MTH141 Quiz 5 Solution

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Question

Find the determinant of the following matrix $|B|$:

$$B = \begin{vmatrix} 2 & 6 & 10 & 4 \\ 0 & -1 & 3 & 4 \\ 2 & 1 & 9 & 6 \\ 3 & 2 & 4 & 8 \end{vmatrix}$$

Solution

To find the determinant of the matrix $|B|$ using row reduction, we'll perform row operations to bring the matrix to upper triangular form and then calculate the determinant.

Starting with matrix B:

$$R_1 \cdot \frac{1}{2}$$

$$B = \begin{vmatrix} \boxed{1} & 3 & 5 & 2 \\ 0 & \boxed{-1} & 3 & 4 \\ 2 & 1 & 9 & 6 \\ 3 & 2 & 4 & 8 \end{vmatrix}$$

Let's perform row operations to get zeros below the main diagonal:

$$R_3 - 2 \cdot R_1$$
$$R_4 - 3 \cdot R_1$$

This gives us:

$$\begin{vmatrix} \boxed{1} & 3 & 5 & 2 \\ 0 & \boxed{-1} & 3 & 4 \\ 0 & -5 & -1 & 2 \\ 0 & -7 & -11 & 2 \end{vmatrix}$$

Next, perform row operations to get zeros in the second column below the main diagonal:

$$R_3 - 5 \cdot R_2 \quad R_4 - 7 \cdot R_2$$

This results in:

$$\begin{vmatrix} \boxed{1} & 3 & 5 & 2 \\ 0 & \boxed{-1} & 3 & 4 \\ 0 & 0 & -16 & -18 \\ 0 & 0 & -32 & -26 \end{vmatrix}$$

One more operation:

$$R_4 - 2 \cdot R_3$$

This gives

$$
\begin{vmatrix}
\boxed{1} & 3 & 5 & 2 \\
0 & \boxed{-1} & 3 & 4 \\
0 & 0 & \boxed{-16} & -18 \\
0 & 0 & 0 & 10
\end{vmatrix}
$$

Now, we can calculate the determinant as the product of the diagonal elements:

$$|B| = 2 \cdot 1 \cdot -1 \cdot -16 \cdot 10 = 320$$

So, the determinant of matrix B is 320.