INTRO TO QUANTUM COMPUTING

Lab #3

VECTORS AND COMPLEX NUMBERS

<insert TA name>

<insert date>

PROGRAM FOR TODAY

- Announcements
- Canvas attendance quiz
- Pre-lab student feedback
- Popular questions from last week
- Lab content
- Post-lab student feedback

Canvas attendance quiz passcode:

ANNOUNCEMENTS

Friday homework review sessions

- Review, ask questions, work through weekly homework problems with an instructor
- Friday 4-5 p.m. EST
- Zoom links posted on Discord (#course-announcements)
- Recordings will be made available if you cannot attend live

Canvas attendance quiz passcode:

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz (using the password posted below and in the chat).
  - This is lab number : **<insert lab number>**
  - Passcode: **<insert passcode>**
- The decimal number 2 is represented in binary as:
  - 11
  - 10
  - 00
  - 01
- **This quiz not graded, but counts for your lab attendance!**

Canvas attendance quiz passcode:

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

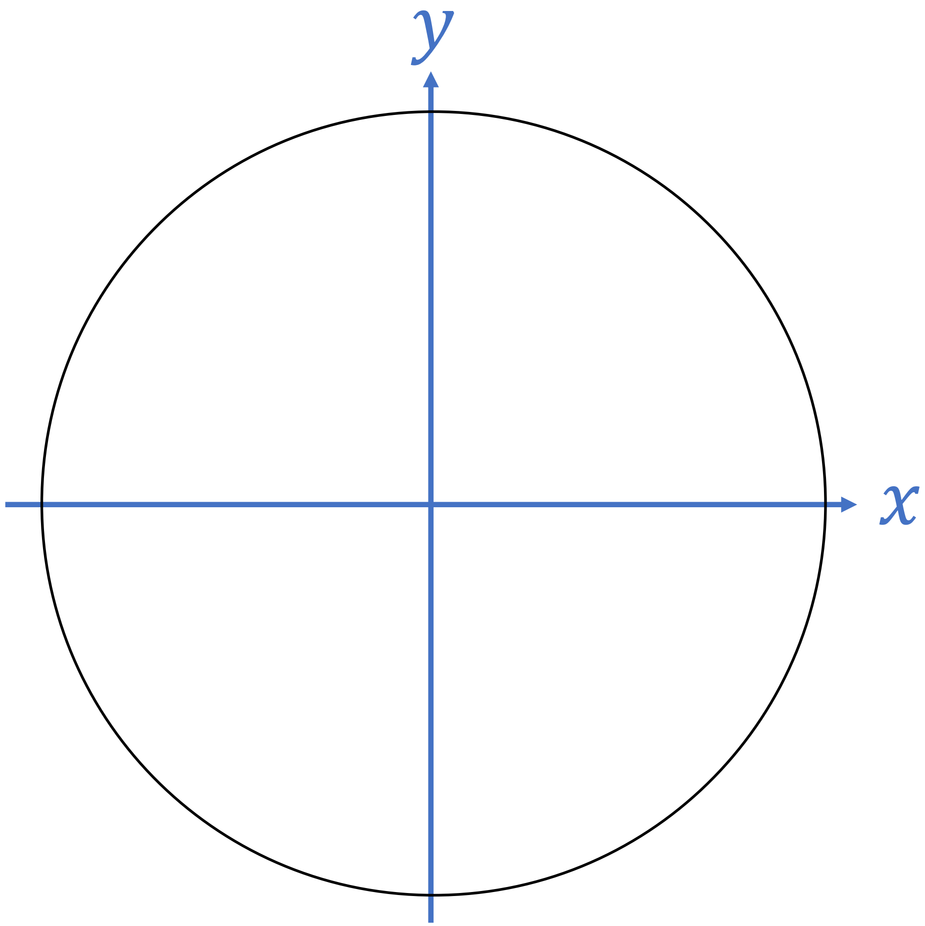
Canvas attendance quiz passcode:

QUESTIONS FROM PAST WEEK

Homework 2, problem 2, part iii)

Convert the following points from Cartesian coordinates to polar coordinates. You may use a calculator for this problem:

x=-2, y=2



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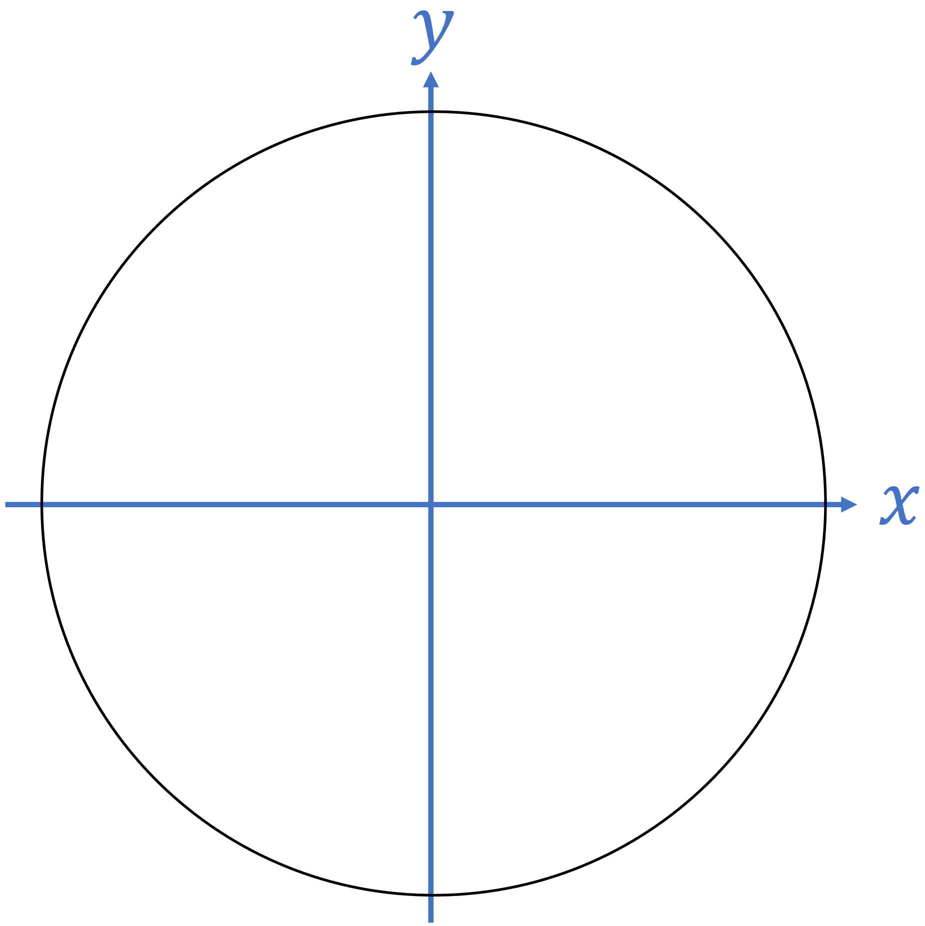
Homework 2, problem 2, part iv)

Convert the following points from Cartesian coordinates to polar coordinates. You may use a calculator for this problem:

x=0, y=1



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LEARNING OBJECTIVES FOR LAB 3

- Solidify understanding of vectors
  - Representation
  - Addition
  - Multiplication
  - Decomposition

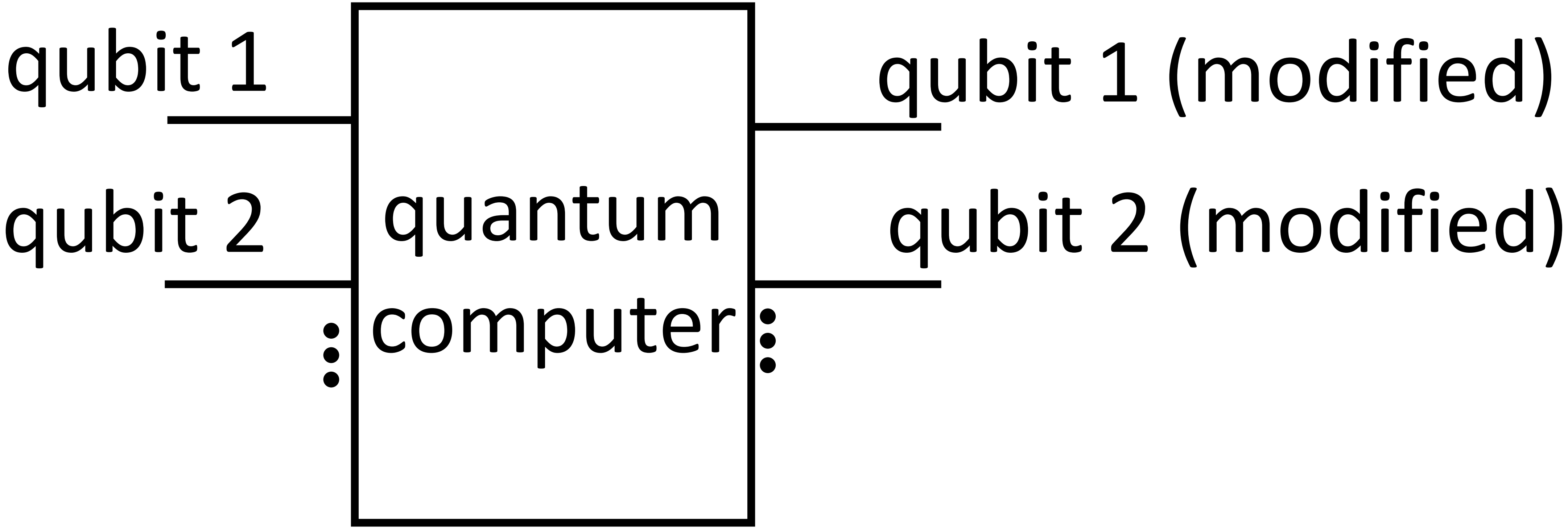
- Perform basic operations on Complex numbers
  - Representation
  - Modulus and Conjugate
  - Multiplication in cartesian form
  - Multiplication in polar form*

- Applying vectors and complex numbers to quantum computing*
  - The Bloch sphere*

*Optional content

Canvas attendance quiz passcode:

SCHEMATIC OF A QUANTUM COMPUTER



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REPRESENTING QUBITS

classical bit

either 0 or 1

REPRESENTING QUBITS

classical bit

qubit

either 0 or 1

superposition of 0 and 1

VECTOR NOTATION

scalar

$$v = 2$$

Canvas attendance quiz passcode:

VECTOR NOTATION

scalar

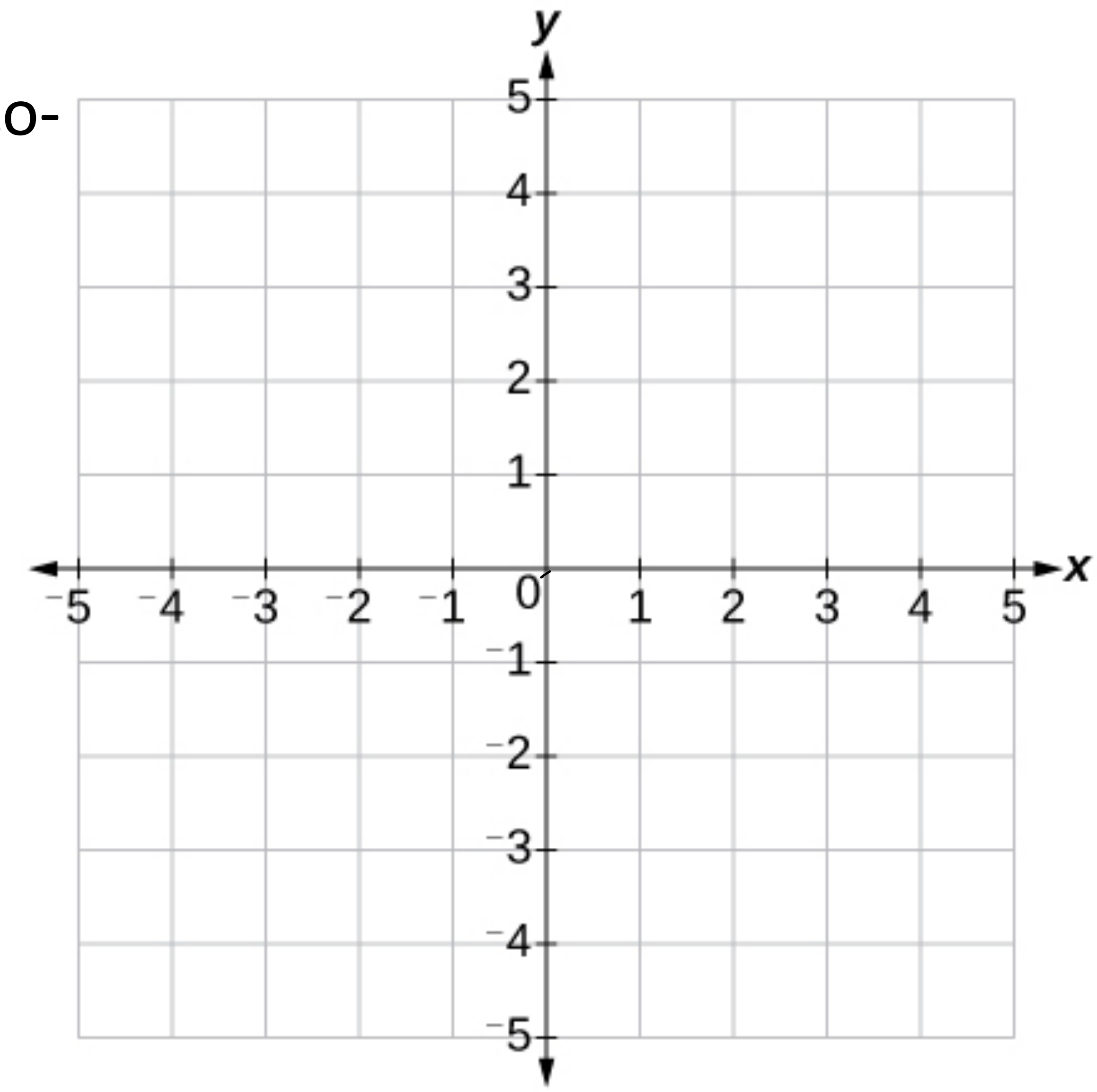
$$v = 2$$

vector

$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system



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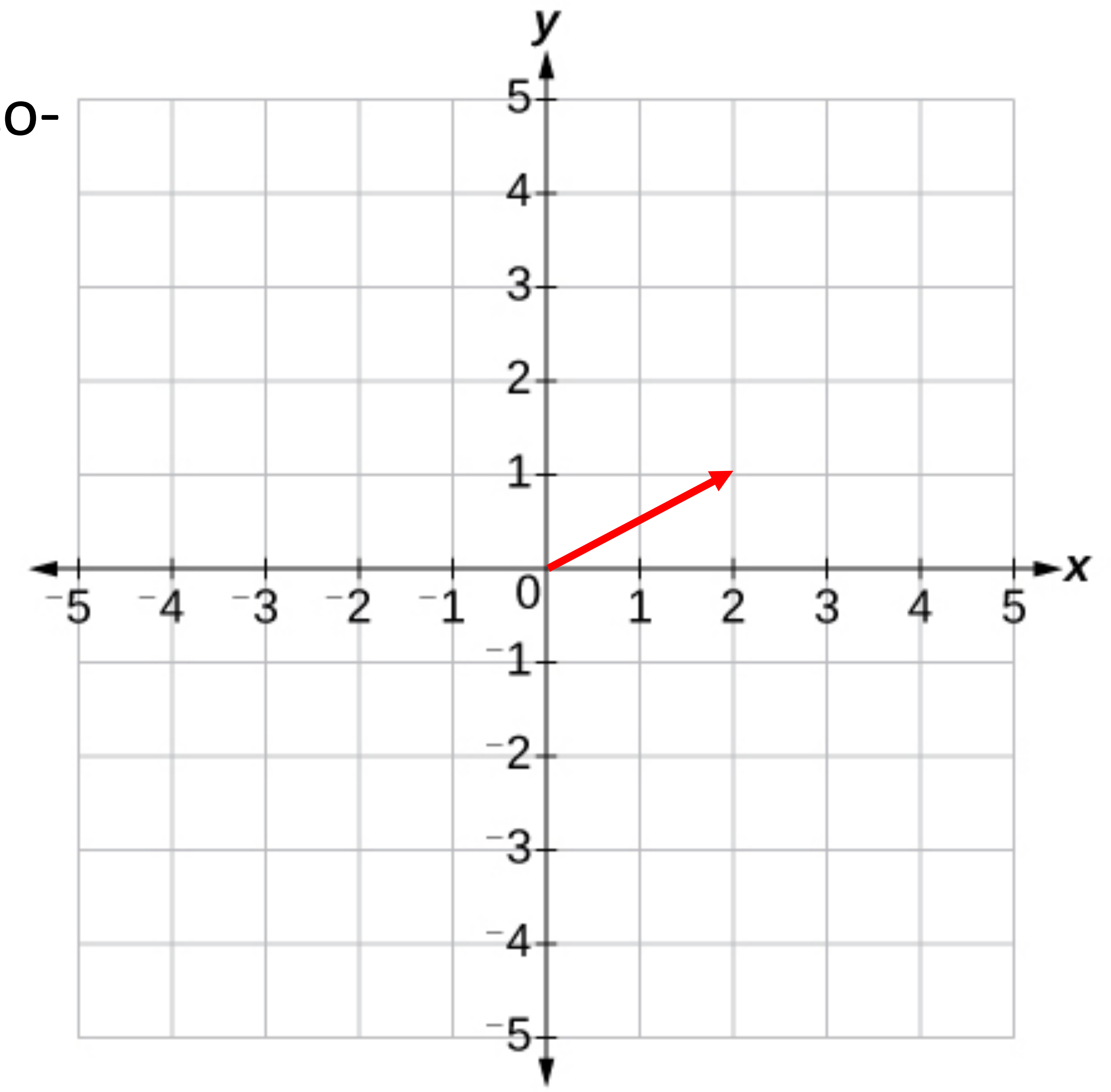
VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

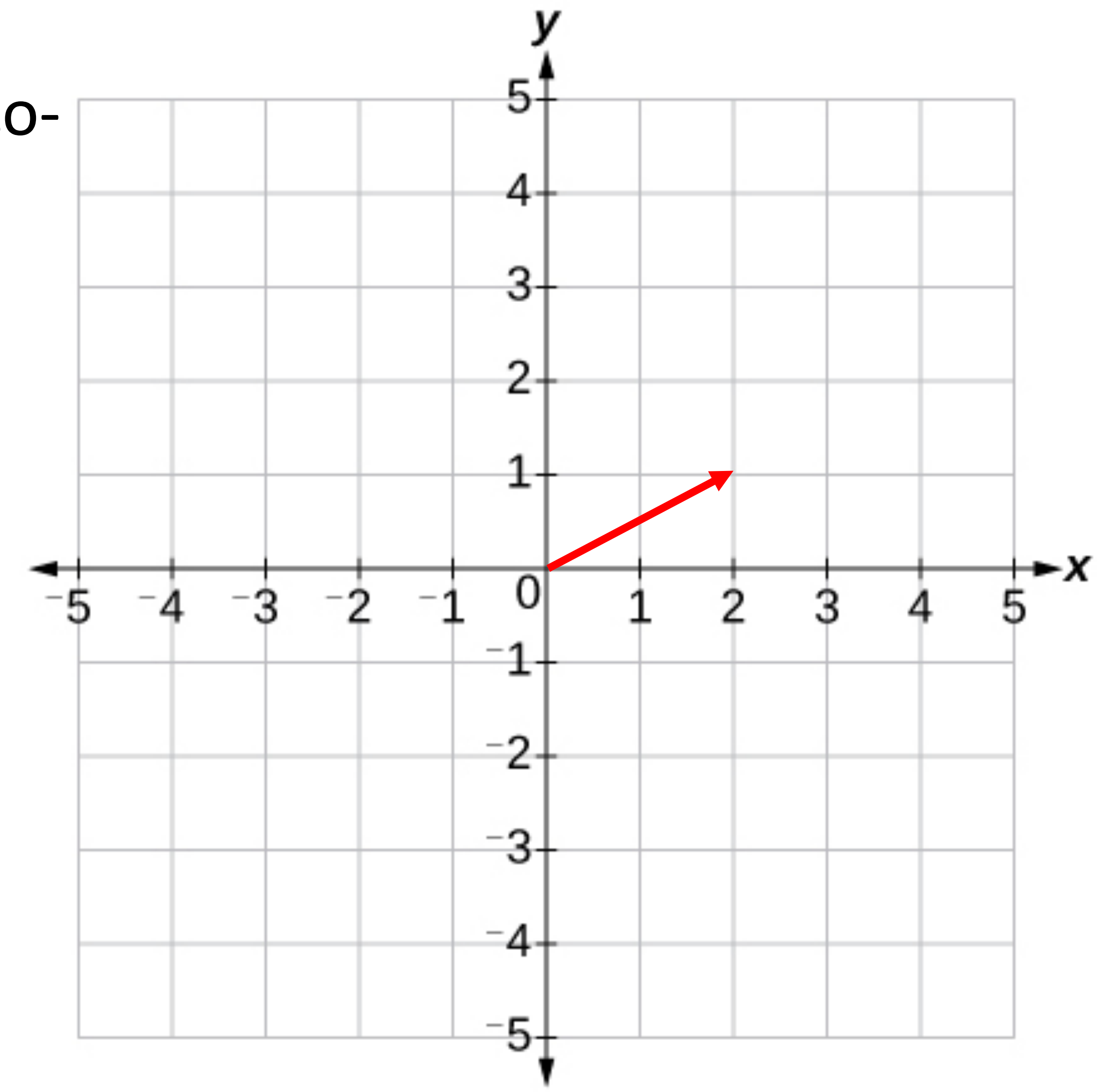


Canvas attendance quiz passcode:

VECTOR NOTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

$$v_x = 2$$
$$v_y = 1$$



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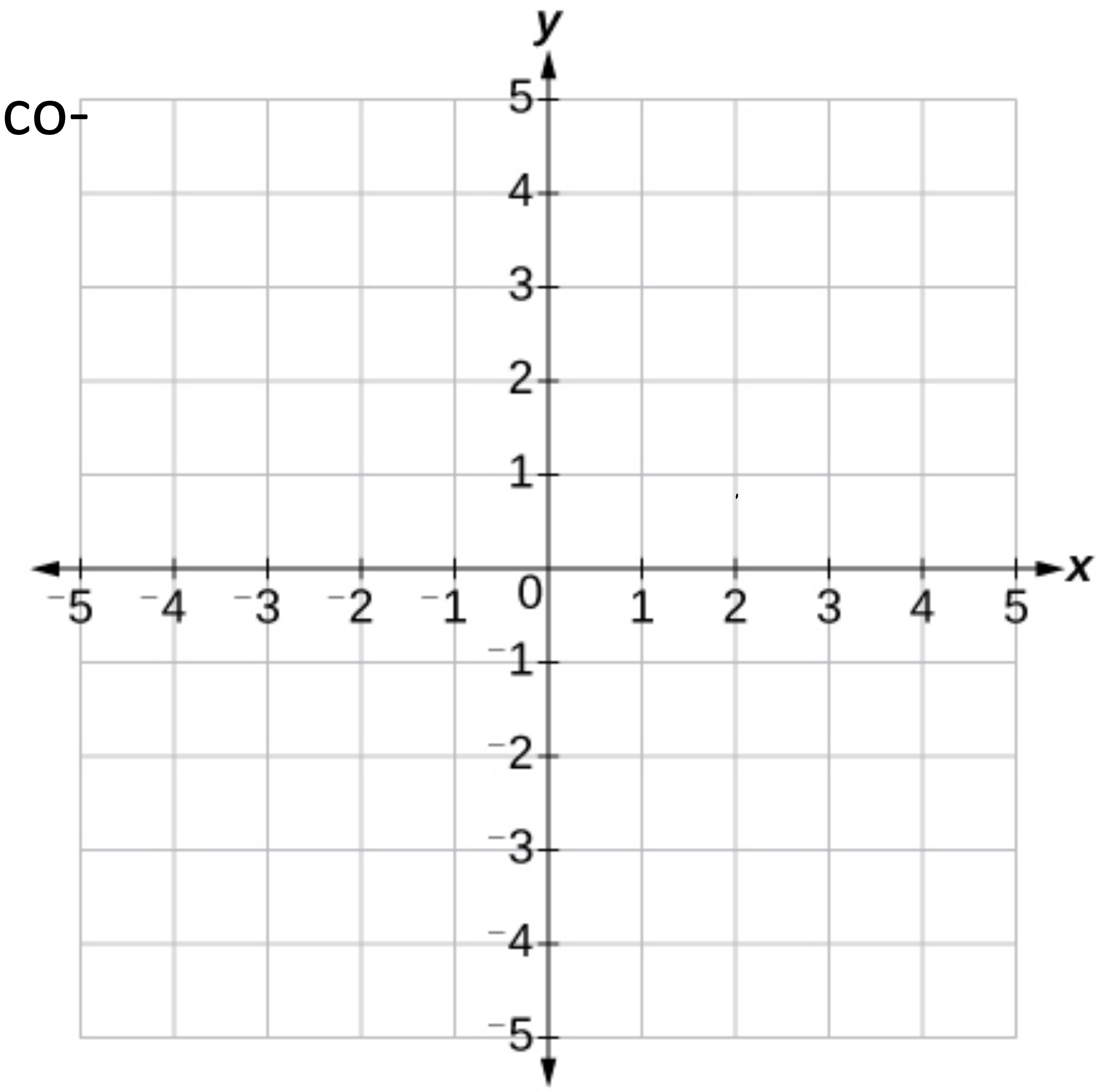
VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?



