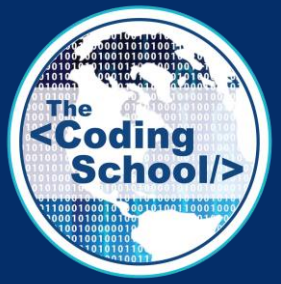

INTRO TO QUANTUM COMPUTING

Week 11 Lab

QUANTUM MECHANICS - I

Corbin McElhanney

January 19, 2021

PROGRAM FOR TODAY

- Logistics and ground rules
- Canvas attendance quiz
- Pre-lab zoom feedback
- Questions from last week
- Lab content
- Post-lab zoom feedback

LOGISTICS

- **Homework:** The week 11 homework is due, like all homework assignments, on Sunday, at 11:59 p.m. ET on Canvas.

- **Canvas:** Contains all required course information and materials
  - TAs will not be able to respond to messages on Canvas
  - If you have a question on logistics, email **student@qubitbyqubit.org** ✓

- **Piazza:** We will not be able to address all content-questions in lecture or lab
  - Look at answers to similar questions on Piazza or Discord
  - Post your question in the relevant folder in Piazza (see Piazza orientation video)
  - We will explore new content and will likely answer your question in future weeks ☺

CANVAS ATTENDANCE QUIZ

- Please log into Canvas and answer your lab section's quiz

Lab Number: 1 | Quiz Password: 9843

- If you participated in the fall semester of QxQ, do you feel more confident in your STEM skills since you started the course?

- **This quiz not graded, but counts for your lab attendance!**

PRE-LAB ZOOM FEEDBACK

On a scale of 1 to 5, how would you rate your understanding of this week's content?

- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

LEARNING OBJECTIVES FOR LAB 11

- Distinguishing between particles and waves
  - Particle and wave properties
  - Ripple on a rope
  - deBroglie wavelength of electrons
- Why do wave-like properties of particles show up at small lengths?
  - Double-slit experiment with light
  - Double-slit experiment with electrons
  - Double-slit experiment with everyday objects
- Double-slit experiment with single electrons*

*Optional content

QUESTIONS FROM LAST WEEK

- Defining numpy arrays

A = np.array([[1, 3 , 4]]) vs A = np.array([1, 3 , 4])

[[1],[3],[4]]

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MOTIVATION

Why do we care about waves and particles in quantum computing?

- Qubits are made of objects that show wave- and particle-like behavior!
- Quantum mechanics is the mathematical description of objects that show such behavior
- Qubits and quantum gates are governed by the rules of quantum mechanics

WAVE OR PARTICLE?

Wave

Particle

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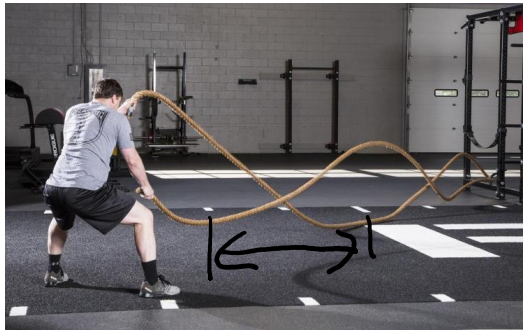
WAVE AND PARTICLE PROPERTIES

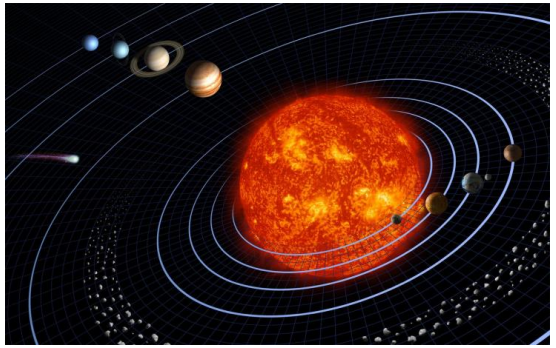
How did you know which was a wave and which was a particle?

Wave properties



- Repetition
  - ↳ frequency
  - ↳ wavelength
- Speed

Particle properties



- Position
- Mass

RIPPLE ON A ROPE - WAVE OR PARTICLE?

RIPPLE ON A ROPE – A BIT OF BOTH!

"Where is the ripple" as well as "how often does the ripple repeat" are meaningful questions!

EVERYTHING IS RIPPLES

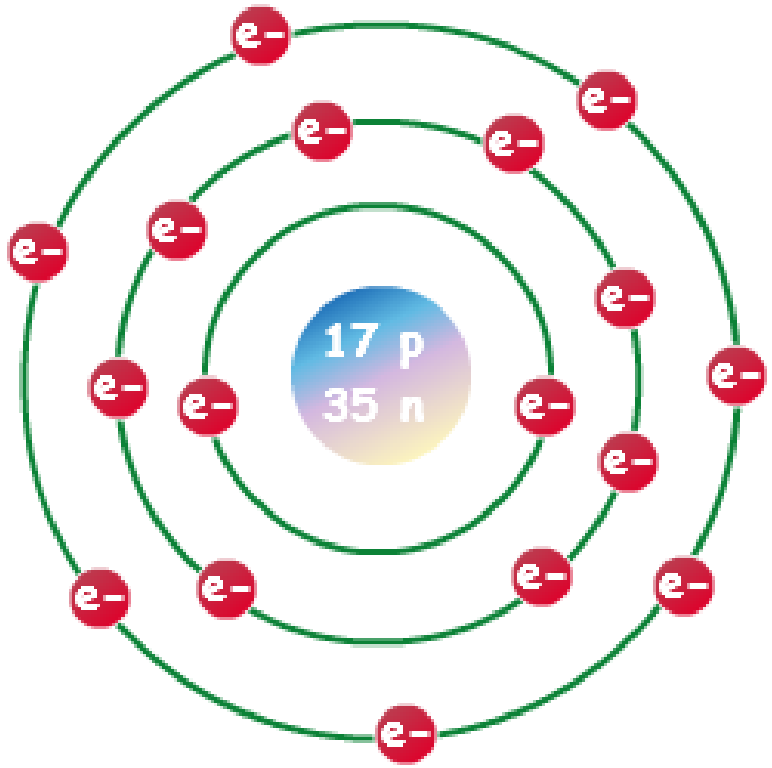
- The distinction between ‘wave’ and ‘particle’ is fuzzy, even in objects we see around us
  - E.g. ripples on a rope

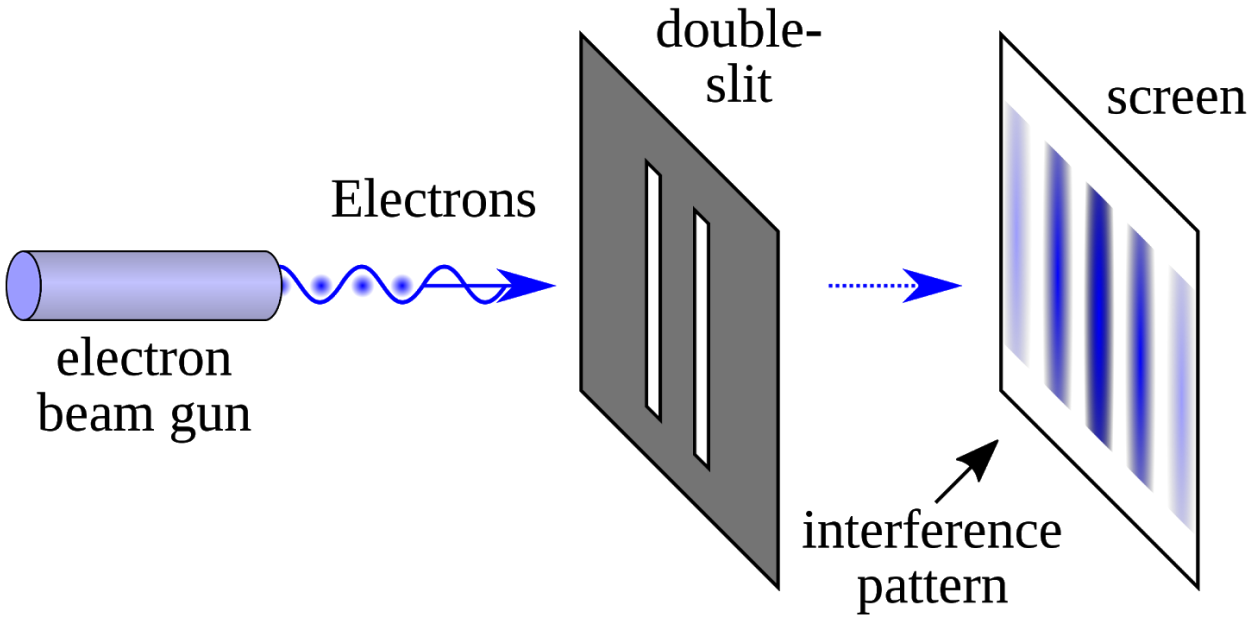
- Early 20th century physicists found that treating atoms, molecules, ions, electrons, protons, neutrons as particles alone was limited and did not explain all their properties
  - At small length scales, **everything** behaves in this fuzzy way!

QUESTIONS?

Questions on content so far?

ELECTRONS – PARTICLES OR WAVES?





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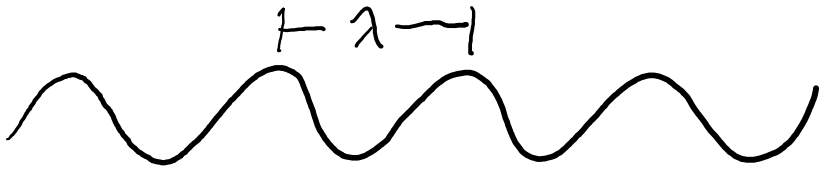
ELECTRONS – A BIT OF BOTH



Planck's constant is *tiny*! It sets the scale of the relationship between particle and wave properties.

Planck's constant
$h = 6.626 \times 10^{-34}\ Js$ ←

wavelength

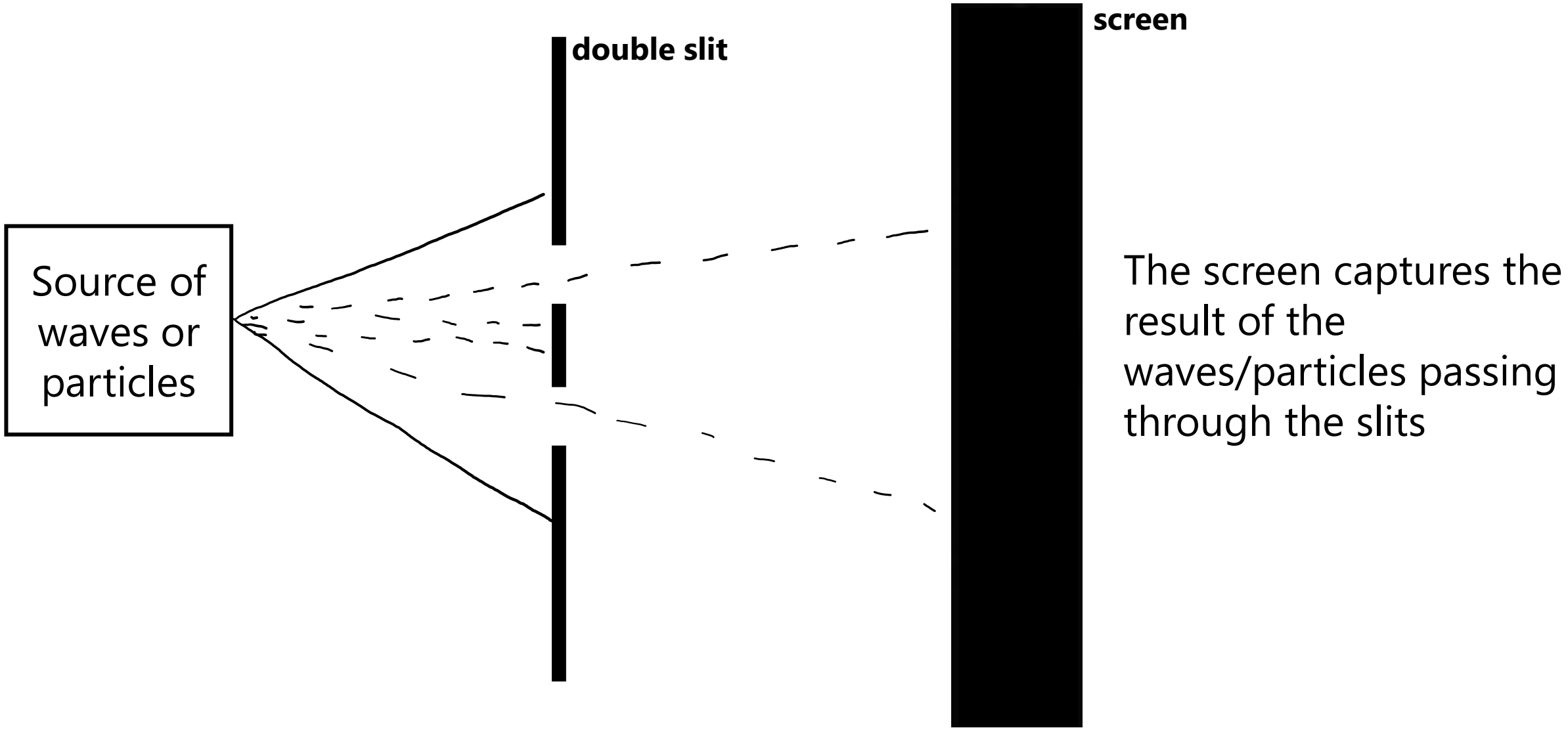
$$\lambda = \frac{h}{p}$$ ← $p = m \cdot v$

momentum
$p = m \cdot v$

Practice: what is λ for electrons, with $m = 9.1 \times 10^{-31}$ kg and $v = 2 \times 10^{8}$ m/s ?

