INTRO TO QUANTUM COMPUTING

LECTURE #12

QUANTUM MECHANICS 2

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01/24/2021

ANNOUNCEMENTS

QUANTUM MECHANICS LECTURE SERIES

Lecture 1 - Principles of Quantum Mechanics

What is quantum and how do things behave on quantum length scales?

Lecture 2 - Quantum Two-Level Systems and Measurement

Objective - What are two-level systems and what can we do with them?

Lecture 3 - Postulates of Quantum Mechanics

Objective - What are the foundational rules of quantum mechanics?

TODAY'S LECTURE

What are two-level systems and what can we do with them?

- ➔ Two-level quantum systems
- ➔ Stern-Gerlach experiment - how do we know electron spin is a two-level system?
- ➔ Quantum measurements - how do we measure a two-level system in different bases?
- ➔ Quantum entanglement - what can we do with two-level systems?

Reminder - Quantum systems have levels

Quantum

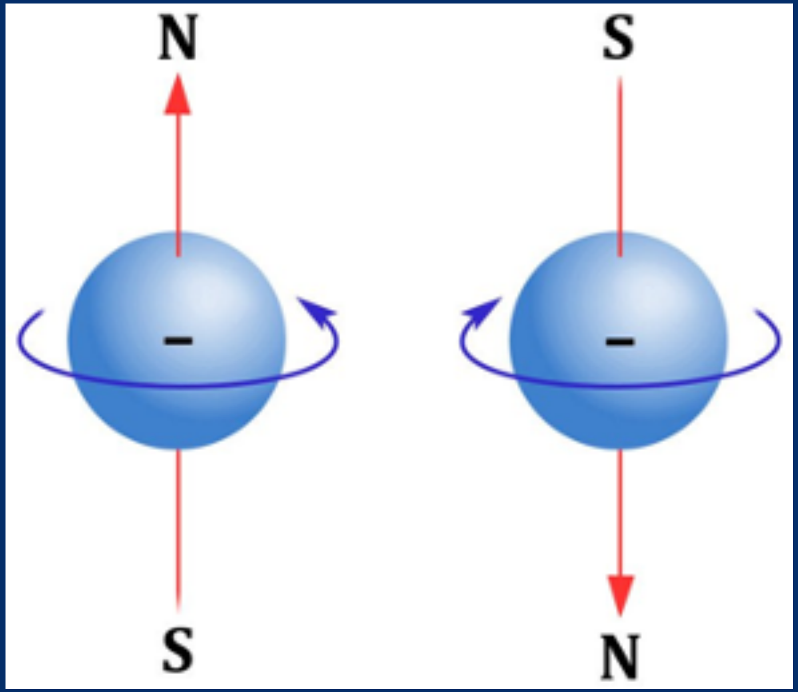
Classical

Two-level systems

A quantum system made out of 2 basis

example:

- Qubits
- Electron spin



Two-level systems

Two-level system:

Qu”bit”: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Three-level system:

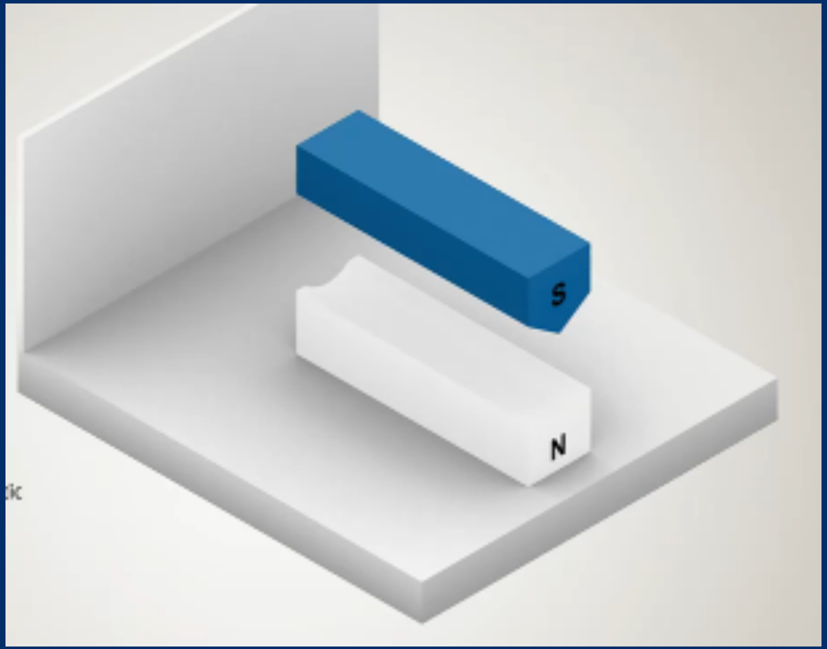
Qu”trit”: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma\ |2\rangle$

Why do we use two level systems for QC?

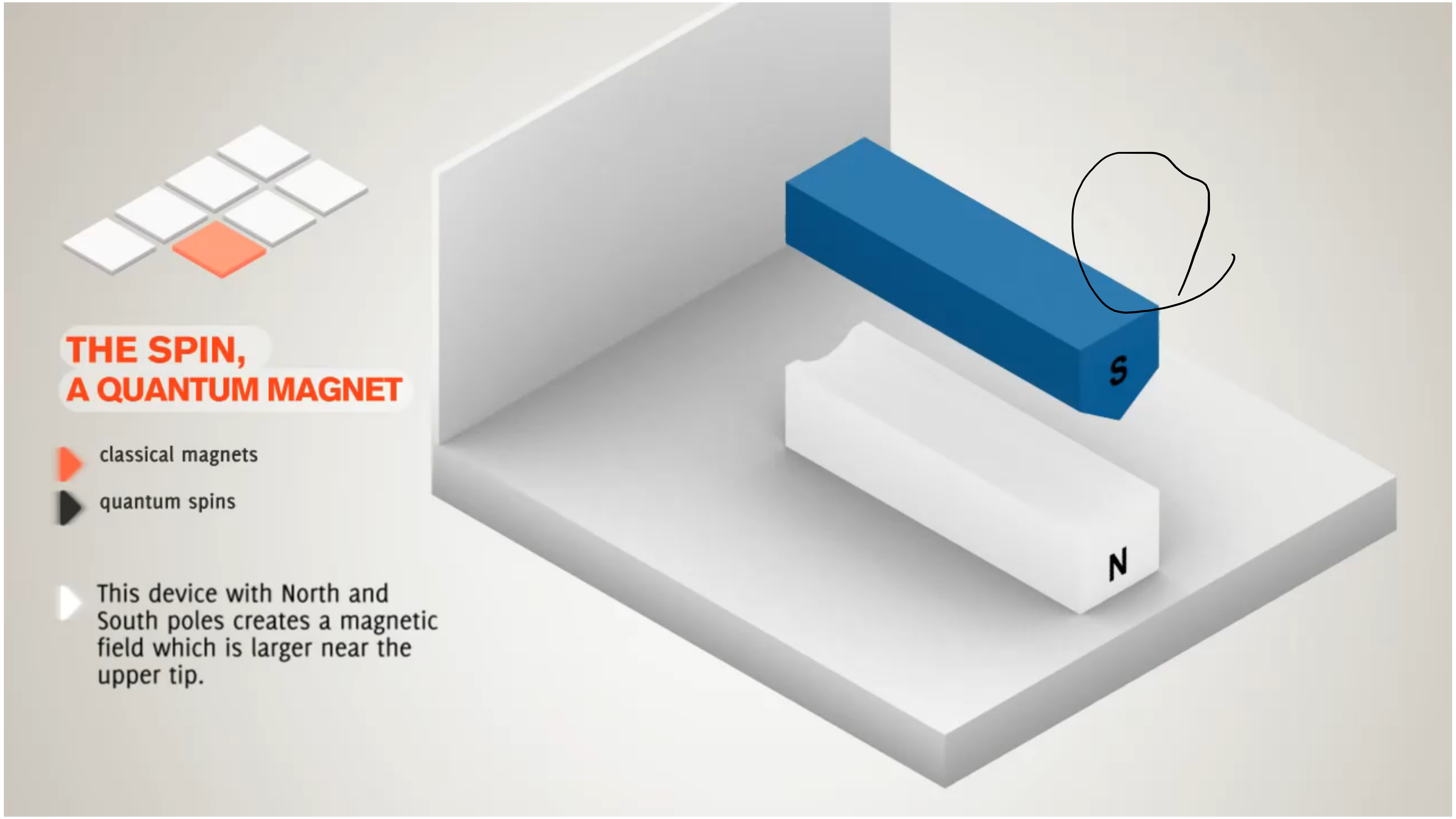
- We can map one level to 0 and another level to 1
- They are easier to control
- Simple linear algebraic properties
- Have been studied for nearly a century

How do we know electron spin is a two-level system?

