MTH140 Lab 6

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October 17-19, 2023

Administrative Items

Structure of Today's Lab

- As usual, 1hr 50 minutes Collaborative Problem-Solving
- Slides to previous labs can be found in **Lab Teaching Sessions** page on D2L.

Recommended Textbook Sections

Please read the following sections from the textbook:

- Section 3.5 Derivatives of Trigonometric Functions
- Section 3.6 The Chain Rule

These sections will provide you with valuable background information and concepts required to solve this week's problem set.

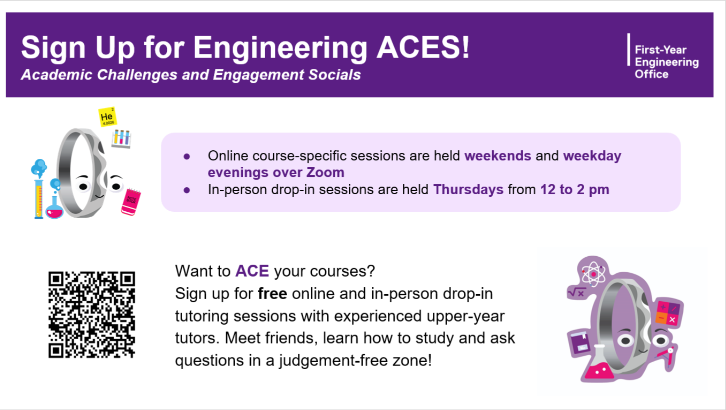
Learning Objectives

This lab will help you achieve the following learning outcomes:

- Calculate first and second derivatives (Q1, Q2).
- Derive using the quotient rule (Q3).
- Use Leibniz notation for chain rule (Q4).
- Decompose functions (Q5).

Announcements

Midterm Test Information: 120 minutes: Friday, Oct. 20, at 6:30 pm, In-person, Paper, and Pen.



In addition to ACES, the **Math and Computer Science Support centre** provides 24/7 drop-in support.

Exercise 1: First Derivative of Trigonometric Equation

Find the first derivative of $y = \frac{1-\cot(x)}{1+\cot(x)}$.

Solution:

To find the first derivative of y, we'll use the quotient rule:

$$\frac{d}{dx}\left(\frac{1-\cot(x)}{1+\cot(x)}\right)$$

$$= \frac{(1+\cot(x))\frac{d}{dx}(1-\cot(x)) - (1-\cot(x))\frac{d}{dx}(1+\cot(x))}{(1+\cot(x))^2}$$

Exercise 1: First Derivative of Trigonometric Equation

Now, we can calculate the derivatives and simplify the expression.

$$\frac{(1+\cot(x))\frac{d}{dx}(1-\cot(x)) - (1-\cot(x))\frac{d}{dx}(1+\cot(x))}{(1+\cot(x))^2}$$

$$= \frac{(1+\cot(x))\cdot csc^2(x)) - ((1-\cot(x))\cdot -csc^2(x))}{(1+\cot(x))^2}$$

$$= \frac{(csc^2(x) + csc^2(x)\cdot \cot(x)) - (-csc^2(x) + csc^2(x)\cdot \cot(x))}{(1+\cot(x))^2}$$

$$= \frac{2\cdot csc^2(x)}{(1+\cot(x))^2}$$

Exercise 2: Second Derivative of Trigonometric Equation

Find the second derivative of $y = x - \frac{1}{2}\sin(x)$.

Solution: To find the second derivative of y, we'll differentiate the function twice. First, find the first derivative:

$$\frac{d}{dx}\left(x - \frac{1}{2}\sin(x)\right) = 1 - \frac{1}{2}\cos(x)$$

Now, find the second derivative:

$$\frac{d}{dx}\left(1 - \frac{1}{2}\cos(x)\right) = \frac{1}{2}\sin(x)$$

So, the second derivative is $\frac{1}{2}\sin(x)$.

Exercise 3: Derivation Using Quotient Rule

Derive $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$ using the quotient rule. **Solution:** To derive $\frac{d}{dx}(\csc(x))$ using the quotient rule, we need to rewrite $\csc(x)$ as a fraction:

$$\csc(x) = \frac{1}{\sin(x)}$$

Now, we can use the quotient rule:

$$\frac{d}{dx} csc(x) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{-cos(x) + sin(x) \cdot 0}{sin^2(x)} = \frac{-\cos(x)}{\sin^2(x)}$$

Finally, we can simplify this to $-\csc(x)\cot(x)$ as follows:

$$\frac{-\cos(x)}{\sin^2(x)} = \frac{-\cos(x)}{\sin(x)} \cdot \frac{1}{\sin(x)} = -cot(x) \cdot csc(x)$$

as wanted.

Exercise 4: Use Leibniz Notation For Chain Rule

Given $y = f(u)$ and $u = g(x)$, find $\frac{dy}{dx}$ using Leibniz notation for the chain rule, with $y = \sqrt{4u+3}$ and $u = x^2 - 6x$. **Solution:** To find $\frac{dy}{dx}$ using Leibniz notation for the chain rule, we'll differentiate y with respect to u and then u with respect to x:

$$\frac{dy}{du} = \frac{d}{du}\left(\sqrt{4u+3}\right) = \frac{1}{2\sqrt{4u+3}} \cdot 4$$

Next, we'll find $\frac{du}{dx}$:

$$\frac{du}{dx} = \frac{d}{dx}\left(x^2 - 6x\right) = 2x - 6$$

Now, we can apply the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{4u+3}} \cdot (2x-6) \cdot 4$$