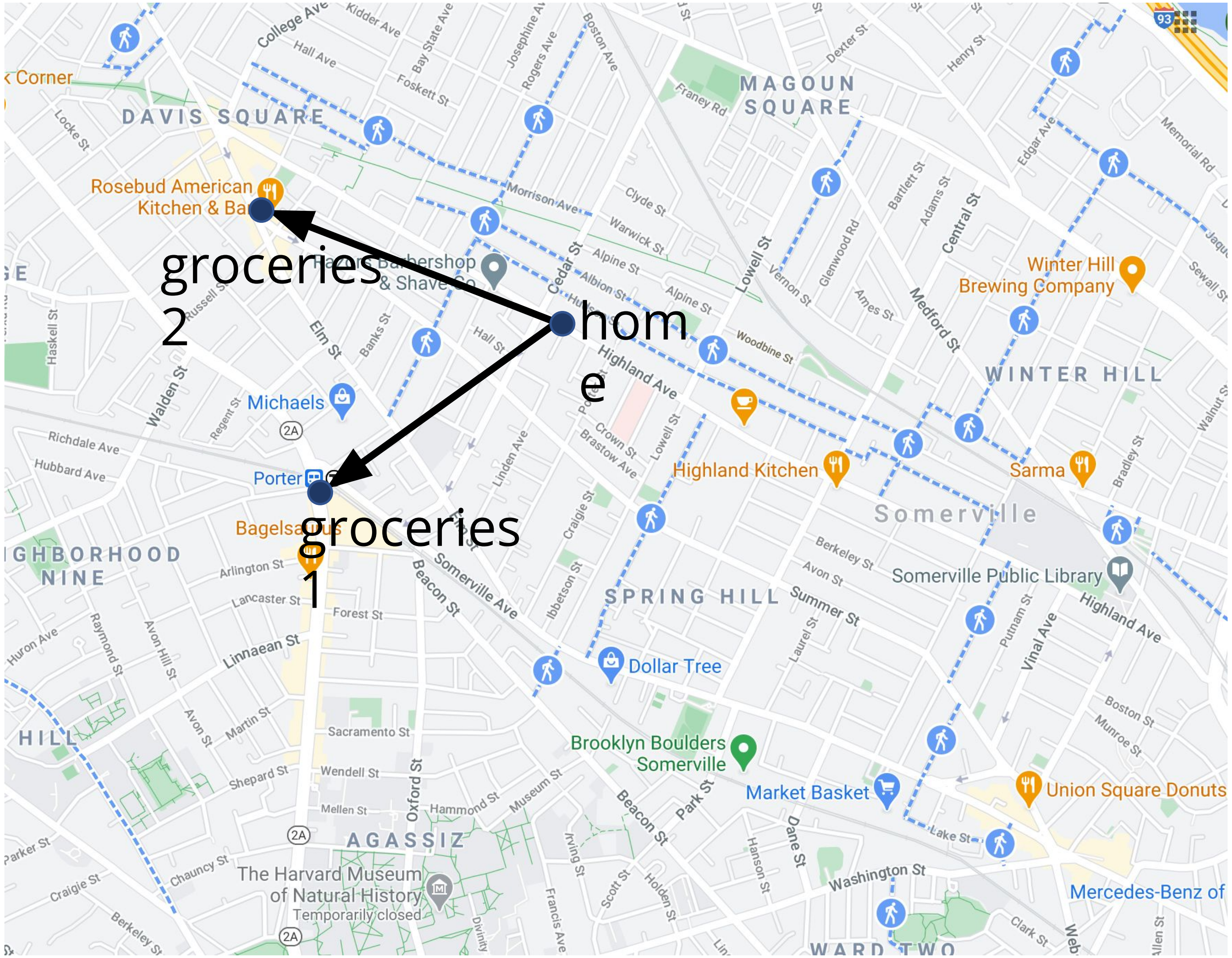
INNER PRODUCT MEANING: REVIEW



inner product corresponds to the angle between vectors

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PRACTICE WITH INNER PRODUCTS

What is $\langle 0|1\rangle$?

$$\begin{aligned}\langle 0|1\rangle &= |0\rangle^{\dagger}|1\rangle \\ &= \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = 0\end{aligned}$$

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SUPERPOSITION QUBIT STATES

Superposition state: a combination of $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Example:

$$|\psi\rangle = i|0\rangle + 2|1\rangle = \begin{pmatrix} i \\ 2 \end{pmatrix}$$

$|\psi\rangle$ has to be **normalized**!

