INTRO TO QUANTUM COMPUTING

Week 9
Lab

MATH FOR QUANTUM CONTD.

<insert TA name>
<insert date>

PROGRAM FOR TODAY

- Logistics
- Attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

Canvas attendance quiz

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number:
  - Passcode:

- If you have NOT attended the Friday Student Assistant Office Hours, why not? Select all that apply.

- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

Canvas attendance quiz

QUESTIONS FROM LAST WEEK

Canvas attendance quiz

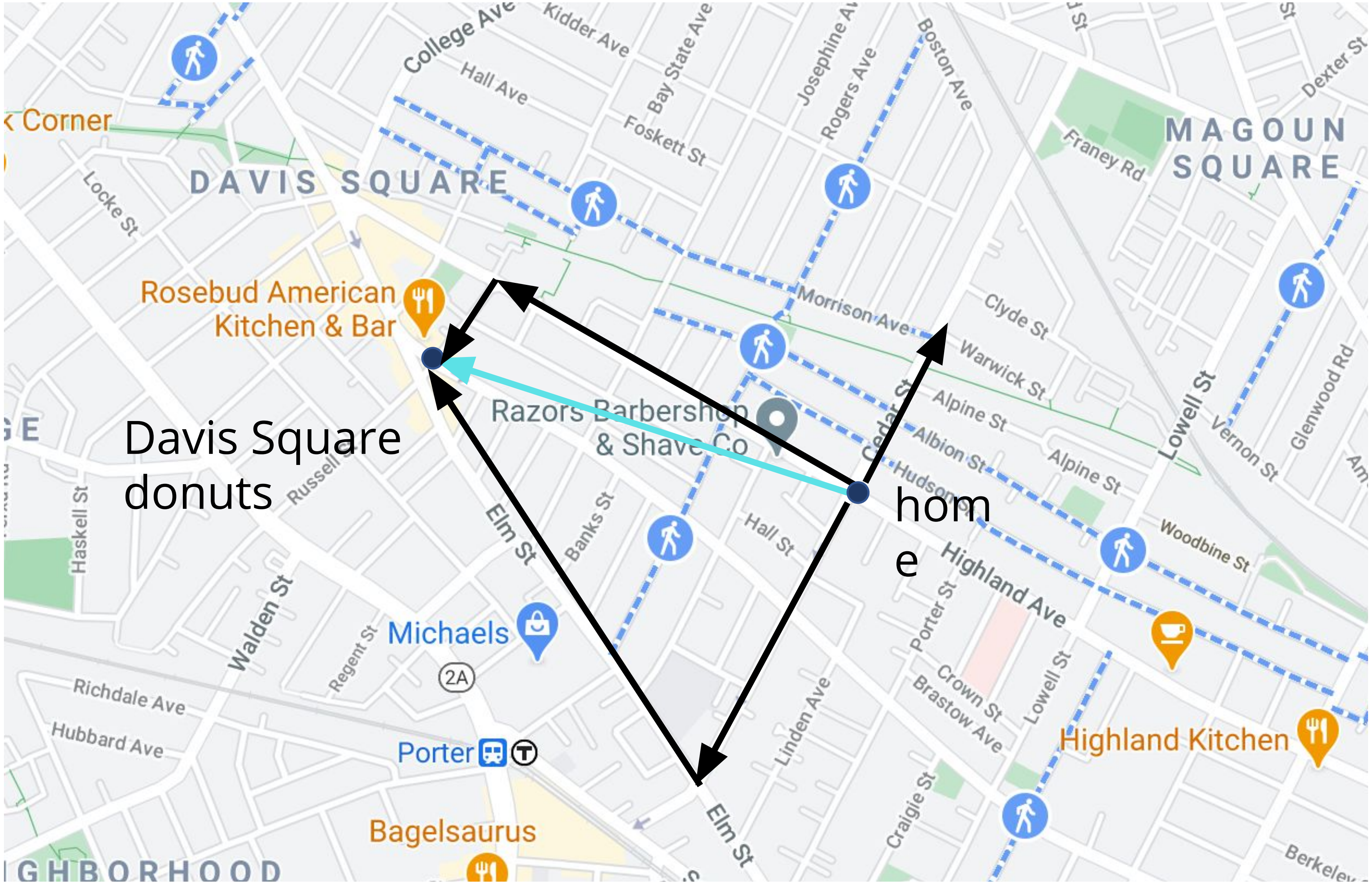
LEARNING OBJECTIVES FOR LAB 5

- Understanding vector spaces and their properties
  - Linear combinations
  - Linear independence
  - Span and Basis vectors
- Demystifying eigenvalues and eigenvectors
  - Applying gates to qubits: Review
  - Eigenvectors for the Z and X matrices
  - Applying gates to qubits using eigenvectors
- TA discussion*

