Partial Wave Analysis studies of $\eta'\pi^0$ system in GlueX

Florida International University 2022

Mariana Khachatryan

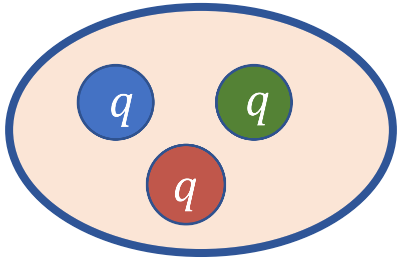
Outline

1. Motivation
2. Implementation and testing of Partial Wave Analysis (PWA) methods.
3. Search for π_1 (1600) signal in $\eta'\pi^0$ system
4. Summary

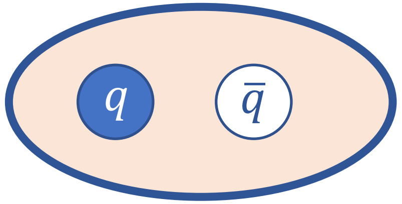
Motivation: Hadrons in Quantum chromodynamics (QCD)

Traditional hadrons:

Baryons

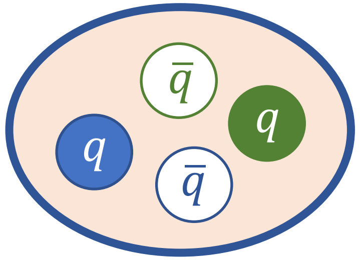


Mesons

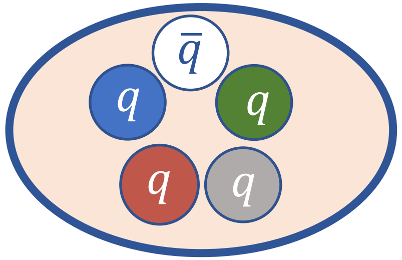


QCD allows “colorless” hadronic states with different quark configurations:

Tetraquarks

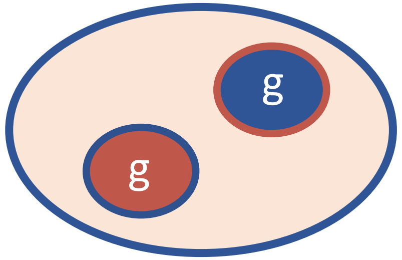


Pentaquarks

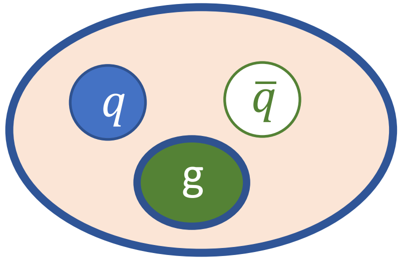


Lattice QCD (LQCD) also predicts a spectrum of bound states beyond the constituent quark model:

Glueball



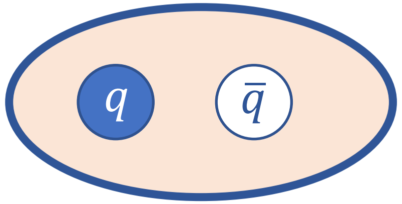
Hybrid mesons



GlueX studies production and structure of hadrons via photoproduction

Motivation: Hadrons in QCD

Mesons in standard quark model

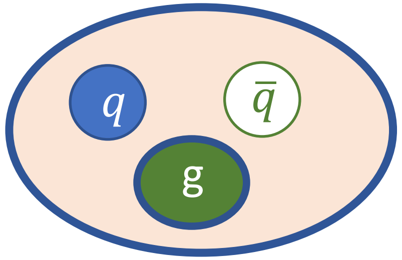


Classified as J^{PC} multiplets:

$\vec{J} = \vec{L} + \vec{S}$,
$P = (-1)^{L+1} \rightarrow$ Spherical harmonics $(-1)^{l}$
$\times$ Product of individual parites of $q, \bar{q}$ (-1)
$C = (-1)^{L+S} \rightarrow$ Orbital angular momentum $(-1)^{l}$
$\times$ Flip of spin wavefunctions $(-1)^{S+1}$
$\times$ interchanging q and $\bar{q}$ (-1)

J- total angular momentum
S- total quark spin
L- orbital angular momentum between $q\bar{q}$ pair
P- parity
C- charge conjugation
$J^{PC}=\mathbf{0}^{--}$, $\mathbf{odd}^{-+}$ and $\mathbf{even}^{+-}$ **"exotic"** quantum numbers are not available.

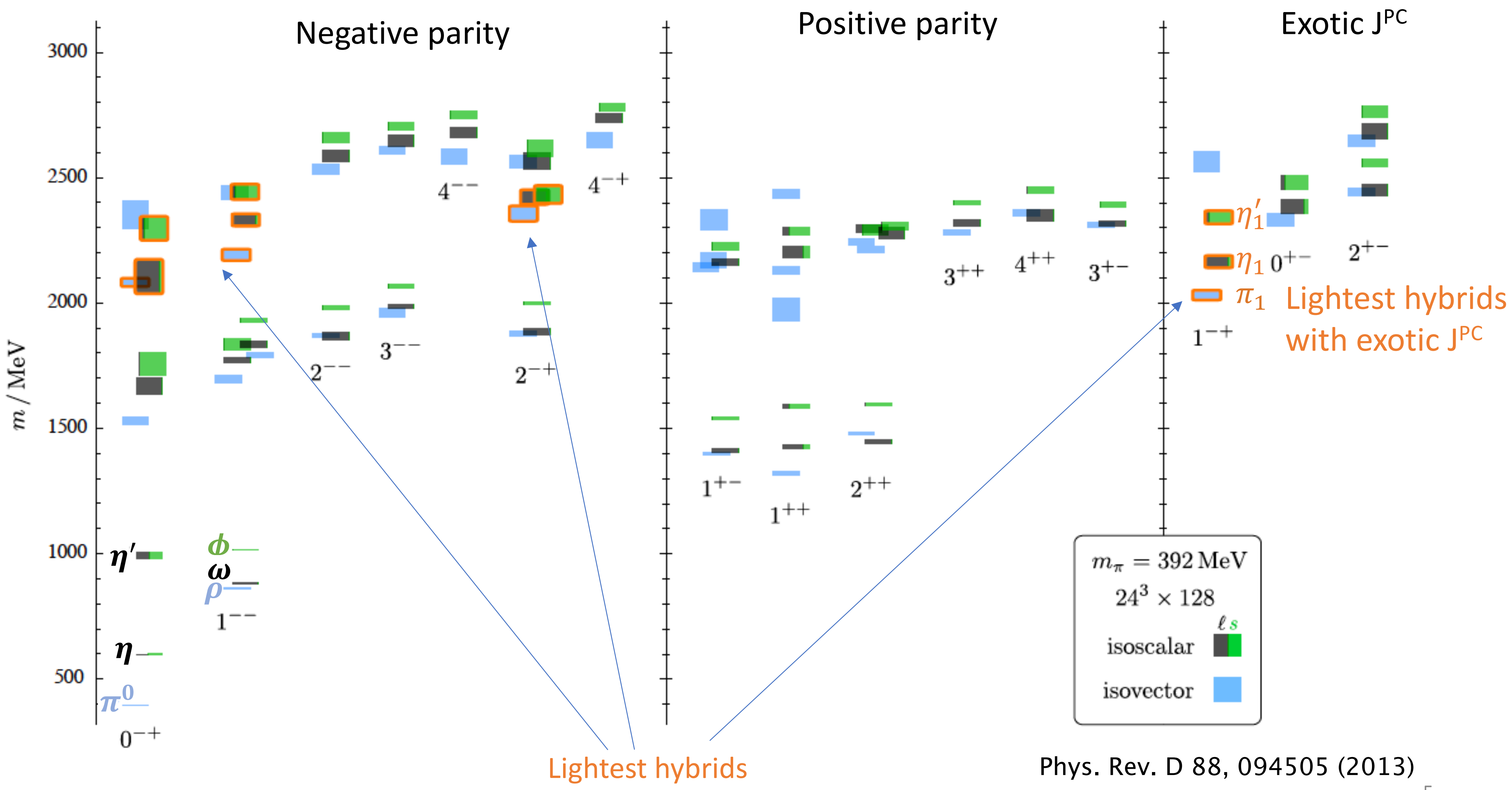
Hybrid mesons



Quark anti-quark pair coupled to valence gluon.
"Exotic" J^{PC} are also available.

Primary motivation of the GlueX is the search for light hybrid mesons.

Motivation: Isoscalar and isovector hybrid spectrum from Lattice QCD



Phys. Rev. D 88, 094505 (2013)

π_1 (1600) results from studies of $\eta^{(\prime)}\pi$ system with π beam incident on a p target

Evidence for exotic $I^G J^{PC} = 1^- 1^{-+}$ state π_1 (1600)

G = C · $(-1)^I$, C operator followed by a rotation in isospin (I)

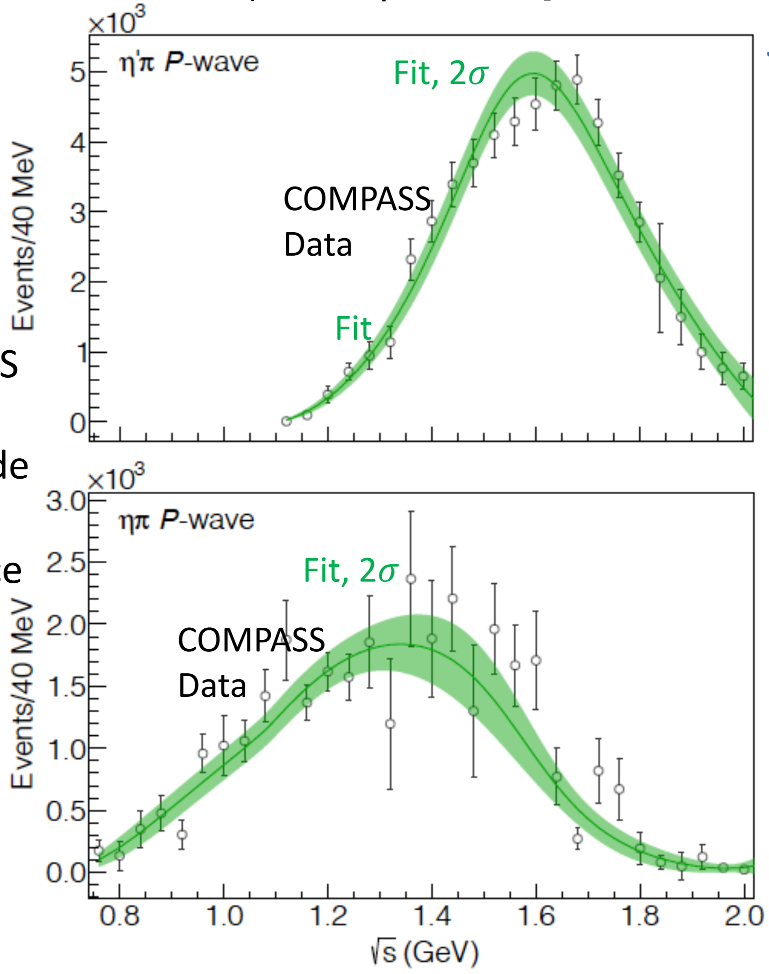
Several experiments suggest existence of π_1:

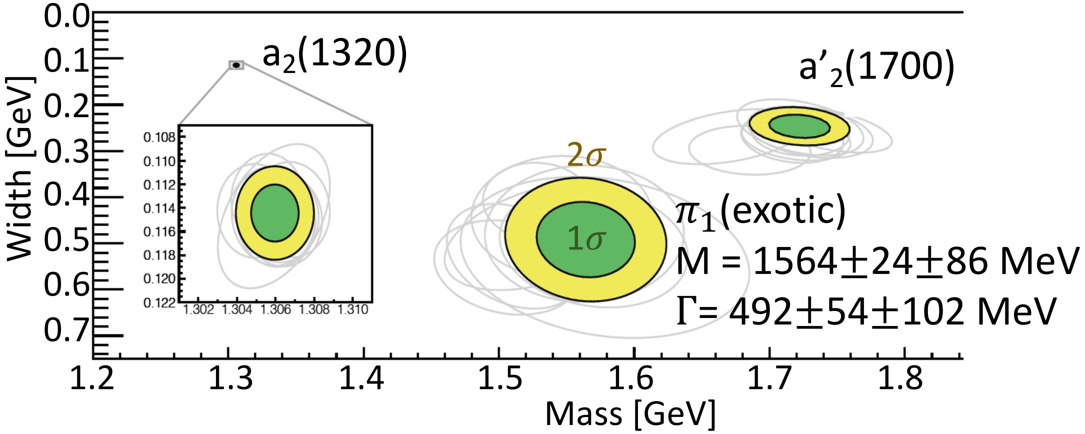
- **VES,** $E_\pi = 37$ GeV/c (D. V. Amelin et al., Phys. Atom. Nucl. 68, 359 (2005))
- **E852,** $E_\pi = 18$ GeV/c (E. I. Ivanov et al. [E852 Collaboration], Phys. Rev. Lett. 86, 3977 (2001))
- **COMPASS,** $E_\pi = 191$ GeV/c (C. Adolph, et al. [COMPASS Collaboration], Phys. Lett. B740, 303 (2015))

Joint Physics Analysis Center (JPAC)

Re-analyze COMPASS data

- Fit with amplitude model
- Extract resonance poles





- Single pole describing the 1^{-+} intensities

A. Rodas et al. [Joint Physics Analysis Center], PRL 122, 042002 (2019)

What can GlueX provide?

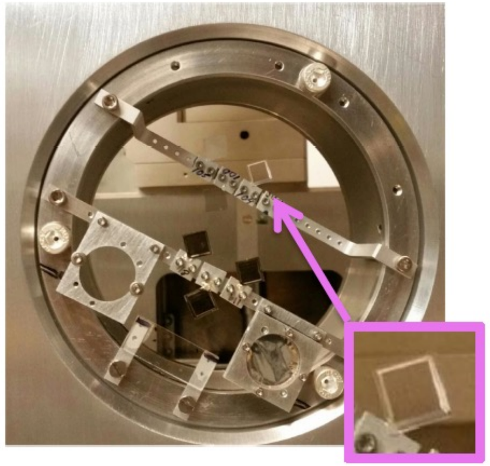
- No photo-production data at GlueX energies allowing PWA
- γ coupling via vector meson dominance (VMD) to wide variety of states (including exotic J^{PC})
- γ beam polarization provides constraints for PWA

The GlueX experiment is in the Hall-D complex at Jlab

Hermetic detector:

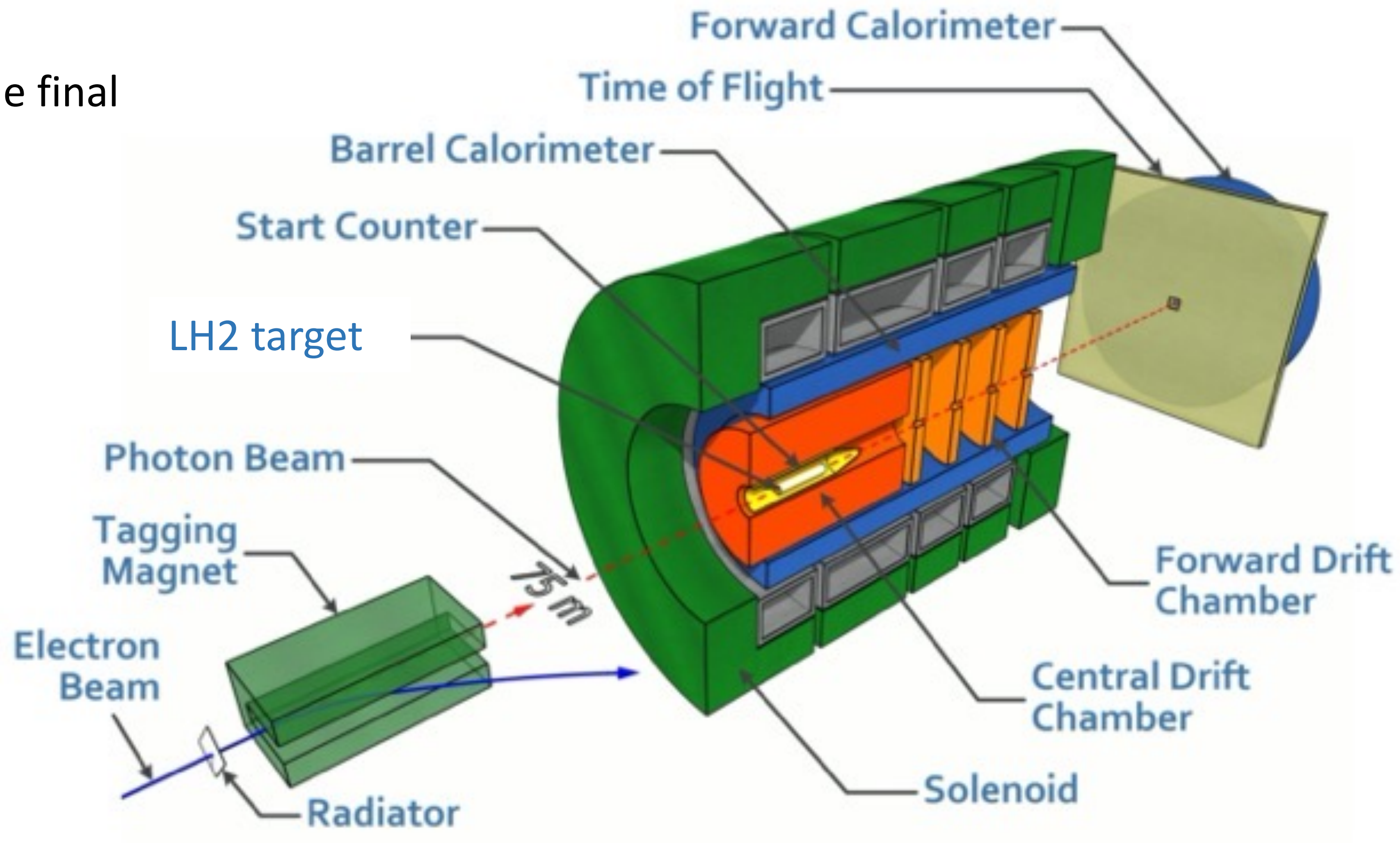
- Optimized for exclusive studies of multiparticle final states

Linearly-polarized photons via coherent bremsstrahlung on 20-60 μm thick diamond crystal radiator using 12 GeV e^- beam.



Radiators on goniometer

The GlueX spectrometer



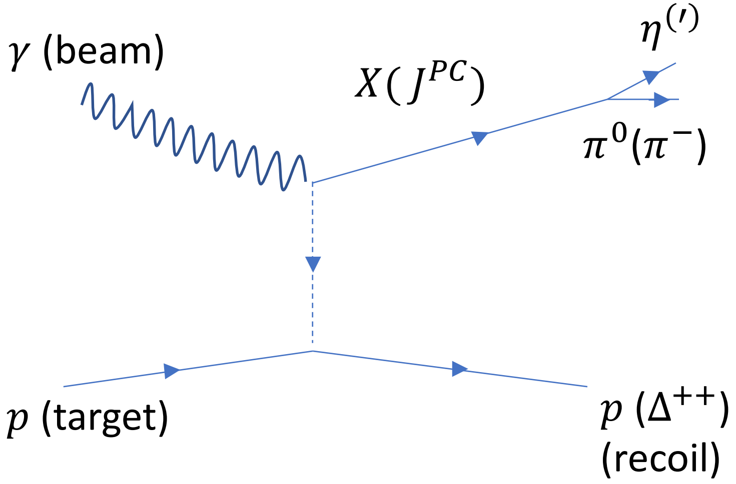
Currently working on

Search for exotic mesons via PWA of $\eta^{(\prime)}\pi$ system

- Focus of GlueX exotic search
- The odd waves (P, F, …) in $\eta^{(\prime)}\pi$ have exotic quantum numbers and the lowest (P) of them corresponds to exotic $\pi_1(1600)$
- Coupling of π_1 to $\eta'\pi$ expected to be larger than to $\eta\pi$
- Different aspects of various decay modes under investigation in parallel:

  - $\gamma p \to p\boldsymbol{\eta\pi^0}$
  - $\gamma p \to \Delta^{++}\boldsymbol{\eta\pi^-}$
  - $\gamma p \to p\boldsymbol{\eta'\pi^0}$
  - $\gamma p \to \Delta^{++}\boldsymbol{\eta'\pi^-}$

$t-$ channel exchange



- Access to different production mechanisms
- Cross-check acceptance, systematics (includes $\eta \to \gamma\gamma$ and $\eta \to 3\pi$)
- Close collaboration with JPAC
- Learn about J^{PC} and production mechanism via PWA

  - New model of Intensity for $\eta^{(\prime)}\pi$ photoproduction at GlueX with polarized beam

Model predicted number of events per unit phase space

Partial wave amplitudes (production of the wave)

Decay into two pseudo-scalars (parity constraints, L cons.)

$$I(\Omega,\Phi) = 2\kappa \sum_k \left\{ (1-P_\gamma)\left|\sum_{l,m} [l]^{(-)}_{m;k} \mathrm{Re}[Z_l^m(\Omega,\Phi)]\right|^2 + (1-P_\gamma)\left|\sum_{l,m} [l]^{(+)}_{m;k} \mathrm{Im}[Z_l^m(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{l,m} [l]^{(+)}_{m;k} \mathrm{Re}[Z_l^m(\Omega,\Phi)]\right|^2 + (1+P_\gamma)\left|\sum_{l,m} [l]^{(-)}_{m;k} \mathrm{Im}[Z_l^m(\Omega,\Phi)]\right|^2 \right\}$$

P_γ - degree of polarization

$Z_l^m(\Omega,\Phi) \equiv Y_l^m(\Omega)e^{-i\Phi}$

$\Omega = (\theta,\varphi)$

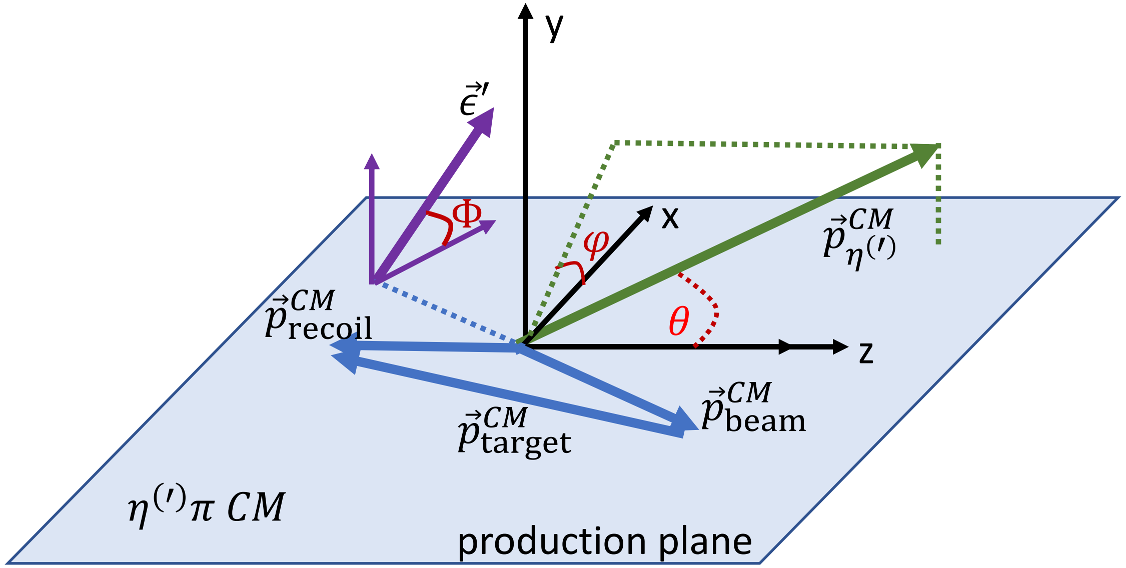
l, m- spin, its projection

$\vec{\epsilon}'$ - γ polarization vector

κ- kinematical factors

Nucleon spin flip k=1, non-flip k=0

Helicity frame (HX)



V. Mathieu et al. Phys. Rev. D 100, 054017 (2019)

- For each set of $m_{\eta^{(\prime)}\pi}$, t and E_γ determine $[l]^{(-)}_{m;k}$, $[l]^{(+)}_{m;k}$ by fitting I_{EXP}
- $[l]^{(-)}_{m;k}$, $[l]^{(+)}_{m;k}$ are fit parameters
- Used extended unbinned (in (θ,φ)) maximum likelihood method (AmpTools package https://github.com/mashephe/AmpTools by Matthew Shepherd)

$$\ln L(l) = \sum_{i=1}^{N} \ln I(l,\theta,\varphi) - \int I(l,\theta,\varphi)\eta(\theta,\varphi)\, d\Omega$$

$\eta(\theta,\varphi)$ -acceptance

- Minimize –lnL to find $[l]^{(-)}_{m;k}$, $[l]^{(+)}_{m;k}$

Determination of moments of angular distribution with simulated $\eta\pi^0$ events

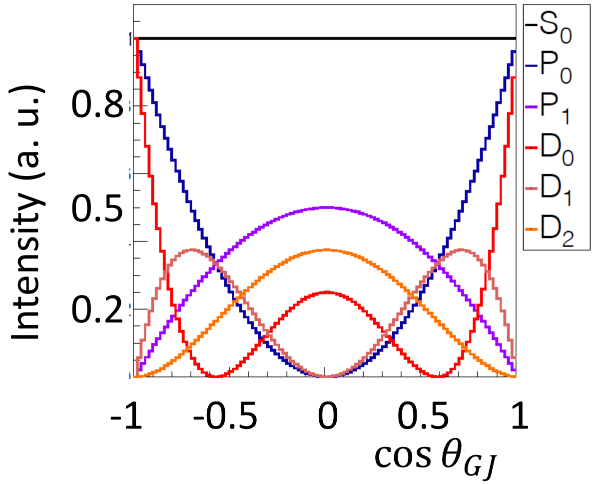
Generated $2*10^6$ $(p\eta\pi^0)$events with AmpTools

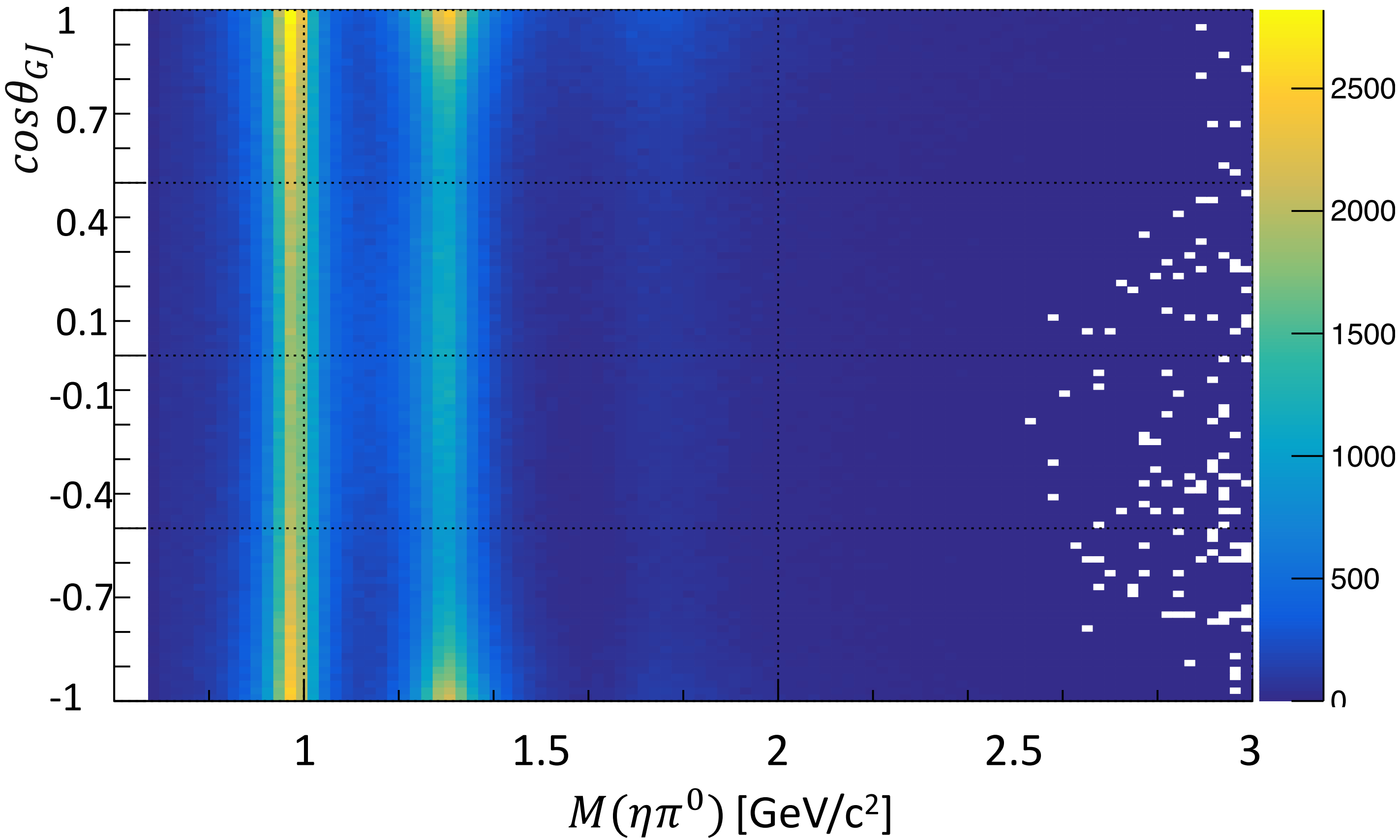
Generated resonances are
- a_0(980 MeV)
- π_1(1600 MeV) (exotic)
- a2 (1320 MeV)
- a2' (1700 MeV)

θ_{pol}=1.77 Deg.
P_γ =0.3

J	M	ε	Real	Imaginary	BW Mass	BW Width
0	0	+1	1000	0	0.980	0.075
1	0, 1	+1	70	70	1.564	0.492
2	0,1,2	+1	150	150	1.306	0.114
2	0,1,2	+1	50	50	1.722	0.247

The wave set: $[l]_{m;k}^{(\epsilon)} = \left\{S_0^{(+)}, P_{0,1}^{(+)}, D_{0,1,2}^{(+)}\right\}_{k=0}$ with M>= 0





PWA procedure

1. Fit intensity to extract partial waves.
    - Challenge: Fewer constraints -> thorough investigation of waveset, ambiguities, leakage, ...

2. Calculate moments of angular distribution using fitted partial waves:
    - Disadvantage:
        - No access to resonance parameters
    - Advantage:
        - Presence of exotic wave → non-zero odd L moments.
        - Interference of exotic wave with even waves in odd L moments amplifies small exotic wave intensity
        - No ambiguities

$$I(\Omega,\Phi) = I^0(\Omega) - P_\gamma I^1(\Omega)\cos 2\Phi - P_\gamma I^2(\Omega)\sin 2\Phi$$

$$I^0(\Omega) = \sum_{LM}\left(\frac{2L+1}{4\pi}\right) H^0(LM) D^{L*}_{M0}(\varphi,\theta,0)$$

$$\boldsymbol{I}(\Omega) = -\sum_{LM}\left(\frac{2L+1}{4\pi}\right) \boldsymbol{H}(LM) D^{L*}_{M0}(\varphi,\theta,0)$$

Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Analysis steps

For the wave set

$$[l]^{(\epsilon)}_{m;k} = \left\{S^{(+)}_0, P^{(+)}_{0,1}, D^{(+)}_{0,1,2}\right\}_{k=0} \text{ with M>= 0}$$

1. M bin 0.051 GeV/c^2, t bin 0.3(GeV/c)2
2. Moment expressions in terms of the $\eta'\pi^0$ SDMEs ${}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$ calculated in reflectivity basis:

$$H^0(LM) = \sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C^{l0}_{l'0L0} C^{lm}_{l'm'LM}\, \rho^{0,ll'}_{mm'}$$

$$H^i(LM) = -\sum_{\substack{ll' \\ mm'}} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C^{l0}_{l'0L0} C^{lm}_{l'm'LM}\, \rho^{i,ll'}_{mm'}$$

i=1, 2, 3

$$\rho^{\alpha,ll'}_{mm'} = \sum_{\epsilon} {}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$$

$${}^{(\epsilon)}\rho^{0,ll'}_{mm'} = \kappa \sum_k \left(|l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{m';k} + (-1)^{m-m'} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{-m';k}\right)$$

$${}^{(\epsilon)}\rho^{1,ll'}_{mm'} = -\epsilon\kappa \sum_k \left((-1)^{m} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} + (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k}\right)$$

$${}^{(\epsilon)}\rho^{2,ll'}_{mm'} = -i\epsilon\kappa \sum_k \left((-1)^{m} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m'} |l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{-m';k}\right)$$

$${}^{(\epsilon)}\rho^{3,ll'}_{mm'} = \kappa \sum_k \left(|l|^{(\epsilon)}_{m;k} |l'|^{(\epsilon)*}_{m';k} - (-1)^{m-m'} |l|^{(\epsilon)}_{-m;k} |l'|^{(\epsilon)*}_{-m';k}\right)$$

where $C^{l0}_{l'0L0}$ and $C^{lm}_{l'm'LM}$ denote the Clebsch-Gordan coeficients, 0$\leq L \leq 4$ (L_{max}=2* l_{max}) and 0$\leq M \leq L$, parity invariance: *H(L,M)=H(L,-M)*

Non-zero odd L moments ↔ presence of exotic wave. If M=0, H^2 and H^3 are 0.

Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Analysis steps

3. Compare moments from previous step to moment distributions obtained by Monte Carlo integration based on the expressions

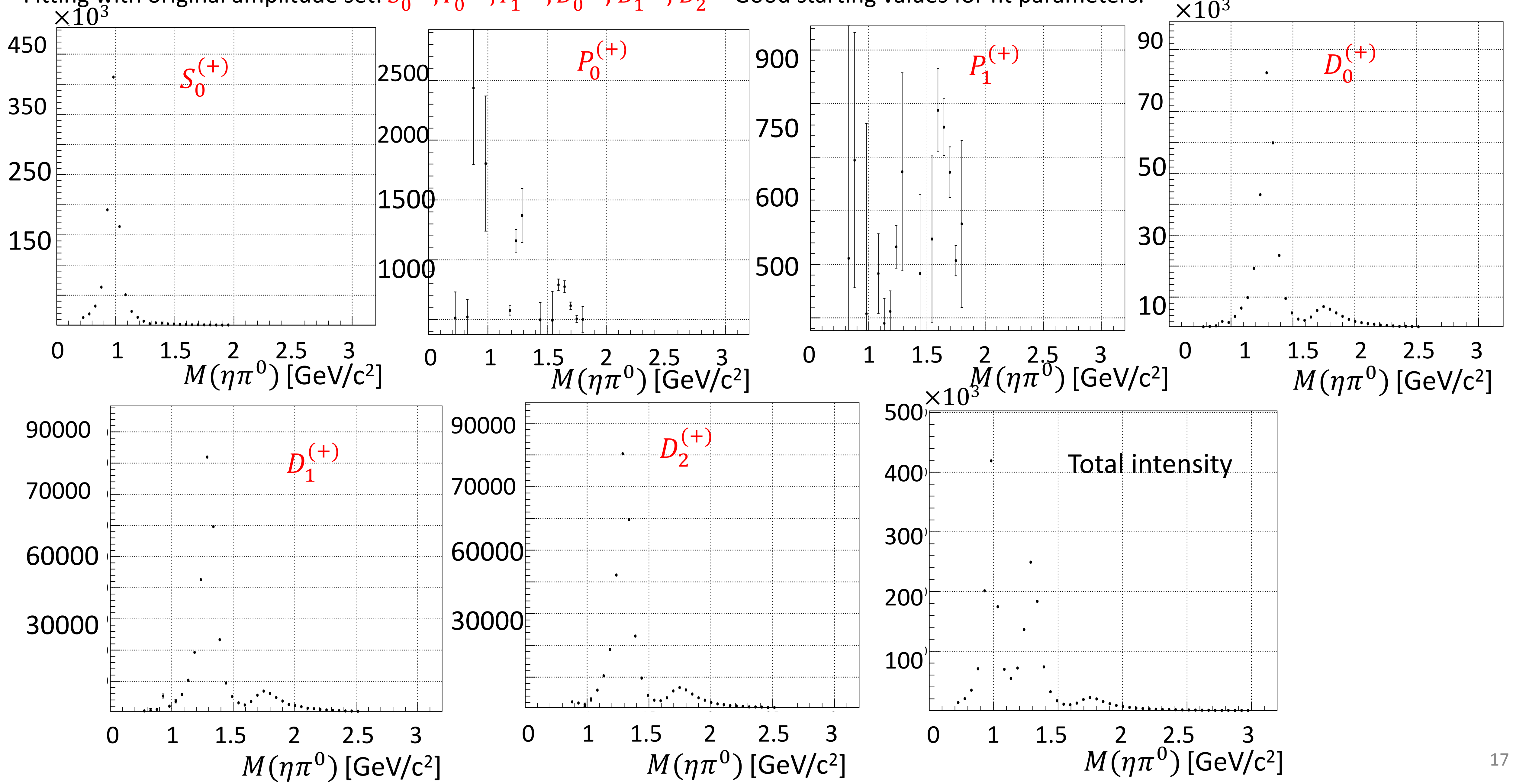
$$H^0(LM)=\frac{P_\gamma}{2}\int_o I(\Omega,\Phi)d^L_{M0}(\theta)cosM\phi,$$
$$H^1(LM)=\int_o I(\Omega,\Phi)d^L_{M0}(\theta)cosM\phi\, cos2\Phi,$$
$$\mathrm{Im}H^2(LM)=-\int_o I(\Omega,\Phi)d^L_{M0}(\theta)sinM\phi\, sin2\Phi,$$

with $\int_o = (1/\pi P_\gamma)\int_0^\pi sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi$ and $d^L_{M0}(\theta)$ denotes Wigner d-function.

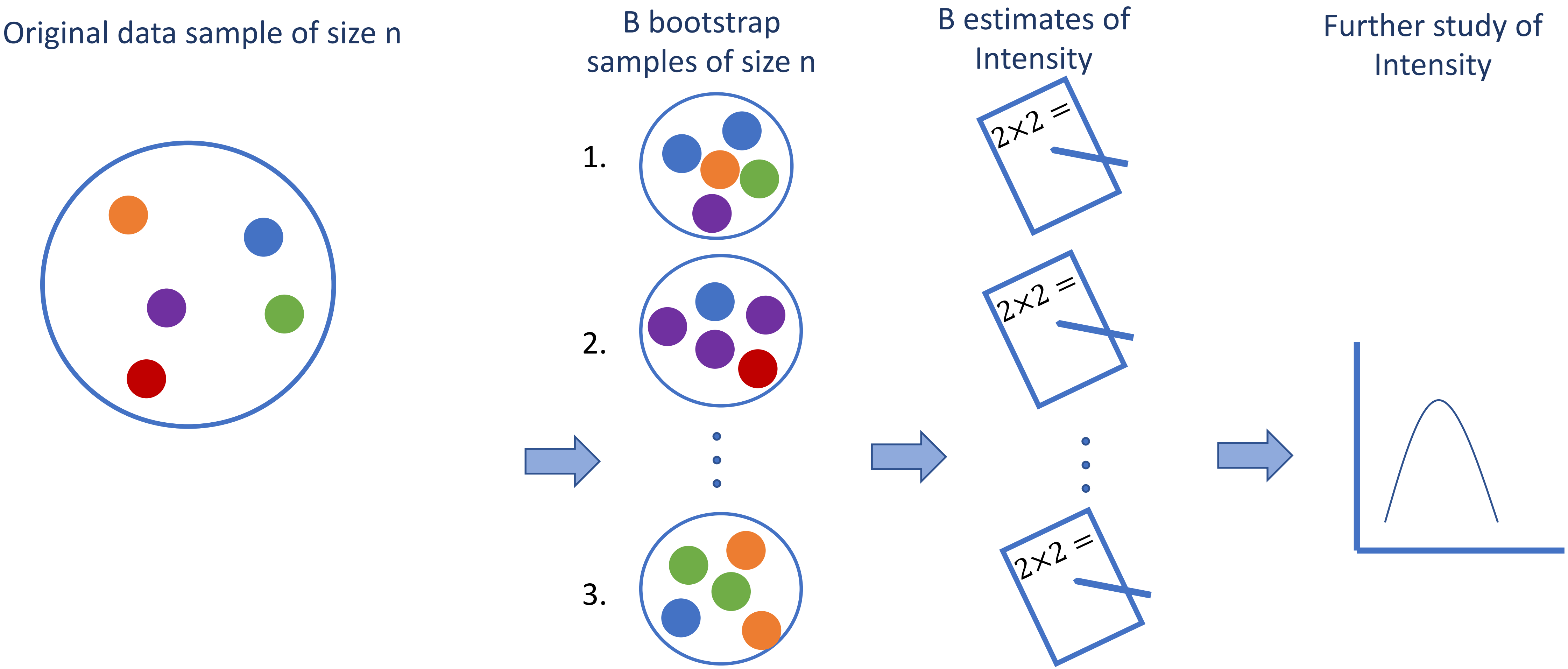
Mathieu et al. Phys. Rev. D 100, 054017 (2019)

Fit 1 results (fitting in M and t bins)

Fitting with original amplitude set: $S_0^{(+)}, P_0^{(+)}, P_1^{(+)}, D_0^{(+)}, D_1^{(+)}, D_2^{(+)}$ Good starting values for fit parameters.



