Tutorial 5: Duality

0.1 Administrative

- **This week**: Reading week. No tutorial or office hours. Study well!
- **Next week:** In-person tutorial and midterm review. No quiz.
- **Your midterm date:** Saturday, March 5.
- **My office hours:** Fridays @ 6:30PM on Zoom

0.2 Recall

By now, you have learned and should be able to recall definitions for the following:

- Forms of Linear Programming Problems (LPP)
- Matrix notation
- Feasible regions
- Feasible solutions and optimal solutions
- Convex sets and combinations
- Graphical solutions to LPPs
- Basic feasible solution algorithm
- Simplex method
- Two-phase method and Big M methos

0.3 Learning Objectives

In this tutorial, you should expect to achieve the following:

- Learn about duality.
- Learn how to convert from primal LP to dual LP, and vice versa.
- Economic interpretation of the dual problem.

Refer to section 3.1 in the textbook:
Elementary Linear Programming with Applications (Kolman, Beck)

Duality

Reading: Section 3.1 Duality

Theorem 3.1 *Given a primal problem as follows*

$$\begin{aligned} \max \quad & z = c^\top x \\ \textit{subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

The dual of its dual is again the primal problem.

Theorem 3.2 *The linear programming problem in canonical form given by*

$$\begin{aligned} \max \quad & z = c^\top x \\ \textit{subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

has for its dual the linear programming problem

$$\begin{aligned} \max \quad & z' = b^\top w \\ \textit{subject to} \quad & A^T w \geq c \\ & w \ \textit{unrestricted} \end{aligned}$$

Summary of Primal and Dual LPs

TABLE 3.1

Primal problem	*Dual problem*
Maximization	Minimization
Coefficients of objective function	Right-hand sides of constraints
Coefficients of ith constraint	Coefficients of ith variable, one in each constraint
ith constraint is an inequality $\leq$	ith variable is ≥ 0
ith constraint is an equality	ith variable is unrestricted
jth variable is unrestricted	jth constraint is an equality
jth variable is ≥ 0	jth constraint is an inequality $\geq$
Number of variables	Number of constraints

Economic Interpretation

The interpretation of the LP problem changes when transformed from the primal LP to the dual LP. For instance, the decision variables are called the **dual variables** and act as prices, or costs, or values of one unit of each of the inputs. A few possible interpretations of the dual problem are as follows:

- **Accounting prices**: The value w_i represents the price of a unit for the particular input i.
- **Marginal value**: The value w_i represents increase in profit per unit for particular input i.
- **Replacement value**: The value w_i represents decrease in profit per unit for particular input i.

Note that the input i represents a resource i and w_i is typically associated with a dollar amount. Read more in pages 161 to 165 of the textbook on economic interpretation of dual LPs.