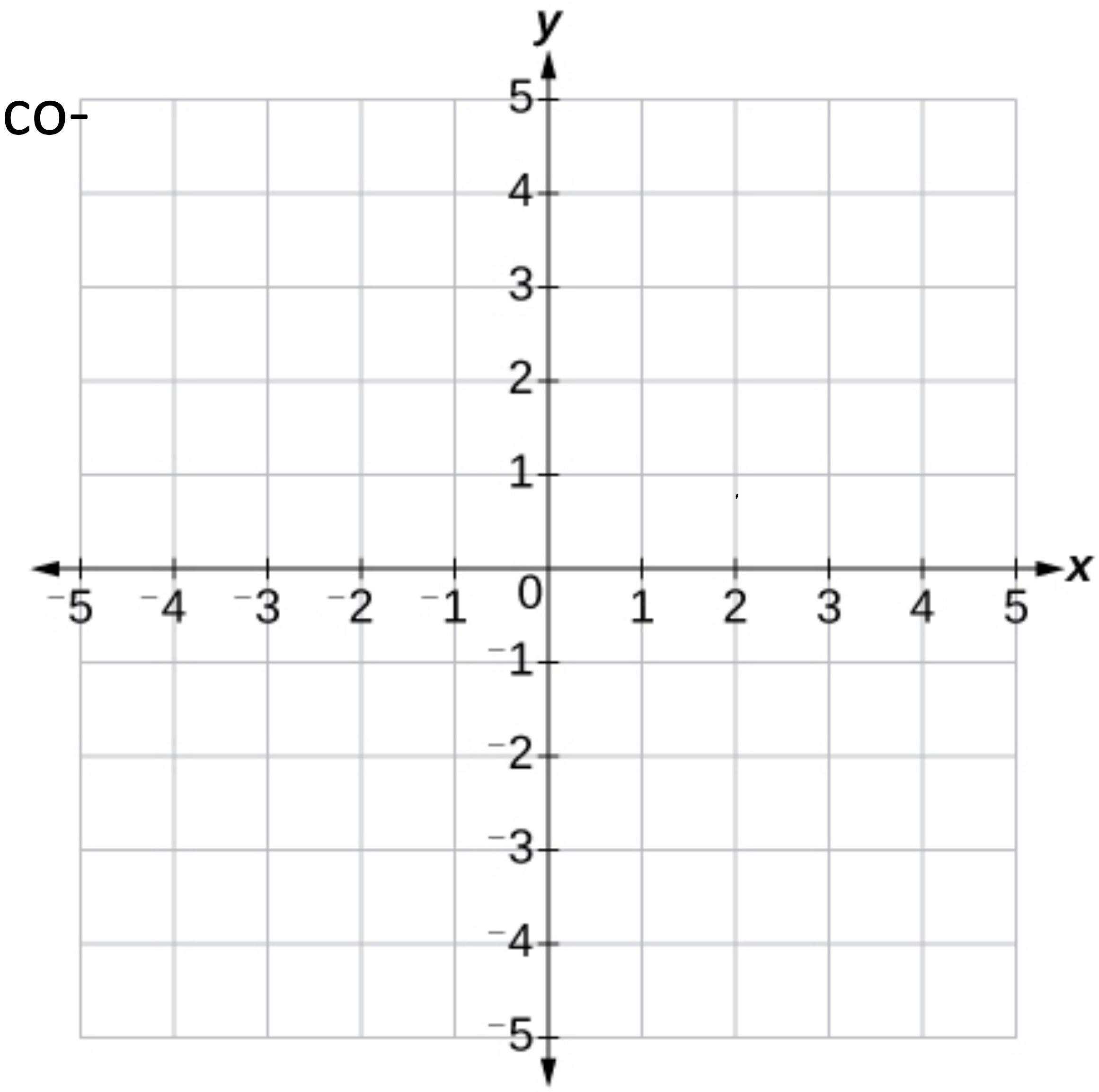
Canvas attendance quiz passcode:

VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$



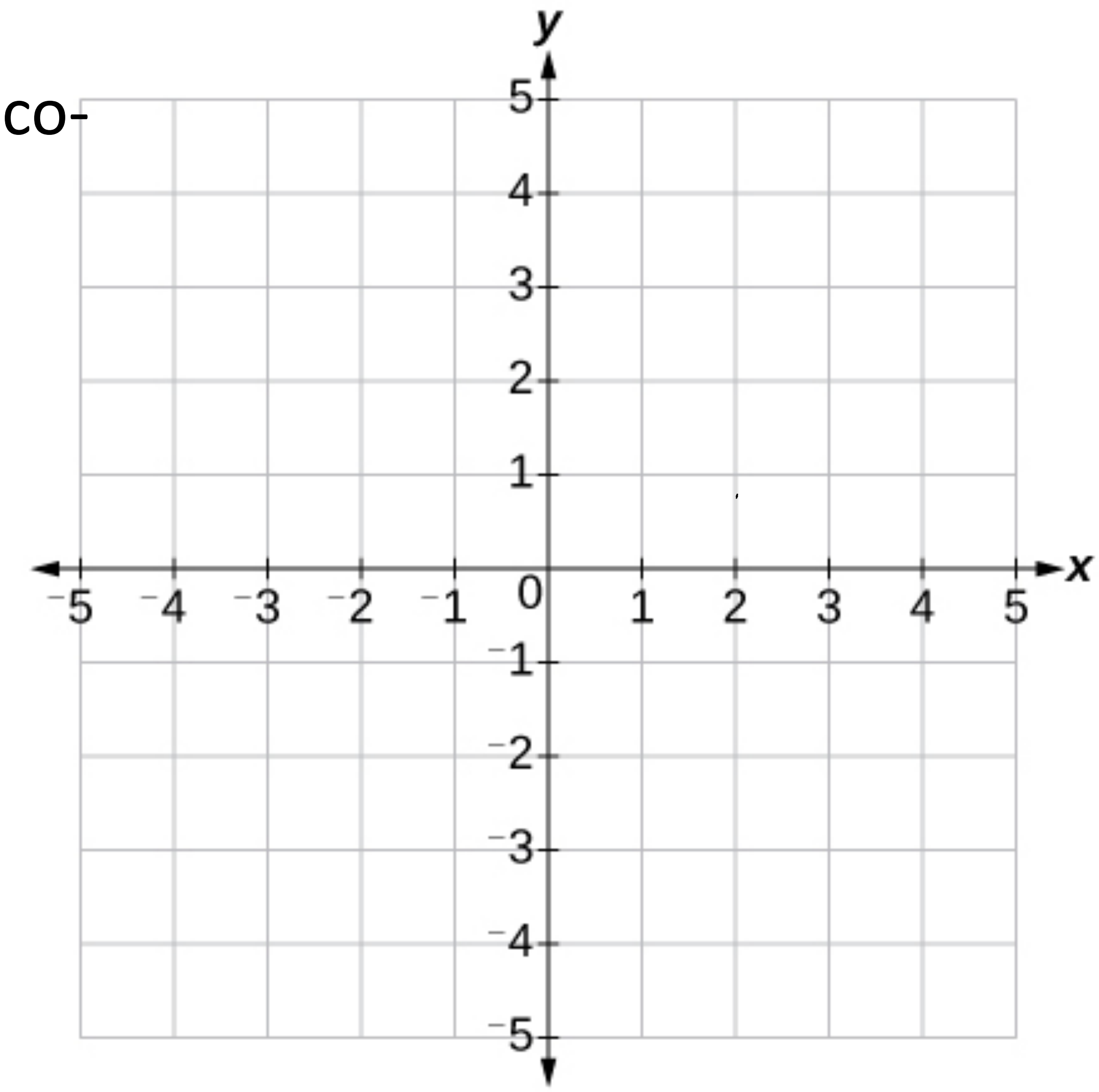
VECTOR REPRESENTATION

Locate the vector $\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ on a cartesian co-ordinate system

- What is the magnitude of this vector?
- What is the direction of this vector?

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

$$\angle\,\vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



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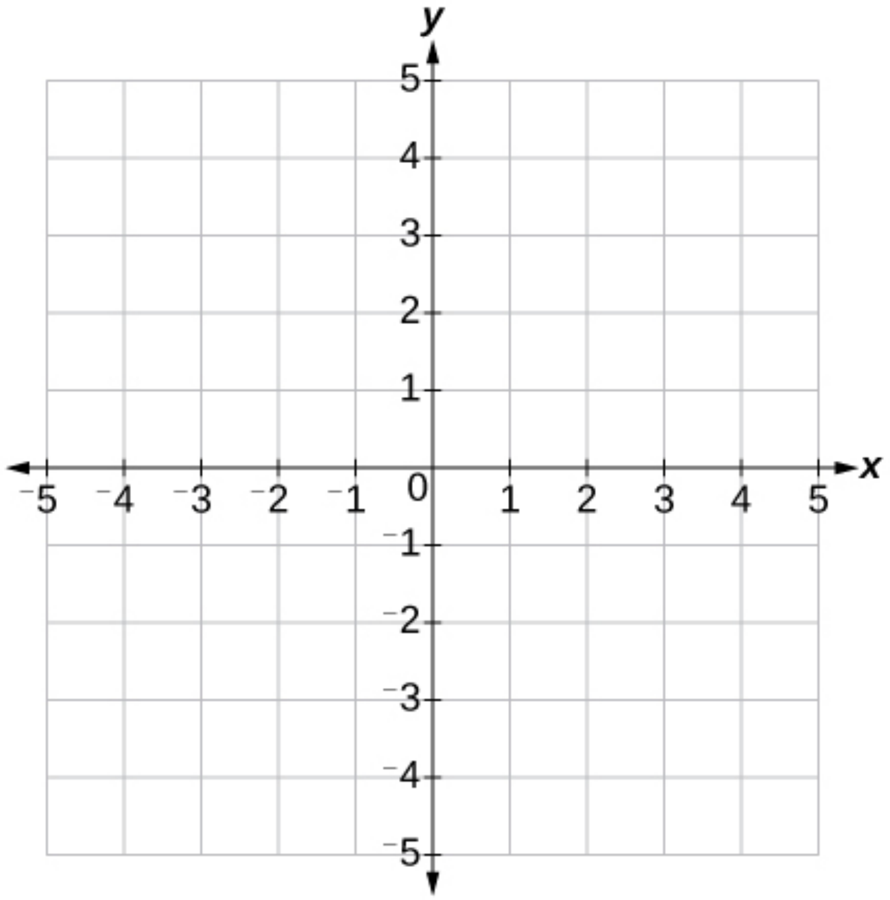
VECTOR ADDITION

Add the vectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$

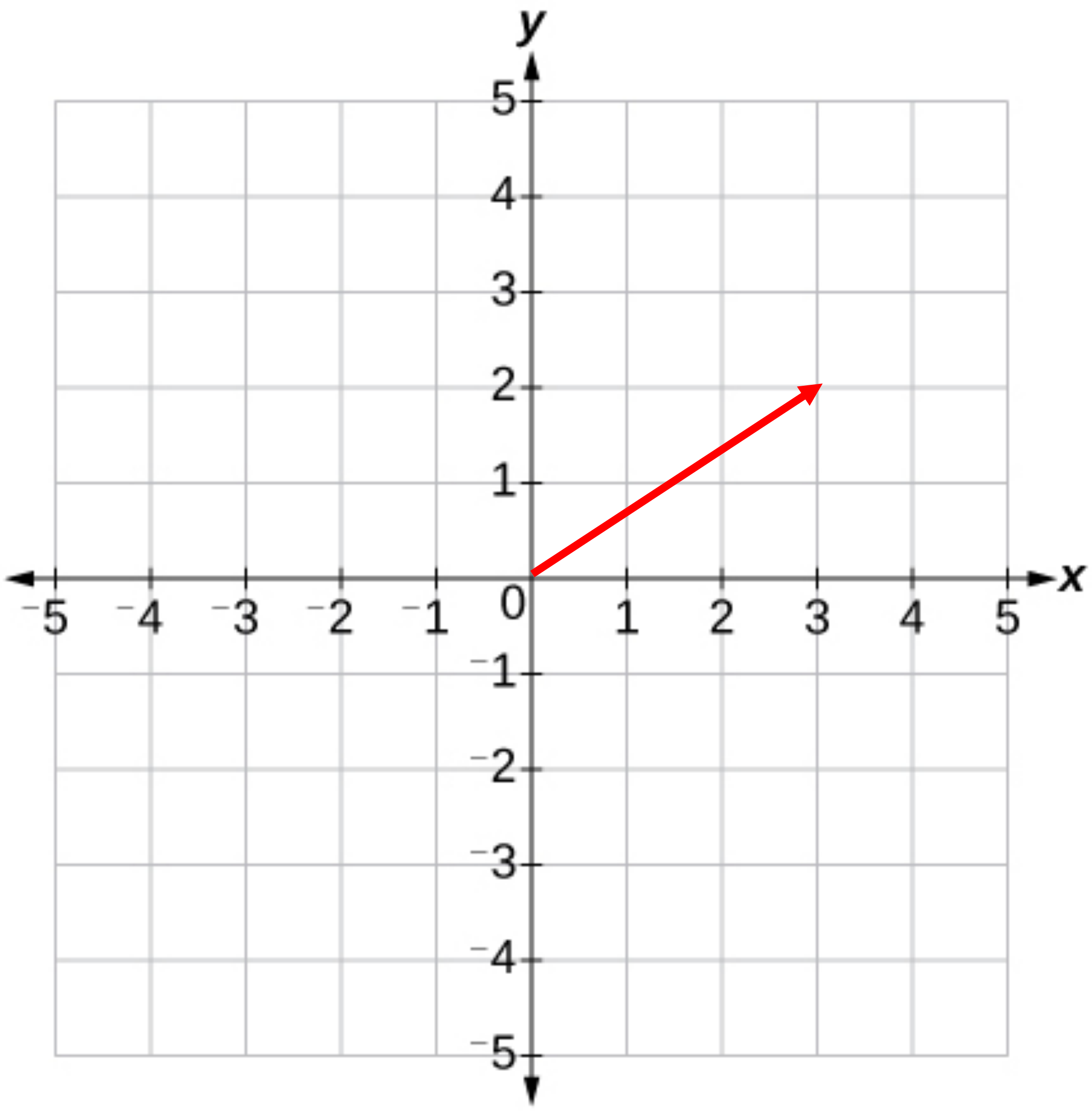
$$\vec{a} + \vec{b} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix}$$



