Midterm 3
May 17, 2011

Name: _____ **UID:** _____

Circle your section: 1 2 3 4 5

- **Show your work to receive full credit.**
- **Calculators are allowed.**
- **Time: 75 minutes.**

Problem	Score	Points
1		13
2		16
3		21
4		20
5		21
6		14
Total		105

1. (13 pts) A 200-liter tank initially full of water develops a leak at the bottom. Given that 20% of the water leaks out in the first five minutes, find the amount of water left in the tank 15 minutes after the leak develops if the water drains off at a rate proportional to the amount of water present.

2. (8 pts each) Differentiate the following functions:

a) $f(x) = \sqrt{9 - x^2} + 3\sin^{-1}(\frac{x}{3})$

b) $f(x) = \sinh(\tan^{-1} e^{2x})$

3. (7 pts each) Evaluate the following indefinite integrals:

a) $\int x^2 \cos x \, dx$

b) $\int \frac{1 + \tanh x}{\cosh^2 x} \, dx$

c) $\int \frac{dx}{\sqrt{4x - x^2}}$

4. (10 pts each) Evaluate the following definite integrals:

a) $\displaystyle\int_1^{e^2} x \ln \sqrt{x}\, dx$

b) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\sqrt{\cos x}}\, dx$

5. (7 pts each) For (a) and (b) evaluate the following limits:

a) $\lim_{x\to 0^+} x^{(x^2)}$

b) (8 pts) $\lim_{x\to\infty} \left(\sqrt{x^2+x} - \sqrt{x^2-x}\right)$

c) What is wrong in the following derivation?

$$\lim_{x\to 0^+} \frac{x^2}{\sin x} = \lim_{x\to 0^+} \frac{2x}{\cos x} = \lim_{x\to 0^+} \frac{2}{-\sin x} = -\infty$$

6. (7 pts each) True or False (Circle one and state your reason):

a) For any real number x we have:

$$\sin(\tan^{-1} x) = \frac{2x}{1+x^2}$$

True False

Reason:

b) For any $x > 0$ we have:

$$2\sinh(\ln x) < x$$

True False

Reason:

Draft: