# MTH141 Quiz 7 Solution

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## Question

Solve the following system of linear equations using the inverse matrix method. Write solution in terms of $x_1$, $x_2$, and $x_3$:

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 1 \\ 2x_1 + 3x_2 + 3x_3 &= -1 \\ x_1 + 6x_3 &= -4 \end{aligned}$$

### Solution

To solve the system of equations using the inverse matrix method, we first represent the system in matrix form and then find the inverse of the coefficient matrix. We can denote the coefficient matrix as $A$ and the right-hand side matrix as $b$:

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 3 & 3 \\ 1 & 0 & 6 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

The augmented matrix set-up is as follows: (2 marks for setting up augmented matrix correctly, 3 marks for correct row operations)

Next, we find the inverse of matrix $A$, denoted as $A^{-1}$.

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 1 & 0 & 0 \\ 2 & 3 & 3 & 0 & 1 & 0 \\ 1 & 0 & 6 & 0 & 0 & 1 \end{array}\right]$$

Reducing the LHS matrix to RREF gives the matrix $A^{-1}$ on the RHS.

$$A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} & -\frac{2}{5} \\ -\frac{3}{10} & \frac{2}{15} & \frac{1}{30} \\ -\frac{1}{10} & -\frac{1}{15} & \frac{7}{30} \end{bmatrix}$$

Once we have the inverse, we can solve for the variables using the equation $X = A^{-1} \cdot B$. The inverse matrix $A^{-1}$ is found to be: (3 marks for attempting to apply correct inverse matrix method)

$$X = A^{-1} \cdot B = \begin{bmatrix} \frac{3}{5} & \frac{2}{5} & -\frac{2}{5} \\ -\frac{3}{10} & \frac{2}{15} & \frac{1}{30} \\ -\frac{1}{10} & -\frac{1}{15} & \frac{7}{30} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}$$

We can now solve for the variables $x_1$, $x_2$, and $x_3$. Calculating the product, we obtain: (1 mark for correct matrix vector multiplication method)

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{9}{5} \\ -\frac{17}{30} \\ -\frac{29}{30} \end{bmatrix}$$

Therefore, the solution to the system of equations is: (1 mark for correct final answer)

$$x_1 = \frac{9}{5}$$
$$x_2 = -\frac{17}{30}$$
$$x_3 = -\frac{29}{30}$$

These values satisfy all three equations in the system.