Bootstrapping method for estimation of uncertainties



1. Draw a Bootstrap Sample from the original sample data with replacement with size n.
2. Evaluate intensity for each Bootstrap Sample which will result in B estimates of intensity.
3. Construct a histogram of B estimates of intensity and use it to make further statistical inference, such as:

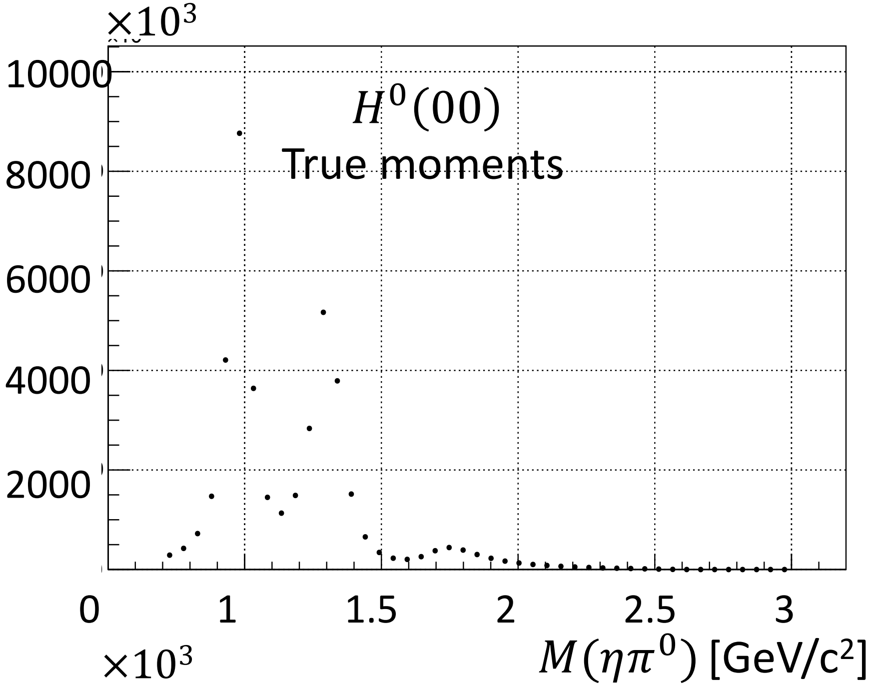
- Estimating the standard error of statistic for Intensity
- Useful when there is no analytical form or normal theory to help estimate the distribution of the statistics of interest

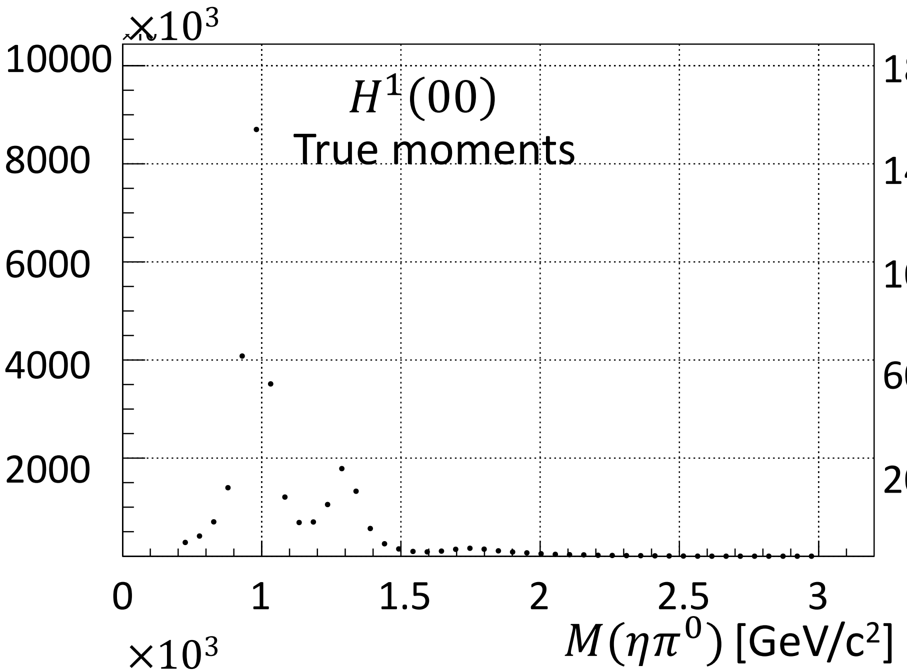
Moments obtained using both methods have similar shapes

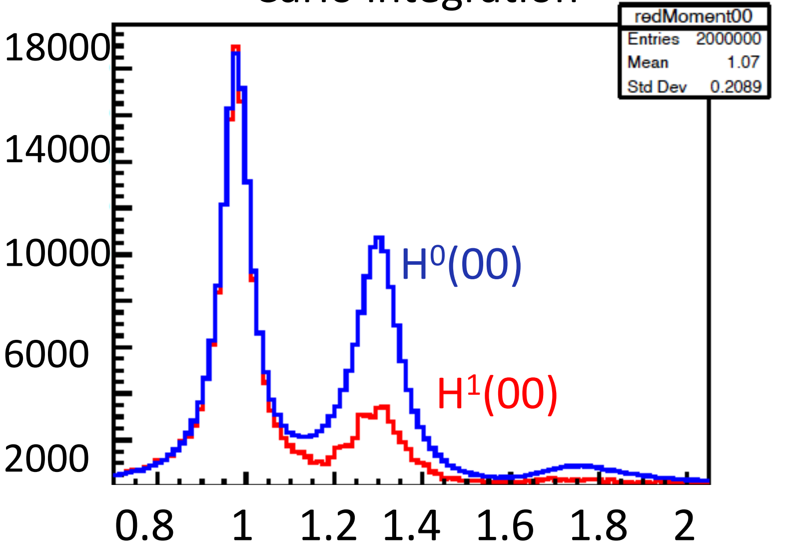
$0<t<0.3\ (\mathrm{GeV}/c)^2$

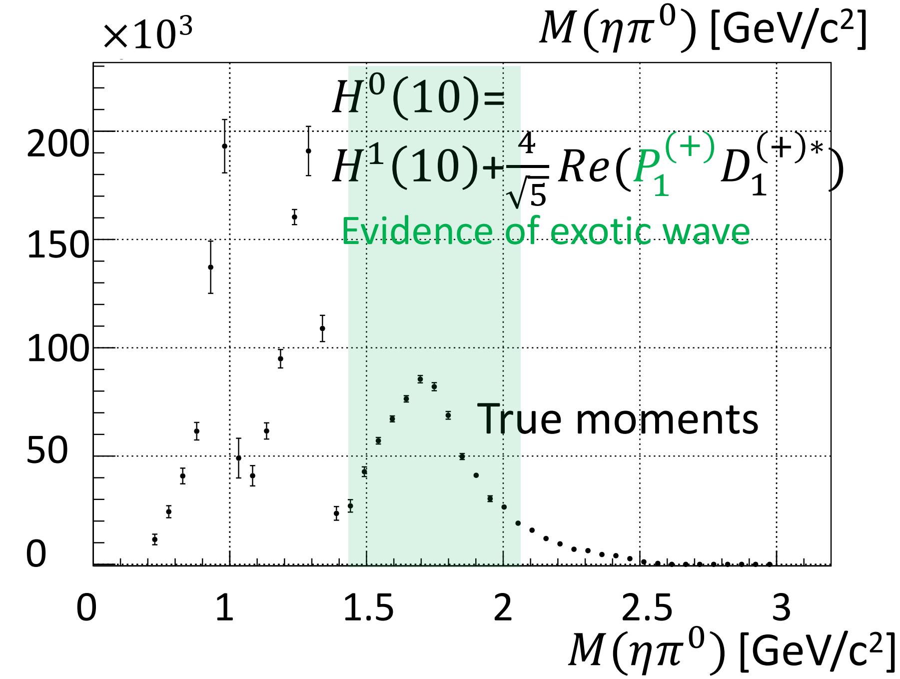
With uncertainties from bootstrapping

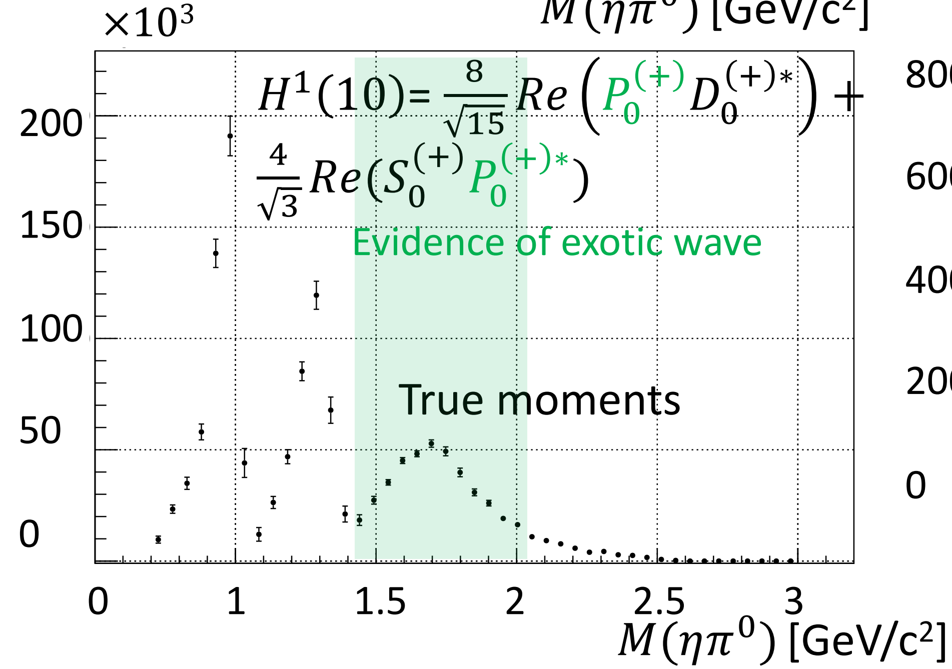
Unnormalized moments from Monte Carlo integration

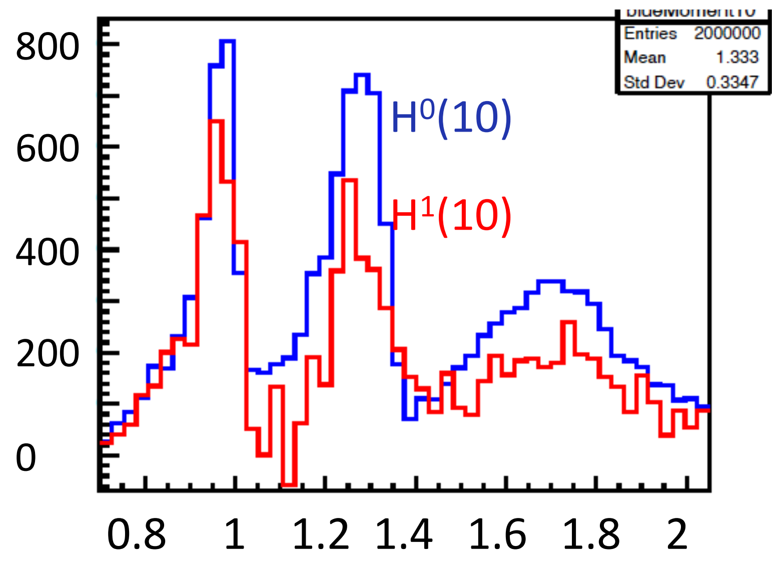








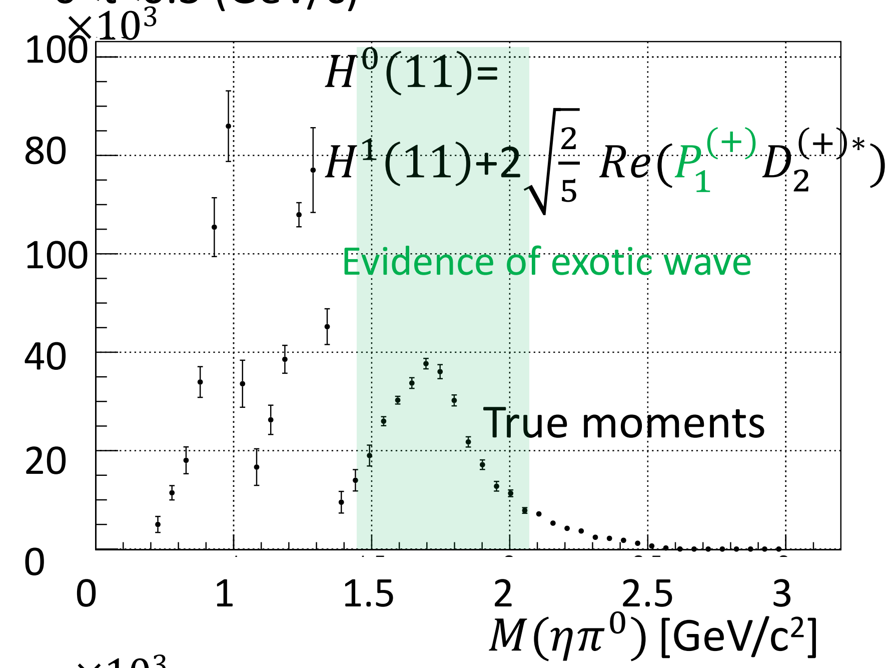


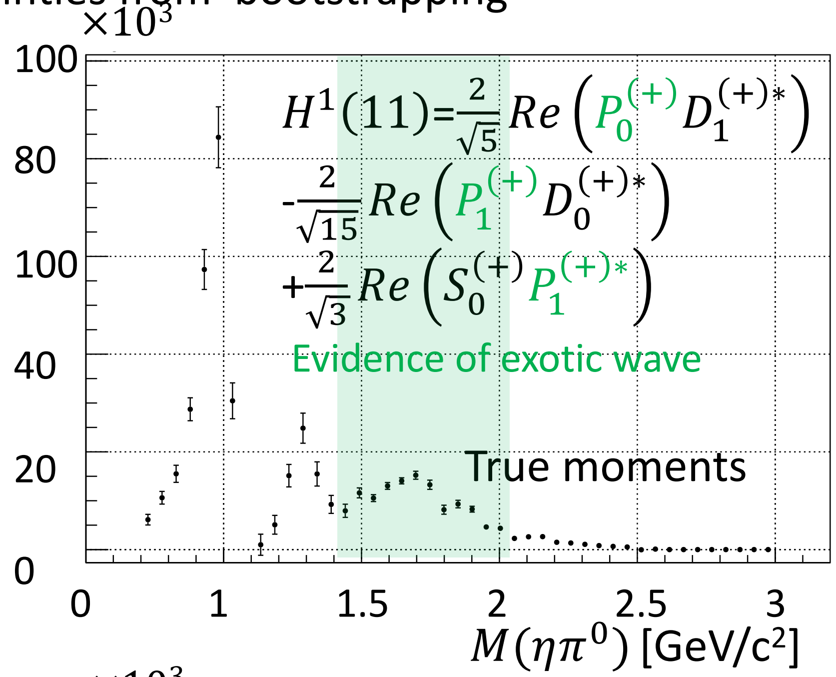


Moments obtained using both methods have similar shapes

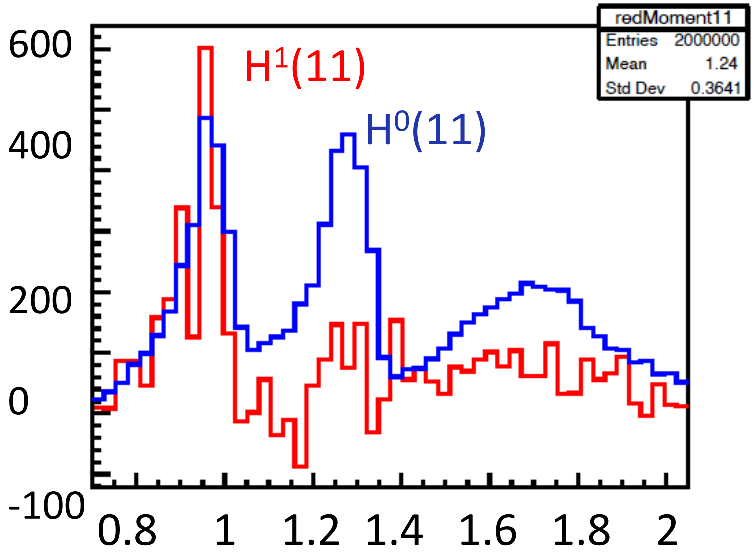
$0<t<0.3$ (GeV/c)^2

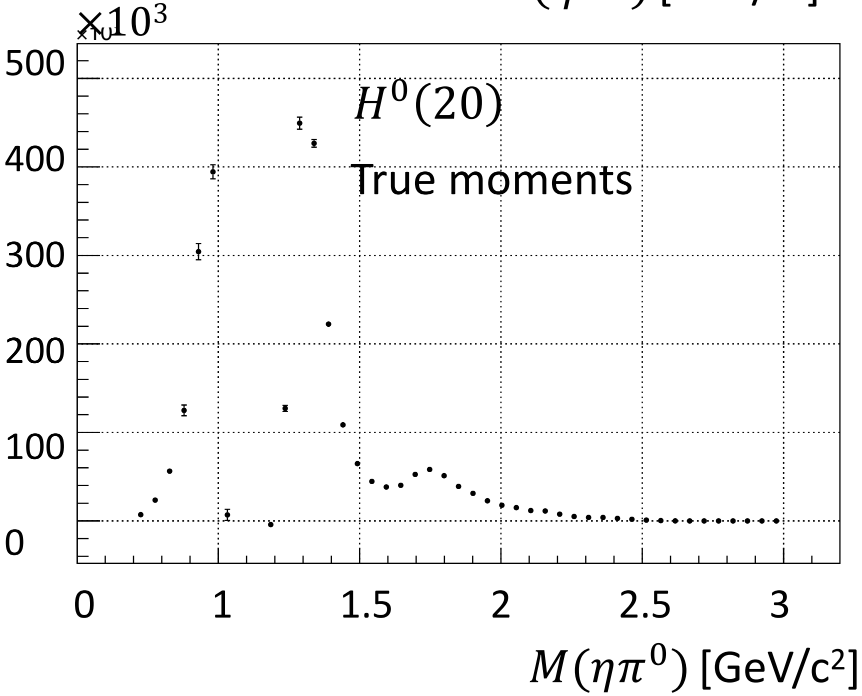
With uncertainties from bootstrapping

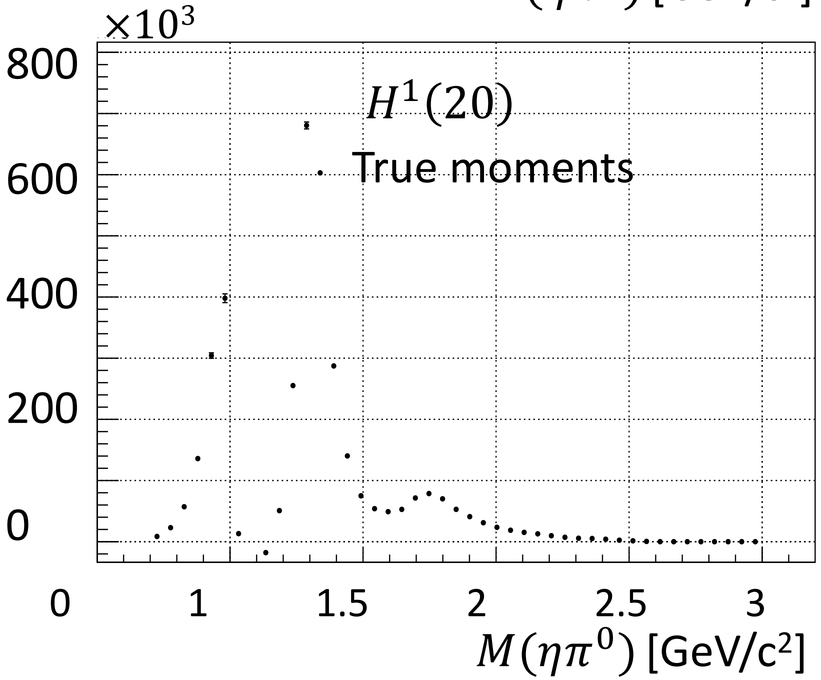


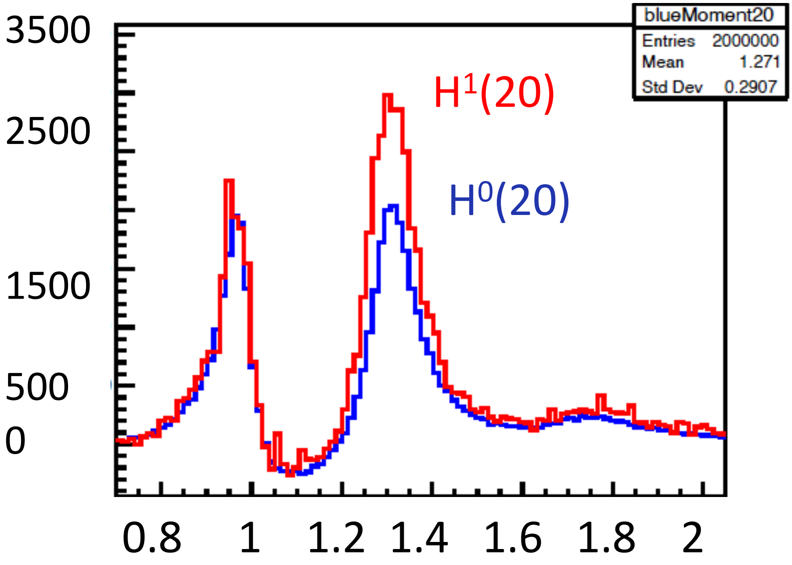


Unnormalized moments from Monte Carlo integration







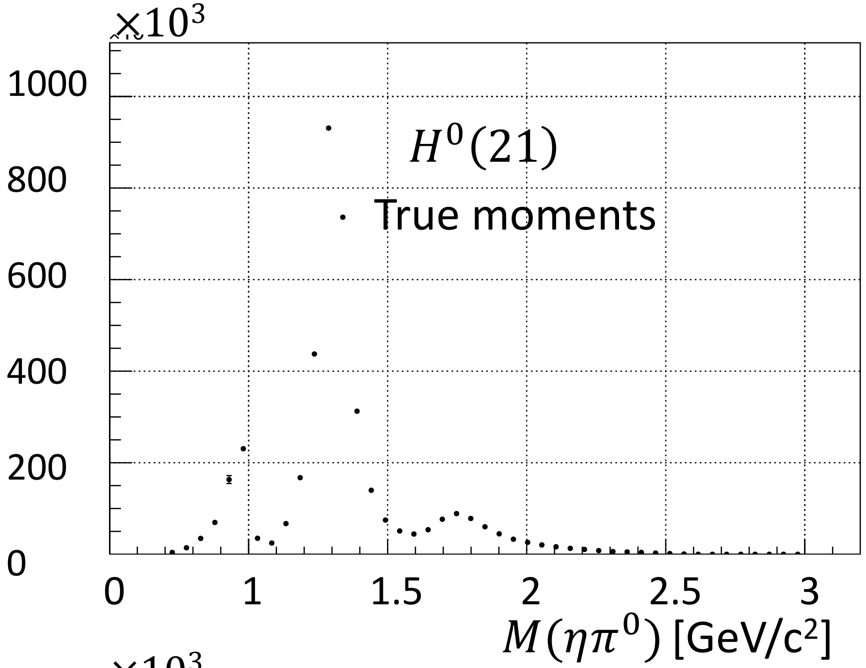


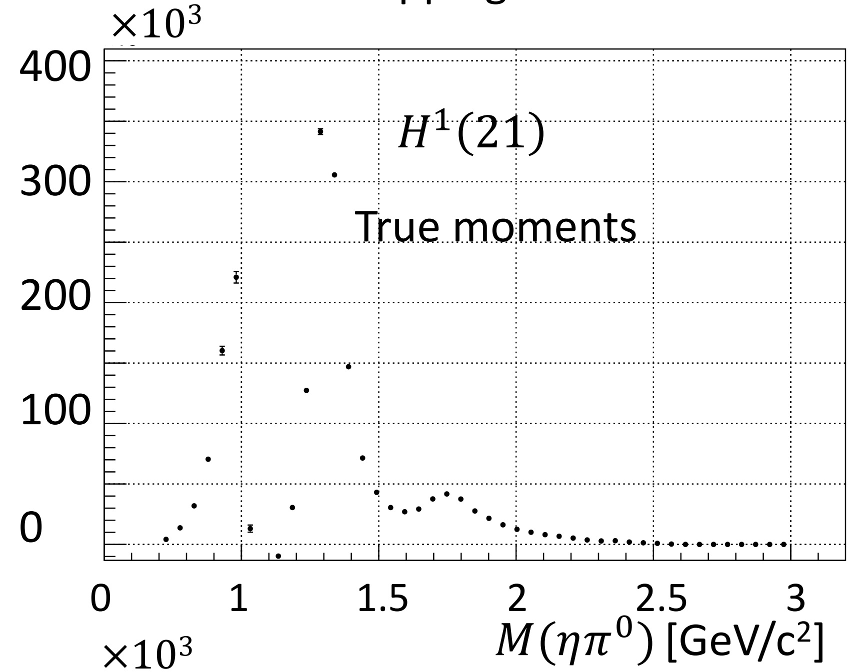
Moments obtained using both methods have similar shapes

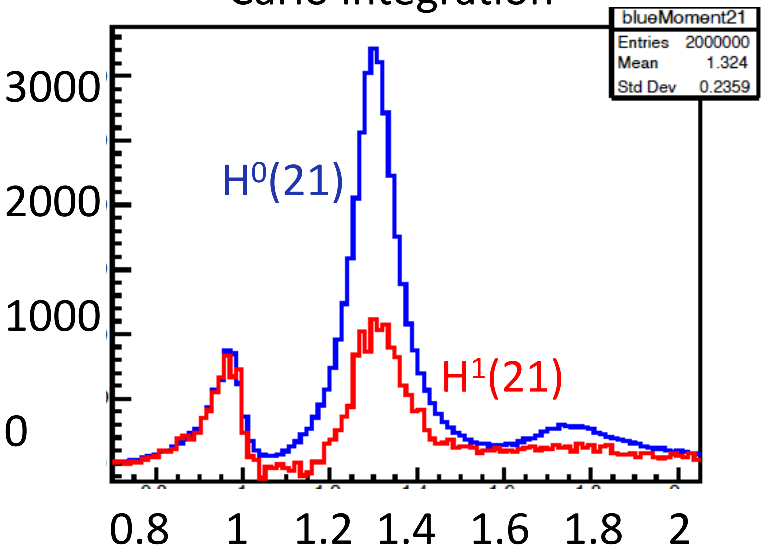
$0<t<0.3\ (\mathrm{GeV}/c)^2$

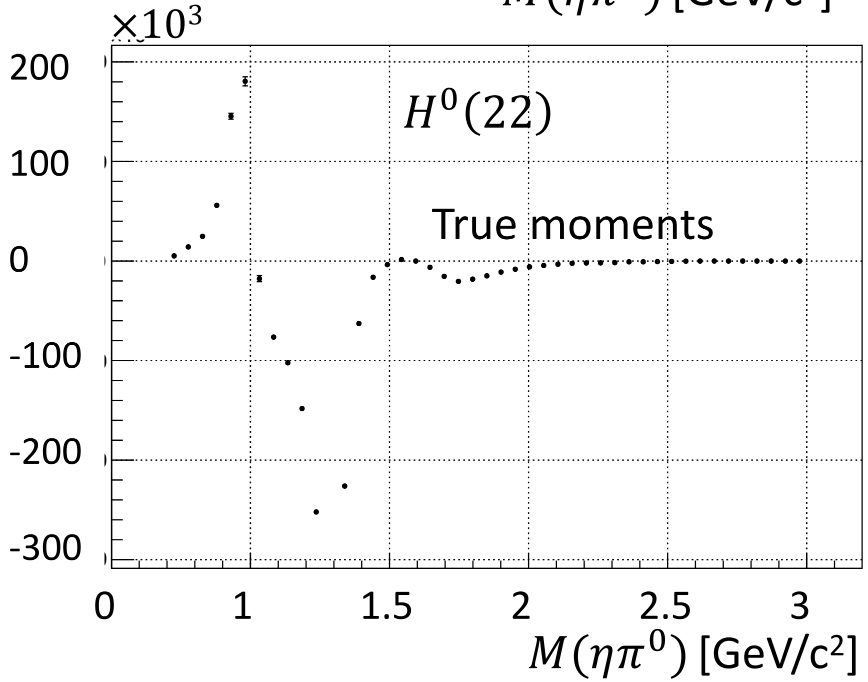
With uncertainties from bootstrapping

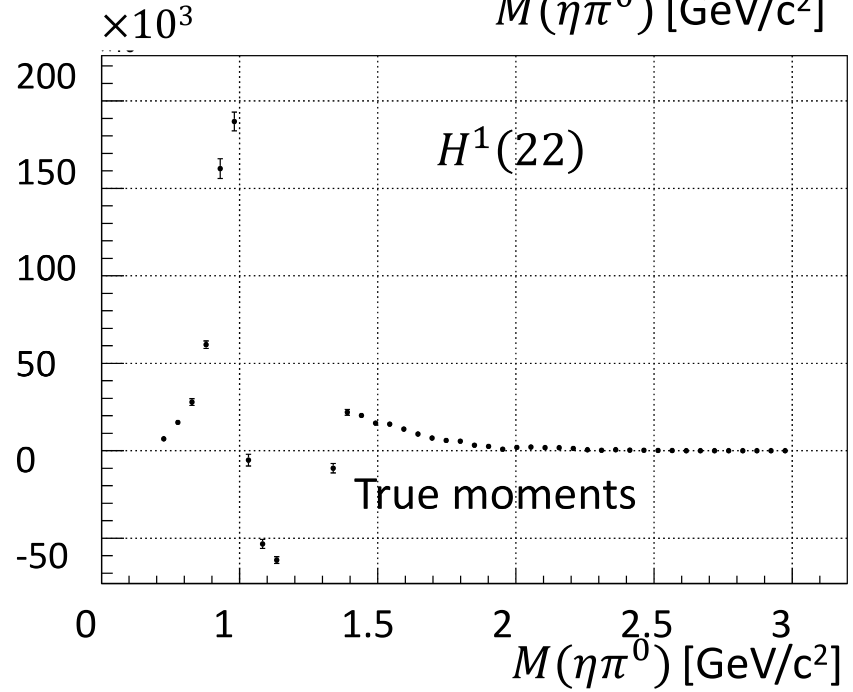
Unnormalized moments from Monte Carlo integration

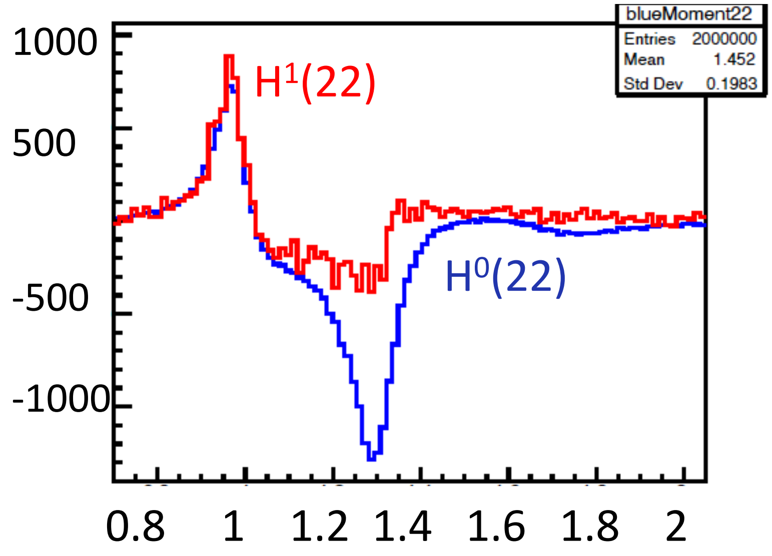












Extraction of polarized moments and exotic signal for generated $(p\eta'\pi^0)$ data including acceptance

Monte Carlo Study Generated ($\eta'\pi^0$)

Generated amplitudes:
- P/π_1(1600 MeV) (BW) (exotic)
- D/a_2 (1320 MeV) (BW)
- D/a_2' (1700 MeV) (BW)

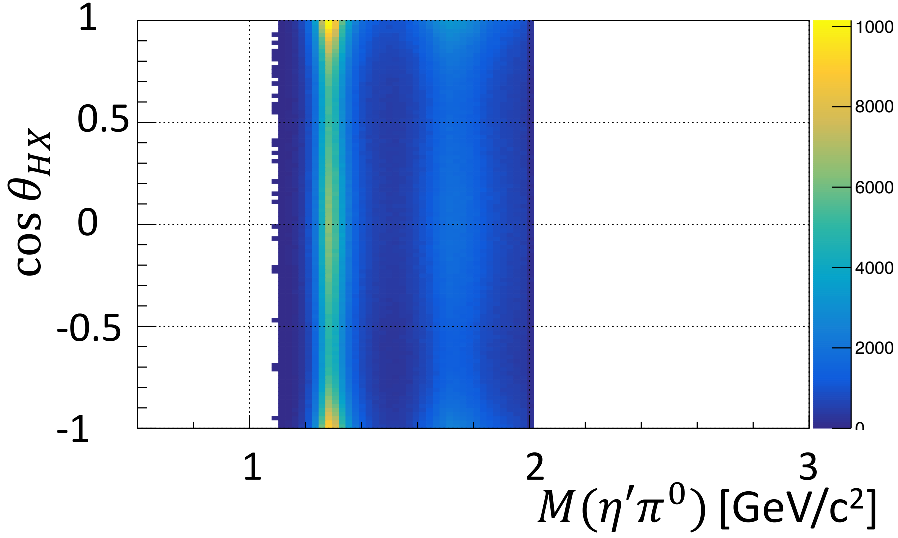
The wave set: $[l]_{m;k}^{(\epsilon)} = \left\{P_0^{(+)}, P_1^{(+)}, D_0^{(+)}, D_1^{(+)}, D_2^{(+)}\right\}_{k=0}$

Small intensity of exotic wave

$\frac{I(a_2)}{I(\pi_1)} \approx 350$ $\frac{I(a'_2)}{I(\pi_1)} \approx 100$

θ_{pol}=1.77 Deg.

P_γ =0.3

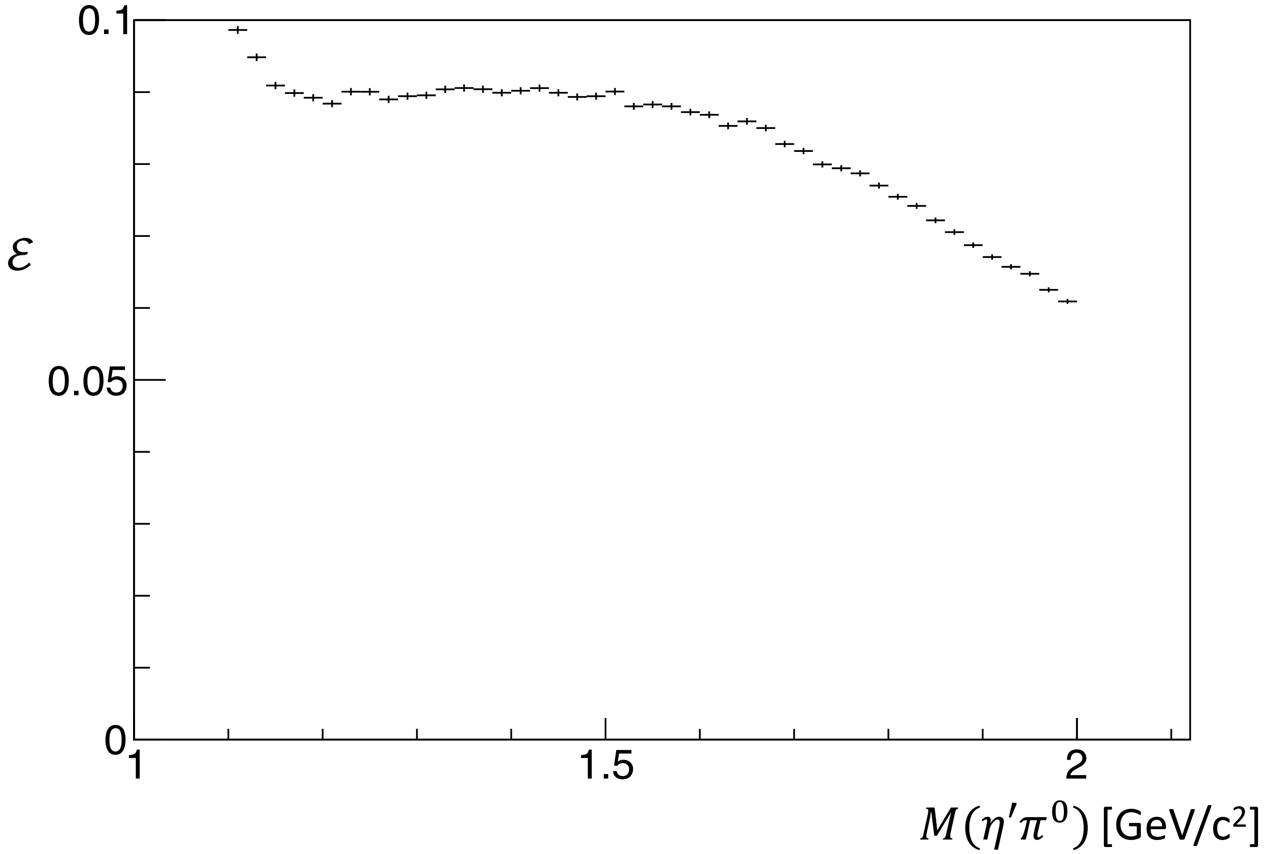


Analysis strategy

- Do PWA studies by assuming perfect acceptance and compare to PWA results with GlueX acceptance.
- Study sensitivity to small exotic signal.

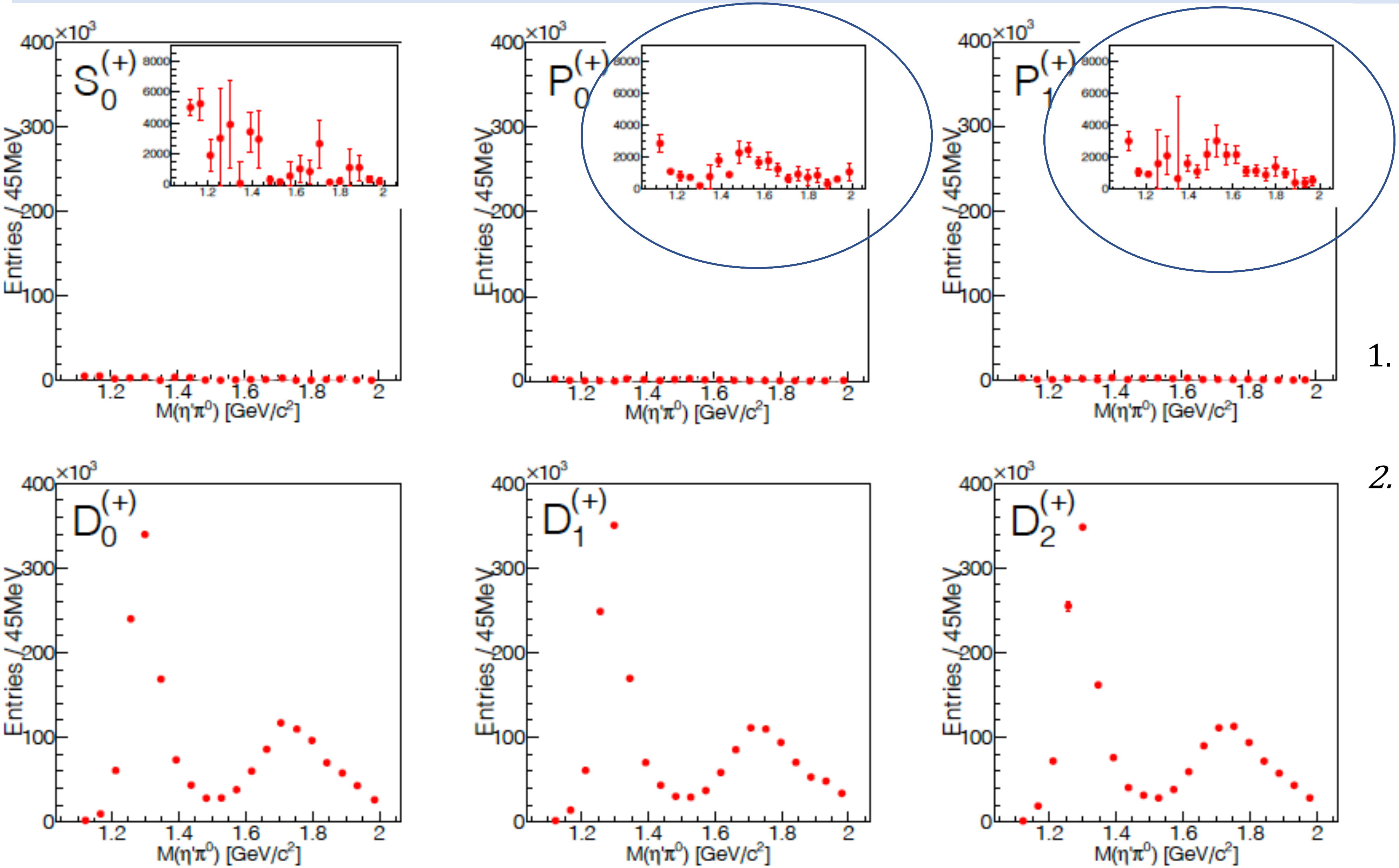
Acceptance with flat data

1. Generate data and pass through the GlueX detector to study acceptance ($\mathcal{E}$) for the reaction $\gamma p \rightarrow p\eta'\pi^0$ (η'-> $\pi^+ \pi^- \eta$, $\eta \rightarrow \gamma\gamma$)
   - Have generated $5*10^6$ events
   - ~8.5% got reconstructed



We select –t >0.1 (GeV/c)^2 to cut events, where p had such low –t , that it couldn't get out of the target.

Fit to reconstructed simulated data: Partial wave intensities



π_1 component

1. Overall good agreement with generated wave set
2. π_1 component two orders of magnitude smaller than a_2

Polarized moments calculated with partial waves and GlueX acceptance agree with generated moments in π_1 mass region, and show exotic signal

Uncertainties from bootstrapping

$0<t<0.3\ (\mathrm{GeV}/c)^2$

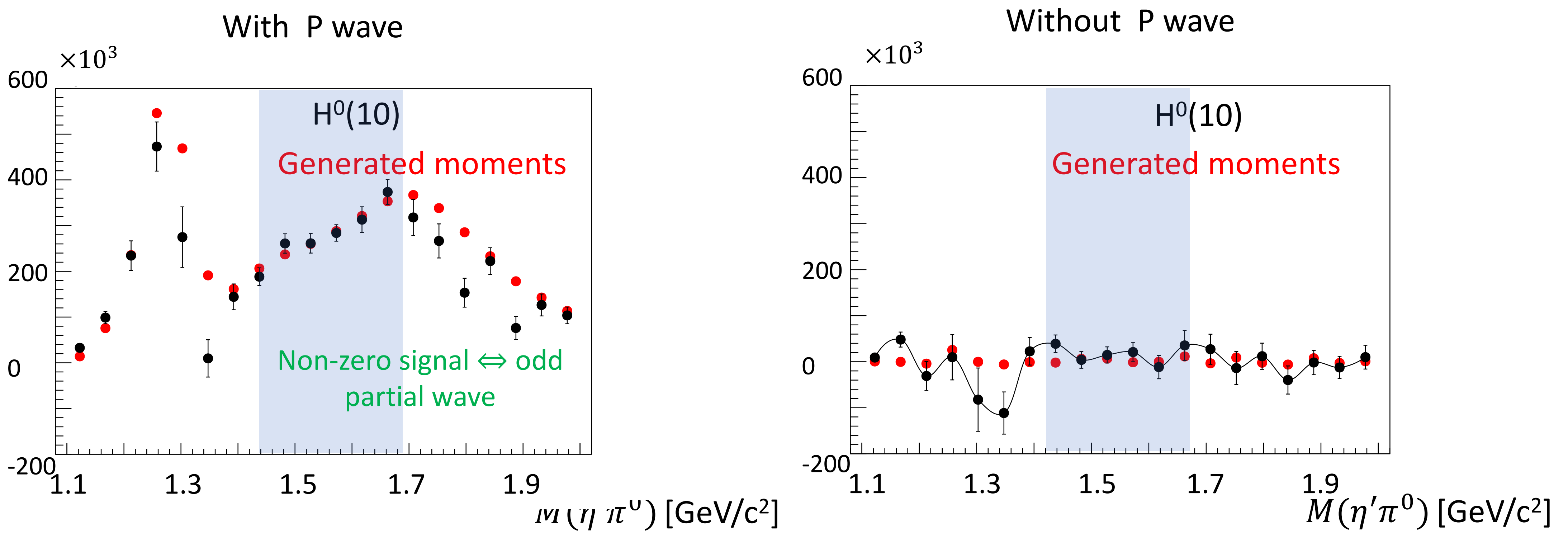
Using generated P4 for reconstructed data

- Agreement with generated moments, especially in region of interest
- Odd L moments are sensitive to presence of odd partial waves

$$H^0(10) = \frac{8}{\sqrt{15}} Re\left(P_0^{(+)} D_0^{(+)*}\right) + \frac{4}{\sqrt{3}} Re\left(S_0^{(+)} P_0^{(+)*}\right) + \frac{4}{\sqrt{5}} Re\left(P_1^{(+)} D_1^{(+)*}\right)$$

- Even with small π_1 intensity we see a clear non-zero signal

Odd L moments are proportional to the interference between P and D waves



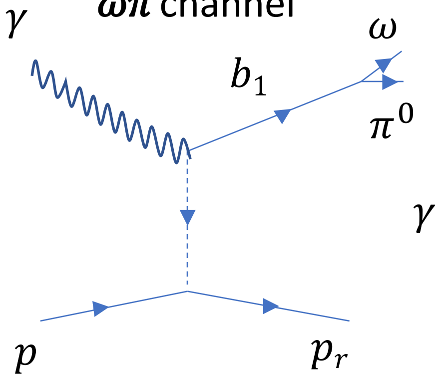
Search for exotic mesons via Partial Wave Analysis (PWA) of $p\eta'\pi^0$ data

Search for exotic π_1 (1600) using the reaction $\gamma p \rightarrow p\eta'\pi^0$ in GLUEX

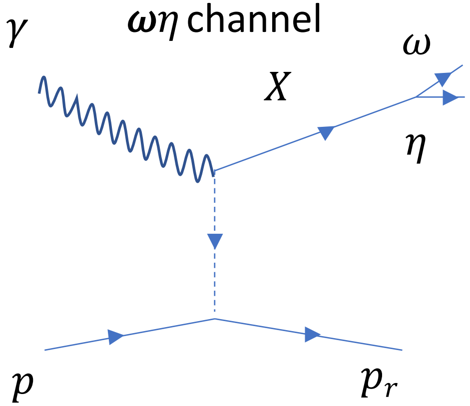
$$\gamma\text{p} \rightarrow \eta'\,\pi^0\text{p} \rightarrow (\pi^+\pi^-\eta)(\gamma\,\gamma)\text{p} \rightarrow \pi^+\pi^-\gamma\,\gamma\,\gamma\,\gamma\ \text{p}$$

Alternate physics processes with same final state (Identified and rejected)

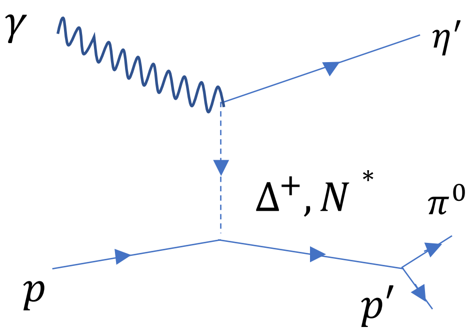
$\omega\pi$ channel



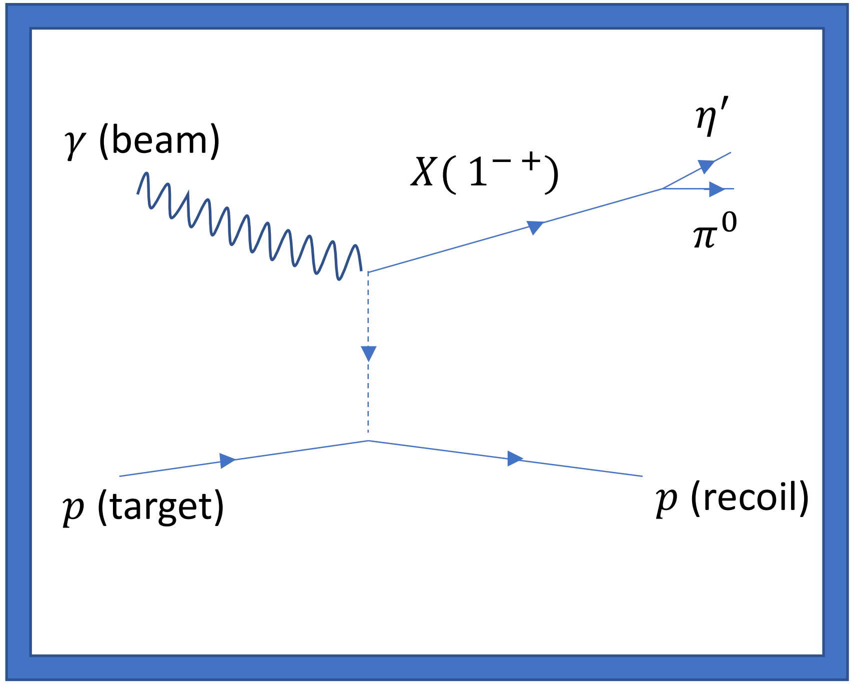
$\omega\eta$ channel



Baryon production

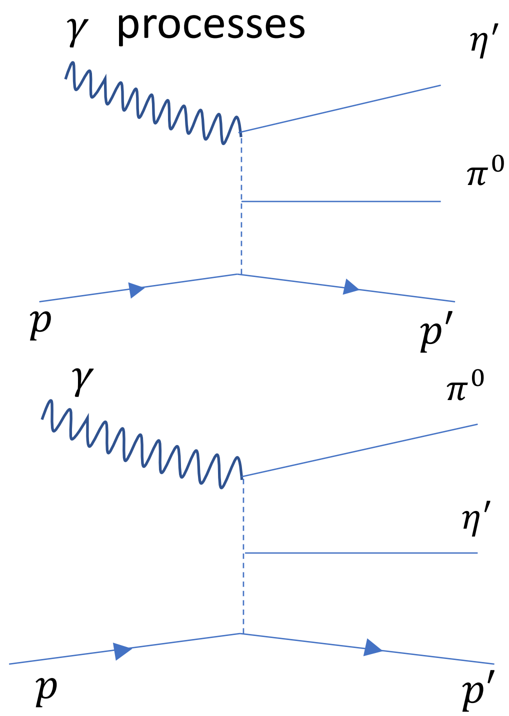


X : Probable resonance in $\eta'\pi^0$ system



Alternate physics processes with same final state (No quantitative analysis yet)

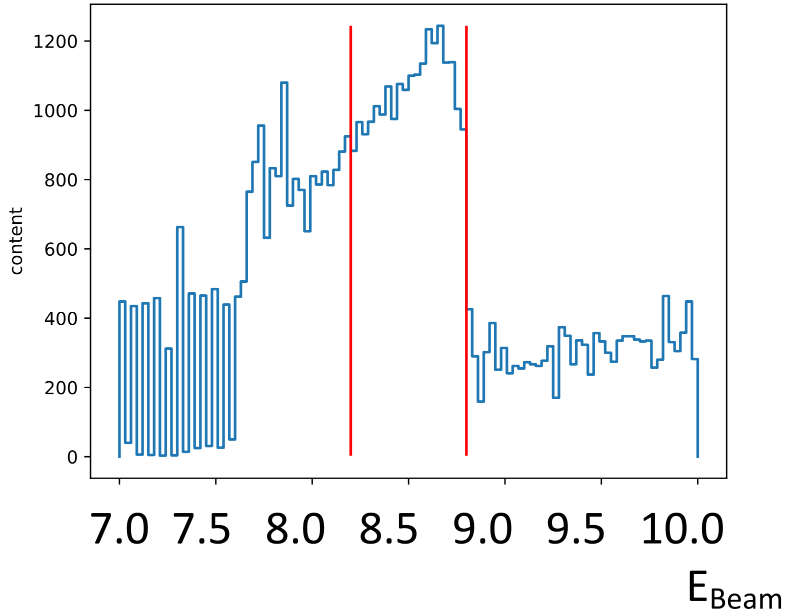
Double-Regge exchange processes



Critical to understand and model these backgrounds→ close collaboration with JPAC 1
Double Regge exchange (Deck effect) can enhance odd partial waves and mimic exotic signal

$\gamma p \rightarrow p\eta'\pi^0$ data set and event selection

Data Set: GlueX Phase I (π^0 and η masses constrained in kinematic fitting)



❑ Selection cuts:
- Kinfit Confidence Level > 0.001
- Missing Mass Squared : -0.02 < MM2 $(GeV/c^2)^2$ < 0.02
- Timing cut : -2 ns < ΔT (Beam - RF) < 2 ns
- Beam Energy : 8.2 < E_{beam} < 8.8
- η' mass window : 0.90 < M($\pi^+ \pi^- \eta$) (GeV/c^2) < 1.02
- Mandelstam t cut : 0.1 < t $(GeV/c)^2$ < 0.7
- FCAL shower quality > 0.5 for all 4 final state photons
- Unique event selection based on (run number, event number) pair

❑ Rejections (Based on alternate channels):
- $\pi^0 \pi^0$ rejection : star shaped polygon cut in 2D histogram of M($\gamma_1\gamma_3$) Vs M($\gamma_2\gamma_4$) OR M($\gamma_1\gamma_4$) Vs M($\gamma_2\gamma_3$)
- Baryon rejection : Cosθ>1 cut in Cosθ Vs M(π^0p); where θ is the polar angle of π^0 in γp rest frame
- ω cut in 1D spectrum: 0.73 < M($\pi^+ \pi^- \pi^0$) (GeV/c^2) < 0.83

❑ Signal Background separation:
- Q-factors

Analysis strategy

1. Fitting entire GlueX phase 1 data (except amorphous) for four γ polarization plane angles relative to horizontal (0, 45, 90, 135^o) using loop statement in AmpTools

2. Do multiple fits with randomized initial parameters (100 fits), to choose good starting parameters

3. Fit intensity with different wave sets:
   - $S_0, P_{0,1}, D_{0,1,2}$ ε=±1 (models predict M≥0 dominance)
   - $S_0, P_{0,\pm1}, D_{0,\pm1,\pm2}$ ε=±1

4. Calculate moments using fitted partial waves:
   - Interference with even waves amplifies exotic signal in moments
   - Presence of exotic wave → non-zero odd L moments.

5. Evaluate phase differences

6. Compare different fit results

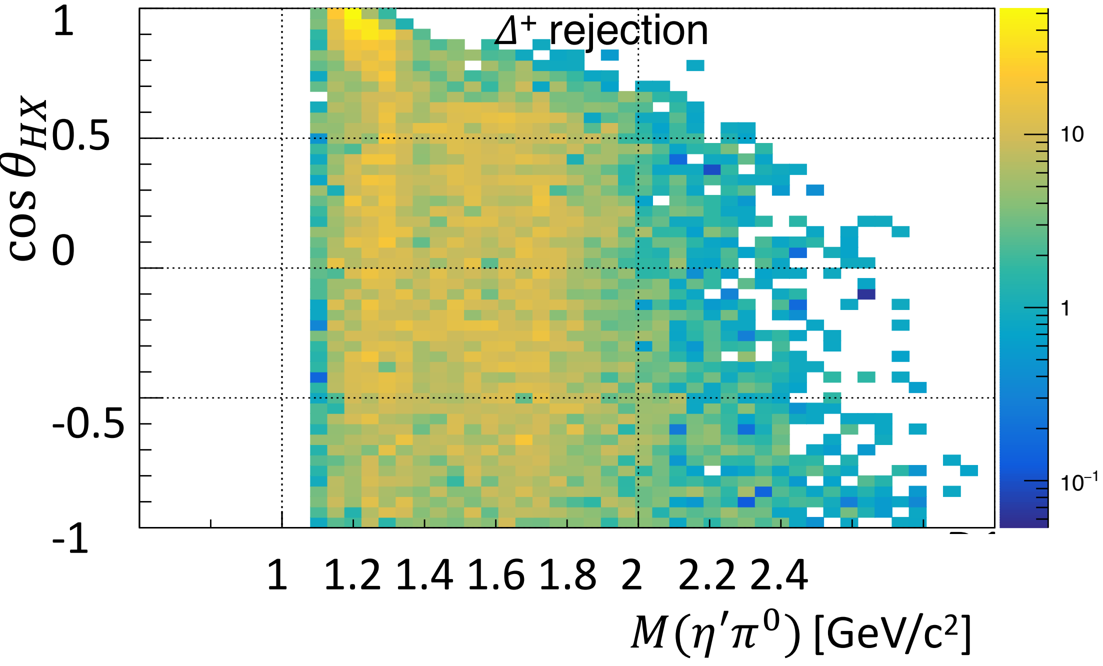
17059 GlueX ($p\eta'\pi^0$) events for 4 γ polarization plane angles relative to horizontal (0, 45, 90, 135^o)

Number of signal events 6777

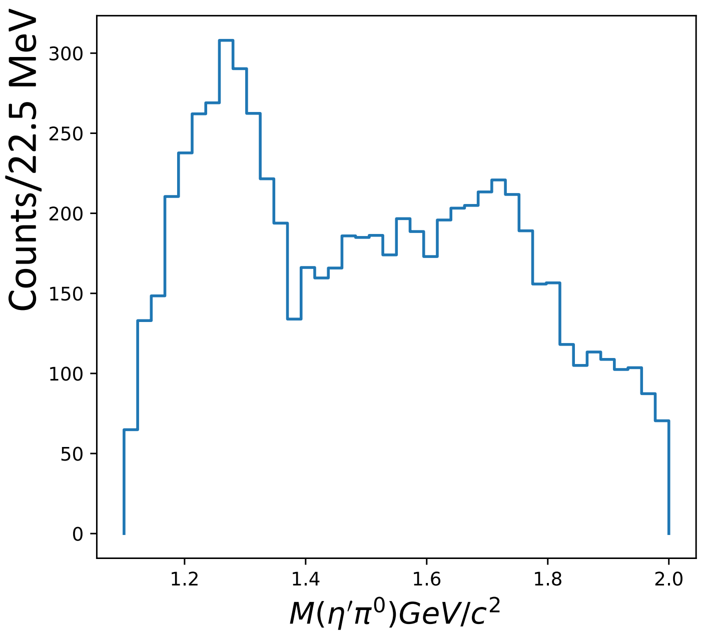
Dataset	Polarization fraction
0 Deg.	$P_\gamma = 0.3519$
45 Deg.	$P_\gamma = 0.3374$
90 Deg.	$P_\gamma = 0.3303$
135 Deg.	$P_\gamma = 0.3375$

Looking for:
P/π_1(1600 MeV) (exotic)
D/a_2 (1320 MeV)
D/a_2' (1700 MeV)

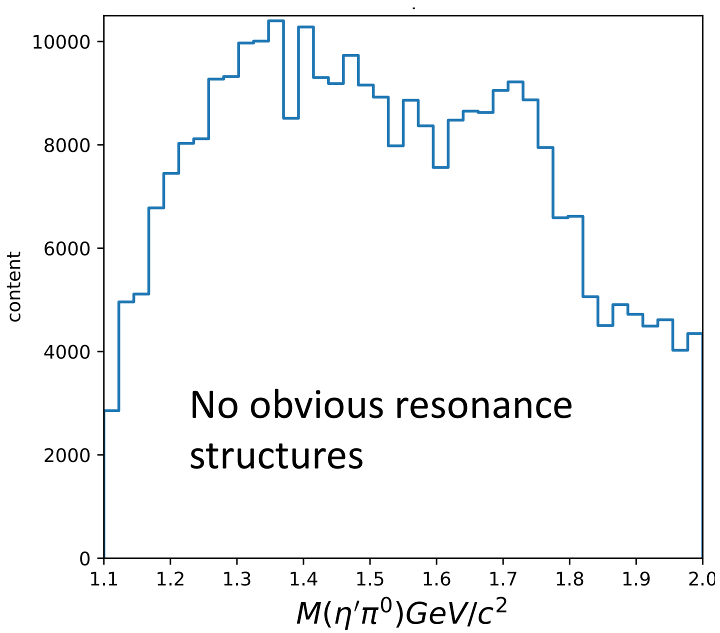
$M(\eta'\pi^0)$



Not corrected for acceptance

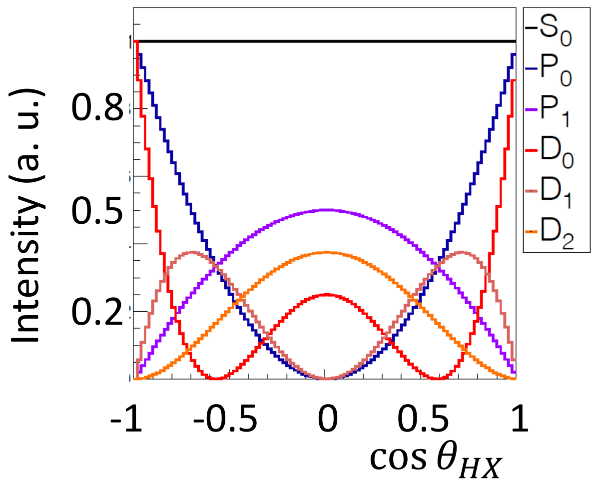


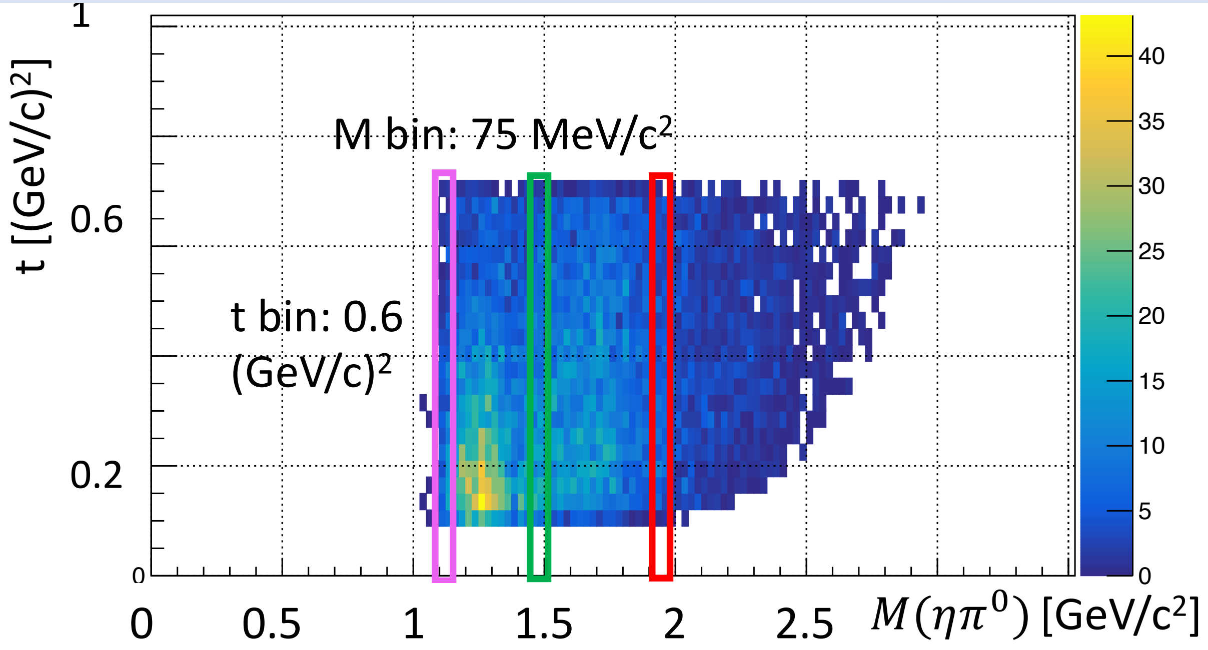
Acceptance corrected

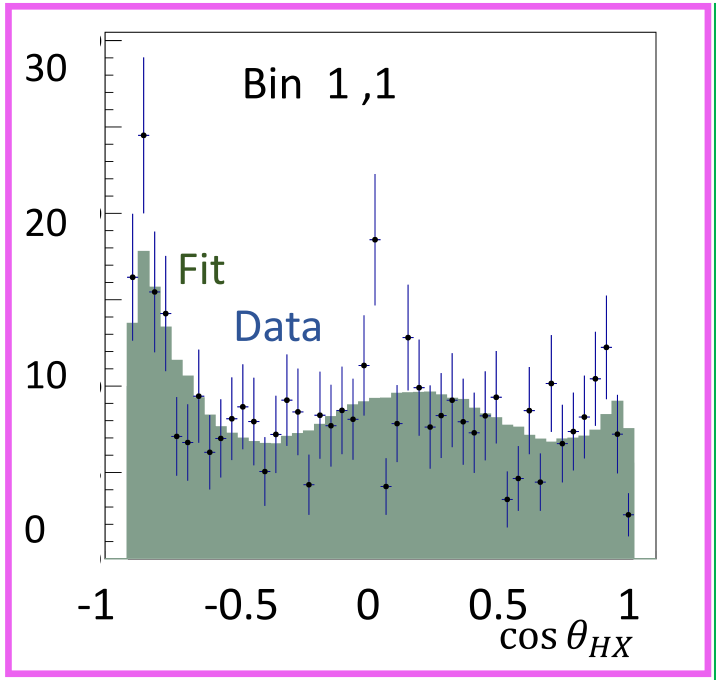


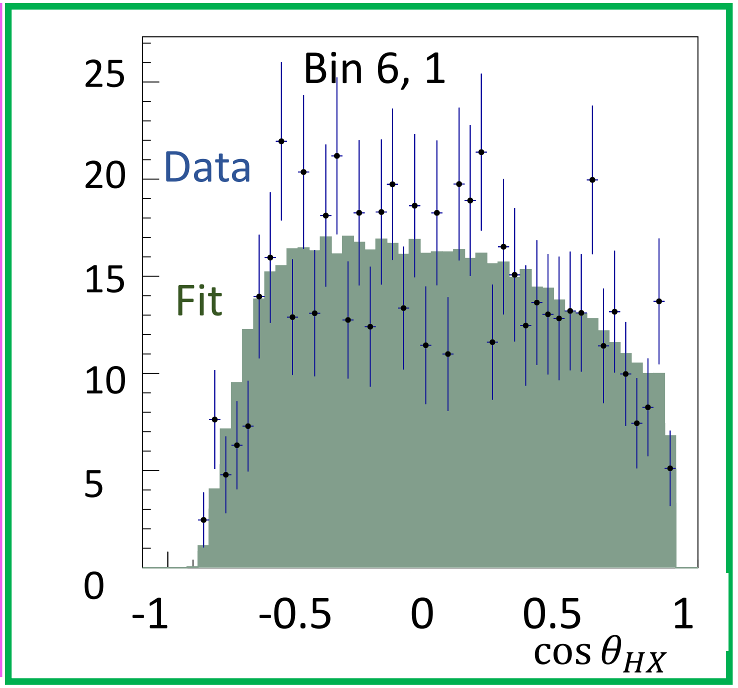
Partial Wave Analysis in bins of M and and t

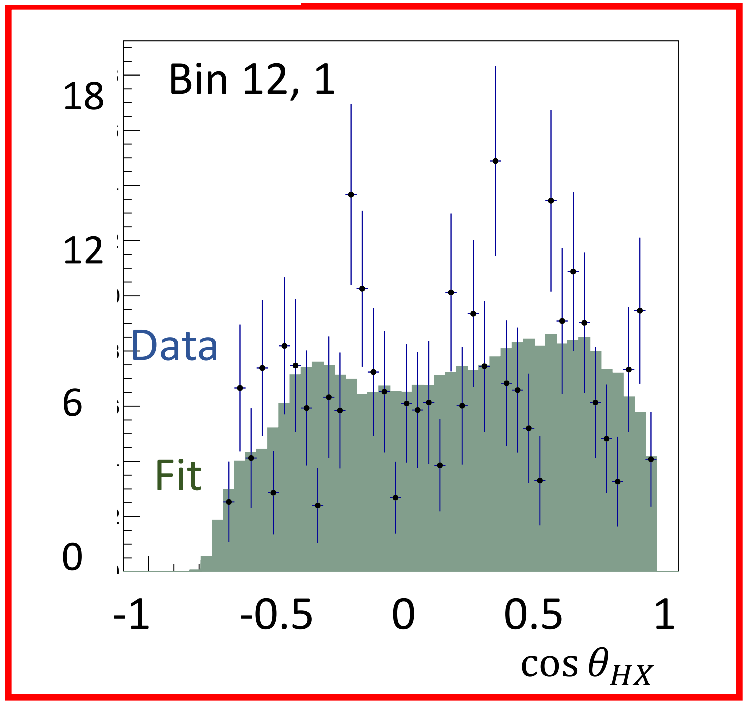
$\cos\theta_{HX}$ distributions in different bins







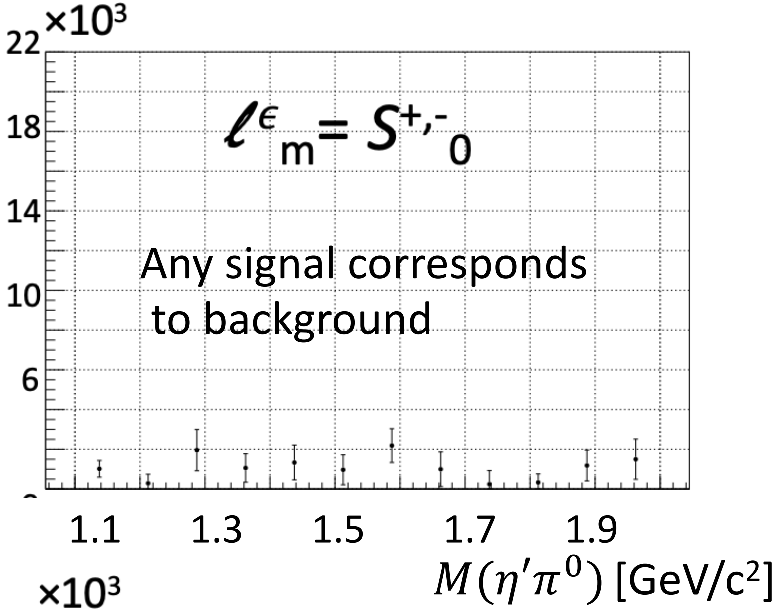


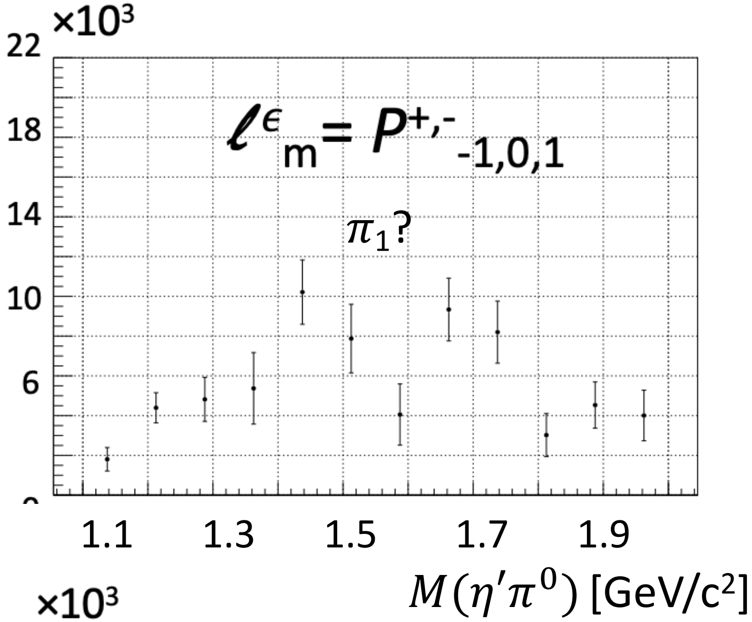


Partial Wave Analysis in bins of M(75 MeV/c²) and and t(0.1<t<0.7 (GeV/c)²)

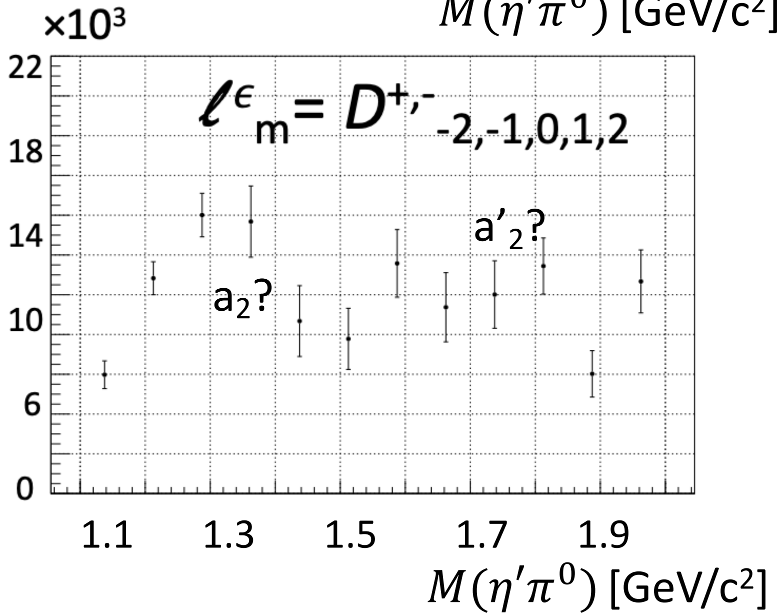
Errors estimated from bootstrap

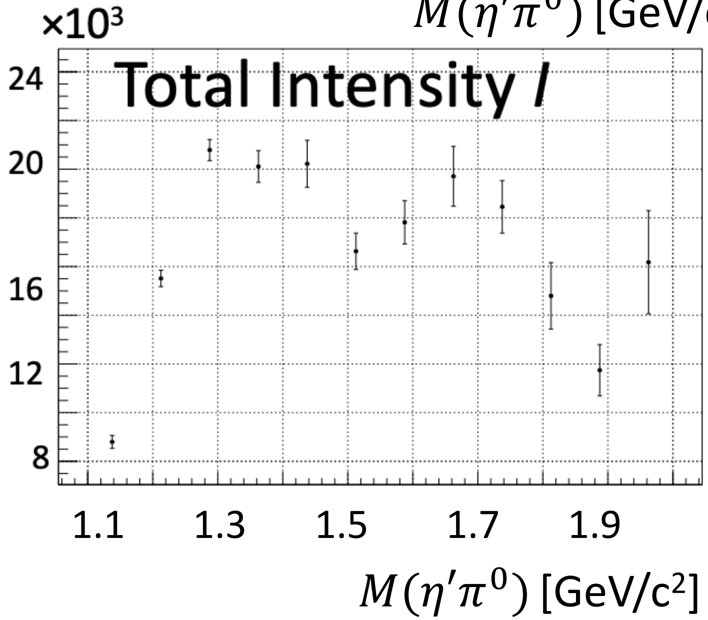
Wave intensities from fit with S, P, D, all M, ε = ±1



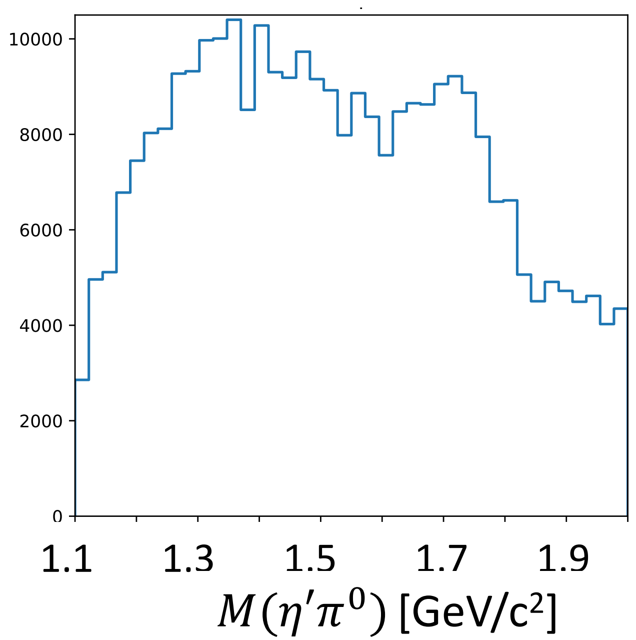


All Histograms are acceptance corrected





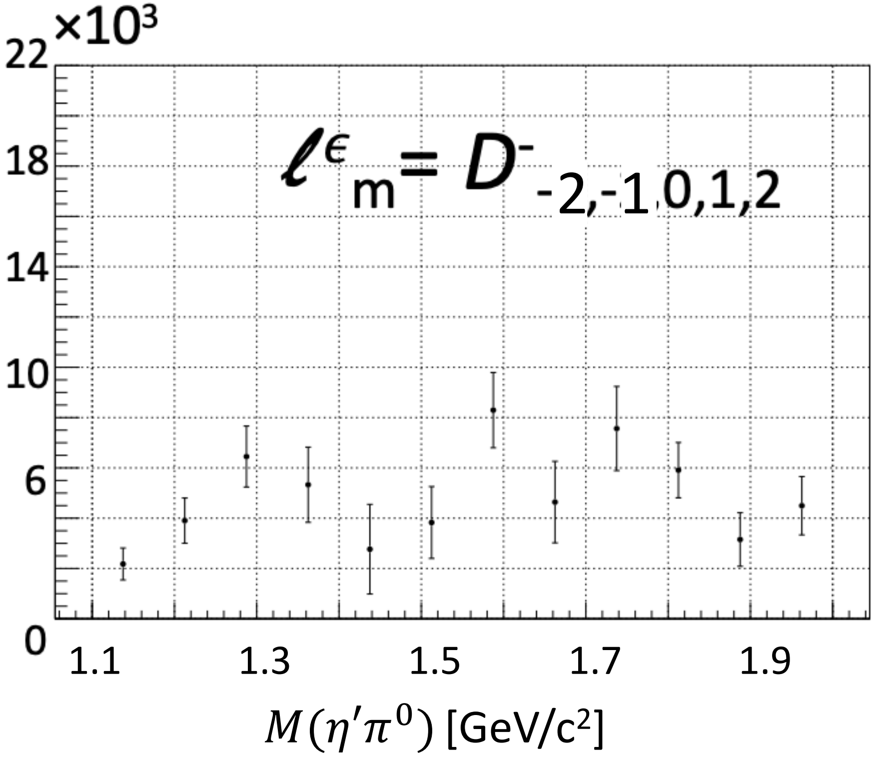
Acceptance corrected

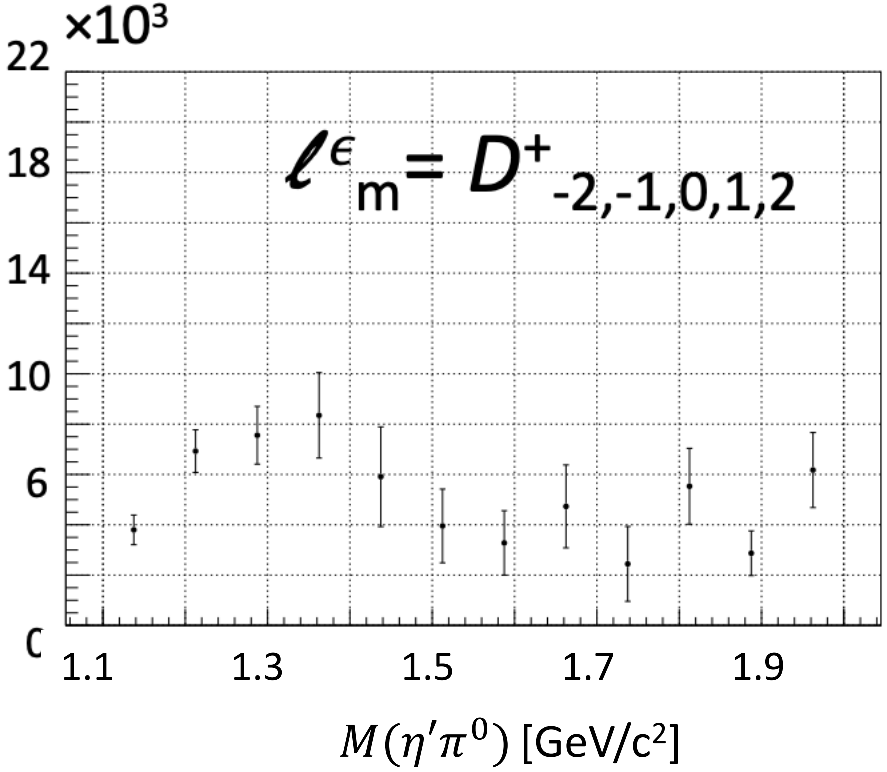


Errors estimated from bootstrap

All Histograms are acceptance corrected

D wave intensities for different reflectivities
from fit with S, P, D, all M, ε = ±1





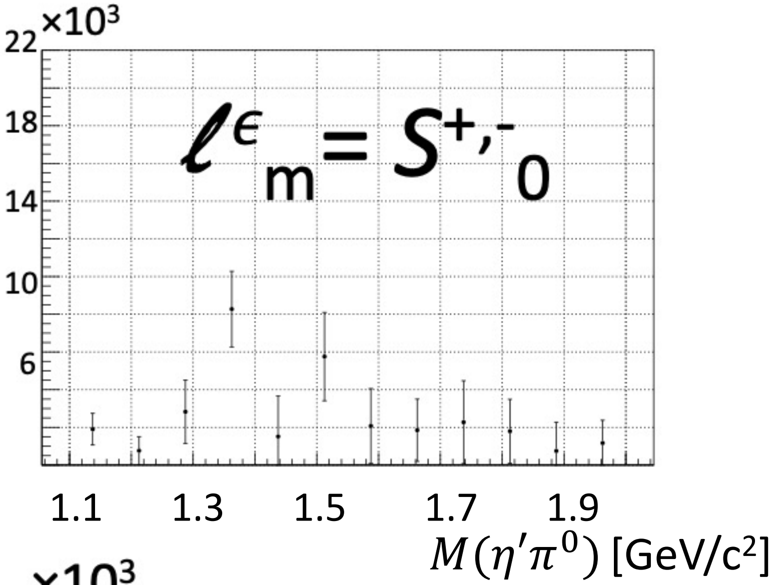
No preference for + or – reflectivity observed

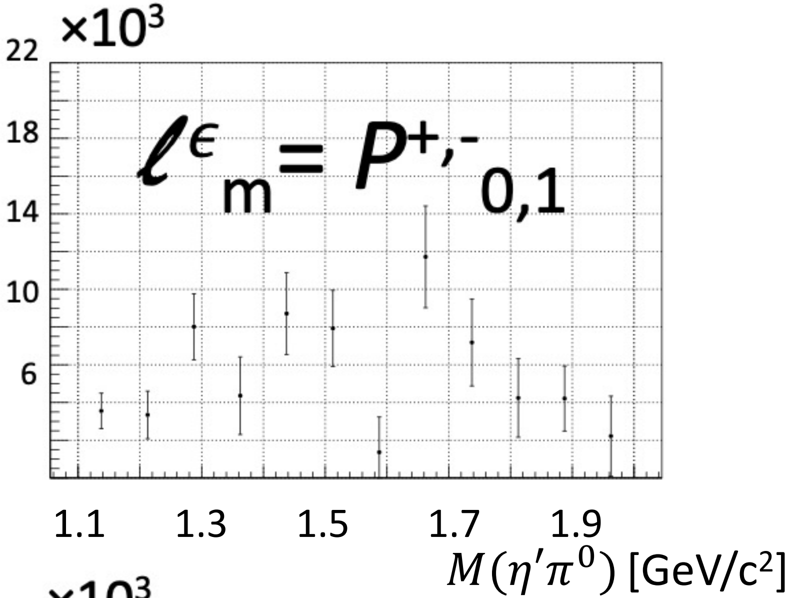
Partial Wave Analysis in bins of M(75 MeV/c^2) and and $t(0.1<t<0.7\ (\mathrm{GeV}/c)^2)$

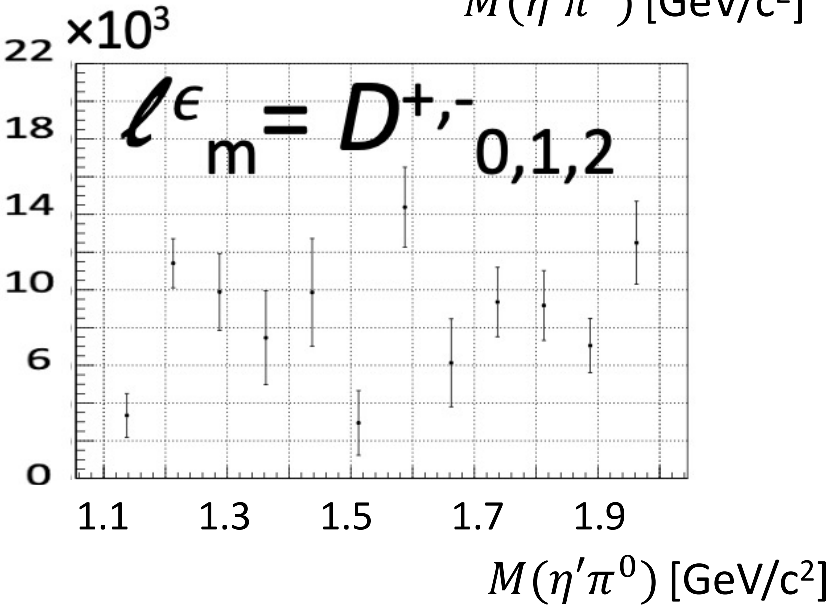
Errors estimated from bootstrap

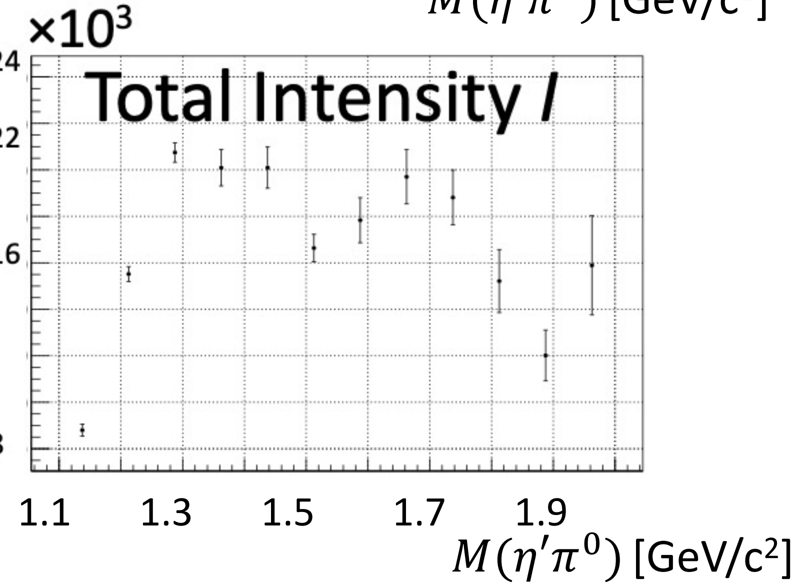
All Histograms are acceptance corrected

Wave intensities from fit with M≥ 0, ε=± 1





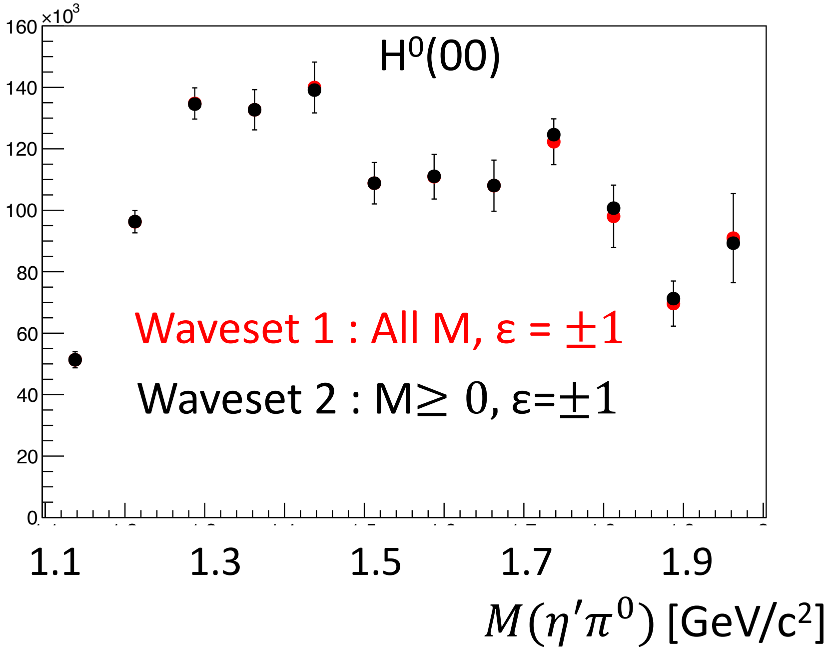


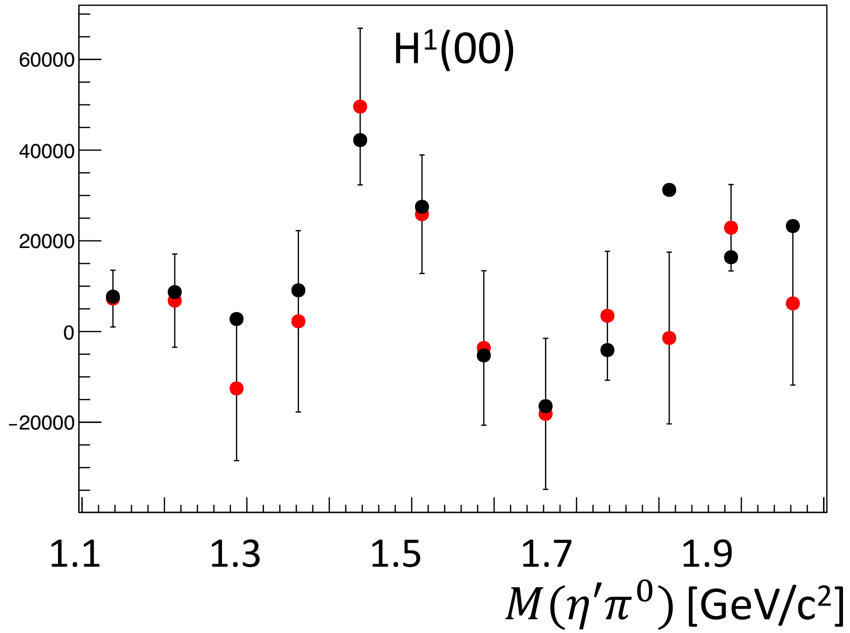


Comparison of moments for two different wavesets

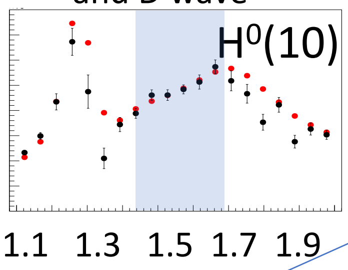
Moments are ambiguities free and interference term in odd moments H(LM) might indicate exotic waves

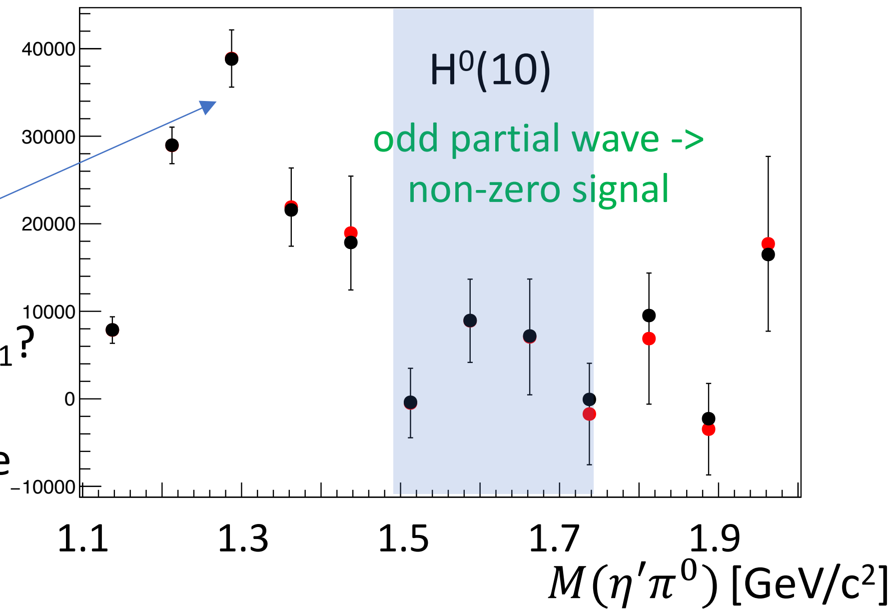
Uncertainties from Bootstrap method

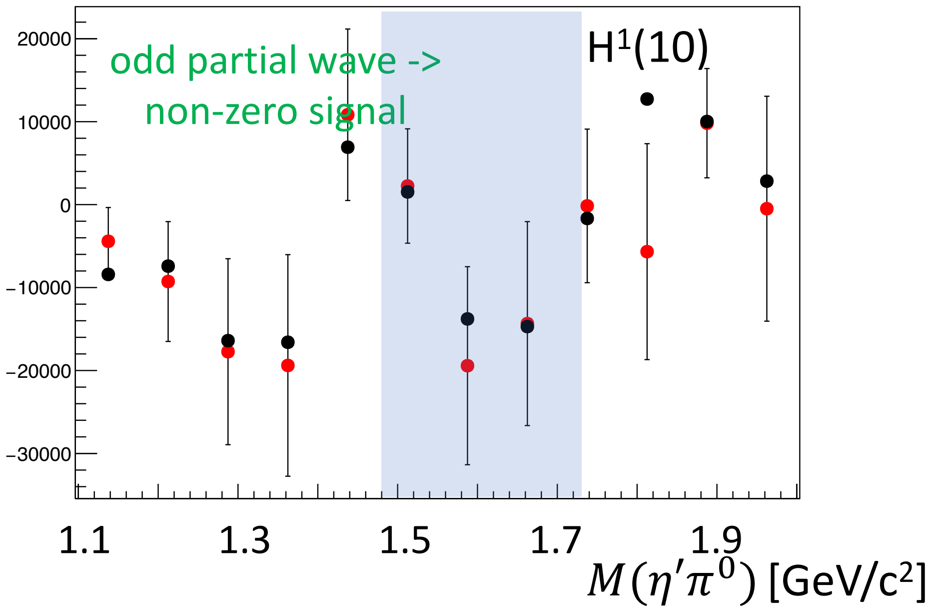




MC studies with P and D wave



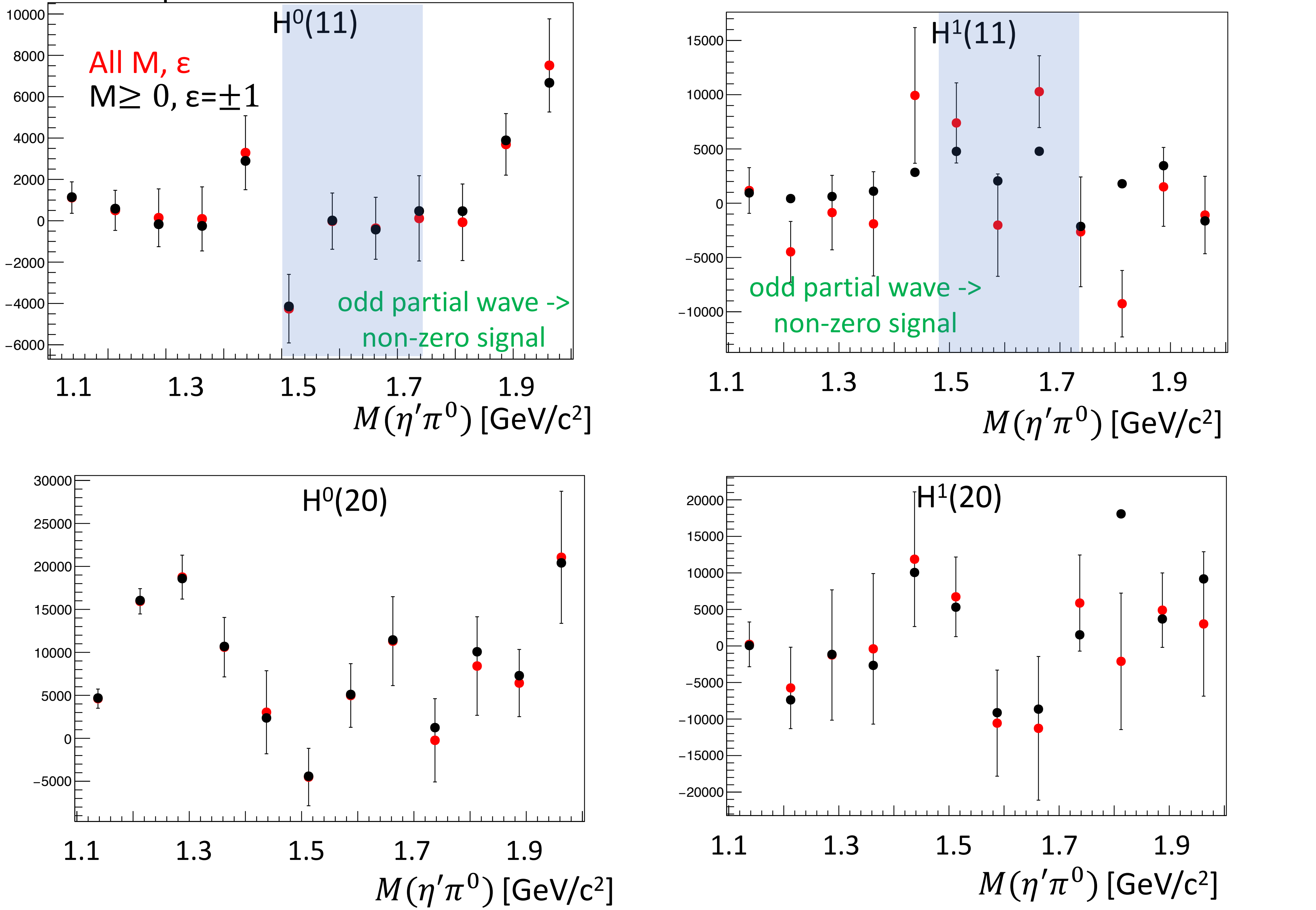




Interference between a_2 and π_1? Search for exotic signal inconclusive due to large uncertainties

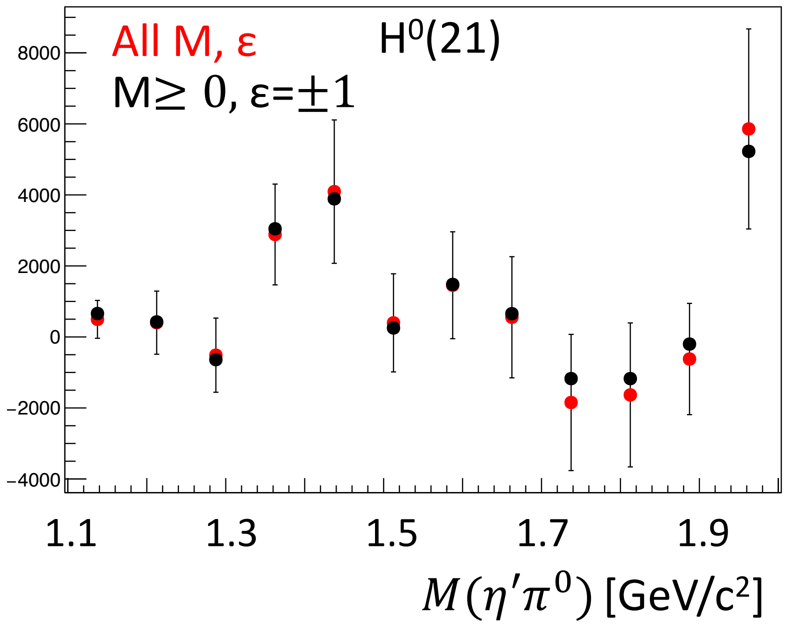
Comparison of moments

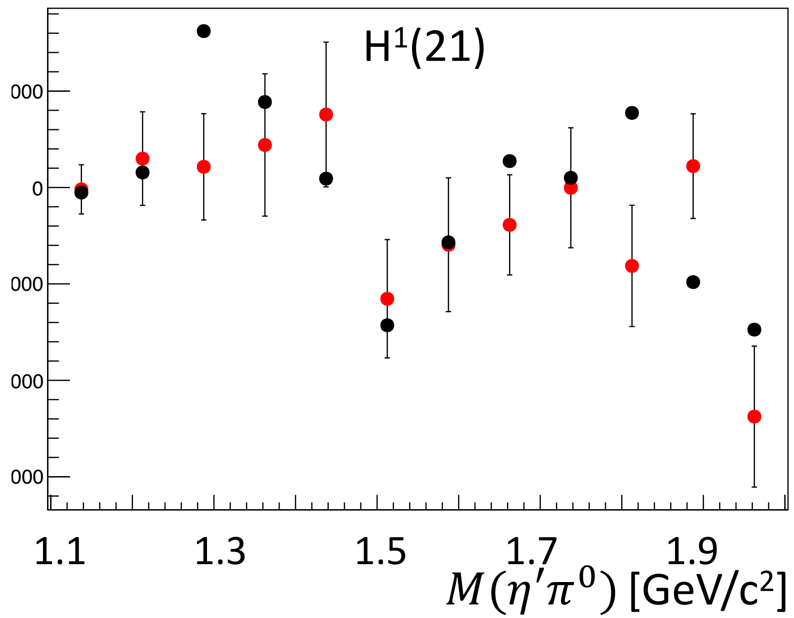
Uncertainties from Bootstrap method

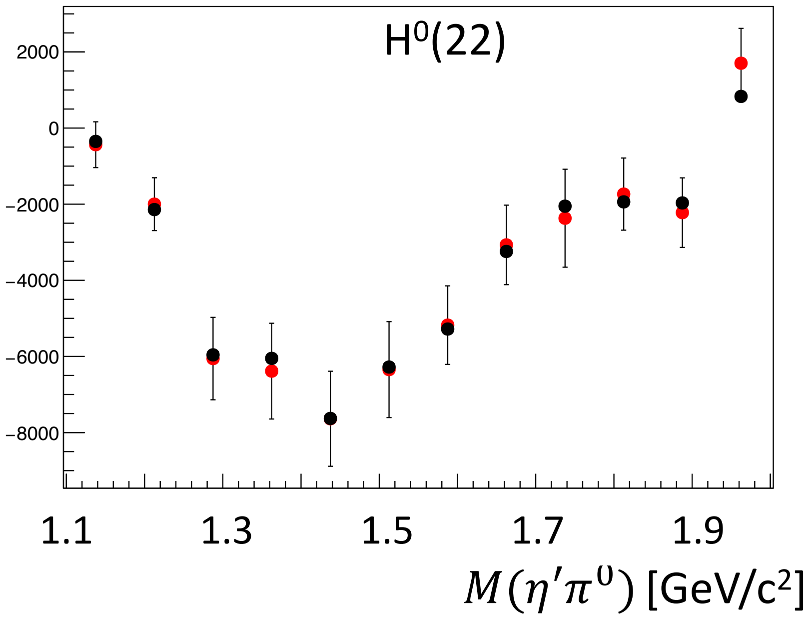


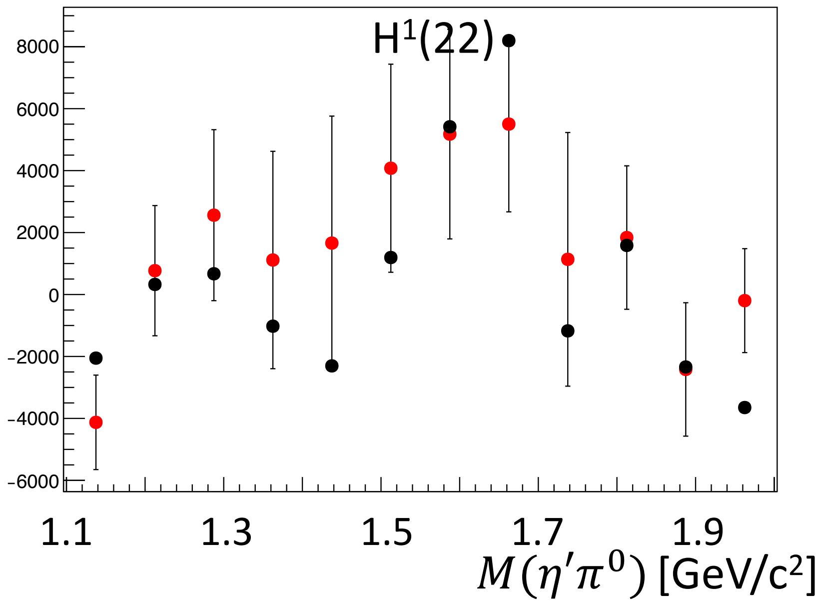
Comparison of moments

Uncertainties from Bootstrap method









Though large uncertainties moments with and without negative M are similar, confirming expectation that M<0 have small contribution

$H^0(00)$ moment agrees with total intensity as expected from model for intensity

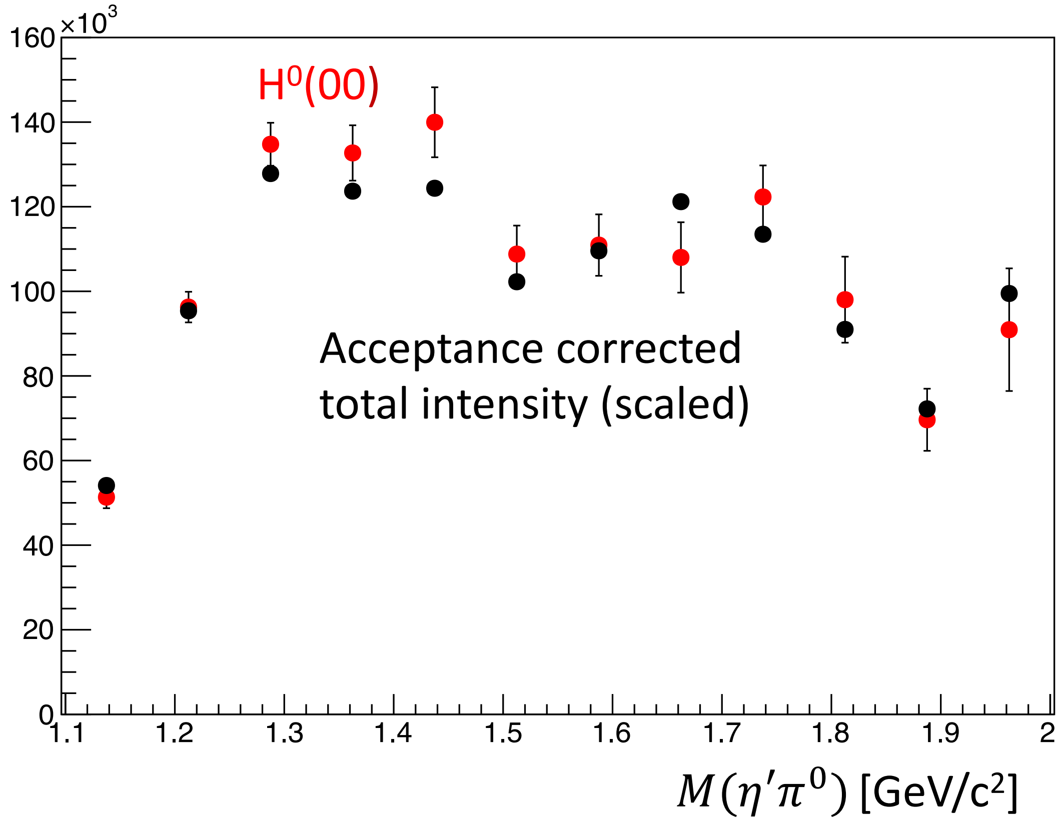
$0.1<|t|<0.7$ $(GeV/c)^2$
Uncertainties from Bootstrap method

$H^0(00)$ moment is equal to total intensity up to normalization factor

$$H^0(LM) = {}^{(+)}H^0(LM) + {}^{(-)}H^0(LM)$$

$${}^{(+)}H^0(00) = 2\left[|S_0^{(+)}|^2 + |P_{-1}^{(+)}|^2 + |P_0^{(+)}|^2 + |P_1^{(+)}|^2 + |D_{-2}^{(+)}|^2 + |D_{-1}^{(+)}|^2 + |D_0^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right]$$

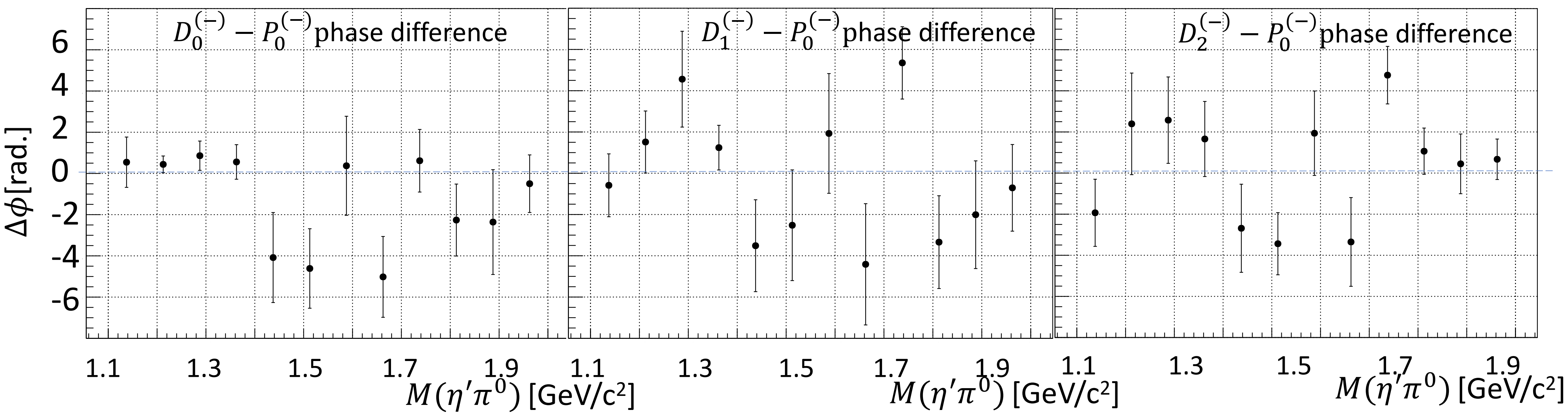
${}^{(-)}H^0(LM)$ are obtained from ${}^{(+)}H^0(LM)$ by replacing the $\varepsilon = +$ waves by the $\varepsilon = -$ waves.



Phase differences from fit with S, P, D waves, M$\geq$ 0, ε=$\pm$1

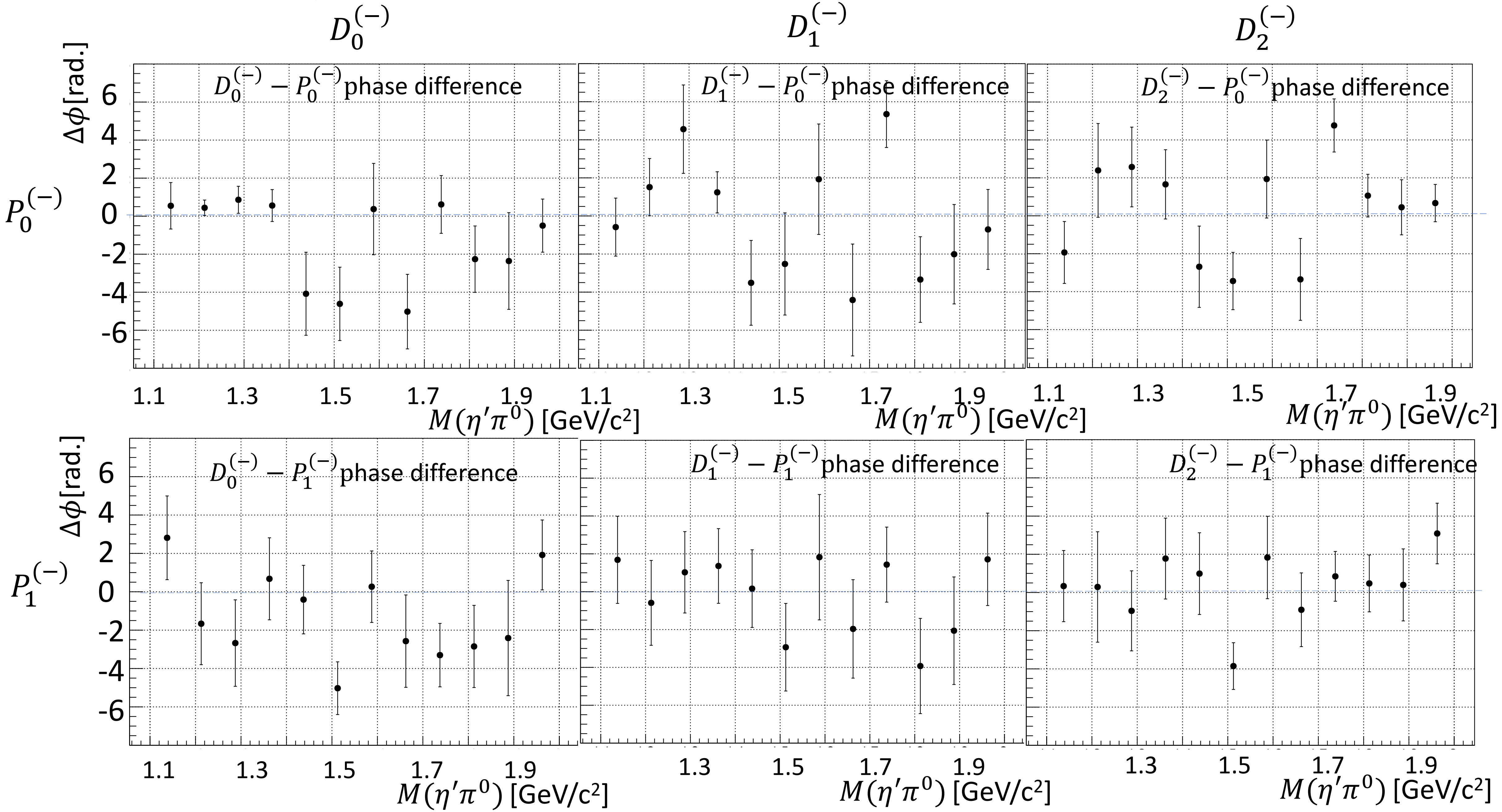
Uncertainties from Bootstrap method

Rapid changes can be indicative of the presence of interfering resonant states. Uncertainties are large to make conclusions



Phase differences from fit with S, P, D waves, M≥ 0, ε=±1

If the wave is dominated by a resonance like the a_2 in the D-wave, it's phase should be the same for all m projections.



Summary

- We have established a pathway for PWA studies
- Have carried out PWA of GlueX $\eta'\pi^0$ data
  - ➢ Acceptance corrected M($\eta'\pi^0$) distribution is almost flat
  - ➢ Low statistics. About 7000 signal events available for partial wave analysis in GlueX-I data set
  - ➢ No statistically significant signal in any of the wave intensities
  - ➢ No conclusive exotic signal from moment analysis
  - ➢ No conclusive signature in phase differences

Future

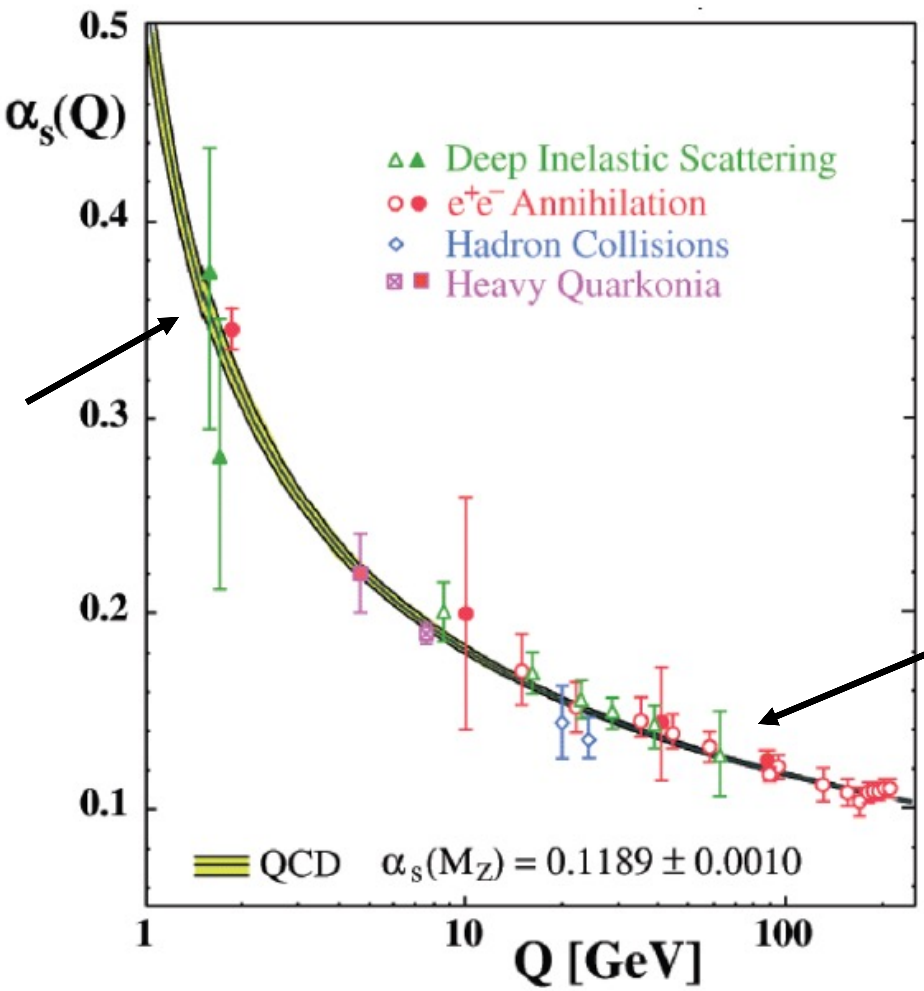
- Add GlueX-II data set (near future double the statistics, full phase two will triple or quadruple the statistics)
- Study the ambiguities in partial wave amplitudes
- Study phase between different waves
- Study fit quality
- Do hybrid fit with mass dependent P-wave
- Do binned PWA for $\eta'\pi^-$ system

Backup slides

Color interactions in QCD

Low energy (long distance limit):

- interactions are strong and increase with distance, so quarks are confined
- use lattice calculations to study QCD in strongly coupled bound states, i.e. hadrons



High energy (short distance limit)
Interactions are weak:

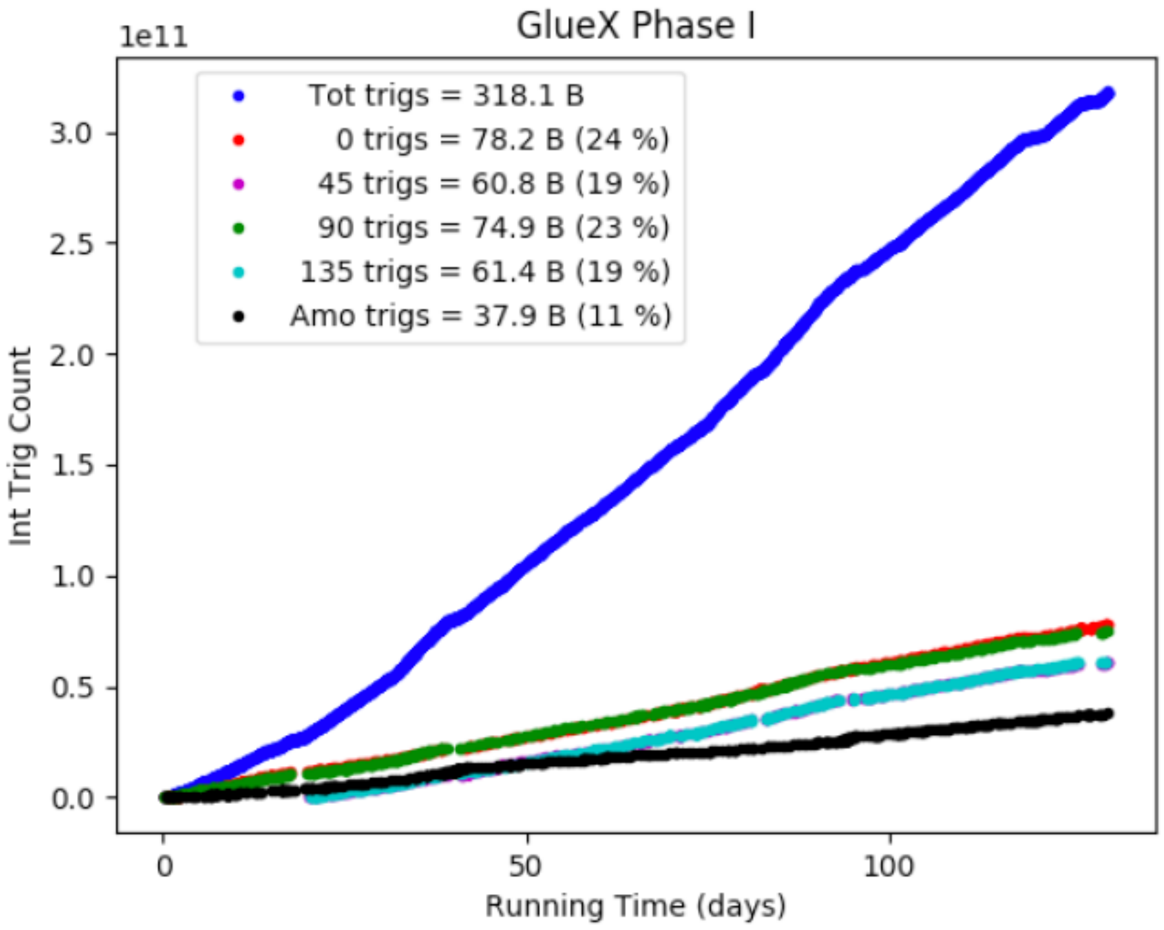
- quarks are "asymptotically free"
- QCD is calculable using perturbation theory

Acquired dataset

GlueX phase I Data (2017, 2018): 3.9 PB (Raw Data)

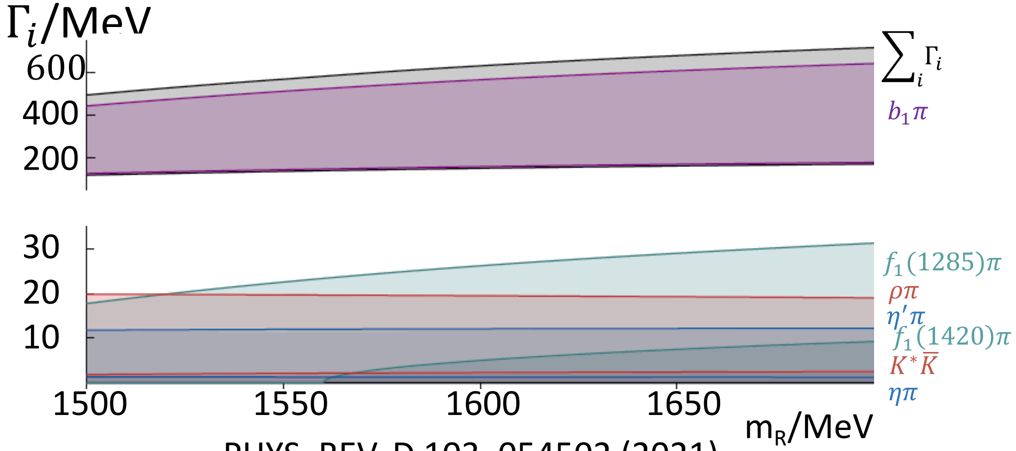
GlueX phase II Data (2020): 14 PB

Approximate GlueX phase I yields			
$\rho(770)$	200 M	$\eta\pi$	2M
$\omega(782)$	25 M	$\omega\pi$	10M
$\phi(1020)$	2 M	J/ψ	2k
$\eta'\pi^0(\eta' \rightarrow \pi^+\pi^-\eta, \eta \rightarrow \gamma\gamma)$	7 K		



Integrated number of triggers versus the number of live days in 2017 and 2018.

