

Método de Lagrange

INTERPOLAÇÃO POLINOMIAL

Método de LAGRANGE

$$P_n(x) = \sum_{k=0}^n y_k L_k(x), \text{ onde}$$

$$L_k(x) = \frac{\prod_{j=0, j \neq k}^n (x - x_j)}{\prod_{j=0, j \neq k}^n (x_k - x_j)}$$

Dado um tabelamento:

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3

$$P_3(x) = ?$$

$$P_3(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x)$$

$$L_0(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)}$$

$$L_2(x) = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)}$$

Ex:

x	x_0	x_1	x_2
y	4	1	-1

$$(a) P_2(x) = ?$$

$$(b) P_2(1) = ?$$

$$P_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

$$P_2(x) = 4 L_0(x) + 1 L_1(x) - 1 L_2(x)$$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0)(x-2)}{(-1-0)(-1-2)} = \boxed{\frac{x^2-2x}{3}}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x+1)(x-2)}{(0+1)(0-2)} = \boxed{\frac{x^2-x-2}{-2}}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x+1)(x-0)}{(2+1)(2-0)} = \boxed{\frac{x^2+x}{6}}$$

$$P_2(x) = 4 \cdot \left[\frac{x^2-2x}{3} \right] + \left[\frac{x^2-x-2}{-2} \right] - \left[\frac{x^2+x}{6} \right]$$

$$P_2(1) = 4 \cdot \left[\frac{1^2-2 \cdot 1}{3} \right] + \left[\frac{1^2-1-2}{-2} \right] - \left[\frac{1^2+1}{6} \right] \approx \underline{\underline{-0,66667}}$$

$$P_2(x) = 4 \left[\frac{x^2-2x}{3} \right] - \left[\frac{x^2-x-2}{2} \right] - \left[\frac{x^2+x}{6} \right]$$

$$= \frac{4x^2-8x}{3} - \frac{x^2-x-2}{2} - \frac{x^2+x}{6}$$

$$= \frac{8x^2-16x-3x^2+3x+6-x^2-x}{6}$$

$$= \frac{4x^2-14x+6}{6} = \boxed{\frac{2}{3}x^2 - \frac{7}{3}x + 1}$$