QC VECTORS AND COMPLEX N

QUBIT X QUBIT

INTRO TO VECTORS

SCALARS VS VECTORS

SCALARS

* A quantity having only magnitude (no direction)
* Written as: $a \in R$

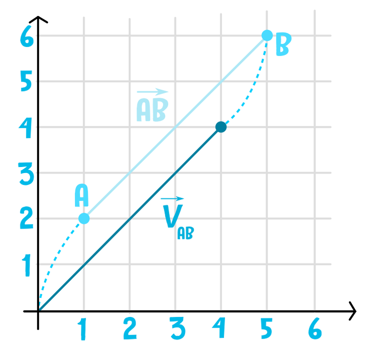
VECTORS

* A quantity with both magnitude and direction
* Written as: $\vec{v} \in R^n$
* Can be described by a list of scalars (cartesian) or a radius and an angle (polar)

VECTOR REPRESENTATION

General 2D vector notation

$$\vec{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

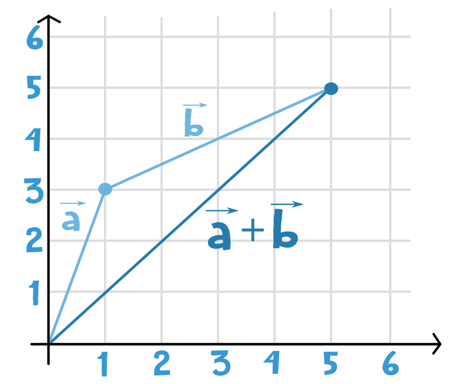


$\vec{AB}$ $A=(1,2)$ $B=(5,6)$ $\vec{V}_{AB}=\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

VECTOR OPERATIONS

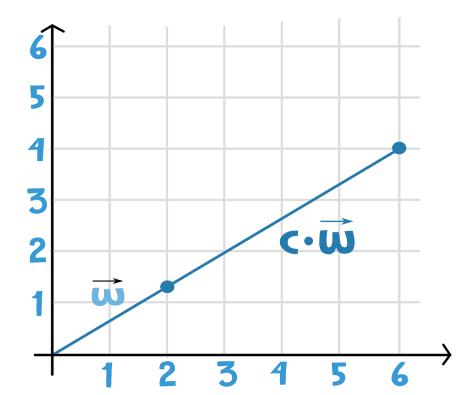
Vector addition

$$\vec{a}+\vec{b}=\begin{pmatrix} a_x+b_x \\ a_y+b_y \end{pmatrix}$$



Vector-scalar mult.

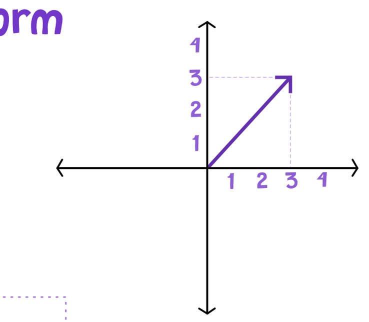
$$c\cdot\vec{w}=\begin{pmatrix} c\cdot w_x \\ c\cdot w_y \end{pmatrix}$$



VECTOR PROPERTIES

Cartesian form

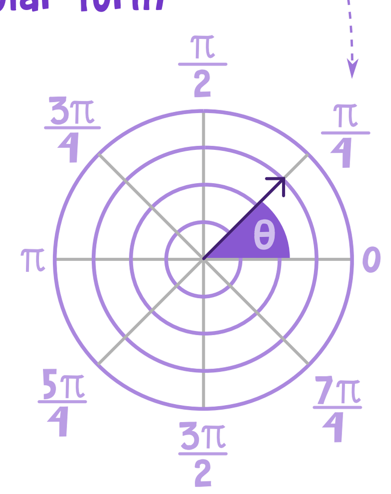
$$\vec{V}=\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$



$r=3$

$\theta=\frac{\pi}{4}$ rad

Polar form



All vectors have

VECTOR MAGNITUDE

$$\|\vec{V}\|=\sqrt{V_x^2+V_y^2}$$

$\vec{V}=\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$\|\vec{V}\|=\sqrt{3^2+6^2}$

