Quiz 1 Solutions

1. **Question:** 5 points

   A container manufacturer is considering the purchase of two different types of cardboard folding machines: model A and model B. Model A can fold 30 boxes per minute and requires 1 attendant, whereas model B can fold 50 boxes per minute and requires 2 attendants. Suppose the manufacturer must fold at least 320 boxes per minute and can not afford more than 12 employees for the folding operation. If a model A machine costs $15,000 and a model B machine costs $25,000, how many machines of each type should be bought to minimize the cost? Formulate the LP model. Please do not attempt to solve the problem.

   - **Solution:**

     Recall the terminology learnt in tutorial 1.

     The decision variables in this problem are as follows:

     - Let X be the number of model A cardboard folding machines.
     - Let Y be the number of model B cardboard folding machines.

     The X and Y variables are unknown and we aim to solve for these in this LP problem.

     The requirements are as follows:

     - At least 320 boxes must be folded per minute.
     - At most 12 employees must be hired for the operation.

     The known parameters are as follows:

     - Model A folds 30 boxes per minute and requires 1 attendant.
     - Model B folds 50 boxes per minute and requires 2 attendants.

     We can combine the requirements and known parameters above to define the constraints in the formulation of the LP problem as follows:

$$
\begin{aligned}
\text{Minimize} \quad & z = 15000x_1 + 25000x_2 \\
\text{Subject to} \quad & 30x_1 + 50x_2 \geq 320 \\
& x_1 + 2x_2 \leq 12 \\
& x_1 \geq 0 \\
& x_2 \geq 0
\end{aligned}
$$

     The **grading scheme** for this question is as follows:

     - Full marks are given for correct model formulation and definition of the decision variables, and correct constraints (including non-negativity).
     - Note that you are not penalized for using different units in the objective function.
     - You are also not required to solve the problem (as stated in the question).

2. **Question:** 5 points

   Write the following LP problem in standard form.

$$\begin{aligned}
\text{Minimize} \quad & z = 4x_1 - 3x_2 - 4x_3 - 12 \\
\text{Subject to} \quad & x_1 + x_2 \leq 4 \\
& x_1 + 2x_2 + 3x_3 \geq 4 \\
& 5 \geq x_1 \geq -1 \\
& x_3 \leq 0
\end{aligned}$$

- **Solution:**

  Note that you first need to take care of the non-negative constraints in this LP problem before advancing to the next step (which is converting the inequalities appropriately). You can do that as follows:

  - First set $x_1' := x_1 + 1$ and then you get $x_1 = x_1' - 1$ which you can substitute in the original model.
  - Next set $x_2' := x_2^+ - x_2^-$ where $x_2^+ \geq 0$ and $x_2^- \geq 0$ since x_2 is an unconstrained variable in this problem. You can then substitute both x_2^+ and x_2^- which are non-negative into the original model
  - Finally set $x_3' := -x_3$ in order to reverse the negativity, and you then have $x_3 = x_3'$ which you can substitute into the original model

  Once you've done the necessary substitution, you will end up with the following LP problem

$$\begin{aligned}
\text{Minimize} \quad & z = 4x_1' - 3x_2^+ + 3x_2^- + 4x_3' - 16 \\
\text{Subject to} \quad & x_1' + x_2^+ - x_2^- \leq 5 \\
& x_1' + 2x_2^+ - 2x_2^- - 3x_3' \geq 5 \\
& x_1' \leq 6 \\
& 5 \geq x_1 \geq -1 \\
& x_3 \leq 0
\end{aligned}$$

  Now we can convert the inequalities and objective function appropriately.

$$\begin{aligned}
\text{Maximize} \quad & z = -4x_1' + 3x_2^+ - 3x_2^- - 4x_3' + 16 \\
\text{Subject to} \quad & x_1' + x_2^+ - x_2^- \leq 5 \\
& -x_1' - 2x_2^+ + 2x_2^- + 3x_3' \leq -5 \\
& x_1' \leq 6 \\
& x_1 \geq 0 \\
& x_2^+, x_2^- \geq 0 \\
& x_3' \geq 0
\end{aligned}$$

  The **grading scheme** for this question is as follows:

- 1 mark was taken off for having an incorrect objective function
- 1 mark was taken off for missing a constraint
- 0.5 mark was taken off for a partially incorrect constraint
- 0.5 mark was taken off for a partially incorrect objective function