MTH141 Quiz 1 Solution

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Question

Let $\mathbf{u} = (-1, a, -2)$ and $\mathbf{v} = (2, 9, b)$. Find values of a and b such that vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel.

Solution

Two vectors are parallel if they are scalar multiples of each other. In other words, vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel if there exist constants k_1 and k_2 such that:

$$\mathbf{u} = k_1 \mathbf{v}$$

In this case, $\mathbf{u} = (-1, a, -2)$ and $\mathbf{v} = (2, 9, b)$.
So, we can write:

$$(-1, a, -2) = k_1(2, 9, b)$$

This means that each component of $\mathbf{u}$ is a multiple of the corresponding component of $\mathbf{v}$. So, we can write three equations:

$$-1 = 2k_1$$
$$a = 9k_1$$
$$-2 = bk_1$$

We must solve these equations.
From the first equation, we have:

$$-1 = 2k_1$$

Solving for k_1:

$$k_1 = -\frac{1}{2}$$

Now, from the second equation, we have:

$$a = 9k_1$$

Substituting the value of k_1 we found:

$$a = 9\left(-\frac{1}{2}\right) = -\frac{9}{2}$$

Finally, from the third equation, we have:

$$-2 = bk_1$$

Substituting the value of k_1 we found:

$$-2 = b\left(-\frac{1}{2}\right)$$

Solving for b:

$$b = -2 \cdot \left(-\frac{1}{2}\right) = 4$$

So, to make vectors $\mathbf{u}$ and $\mathbf{v}$ parallel, you should have:

$$a = -\frac{9}{2} \quad \text{and} \quad b = 4$$