Canvas attendance quiz passcode:

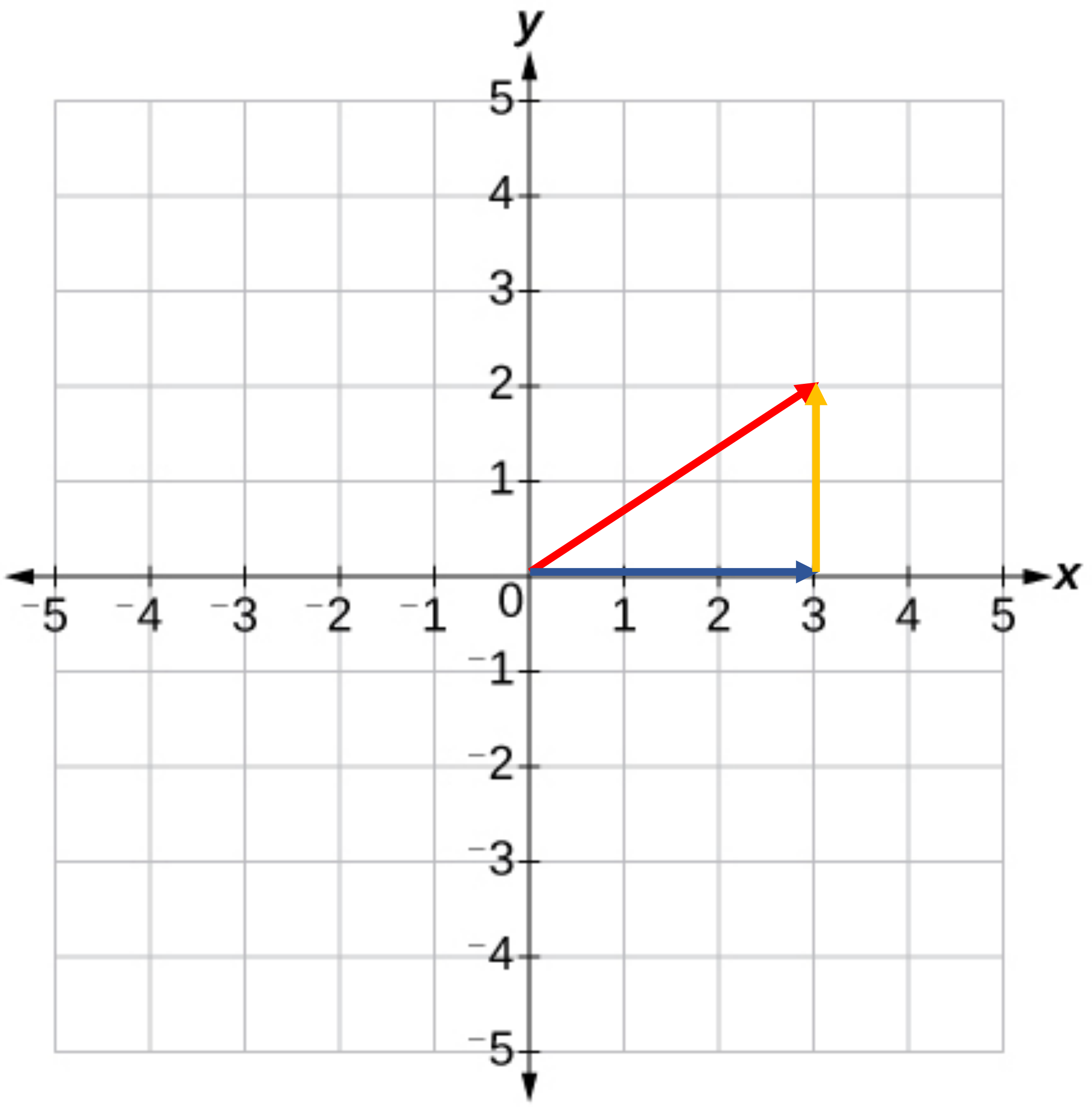
VECTOR DECOMPOSITION

$$\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix}$$



VECTOR DECOMPOSITION

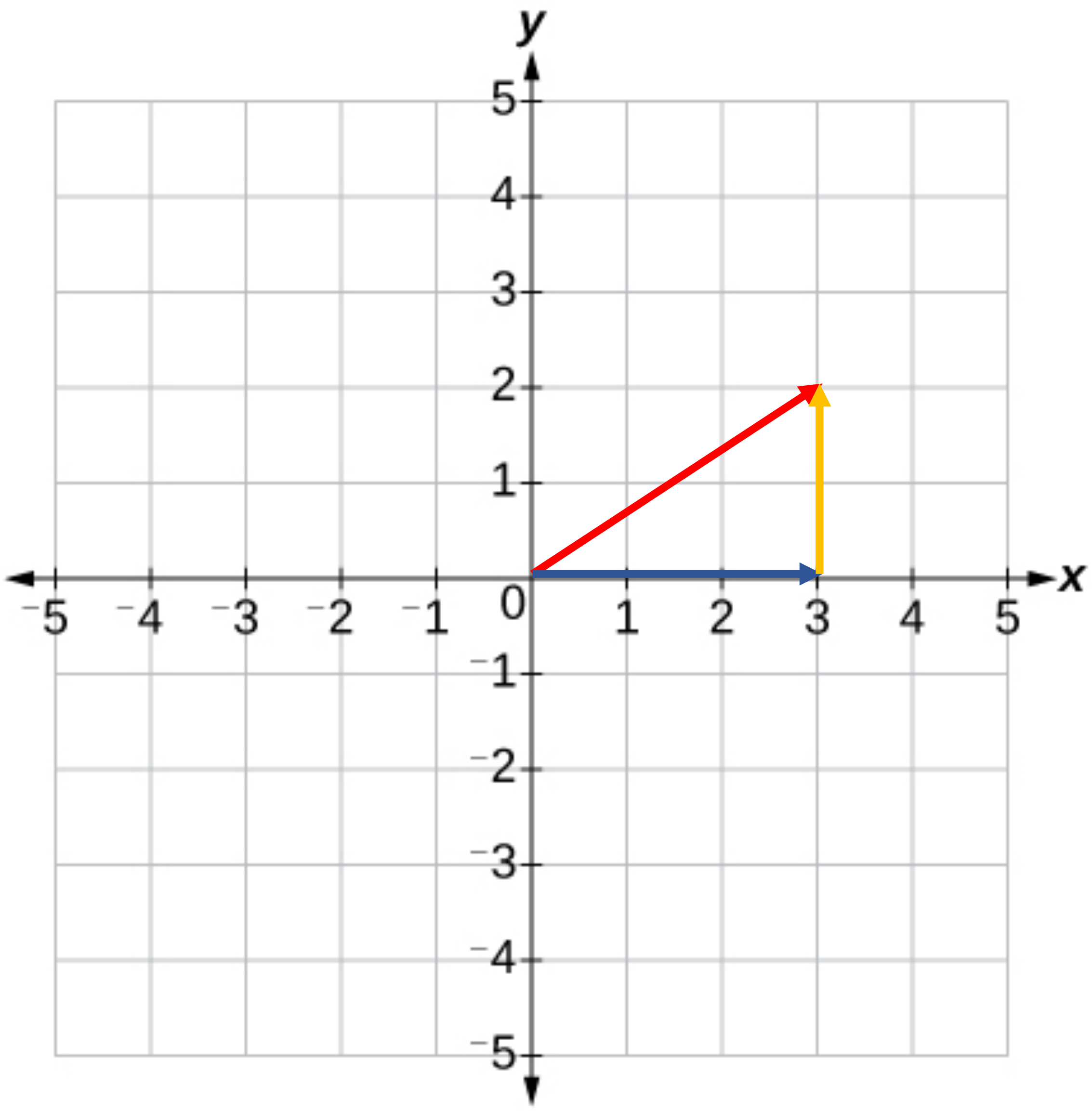
$$\begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix}$$



Canvas attendance quiz passcode:

VECTOR DECOMPOSITION

$$\begin{pmatrix}3\\2\end{pmatrix} = \underbrace{\begin{pmatrix}3\\0\end{pmatrix}}_{\text{along x-axis}} + \underbrace{\begin{pmatrix}0\\2\end{pmatrix}}_{\text{along y-axis}}$$



- x-component: 3
- y-component: 2

QUESTIONS

Questions on content so far?

Canvas attendance quiz passcode:

COMPLEX NUMBERS

A complex number consists of both a ***real*** and ***imaginary*** component.

$$z = a + i\,b$$

Complex Number

Real Component

Imaginary Unit

Imaginary Coefficient

Imaginary Component

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COMPLEX NUMBER REPRESENTATION

Complex numbers can additionally be represented as ***vectors***,
in the 2D ***complex plane***!

$$z = a + i\,b$$

Canvas attendance quiz passcode:

COMPLEX NUMBER REPRESENTATION

Complex numbers can additionally be represented as ***vectors***, in the 2D ***complex plane***!

$$z = a + i\,b$$

$$\vec{z} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Real Component

Imag. Component

Canvas attendance quiz passcode:

COMPLEX NUMBER REPRESENTATION

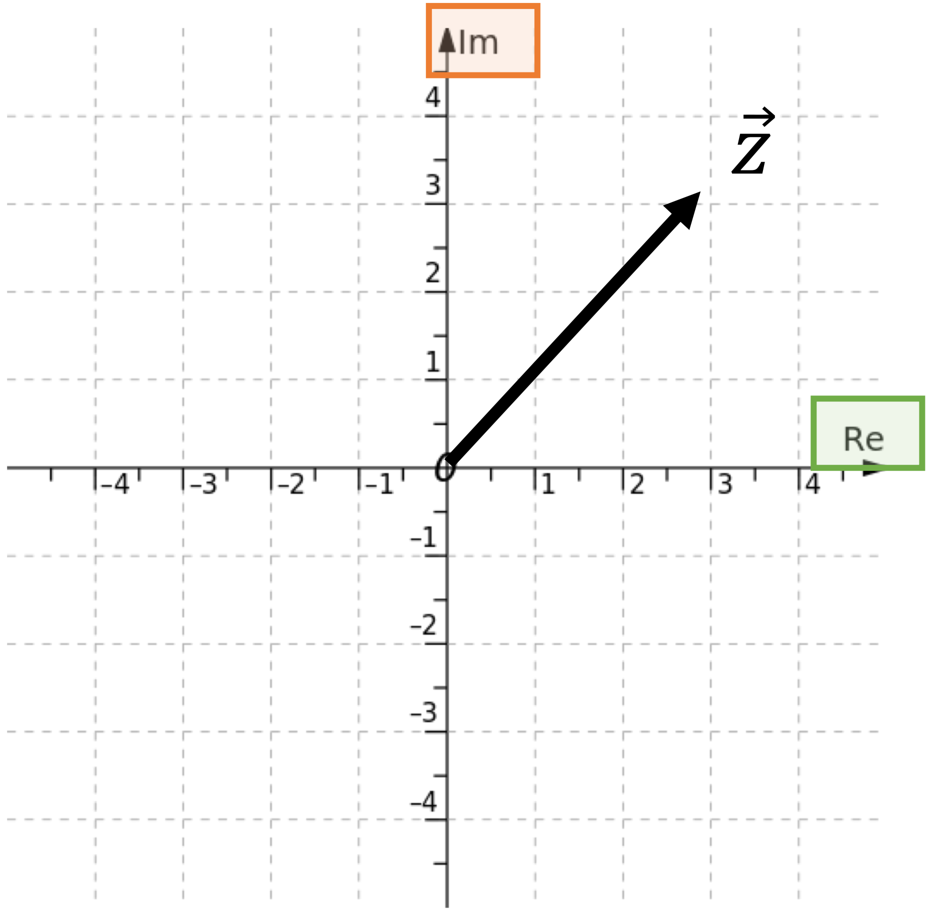
Complex numbers can additionally be represented as ***vectors***, in the 2D ***complex plane***!

