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Quiz 2 Solutions

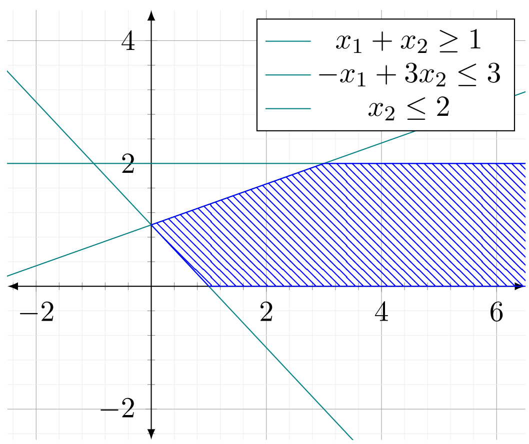
1. **Question:** 5 points

   Graphically solve the following LP problem

$$\begin{aligned} \text{Maximize} \quad & z = -x_1 + 3x_2 \\ \text{Subject to} \quad & x_1 + x_2 \geq 1 \\ & -x_1 + 3x_2 \leq 3 \\ & x_1 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

   - **Solution:**

     The feasible region for this LP problem can be seen below.



     It is clear that the feasible region is unbounded but the optimal solution exists. In fact, an infinite number of optimal solutions exist on the line segment between the two extreme points (0,1) and (3,2).

     The **grading scheme** for this question is as follows:

     – Full marks are given for correctly graphing the constraints, correctly finding the feasible region, and identifying the line segment in which all points are optimal points.

     – No marks are taken off for this, but please be careful in determining which variable to use on the x-axis and which variable to use on the y-axis.

2. **Question:** 5 points

   Prove that the following region is a convex set

$$
\begin{aligned}
\text{Minimize} \quad & z = 4x_1 - 3x_2 - 4x_3 - 12 \\
\text{Subject to} \quad & x_1 + x_2 \leq 4 \\
& x_1 + 2x_2 + 3x_3 \geq 4 \\
& 5 \geq x_1 \geq -1 \\
& x_3 \leq 0
\end{aligned}
$$

- **Solution:**

  There are multiple ways to solve this question:

  - Use the known theorems as follows:
    * The shaded areas in Figure 1.4(a) and Figure 1.4(b) are both **closed half-spaces**.
    * By Theorem 1.1, a closed half-space is a convex set.
    * By Theorem 1.3, the intersection of a finite set of convex sets is a convex set.
    * Therefore, the region in Figure 1.4(c) is a convex set.
  - Prove directly using convexity
    * Define the LP problem for the given feasible region
    * Take two arbitrary points satisfying the LP problem
    * Show that any convex combination of those two points also satisfies the LP problem

  The **grading scheme** for this question is as follows:

  - 1.5 marks are taken off for incorrect LP set-up
  - 2-3 marks are taken off for not showing a convex combination satisfies LP problem
  - 0.5 marks are taken off for minor details missing