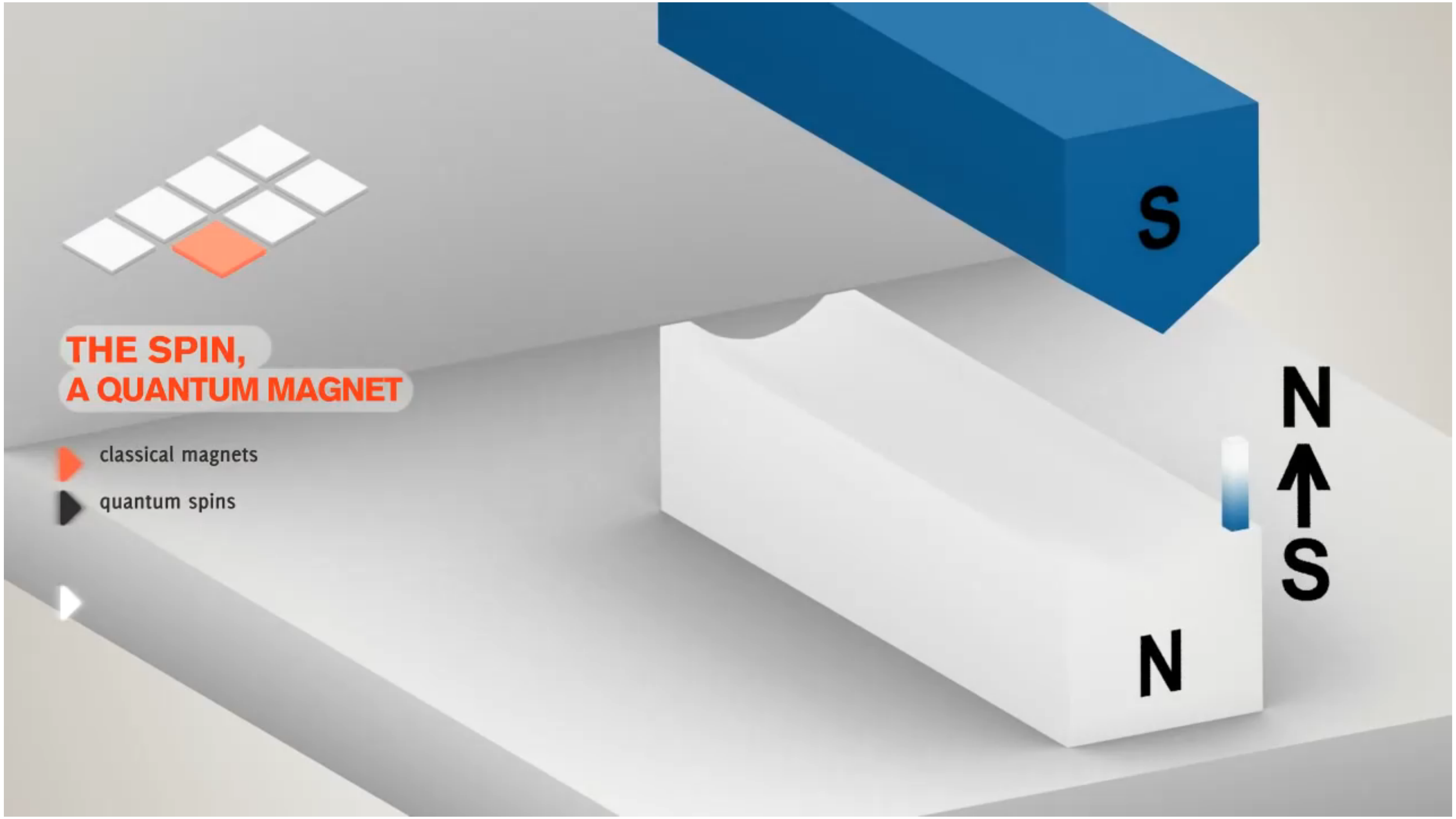
The Stern-Gerlach experiment



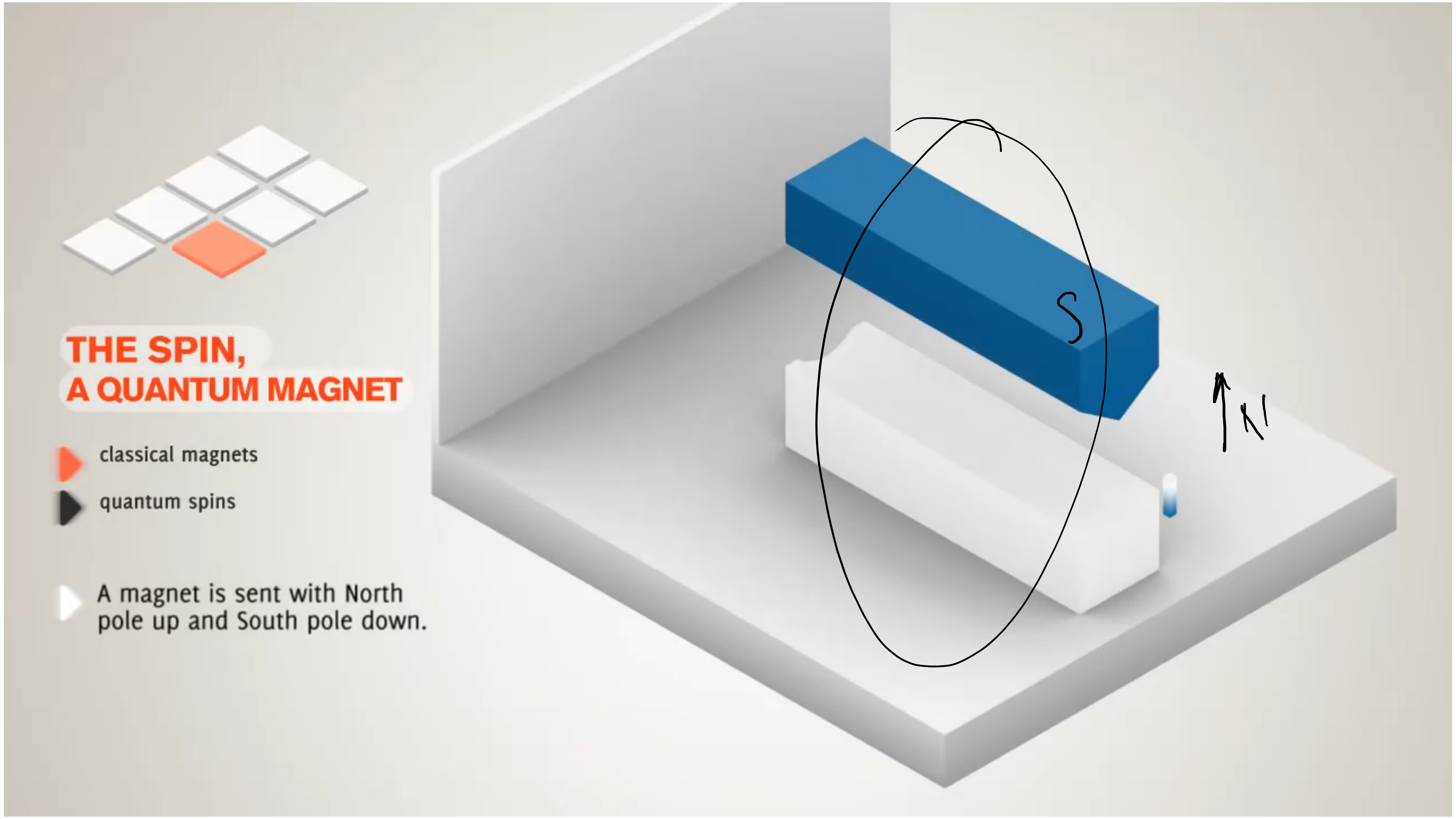
The Stern-Gerlach Experiment



The Stern-Gerlach Experiment

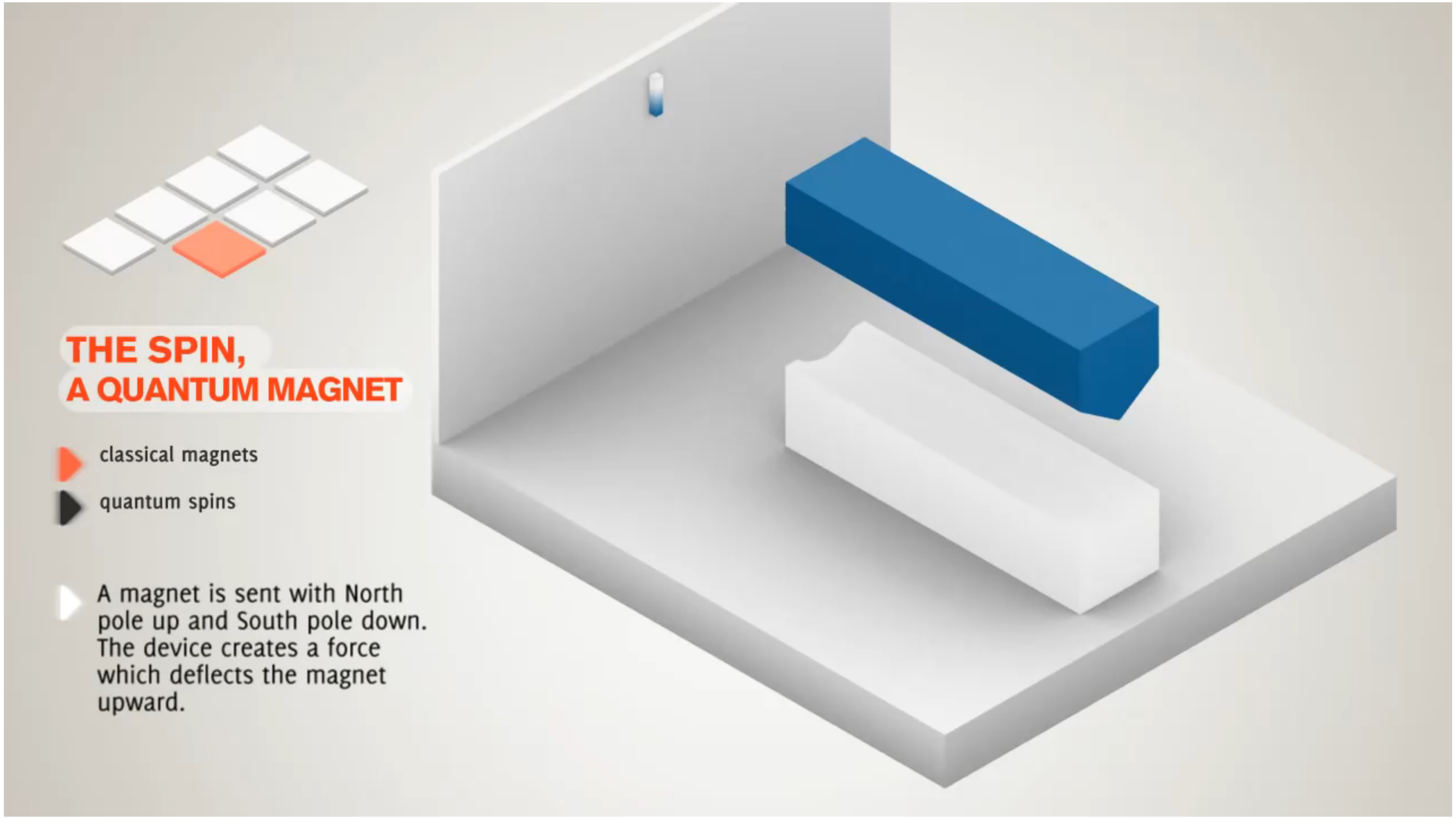


The Stern-Gerlach Experiment

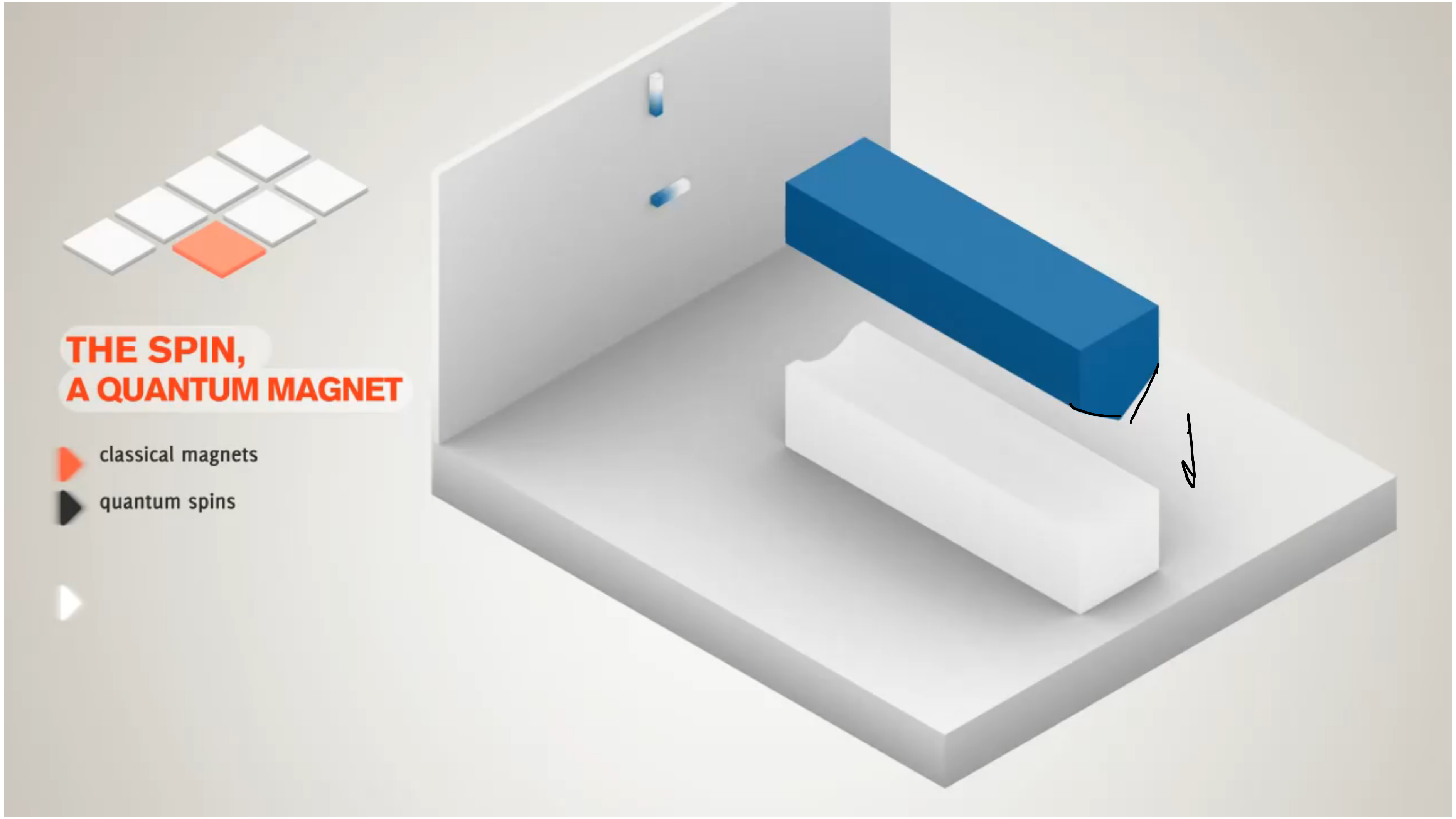


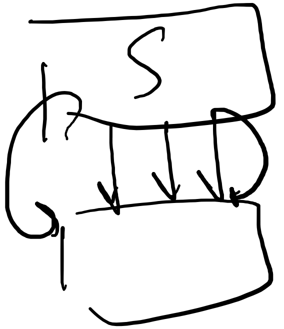
Let’s start by putting a regular magnet through the device.