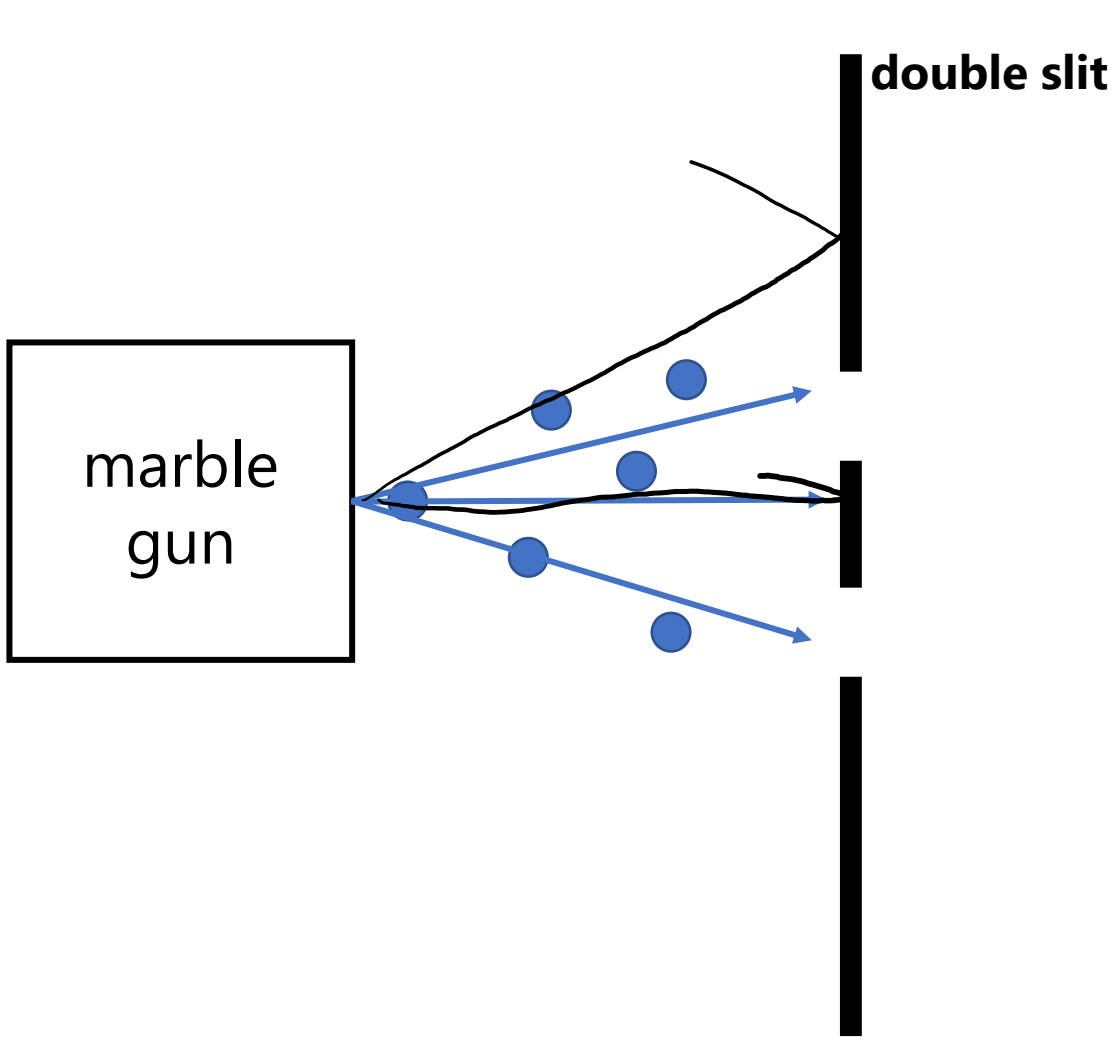
$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ J \cdot s}{(9.1 \times 10^{-31}\ kg)(2 \times 10^{8}\ m/s)} = 3.64 \times 10^{-12}\ m$$ ← 100m human hair

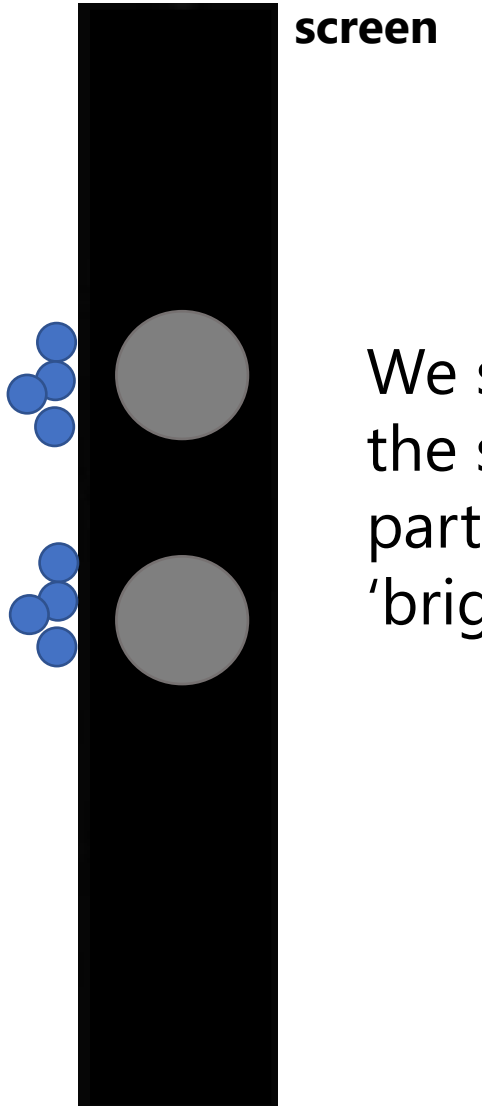
THE DOUBLE SLIT EXPERIMENT



The screen captures the result of the waves/particles passing through the slits

THE DOUBLE SLIT EXPERIMENT - PARTICLES





We see two spots on the screen with lots of particles, i.e., two 'bright' spots

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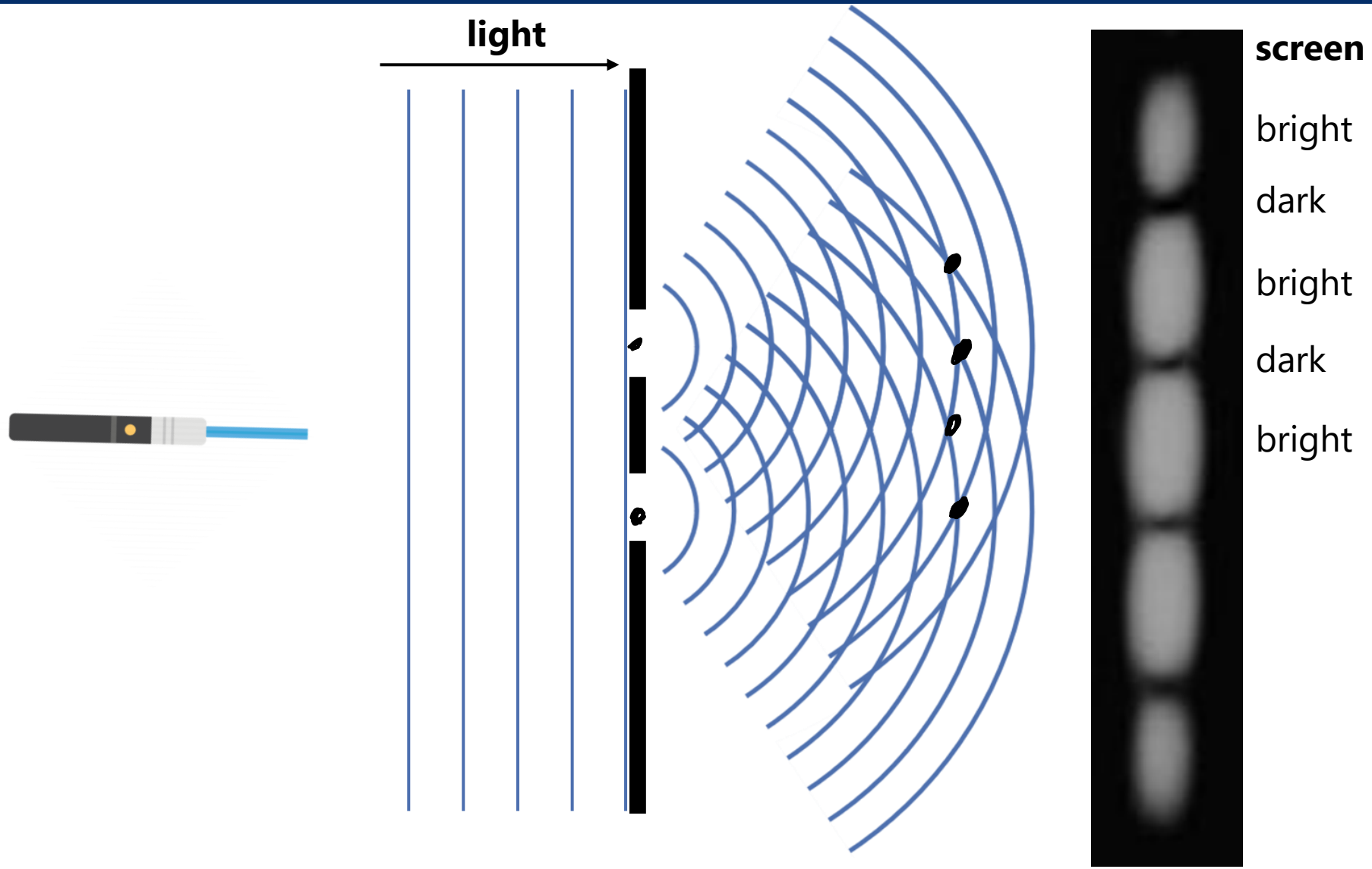
THE DOUBLE SLIT EXPERIMENT - WAVES



