MTH141 Quiz 3 Solution

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Question

Let $\mathbf{a}_1 = (1, 2)$, $\mathbf{a}_2 = (2, -1)$, and $\mathbf{b} = (4, 3)$. Show that $\mathbf{b}$ is in the span of $\mathbf{a}_1$ and $\mathbf{a}_2$.

Solution

To show that $\mathbf{b}$ is in the span of $\mathbf{a}_1$ and $\mathbf{a}_2$, we need to find scalars c_1 and c_2 such that

$$c_1\mathbf{a}_1 + c_2\mathbf{a}_2 = \mathbf{b}.$$

Let's set up a system of equations using the components of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{b}$:

$$\begin{aligned} c_1(1, 2) + c_2(2, -1) &= (4, 3) \\ (c_1 + 2c_2, 2c_1 - c_2) &= (4, 3). \end{aligned}$$

Now, we can write this system as two equations:

$$\begin{aligned} c_1 + 2c_2 &= 4 \quad \text{(Equation 1)} \\ 2c_1 - c_2 &= 3 \quad \text{(Equation 2)} \end{aligned}$$

We can solve this system of equations using any method of your choice. Here, we will use the method of substitution.

From Equation 2, we can express c_2 in terms of c_1:

$$c_2 = 2c_1 - 3.$$

Now, substitute this expression for c_2 into Equation 1:

$$\begin{aligned} c_1 + 2(2c_1 - 3) &= 4 \\ c_1 + 4c_1 - 6 &= 4 \\ 5c_1 &= 10 \\ c_1 &= 2. \end{aligned}$$

Now that we have found c_1, we can find c_2:

$$c_2 = 2c_1 - 3 = 2(2) - 3 = 4 - 3 = 1.$$

So, we have found that $c_1 = 2$ and $c_2 = 1$. Therefore, $\mathbf{b}$ can be expressed as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$:

$$2\mathbf{a}_1 + \mathbf{a}_2 = 2(1, 2) + (2, -1) = (2, 4) + (2, -1) = (4, 3) = \mathbf{b}.$$

Since we have found values of c_1 and c_2 such that $\mathbf{b}$ is expressed as a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$, we have shown that $\mathbf{b}$ is in the span of $\mathbf{a}_1$ and $\mathbf{a}_2$.