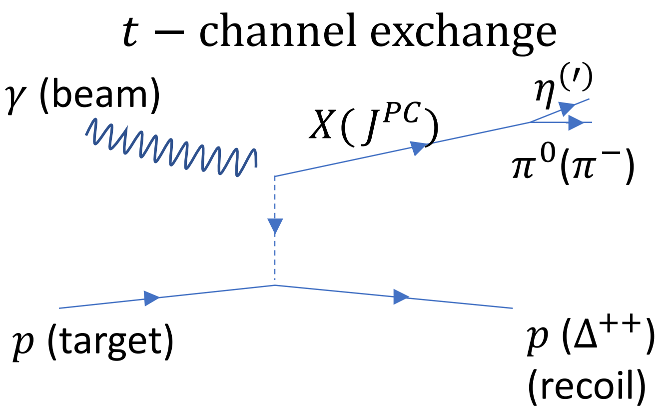
π_1 exotic dominant decay - $b_1\pi$

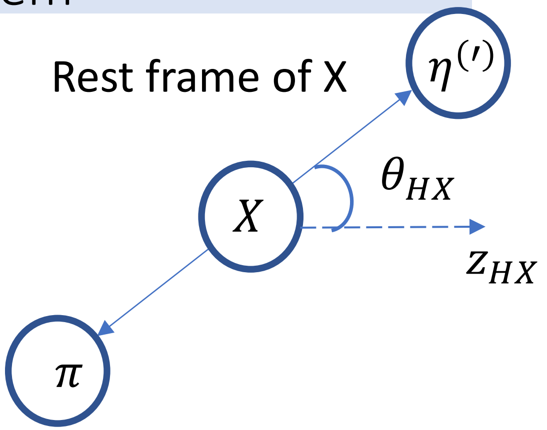


PHYS. REV. D 103, 054502 (2021)

Want to learn about J^{PC} and production mechanism

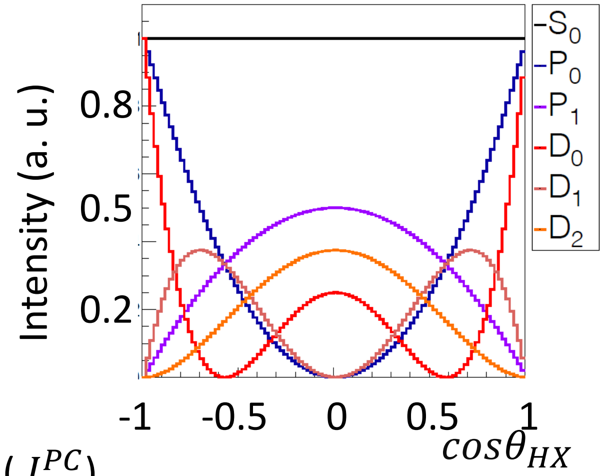
Look for decays of meson $X \to \eta^{(\prime)}\pi$





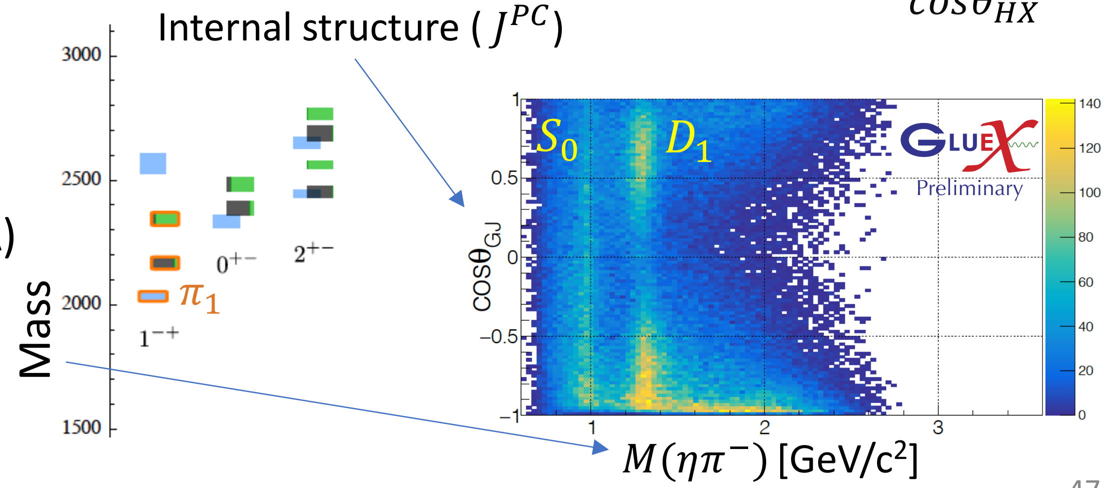
Choose axis so $cos\theta_{HX}$ aligns with spherical harmonics

- The odd waves (P, F, …) in $\eta^{(\prime)}\pi$ system have exotic quantum numbers,



For all events plot of $cos\theta_{HX}$ vs $M(\eta^{(\prime)}\pi)$ and look for signatures of different waves

- Fit intensity to extract wave (S, P, D …) contributions (do PWA)



Moment studies with simulated $\eta^{(\prime)}\pi^0$ events

Implementation of calculation of moments

1. Implement and test calculation of moments in terms of partial waves using the following expressions in terms of the $\eta'\pi^0$ SDMEs calculated in reflectivity basis:

$$H^0(LM) = \sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\, \rho^{\alpha,\ell\ell'}_{mm'} \qquad \rho^{\alpha,ll'}_{mm'} = \sum_{\epsilon} {}^{(\epsilon)}\rho^{\alpha,ll'}_{mm'}$$

$$H(LM) = -\sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\, \rho^{\ell\ell'}_{mm'}$$

$$\begin{aligned}
{}^{(\epsilon)}\rho^{0,\ell\ell'}_{mm'} &= \kappa \sum_k \Big([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\qquad + (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{1,\ell\ell'}_{mm'} &= -\epsilon\kappa \sum_k \Big((-1)^m [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\qquad + (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{2,\ell\ell'}_{mm'} &= -i\epsilon\kappa \sum_k \Big((-1)^m [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\qquad - (-1)^{m'} [\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) , \\
{}^{(\epsilon)}\rho^{3,\ell\ell'}_{mm'} &= \kappa \sum_k \Big([\ell]^{(\epsilon)}_{m;k} [\ell']^{(\epsilon)*}_{m';k} \\
&\qquad - (-1)^{m-m'} [\ell]^{(\epsilon)}_{-m;k} [\ell']^{(\epsilon)*}_{-m';k} \Big) .
\end{aligned}$$

The calculation is implemented by me in "**project_moments_polarized**"
Another version of the calculation based on Vincents codes is called "**Pol_moments_viafittedPW**"
The codes can be found in **halld_sim/src/programs/AmplitudeAnalysis/**

3. I have also added scripts and codes for plotting moments in **hd_utilities/PWA_scripts/Polarized_moments_viaPW**

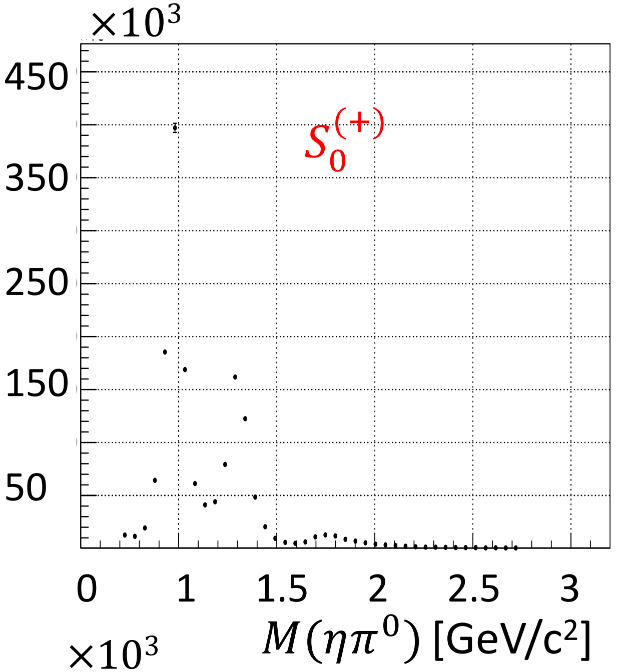
4. Uncertainties on moments from bootstrap method can be found in **hd_utilities/PWA_scripts/Polarized_moments_viaPW/Bootstrapping_M_t_bins_polarized_moments/**

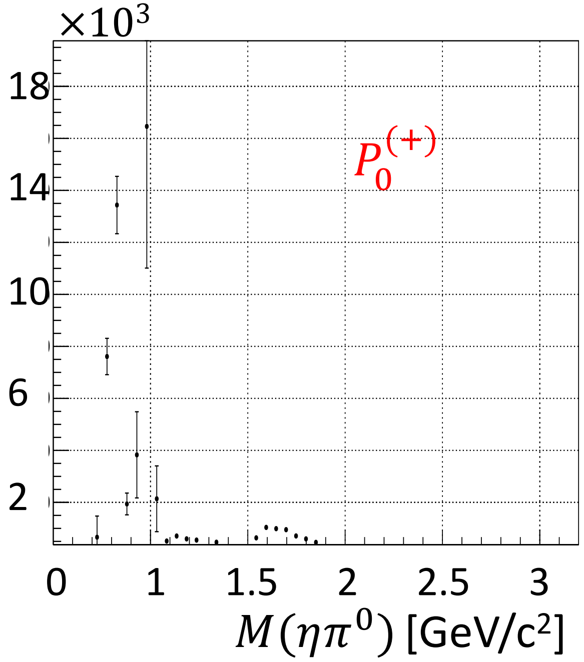
Fitting data with $S_0^{(+)}, P_0^{(+)}, P_1^{(+)}, D_0^{(+)}, D_1^{(+)}, D_2^{(+)}$ amplitude set

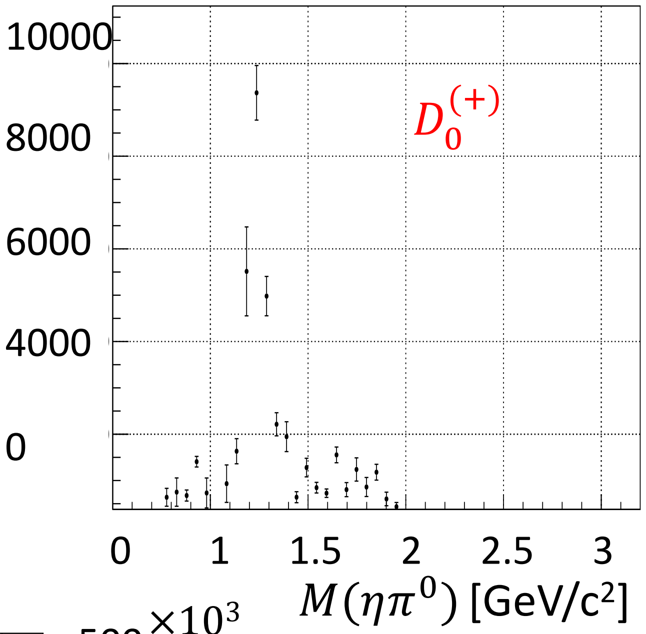
with $S_0^{(+)}, P_0^{(+)}, D_0^{(+)}, D_1^{(+)}, G_0^{(+)}, G_1^{(+)}$.

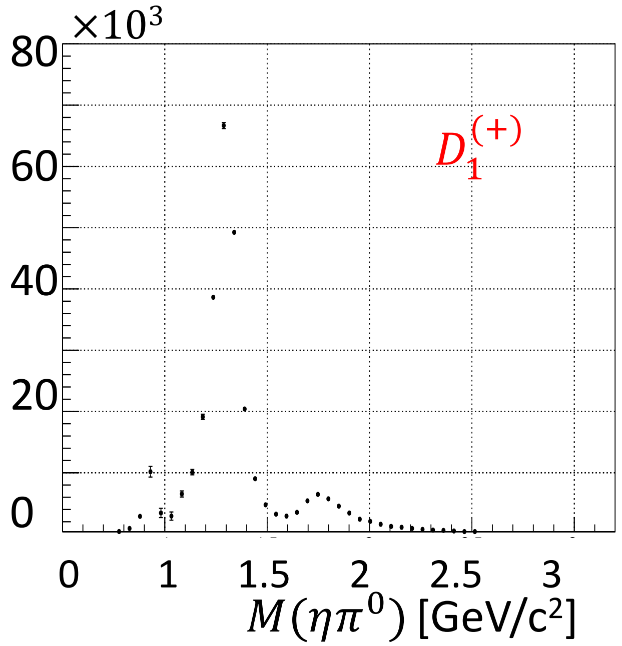
Fit 2 results (fitting in M and t bins)

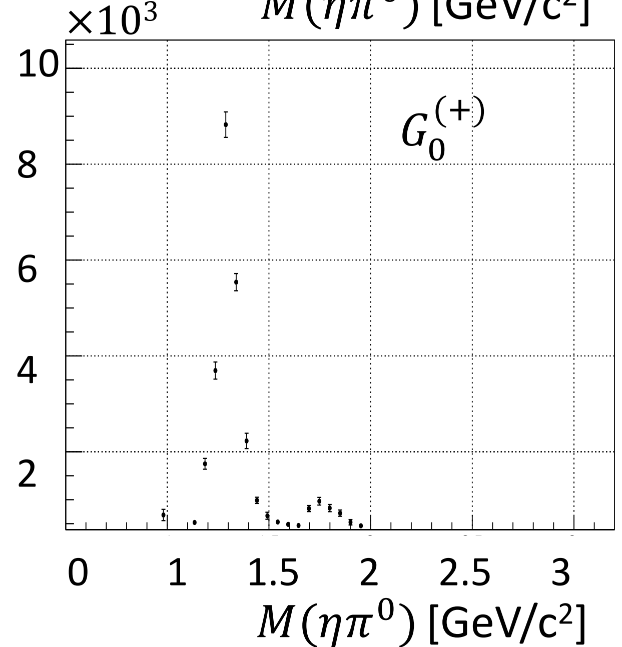
Good starting values for fit parameters

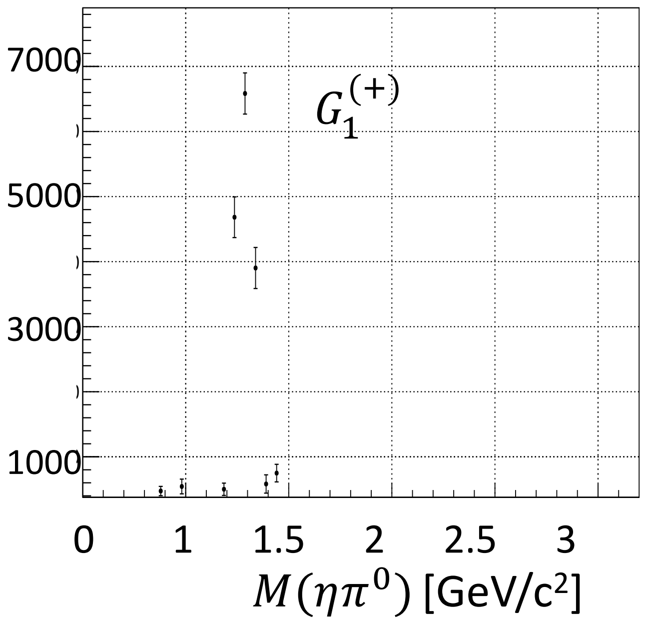


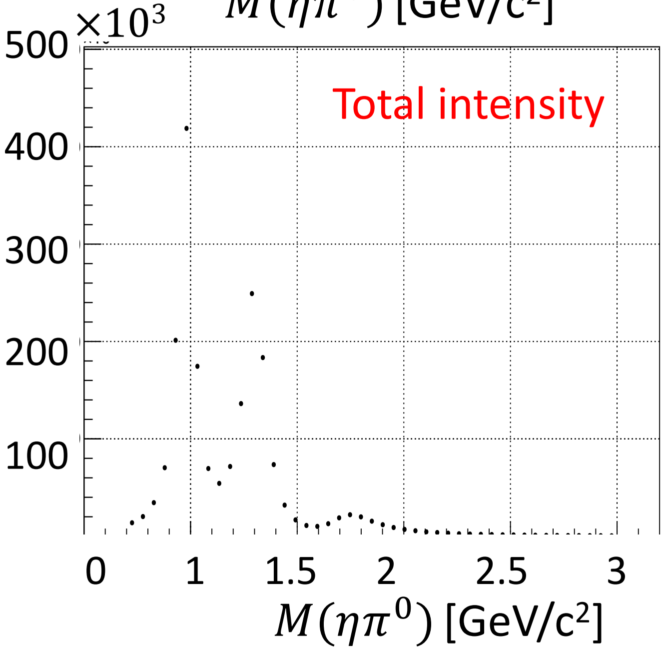








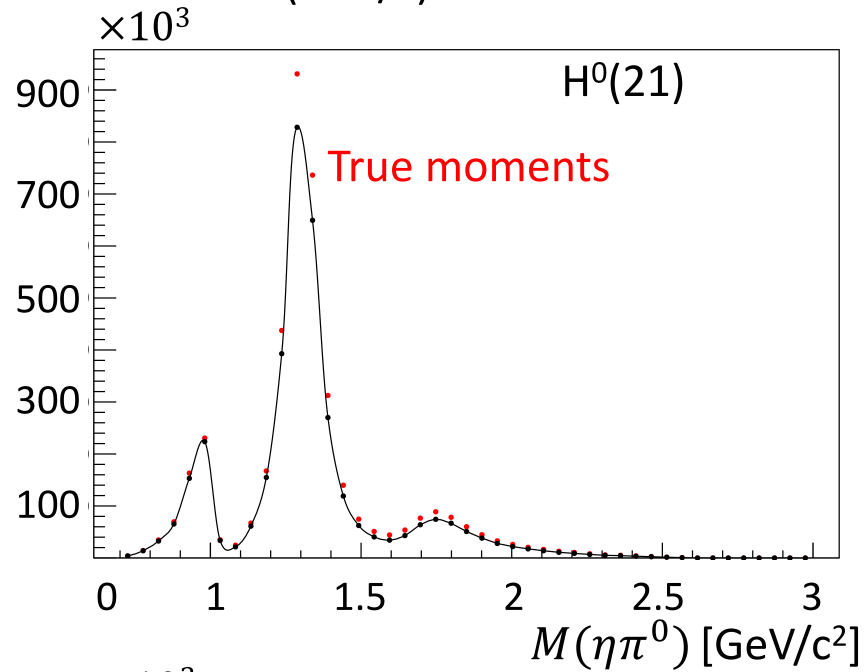


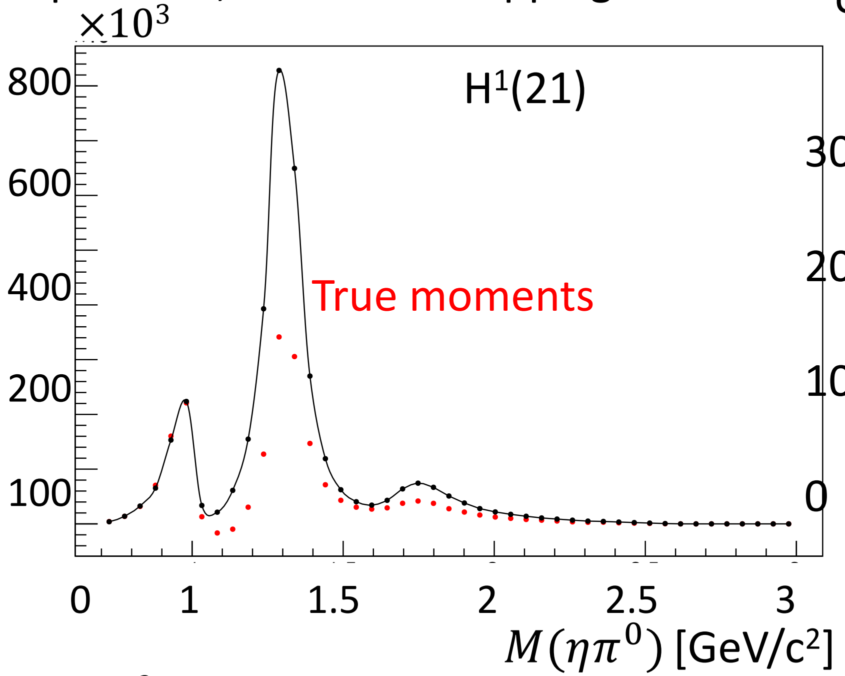


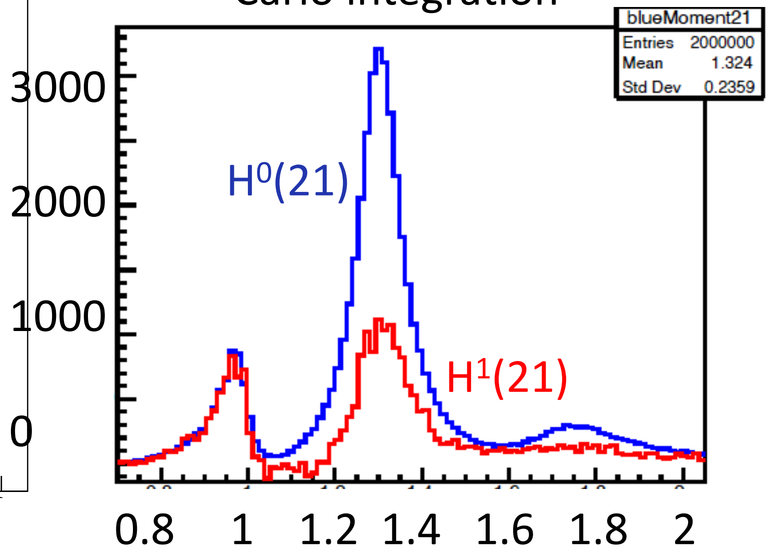
Leaving out any of the waves from original wave set results in poor extracted moments

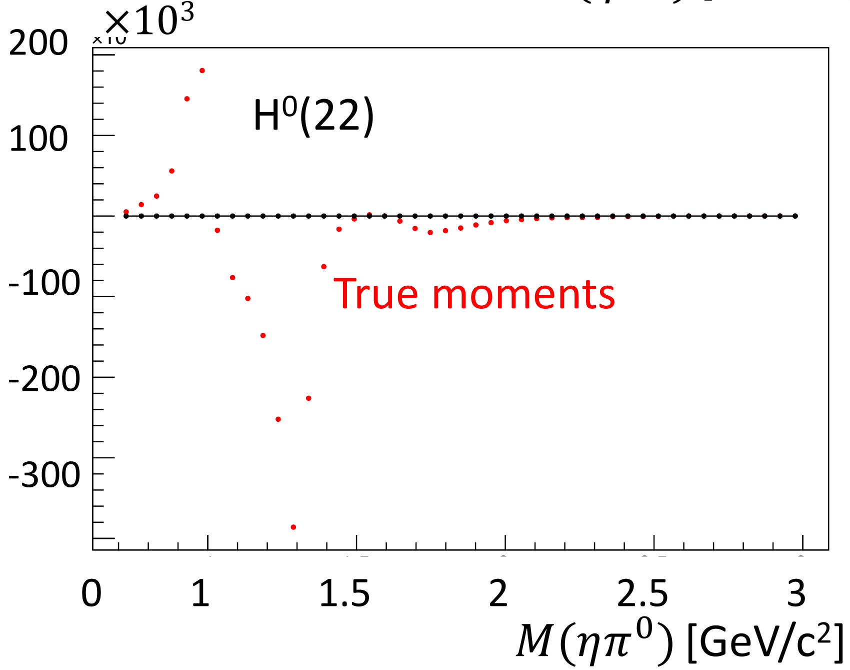
0<t<0.3 (GeV/c)^2 Calculated rom fitted amplitudes , with bootstrapping uncert.

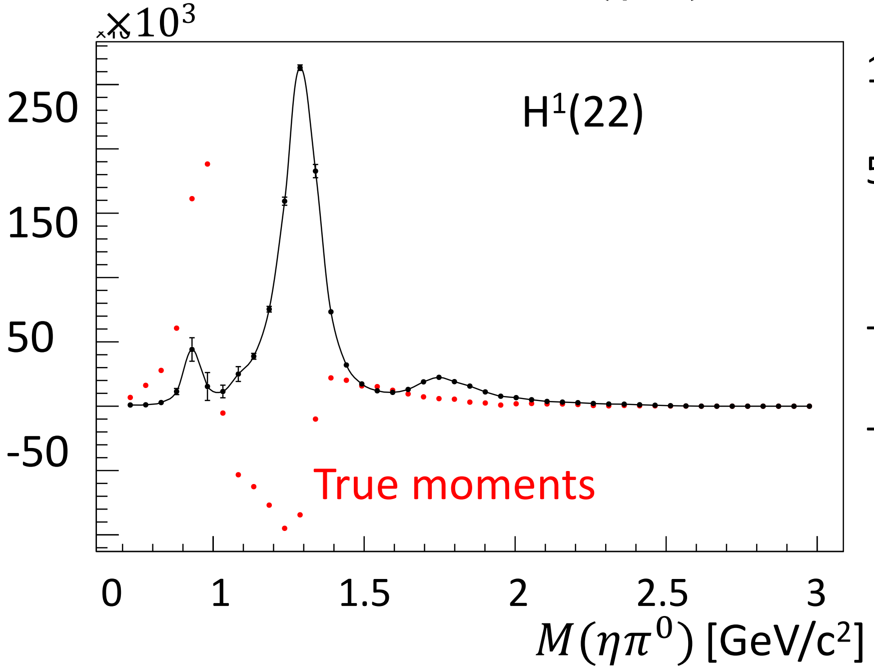
Unnormalized moments from Monte Carlo integration

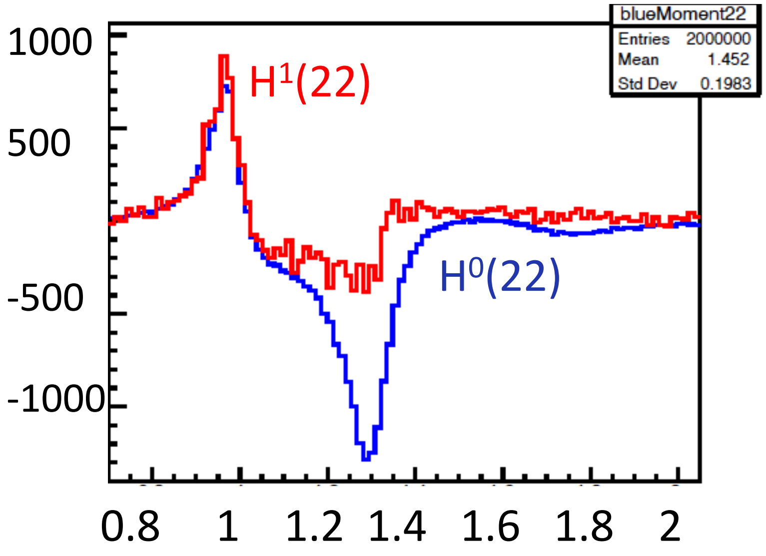










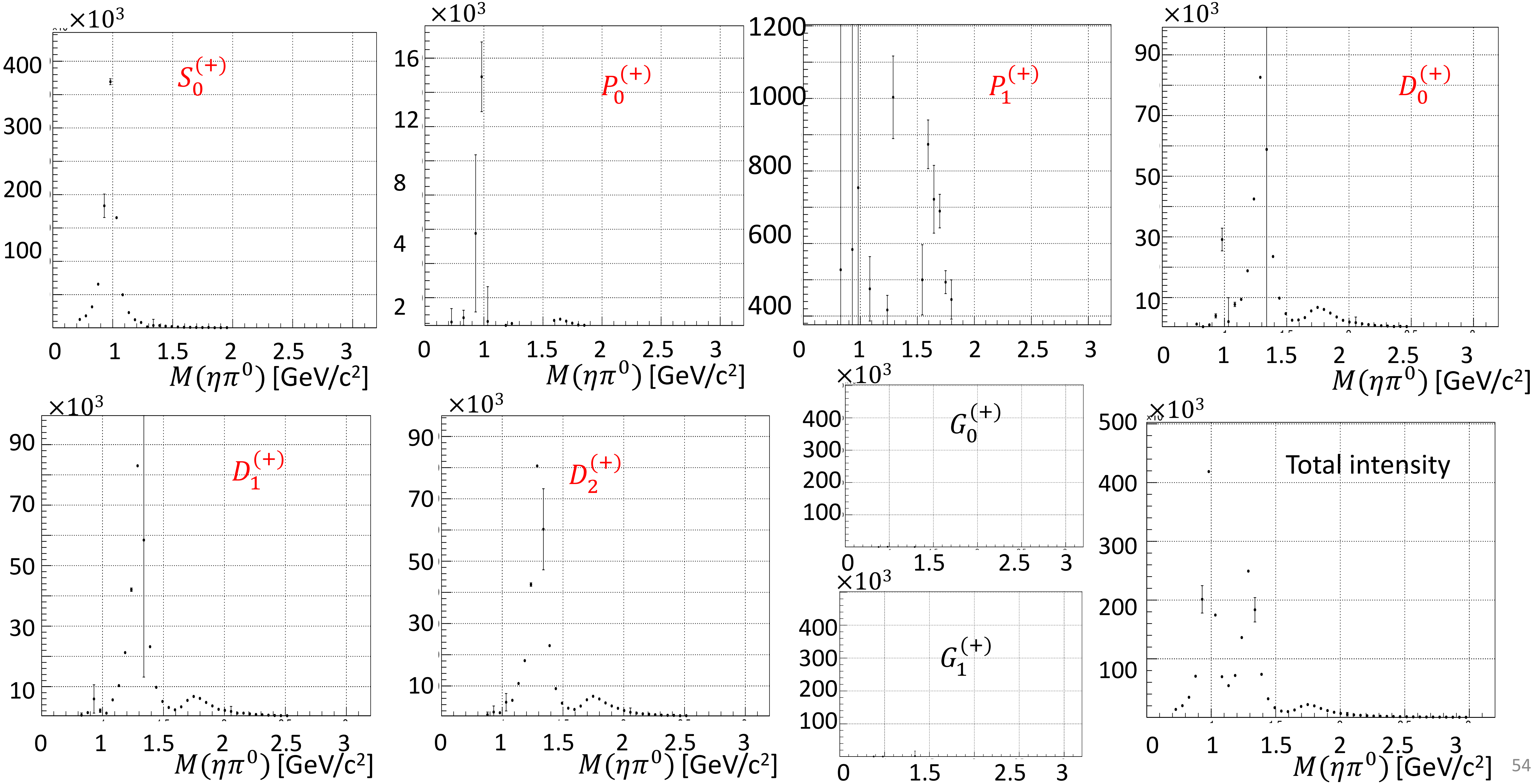


Fitting data with $S_0^{(+)}, P_0^{(+)}, P_1^{(+)}, D_0^{(+)}, D_1^{(+)}, D_2^{(+)}$ amplitude set

with $S_0^{(+)}, P_0^{(+)}, P_1^{(+)}, D_0^{(+)}, D_1^{(+)}, D_2^{(+)}, G_0^{(+)}, G_1^{(+)}$.

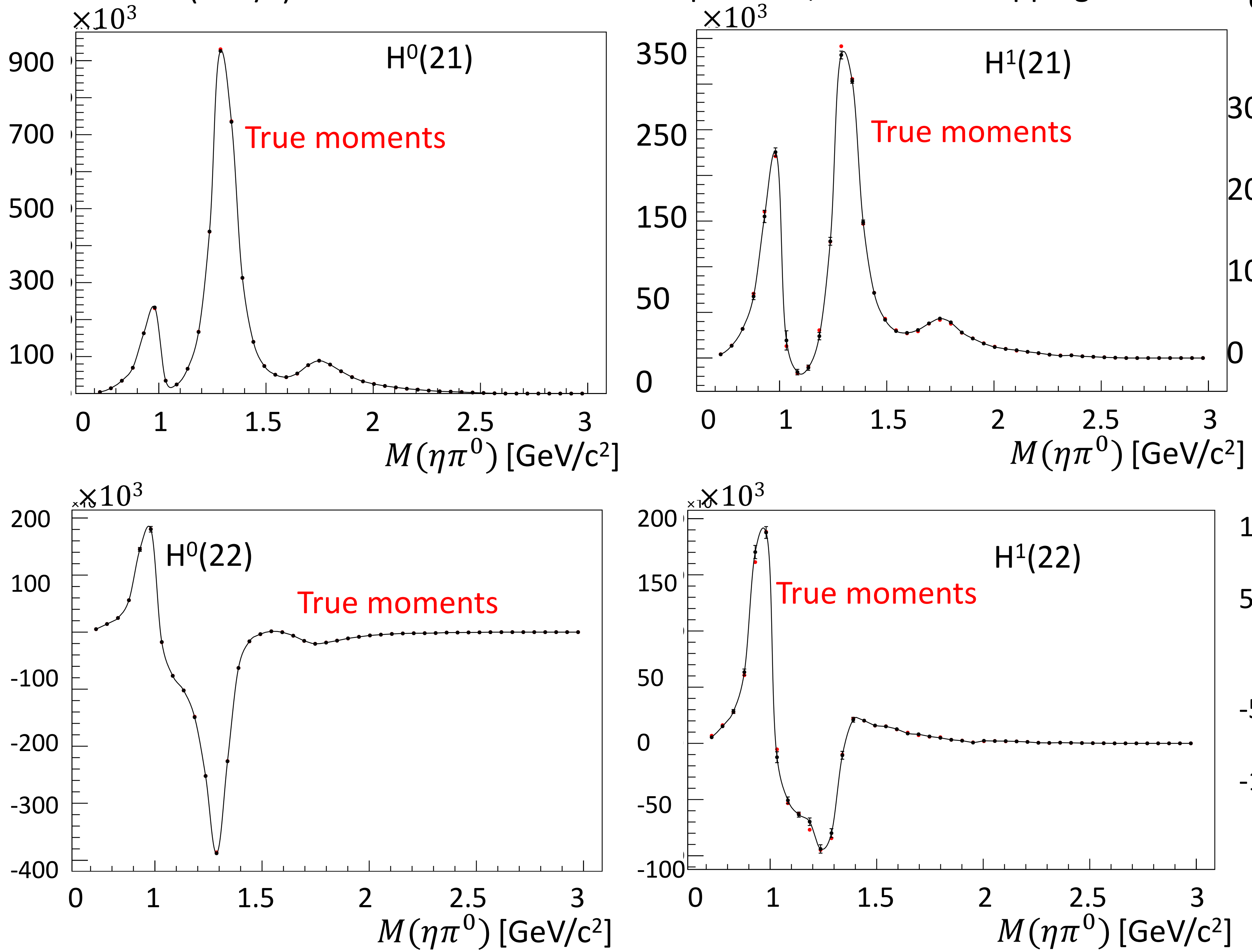
Fit 3 results (fitting in M and t bins)

Good starting values for fit parameters

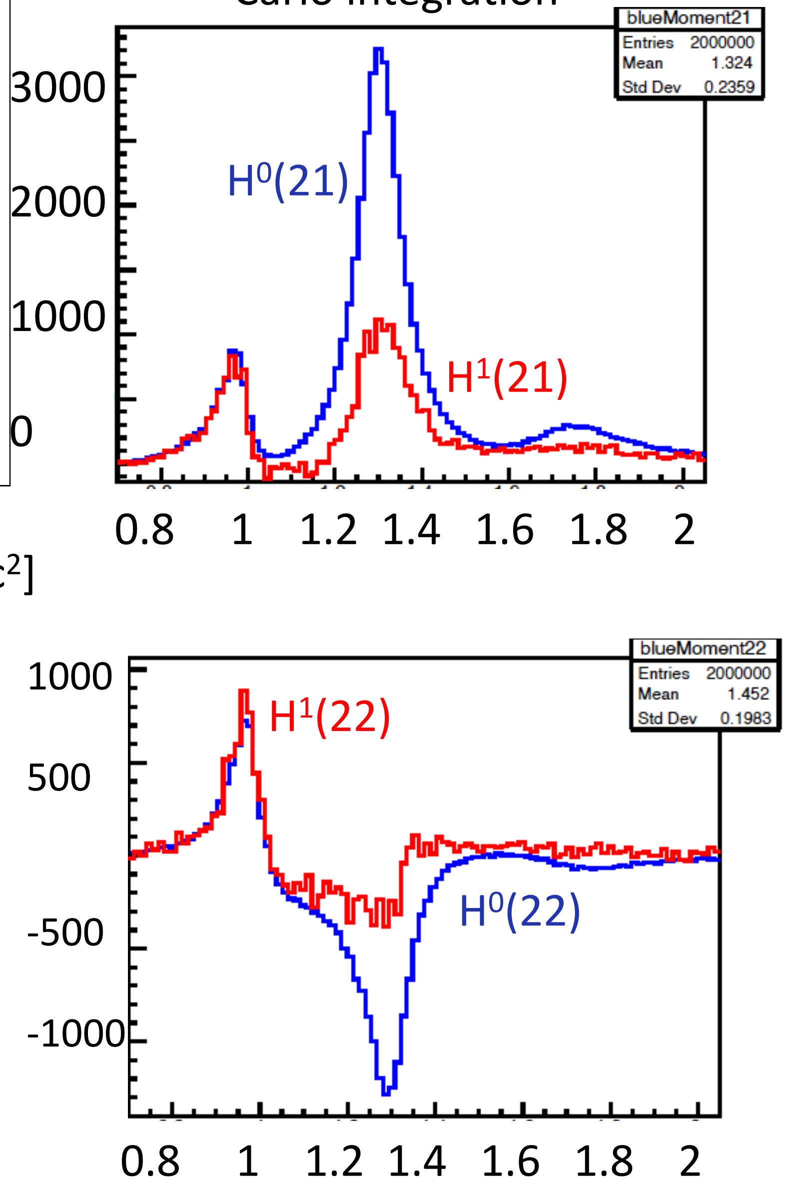


Adding additional amplitudes leaves the result unchanged

$0<t<0.3$ (GeV/c)^2 Calculated rom fitted amplitudes , with bootstrapping uncert.