$=\sqrt{45}$

$=9$

VECTOR DIRECTION

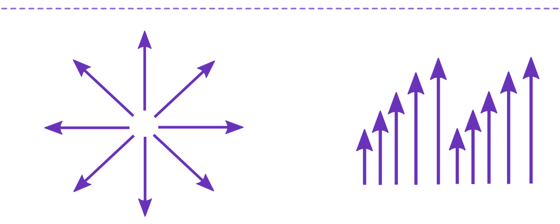
$$\angle\vec{V}=\tan^{-1}\left(\frac{V_y}{V_x}\right)$$

$\vec{V}=\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$\tan\theta=\frac{6}{3}$

$\theta=\tan^{-1}\left(\frac{6}{3}\right)$

$=1.1$ rad



All vectors have the same magnitude but diff direction

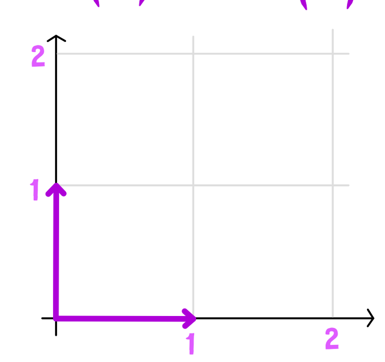
All vectors have the same direction but diff magnitude

VECTOR DECOMPOSITION

"Every vector in R^2 can be expressed as a linear combination of $\hat{x}$ and $\hat{y}$"

Define vectors:

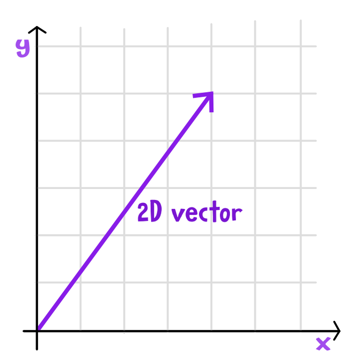
$$\hat{x}=\begin{pmatrix} 1 \\ 0 \end{pmatrix},\ \hat{y}=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

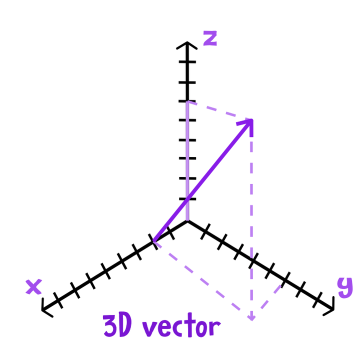


$$\vec{V}=\begin{pmatrix} V_x \\ V_y \end{pmatrix}=V_x\hat{x}+V_y\hat{y}$$

VECTOR GENERALIZATION

* Scalars: $a \in R^1$
* 2D vectors: $\vec{a} \in R^2$
* 3D vectors: $\vec{a} \in R^3$
* 4D vectors: $\vec{a} \in R^4$
* ND vectors: $\vec{a} \in R^n$





INTRO TO COMPLEX NUMBERS

WHY COMPLEX N

QUADRATIC EQUATION

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\sqrt{-1}=???$

IMAGINARY UNIT

$i=\sqrt{-1}$ $i^2=-1$

* $(bi)^2=b^2i^2=-b^2$, $b\in R$
* $\sqrt{-a}=\sqrt{a}i$, $a\in R$

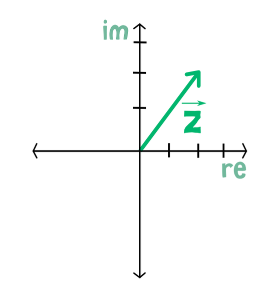
COMPLEX NUMBERS DEFINITION

$$z=\underbrace{a}_{\text{real component}}+\underbrace{\overset{\text{im. unit}}{i}\overset{\text{im. coeff}}{b}}_{\text{imaginary component}}$$

VECTOR REPRESENTATION

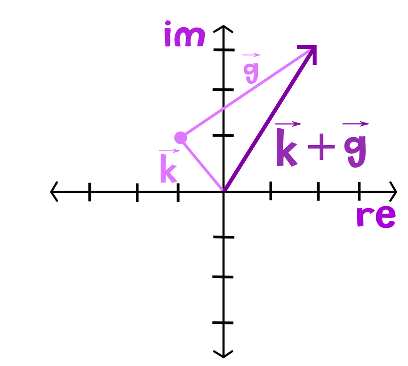
$z=a+ib$

$\vec{z}=\begin{pmatrix} a \\ b \end{pmatrix}$ re. comp. / im. comp.