$$z = a + i\,b$$

$$\vec{z} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Real Component

Imag. Component



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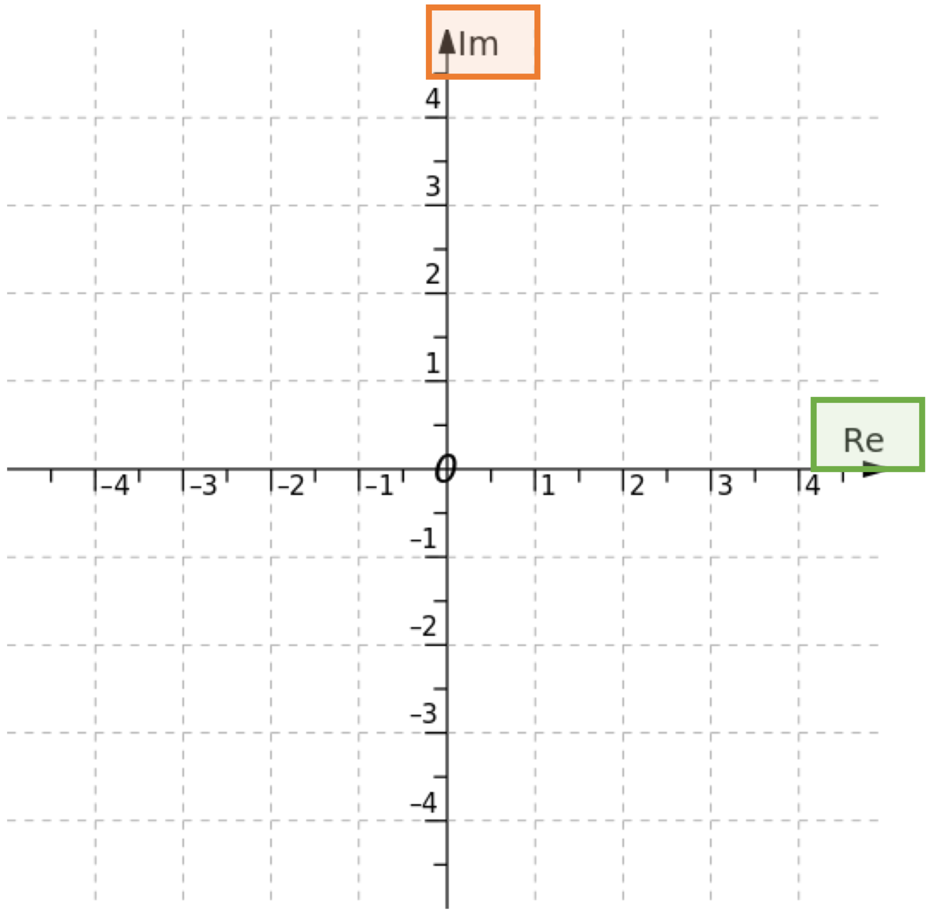
COMPLEX NUMBER REPRESENTATION

$$z = 2 + 3i$$

$$\vec{z} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$



Canvas attendance quiz passcode:

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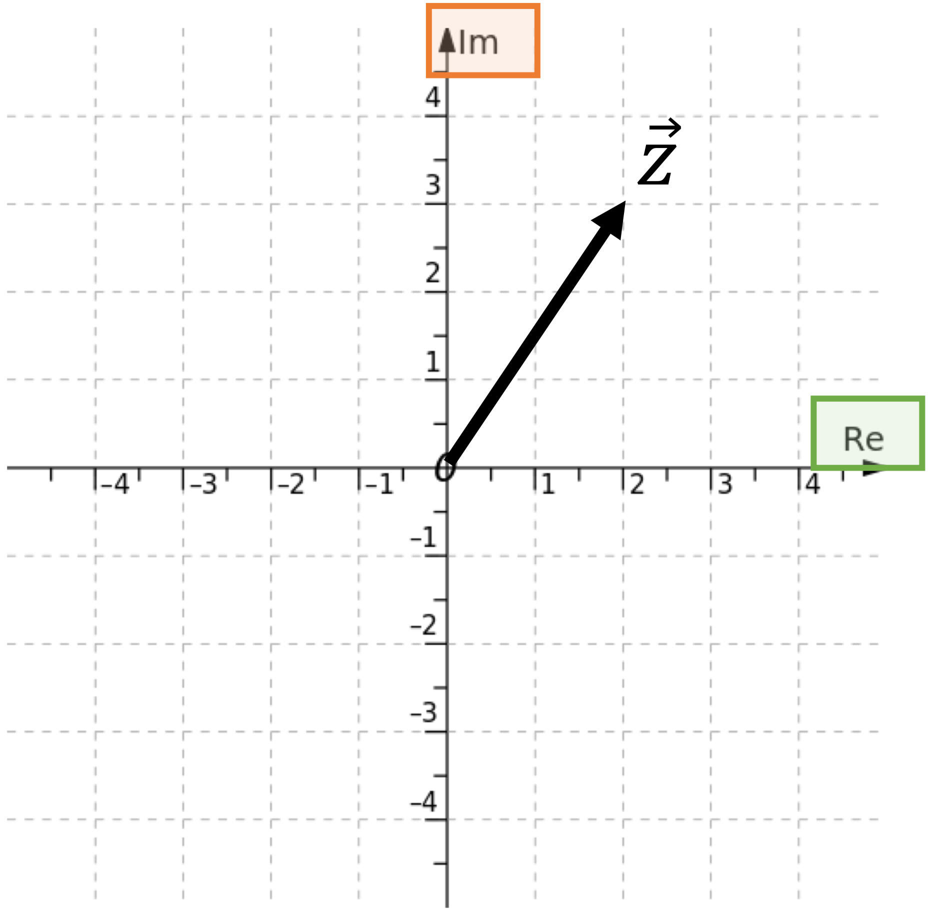


Canvas attendance quiz passcode:

MODULUS AND CONJUGATE NOTATION

$$z = a + \ i\, b$$

Canvas attendance quiz passcode:

MODULUS AND CONJUGATE NOTATION

$$z = a + \ i\, b$$

Modulus: $|z| = \sqrt{a^2 + b^2}$

Conjugate: $\bar{z} = a - ib$

Canvas attendance quiz passcode:

MODULUS AND CONJUGATE

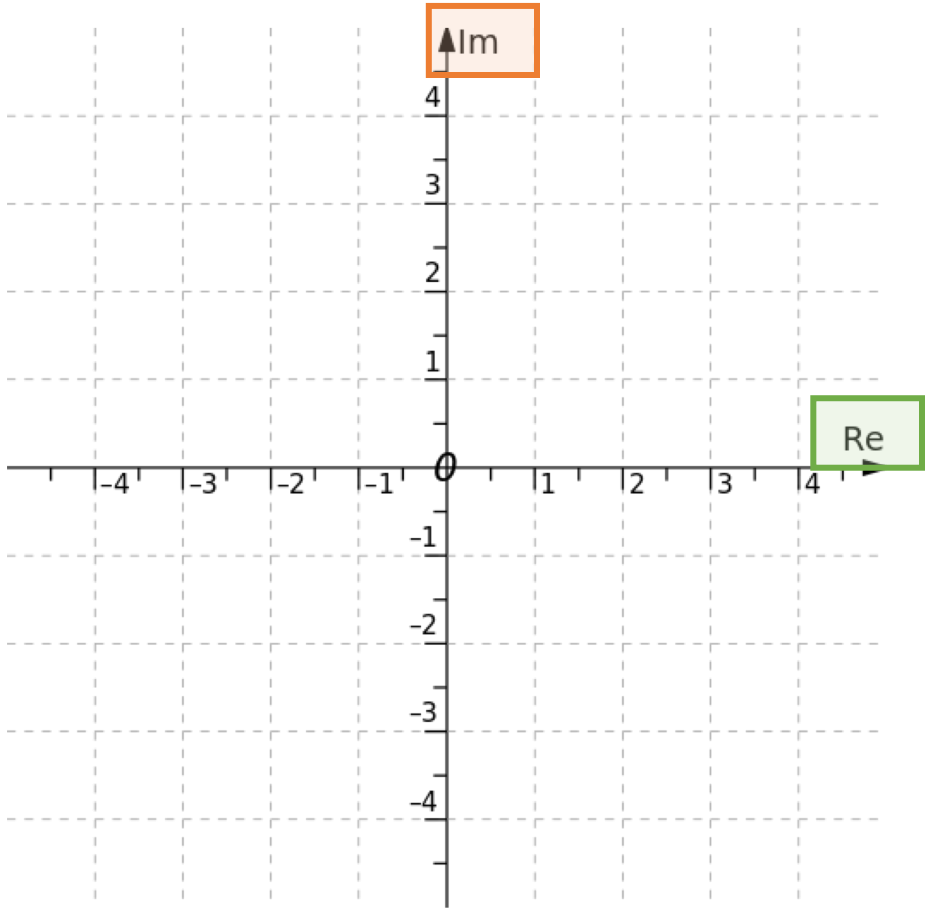
$$|a + ib| = \sqrt{a^2 + b^2}$$

$$\overline{(a + i\,b)} = (a - ib)$$

$$z = 2 + 3i$$

$$|z| =$$

$$\bar{z} =$$



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COMPLEX NUMBER ADDITION

$$\boxed{(a + i\,b) + (c + id) = (a + c) + i(b + d)}$$

$$z_1 = 2 + 3i$$

$$z_2 = 3 - 2i$$

$$z_1 + z_2 =$$

Canvas attendance quiz passcode:

COMPLEX NUMBER MULTIPLICATION

$$(a + i\,b) * (c + id) = (ac - bd) + i(ad + bc)$$

$$z_1 = 2 + 3i$$

$$z_2 = 3 - 2i$$

$$z_1 . z_2 =$$

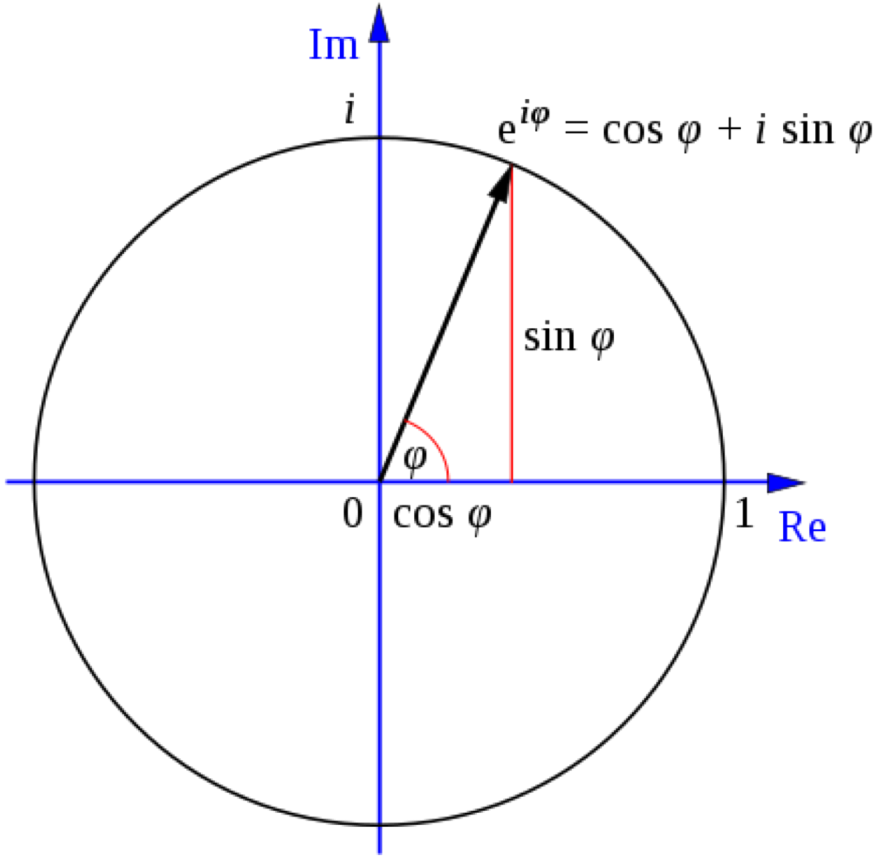
Canvas attendance quiz passcode:

POLAR FORM OF COMPLEX NUMBERS

Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

POLAR FORM OF COMPLEX NUMBERS



Euler's formula:

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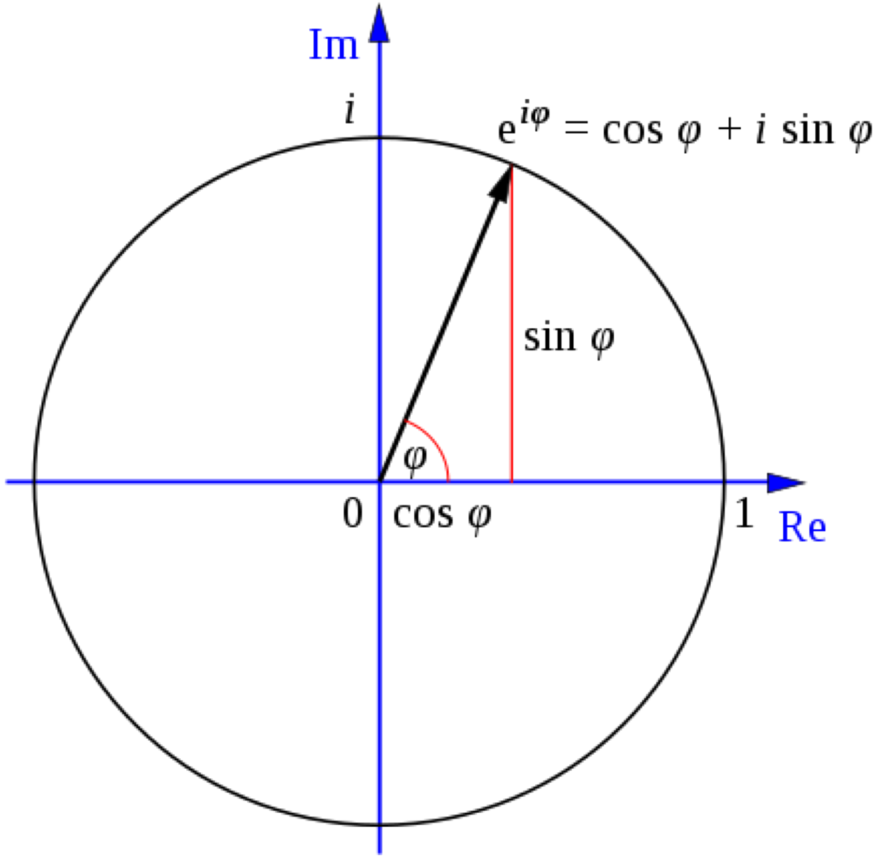
Canvas attendance quiz passcode:

POLAR FORM OF COMPLEX NUMBERS



Source: Wikipedia (CC)

Euler's formula:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

Polar representation of complex numbers!

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

Canvas attendance quiz passcode:

POLAR FORM OF COMPLEX NUMBERS

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$z = 2 + 3i$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

$$r =$$

$$\varphi =$$

$$z \text{ (Polar form)} =$$

Canvas attendance quiz passcode:

POLAR FORM OF COMPLEX NUMBERS

Steps to find the polar form of $z = x + iy$:

1. Find the vector radius $r = |z| = \sqrt{x^2 + y^2}$

2. Find the vector angle $\varphi = \tan^{-1}\left(\frac{y}{x}\right)$

3. Combine radius and angle to get $z = re^{i\varphi}$

Canvas attendance quiz passcode:

QUESTIONS

Questions on content so far?

POST-LAB CANVAS FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

Canvas attendance quiz passcode:

OPTIONAL CONENT

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LOCATING A PLACE ON EARTH

- How would you tell me where your hometown is?

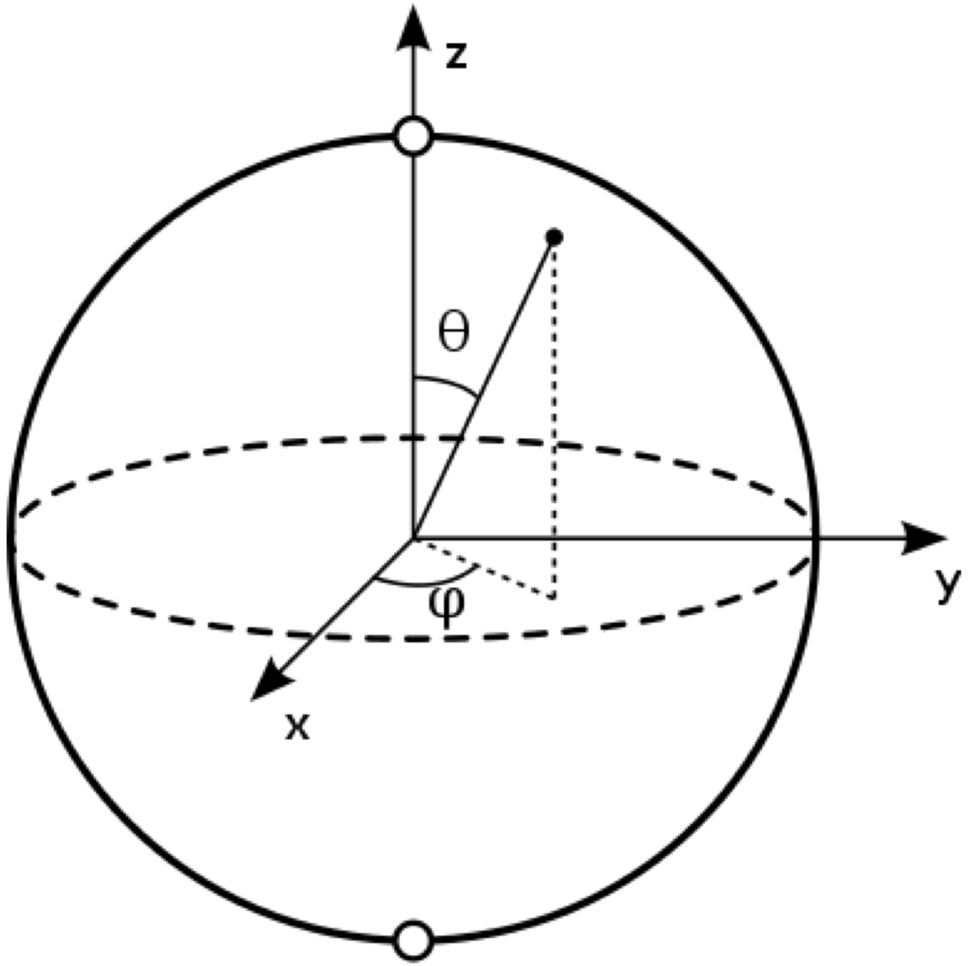
Canvas attendance quiz passcode:

LOCATING A PLACE ON EARTH

- How would you tell me where your hometown is?
- Latitude and longitude! Two angles

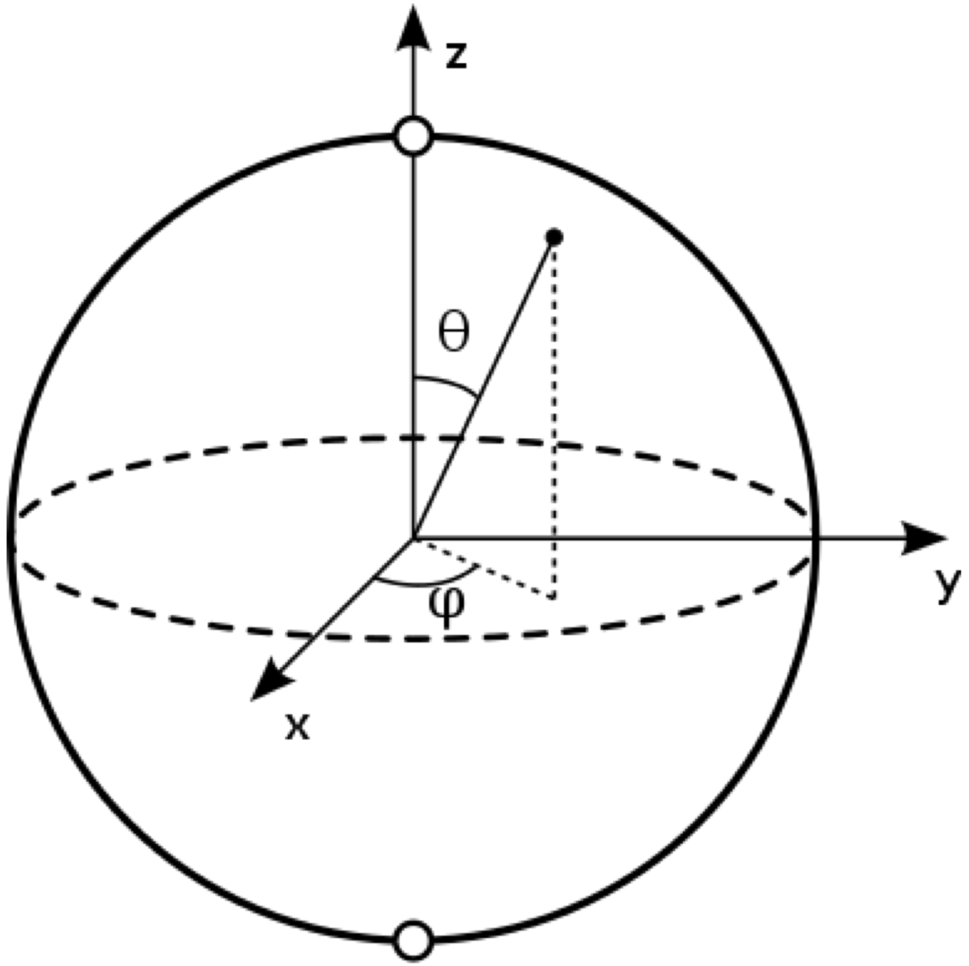
Canvas attendance quiz passcode:

LOCATING POINTS ON A SPHERE



Canvas attendance quiz passcode:

LOCATING POINTS ON A SPHERE

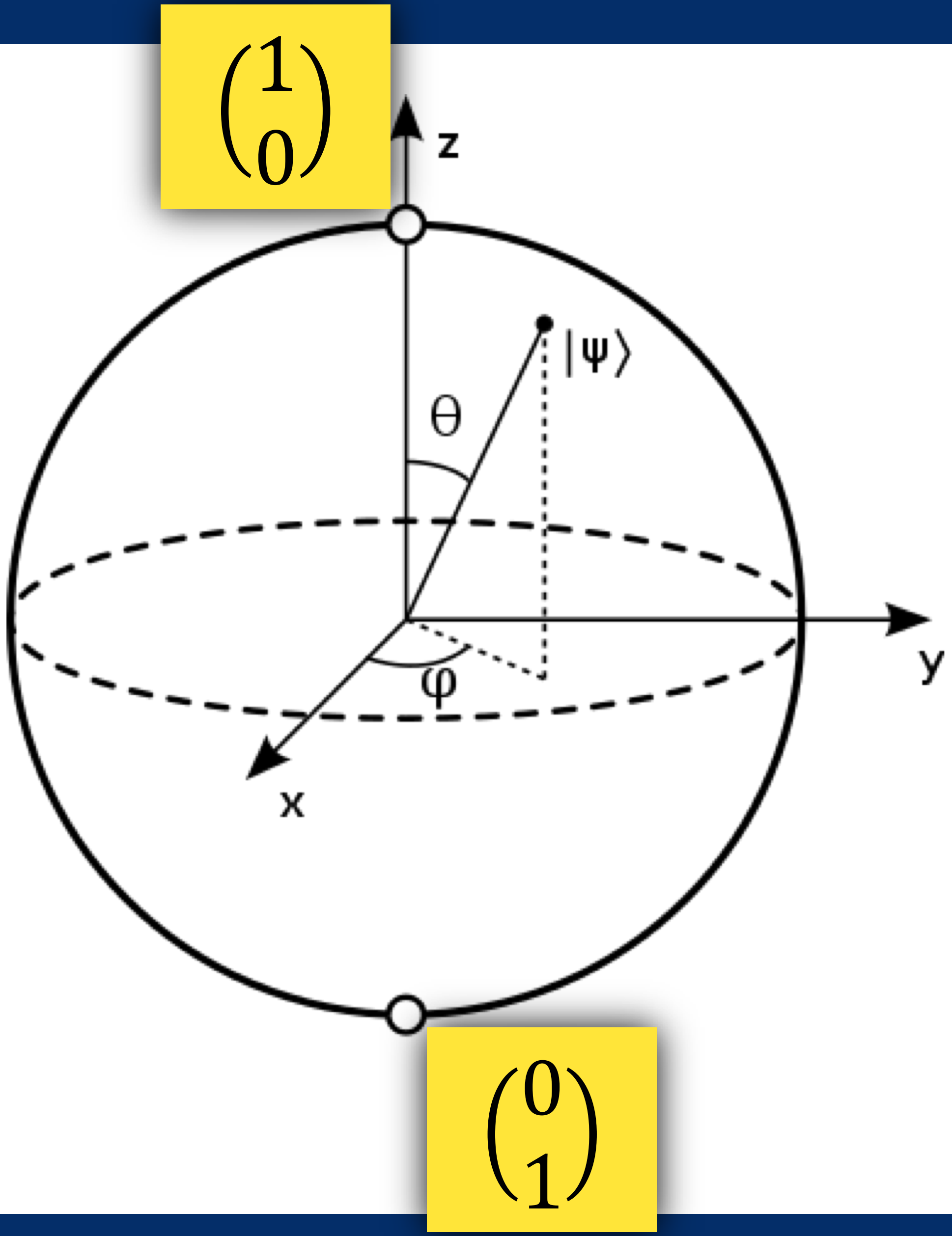


$$\theta \in \{0, \pi\}$$

$$\phi \in \{0, 2\pi\}$$

Canvas attendance quiz passcode:

THE BLOCH SPHERE



$$\psi(\theta,\phi) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$\theta \in \{0,\pi\}$$

$$\phi \in \{0,2\pi\}$$

Canvas attendance quiz passcode:

MULTIPLICATION IN THE POLAR FORM

$$z_1 = 1e^{i\pi/4}$$

$$z_2 = 2e^{i\pi/4}$$

$$z_1 . z_2 =$$

$$z = re^{i\varphi}$$

$$\bar{z} = re^{-i\varphi}$$

$$|z| = r$$

$$z_x = r * \cos(\varphi)$$

$$z_y = r * \sin(\varphi)$$

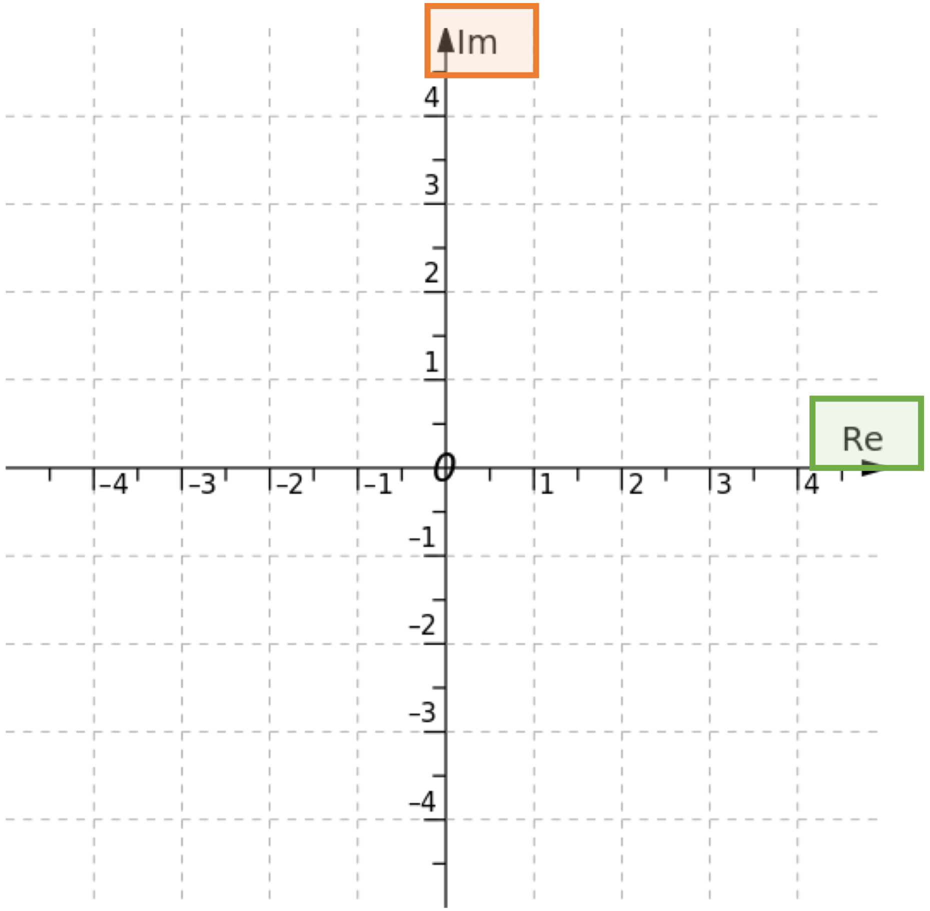
Canvas attendance quiz passcode:

MULTIPLICATION IN THE POLAR FORM

$$z_1 = 1e^{i\pi/4}$$

$$z_2 = 2e^{i\pi/4}$$

$$z_1 . z_2 =$$

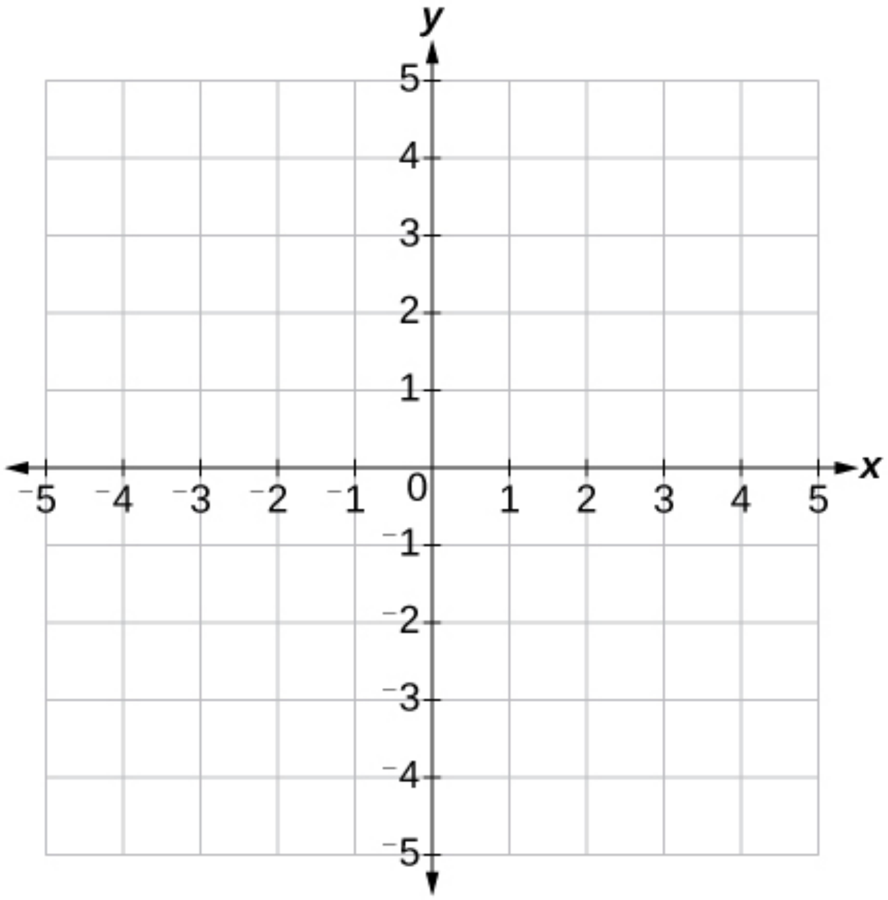


Canvas attendance quiz passcode:

EXTRA PROBLEMS

SCALAR MULTIPLCATION

Find $2 * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $-2 * \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

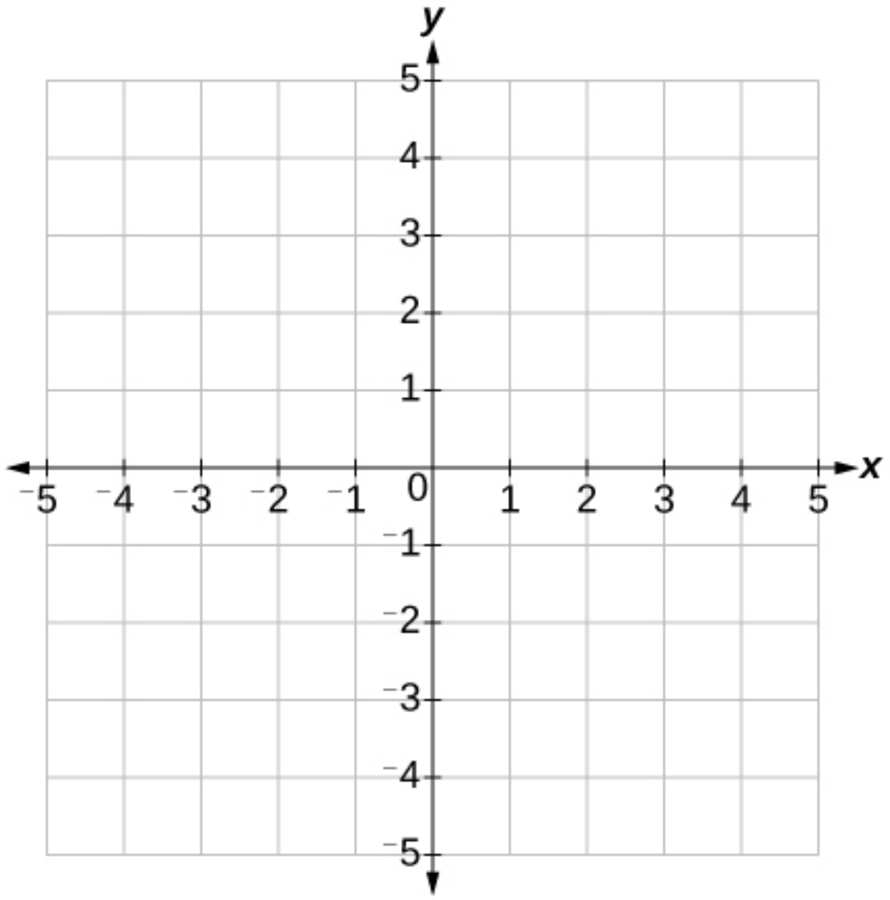


$$c * \vec{a} = \begin{pmatrix} c * w_x \\ c * w_y \end{pmatrix}$$

Canvas attendance quiz passcode:

VECTOR ADDITION

Add the vectors $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

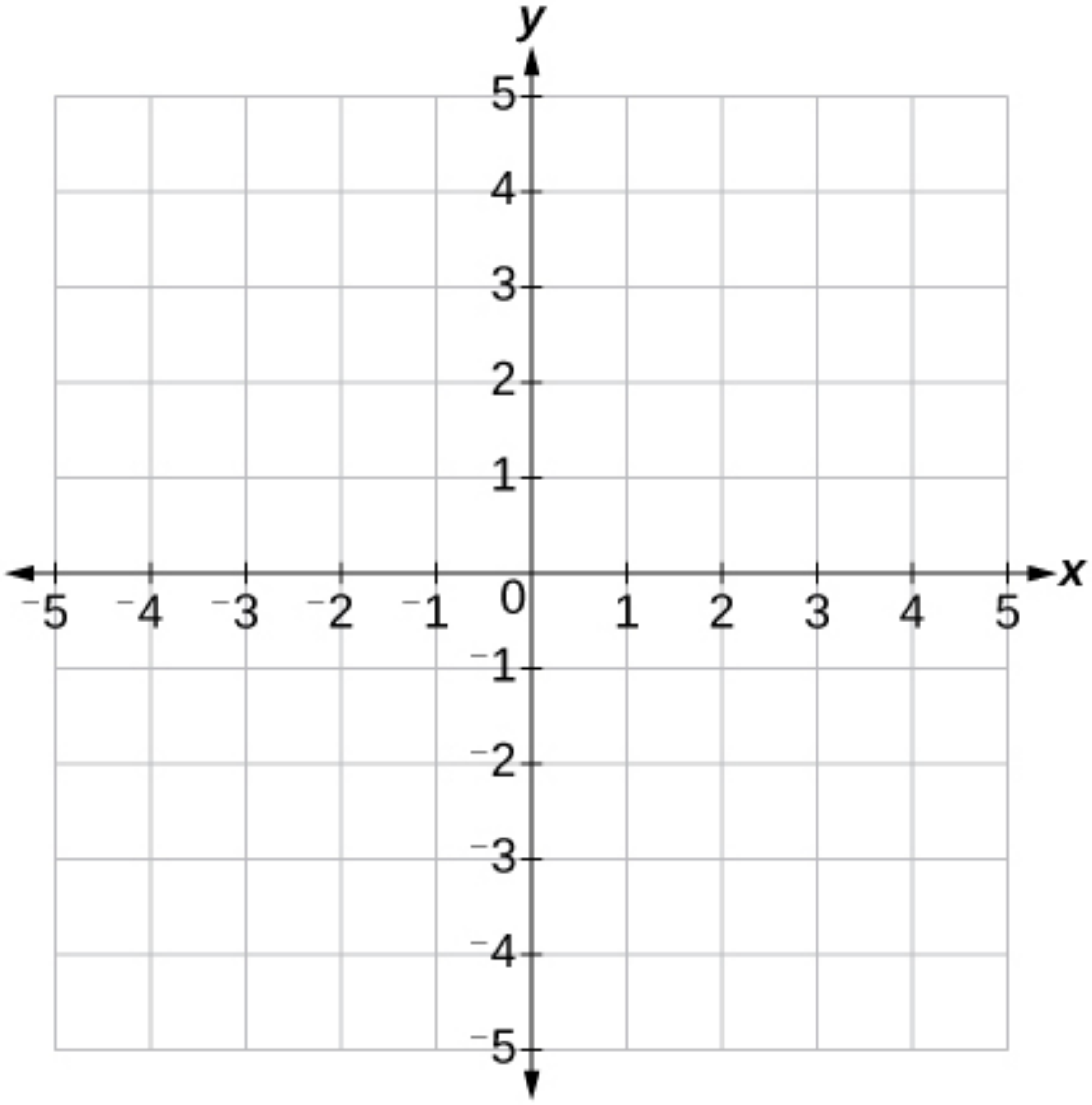


$$\vec{a} + \vec{b} = \begin{pmatrix} a_x + b_x \\ a_y + b_y \end{pmatrix}$$

VECTOR DECOMPOSITION

Find the x and y components of $\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