*Optional content

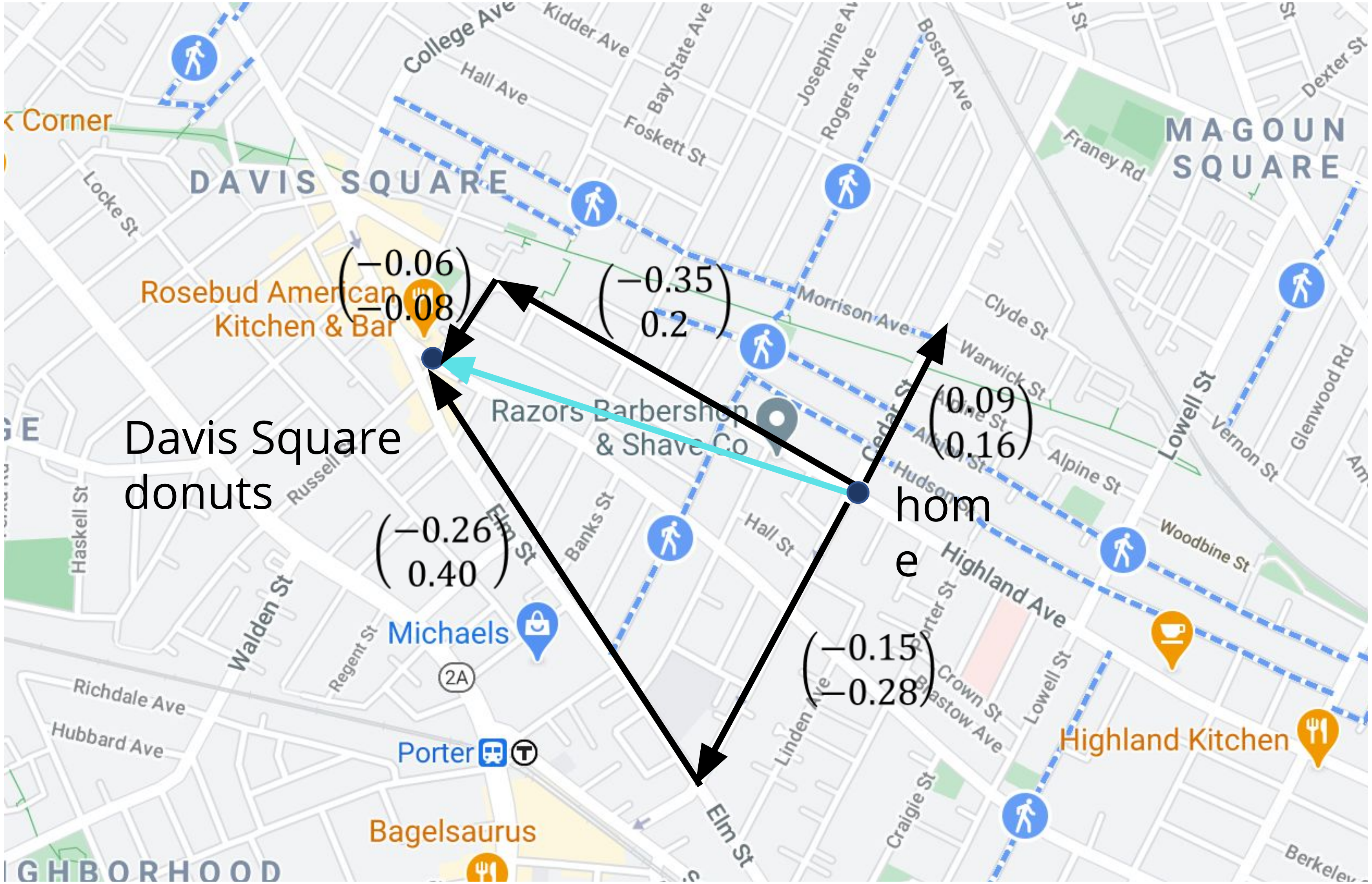
Canvas attendance quiz

GETTING DONUTS



Canvas attendance quiz

GETTING DONUTS



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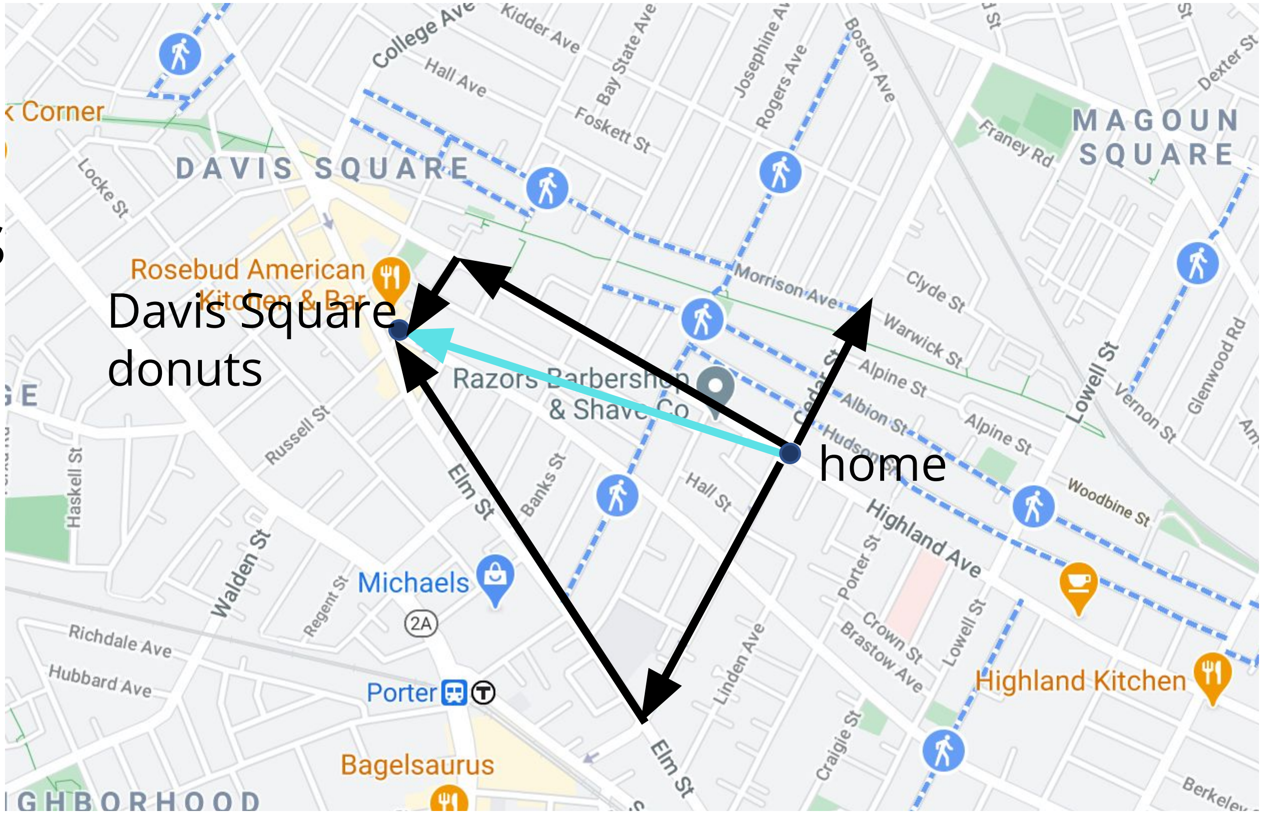
LINEAR COMBINATIONS

Linear combinations of vectors

$$y = x + x^2 + x^3 + \cdots$$



EXPRESSING SUPERPOSITION STATES

Superposition state: a **linear combination** of $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

