Método de Newton

INTERPOLAÇÃO POLINOMIAL

Método de Newton
(diferenças divididas)

$$P_n(x) = d_0 + d_1(x-x_0) + d_2(x-x_0)(x-x_1) + \dots + d_n(x-x_0)(x-x_1)\cdot(\dots)\cdot(x-x_{n-1})$$

onde d_i, $0 \le i \le n$ é o operador diferença dividida de ordem i.

Operador diferenças divididas

$f[x_0] = y_0$ ordem 0

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} \quad \text{ordem 1}$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \quad \text{ordem 2}$$

(...)

$$f[x_0, x_1, \dots, x_n] = \frac{f[x_1, x_2, \dots, x_n] - f[x_0, x_1, \dots, x_{n-1}]}{x_n - x_0}$$

ordem n

x	y
x_0	y_0
x_1	y_1
$\vdots$	$\vdots$
x_n	y_n

Dado um tabelamento:

x	x_0	x_1	x_2
y	y_0	y_1	y_2

$$P_2(x) = d_0 + d_1(x - x_0) + d_2(x-x_0)(x-x_1)$$

x	$\theta(0)$	$\theta(1)$	$\theta(2)$
x_0	$y_0 = f[x_0]$ d_0	$\frac{f[x_1]-f[x_0]}{x_1-x_0} = f[x_0, x_1]$ d_1	$\frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$ d_2
x_1	$y_1 = f[x_1]$	$\frac{f[x_2]-f[x_1]}{x_2-x_1} = f[x_1,x_2]$	$\boxed{f[x_0,x_1,x_2]}$
x_2	$y_2 = f[x_2]$		

Ex:

x	-1 (x_0)	0 (x_1)	2 (x_2)
y	4 (y_0)	1 (y_1)	-1 (y_2)

(a) $P_2(x) = ?$

(b) $P_2(1) = ?$

$$\boxed{P_2(x) = d_0 + d_1(x-x_0) + d_2(x-x_0)(x-x_1)}$$

$$\boxed{P_2(x) = d_0 + d_1(x+1) + d_2(x+1)(x-0)}$$

x	$\theta(0)$	$\theta(1)$	$\theta(2)$
$-1 = x_0$	4 d_0	$\frac{1-4}{0+1} = -3$ d_1	$\frac{-1+3}{2+1} = \frac{2}{3}$ d_2
$0 = x_1$	1	$\frac{-1-1}{2-0} = -1$	
$2 = x_2$	-1		

$$\boxed{P_2(x) = d_0 + d_1(x+1) + d_2(x+1)(x-0)}$$

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$$\boxed{P_2(x) = 4 + (-3)(x+1) + \frac{2}{3}(x+1)(x)}$$

$$P_2(1) = 4 - 3(1+1) + \frac{2}{3}(1+1)(1) \cong -0,66667$$

$2 = x_2$	-1	$\frac{}{2-0}$

$$\boxed{P_2(x) = 4 + (-3)(x+1) + \frac{2}{3}(x+1)(x)}$$

$$P_2(1) = 4 - 3(1+1) + \frac{2}{3}(1+1)(1) \cong -0,66667$$

$$P_2(x) = 4 - 3x - 3 + \frac{2}{3}x^2 + \frac{2}{3}x$$

$$\boxed{P_2(x) = \frac{2}{3}x^2 - \frac{7}{3}x + 1}$$