The Stern-Gerlach Experiment

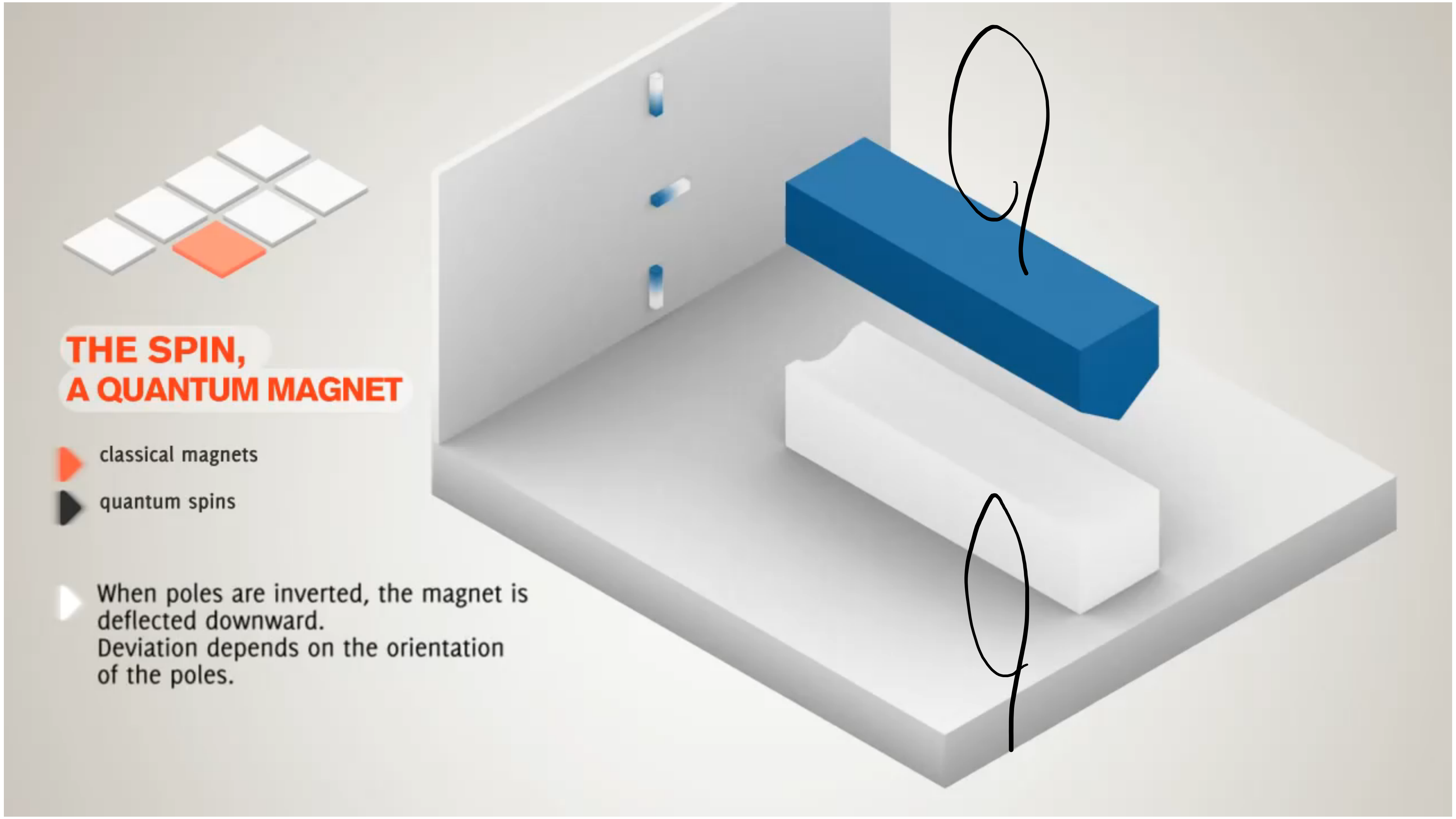


The Stern-Gerlach Experiment

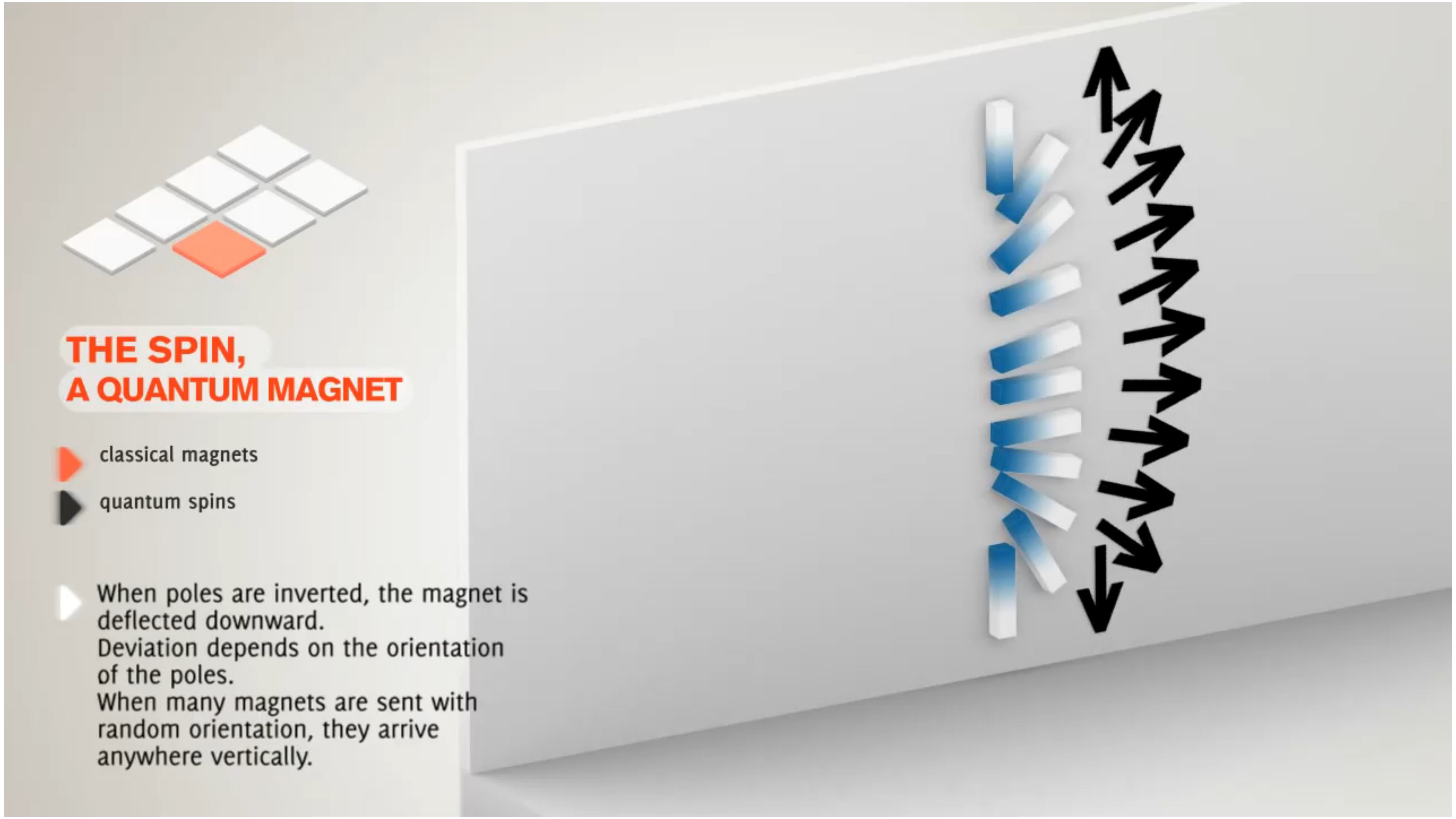




The Stern-Gerlach Experiment

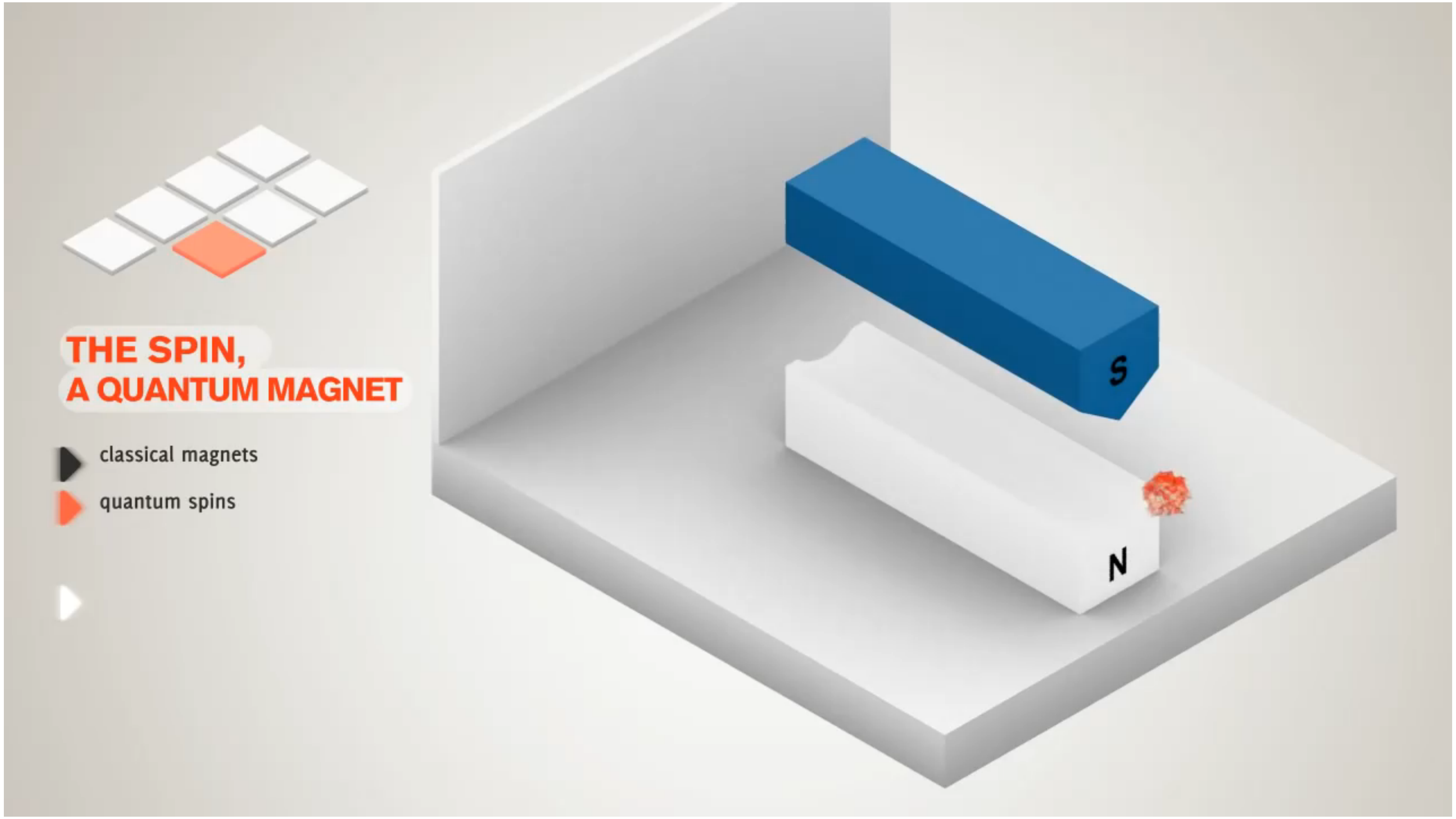


The Stern-Gerlach Experiment



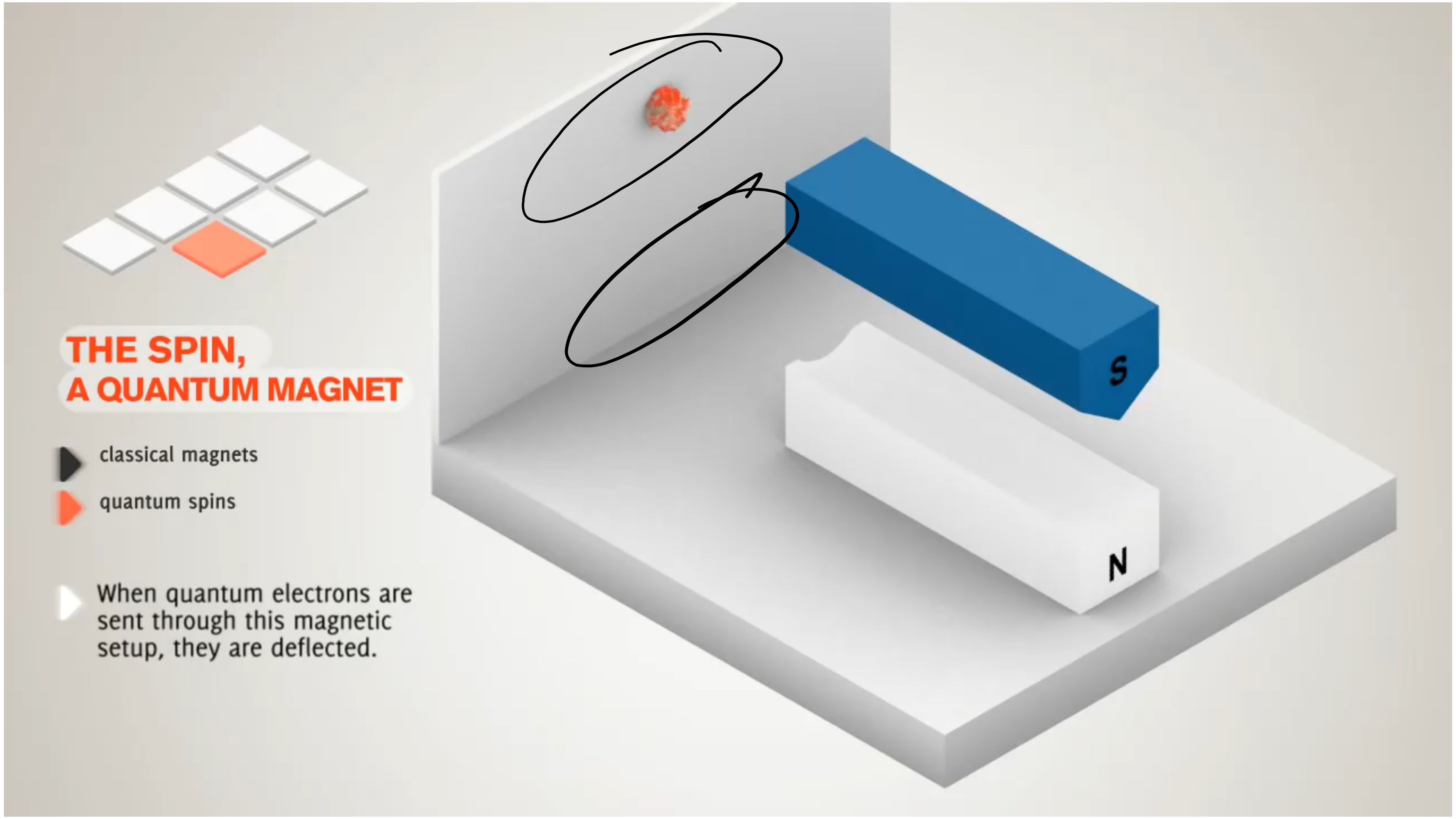
We end up with a **continuum** of positions. The magnets can appear at the top, bottom, or anywhere in between.

