INTRO TO QUANTUM COMPUTING

LECTURE #13

QUANTUM MECHANICS 3

Amir Karamlou

01/31/2021

ANNOUNCEMENTS

QUANTUM MECHANICS LECTURE SERIES

Lecture 1 - Principles of Quantum Mechanics

What is quantum and how do things behave on quantum length scales?

Lecture 2 - Quantum Two-Level Systems and Measurement

Objective - What are two-level systems and what can we do with them?

Lecture 3 - Postulates of Quantum Mechanics

Objective - What are the foundational rules of quantum mechanics?

TODAY'S LECTURE

➔ The six postulates of quantum mechanics

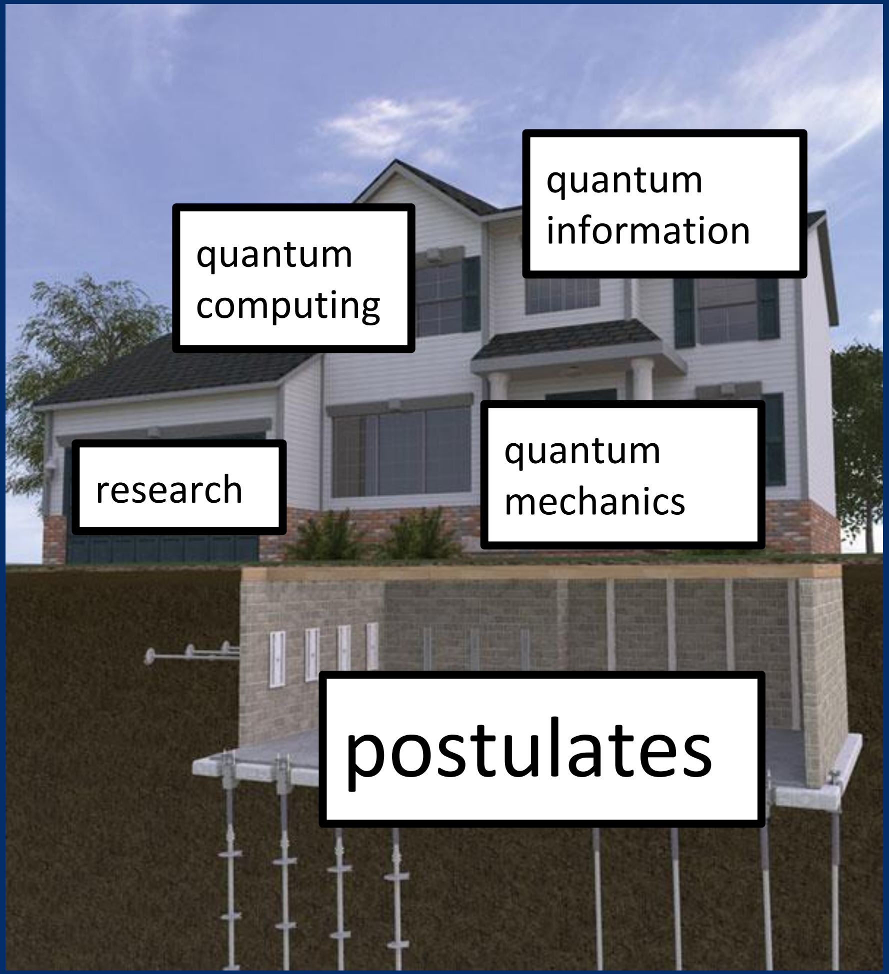
- 1 - Describing a quantum state
- 2 & 3 - Observing and measuring a quantum state
- 4 & 5 - What happens after a measurement
- 6 - How does a quantum state change with time

➔ How do these postulates relate to quantum computing?

Postulates of Quantum mechanics

Postulate:

an assumption used as a basis for mathematical reasoning



First Postulate:

A quantum state is represented by a ket $|\psi\rangle$ in the state space

Classical analogy:

If I’m running, you can create a function that describes my path on a coordinate system.

First postulate

Consequence: The superposition of any two quantum states is also a quantum state

No matter what path I take while running, you can always represent my location as a point on the coordinate system.

Second Postulate:

Classical observables are introduced into quantum mechanics using operators. Specifically, every observable (measurable property) of a physical system is described by an operator that acts on state kets

Classical analogy:

If I'm running, you can observe my:

- Speed
- Direction
- Position
- Energy
- Momentum
- ...