COMPLEX ADDITION

$$(a+ib)+(c+id)=(a+c)+i(b+d)$$



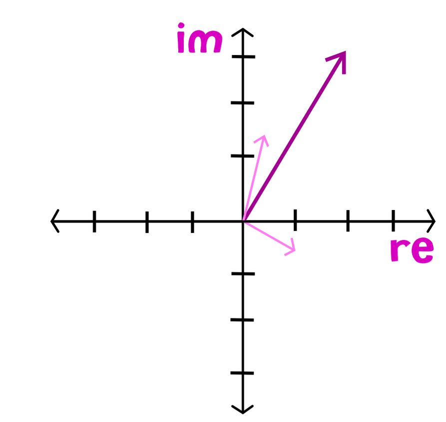
COMPLEX MULTIPLICATION

$$(a+ib)(c+id)=(ac-bd)+i(ad+bc)$$

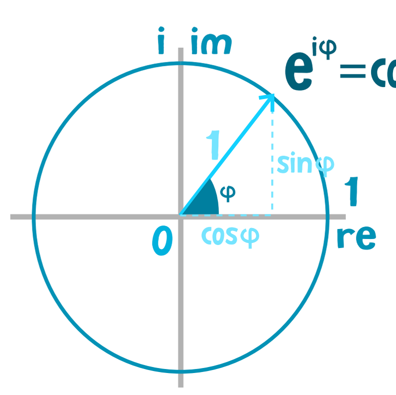
$(a+ib)(c+id)=ac+iad+ibc+i^2bd$

$=ac+i(ad+bc)+(-1)bd$

$=(ac-bd)+i(ad+bc)$



EULER'S FORMULA AND COMPLEX EXPONENTIALS



$e^{i\varphi}=\cos\varphi+i\sin\varphi$

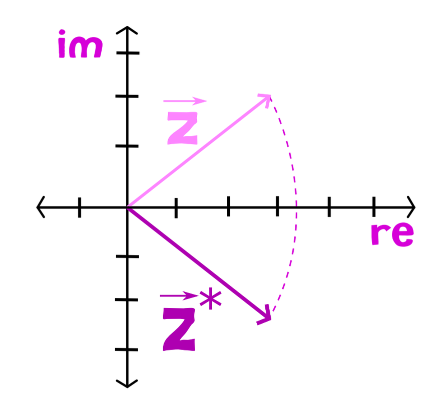
Euler's formula: $e^{i\varphi}=\cos\varphi+i\sin\varphi$

Polar representation: $z=x+iy=|z|(\cos\varphi+i\sin\varphi)=re^{i\varphi}$

$r=|z|=\sqrt{x^2+y^2}$ $\varphi=\tan^{-1}\left(\frac{y}{x}\right)$

COMPLEX CONJUGATION

$$\overline{(a+ib)}=(a-ib)$$



COMPLEX MODULUS

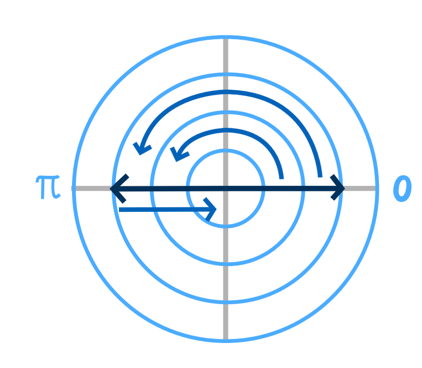
$$|a+ib|=\sqrt{a^2+b^2}$$

$|z|=|a+ib|=\sqrt{z\cdot\bar{z}}$

$=\sqrt{(a+ib)(a-ib)}$

$=\sqrt{a^2+b^2}$

EULER'S INDENTITY



$e^{i\pi}+1=0$

$=\cos(\pi)+i\sin(\pi)+1$

$=-1+1=0$

COMPLEX OPERATIONS

Complex exp. addition

It's good to know the following key identites"

$$\cos\varphi=\frac{e^{i\varphi}+e^{-i\varphi}}{2} \qquad \sin\varphi=\frac{e^{i\varphi}-e^{-i\varphi}}{2}$$

Complex exp. multiplication

$$e^{i\varphi}e^{i\theta}=e^{i(\varphi+\theta)}$$

Complex exp. modulus

$$|z|=|re^{i\varphi}|=r$$

Complex exp. conjugation

$z=x+iy=|z|(\cos\varphi+i\sin\varphi)=re^{i\varphi}$

$$\overline{re^{i\varphi}}=re^{-i\varphi}$$

LECTURE 3