The Stern-Gerlach Experiment

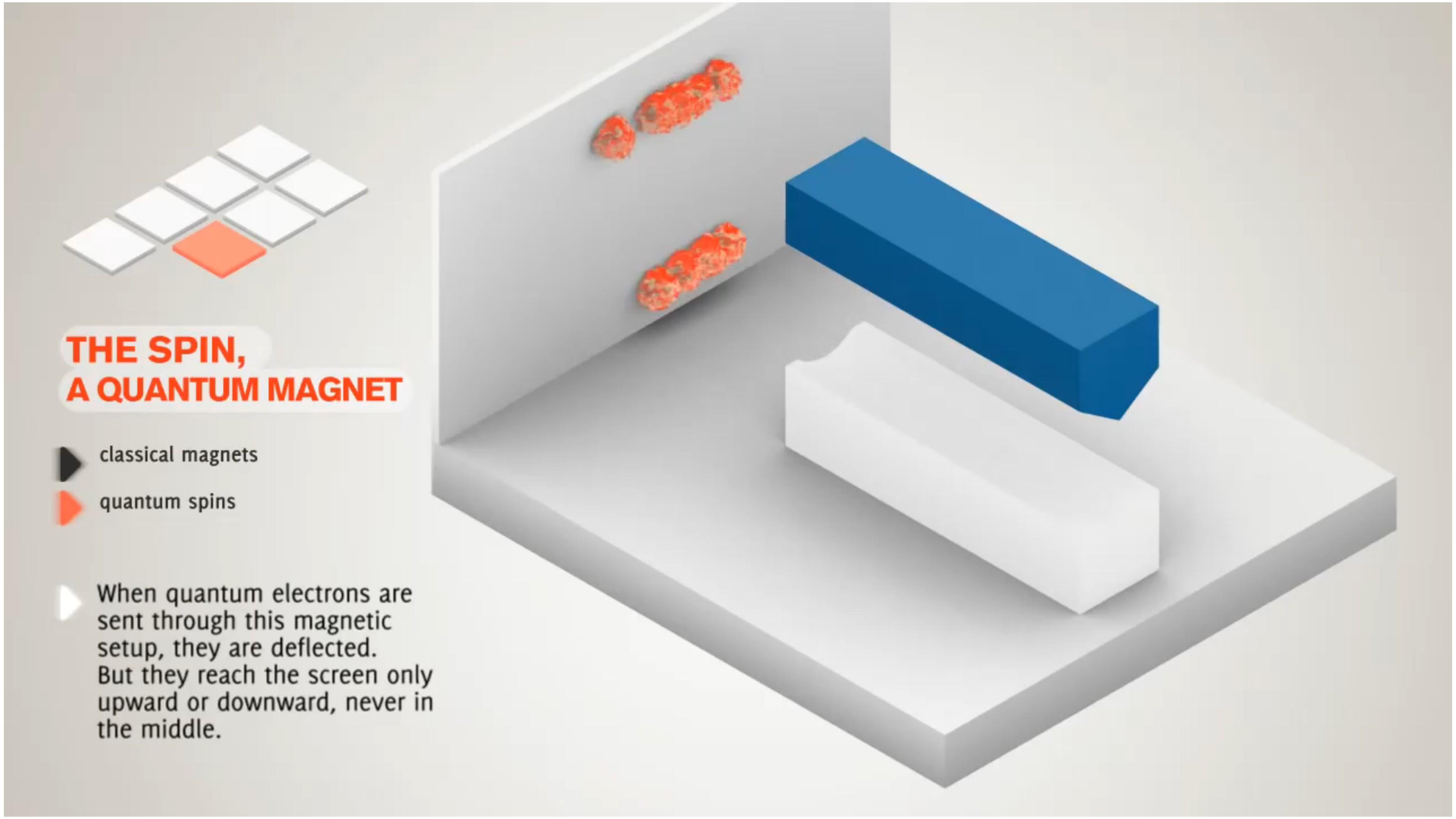


What happens when we use an electron instead of a magnet?

The Stern-Gerlach Experiment

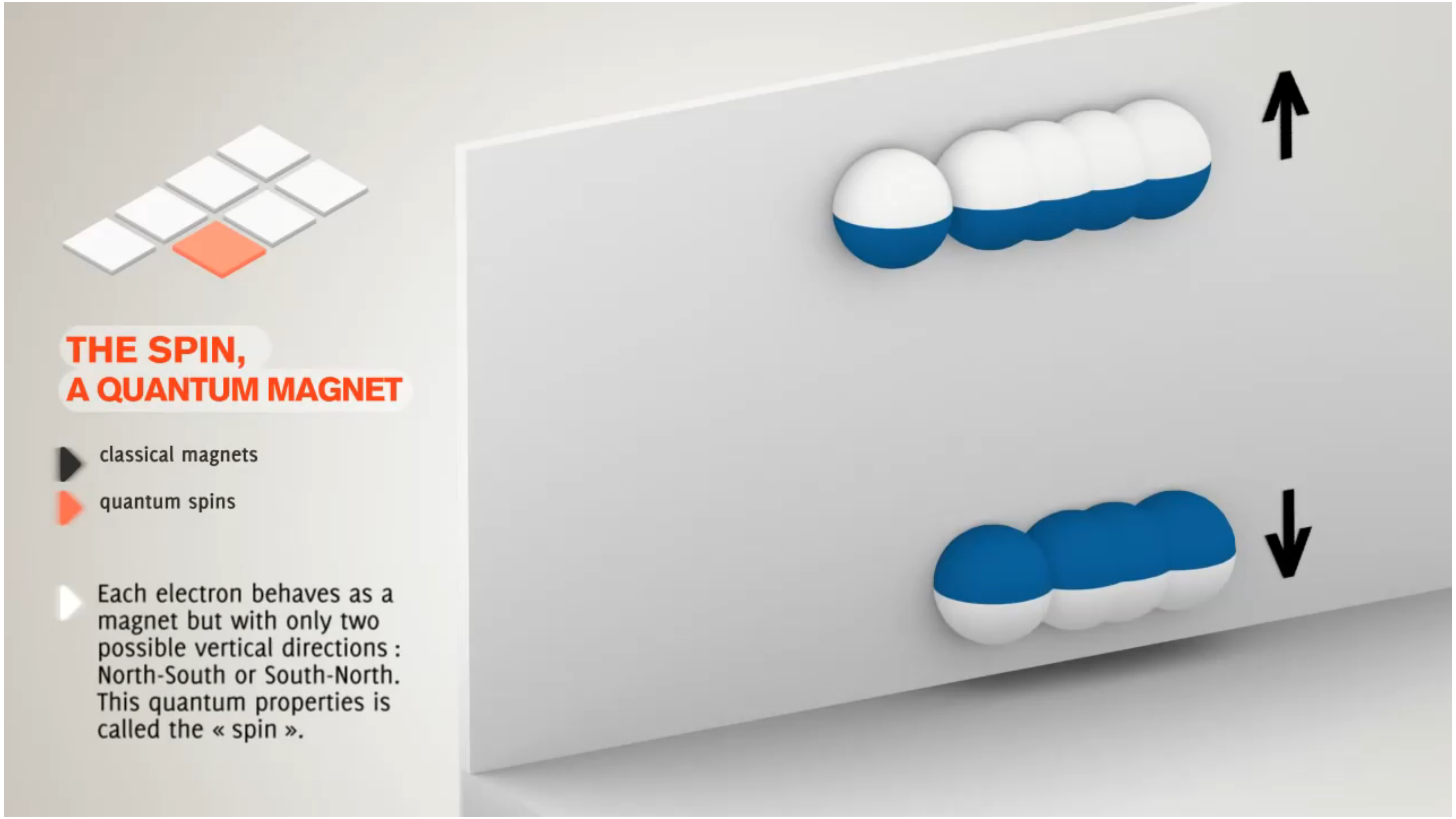


The Stern-Gerlach Experiment



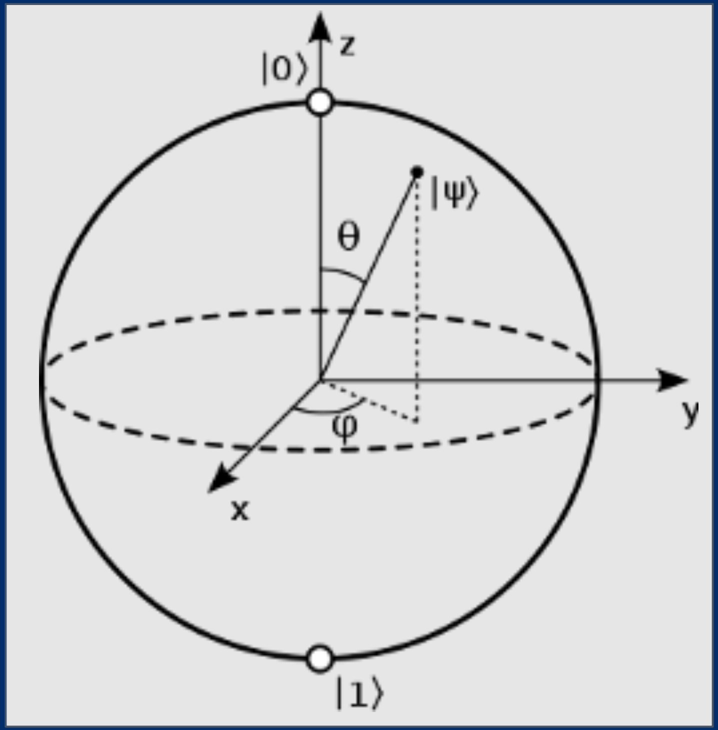
When we used magnets, they could show a continuum of deflections, but electrons can only be deflected to two different levels!