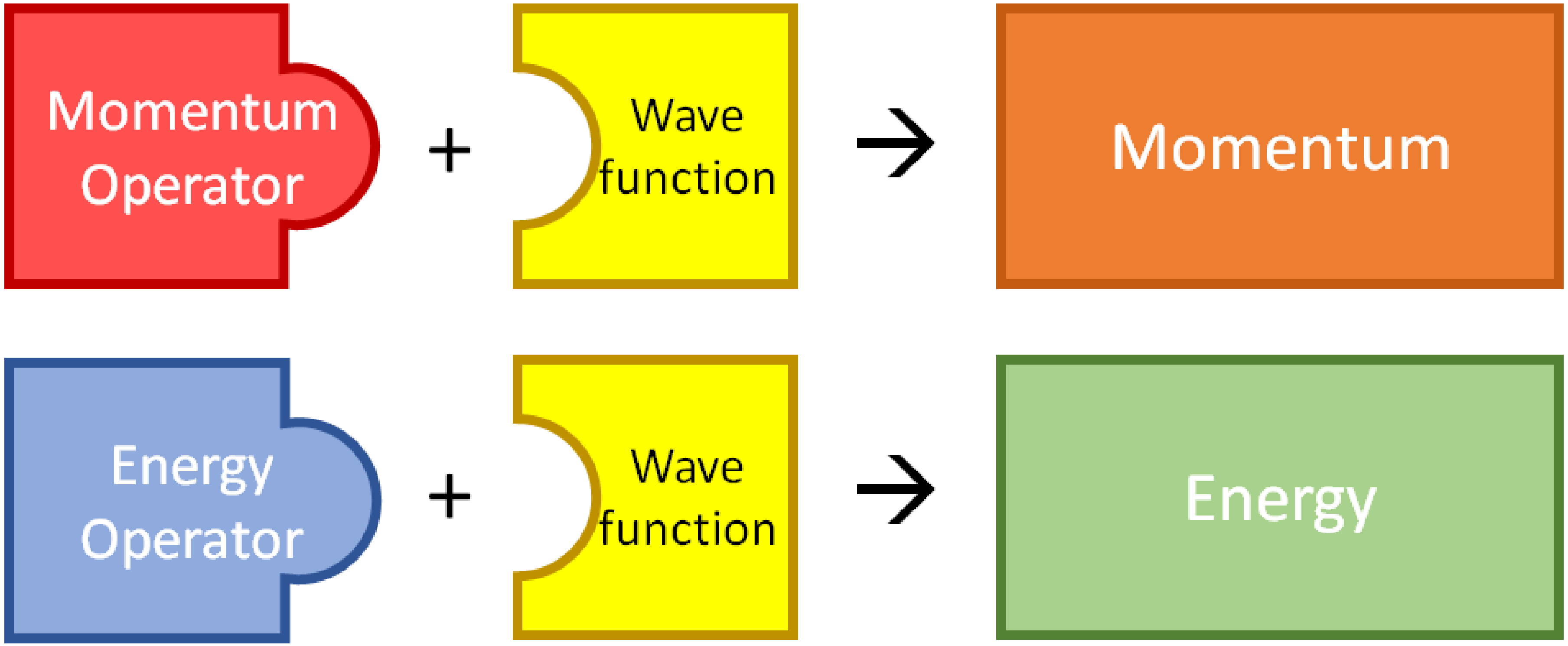
Classically, we can find out these values by measuring them. For example, we can measure energy.

In quantum, we apply an **operator** to the wavefunction to get these values:

Kinetic energy = Kinetic energy operator * $|\psi\rangle$

Second postulate

What’s an operator?



Second postulate

Consequence: Relates observables that we can “measure” to the quantum world.

Third Postulate:

The result of a measurement of an observable with an operator $\hat{A}$ will only ever be an eigenvalue of $\hat{A}$.

Classical analogy:

We don’t really have a real-life example, because this is explicitly a quantum concept. The speed of a runner can only take on certain values.

10 m/s

8 m/s

6 m/s

4 m/s

2 m/s

Eigenvalue Review

Our matrix-vector multiplication simplifies to a scalar-vector multiplication!

$$\boxed{A\vec{v} = \lambda\vec{v}}$$

In German, "eigen" means *proper*, *characteristic*, or *own*.