- Result of adding:
- x-component:
- y-component:

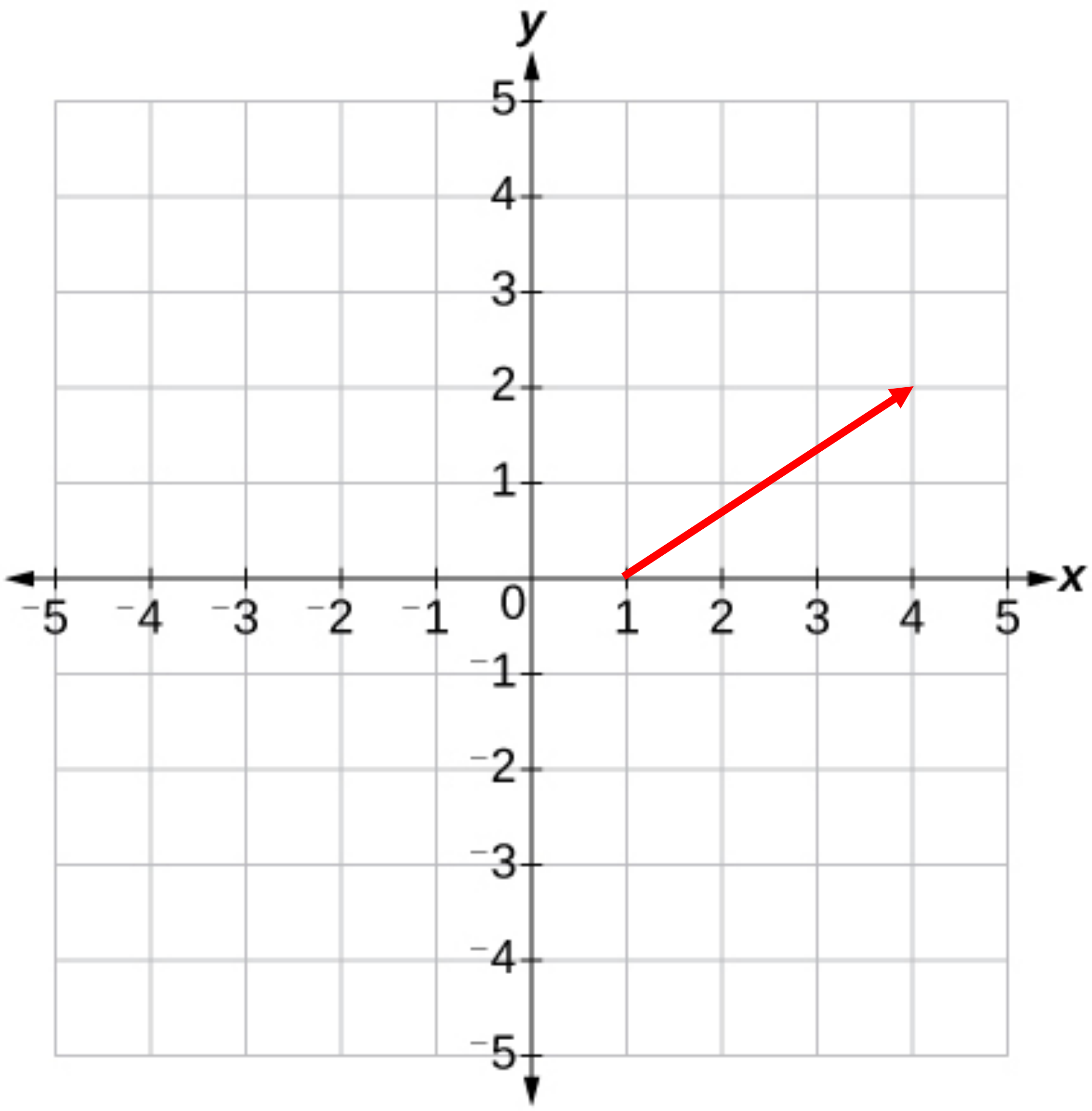


VECTOR REPRESENTATION

- What is this vector?
- What is the magnitude of this vector?
- What is the direction of this vector?

$$\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$$

$$\angle\,\vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$



POLAR FORM OF COMPLEX NUMBERS

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$z = 3/5 + 4/5i$$

$$r = |z| = \sqrt{x^2 + y^2}$$

(vector radius)

$$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$$

(vector angle)

$$r =$$

$$\varphi =$$

$$z \text{ (Polar form)} =$$

Canvas attendance quiz passcode:

WORKING WITH THE POLAR FORM

$$z = 1\, e^{i\pi/4}$$

$$\bar{z} =$$

$$|z| =$$

$$z(\text{Cartesian form}) =$$

$$z = x + iy = |z|(\cos\varphi + i\sin\varphi) = re^{i\varphi}$$

$$\bar{z} = re^{-i\varphi}$$

$$|z| = r$$

$$x = r * \cos\varphi$$

$$y = r * \sin\varphi$$

Canvas attendance quiz passcode:

MODULUS AND CONJUGATE

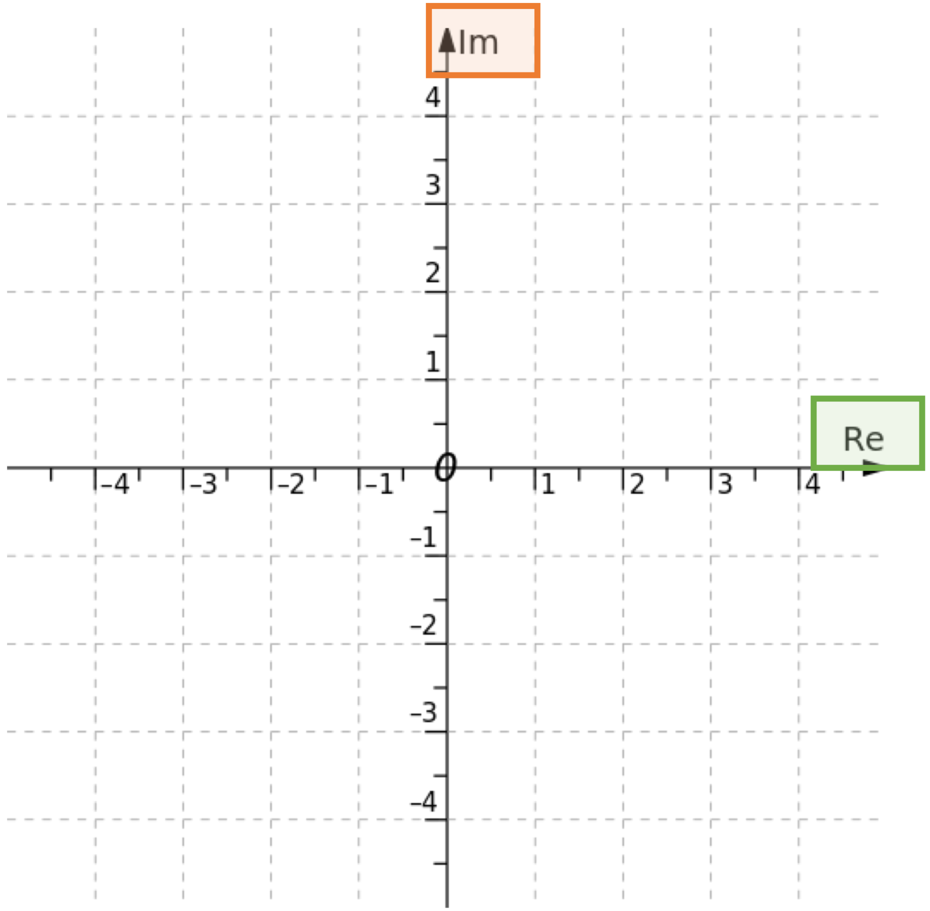
$$|a + ib| = \sqrt{a^2 + b^2}$$

$$\overline{(a + i\,b)} = (a - ib)$$

$$z = 3/5 + 4/5i$$

$$|z| =$$

$$\bar{z} =$$



Canvas attendance quiz passcode:

COMPLEX NUMBER ADDITION

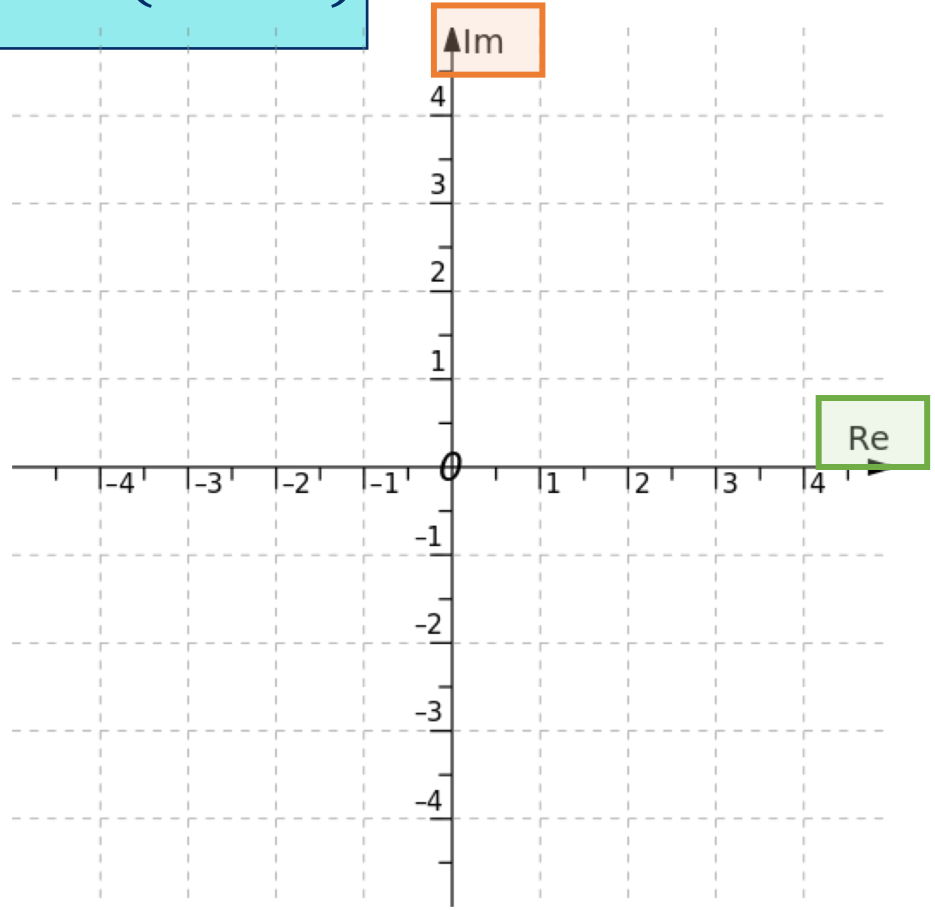
$$(a + i\,b) + (c + id) = (a + c) + i(b + d)$$

$$z = 2 + 3i$$

$$\bar{z} = 2 - 3i$$

$$z + \bar{z} =$$

$$z - \bar{z} =$$



COMPLEX NUMBER MULTIPLICATION

$$(a + i\, b) * (c + id) = (ac - bd) + i(ad + bc)$$

$$z = 2 + 3i$$

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$$z.\bar{z} =$$

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