The Stern-Gerlach Experiment



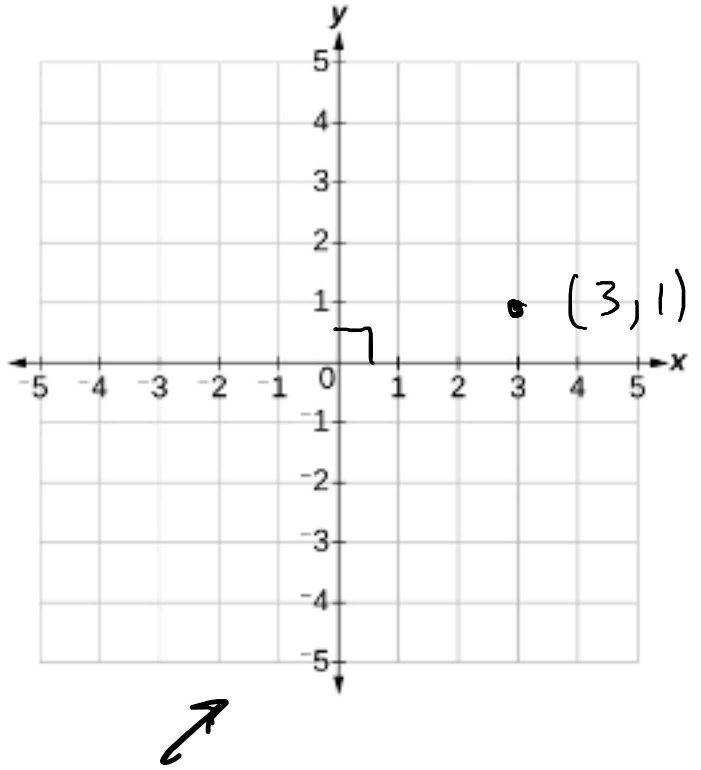
This means electrons can be either "spin up" or "spin down" when they are measured, and nowhere in between- they are two-level systems!

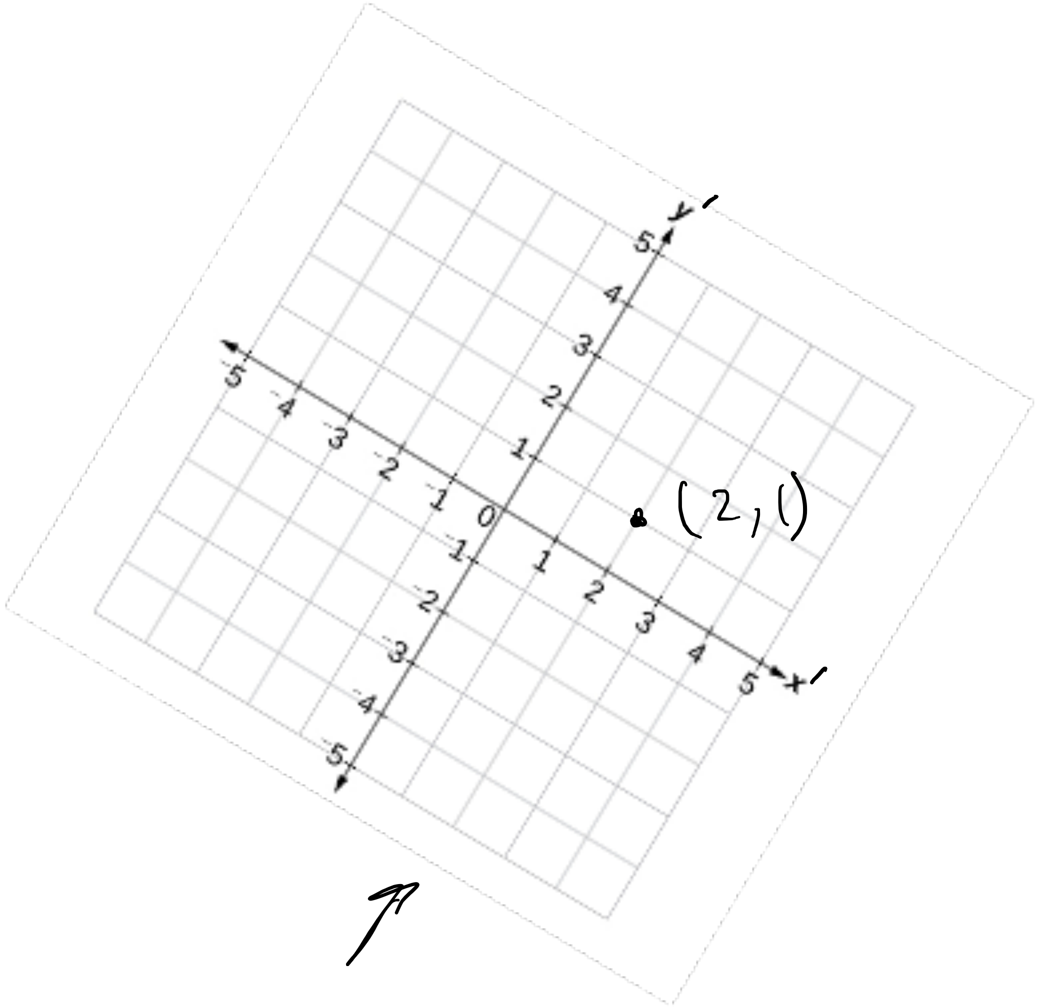
What does measurement of electron spin look like in different bases?



Review - what is a basis?

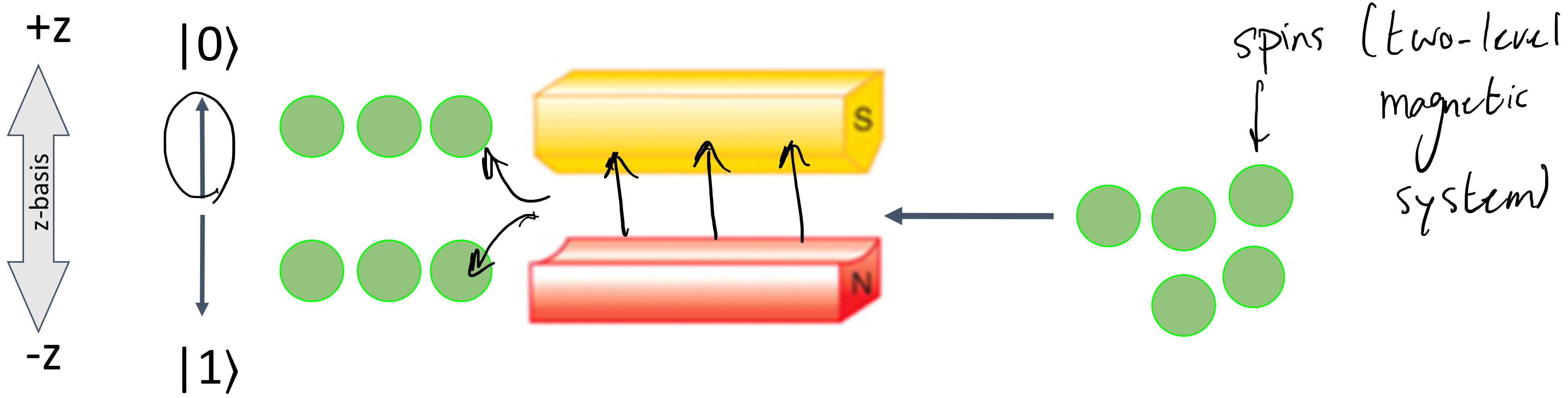
It’s the system you are measuring in.





The Stern-Gerlach Experiment

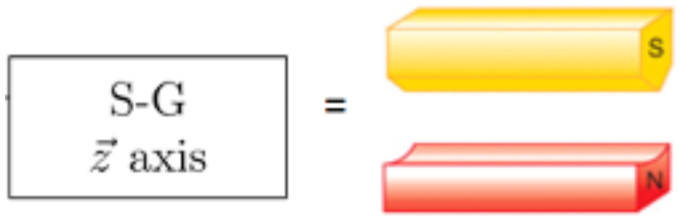
- Measurement in the z-basis:



This is pretty straightforward because the electrons line up along the z direction.

Measurement in z-basis

- Measurement in the z-basis:



Qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

measurement: collapses the quantum state of the qubit $|\psi\rangle$ to either $|0\rangle$ or $|1\rangle$

probability of measuring $|0\rangle$: $|\alpha|^2$

$$P(|0\rangle) = |\langle 0|\psi\rangle|^2 = |\langle 0|(\alpha|0\rangle + \beta|1\rangle)|^2$$
$$= |\alpha \cancelto{1}{\langle 0|0\rangle} + \beta \cancelto{0}{\langle 0|1\rangle}|^2 = |\alpha|^2$$

probability of measuring $|1\rangle$: $|\beta|^2$

$$P(|1\rangle) = |\langle 1|\psi\rangle|^2 = |\langle 1|(\alpha|0\rangle + \beta|1\rangle)|^2$$
$$= |\alpha \cancelto{0}{\langle 1|0\rangle} + \beta \cancelto{1}{\langle 1|1\rangle}|^2 = |\beta|^2$$

Measurement of a quantum system collapses it to one state

Before measurement: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

After measurement:

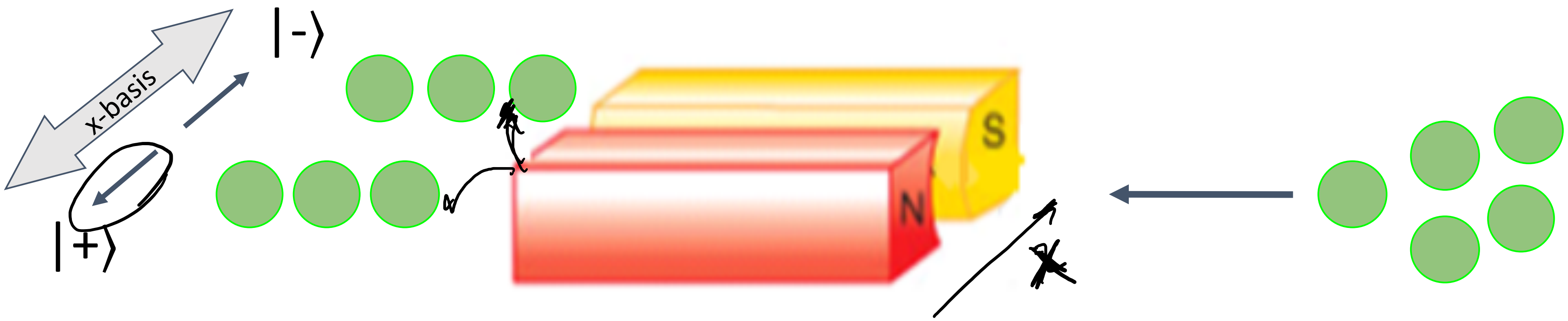
$|\psi\rangle = |0\rangle$

OR

$|\psi\rangle = |1\rangle$

Measurement in other bases

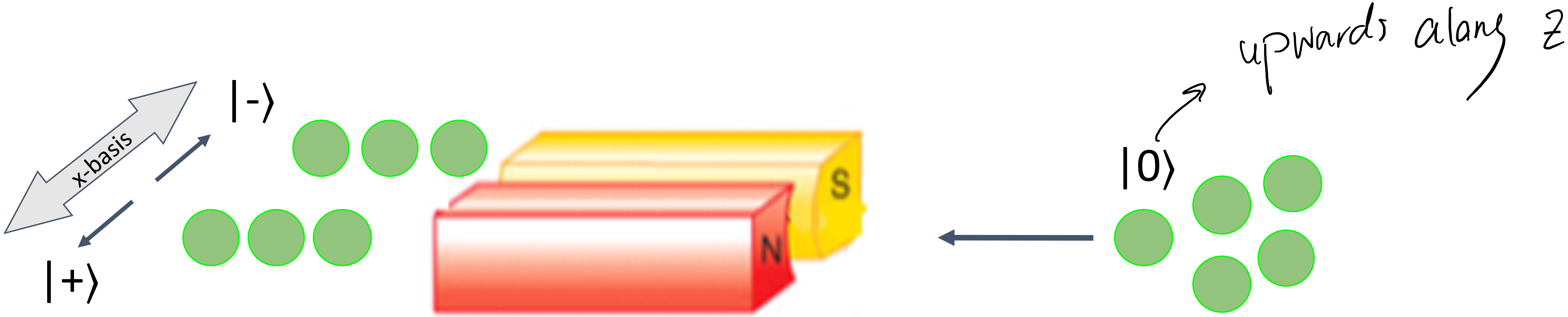
We could have aligned the experiment like this:



Now, the electrons would go to the two different sides instead of up and down.
How do we represent the + and - states?

Measurement in other bases

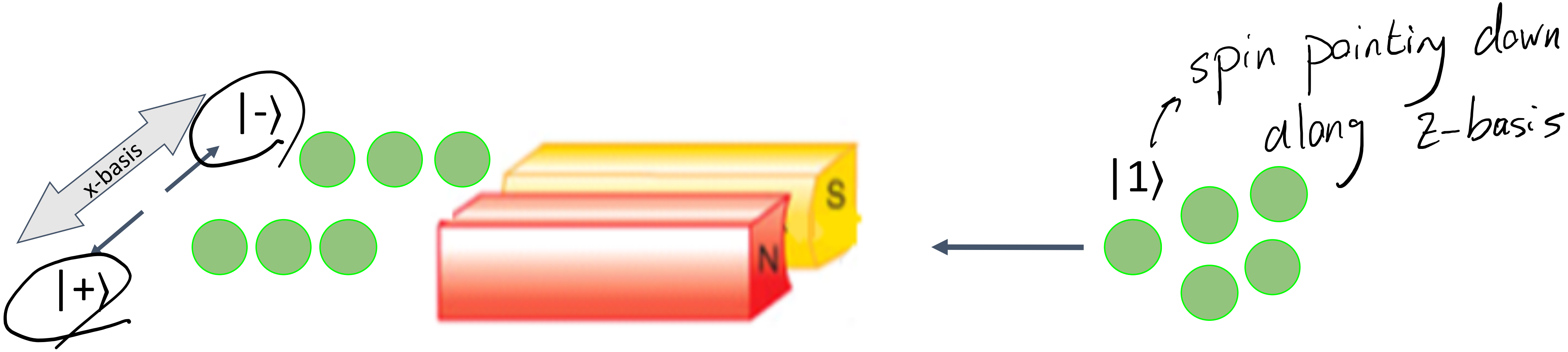
We could have aligned the experiment like this:



If the incoming electrons were all in the $|0\rangle$ (up) state, then we will observe that half of them will go to $|+\rangle$ and half will go to $|-\rangle$.

Measurement in other bases

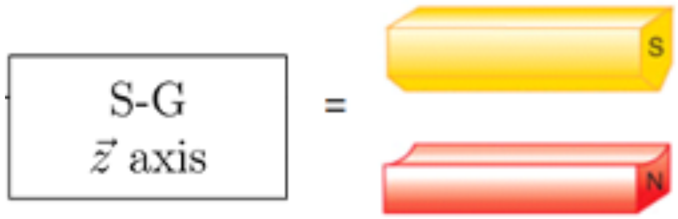
We could have aligned the experiment like this:

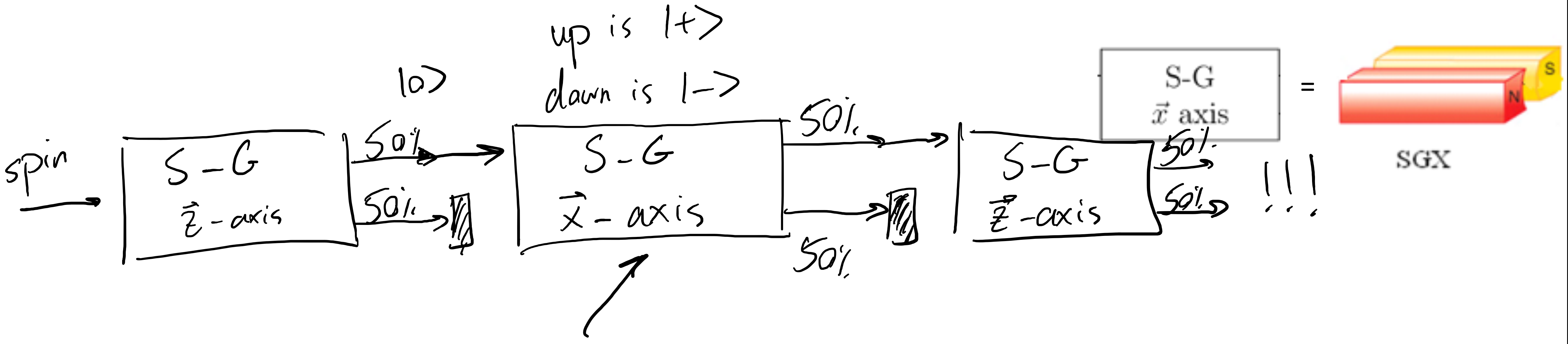


If the incoming electrons were all in the |-⟩ (up) state, then we also observe that half of them will go to |+⟩ and half will go to |-⟩.

So we can represent |+⟩ and |-⟩ as combinations of |0⟩ and |1⟩.

Stern-Gerlach in multiple basis:

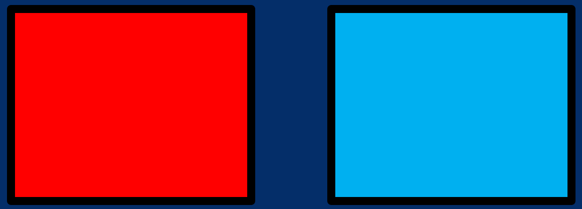




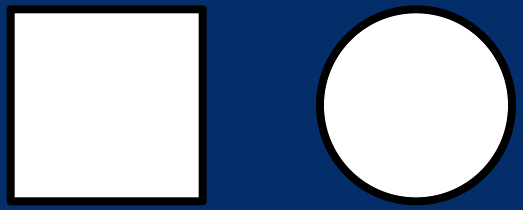
Classical analogy

Let's assume an object has two properties: ***Color*** and ***Shape***

Color: red or blue

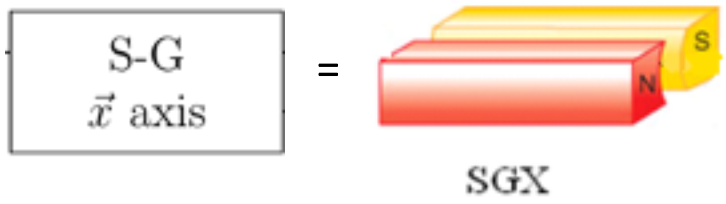


Shape: square or circle



Measurement in x-basis

- Measurement in the x-basis:



Qubit: $|\psi\rangle = |0\rangle$

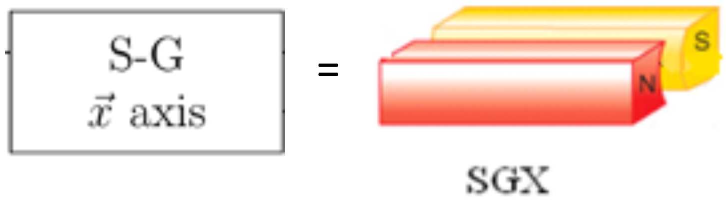
measurement in x basis: collapses the quantum state of the qubit $|\psi\rangle$ to either $|+\rangle$ or $|-\rangle$

$|+\rangle : (|0\rangle + |1\rangle)/\sqrt{2}$ $\qquad |+\rangle = H|0\rangle$

$|-\rangle : (|0\rangle - |1\rangle)/\sqrt{2}$ $\qquad |-\rangle = H|1\rangle$

Measurement in x-basis

- Measurement in the x-basis:



Qubit: $|\psi\rangle = |0\rangle$

measurement in x basis: collapses the quantum state of the qubit $|\psi\rangle$ to either $|+\rangle$ or $|-\rangle$

probability of measuring $|+\rangle$: $P(|+\rangle) = |\langle + | \psi \rangle|^2 = |\langle + | 0 \rangle|^2 = \left|\left(\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)|0\rangle\right|^2$

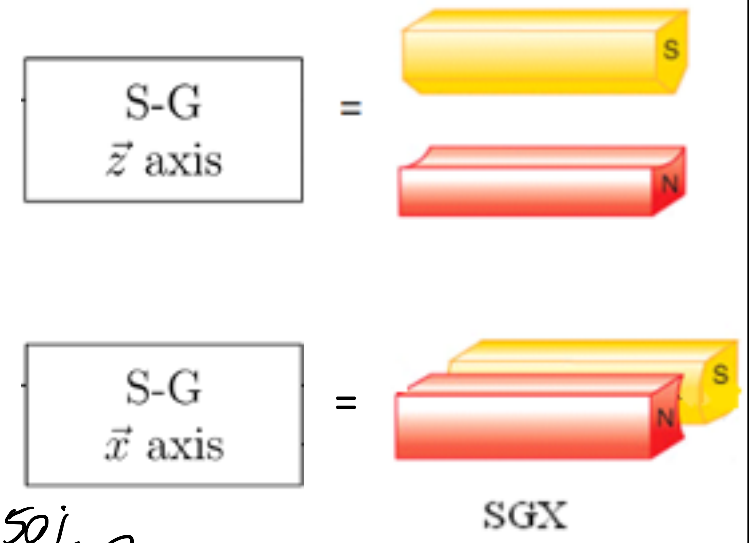
$= \frac{1}{2} \left| \underbrace{\langle 0|0\rangle}_{1} + \underbrace{\langle 1|0\rangle}_{0} \right| = \frac{1}{2}$

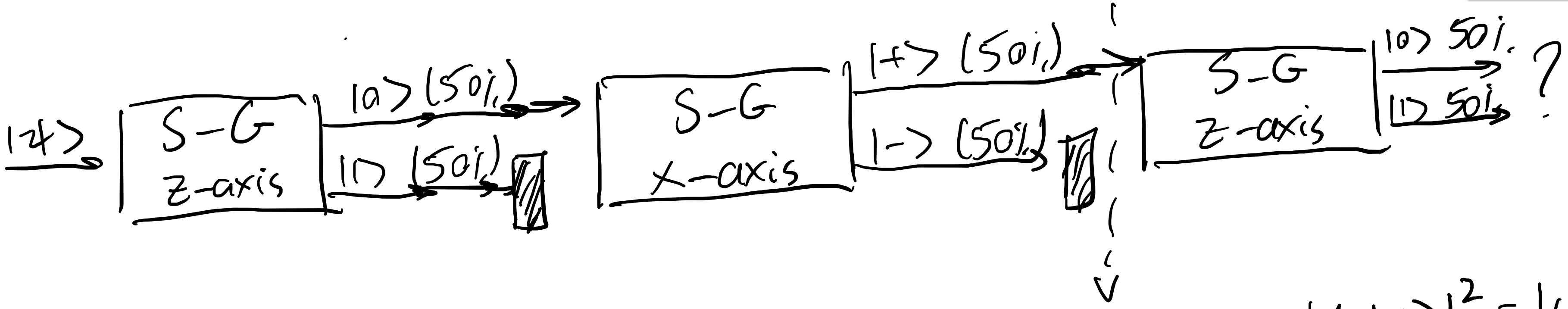
probability of measuring $|-\rangle$: $P(|-\rangle) = |\langle - | \psi \rangle|^2 = |\langle - | 0 \rangle|^2 = \left|\left(\frac{\langle 0| - \langle 1|}{\sqrt{2}}\right)|0\rangle\right|^2$

$= \frac{1}{2} \left| \underbrace{\langle 0|0\rangle}_{1} - \underbrace{\langle 1|0\rangle}_{0} \right|^2 = \frac{1}{2}$

The Stern-Gerlach Experiment

- Revisiting Stern-Gerlach in multiple basis:





$$P(|0\rangle) = |\langle 0|+\rangle|^2 = 1/2$$

$$P(|1\rangle) = |\langle 1|+\rangle|^2 = 1/2$$

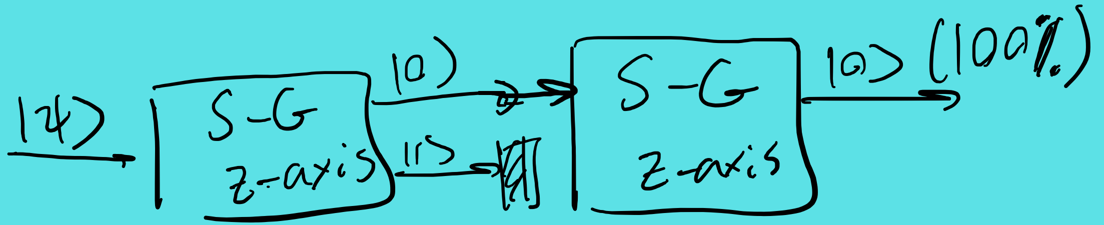
So far...

- We know what two-level systems are and why we use them
- We know electron spin is a two level system
- We know how to measure electron spin in different bases

Next...

- We will see an interesting property that arises out of two-level systems

10 MIN BREAK!



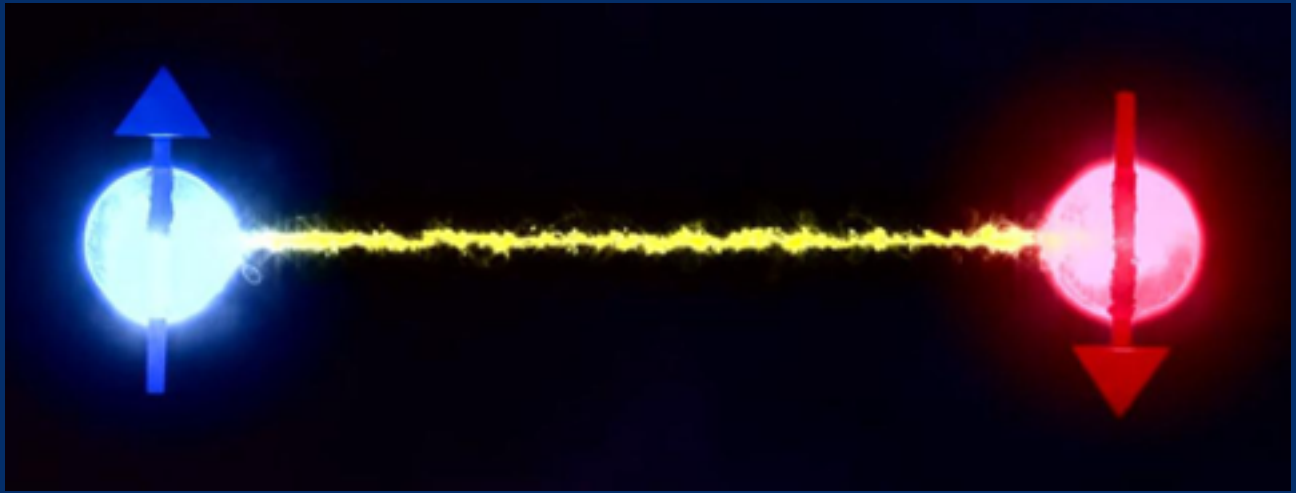
$$P(|0\rangle) = |\langle 0|\psi\rangle|^2 = |\langle 0|0\rangle|^2 = 1$$

$$P(|1\rangle) = |\langle 1|\psi\rangle|^2 = |\langle 1|0\rangle|^2 = 0$$

What can we do with two-level systems?

Quantum Entanglement

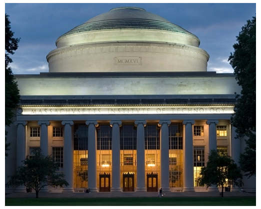
Quantum correlation between two (or more) objects where each of their states will depend on the state of the other



Entanglement

Oxford, UK

Entangled pair



Cambridge, MA

Amin

Theoretical Quantum Computing

Theoretical Quantum Computing
or
Experimental Quantum Computing

Amir

Experimental Quantum Computing

or

or

or

Entanglement

QBA QBB QBA QBB

Entangled state: $|\psi\rangle = \sqrt{0.5}\,\underline{|01\rangle} + \sqrt{0.5}\,\underline{|10\rangle}$

what if we only measure qubit A?

50%

- If qubit A is 0 → the quantum state of qubit B is immediately set to $|1\rangle$

50%

- If qubit A is 1 → the quantum state of qubit B is immediately set to $|0\rangle$

Entanglement

Entanglement is preserved under local operations:

if I apply an operator to one of the qubits

$$|\phi\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}} \xrightarrow[\text{qubit}]{X \text{ to the first}} \frac{|11\rangle + |00\rangle}{\sqrt{2}}$$

if QB "A" is 1, → QB "B" is also 1

if QB "A" is 0, → QB "B" is also 0

Take away message: Entangled qubits will stay entangled if we only apply an operator to a single one of them!

Bell states

There are different ways we can arrive at an entangled state.

$$|\beta_{00}\rangle = \sqrt{0.5}|00\rangle + \sqrt{0.5}|11\rangle$$

$$|\beta_{01}\rangle = \sqrt{0.5}|01\rangle + \sqrt{0.5}|10\rangle$$

$$|\beta_{10}\rangle = \sqrt{0.5}|00\rangle - \sqrt{0.5}|11\rangle$$

$$|\beta_{11}\rangle = \sqrt{0.5}|01\rangle - \sqrt{0.5}|10\rangle$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|+\rangle \neq |-\rangle$$

$$|\beta_{00}\rangle \neq |\beta_{10}\rangle$$

$$|\beta_{11}\rangle \neq |\beta_{01}\rangle$$

Applications of Entanglement

- Quantum Teleportation
- Quantum Cryptography
- Superdense Coding
- Quantum speedups

QKD:
(BB84)

Quantum teleportation

Quantum speedups