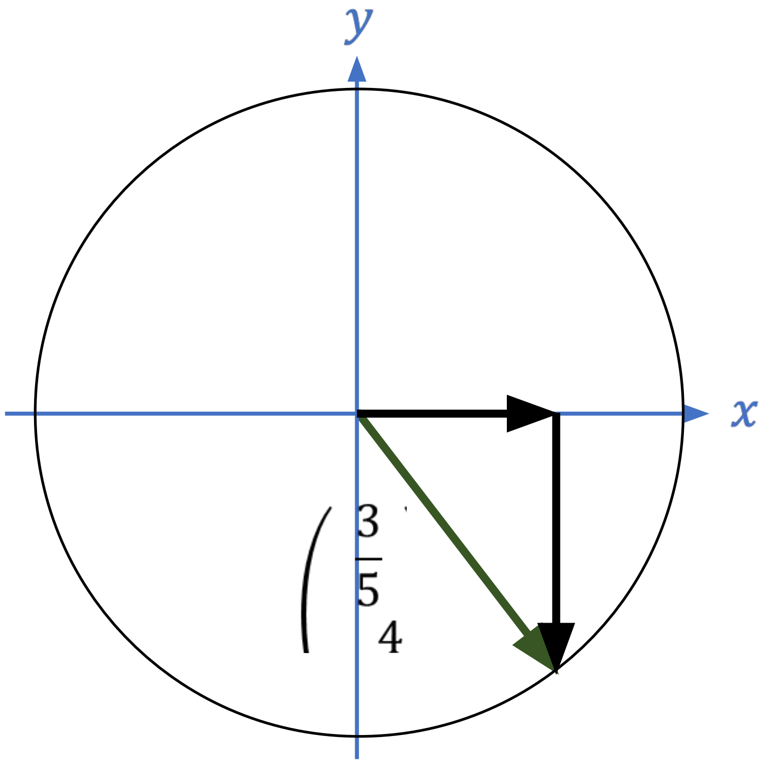
$$|\psi_3\rangle = \frac{3i}{\sqrt{10}}|+\rangle - \frac{1}{\sqrt{10}}|-\rangle$$

Can make linear combinations from any vectors!

Canvas attendance quiz

WRITING STATES AS LINEAR COMBINATIONS

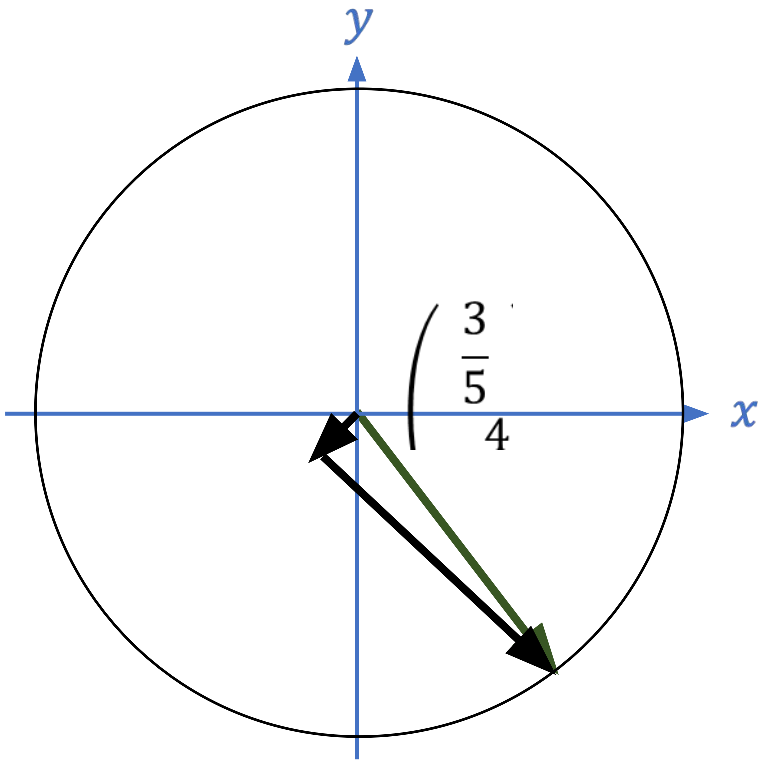
- Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



Canvas attendance quiz

WRITING STATES AS LINEAR COMBINATIONS

- Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$and $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$



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WRITING STATES AS LINEAR COMBINATIONS

Can we do this with any two vectors?

Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$and $\frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Canvas attendance quiz

LINEAR INDEPENDENCE

- We need the two (or more) vectors to be in **different** directions!

Write the state $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$ as a combination of $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$and $\frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix}$

$$\frac{1}{\sqrt{8}}\begin{pmatrix} -2 \\ -2 \end{pmatrix} = -\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Linearly dependent (along the same direction)

Canvas attendance quiz

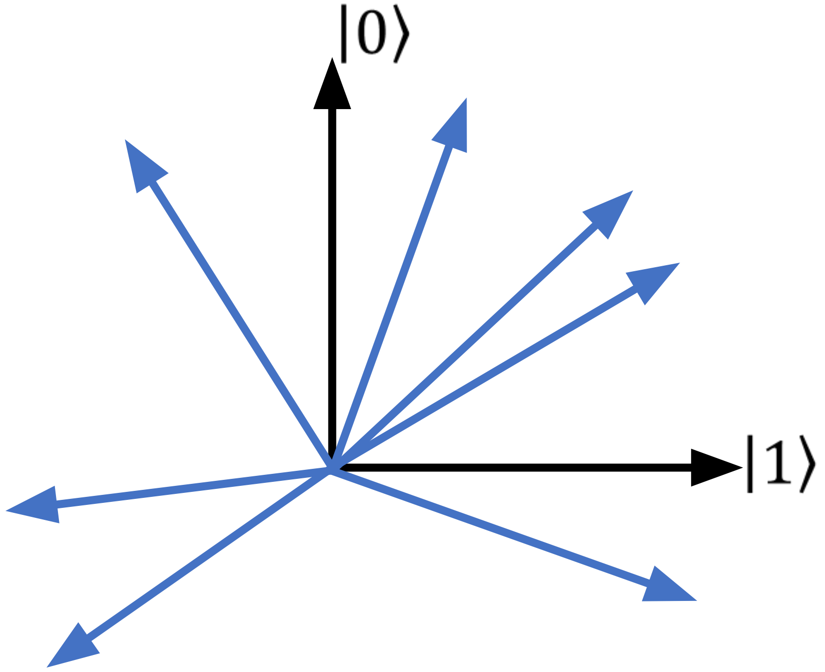
LINEAR INDEPENDENCE

- **Linearly independent** vectors – Vectors that cannot be written as combinations of each other

SPAN

- Span: All the vectors that can be written as combinations of the given vector set