Unnormalized moments from Monte Carlo integration



5. Compare to the moments calculated using Vincent's codes.

1st method

$$H^0(LM) = \sum_{\ell\ell'} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\ \rho^{\alpha,\ell\ell'}_{mm'}$$

$$H(LM) = -\sum_{\substack{\ell\ell' \\ mm'}} \left(\frac{2\ell'+1}{2\ell+1}\right)^{1/2} C^{\ell 0}_{\ell' 0 L 0} C^{\ell m}_{\ell' m' L M}\ \rho^{\ell\ell'}_{mm'}$$

$$\rho^{0,\ell\ell'}_{mm'} = \frac{\kappa}{2} \sum_{\lambda,\lambda_1,\lambda_2} T^{\ell}_{\lambda m;\lambda_1\lambda_2} T^{\ell' *}_{\lambda m';\lambda_1\lambda_2},$$

$$\rho^{1,\ell\ell'}_{mm'} = \frac{\kappa}{2} \sum_{\lambda,\lambda_1,\lambda_2} T^{\ell}_{-\lambda m;\lambda_1\lambda_2} T^{\ell' *}_{\lambda m';\lambda_1\lambda_2}$$

$$\rho^{2,\ell\ell'}_{mm'} = i\frac{\kappa}{2} \sum_{\lambda,\lambda_1,\lambda_2} \lambda T^{\ell}_{-\lambda m;\lambda_1\lambda_2} T^{\ell' *}_{\lambda m';\lambda_1\lambda_2},$$

$$\rho^{3,\ell\ell'}_{mm'} = \frac{\kappa}{2} \sum_{\lambda,\lambda_1,\lambda_2} \lambda T^{\ell}_{\lambda m;\lambda_1\lambda_2} T^{\ell' *}_{\lambda m';\lambda_1\lambda_2}$$

2nd method

$$H^0(00) = H^1(00) + 2\left[|P_1^{(+)}|^2 + |D_1^{(+)}|^2 + |D_2^{(+)}|^2\right]$$

$$\vdots$$

$$H^1(22) = H^0(22) + \frac{\sqrt{6}}{7}|D_1^{(+)}|^2 + \frac{\sqrt{6}}{5}|P_1^{(+)}|^2$$

Neglect t dependence due to barrier factor $(\sqrt{-t})^{|m-1|}$

$$[\ell]^{(+)}_{m;0} = N_0 N_R \cancel{\left(\delta_R \frac{\sqrt{-t}}{m_R}\right)^{|m-1|}} \Delta_R(m_{\eta\pi}) P_V(s,t)$$

$$\Delta_R(m_{\eta\pi}) = \frac{m_R\Gamma_R}{m_R^2 - m_{\eta\pi}^2 - i m_R\Gamma_R}$$

$$P_V(s,t) = \Gamma[1-\alpha(t)]\left(1 - e^{-i\pi\alpha(t)}\right) s^{\alpha(t)} \qquad \alpha(t) = 0.5 + 0.9t$$

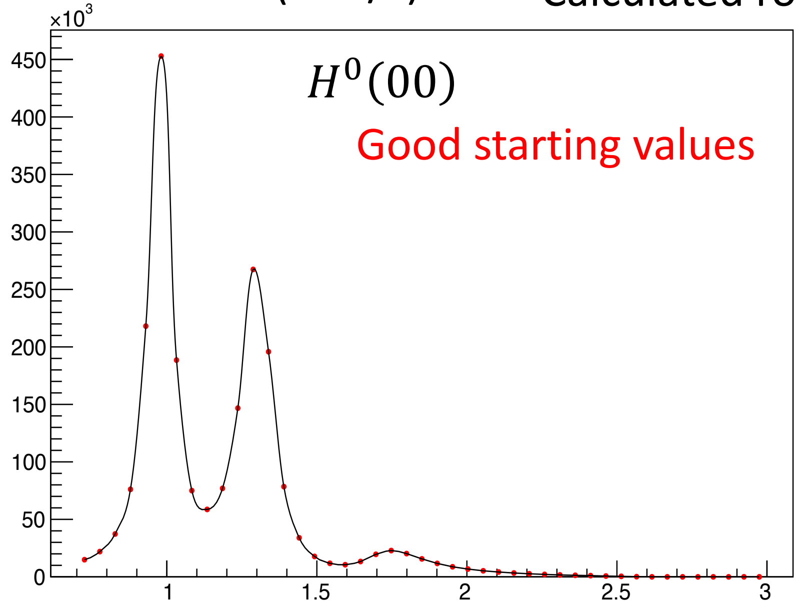
N_0-overall normalization, N_R-relative normalization, δ_R- helicity-flip coupling

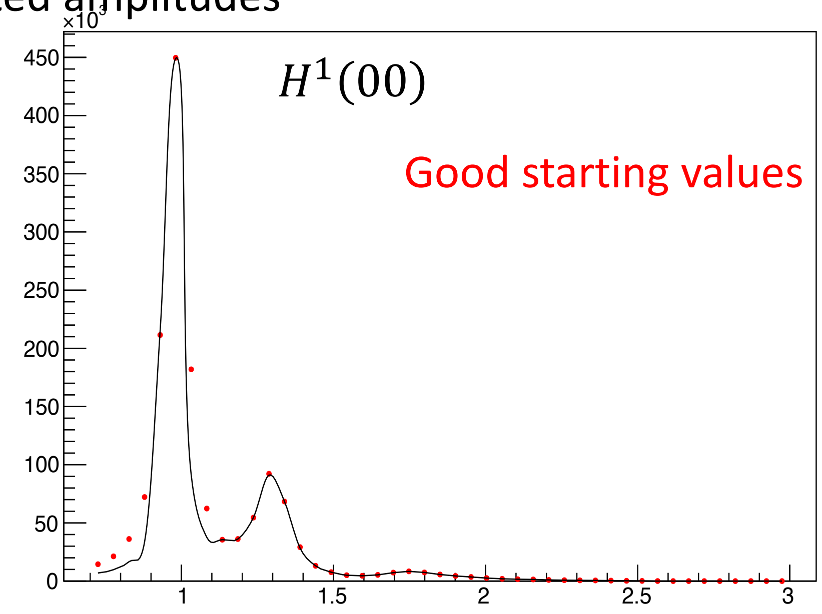
Polarized moments calculated with partial waves

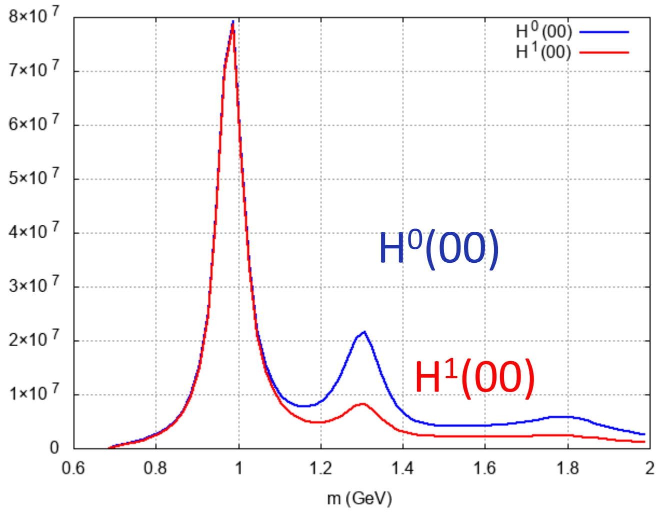
$0<t<0.3\ (GeV/c)^2$

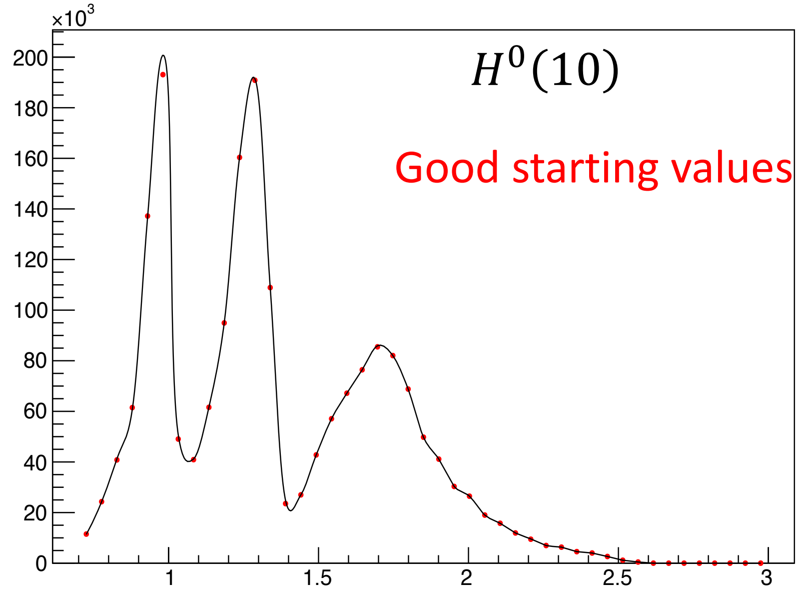
Calculated rom fitted amplitudes

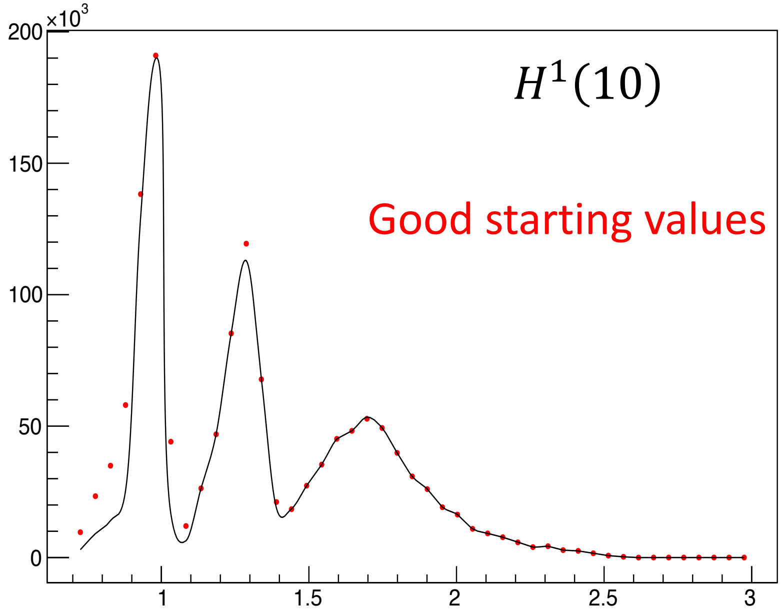
Calculated using Vincent's codes

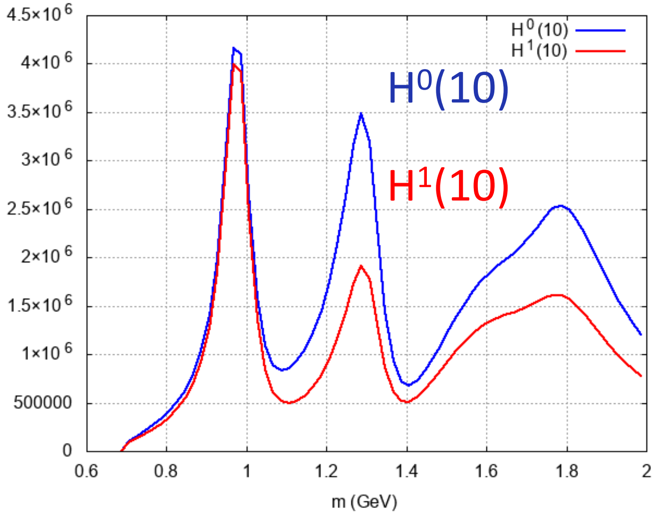










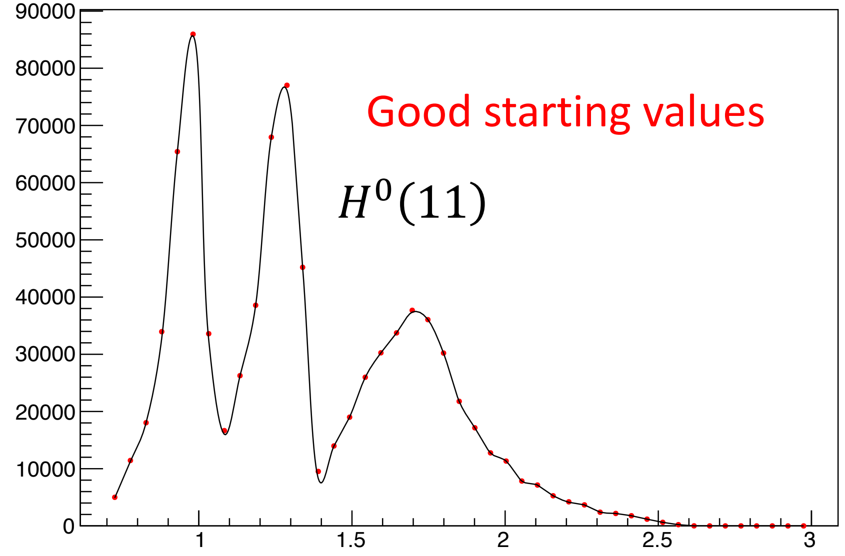


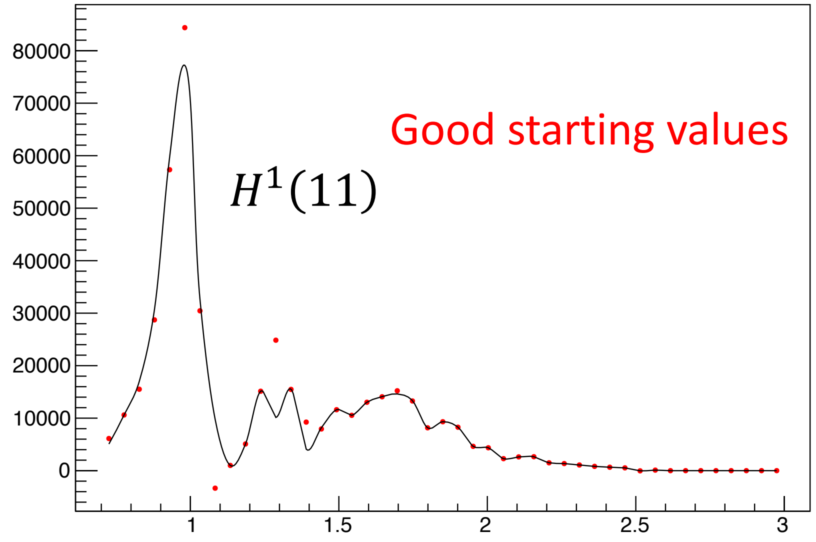
Polarized moments calculated with partial waves

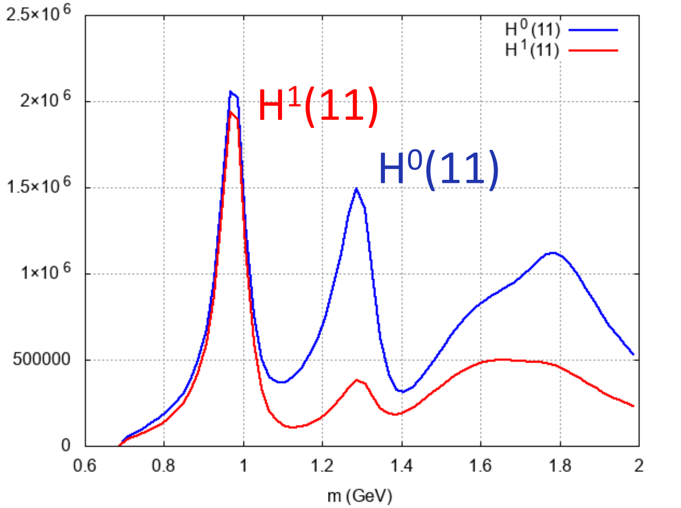
0<t<0.3 (GeV/c)^2

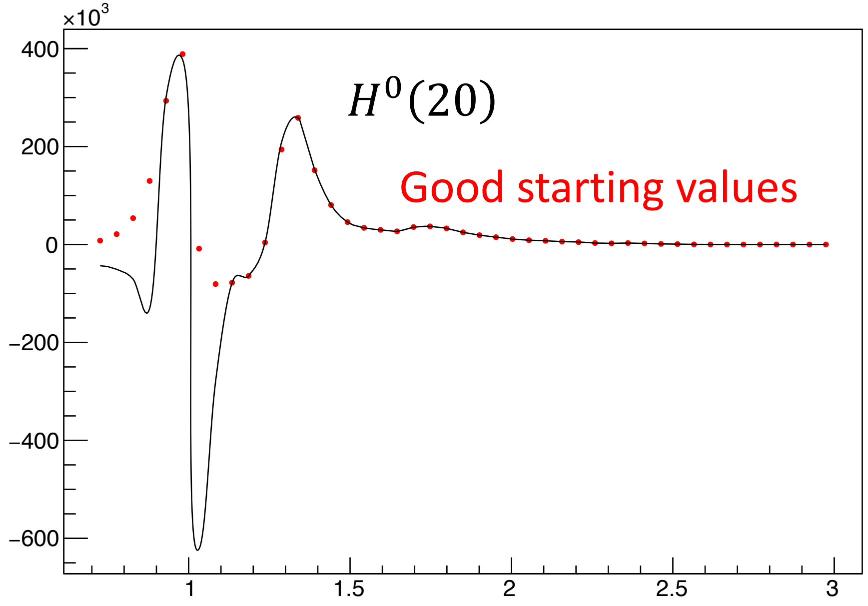
Calculated rom fitted amplitudes

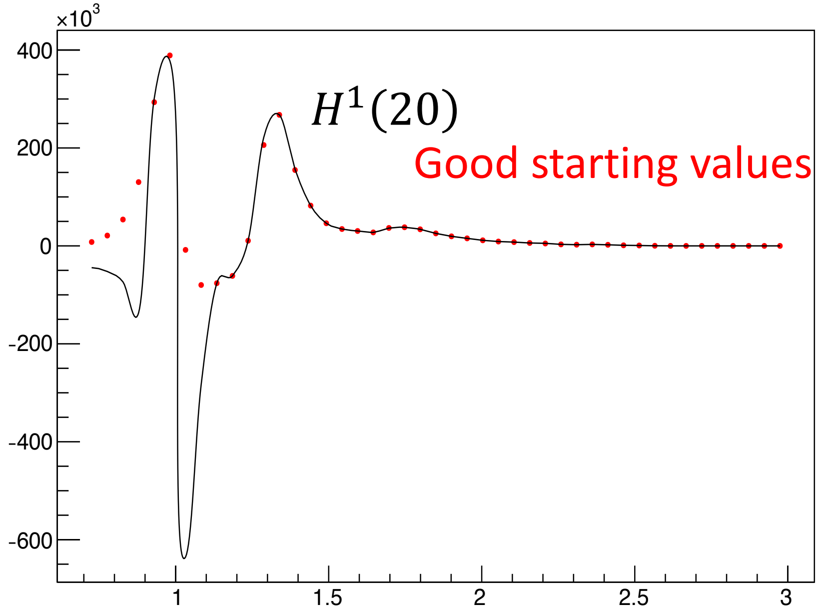
Calculated using Vincent's codes

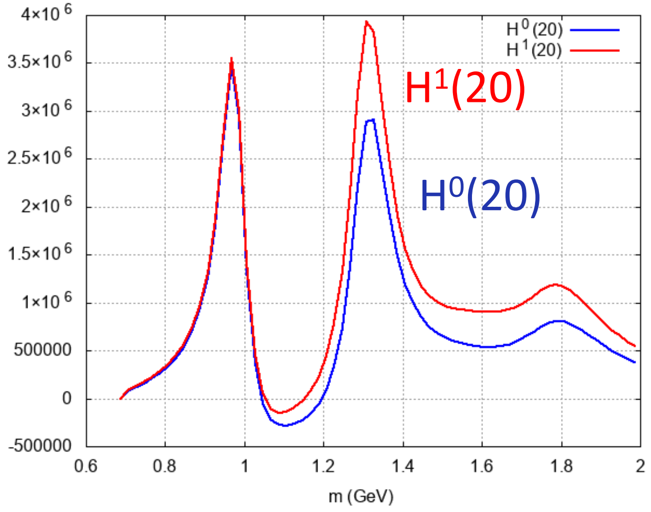










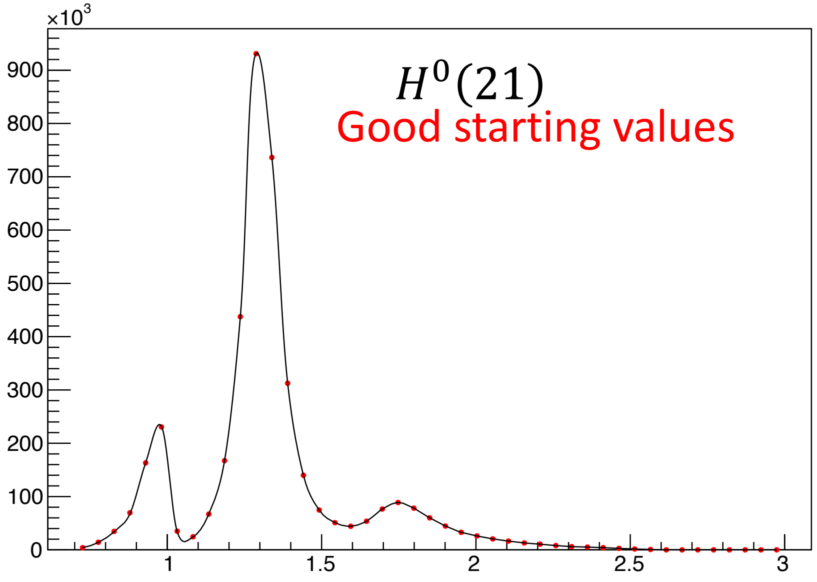


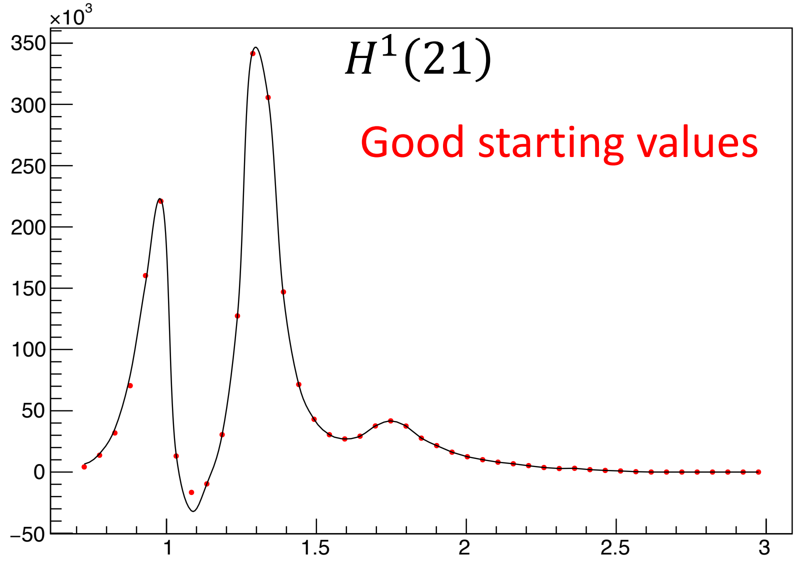
Polarized moments calculated with partial waves

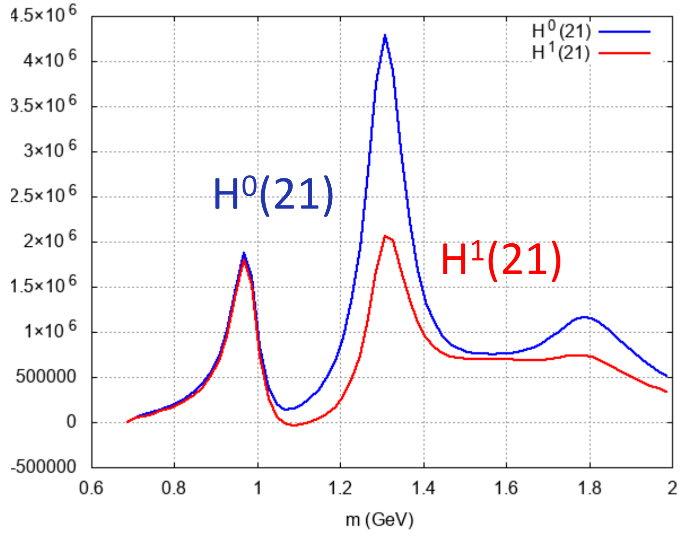
0<t<0.3 (GeV/c)^2

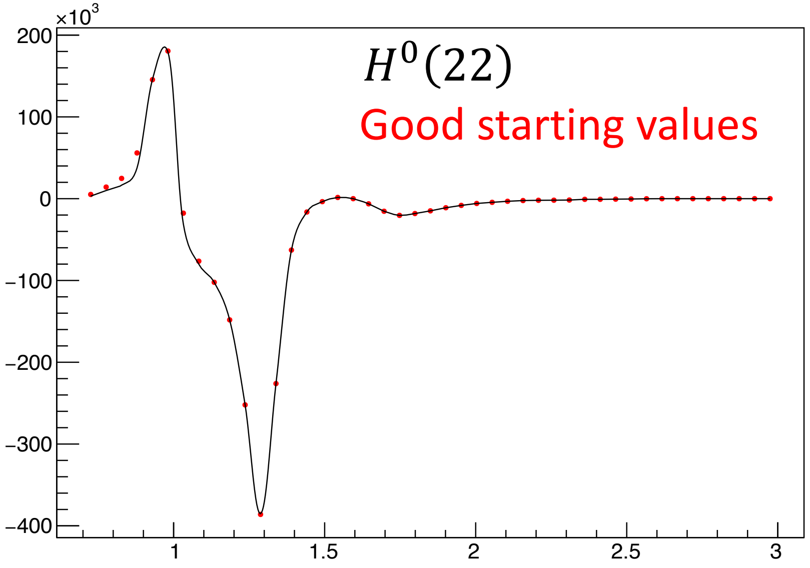
Calculated rom fitted amplitudes

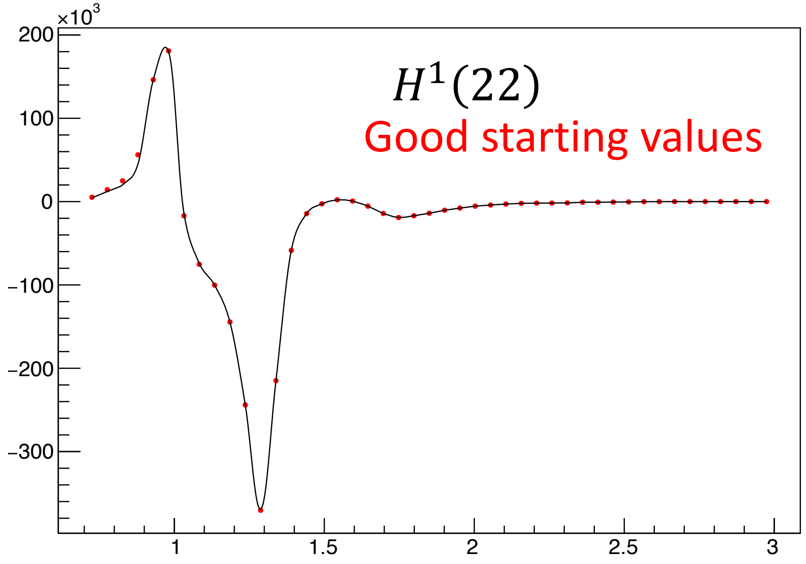
Calculated using Vincent's codes

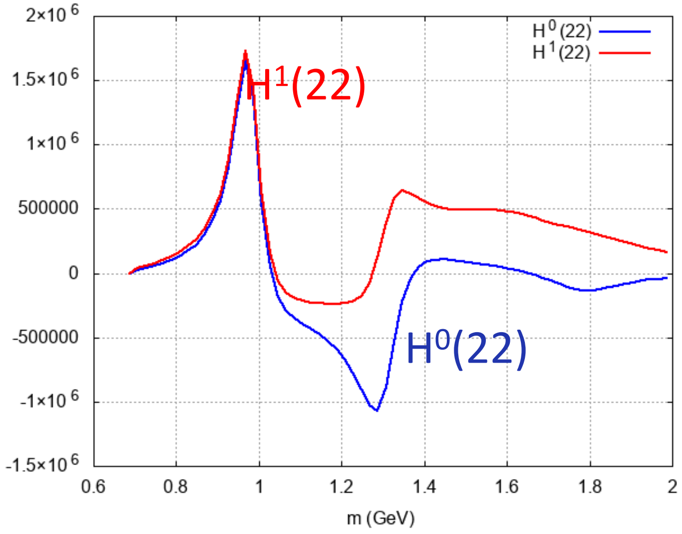








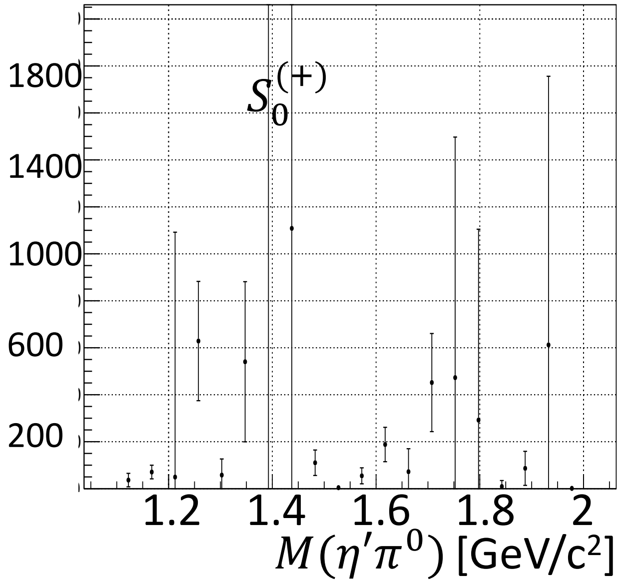


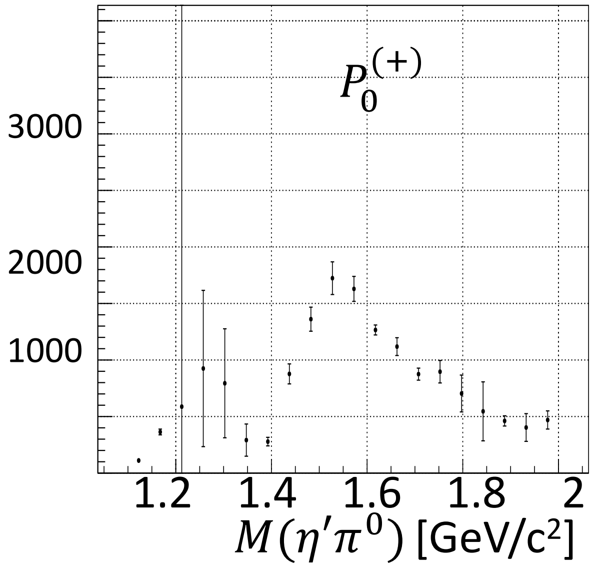


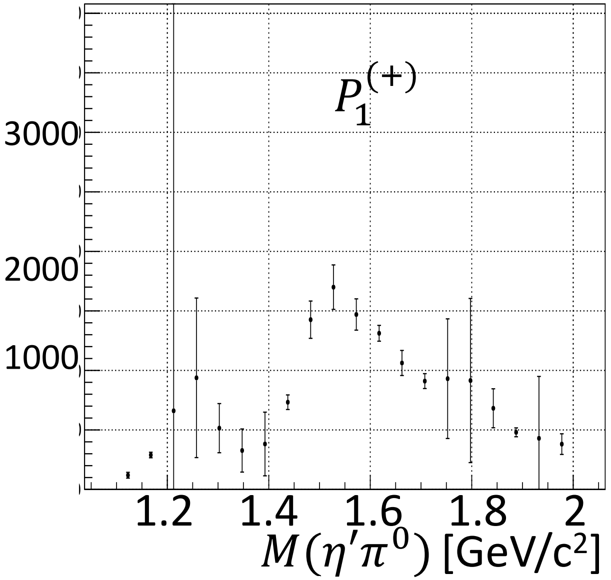
Extraction of polarized moments and exotic signal for generated $(p\eta'\pi^0)$ data with GlueX acceptance

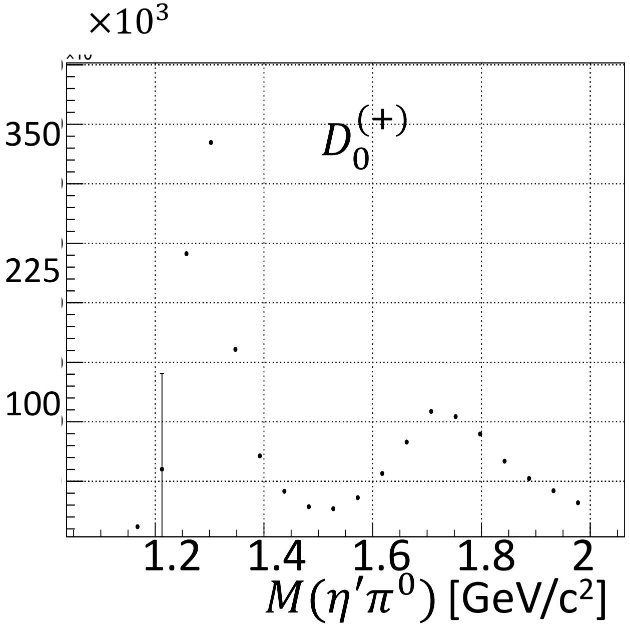
Fit results for generated data (no acceptance)

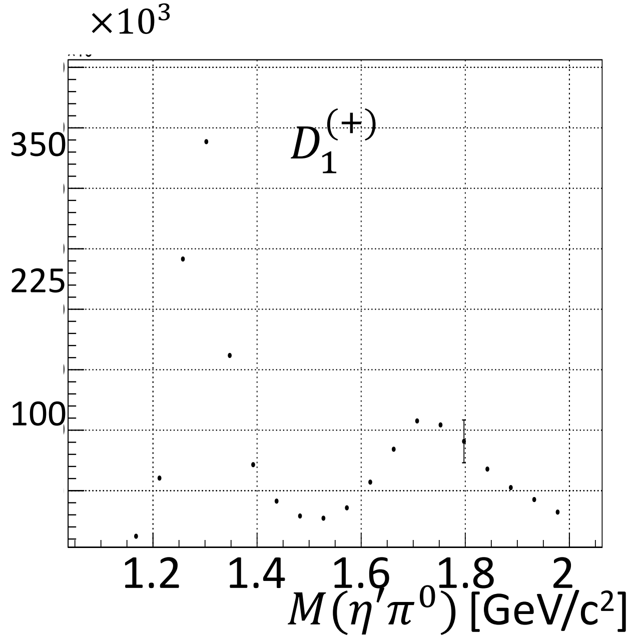
Fitting with amplitude set: $S_0^{(+)}$, $P_0^{(+)}$, $P_1^{(+)}$, $D_0^{(+)}$, $D_1^{(+)}$, $D_2^{(+)}$

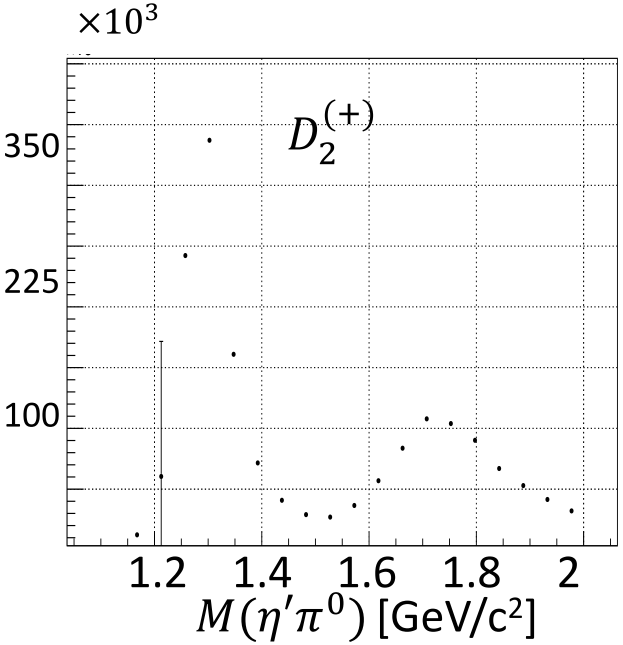


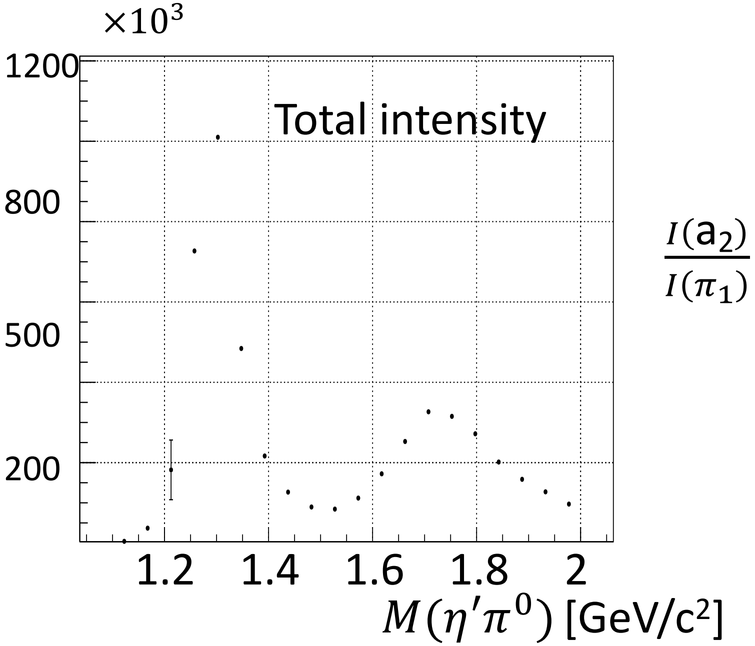












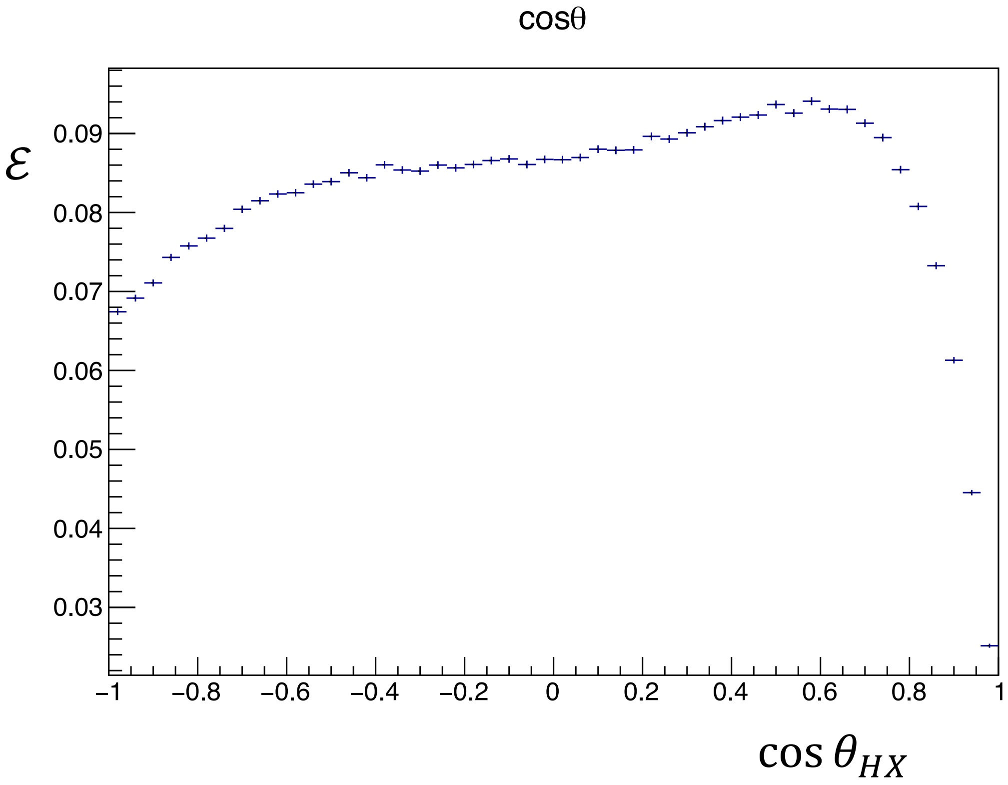
$\frac{I(\mathrm{a}_2)}{I(\pi_1)} \approx 350$ $\frac{I(\mathrm{a'}_2)}{I(\pi_1)} \approx 100$

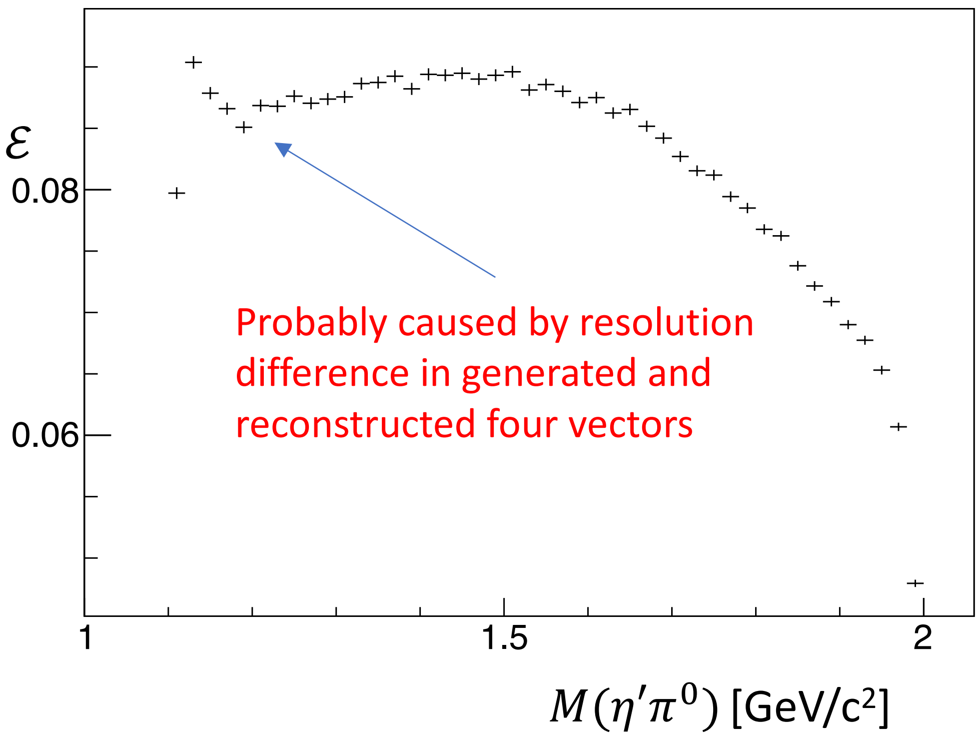
Cuts applied on reconstructed data (Rupeshes analysis cuts)

1. Kinfit confidence level > 10^{-3}
2. Check if combo has already been used with all the particles in it, if so skip it
3. Missing mass squared (-0.02, 0.02) GeV/c^2, coherent beam energy (8, 9) GeV and timing selection (select prompt peak $|\Delta t| < 2.004$ ns)
4. Fcal shower quality cut (>0.5)
5. Select eta ($\gamma_3\gamma_4$) and pi0 ($\gamma_1\gamma_2$) mass region in the $M_{\gamma\gamma}$
6. Reject major $\pi^0\ \pi^0$ events (0.11 GeV/c^2 < $M_{\gamma\gamma}$ < 0.17 GeV/c^2)
7. Analyze only events with one combo (not applied)
8. Select η' mass window in the $M_{\pi^+\pi^-\eta}$

Acceptance with flat data

1. Generate data and pass through the GlueX detector to study acceptance ($\mathcal{E}$) for the reaction $\gamma p \rightarrow p\eta'\pi^0$ (η'-> $\pi^+\pi^-\eta$, $\eta \rightarrow \gamma\gamma$)
   - Have generated 5*10^6 events
   - ~8.5% got reconstructed and have passed analysis cuts (have used Rupeshes analysis code)

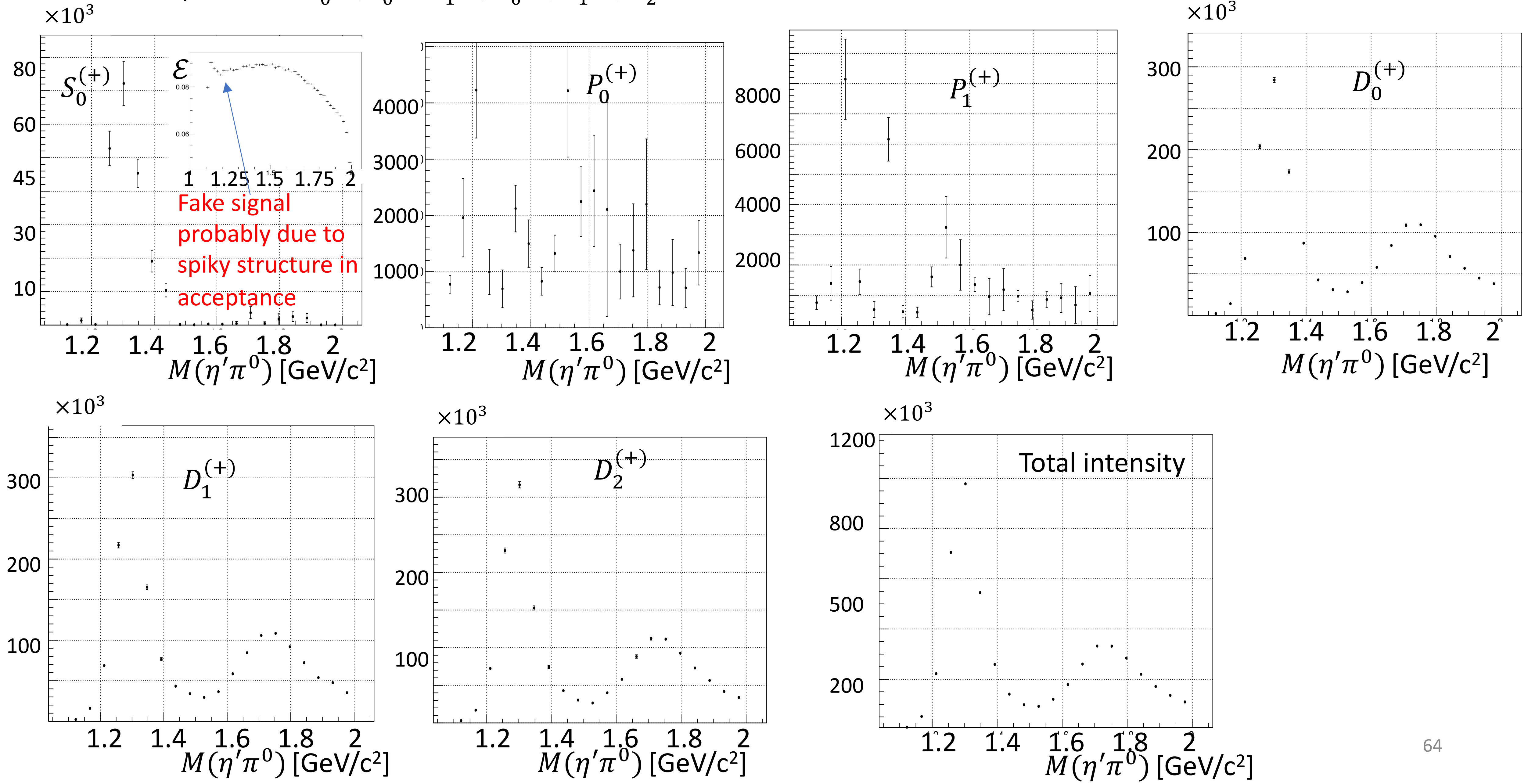




We select -t>0.1 (GeV/c)^2 to cut events, where p had such low –t , that it couldn't get out of the target.

