the Quantum Stack & Math Review

By Meteorite
QxQ | TCS

The Quantum Stack

FULL-STACK: the entirety of a computer system or application, comprising both the front-end and the back-end

STACK

* How do computers work, from the highest level of abstraction to the lowest level of abstraction

THE CLASSICAL COM. STACK

classical software
classical compiler
classical hardware

HARDWARE AND SOFTWARE

how computers work

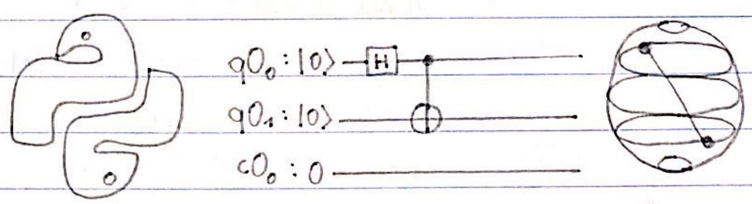
circuits
chips
wires
speakers
plugs
stuff

```
printf("Hello, quantum!");
cout << "Hello, leaders!";
print ("Hello, everyone!");
System.out.println ("Hello");
```

COMPILERS: is a program that translates computer code written in a source programming language to a target programming language. Oftentimes, the target language is machine code.

THE QUANTUM COMPUTING STACK.

quantum software - classical software - quantum program compiler
classical control hardware - quantum hardware



MATH REVIEW

VECTORS ⇒ every vector has a direction and a magnitude

VECTOR REPRESENTATION

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

2D VECTOR DIRECTION

$$\angle \vec{v} = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

VECTOR MAGNITUDE

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

VECTOR ADDITION

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}$$

VECTOR-SCALAR MULTIPLICATION

$$c * \vec{v} = \begin{pmatrix} c * v_1 \\ c * v_2 \\ \vdots \\ c * v_n \end{pmatrix}$$

COMPLEX NUMBERS

IMAGINARY UNIT ⟶ $i = \sqrt{-1}$, $i^2 = -1$

COMPLEX NUMBER ⟶ $a + ib$

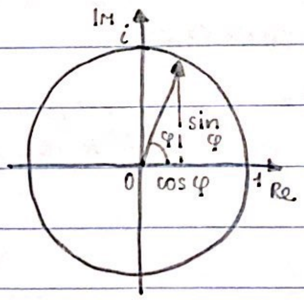
COMPLEX CONJUGATION ⟶ $\overline{(a+ib)} = (a+ib)^* = (a-ib)$

COMPLEX MODULUS ⟶ $|a+ib| = \sqrt{a^2+b^2}$

COMPLEX ADDITION ⟶ $(a+ib)+(c+id) = (a+c) + i(b+d)$

COMPLEX MULTIP. ⟶ $(a+ib)*(c+id) = (ac-bd) + i(ad+bc)$

COMPLEX EXPONENTIALS



EULER'S FORMULA ⟶ $e^{i\varphi} = \cos\varphi + i\sin\varphi$

POLAR REPRESENT. ⟶ $z = x + iy = re^{i\varphi}$

MULTIPLICATION ⟶ $e^{i\varphi}e^{i\theta} = e^{i(\varphi+\theta)}$

COMPLEX CONJUGAT. ⟶ $\overline{e^{i\varphi}} = e^{-i\varphi}$

MODULUS ⟶ $|z| = |re^{i\varphi}| = |r|$

VECTOR RADIUS

$$r = |z| = \sqrt{x^2+y^2}$$

VECTOR ANGLE

$$\varphi = \tan^{-1}(y/x)$$

TRIG RELATIONS

$$\cos\varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}$$

$$\sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$$

VECTOR SHAPE = (# rows x # cols)

→ column vector → $(n \times 1)$

→ row vector → $(1 \times m)$

TRANSPOSE → $\vec{v}_1 = (n \times 1)$, $\vec{v}_1^{\,T} = (1 \times n)$

$\vec{v}_2 = (1 \times m)$, $\vec{v}_2^{\,T} = (m \times 1)$

INNER PRODUCT

THE INNER PRODUCT IS:

$$\langle \vec{v}, \vec{w} \rangle = \vec{v}\,\vec{w}^{T} = \sum_{i=1}^{n} v_i w_i$$

1. vector x vector to scalar mapping
2. tool for calculating vector magnitude
3. tool for vector normal.
4. tool for geometrically comparing vectors
5. tool for determining vector orthogonality

MAGNITUDE ⟶ $\langle \vec{v}, \vec{v} \rangle = \|\vec{v}\|^2$

NORMALIZATION → $\dfrac{\vec{v}}{\sqrt{\langle \vec{v}, \vec{v} \rangle}} = \dfrac{\vec{v}}{\|\vec{v}\|}$

GEOM. COMPARISON & VECT. ORTHOG

$$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\|\,\|\vec{y}\| \cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{\langle \vec{x}, \vec{y} \rangle}{\|\vec{x}\|\,\|\vec{y}\|} \right)$$

LINEAR COMBINATIONS

$$\vec{v} = a_1 \vec{v}_1 + a_2 \vec{v}_2 + \dots + a_n \vec{v}_n = \sum_{i=1}^{n} a_i \vec{v}_i$$

COMPLEX VECTOR MANIPULATION

CONJUGATE TRANSPOSE → $\vec{v}^{\dagger} = (\vec{v}^{T})^{*} = (\vec{v}^{*})^{T}$

COMPLEX INNER PRODUCT → $\langle \vec{v}, \vec{w} \rangle = \vec{v}^{\dagger} \vec{w} = \sum_{i=1}^{n} v_i^{*} w_i$