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

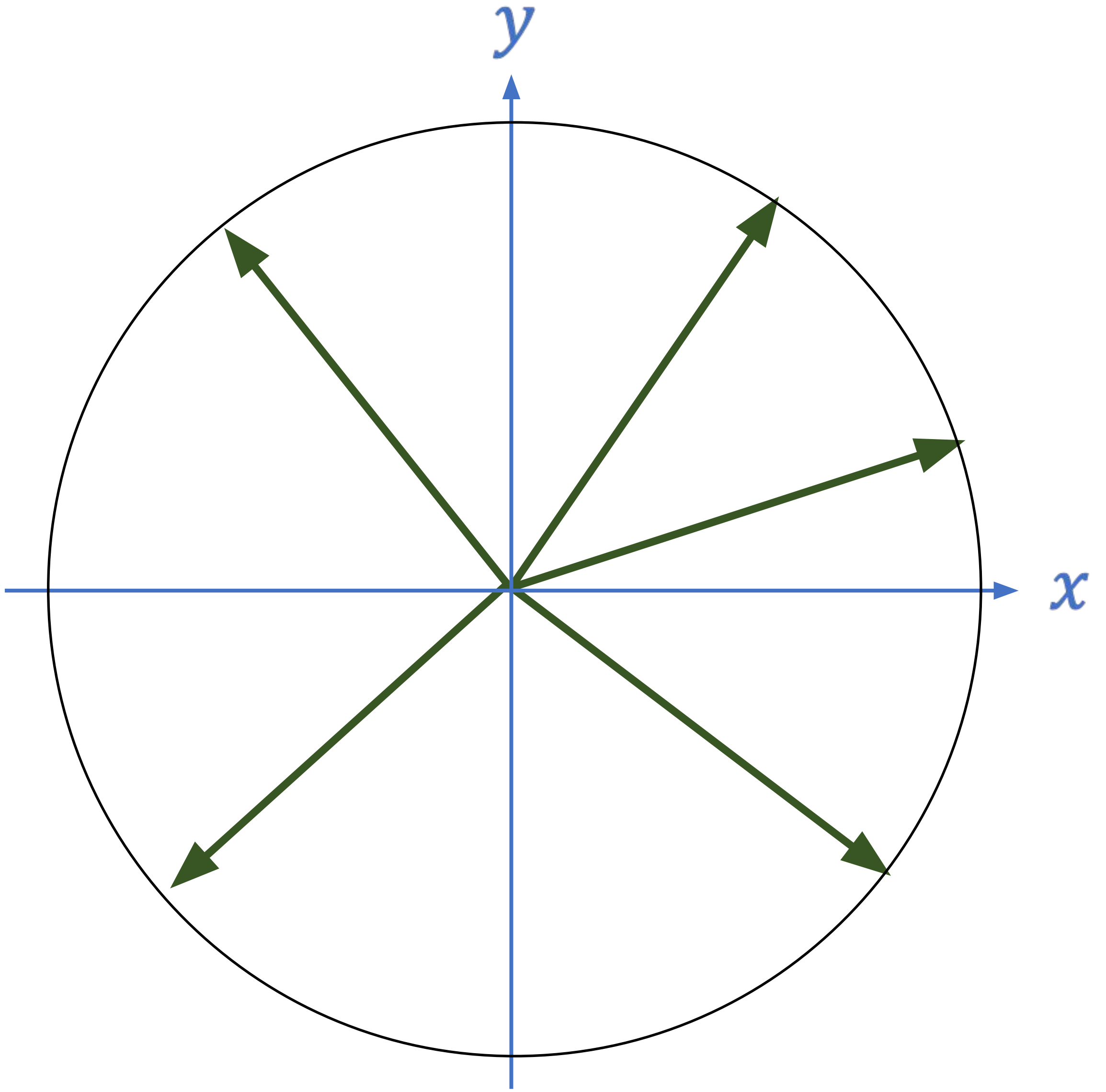


....and infinitely more

VECTOR SPACE

- **Vector space:** A collection of vectors (states)

- **Examples**
  - All 2-D vectors that line on the unit circle
  - All vectors on the surface of the Bloch sphere