Fit with GlueX acceptance

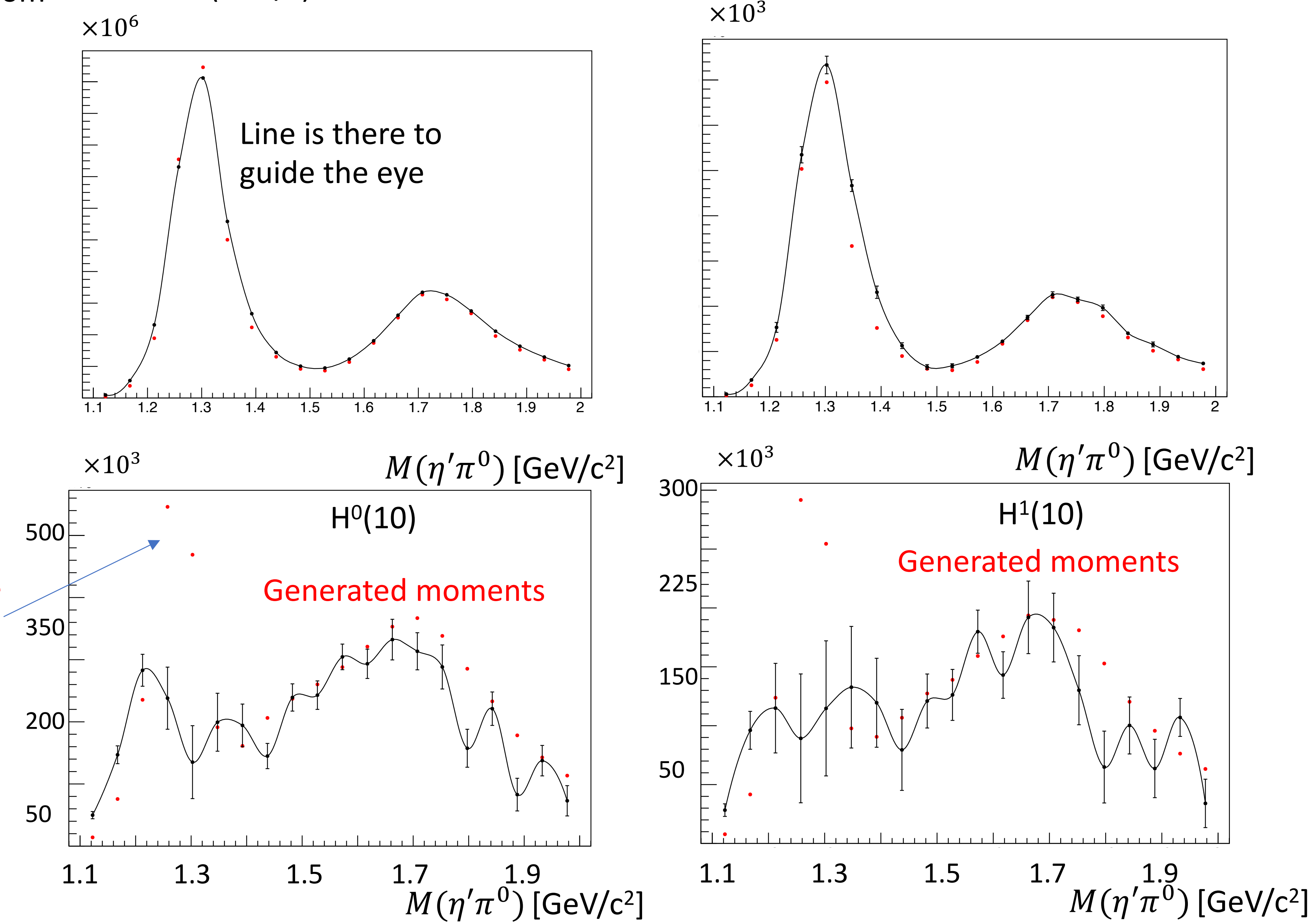
Fitting with amplitude set: $S_0^{(+)}$, $P_0^{(+)}$, $P_1^{(+)}$, $D_0^{(+)}$, $D_1^{(+)}$, $D_2^{(+)}$



Polarized moments calculated with partial waves and GlueX acceptance don’t agree with generated moments at M~1.25

Uncertainties from bootstrapping

0<t<0.3 $(GeV/c)^2$

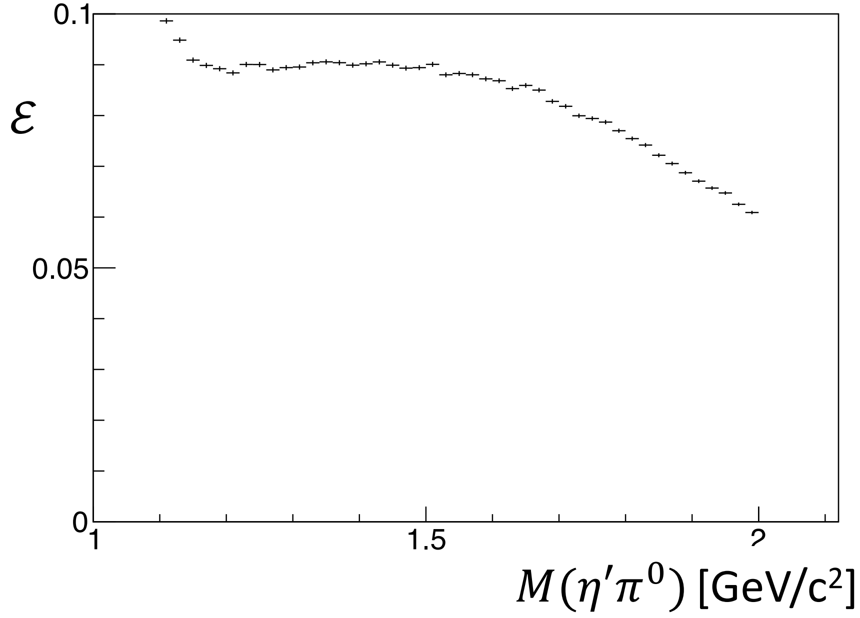


Discrepancy probably due to fake $S_0^{(+)}$ signal

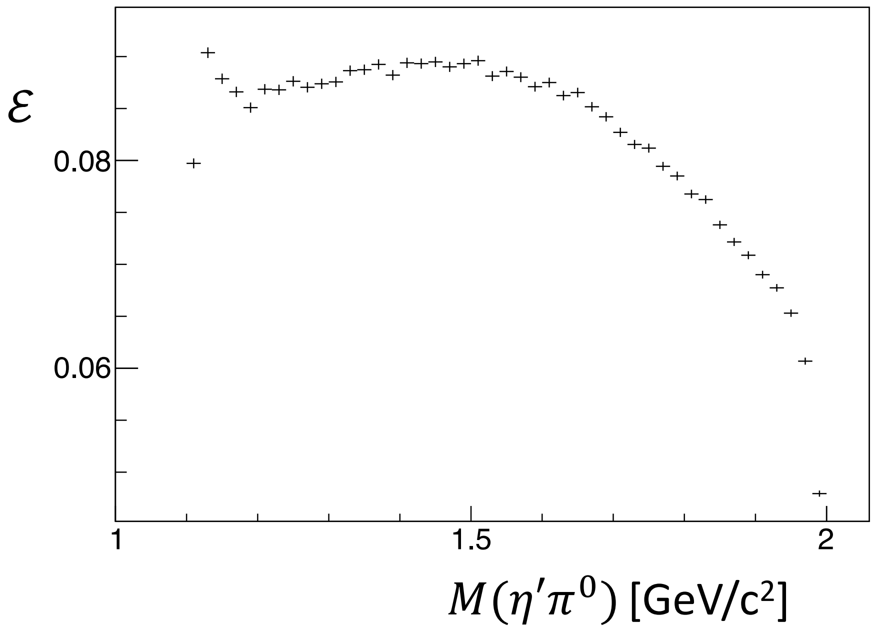
Acceptance with flat data for 0.1<-t<1 $(GeV/c)^2$ with uncertainties: Sensitivity to resolution difference

We select -t>0.1 $(GeV/c)^2$ to cut events, where p had such low –t , that it couldn't get out of the target.

Using generated P4 for reconstructed data



Using reconstructed P4 for reconstructed data



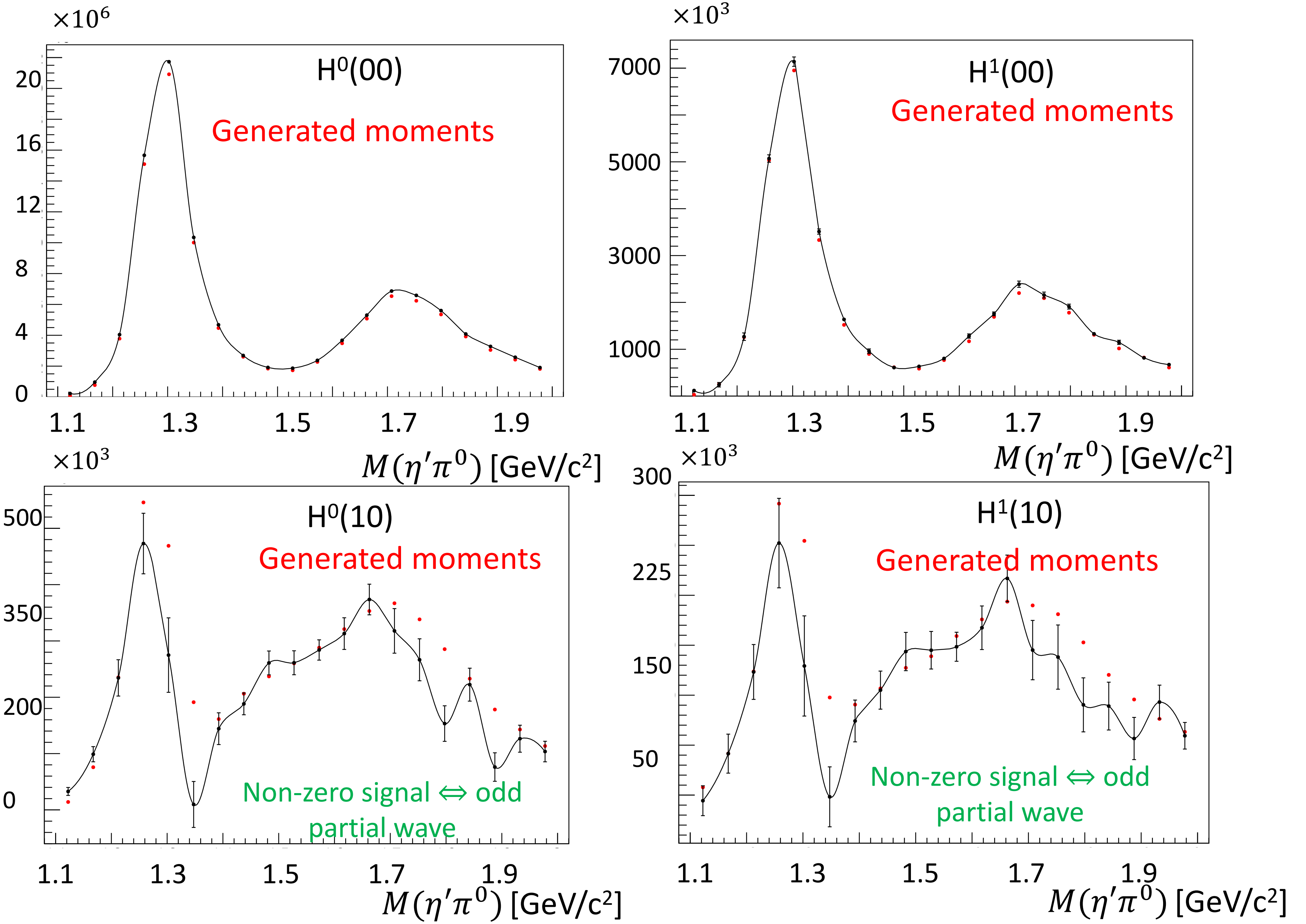
The spiky structure seen in the acceptance at lower M is due to resolution difference between generated and accepted data and wrong reconstructed combos.

Polarized moments agree with generated moments

Uncertainties from bootstrapping

$0<t<0.3$ (GeV/c)^2

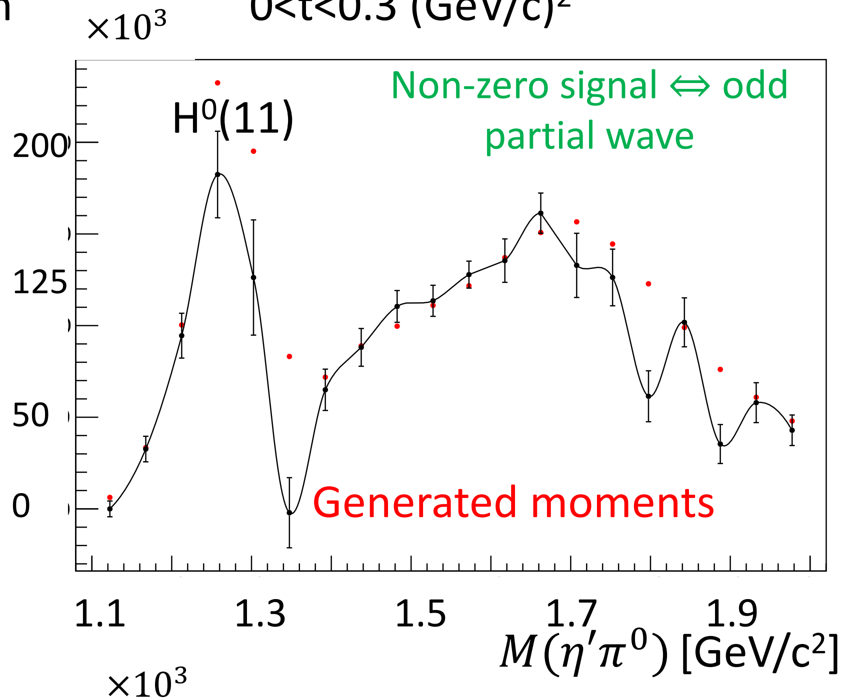
Using generated four vectors for reconstructed MC

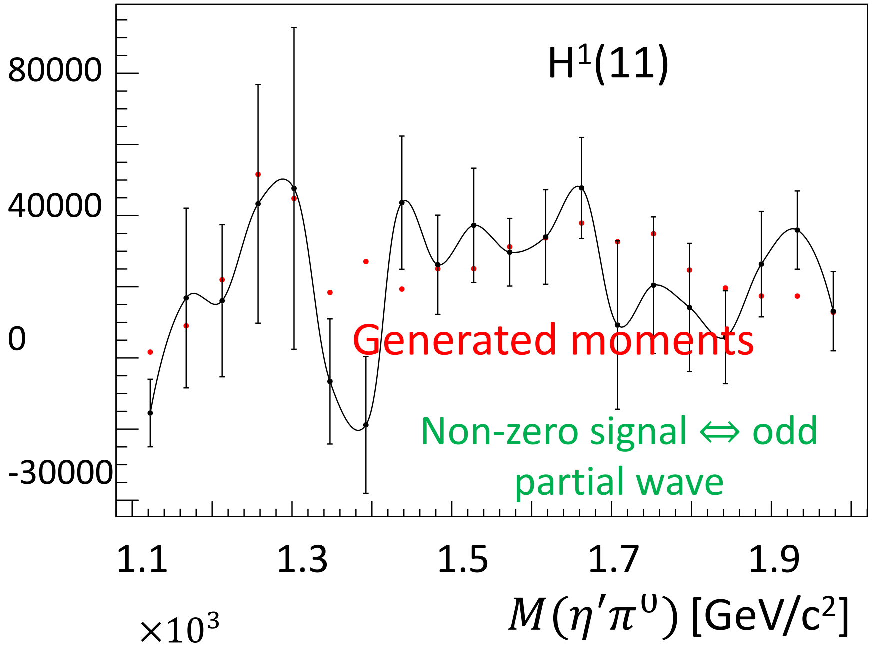


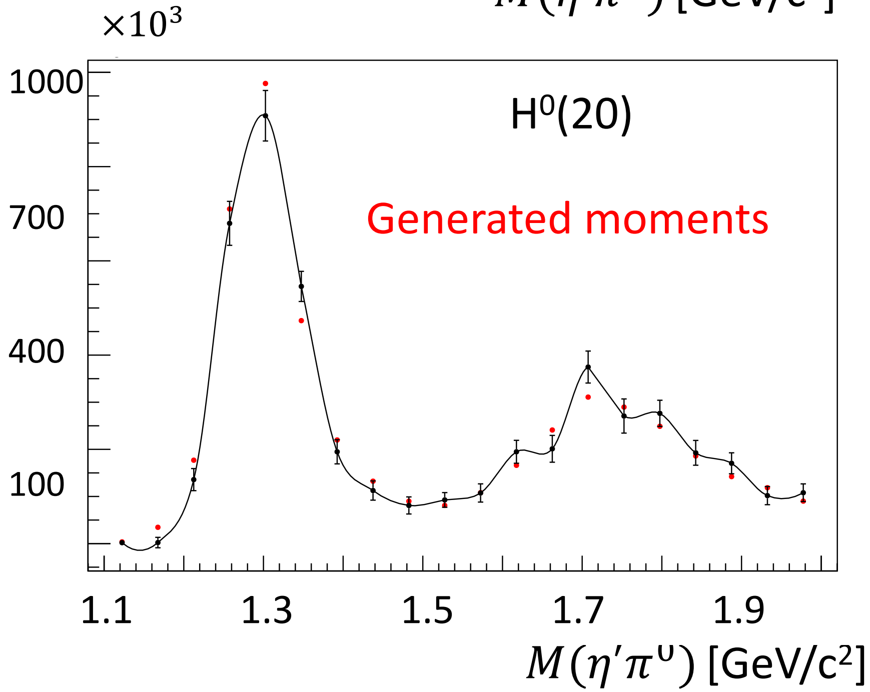
Uncertainties from bootstrapping

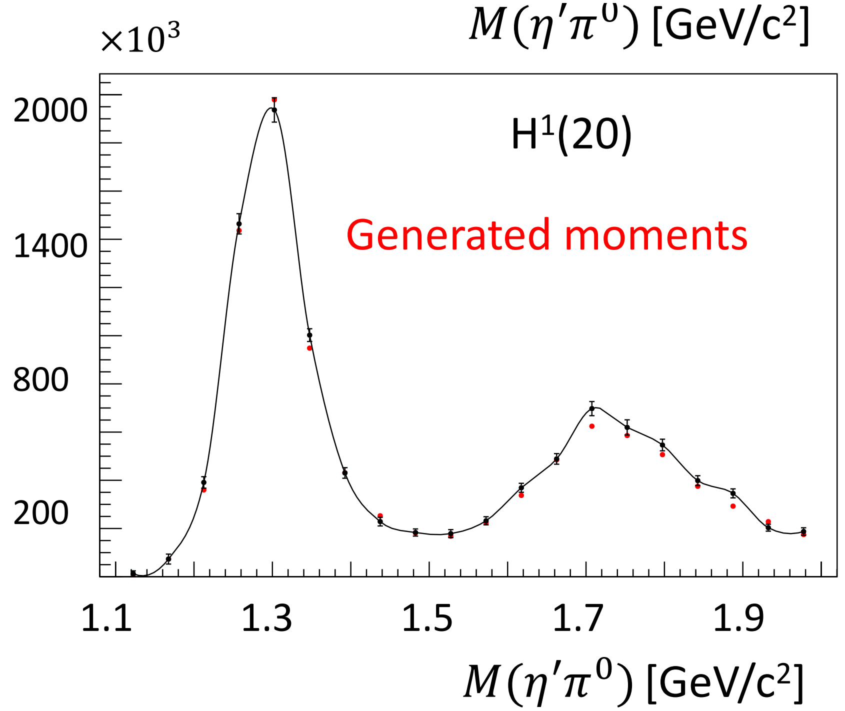
0<t<0.3 (GeV/c)^2

Using generated four vectors for reconstructed MC







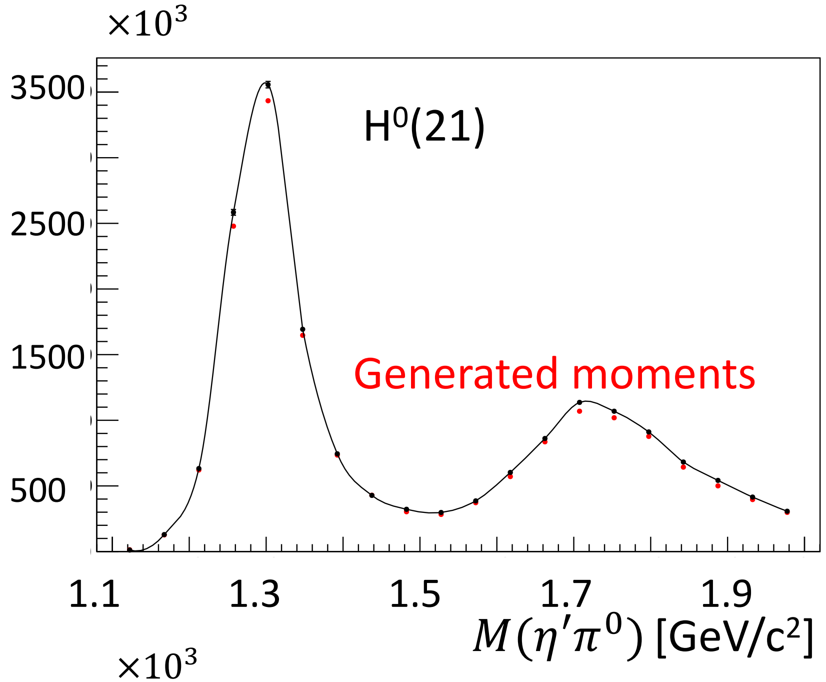


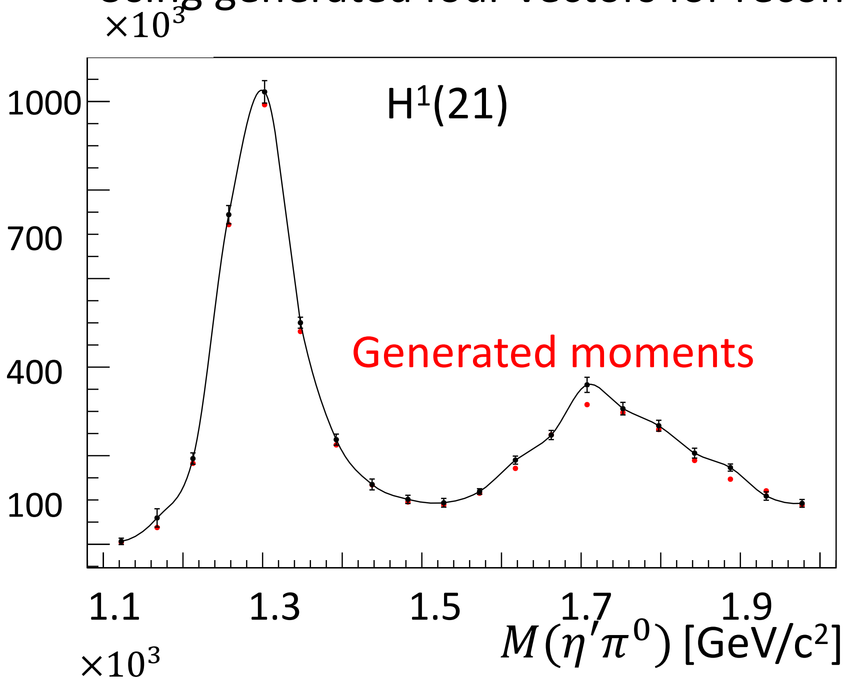
Polarized moments agree with generated moments

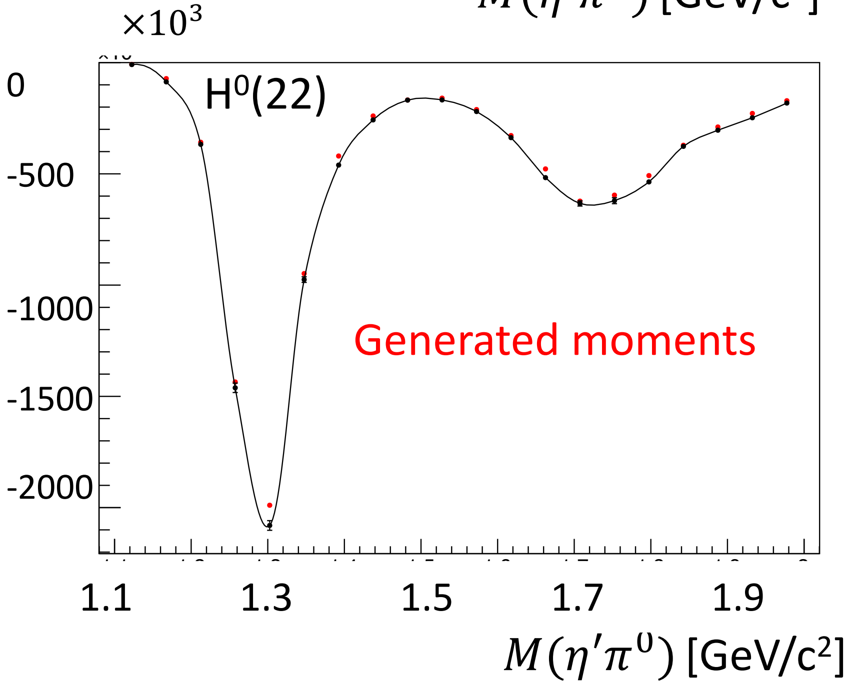
Uncertainties from bootstrapping

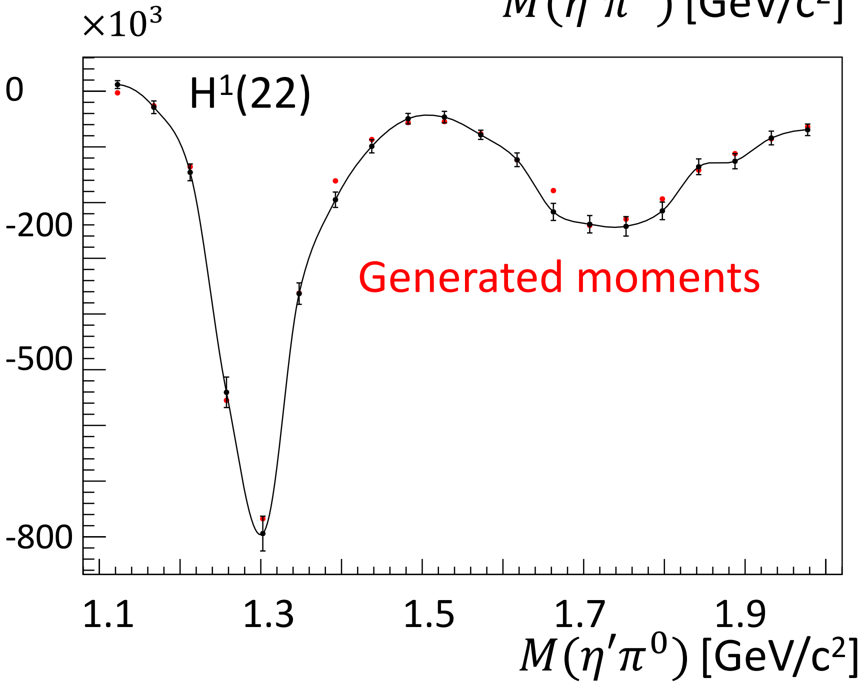
$0<t<0.3\ (\mathrm{GeV}/c)^2$

Using generated four vectors for reconstructed MC







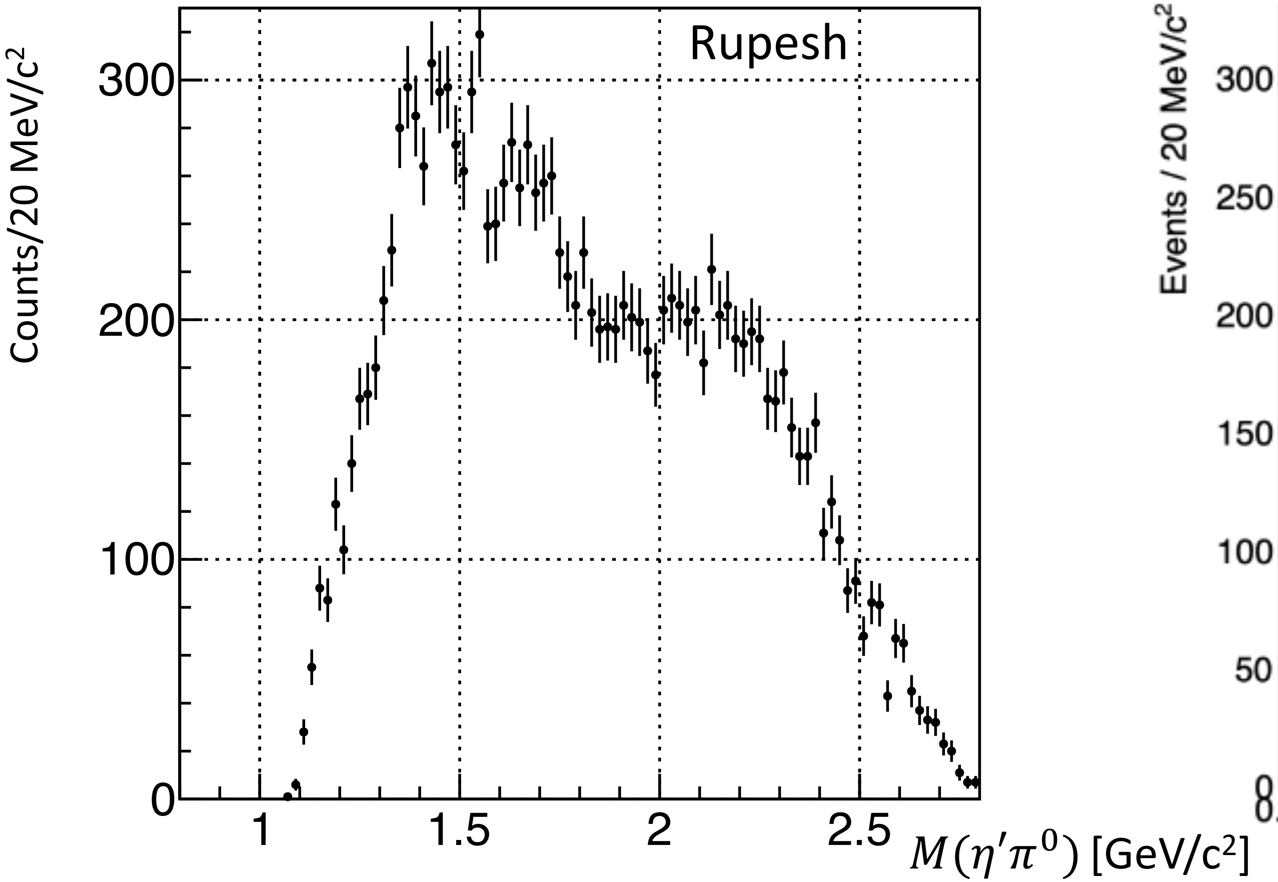


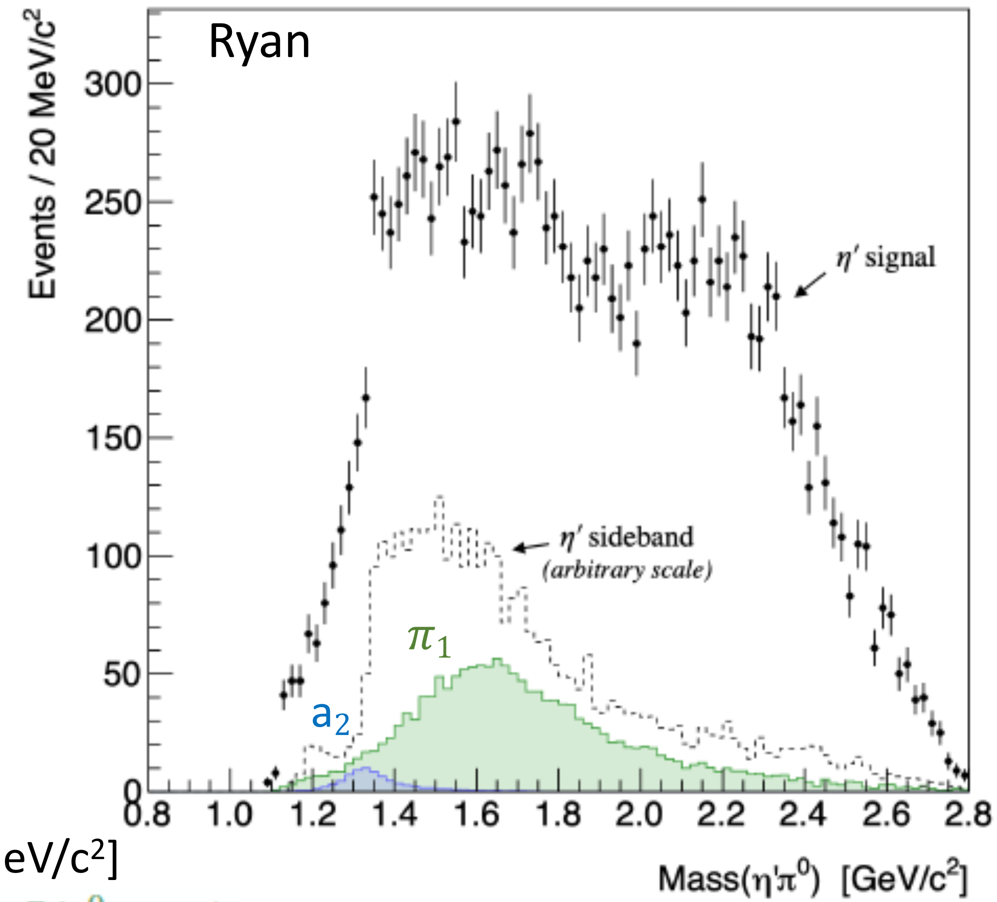
Search for exotic mesons via Partial Wave Analysis (PWA) of $p\eta'\pi^0$ data

Comparison of $p\eta'\pi^0$ yields

Similar distributions

Not corrected for acceptance





For the $\pi_1(1600)$: $\sigma B = \sigma(\gamma p \to \pi_1 p) \times B(\pi_1 \to \eta'\pi^0) \times B(\eta' \to \pi^+\pi^-\eta) \times B(\eta \to \gamma\gamma) \times B(\pi^0 \to \gamma\gamma)$
$= (20\text{nb}) \times \quad (10\%)_{\text{LQCD}} \times \ldots \quad = 2\text{nb} \times \ldots$

Against a proton, use:
$\sigma(\gamma p \to \pi_1^0 p) = 20\text{nb}$
$\sigma(\gamma p \to a_2^0 p) = 20\text{nb}$
(remember this is for $0.2 < -t < 0.5$ $(\text{GeV}/c^2)^2$*)*

$\sigma(\pi_1^0)$ obtained assuming *all* of the I=1 $\omega\pi^+\pi^-$ comes via: π_1 -> b_1 π -> ω π π. This puts an upper limit on the cross section for the production of π_1 times the branching fraction for π_1-> b_1 π. $\sigma(\pi_1^-)$ obtained from ω $\pi^-\pi^0$.