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STATE NORMALIZATION

Example:

$$|\psi\rangle = i|0\rangle + 2|1\rangle = \begin{pmatrix} i \\ 2 \end{pmatrix}$$

What is the normalized form of $|\psi\rangle$?

To ensure that the answers we get from the math are correct, qubit states **must** be normalized!

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INNER PRODUCT WITH SUPERPOSITION STATES

$$|+\rangle = \frac{1}{\sqrt{2}}\,|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\,|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

What is $\langle+|-\rangle$?

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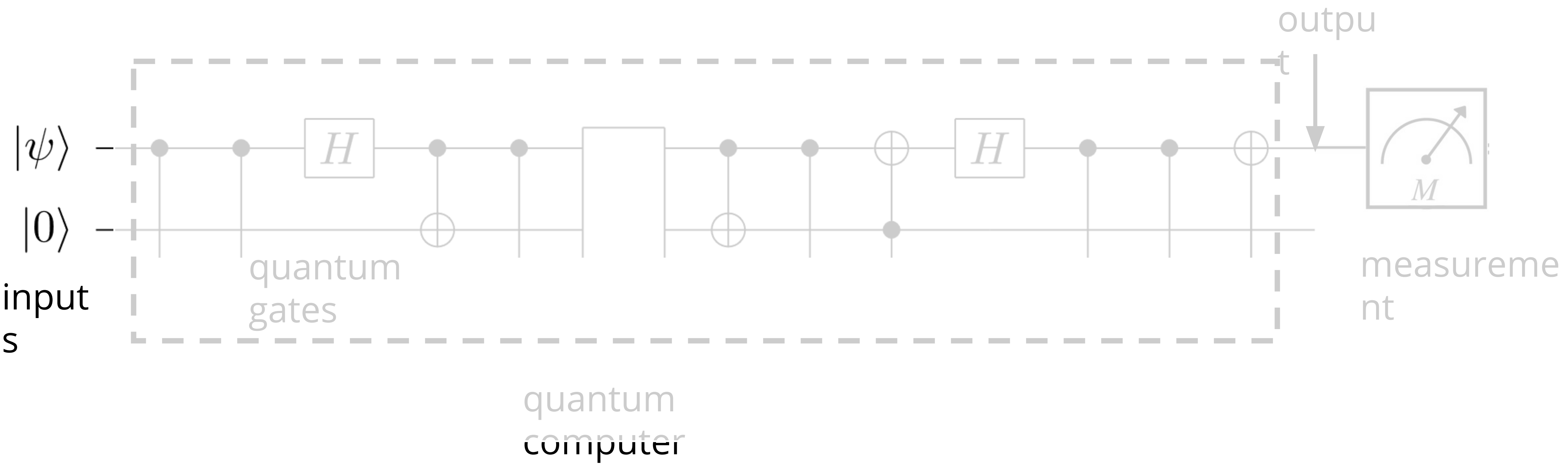
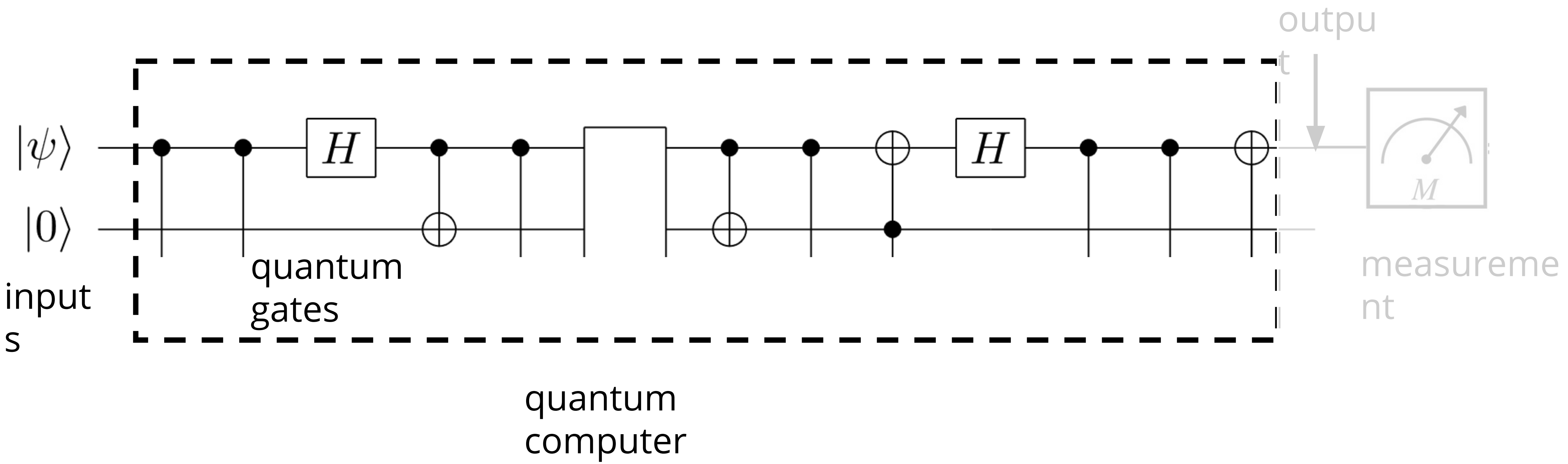
EXPRESSING QUBITS