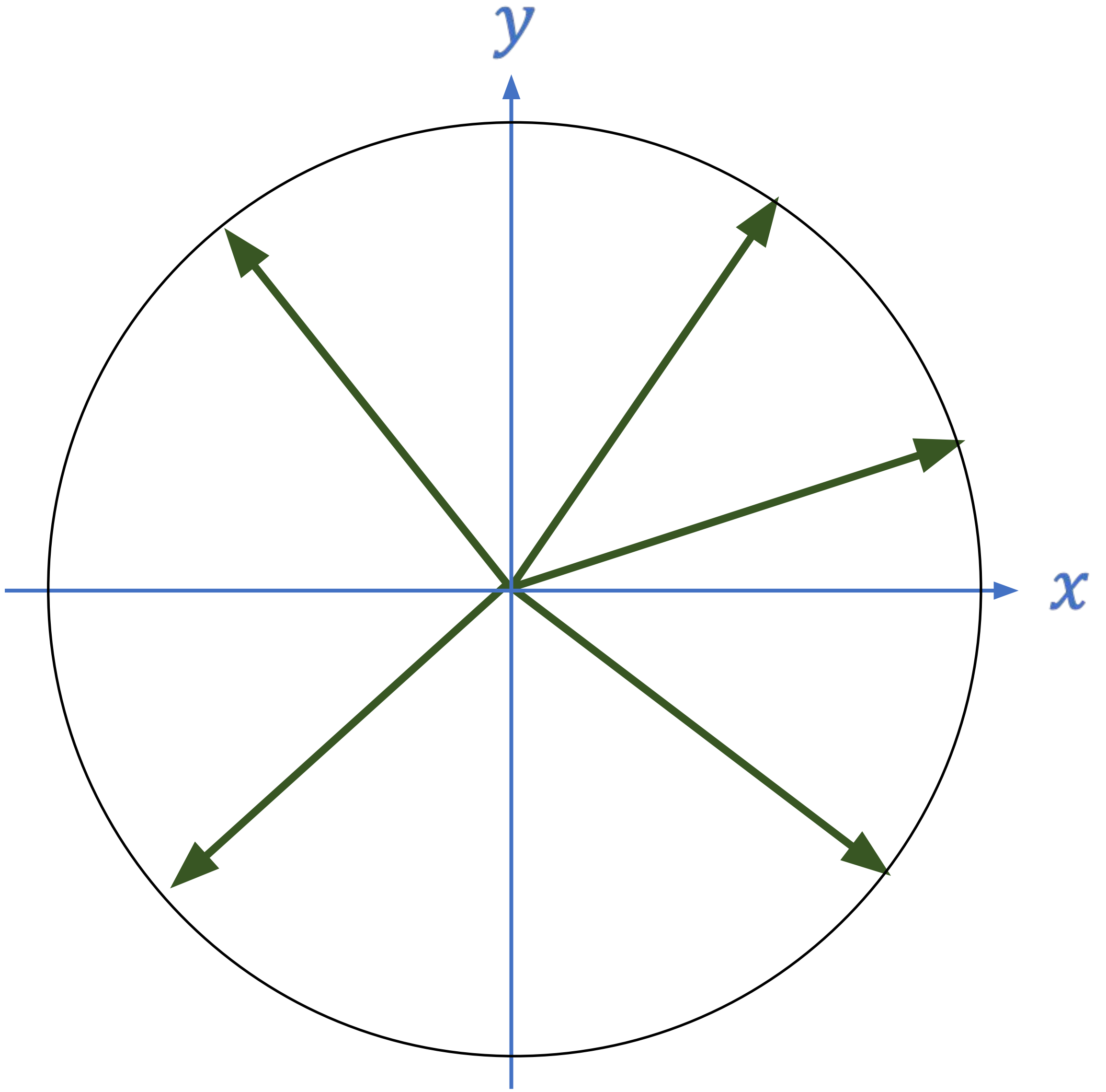


BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- Yes!

- These two vectors form a “basis” for this vector space – they are called **basis vectors**

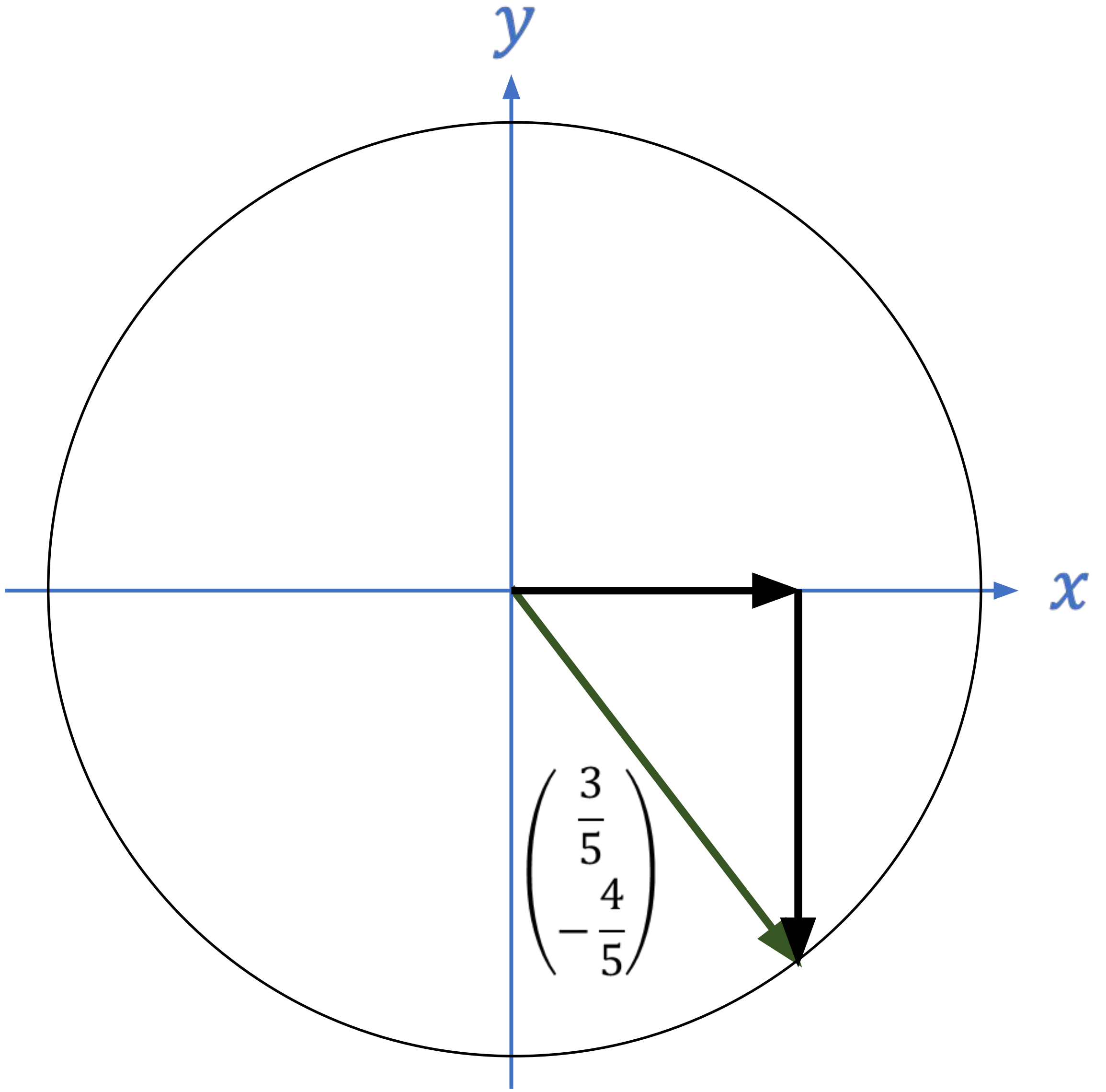


Canvas attendance quiz

BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- Example: $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = \frac{3}{5}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{4}{5}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$