- With waves, we see multiple bright and dark spots due to wave interference
- This experiment tells us that light behaves like a wave!

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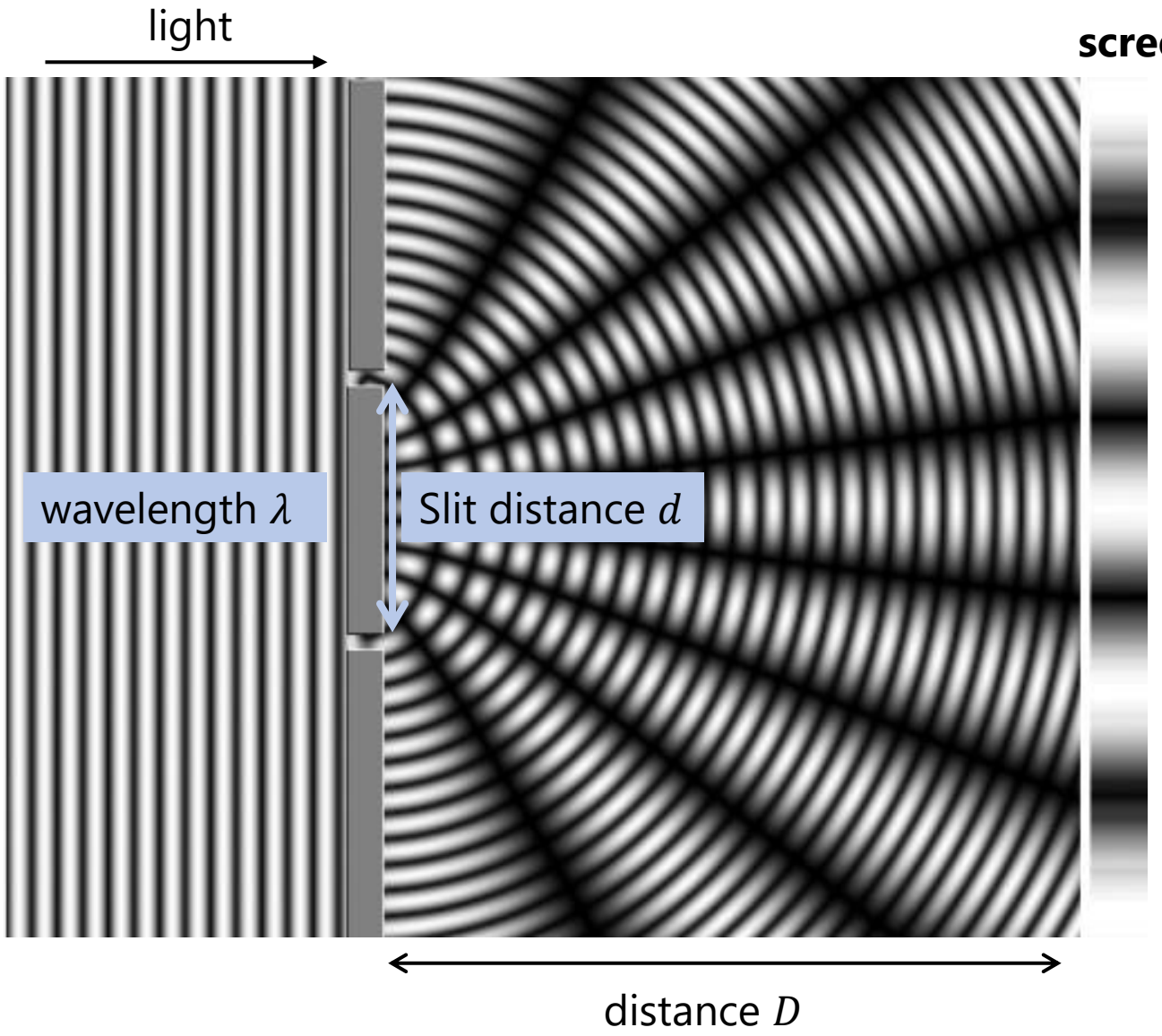
THE DOUBLE SLIT EXPERIMENT



spacing $s = \frac{D\lambda}{d}$

spacing depends on d!
As d increases, the spacing gets smaller and smaller and harder to see

Canvas attendance quiz passcode: 9843

VARYING THE SLIT DISTANCE

$$\text{spacing } s = \frac{D\lambda}{d}$$

$1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$
$1\ \mu\text{m} = 1 \times 10^{-6}\ \text{m}$
$1\ \text{nm} = 1 \times 10^{-9}\ \text{m}$

Problem 1: What is the spacing s with distance $D = 1$ m, slit distance $d = 1$ mm and using red light $\lambda = 800$ nm ?

$$s = \frac{D\lambda}{d} = \frac{(1\text{m})(800\times10^{-9}\text{m})}{(1\times10^{-3}\text{m})} = 800\times10^{-6}\text{m} = 800\,\mu\text{m}$$