EXPRESSING QUANTUM GATES



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EXAMPLES OF QUANTUM GATES

$$Z \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$R\phi \quad \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$X \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sqrt{\neg} \quad \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$

$$Y \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$H \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

You don’t need to memorize these! Just look them up

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- Find the resulting state from applying a Z gate followed by an H gate to the initial state

$$|\psi\rangle = \frac{i}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

Step 1: Find the column vector form $|\psi\rangle$

APPLYING GATES TO QUBITS

- Find the resulting state from applying a Z gate followed by an H gate to the initial state

$$|\psi\rangle = \frac{i}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

Step 2: Multiply the matrices of the H gate and Z gate

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APPLYING GATES TO QUBITS

- Find the resulting state from applying a Z gate followed by an H gate to the initial state

$$|\psi\rangle = \frac{i}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

Step 3: Multiply the resulting matrix with column vector

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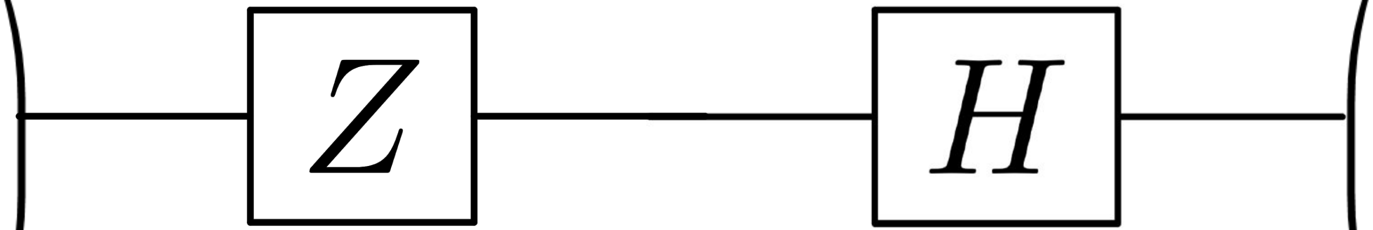
- Find the resulting state from applying a Z gate followed by an H gate to the initial state

$$|\psi\rangle = \frac{i}{\sqrt{5}}|0\rangle + \frac{2}{\sqrt{5}}|1\rangle$$

$$\begin{pmatrix} \frac{i}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$



$$\begin{pmatrix} \frac{i-2}{\sqrt{10}} \\ \frac{i+2}{\sqrt{10}} \end{pmatrix}$$

input qubit state

output qubit state

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QUESTIONS

Questions on content so far?