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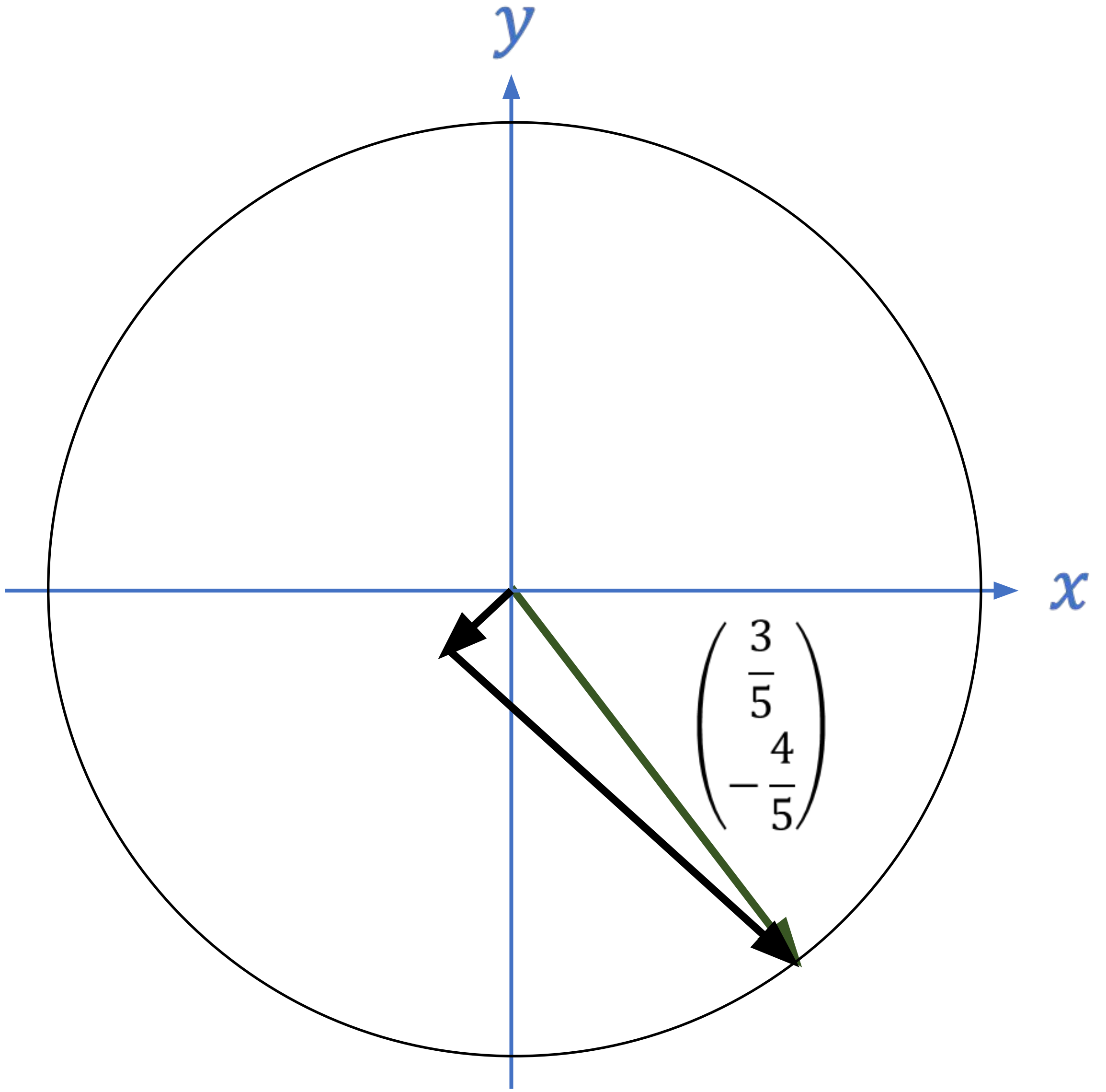
ANOTHER BASIS

- Can we find a way to describe each vector in this space as a combination of two vectors?

- There are other options!
- Example: $\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = -\frac{\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{7\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

- There are an infinite number of choices for basis!



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QUESTIONS

Questions on content so far?

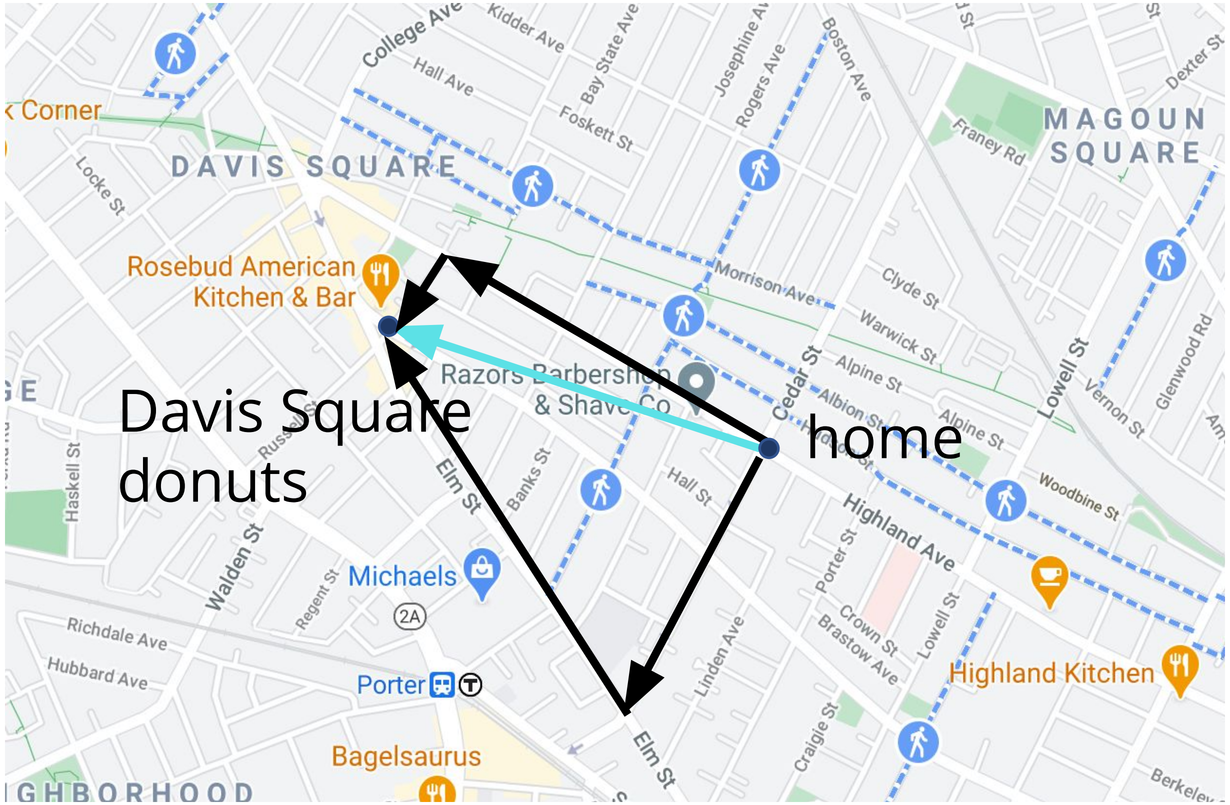
Canvas attendance quiz

WHY USE DIFFERENT BASES?