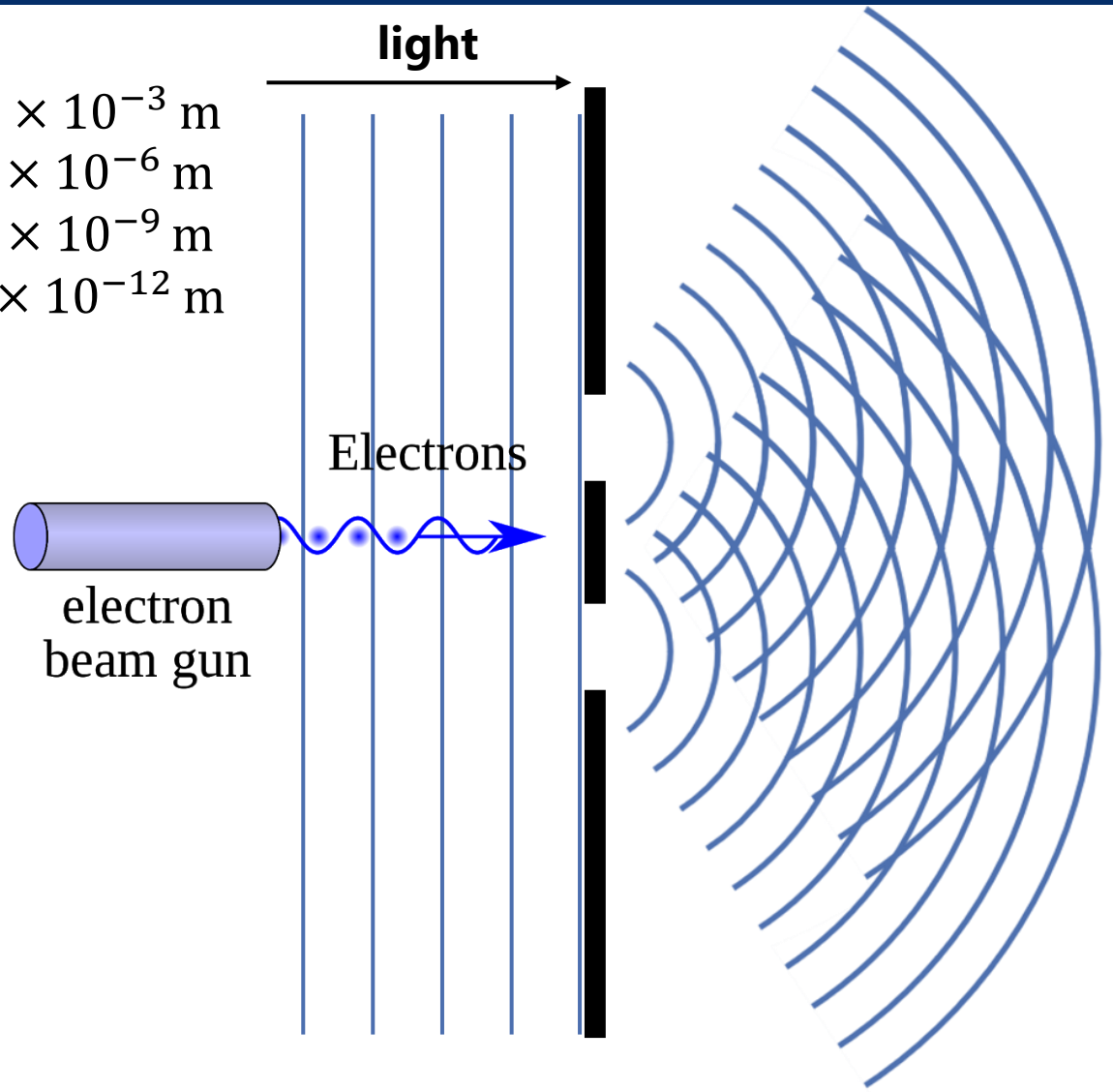
Problem 2: What is the spacing s with distance $D = 1$ m, slit distance $d = 10$ mm and using red light $\lambda = 800$ nm ?

$$s = \frac{(1\text{m})(800\times10^{-9}\text{m})}{10\times10^{-3}\text{m}} = 80\,\mu\text{m}$$

Human eye can resolve features $\sim 300\ \mu\text{m}$ or larger at a distance $D = 1$ m

ELECTRON DOUBLE SLIT EXPERIMENT

$1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$
$1\ \mu\text{m} = 1 \times 10^{-6}\ \text{m}$
$1\ \text{nm} = 1 \times 10^{-9}\ \text{m}$
$1\ \text{pm} = 1 \times 10^{-12}\ \text{m}$



$\rightarrow$ spacing $s = \frac{D\lambda}{d}$

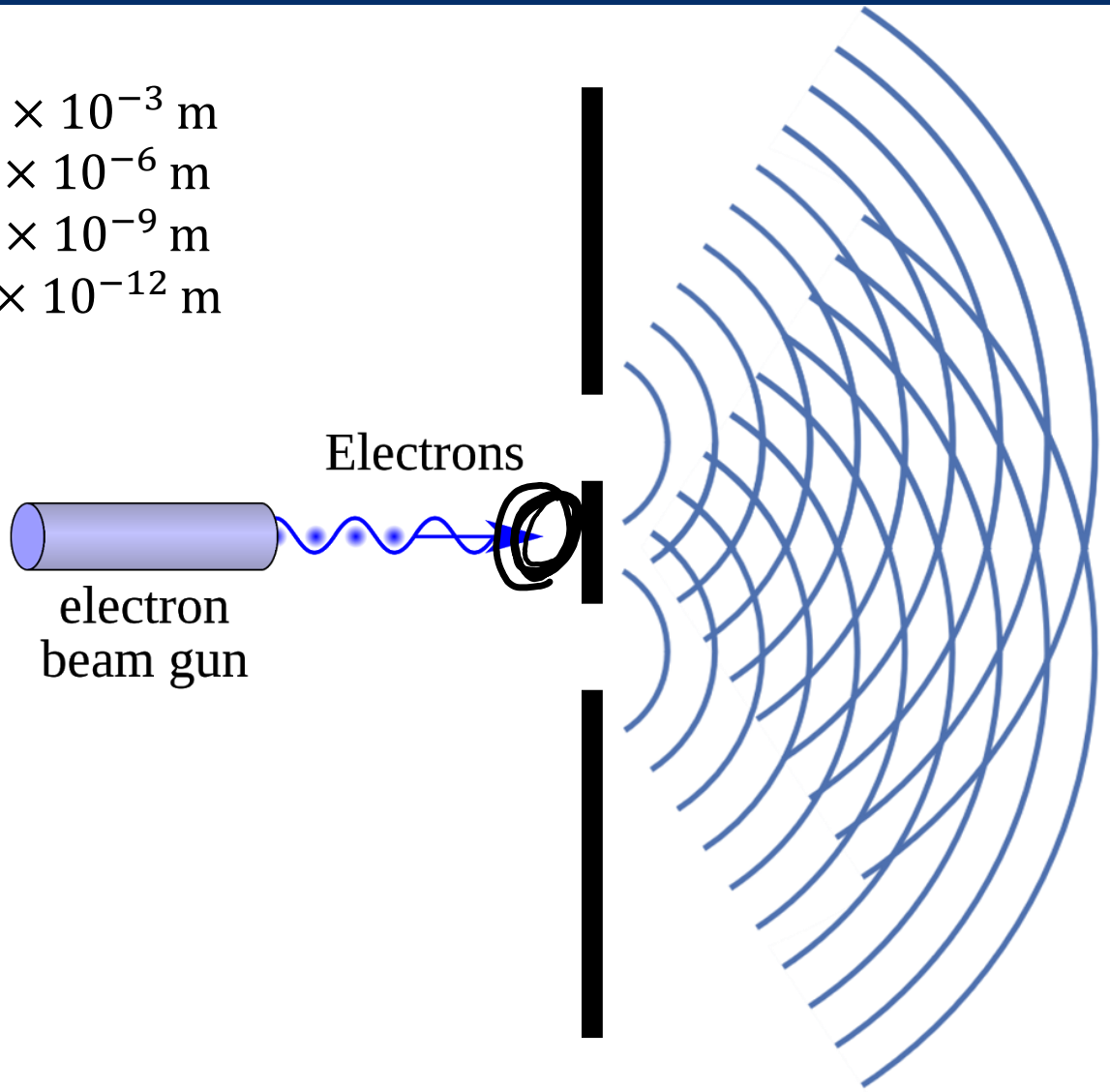
Problem: What is the spacing s with distance $D = 1$ m, slit distance $d = 1$ mm and using electrons with $\lambda = 3.6$ pm ?

$$S = \frac{(1\text{m})(3.6\times10^{-12}\text{m})}{1\times10^{-3}\text{m}} \approx 3.6\text{nm}$$

300 μm

ELECTRON DOUBLE SLIT EXPERIMENT

$1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$
$1\ \mu\text{m} = 1 \times 10^{-6}\ \text{m}$
$1\ \text{nm} = 1 \times 10^{-9}\ \text{m}$
$1\ \text{pm} = 1 \times 10^{-12}\ \text{m}$



spacing $s = \frac{D\lambda}{d}$

Can we change the slit distance d to make the spacing visible? $D = 1$ m, $\lambda = 3.6$ pm, and we want $s = 300\ \mu$m ?