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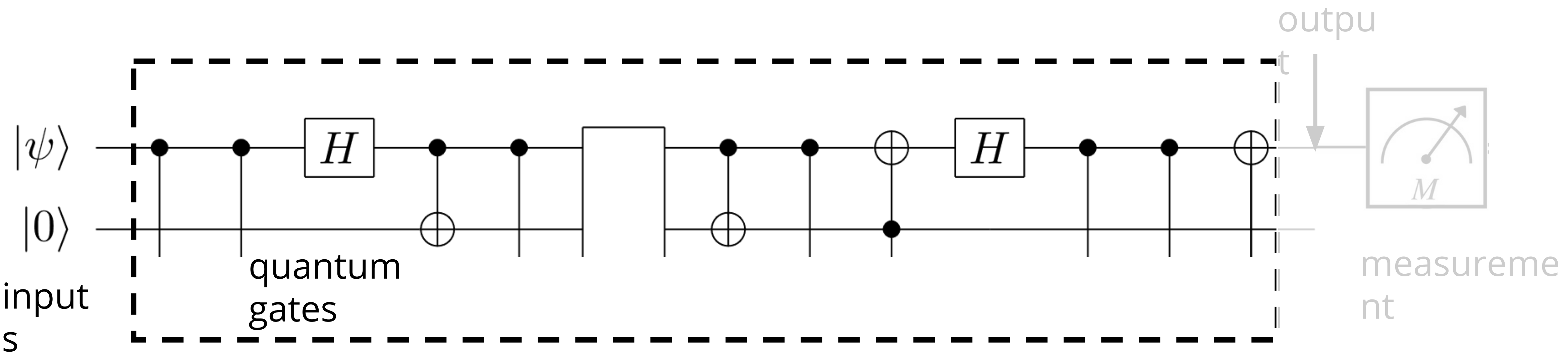
EXPRESSING QUANTUM GATES