Same vector, two bases:

$$\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = \frac{3}{5}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{4}{5}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = -\frac{\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{7\sqrt{2}}{10}\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$$



But why?

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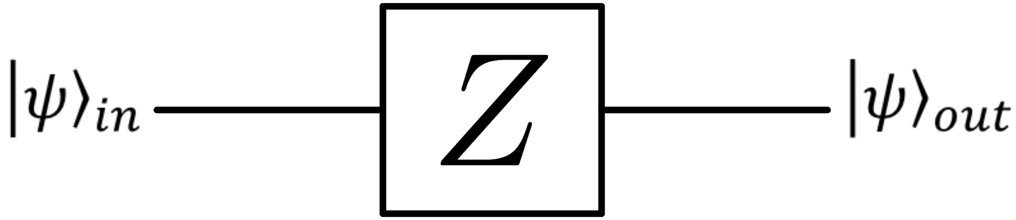
APPLYING GATES TO QUBITS: REVIEW



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\psi\rangle_{out} = Z|\psi\rangle_{in}$$

Multiply the matrix of Z with the column vector $|\psi\rangle_{in}$ to get $|\psi\rangle_{out}$

- Example: Find $Z\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{pmatrix}$

Canvas attendance quiz

APPLYING GATES TO QUBITS: REVIEW

Find $Z\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$ and $Z\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

EIGENVALUES AND EIGENVECTORS

$$Z\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 1 \cdot \begin{pmatrix}1\\0\end{pmatrix}$$

eigenvalue
s

Z-gate
eigenvectors

$$Z\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -1 \cdot \begin{pmatrix}0\\1\end{pmatrix}$$

Takeaway: To find if a given vector is an eigenvector of a matrix, multiply the vector with the matrix and check if the result is a scalar times the same vector

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WHY USE EIGENVECTORS?

Find $Z\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix}$

$$\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}\begin{pmatrix}0\\1\end{pmatrix}$$

$$Z\begin{pmatrix}\frac{3}{5}\\-\frac{4}{5}\end{pmatrix} = \frac{3}{5}Z\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}Z\begin{pmatrix}0\\1\end{pmatrix} = \frac{3}{5}(1)\begin{pmatrix}1\\0\end{pmatrix} - \frac{4}{5}(-1)\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \frac{3}{5}\begin{pmatrix}1\\0\end{pmatrix} + \frac{4}{5}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\frac{3}{5}\\\frac{4}{5}\end{pmatrix}$$

Takeaway: Applying a gate to qubit states is really easy, if you can write the vector as a linear combination of the eigenvectors of that gate!

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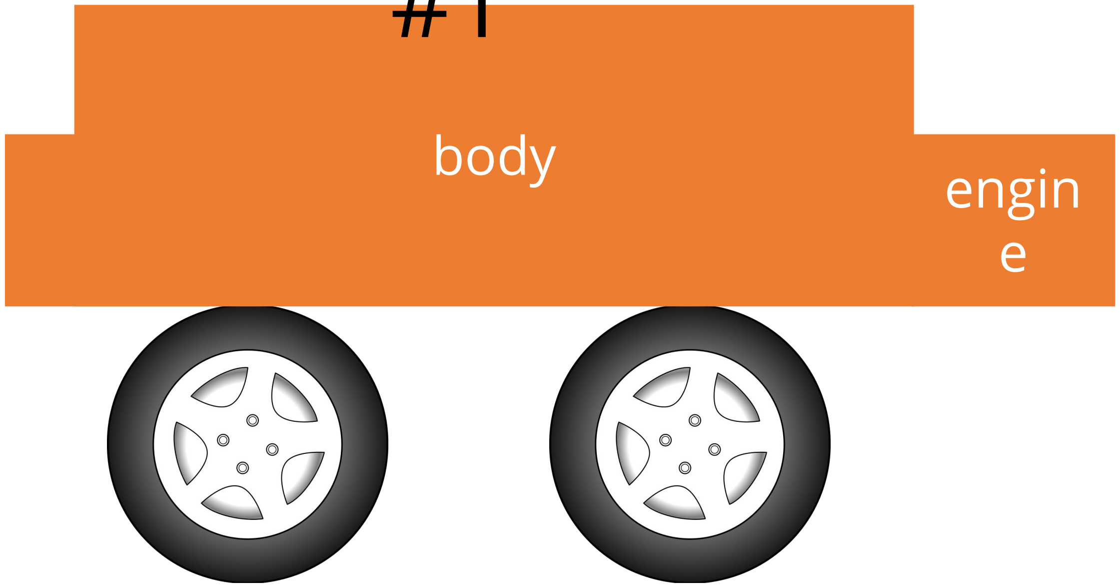
CARS VS EIGENVECTORS

Car #1



Car #1 = 4 * wheel +

y

\+ engine

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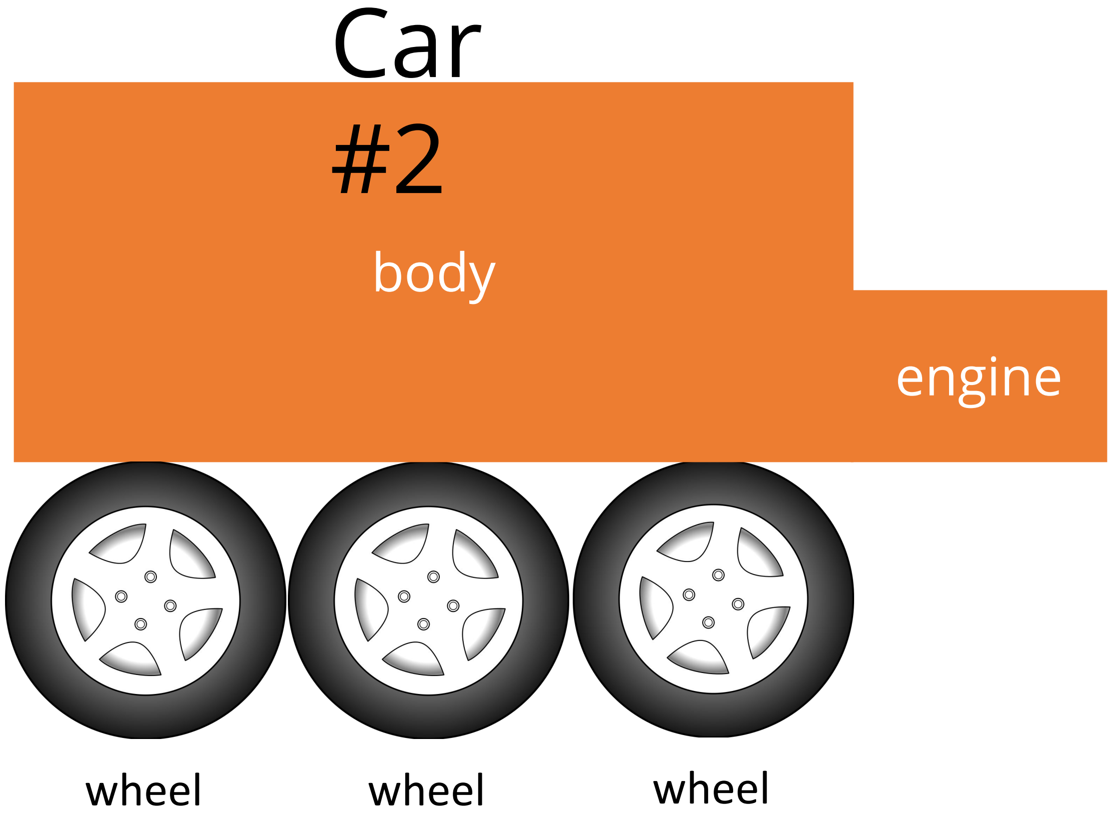
CARS VS EIGENVECTORS