MATRICES

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{pmatrix}$$

SHAPE: (# rows x # cols)

$$X^{T} = \begin{pmatrix} x_{11} & x_{21} & \cdots & x_{n1} \\ x_{12} & x_{22} & \cdots & x_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1m} & x_{2m} & \cdots & x_{nm} \end{pmatrix} \qquad X^{\dagger} = \begin{pmatrix} x_{11}^{*} & x_{21}^{*} & \cdots & x_{n1}^{*} \\ x_{12}^{*} & x_{22}^{*} & \cdots & x_{n2}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ x_{1m}^{*} & x_{2m}^{*} & \cdots & x_{nm}^{*} \end{pmatrix}$$

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MATRICES

$$A+B=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1M}+b_{1M} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2M}+b_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}+b_{n1} & a_{n2}+b_{n2} & \cdots & a_{nM}+b_{nM} \end{pmatrix} \qquad A\vec{x}=\begin{pmatrix} \langle \vec{a}_1,\vec{x}\rangle \\ \langle \vec{a}_2,\vec{x}\rangle \\ \vdots \\ \langle \vec{a}_n,\vec{x}\rangle \end{pmatrix}$$

$$c*A=\begin{pmatrix} c*a_{11} & c*a_{12} & \cdots & c*a_{1M} \\ c*a_{21} & c*a_{22} & \cdots & c*a_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ c*a_{n1} & c*a_{n2} & \cdots & c*a_{nM} \end{pmatrix} \qquad AB=\begin{pmatrix} \langle \vec{a}_1,\vec{b}_1\rangle & \langle \vec{a}_1,\vec{b}_2\rangle & \cdots & \langle \vec{a}_1,\vec{b}_K\rangle \\ \langle \vec{a}_2,\vec{b}_1\rangle & \langle \vec{a}_2,\vec{b}_2\rangle & \cdots & \langle \vec{a}_2,\vec{b}_K\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \vec{a}_n,\vec{b}_1\rangle & \langle \vec{a}_n,\vec{b}_2\rangle & \cdots & \langle \vec{a}_n,\vec{b}_K\rangle \end{pmatrix}$$

IDENTITY MATRIX: $\mathbb{I}=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$ MATRIX INVERSION: $XX^{-1}=X^{-1}X=\mathbb{I}$

SETS

SET A IS A SUBSET OF SET B $A\subset B$

SET A IS EQUAL TO SET B $A=B$

SET A IS A SUPERSET OF SET B $A\supset B$

UNION $\longrightarrow A\cup B$

INTERSECT $\rightarrow A\cap B$

COMPLEMENT $\rightarrow A^c$

PROBABILISTIC MODEL: A probabilistic model is a way to mathematically describe an unknown situation

There are two key components: sample space / probability law

AXIONS OF PROBABILITY

1. NONNEGATIVITY $\mathbb{P}(A)\geq 0$, for every event A
2. ADDITIVITY $\mathbb{P}(A\cup B)=\mathbb{P}(A)+\mathbb{P}(B)$
3. NORMALIZATION $\mathbb{P}(\Omega)=1$

RANDOM VARIABLES

$\langle X\rangle=\mathbb{E}[X]=\sum_x x\,\mathbb{P}(X=x)$ $\quad$ $var(X)=\mathbb{E}[(X-\mathbb{E}[X])^2]=\sum_x (x-\mathbb{E}[X])^2\,\mathbb{P}(X=x)$

DIRAC NOTATION & QUANTUM STATES

* QUANTUM STATES inputs and outputs of a quantum computer
* A KET is simply a column vector
* A BRA is the conjugate traspose of a ket

SUPERPOSITION $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

BRAKET $\langle\psi|\phi\rangle$

EXPECTATION $\langle\psi|A|\psi\rangle$

QUANTUM OPERATIONS & GATES

QUANTUM OPERATIONS: perform the computation in a q. computer

PAULI OPERATORS

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ All observable operators are HERMITIAN → $A = A^\dagger$

→ All reversible operations are UNITARY → $AA^\dagger = A^\dagger A = I$

VECTOR & HILBERT SPACES

A vector space is a collection of vectors which can be added and multiplied, adhering to the following axioms:

For all $\vec{x}, \vec{y}, \vec{z} \in \mathcal{V}$, where $\mathcal{V}$ = vector space ; $a, b \in \mathbb{C}$

1. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$
2. $\vec{x} + (\vec{y} + \vec{z}) = (\vec{x} + \vec{y}) + \vec{z}$
3. There is a unique zero vector $\vec{0}$, such that $\vec{x} + \vec{0} = \vec{x}$
4. For each $\vec{x}$ there is a unique $-\vec{x}$, such that $\vec{x} + (-\vec{x}) = \vec{0}$
5. $1\vec{x} = \vec{x}$
6. $(ab)\vec{x} = a(\vec{x}b)$
7. $a(\vec{x} + \vec{y}) = a\vec{x} + a\vec{y}$
8. $(a+b)\vec{x} = a\vec{x} + b\vec{x}$

HILBERT SPACES are just vector space equipped with an inner prod.

BASES

A BASIS for the vector space $\mathcal{V}$ is the set of vectors $\{\vec{v}_1, \vec{v}_2 \dots \vec{v}_n\}$ having the following two properties at once.

1. Linearly independent: $c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_k\vec{v}_k = 0$ only when $c_1 = c_k = 0$
1. Span $\mathcal{V}$ every vector $\vec{v} \in V$ can be expressed as $\vec{v} = c_1\vec{v}_1 + \dots + c_k\vec{v}_k$

Thus, every vector in the space $\mathcal{V}$ is a unique linear combination of the basis vectors.

EIGENVECTORS & EIGENVALUES

$A\vec{v} = \lambda\vec{v}$ If v is an eigenvector of A, our matrix-vector multiplication simplifies to a scalar-vector multiplication