quantum computer

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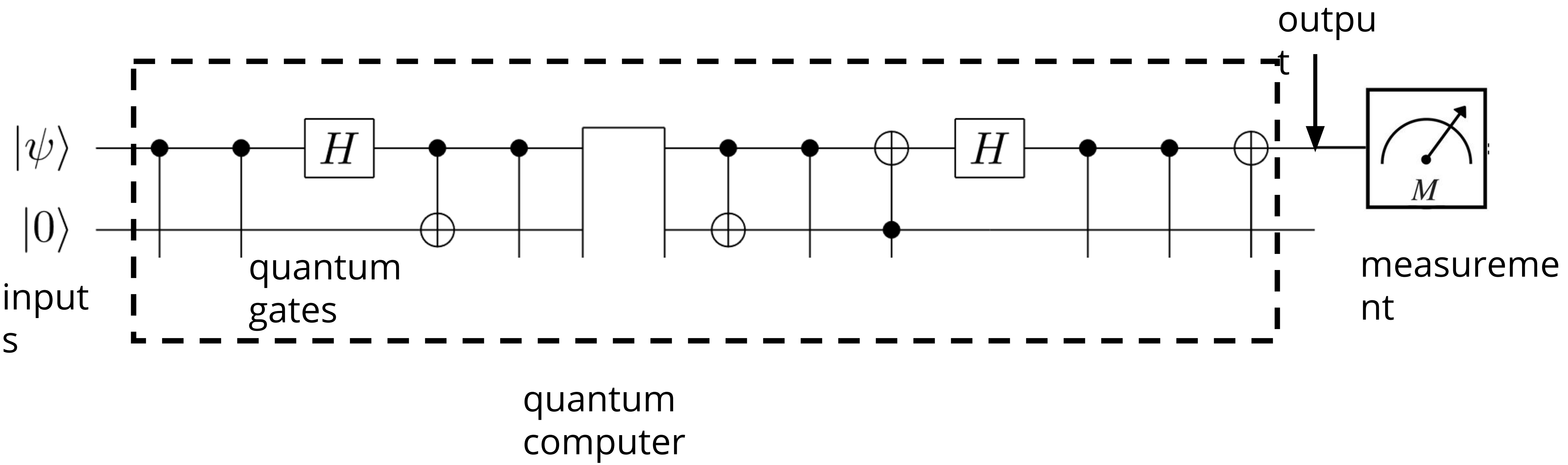
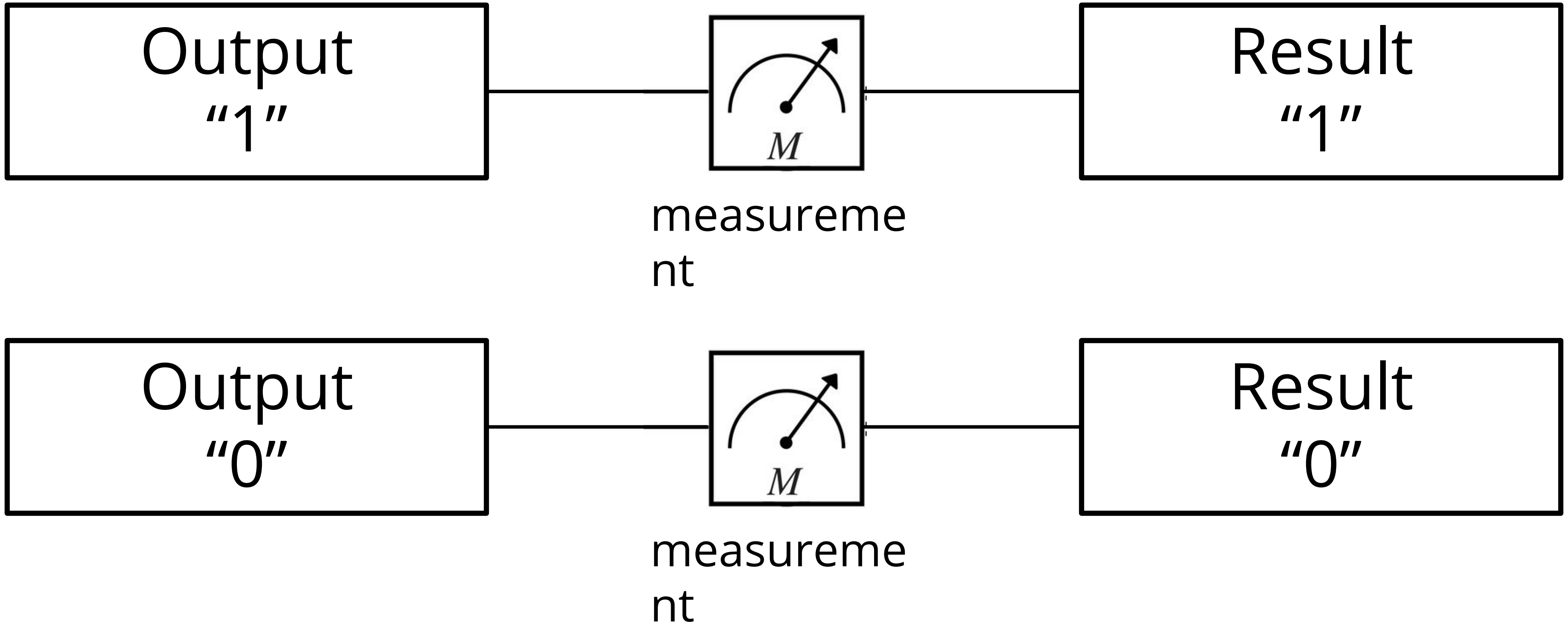
EXPRESSING QUANTUM GATES