Car #2 = 6 * wheel +

y

+ engine

- We know how to work with wheels, engines and bodies, and we combine them in different ways to make different cars
- We know how to work with eigenvectors, and we combine them in different ways to make different qubit states

EIGENVECTORS FOR THE X GATE

Example: Find $X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$

Canvas attendance quiz

EIGENVECTORS FOR THE X GATE

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}=1\cdot\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

eigenvalues

X-gate eigenvectors

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}=-1\cdot\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{-1}{\sqrt{2}}\end{pmatrix}$$

USING EIGENVECTORS OF THE X GATE

Find $X\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix}$

$$\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} + \frac{7\sqrt{2}}{10}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix}$$

$$\begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} + \frac{7\sqrt{2}}{10}X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{3}{5}\\ -\frac{4}{5}\end{pmatrix} = -\frac{\sqrt{2}}{10}(1)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} + \frac{7\sqrt{2}}{10}(-1)\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{-4}{5}\\ \frac{3}{5}\end{pmatrix}$$

Takeaway: Applying a gate to qubit states is really easy, if you can write the vector as a linear combination of the eigenvectors of that gate!

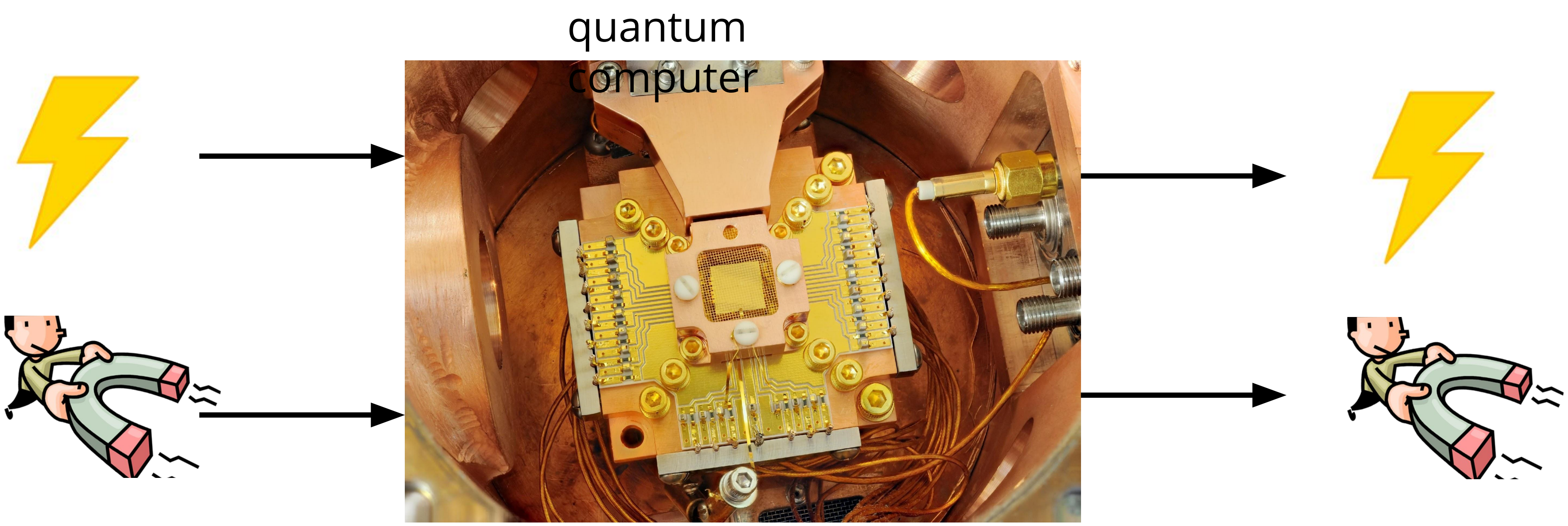
Canvas attendance quiz

IMPORTANT TAKEAWAYS

- **Vector space**: A collection of vectors
  - A given qubit state in a vector space can be expressed as a combination of two (or more) states in the space
  - **Linear independence:** These states cannot themselves be written as combinations of one another
  - **Basis**: These states can be combined to express every other state in the vector space
- Applying a gate to its **eigenvector** results in the same vector times a constant (called the **eigenvalue**)
- Applying a gate to any other vector can be broken down into two steps
  - Express the vector as a linear combination of the eigenvectors of the gate
  - Apply the gate to the eigenvectors to find the result

Canvas attendance quiz

WHY ALL THE MATH?



- We use currents and magnetic fields to control the quantum computer, and get currents and magnetic fields out of it
- **The math is our attempt at describing the physics in a quantum computer**
- **Why use vectors for qubits?** They can't be described by just a single current or magnetic field number
- **Why use matrices for quantum gates?** Gates change qubits in the same way as matrices change vectors

Canvas attendance quiz

QUESTIONS?

Questions on content so far?

POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

OPTIONAL CONTENT

TA discussion!

Canvas attendance quiz