Quiz 3 Solutions

1. **Question:** 5.5 points

   Given the LP program:

$$\begin{aligned}
\text{Maximize} \quad & z = x_1 - 2x_2 + x_3 \\
\text{Subject to} \quad & x_1 + 2x_2 + 3x_3 \leq 12 \\
& -x_1 + 3x_2 \leq 9 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

   Identify whether each of the following is a basic solution, an extreme point, or neither. Show your steps.

$$(a) \begin{bmatrix} 1 \\ 3 \\ 5/3 \end{bmatrix} \quad (b) \begin{bmatrix} -1 \\ 8/3 \\ 0 \end{bmatrix} \quad (c) \begin{bmatrix} 3 \\ 4 \\ 1/3 \end{bmatrix}$$

   - **Solution:**

     (a) Neither. There are $< n - m$ zeros.

     (b) Neither. Note that while there are $n-m$ zeros in this example, this does not imply that this is a basic solution. This is, however, a feasible solution as it satisfies the LP problem.

     (c) Neither. There are $< n - m$ zeros.

     The **grading scheme** for this question is as follows:

     – Full marks are given for proper response and justification.

2. **Question:** 4.5 points

   Which of the following statements are true?

   (a) The number of basic feasible solutions of a canonical LP is $\frac{n!}{m!(m-n)!}$, where in $Ax = b$, A is a full rank $m x n$ matrix and $m \leq n$.

   (b) An extreme point is a basic solution.

   (c) The set of solutions to an inequality constraint is a hyperplane.

   - **Solution:**

     (a) FALSE. The number of BFS can be up to $\binom{n}{m}$ but not necessarily $\binom{n}{m}$.

     (b) TRUE. An extreme point is a basic feasible solution.

     (c) FALSE. The set of solutions to an inequality constraint is a closed half-space, whereas the set of solutions to an equality constraint is a hyperplane.

     The **grading scheme** for this question is as follows:

     – 1.5 marks for correct answer (no reasoning required) per question