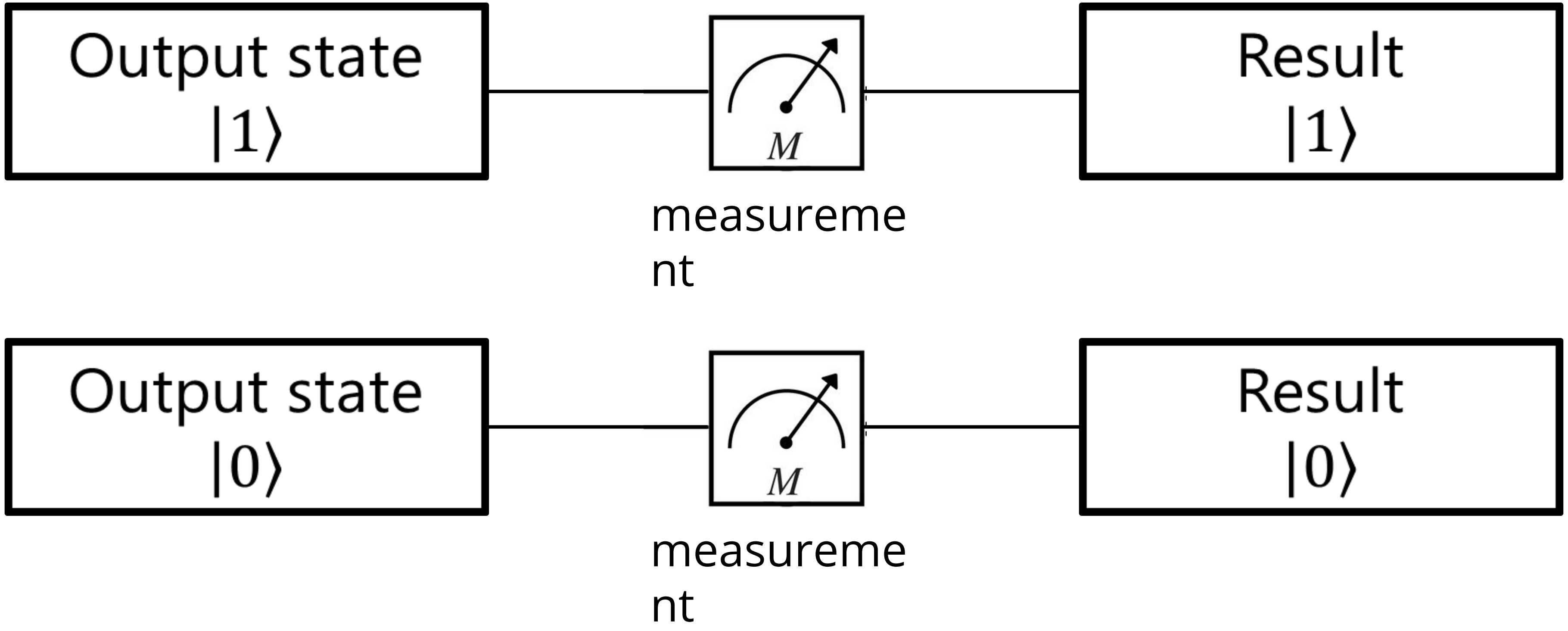
MEASURING CLASSICAL BITS

Classical computing



MEASURING QUANTUM BITS

Quantum computing



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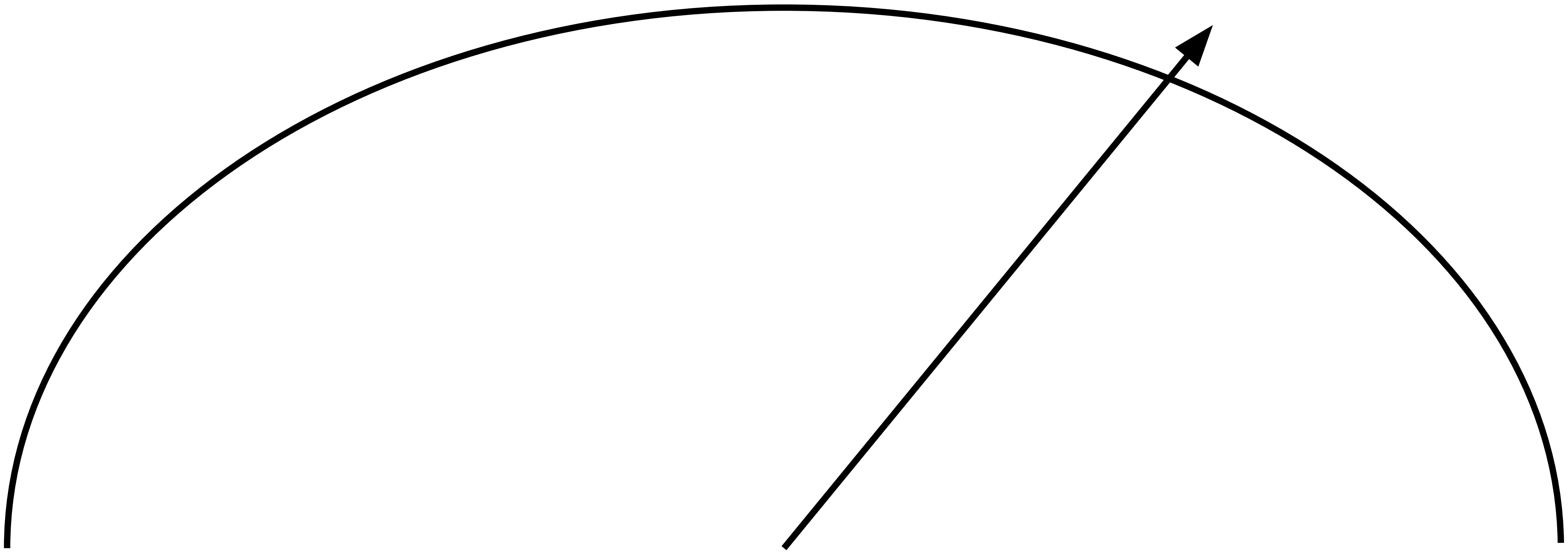
MEASURING SUPERPOSITION STATES



