MTH140 Lab 4

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Administrative Items

Structure of Today's Lab

- Collaborative Problem Solving & Take-up: As usual, 1hr 50 minutes.
- Please participate.

This lab's problem set will help you achieve the following learning outcomes:

- Recognize the basic limit laws.
- Use the limit laws to evaluate the limit of a function.
- Evaluate the limit of a function by factoring.
- Use limit laws to evaluate the limit of a polynomial or rational function.
- Evaluate the limit of a function by factoring or by using conjugates.
- Evaluate the limit of a function by using the squeeze theorem.
- Explain the three conditions for continuity at a point.
- Describe three kinds of discontinuities.
- Define continuity on an interval.

Recommended Textbook Sections

Please read the following sections from the textbook:

- 2.1 A Preview of Calculus
- 2.2 The Limit of a Function
- 2.3 The Limit Laws

These sections will provide you with valuable background information and concepts required to solve this week's problem set.

Exercise 3: Using Limit Laws to Evaluate a Limit

Use limit laws to evaluate the following limit and justify each step by stating the appropriate limit law(s):

$$\lim_{x \to 1} \frac{x^3 + 3x^2 + 5}{4 - 7x}$$

Exercise 3: Using Limit Laws to Evaluate a Limit

Solution: We can evaluate the given limit step by step using limit laws.

$$\lim_{x \to 1} \frac{x^3 + 3x^2 + 5}{4 - 7x} = \lim_{x \to 1} \frac{x^3}{4 - 7x} + \lim_{x \to 1} \frac{3x^2}{4 - 7x} + \lim_{x \to 1} \frac{5}{4 - 7x}$$

Now, let's apply the limit laws:

- The limit of a sum is the sum of the limits.
- The limit of a constant times a function is the constant times the limit of the function.
- The limit of a constant is the constant itself.

Exercise 3: Using Limit Laws to Evaluate a Limit

Solution (Continued): Continuing from the previous slide:

$$= \frac{\lim_{x\to 1} x^3}{\lim_{x\to 1}(4-7x)} + \frac{\lim_{x\to 1} 3x^2}{\lim_{x\to 1}(4-7x)} + \frac{\lim_{x\to 1} 5}{\lim_{x\to 1}(4-7x)}$$

Now, evaluate the limits individually:

$$\lim_{x\to 1} \frac{x^3}{4-7x} = \frac{1^3}{4-7\cdot 1} = \frac{1}{-3} = -\frac{1}{3}$$
$$\lim_{x\to 1} \frac{3x^2}{4-7x} = \frac{3\cdot 1^2}{4-7\cdot 1} = \frac{3}{-3} = -1$$
$$\lim_{x\to 1} \frac{5}{4-7x} = \frac{5}{4-7\cdot 1} = \frac{5}{-3} = -\frac{5}{3}$$

Therefore, the overall limit is:

$$-\frac{1}{3} - 1 - \frac{5}{3} = -\frac{9}{3} = -3$$

Exercise 4: Using Direct Substitution to Evaluate a Limit

Use direct substitution to evaluate the following limit:

$$\lim_{x \to 3} \ln(e^{3x})$$

Exercise 4: Using Direct Substitution to Evaluate a Limit

Solution:
To evaluate the limit $\lim_{x \to 3} \ln(e^{3x})$, we can use direct substitution.
So, substitute $x = 3$:

$$\lim_{x \to 3} \ln(e^{3x}) = \ln(e^{3 \cdot 3}) = \ln(e^9)$$

Since $\ln(e^x) = x$ for all x, we have:

$$\ln(e^9) = 9$$

Therefore, the limit is:

$$\lim_{x \to 3} \ln(e^{3x}) = 9$$

Exercise 5: Evaluating the Limit by Rationalizing

Determine whether the following limit leads to the indeterminate form $\frac{0}{0}$, and if so, evaluate the limit.

$$\lim_{x \to -3} \frac{\sqrt{x+4} - 1}{x+3}$$

Exercise 5: Evaluating the Limit by Rationalizing

Solution:
We have confirmed that the limit is of the indeterminate form $\frac{0}{0}$. To evaluate it without using L'Hôpital's Rule, we can rationalize the expression.

$$\lim_{x \to -3} \frac{\sqrt{x+4} - 1}{x+3}$$

Multiply both the numerator and denominator by the conjugate of the numerator:

$$\lim_{x \to -3} \frac{(\sqrt{x+4} - 1)(\sqrt{x+4} + 1)}{(x+3)(\sqrt{x+4} + 1)}$$

Exercise 5: Evaluating the Limit by Rationalizing

Solution (contd):

Continuing from the previous slide:

Simplify the expression:

$$\lim_{x \to -3} \frac{(\sqrt{x+4})^2 - 1^2}{(x+3)(\sqrt{x+4}+1)}$$

$$\lim_{x \to -3} \frac{x+4-1}{(x+3)(\sqrt{x+4}+1)}$$

$$\lim_{x \to -3} \frac{x+3}{(x+3)(\sqrt{x+4}+1)}$$

Exercise 5: Evaluating the Limit by Rationalizing

Solution (contd):
Continuing from the previous slide:
Now, we can simplify further, canceling out the common factor $(x+3)$ from the numerator and denominator:

$$\lim_{x \to -3} \frac{x+3}{(x+3)(\sqrt{x+4}+1)}$$

$$\lim_{x \to -3} \frac{1}{\sqrt{x+4}+1}$$

Substitute $x = -3$:

$$\frac{1}{\sqrt{-3+4}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2}$$

Therefore, the limit is:

$$\lim_{x \to -3} \frac{\sqrt{x+4}-1}{x+3} = \frac{1}{2}$$

Exercise 6: Evaluate Indeterminate Form Using Limit Laws

Using Direct Substitution and Simplifying

Determine an undefined expression by using direct substitution. Then, simplify the function by factoring out the part of the denominator that becomes zero when substituting the limit.

$$\lim_{x \to 1^+} \frac{2x^2 + 7x - 4}{x^2 + x - 2}$$

Exercise 6: Evaluate Indeterminate Form Using Limit Laws

Solution:
Let's start by using direct substitution to evaluate the limit:

$$\lim_{x \to 1^+} \frac{2x^2 + 7x - 4}{x^2 + x - 2}$$

Substitute $x = 1^+$ (from the positive side):

$$\frac{2(1)^2 + 7(1) - 4}{(1)^2 + (1) - 2} = \frac{2 + 7 - 4}{1 + 1 - 2} = \frac{5}{0}$$

So, the initial expression is undefined.

Exercise 6: Evaluate Indeterminate Form Using Limit Laws

Solution (contd):
To simplify the expression further, let's factor out the part of the denominator that becomes zero when substituting the limit.
The denominator $x^2 + x - 2$ becomes zero at $x = 1$. Therefore, we can factor it as follows:

$$x^2 + x - 2 = (x - 1)(x + 2)$$

Now, we can rewrite the original expression as:

$$\frac{2x^2 + 7x - 4}{(x - 1)(x + 2)}$$

Exercise 6: Evaluate Indeterminate Form Using Limit Laws

Solution (contd):
Now that we have factored the denominator, we can rewrite the limit expression as:

$$\lim_{x \to 1^+} \frac{1}{(x-1)} \cdot \frac{2x^2 + 7x - 4}{(x+2)}$$

Substitute $x = 1^+$:

$$\frac{2(1)^2 + 7(1) - 4}{(1-1)(1+2)} = \frac{1}{0} \cdot \frac{2+7-4}{3} = +\infty \cdot \frac{5}{3} = +\infty$$

Therefore, the limit is $+\infty$.