MTH141 Quiz 3 Solution

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Question

Find the reduced row echelon form of the matrix F below. Clearly mark every leading entry with the symbol $\boxed{a}$ and use the symbols $\uparrow$ and $\downarrow$ to show the directions of eliminating nonzero entries above or below leading entries.

$$F = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & -2 \\ 2 & 2 & 1 & 1 \end{bmatrix}$$

Solution

To find the reduced row echelon form (RREF) of matrix F, we will perform row operations to create leading entries ($\boxed{a}$) in each row and eliminate all nonzero entries below and above them.

Enumerate each row i as $R_i, i = 1, 2, 3$. Notice that there are two leading entries in R_1. First, we will perform row operations to remove non-zero entries below the leading entries of R_1.

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & -2 \\ 2 & 2 & 1 & 1 \end{bmatrix} \rightarrow \begin{matrix} \xrightarrow{R_2 - R_1} \\ \xrightarrow{R_3 - 2R_1} \end{matrix} \begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -1 \end{bmatrix}$$

Next, we will perform row operations to create leading entries in rows 2 and 3 and eliminate the nonzero entries above and below them.

$$\begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -1 \end{bmatrix} \xrightarrow{R_2 \cdot (-1)} \begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & -3 & -1 \end{bmatrix}$$

Now, we will create a leading entry in R_2, and eliminate the nonzero entries in row 3 above and below the leading entry.

$$\begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & -3 & -1 \end{bmatrix} \begin{matrix} \xrightarrow{R_1 - 2R_2} \\ \xrightarrow{R_3 + 3R_2} \end{matrix} \begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

Finally, we'll create a leading entry in R_1 and eliminate the nonzero entry below it.

$$\begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix} \xrightarrow{R_3 \cdot \frac{1}{8}} \begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

Note that the matrix is currently in row echelon form (*not* reduced row echelon form) as there non-zero entries in the upper triangular matrix of F. (It is also possible for F to be in non-reduced row echelon form if some leading entries are non-zero but not 1.

After the final row operations $R_2 - R_3 \cdot 3$ and $R1 + R_3 * 5$, the reduced row echelon form of matrix F is:

$$\begin{bmatrix} \boxed{1} & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$