$|0\rangle$ $|1\rangle$

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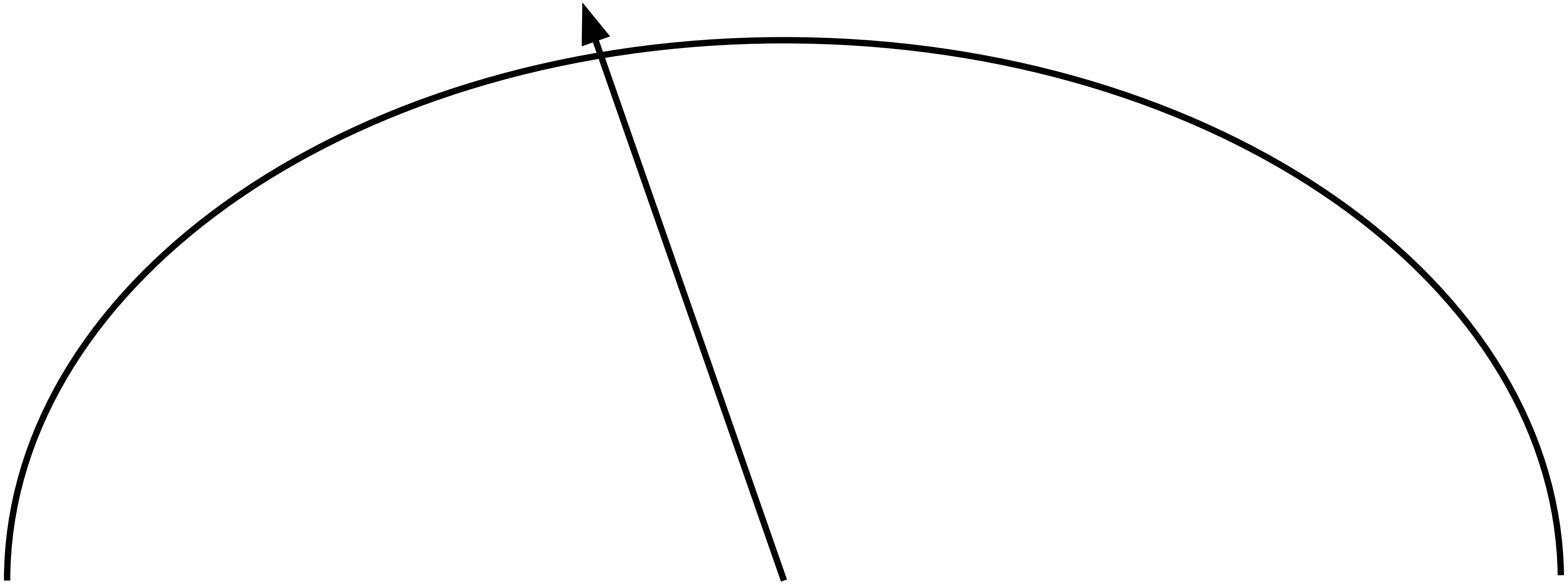
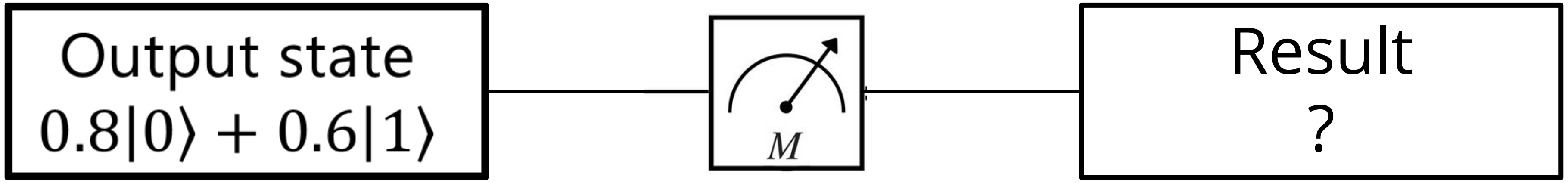
MEASURING SUPERPOSITION STATES