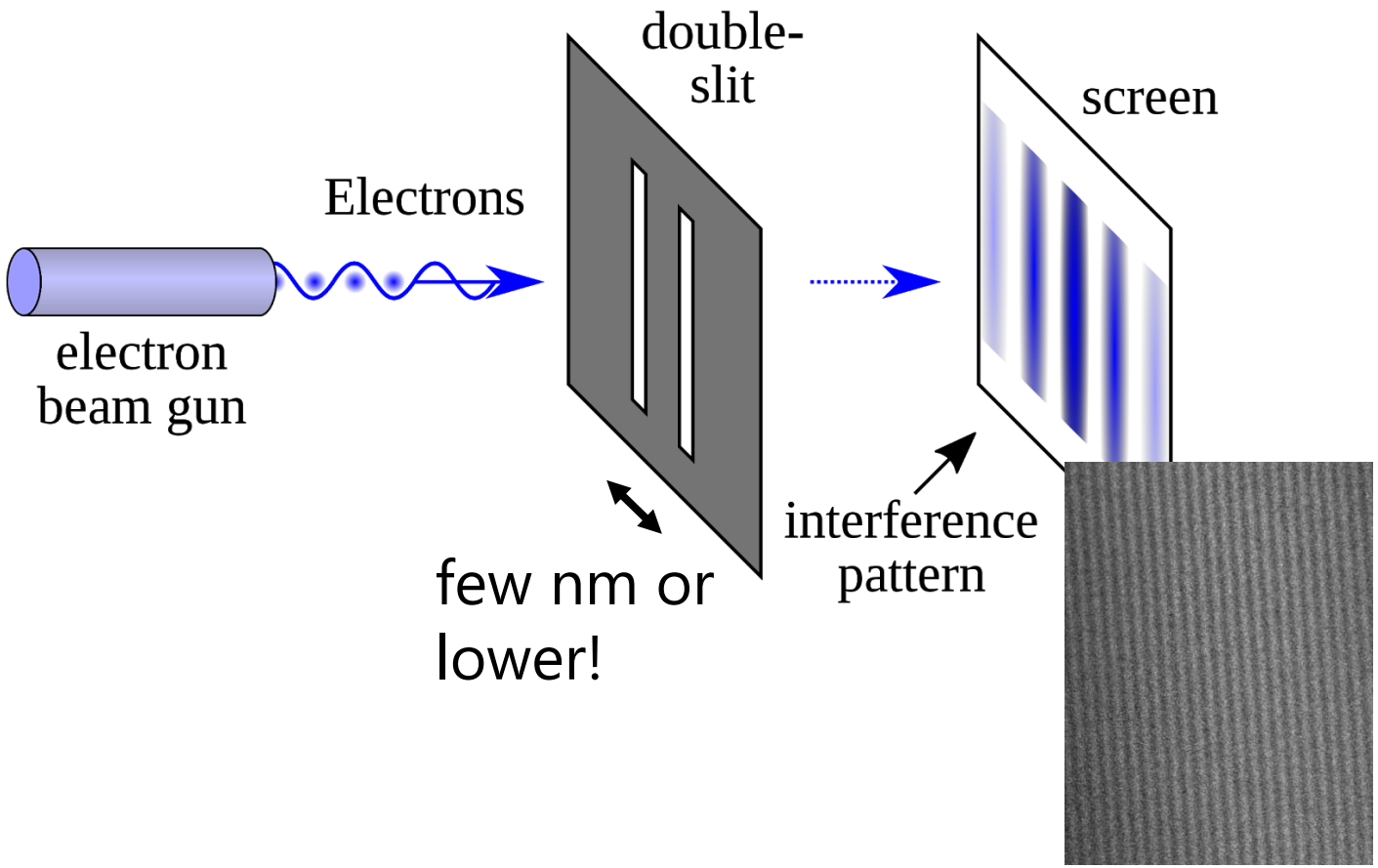
$$s = \frac{D\lambda}{d}$$

$$d = \frac{D\lambda}{s} = \frac{(1\text{m})(3.6\times10^{-12}\text{m})}{300\times10^{-6}\text{m}}$$

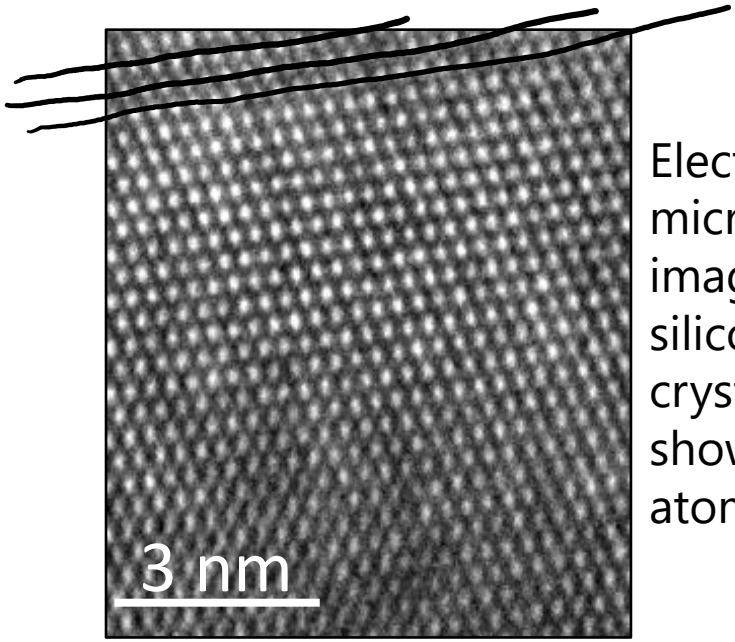
$$= 1.2\times10^{-8}$$

$$= 12\text{nm} \leftarrow \text{width DNA}$$

ELECTRON DOUBLE SLIT EXPERIMENT



- How can we make such small slits?
  - Gaps between atoms in crystals!



Electron microscope image of silicon crystal, showing atoms

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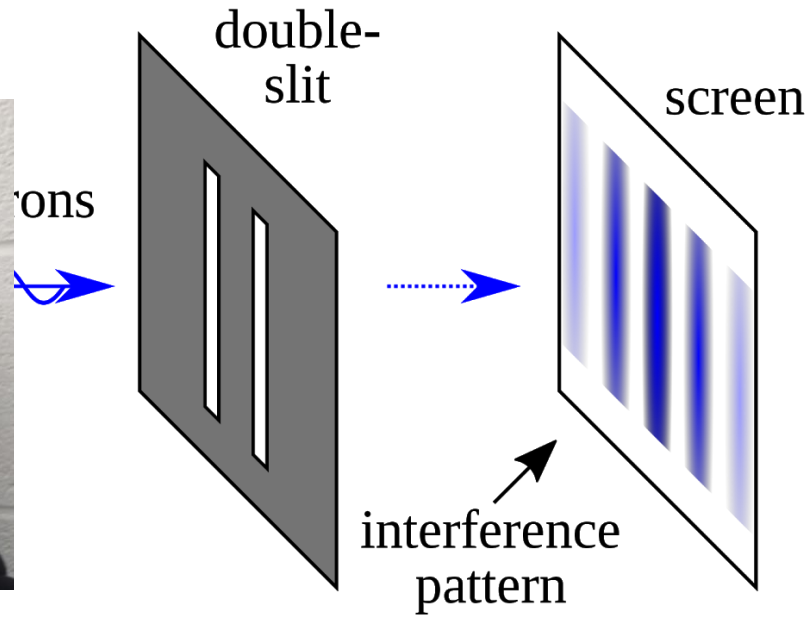
HUMAN DOUBLE SLIT EXPERIMENT(?)