Third postulate

Consequence: Determines the possible outcomes of an observable measurement

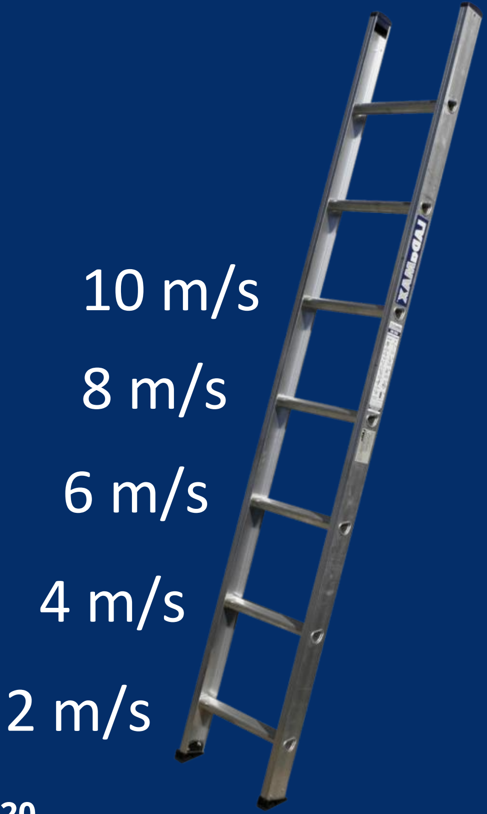
10 MIN BREAK!

Fourth Postulate:

When a measurement of an observable with operator $\hat{A}$ is made on a generic state $|\psi\rangle$, the probability of obtaining a certain eigenvalue a_i is given by the square of the inner product of $|\psi\rangle$ with the corresponding eigenstate: $|\langle a_i|\psi\rangle|^2$

Analogy:

This would be like saying that there's a certain probability that you will be running at a certain speed. If your coach was measuring your speed, there is a certain probability that they would measure different values.



Fourth postulate

Consequence: Gives the probabilities of measuring the different outcomes of an observable measurement

Fifth Postulate:

immediately after the measurement of an observable **A** with a value $\mathbf{a_n}$, the state of the system is the normalized eigenstate $|a_n\rangle$

Analogy:

This would be like saying that before your coach measured your speed, it was not well-defined (we don't know for sure what speed you are at). But after the measurement, the speed is well-defined (we know the value).

Fifth postulate

Consequence: The superposition collapses into one state after the measurement

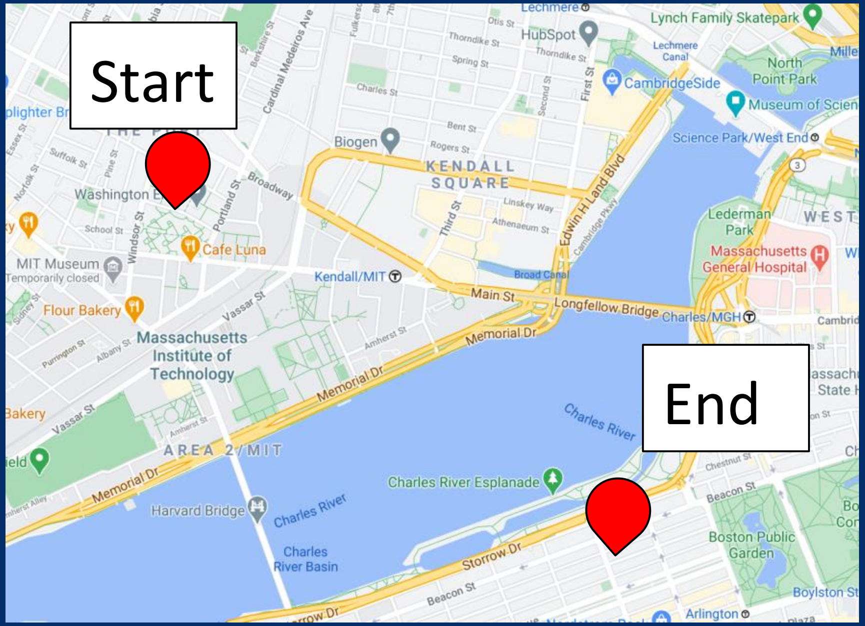
Sixth Postulate:

Quantum states generally change (evolve) with time. This time evolution preserves the normalization of the state. The time evolution of the state of a quantum system is described by:

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle \text{, for some unitary operator } \hat{U}$$

Analogy:

This is like saying that if I went for a run and you knew where I ended up and the conditions I was running in (what I ate, weather, who I'm running with), you could figure out what my starting point was.



Sixth postulate

Consequence: The evolution of the quantum state with time is reversible.

How do the postulates relate to quantum computing?

- Give us instructions on how to operate the quantum hardware
- Tells us what happens after we measure our quantum circuit
- Allow us to calculate the probability of getting the right answer based on the set of gates that we run in our algorithm