Quiz 7 Solutions

1. **Question:** 3 points

   Refer to the quiz sheet for the problem statement.

   - **Solution:**

     The cutting plane for x_1 in the given problem can be found as follows.
     Using the cutting plane algorithm:

     (a) Pick row corresponding to b_i with largest fractional part.
     (b) Find floor of each coefficient for equation corresponding to row.
     (c) Take original equation corresponding to row and subtract from equation in (b).

     Version A
     Solution: $-\frac{1}{8}x_3 - \frac{7}{8}x_4 + u_1 = -\frac{1}{4}$
     See page 265 of textbook.

     Version B
     Solution: $-\frac{1}{2}x_1 - \frac{1}{2}x_3 - \frac{1}{2}x_4 + u_1 = -\frac{1}{2}$

   The **grading scheme** for this question is as follows:

   - 3 marks for correct final answer
   - 2.5 marks for correct methodology and calculation error
   - 1.5 marks for partially correct answer

2. **Question:** 7 points

   Refer to the quiz sheet for the problem statement.

   - **Solution:**

     Solve the given integer programming problem.
     Using the Branch and Bound algorithm:

     (a) **Initial solution:** Solve LP as usual. If solution is not integral, continue Dakin's method.
     (b) **Branching Variable Selection:** Choose, from optimal solution, x_j with non-integral value to form 2 branching constraints: $x_j \leq [x_{Bi}]$ and $x_j \geq [x_{Bi}] + 1$.
     (c) **Formation of New Nodes:** Create two new mixed integer problems each corresponding to one of the 2 branching constraints, and solve using the dual simplex method.
     (d) **Test for Terminal Node:** A node is a terminal node if no feasible solution to the dual simplex method exists, or if values of x_j are all integers. In the latter case, compare with objective function value to determine optimal solution.

     Version A

$$\left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & \\ \hline 2 & 9 & 1 & 0 & 40 \\ 11 & -8 & 0 & 1 & 82 \\ \hline -3 & -13 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\ \left[\begin{array}{cccc|c} \frac{2}{9} & 1 & \frac{1}{9} & 0 & \frac{40}{9} \\ \frac{115}{9} & 0 & \frac{8}{9} & 1 & \frac{1258}{9} \\ \hline \frac{-1}{9} & 0 & \frac{13}{9} & 0 & \frac{520}{9} \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\ \left[\begin{array}{cccc|c} 0 & 1 & \frac{11}{115} & -\frac{2}{115} & \frac{12}{5} \\ 1 & 0 & \frac{8}{115} & \frac{9}{115} & \frac{46}{5} \\ \hline 0 & 0 & \frac{167}{115} & \frac{1}{115} & \frac{294}{5} \end{array}\right] \end{array}$$

Optimal solution: $(\frac{46}{5}, \frac{12}{5}, 0, 0)$ @ $z = \frac{294}{5}$
The variable x_j with largest fractional part is $x_2 = \frac{12}{5}$. Therefore, we work with $x_2 \leq 2$ and $x_2 \geq 3$ as the branching constraints. The resulting optimal solution is $(2, 4, 0, 0)$ at $z = 58$.

Version B

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\ \left[\begin{array}{cccc|c} 2 & 1 & 1 & 0 & 5 \\ -4 & 4 & 0 & 1 & 5 \\ \hline 1 & -2 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\ \left[\begin{array}{cccc|c} 3 & 0 & 0 & -\frac{1}{4} & \frac{15}{4} \\ -1 & 1 & 0 & \frac{1}{4} & \frac{5}{4} \\ \hline -1 & 0 & 0 & \frac{1}{2} & \frac{5}{2} \end{array}\right] \end{array}$$

$$\begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & s_1 & s_2 & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{12} & \frac{5}{4} \\ 0 & 1 & 0 & \frac{1}{6} & \frac{5}{2} \\ \hline 0 & 0 & 0 & \frac{5}{12} & \frac{15}{4} \end{array}\right] \end{array}$$

Optimal solution: $(\frac{5}{4}, \frac{5}{2}, 0, 0)$ @ $z = \frac{15}{4}$
The variable x_j with largest fractional part is $x_2 = \frac{5}{2}$. Therefore, we work with $x_2 \leq 2$ and $x_2 \geq 3$ as the branching constraints. The resulting optimal solution is $(1, 2, 0, 0)$ at $z = -3$.

The **grading scheme** for this question is as follows:

- 2.5 marks for correct initial tableau
- 1.5 marks for finding remaining tableaus
- 3 marks for correctly implementing Dakin's algorithm