[redacted]



$$\lambda = \frac{6.626\times10^{-34}\ Js}{(70kg)(8m/s)} = 1.18\times10^{-36}\ m$$

Let's find Rahul's wavelength if he runs towards the slits at a speed of $v = 8$ m/s, assuming he weighs $m = 70$ kg.

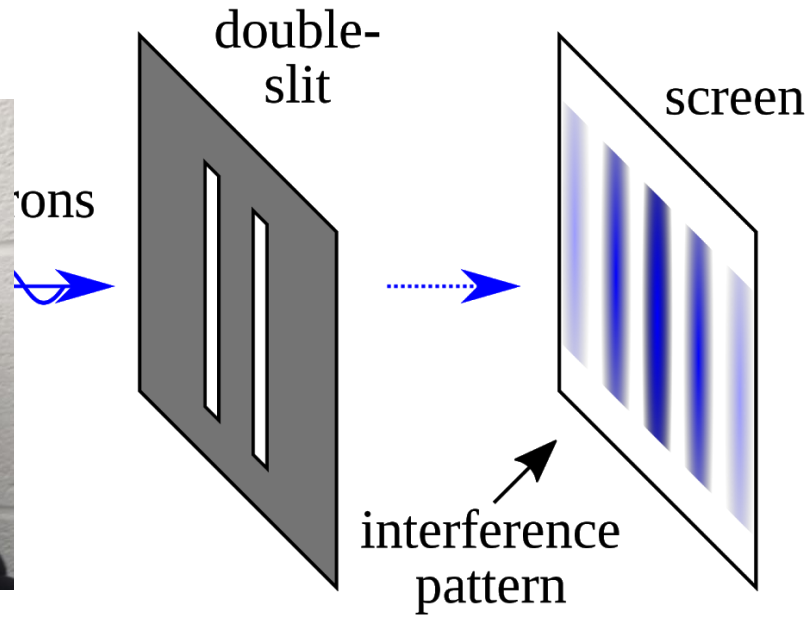
Planck's constant
$h = 6.626 \times 10^{-34}\ Js$

wavelength

$$\lambda = \frac{h}{p}$$

momentum
$p = m \cdot v$

HUMAN DOUBLE SLIT EXPERIMENT(?)



Now let's find the slit distance required to see the interference pattern

$$\text{spacing } s = \frac{D\lambda}{d}$$

$$d = \frac{D\lambda}{s} = \frac{(1\,m)(1.18\times10^{-36}\,m)}{300\times10^{-6}\,m}$$

$$\approx 4\times10^{-33}\,m$$

$\lambda = 1.18\times10^{-36}\,m$ $\quad s = 300\mu m$

Radius of a proton $\sim 1 \times 10^{-15}$ m $= 300\times10^{-6}\,m$

KEY TAKEAWAYS

- Particles and waves are human-defined buckets to sort phenomena we see around us
  - Particles have mass, position, speed, etc.
  - Waves have wavelength, frequency, speed, etc.

wave $\lambda = \frac{h}{p}$ particles

- Pulses and ripples can possess both particle and wave-like properties
  - At the nanoscale, everything behaves like pulses and ripples!

- The wave-like nature of particles is expressed by their deBroglie wavelength ✓
  - For the wave-like nature of particles to become observable, we have to manipulate them on the scale close to their deBroglie wavelength

QUESTIONS?

Questions on content so far?

POST-LAB ZOOM FEEDBACK

After this lab, on a scale of 1 to 5, how would you rate your understanding of this week's content?

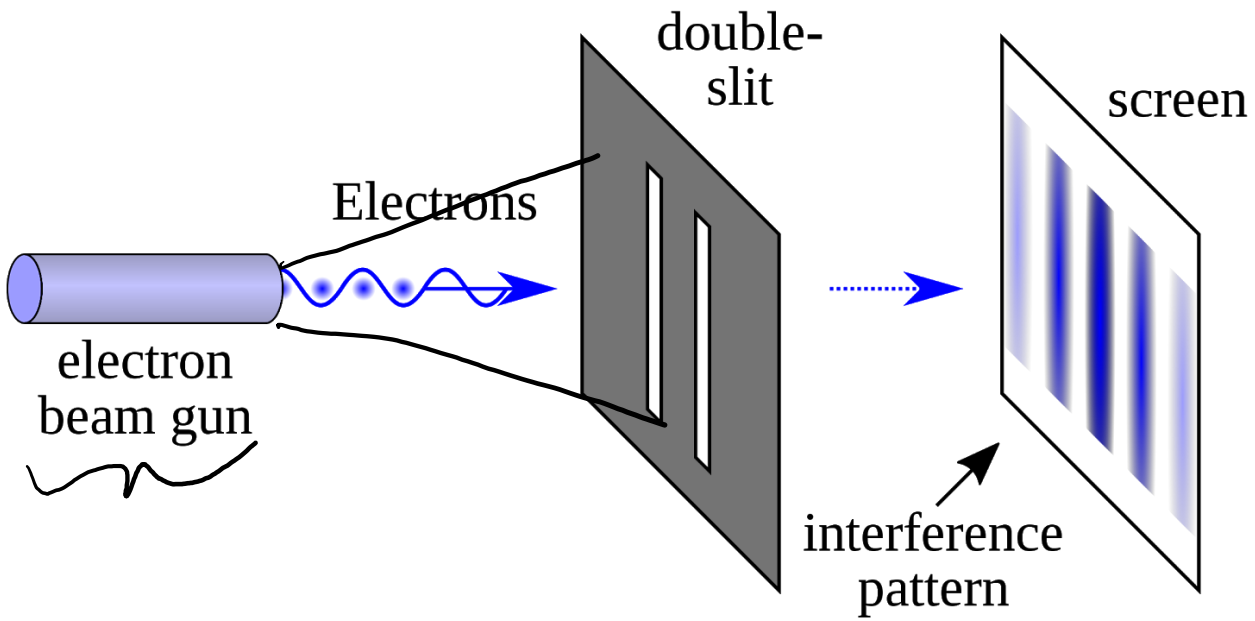
- 1 –Did not understand anything
- 2 – Understood some parts
- 3 – Understood most of the content
- 4 – Understood all of the content
- 5 – The content was easy for me/I already knew all of the content

FURTHER READING AND RESOURCES

- Dr. Quantum's guide to the double-slit experiment
- https://www.hitachi.com/rd/research/materials/quantum/doubleslit/index.html – A description of a double-slit experiment with electrons from Hitachi, a major manufacturer of electron microscopes
- https://physicsworld.com/a/the-double-slit-experiment/ - History of the double slit experiment with electrons, with links to important papers and relevant articles

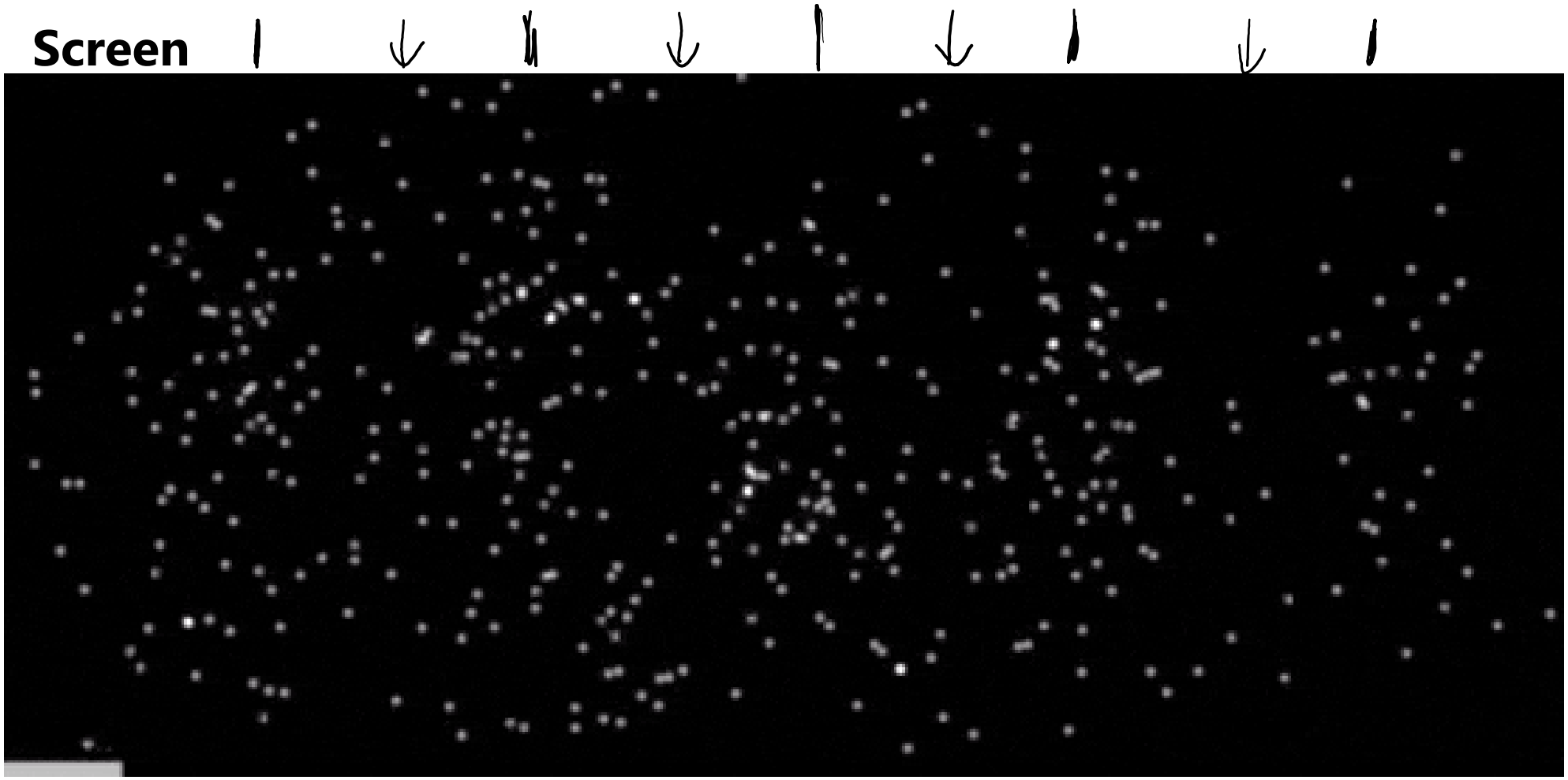
OPTIONAL CONTENT

MORE ON THE DOUBLE SLIT EXPERIMENT



Could we shoot the electrons one-by-one? What do you expect to happen?

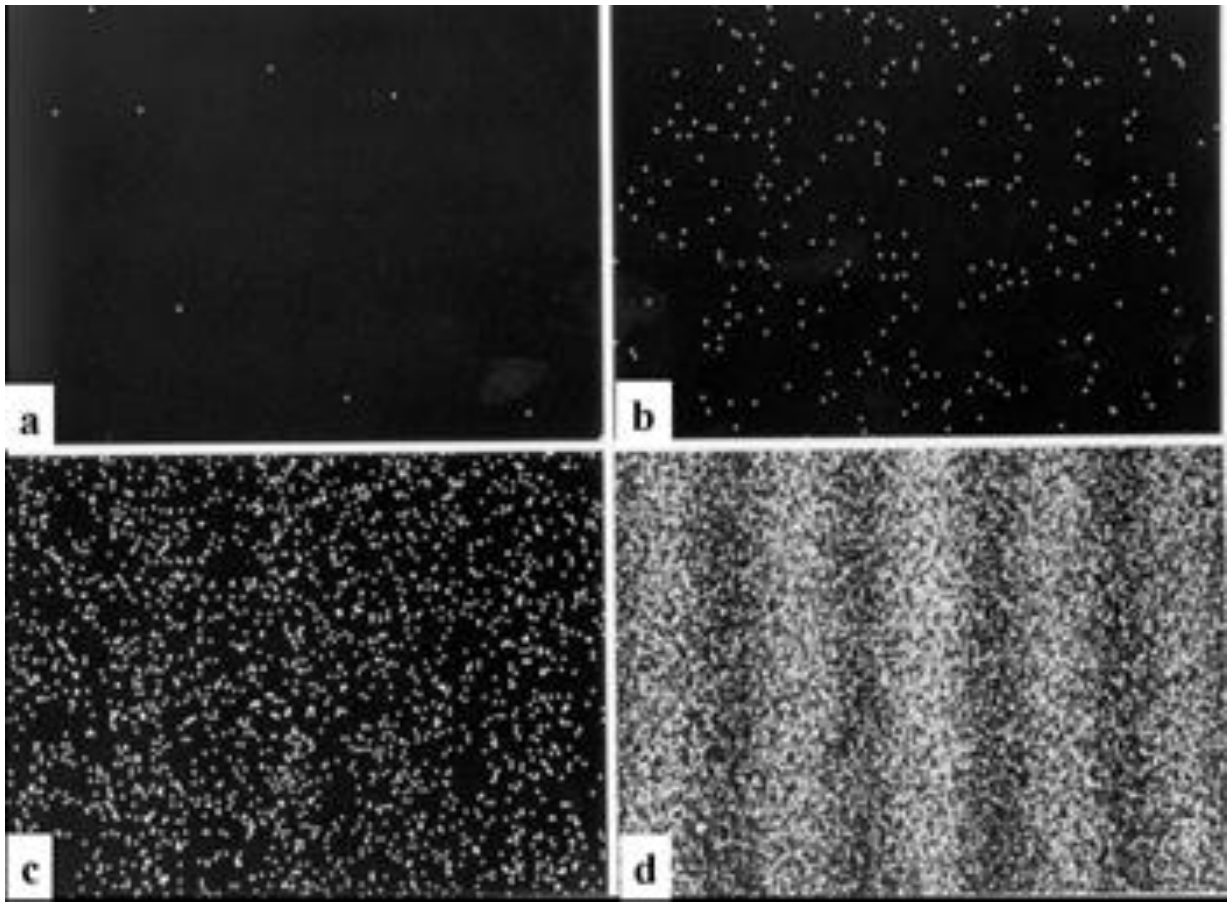
EXPERIMENT WITH SINGLE ELECTRONS



What do you notice?

Adapted from A Tonomura, J Endo, T Matsuda, T Kawasaki and H Ezawa 1989 Demonstration of single-electron build-up of an interference pattern *American Journal of Physics* **57** 117-120

‘THE MOST BEAUTIFUL EXPERIMENT IN PHYSICS’



$\lambda \approx 3.6 \times 10^{-12}$

- Each electron registers as a single spot (a ‘particle’)
- The double-slit interference pattern ‘emerges’ as more and more electrons hit the screen

$|\langle 0|\psi\rangle|^2$

- The math gives us the probability that each electron ends up at a certain point on the screen

https://www.hitachi.com/rd/research/materials/quantum/doubleslit/index.html