

MTH141 Quiz 4 Solution

TA & Solution Author: Mariam Walaa

October 10, 2023

Question

Find the reduced row echelon form of the matrix F below. Clearly mark every leading entry with the symbol \boxed{a} and use the symbols \uparrow and \downarrow to show the directions of eliminating nonzero entries above or below leading entries.

$$F = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & -2 \\ 2 & 2 & 1 & 1 \end{bmatrix}$$

Solution

To find the reduced row echelon form (RREF) of matrix F , we will perform row operations to create leading entries (\boxed{a}) in each row (-2 marks deducted for not indicating leading ones). and eliminate all nonzero entries below and above them. (-2 marks deducted for having non-zero entries below a leading entry).

Enumerate each row i as $R_i, i = 1, 2, 3$. Notice that there are two leading entries in R_1 . First, we will perform row operations to remove non-zero entries below the leading entries of R_1 .

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & -2 \\ 2 & 2 & 1 & 1 \end{bmatrix} \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -1 \end{bmatrix}$$

Next, we will perform row operations to create leading entries in rows 2 and 3 and eliminate the nonzero entries above and below them.

$$\begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -1 \end{bmatrix} \xrightarrow{R_2 \cdot (-1)} \begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & -3 & -1 \end{bmatrix}$$

Now, we will create a leading entry in R_2 , and eliminate the nonzero entries in row 3 above and below the leading entry.

$$\begin{bmatrix} \boxed{1} & 1 & 2 & 1 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & -3 & -1 \end{bmatrix} \xrightarrow[R_3 + 3R_2]{R_1 - 2R_2} \begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

Finally, we'll create a leading entry in R_1 and eliminate the nonzero entry below it.

$$\begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix} \xrightarrow{R_3 \cdot \frac{1}{8}} \begin{bmatrix} \boxed{1} & 1 & 0 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

Note that the matrix is currently in row echelon form (not reduced row echelon form) as there non-zero entries in the upper triangular matrix of F . (-2 marks deducted for having non-zero entries in the upper triangular matrix of F).

It is also possible for F to be in non-reduced row echelon form if some leading entries are non-zero but not 1. (-2 marks deducted for having non-one leading entries in the diagonal of F).

After the final row operations $R_2 - R_3 \cdot 3$ and $R_1 + R_3 \cdot 5$, the reduced row echelon form of matrix F is:

$$\begin{bmatrix} \boxed{1} & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$

(-2 marks deducted for having incorrectly placed leading ones).