Generated 59 M $(p\eta'\pi^0)$ flat events with AmpTools

$p\gamma \rightarrow p\eta'\pi^0,$
$\eta' \rightarrow \pi^+\pi^-\eta,$
$\eta \rightarrow \gamma\gamma$

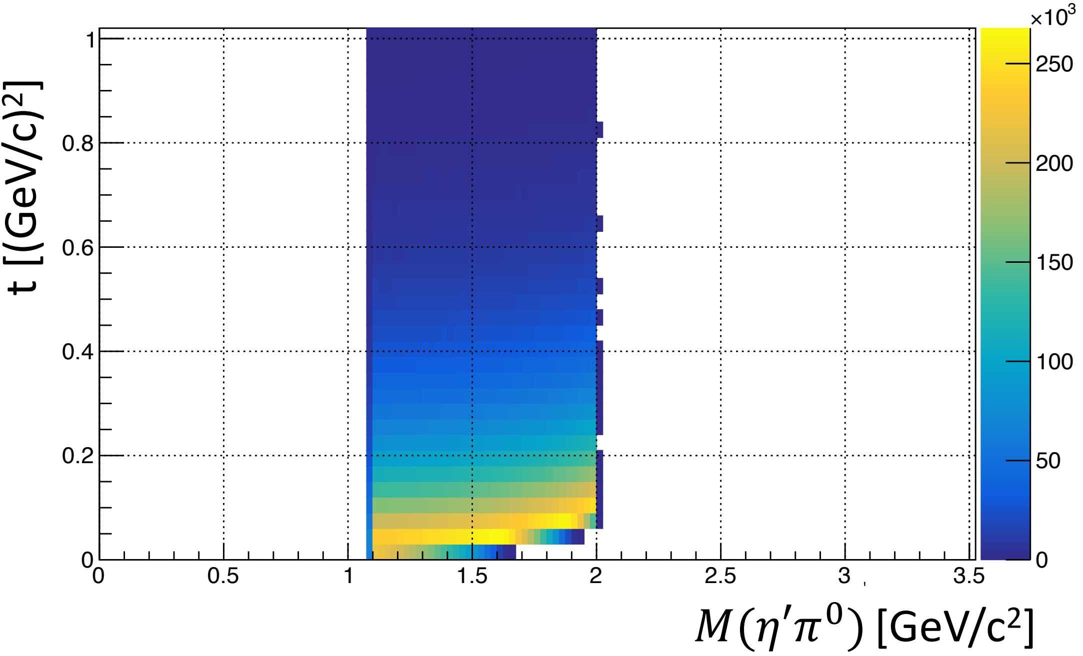
Generated for three run periods, proportion of each picked based on integrated luminosity to better description of acceptance :

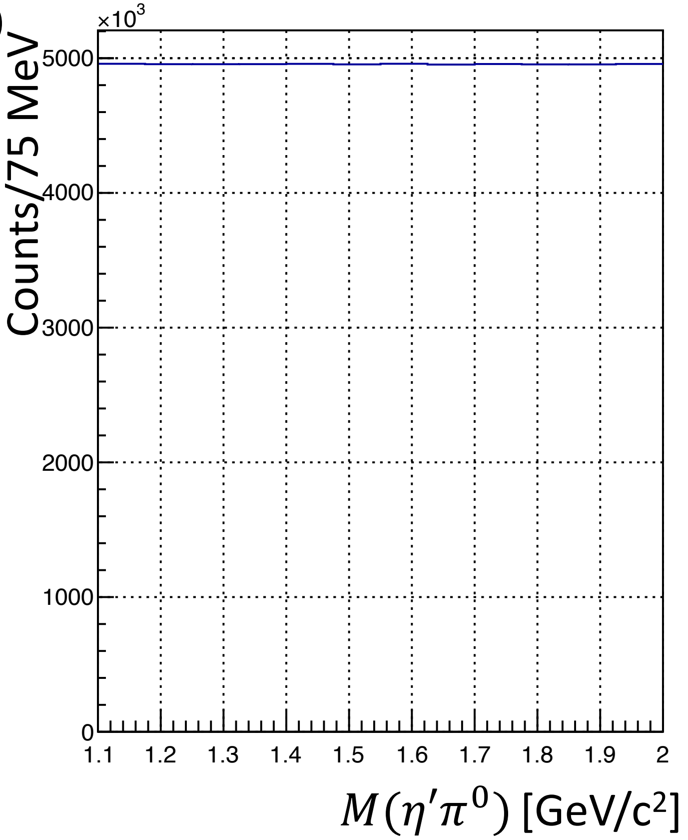
spring 2017: 10M

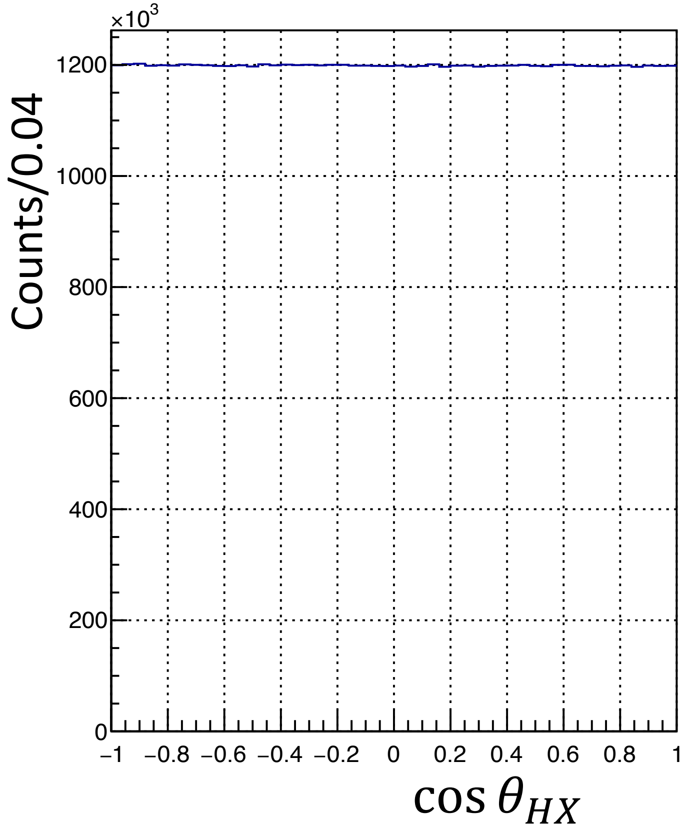
spring 2018: 30M

Fall 2018: 19M

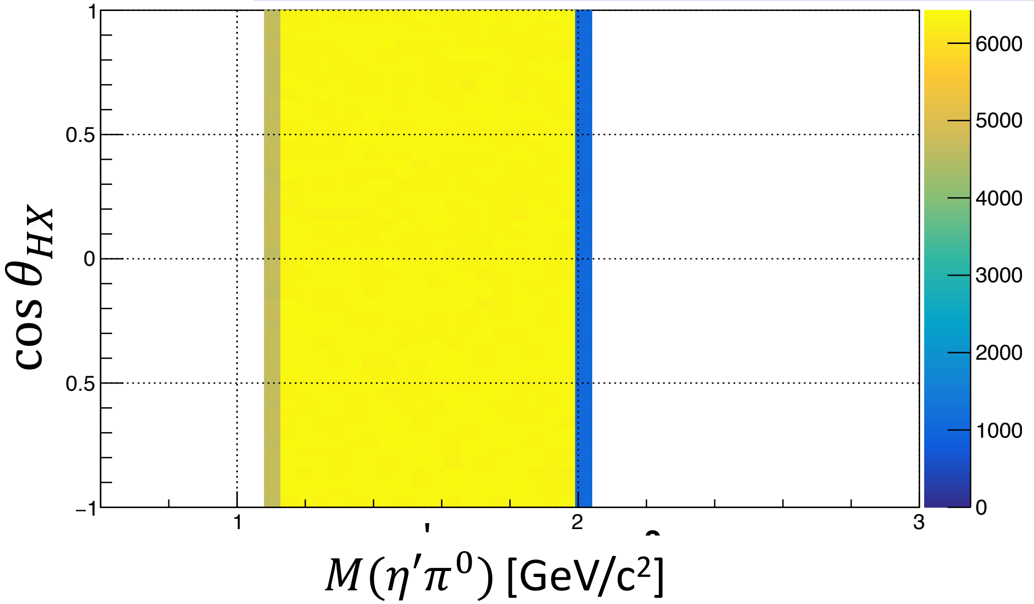
- Flat in $\cos\theta_{GJ}$
- Flat in $M(\eta\pi^0)$

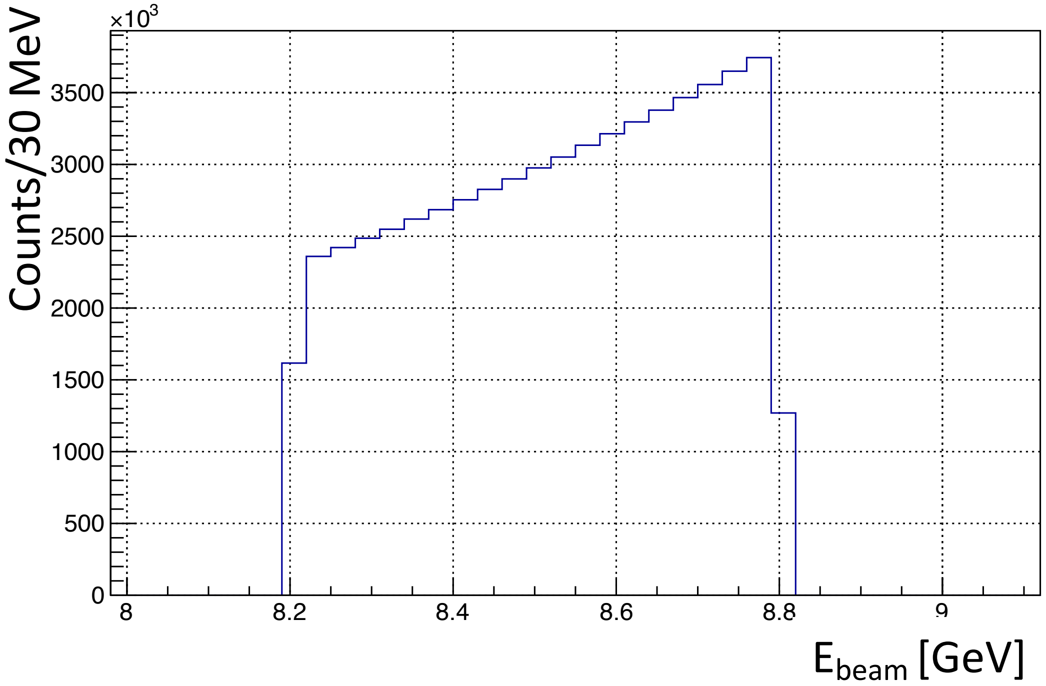


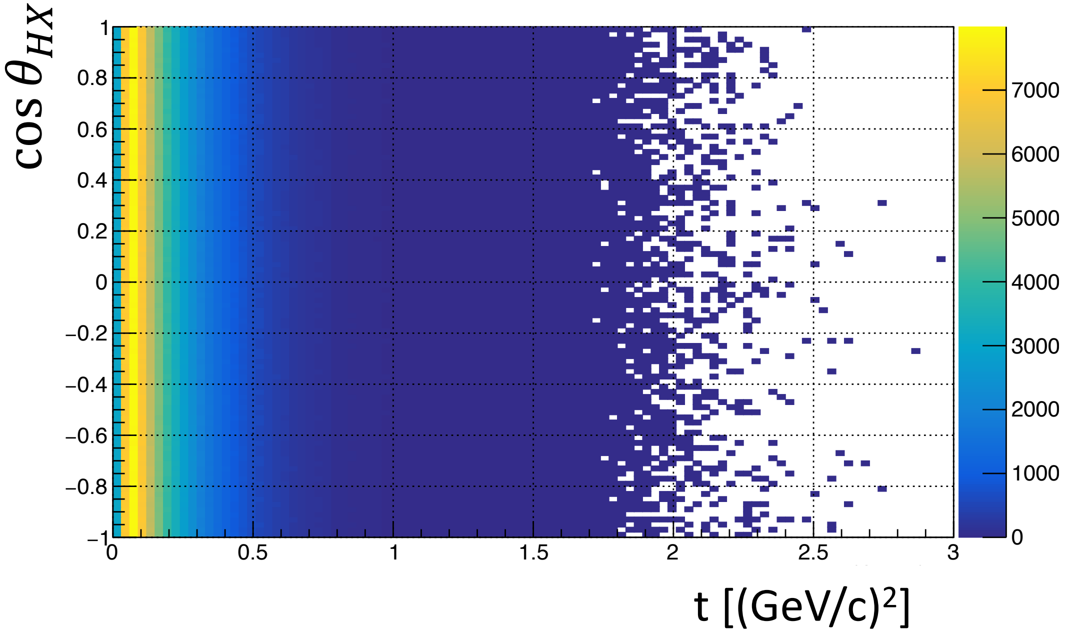




Generated 59 M ($p\eta'\pi^0$) flat events with AmpTools

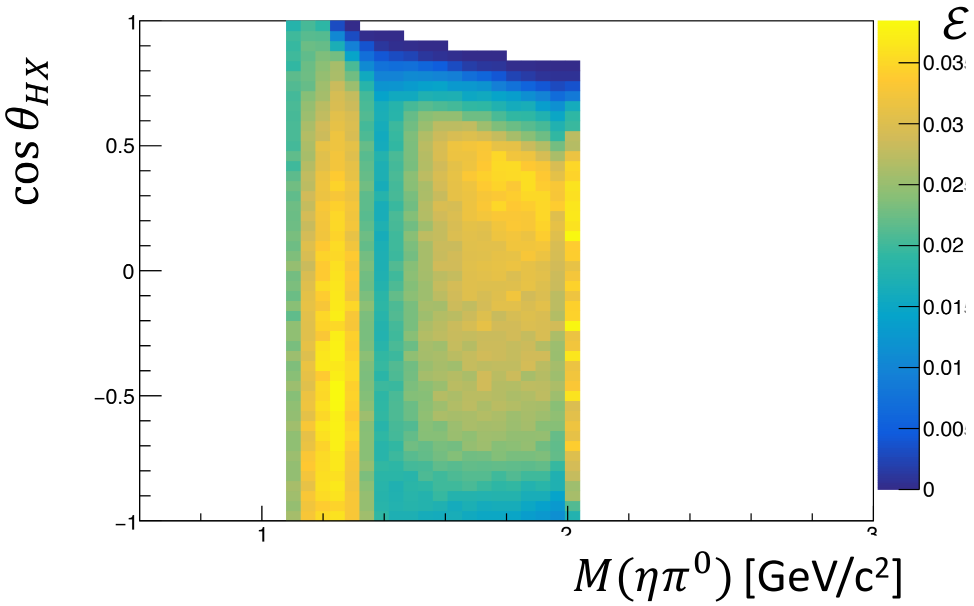


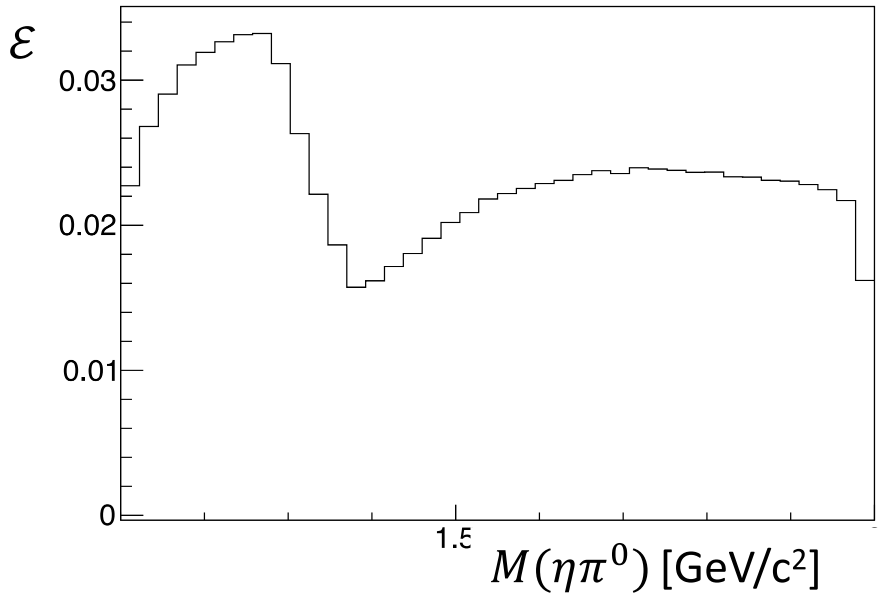


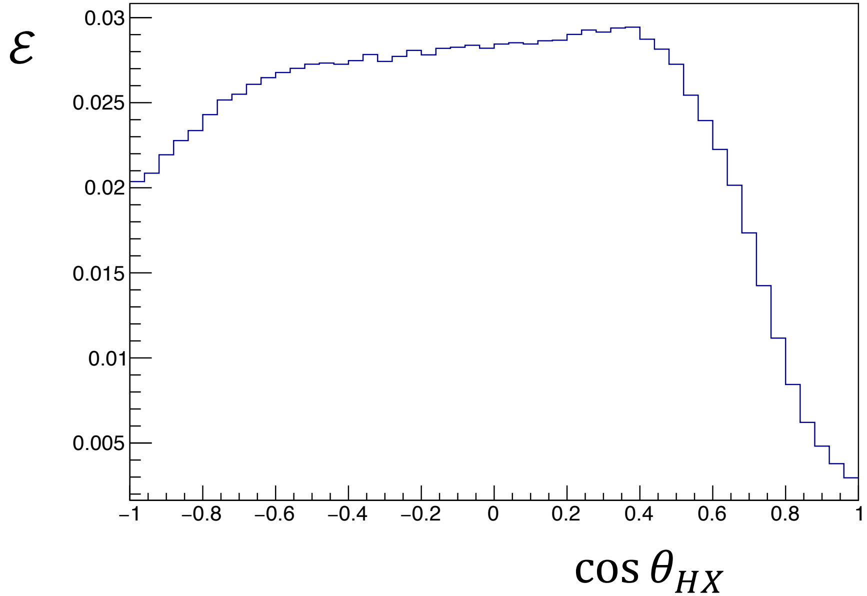


Acceptance with flat data for 0.1<-t<0.7 $(GeV/c)^2$ with uncertainties

We select $-t>0.1$ $(GeV/c)^2$ to cut events, where p had such low –t , that it couldn't get out of the target.



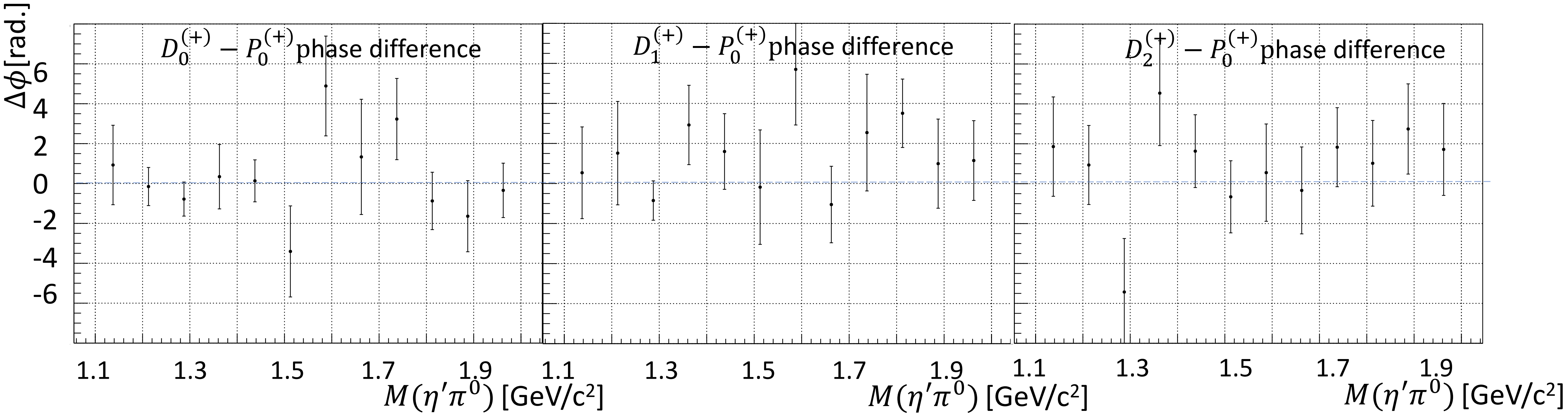




Phase differences from fit with S, P, D waves, M≥ 0, ε=± 1

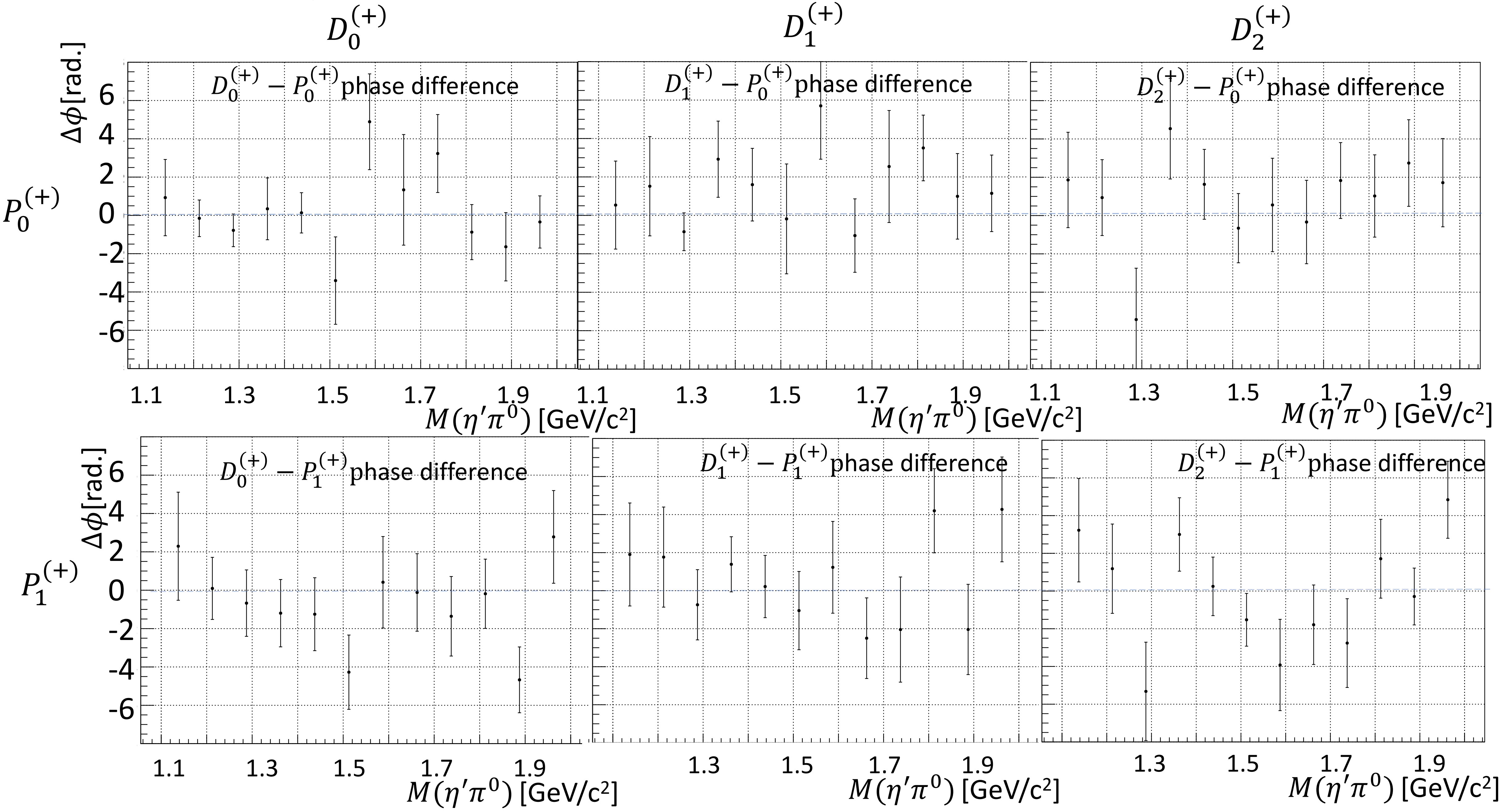
Uncertainties from Bootstrap method

Rapid changes can be indicative of the presence of interfering resonant states. Uncertainties are large to make conclusions



Phase differences from fit with S, P, D waves, M$\geq$ 0, ε=±1

If the wave is dominated by a resonance like the a_2 in the D-wave, it's phase should be the same for all m projections.

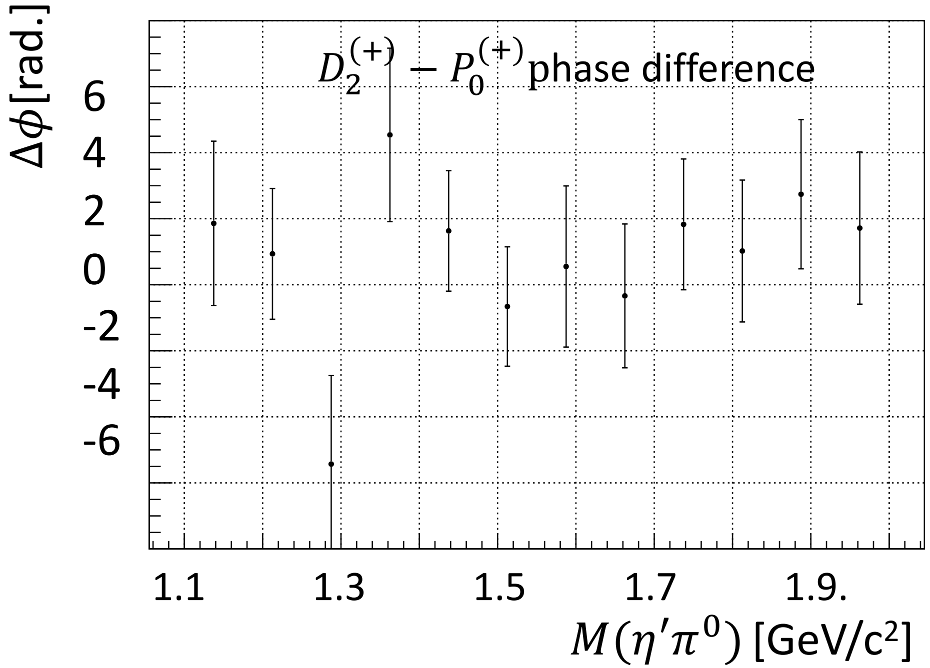


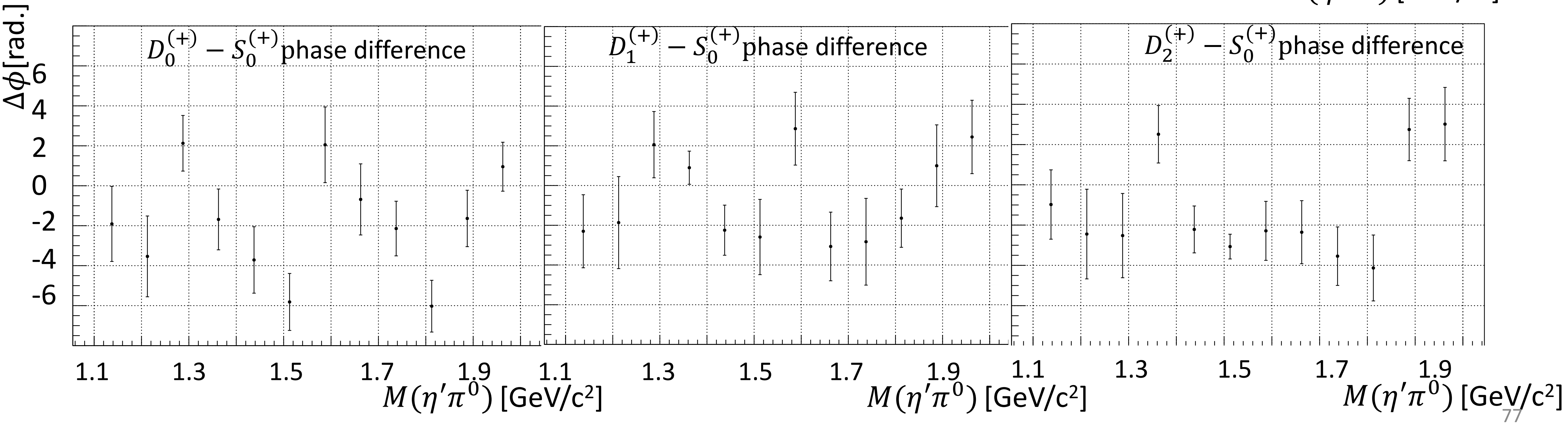
Phase differences from fit with S, P, D waves, M$\geq$ 0, ε=$\pm$1

Uncertainties from Bootstrap method

Rapid changes can be indicative of the presence of interfering resonant states. Uncertainties are large to make conclusions

If the wave is dominated by a resonance like the a_2 in the D-wave, it's phase should be the same for all m projections.

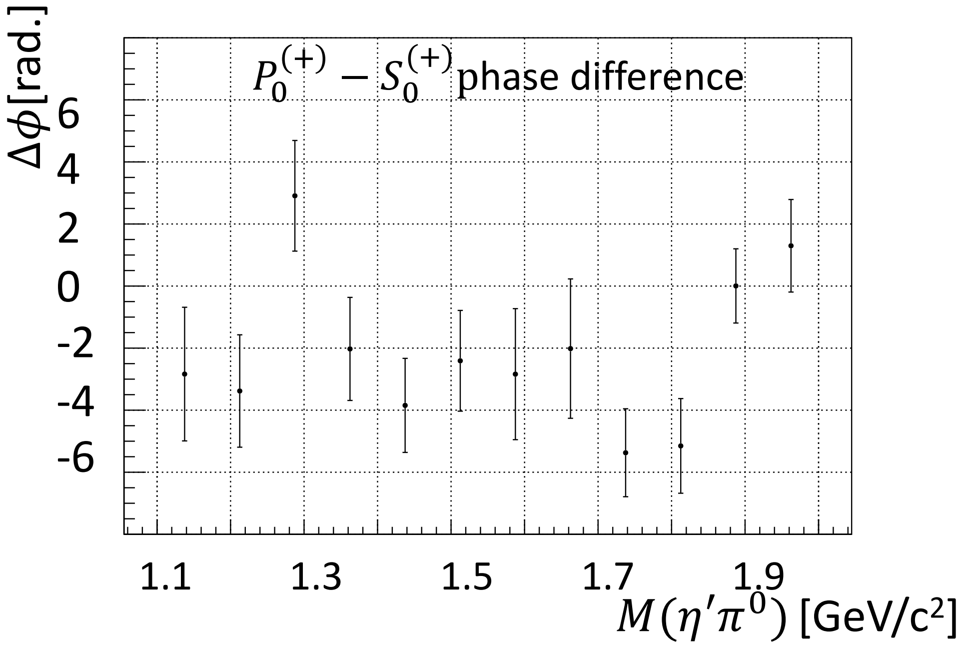


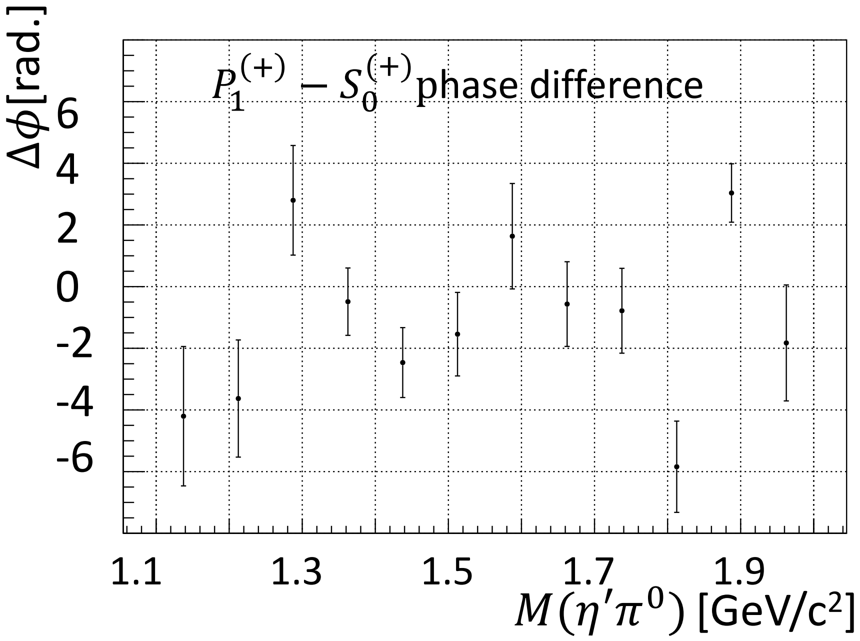


Phase differences from fit with S, P, D waves, M≥ 0, ε=± 1

Uncertainties from Bootstrap method

Rapid changes can be indicative of the presence of interfering resonant states. Uncertainties are large to make conclusions





AmpTools uses extended unbinned maximum likelihood method to estimate the values of model parameters used to describe the data

$$\mathcal{L}(\boldsymbol{\theta}) = \frac{e^{-\mu}\mu^N}{N!}\prod_{i=1}^{N}\mathcal{P}(\mathbf{x}_i;\boldsymbol{\theta})$$

x_i- is a vector of dimension n that describes the kinematics of the reaction
N- number of independent observations of x_i
θ- parameters of the physics model that is being fit to the data (the physics model depends on x_i and θ)
$\frac{e^{-\mu}\mu^N}{N!}$ - is the Poisson probability of obtaining N events from a mean of μ

$$\mathcal{P}(\mathbf{x};\boldsymbol{\theta}) = \frac{1}{\mu}\mathcal{I}(\mathbf{x};\boldsymbol{\theta})\eta(\mathbf{x})$$

$I(x_i,\theta)$- model predicted number of signal events per unit phase space

$$\mu = \int \mathcal{I}(\mathbf{x};\boldsymbol{\theta})\eta(\mathbf{x})\ d\mathbf{x}$$

$$-2\ln\mathcal{L}(\boldsymbol{\theta}) = -2\left(\sum_{i=1}^{N}\ln\mathcal{I}(\mathbf{x}_i;\boldsymbol{\theta}) - \int \mathcal{I}(\mathbf{x};\boldsymbol{\theta})\eta(\mathbf{x})\ d\mathbf{x}\right) + c_1$$

The Intensity

$$\mathcal{I}(\mathbf{x}) = \sum_{\sigma} \left| \sum_{\alpha} s_{\sigma,\alpha} V_{\sigma,\alpha} A_{\sigma,\alpha}(\mathbf{x}) \right|^2$$

$A_{\sigma,\alpha}(x)$ - complex valued function that describes particular amplitude (decay through particular resonance)
$V_{\sigma,\alpha}$ - are the "production coefficients", complex-valued numbers that represent the magnitude and the phase of the production of a particular amplitude $A_{\sigma,\alpha}(x)$
$s_{\sigma,\alpha}(x)$ - are real valued "scale factors"

$A_{\sigma,\alpha}(x)$ is factorized the following way:

$$A_{\sigma,\alpha}(\mathbf{x}) = \prod_{\gamma=1}^{n_{\sigma,\alpha}} a_{\sigma,\alpha,\gamma}(\mathbf{x})$$

$a_{\sigma,\alpha,\gamma}(\mathrm{x})$ – functions are defined by a user
$n_{\sigma,\alpha}$ - defined in the configuration file

Integration of intensity

It is possible to write the integral of a function f(x) in terms of its average value as:

$$\int_R f(x)\,dx = R\langle f(x)\rangle$$

For a sample of M_g generated Monte-Carlo events, we can compute the average value

$$\langle \mathcal{I}(\mathbf{x};\boldsymbol{\theta})\eta(\mathbf{x})\rangle = \frac{1}{M_g}\sum_{i=1}^{M_a}\mathcal{I}(\mathbf{x}_i;\boldsymbol{\theta})$$

M_a- events that pass all of the analysis criteria

This is equivalent to setting the value of the function η(x) to 1 for accepted events and 0 for others

$$\begin{aligned}\langle \mathcal{I}(\mathbf{x};\boldsymbol{\theta})\eta(\mathbf{x})\rangle &= \sum_{\sigma}\left|\sum_{\alpha} s_{\sigma,\alpha}V_{\sigma,\alpha}\left\{\frac{1}{M_g}\sum_{i=1}^{M_a}A_{\sigma,\alpha}(\mathbf{x}_i)\right\}\right|^2\\ &= \sum_{\sigma}\sum_{\alpha,\alpha'} s_{\sigma,\alpha}s_{\sigma,\alpha'}V_{\sigma,\alpha}V^*_{\sigma,\alpha'}\left\{\frac{1}{M_g}\sum_{i=1}^{M_a}A_{\sigma,\alpha}(\mathbf{x}_i)A^*_{\sigma,\alpha'}(\mathbf{x}_i)\right\}\end{aligned}$$

Weigting MC to obtain fit results

Model predicted number of observed events is calculated using MC integration:

$$\mu = \int I(\theta, \varphi)\eta(\theta, \varphi)\, d\Omega \approx \frac{4\pi}{N_{Gen}} \sum_{N_{Acc}} I(\theta, \varphi)$$

If we weight each MC event with following weight, we will obtain fit results to be compared to data:

$$\omega_i = \frac{4\pi}{N_{MC}} I(\theta_i, \varphi_i)$$

Config file for fitting with generated amplitudes in M and t bins

```
define polVal 0.3
fit FITNAME
reaction EtaPrimePi0 Beam Proton Eta Pi0

genmc  EtaPrimePi0 ROOTDataReader GENMCFILE
accmc  EtaPrimePi0 ROOTDataReader ACCMCFILE
data EtaPrimePi0 ROOTDataReader DATAFILE

sum EtaPrimePi0 PositiveRe
sum EtaPrimePi0 PositiveIm
parameter polAngle 1.77 fixed

# a0(980)
amplitude EtaPrimePi0::PositiveIm::S0+ Zlm 0 0 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::S0+ Zlm 0 0 +1 +1 polAngle polVal
# a2(1320)a2'(1700)
amplitude EtaPrimePi0::PositiveIm::D0+ Zlm 2 0 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D0+ Zlm 2 0 +1 +1 polAngle polVal
amplitude EtaPrimePi0::PositiveIm::D1+ Zlm 2 1 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D1+ Zlm 2 1 +1 +1 polAngle polVal
amplitude EtaPrimePi0::PositiveIm::D2+ Zlm 2 2 -1 -1 polAngle polVal
amplitude EtaPrimePi0::PositiveRe::D2+ Zlm 2 2 +1 +1 polAngle polVal
# pi1(1600)        • • •

initialize EtaPrimePi0::PositiveIm::S0+ cartesian 1000.0 0.0 real
initialize EtaPrimePi0::PositiveRe::S0+ cartesian 1000.0 0.0 real
initialize EtaPrimePi0::PositiveIm::D0+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D0+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveIm::D1+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D1+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveIm::D2+ cartesian 70.0 70.0
initialize EtaPrimePi0::PositiveRe::D2+ cartesian 70.0 70.0
        • • •

constrain EtaPrimePi0::PositiveIm::S0+ EtaPrimePi0::PositiveRe::S0+
        • • •
```

Typically refers to unique set of initial and final state particles
Can also refer to multiple decay modes of the same set of final state particles

Reaction, data reader class, argument

Events to fit intensity to

All amplitudes within a given sum are added coherently

User defined classes

Keywords

Reaction, Sum, amplitude name, amplitude class, arguments

```
Zlm as suggested in GlueX doc-4094 (M. Shepherd)
argument 1 : j
argument 2 : m
argument 3 : real (+1) or imaginary (-1) part
argument 4 : 1 + (+1/-1) * P_gamma
argument 5 : polarization angle (in Deg.)
argument 6 : beam properties config file or fixed polarization
```

Initial value of partial wave amplitudes $[l]_{m;k}^{(\epsilon)}$ in cartesian coordinate system

Factors with the same reaction sum and amplitude name are multiplied together

Same amplitudes corresponding to different sums should be equal

Config file for fitting with generated amplitudes, mass dependent fit

```
# resonance parameters
parameter azeromass 0.980 fixed
parameter azerowidth 0.075 fixed
parameter atwomass 1.318 fixed
parameter atwowidth 0.111 fixed
parameter afourmass 1.995 fixed
parameter afourwidth 0.257 fixed
parameter pionemass 1.354 fixed
parameter pionewidth 0.330 fixed

fit etapi0

reaction EtaPi0 Beam Proton Eta Pi0

genmc  EtaPi0 ROOTDataReader flat.root
accmc  EtaPi0 ROOTDataReader flat.root
data EtaPi0 ROOTDataReader sample1.root

normintfile EtaPi0 etapi0_ni.txt

sum EtaPi0 Negative
sum EtaPi0 Positive

amplitude EtaPi0::Negative::S0- TwoPSAngles 0 0 -1
amplitude EtaPi0::Negative::S0- BreitWigner [azeromass] [azerowidth] 0 2 3
amplitude EtaPi0::Positive::P1+ TwoPSAngles 1 1 1
amplitude EtaPi0::Positive::P1+ BreitWigner [pionemass] [pionewidth] 1 2 3
amplitude EtaPi0::Positive::D1+ TwoPSAngles 2 1 1
amplitude EtaPi0::Positive::D1+ BreitWigner [atwomass] [atwowidth] 2 2 3
amplitude EtaPi0::Positive::G1+ TwoPSAngles 4 1 1
amplitude EtaPi0::Positive::G1+ BreitWigner [afourmass] [afourwidth] 4 2 3

initialize EtaPi0::Negative::S0- cartesian 10.0 0.0 real
initialize EtaPi0::Positive::P1+ cartesian 10.0 10.0
initialize EtaPi0::Positive::D1+ cartesian 10.0 0.0 real
initialize EtaPi0::Positive::G1+ cartesian 10.0 10.0
```

Keywords

User defined classes

Typically refers to unique set of initial and final state particles
Can also refer to multiple decay modes of the same set of final state particles

Reaction, data reader class, argument

Events to fit intensity to

If the amplitudes don't have free parameters, the normalization integrals will not change, and we can store the integrals after the first fit and utilize them for subsequent.

All amplitudes within a given sum are added coherently

Factors with the same reaction sum and amplitude name are multiplied together

Reaction, Sum, amplitude name, amplitude class, J, M, ϵ
Mass, BW width, L, particle1, particle2

Initial value of production coefficient ($V_{\sigma,\alpha}$) in cartesian coordinate system

Config file for fitting with loop statement

```
#######################################
# GLOBAL VARIABLES
#######################################
fit FITNAME
define flat 0

define polVal_00 0.3519
…

define polAngle_00 0.0
...

parameter parScale00 1.0 fixed
…

#######################################
# LOOP STATEMENTS
#######################################
loop LOOPREAC EtaPi0_00 EtaPi0_45 EtaPi0_90 EtaPi0_135
loop LOOPDATA DATAFILE_00 DATAFILE_45 DATAFILE_90 DATAFILE_135
…

loop LOOPPOLANG polAngle_00 polAngle_45 polAngle_90 polAngle_135
loop LOOPPOLVAL polVal_00 polVal_45 polVal_90 polVal_135
loop LOOPSCALE [parScale00] [parScale45] [parScale90] [parScale135]

#######################################
# SETUP INPUT, REACTIONS, SUMS
#######################################
data LOOPREAC ROOTDataReader LOOPDATA
…
reaction LOOPREAC Beam Proton EtaPrime Pi0
sum LOOPREAC NegativeRe
sum LOOPREAC NegativeIm
sum LOOPREAC PositiveRe
sum LOOPREAC PositiveIm

#######################################
# DEFINE AMPLITUDES
#######################################
#Positive reflectivity
# S-wave amplitudes
amplitude LOOPREAC::PositiveIm::S0+ Zlm 0 0 -1 -1 LOOPPOLANG LOOPPOLVAL
amplitude LOOPREAC::PositiveRe::S0+ Zlm 0 0 +1 +1 LOOPPOLANG LOOPPOLVAL

#######################################
# INITIALIZE PARAMETERS
#######################################
initialize LOOPREAC::NegativeRe::S0- cartesian 5.0 5.0 real
initialize LOOPREAC::PositiveIm::S0+ cartesian 500.0 0.0 real

#######################################
# SET CONSTRAINS
#######################################
# Constrain same amplitudes in different incoherent sums
constrain LOOPREAC::PositiveIm::S0+ LOOPREAC::PositiveRe::S0+
constrain LOOPREAC::NegativeRe::S0- LOOPREAC::NegativeIm::S0-

# Constrain all other 'reactions' to the first one:
constrain EtaPi0_00::NegativeRe::S0- LOOPREAC::NegativeRe::S0-
constrain EtaPi0_00::NegativeIm::S0- LOOPREAC::NegativeIm::S0-
constrain EtaPi0_00::PositiveIm::S0+ LOOPREAC::PositiveIm::S0+
constrain EtaPi0_00::PositiveRe::S0+ LOOPREAC::PositiveRe::S0+

#######################################
# SETUP SCALING
#######################################
scale LOOPREAC::NegativeRe::S0- LOOPSCALE
scale LOOPREAC::NegativeIm::S0- LOOPSCALE
scale LOOPREAC::PositiveIm::S0+ LOOPSCALE
scale LOOPREAC::PositiveRe::S0+ LOOPSCALE
```