$|0\rangle$ $|1\rangle$

MEASURING SUPERPOSITION STATES

Quantum computing



measureme
nt

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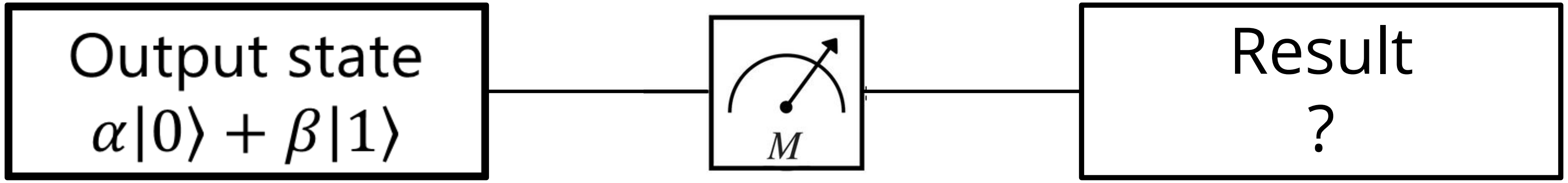
MEASURING SUPERPOSITION STATES

Quantum computing



measureme
nt

Probability of the result of measurement being $|0\rangle$: $|\alpha|^2$

Probability of the result of measurement being $|1\rangle$: $|\beta|^2$

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UNANSWERED QUESTIONS ABOUT MEASUREMENT

- Why do we only measure for a 0 or 1?
- Can we do a different kind of measurement?
- What is the state of the qubit right after the measurement?
- What would we get if we measured the qubit twice?

Coming up next semester!

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IMPORTANT TAKEAWAYS

- Bra-ket notation
  - **Ket:** column vector
  - **Bra:** row vector
  - **Inner product:** Bra-ket

- Quantum gates are represented by matrices
  - Applying quantum gates to qubits → Multiplying the gate matrix by the qubit column vector

- The result of measuring a qubit state is probabilistic
  - $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, we get $|0\rangle$ with probability $|\alpha|^2$ and $|1\rangle$ with probability $|\beta|^2$

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QUESTIONS

Questions on content so far?

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POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
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Canvas attendance quiz

OPTIONAL CONTENT

INNER PRODUCT OF SUPERPOSITION STATE

- Calculate the inner product $\langle 0|\psi\rangle$, for $|\psi\rangle = \frac{3i}{5}|0\rangle - \frac{4}{5}|1\rangle$

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RELATING INNER PRODUCTS WITH MEASUREMENTS

- Calculate the inner product $\langle 0|\psi\rangle$, for $|\psi\rangle = \frac{3i}{5}|0\rangle - \frac{4}{5}|1\rangle$

$$\langle 0|\psi\rangle = \frac{3\mathrm{i}}{5}$$

$$\text{Prob. of measuring } |0\rangle = \left|\frac{3i}{5}\right|^2 = |\langle 0|\psi\rangle|^2$$

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MEASUREMENTS AND BRA-KET NOTATION

- For $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$\text{Prob. of measuring } |0\rangle = |\alpha|^2 = |\langle 0|\psi\rangle|^2$$

$$\text{Prob. of measuring } |1\rangle = |\beta|^2 = |\langle 1|\psi\rangle|^2$$