Quiz 4 Solutions

1. **Question:** 5.5 points

   Given the LP program:

$$\begin{aligned} \text{Minimize} \quad & p = 3x - 2y + z \\ \text{Subject to} \quad & -x - y + z \leq 1 \\ & x + 2y - z \leq 5 \\ & x, y, z \geq 0 \end{aligned}$$

   Set up the initial simplex tableau.

   - **Solution:**

$$\begin{array}{c} \\ u \\ v \\ \end{array} \left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & u & v & z & \\ \hline -1 & -1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 2 & -1 & 0 & 1 & 0 & 5 \\ \hline 3 & -2 & 1 & 0 & 0 & 1 & 0 \end{array}\right]$$

     The **grading scheme** for this question is as follows:
     - 1 mark off for incorrect objective function
     - 0.5 mark off for incorrect objective row (check signs)
     - 0.5 mark off for no z or P column present in tableau

2. **Question:** 4.5 points

   Consider the following tableau for a maximization problem:

$$\left[\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & s_2 & s_3 & \\ \hline 0 & 1/3 & 1 & 2 & 1 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & 1/2 & 1 & 1 & -1/3 & -1/2 & 2 \\ 1 & -1 & 0 & 0 & 1/3 & 0 & 0 & 1 & 0 \\ \hline 0 & -6 & 0 & -3 & -1 & 0 & 3 & 9 & 24 \end{array}\right]$$

   (a) Identify the basic variables in the above tableau.

   (b) What basic feasible solution does the tableau represent?

   (c) Indicate which variable should be the entering variable and which variable should be the departing variable.

   (d) Compute the next tableau using Simplex method, and what can you tell from the new tableau?

   - **Solution:**

     (a) x_1, x_3, s_1

     (b) $(0, 0, 1, 0, 0, 2, 0, 0)$

(c) x_2 enters, s_1 departs

(d) The next tableau is as follows

$$\left[\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & s_1 & s_2 & s_3 & \\ \hline 0 & 0 & 1 & 11/6 & 2/3 & -1/3 & 28/9 & -17/6 & 1/3 \\ 0 & 1 & 0 & 1/2 & 1 & 1 & -1/3 & -1/2 & 2 \\ 1 & 0 & 0 & 1/2 & 4/3 & 1 & -1/3 & -1/2 & 2 \\ \hline 0 & 0 & 0 & 0 & 5 & 6 & 1 & 6 & 36 \end{array}\right]$$

The **grading scheme** for this question is as follows:

- 1 mark for correct answer to each of (a) to (c)
- 2 marks for correct answer to (d): 1 mark for correct next tableau and 1 mark for correct analysis of tableau