

# Linear Mixed Models

# Longitudinal data

- ▶ Repeated measures
  - ▶ measurements are taken multiple times on each participant

*measure the lengths of both arms on each participants*
- ▶ Longitudinal data:
  - ▶ Repeated measurements over time

*measure the length of the right arm at 3, 4, 5, and 6 years old of each participant*

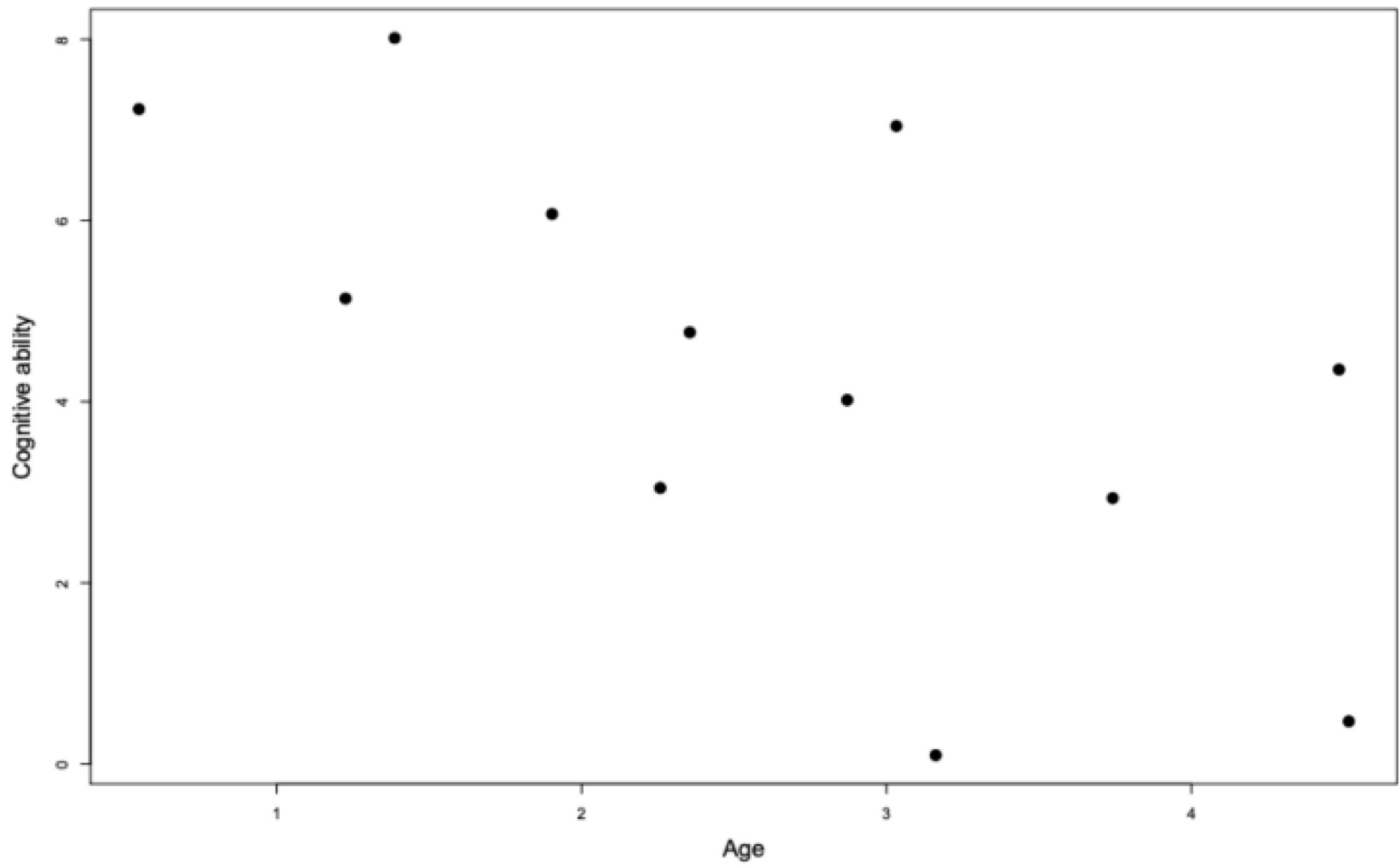
# Longitudinal data

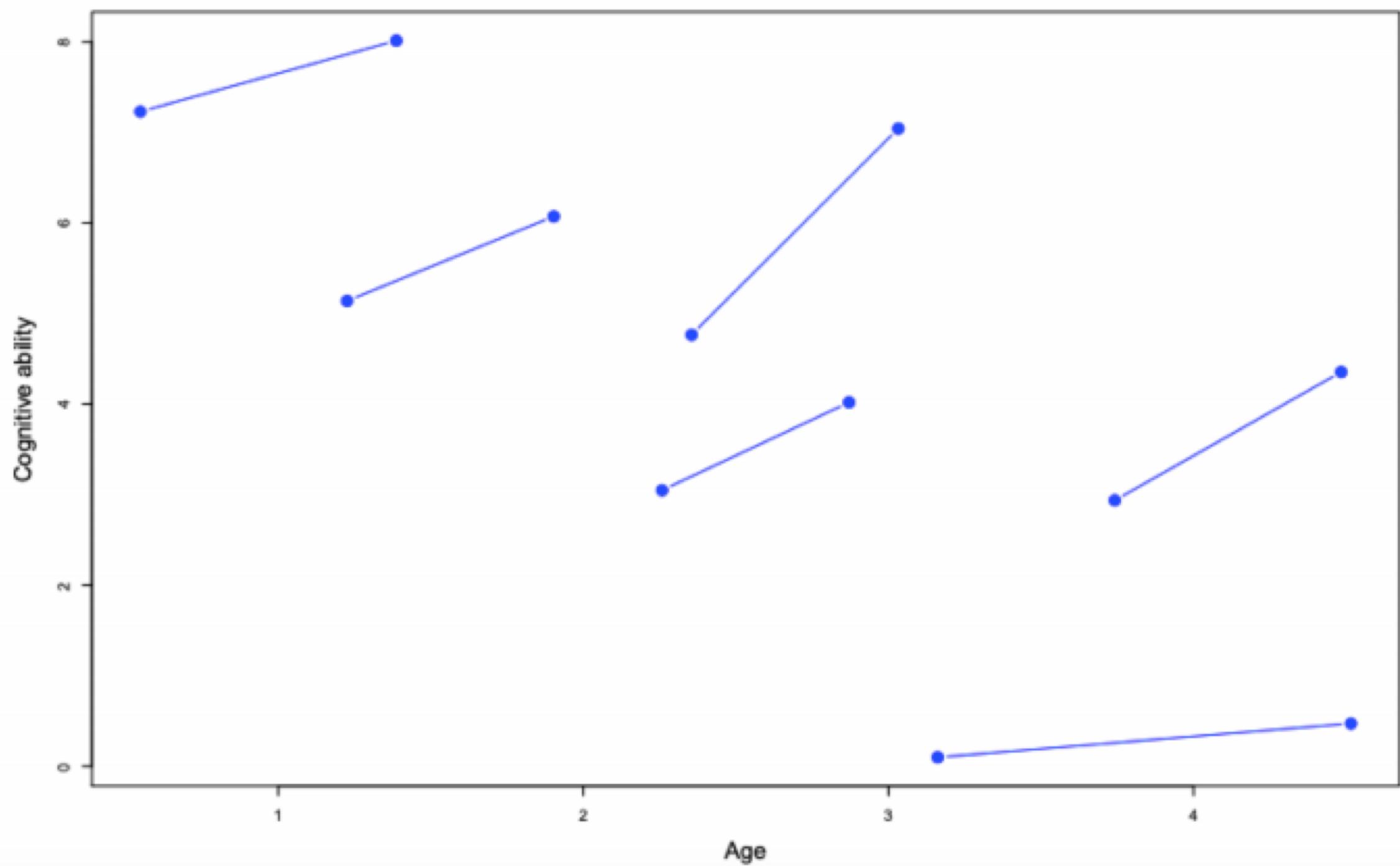
- ▶ **longitudinal data have strong intraindividual correlations:**  
the value observed at time t depends on the values observed before

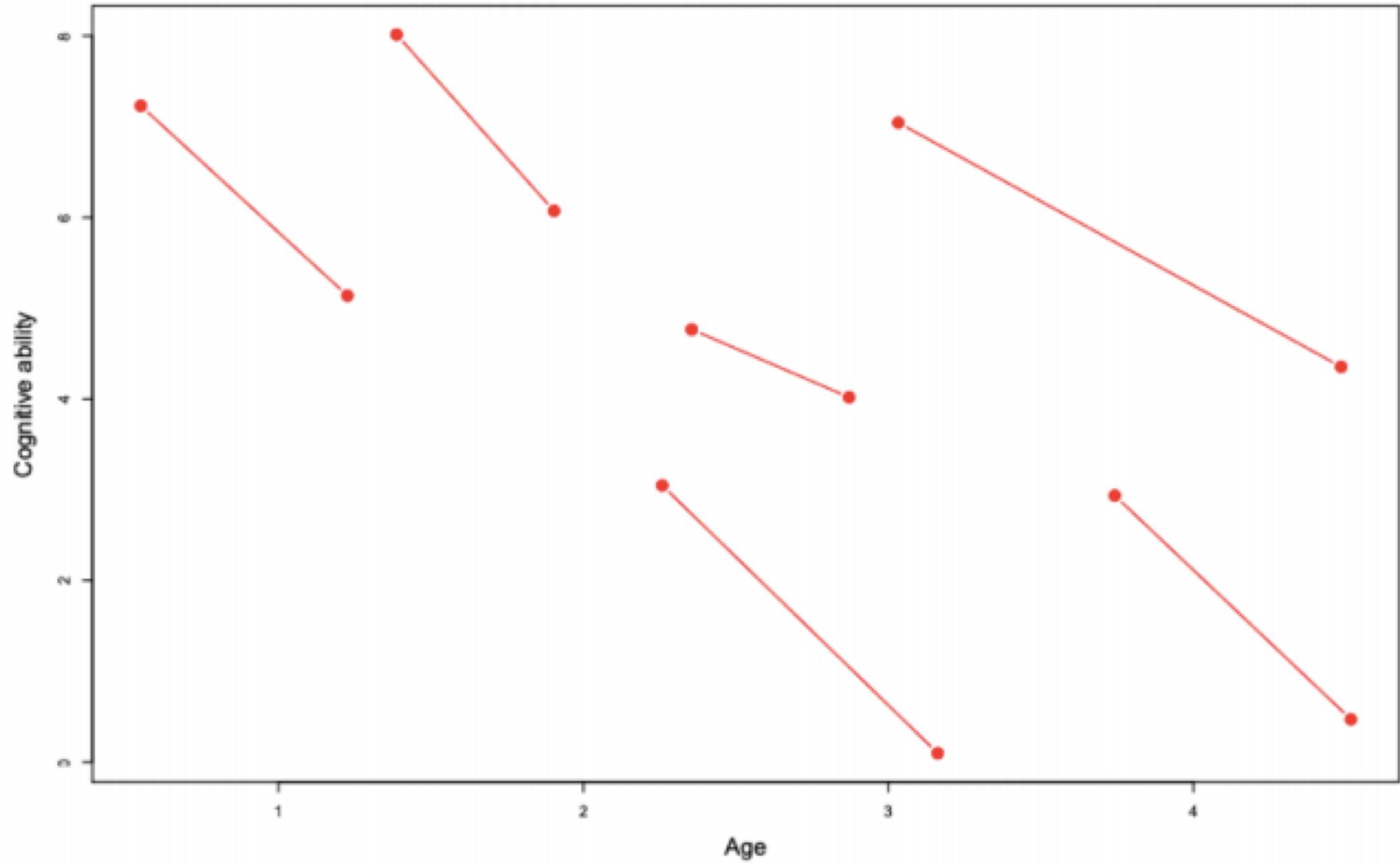
# Children

- ▶ Cognitive ability was measured in 6 children with syndrome A twice, once at 6 and once at 7 years old
- ▶ We have 12 observations









# Linear Model

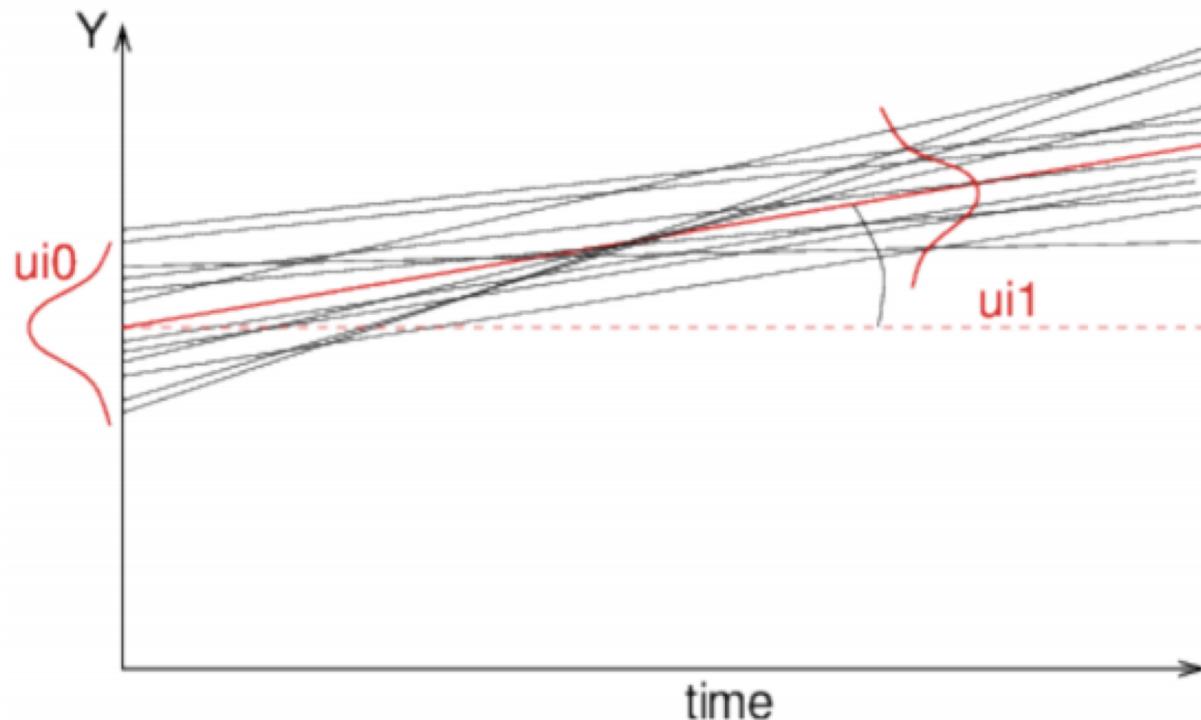
- linear model for the  $j^{\text{th}}$  observation of individual  $i$ :
- $Y_{ij} = \alpha + \beta X_{ij} + \epsilon_{ij}$ ,
- $Y_{ij}$  - outcome ( $j^{\text{th}}$  cognitive ability measure of child  $i$ )
- $X_{ij}$  - predictive value (age on the  $i^{\text{th}}$  child at the  $j^{\text{th}}$  measurement)
- $\alpha$  - intercept (average cognitive ability at birth)
- $\beta$  - regression coefficient defining how much the outcome should vary for a one-unit increase in  $X$
- $\epsilon_{ij}$  - residual error (luck when child  $i$  took his/her  $j^{\text{th}}$  cognitive test)

# Linear Mixed Model

- linear **mixed** model for the  $j^{\text{th}}$  observation of individual  $i$ :
- $Y_{ij} = (\alpha + u_{0i}) + (\beta + u_{1i}) X_{ij} + \epsilon_{ij}$ ,
- $u_{0i}$  - random intercept: modelling baseline individual heterogeneity (difference in cognitive ability at birth)
- $u_{1i}$  - random slope: modelling individual heterogeneity in the X-Y relationship (differences in the age-related evolution of cognitive ability)

# Linear Mixed Model

- linear **mixed** model for the  $j^{\text{th}}$  observation of individual  $i$ :
- $Y_{ij} = (\alpha + u_{0i}) + (\beta + u_{1i}) X_{ij} + \epsilon_{ij}$ ,



# Sleep deprivation

- ▶ Effect of sleep deprivation on the cognitive performance of drivers.
- ▶ Several truck drivers were subjected to only 3 hours of sleep a night.
- ▶ Their cognitive performance (Reaction) was assessed each day using the average reaction time (in ms)
- ▶ Does the reaction time vary over time?

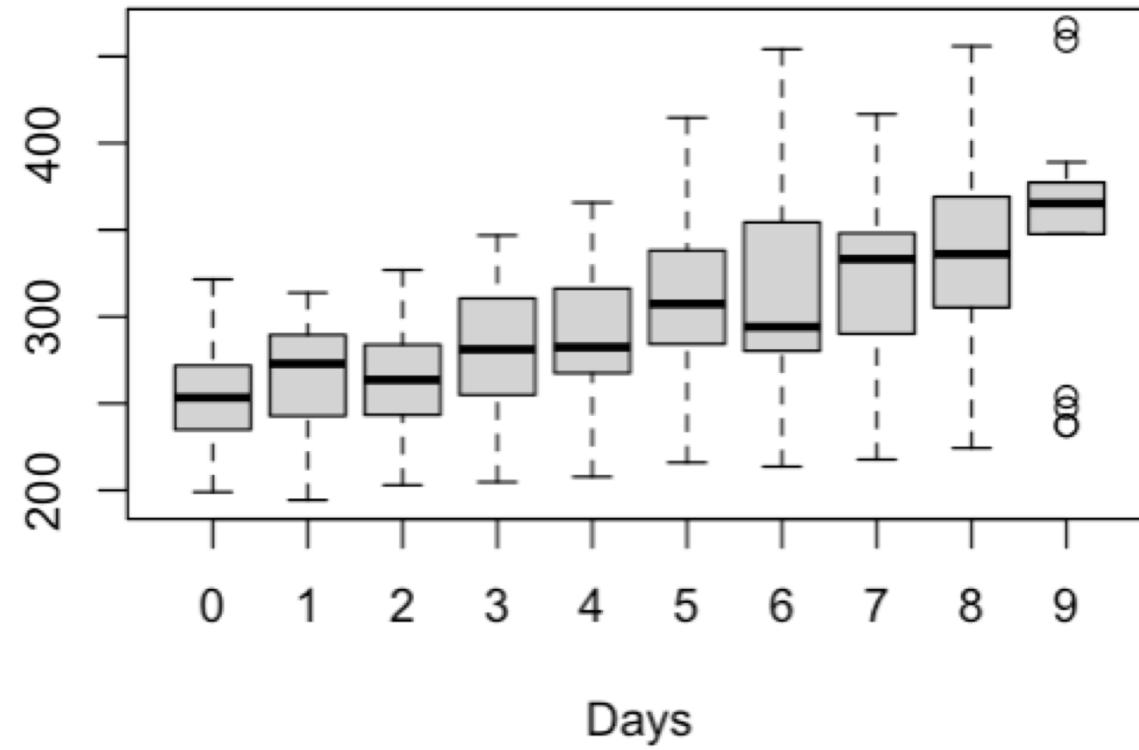


# Sleep deprivation

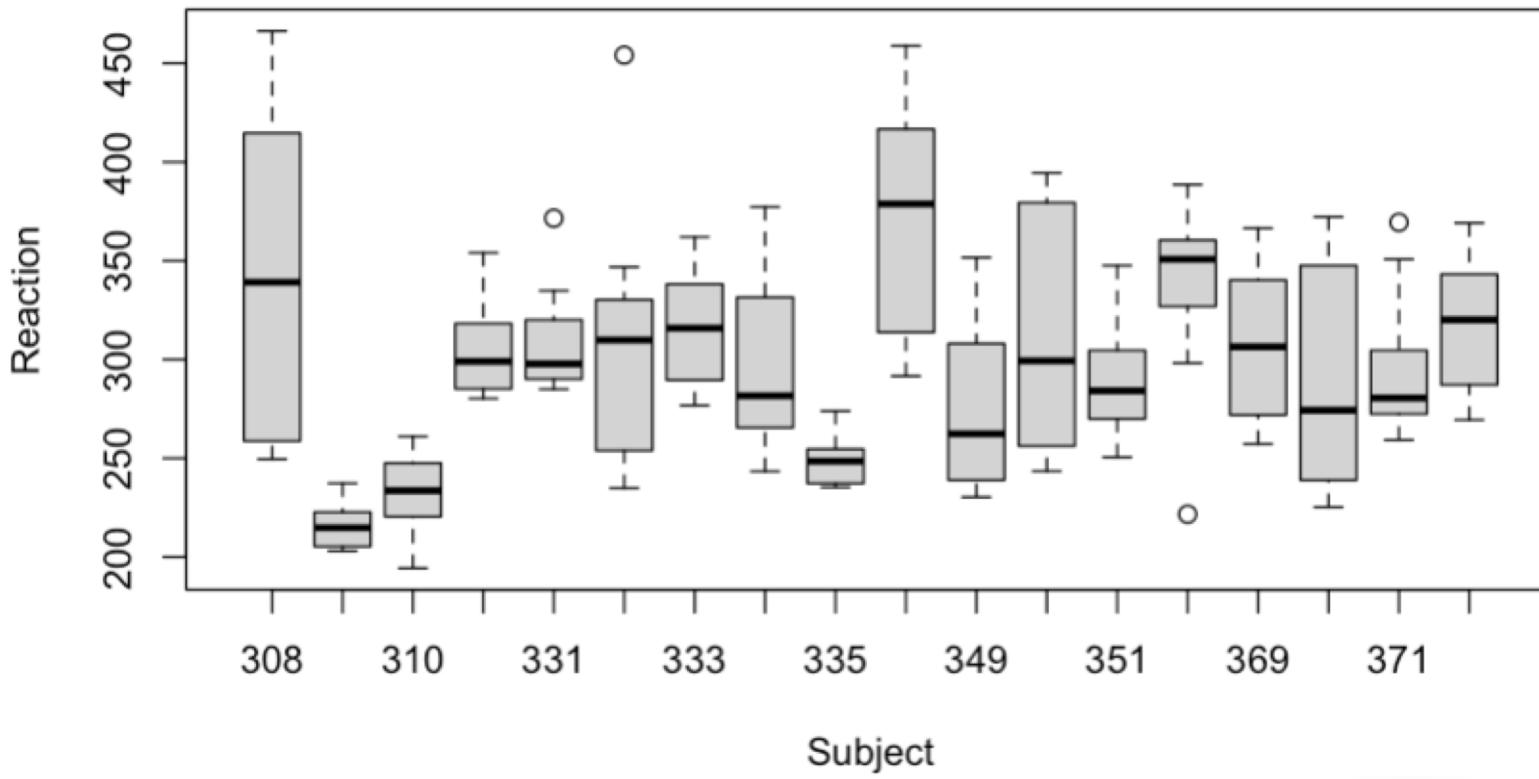


	Reaction	Days	Subject
1	249.5600	0	308
2	258.7047	1	308
3	250.8006	2	308
4	321.4398	3	308
5	356.8519	4	308
6	414.6901	5	308
7	382.2038	6	308
8	290.1486	7	308
9	430.5853	8	308
10	466.3535	9	308
11	222.7339	0	309
12	205.2658	1	309
13	202.9778	2	309
14	204.7070	3	309
15	207.7161	4	309
16	215.9618	5	309
17	213.6303	6	309
18	217.7272	7	309
19	224.2957	8	309
20	237.3142	9	309
21	199.0539	0	310
22	194.3322	1	310

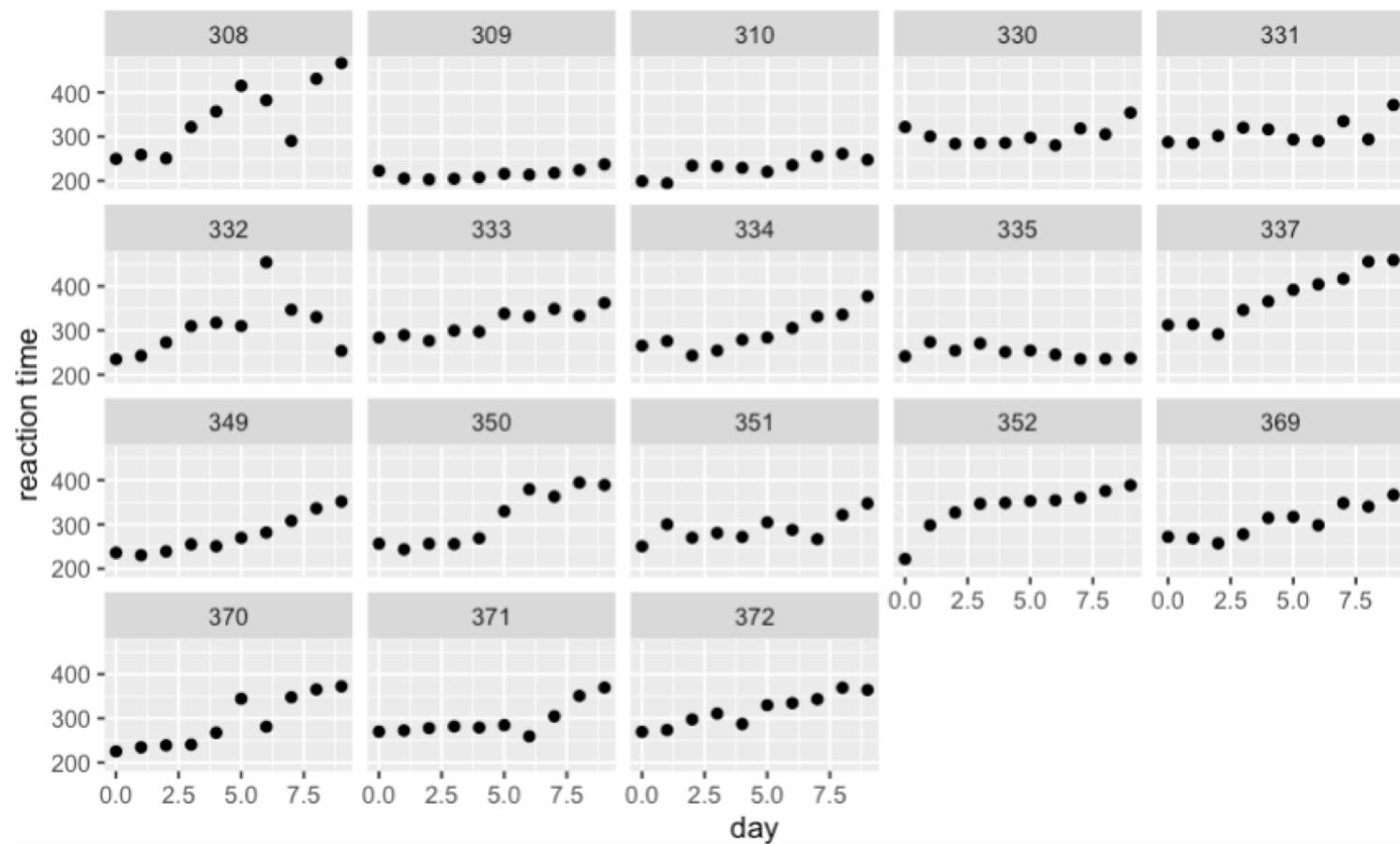
# Sleep deprivation



# Sleep deprivation



# Sleep deprivation



# Linear mixed model

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	251.405	6.825	36.838
Days	10.467	1.546	6.771

average reaction time at start = 251.4 ms

average reaction time increase of 10.5ms/day of sleep deprivation

# Linear mixed model

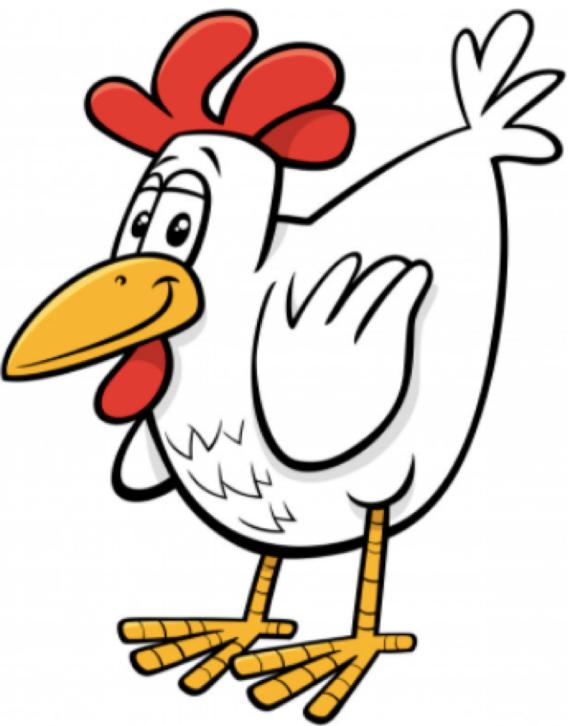
\$Subject	(Intercept)	Days
308	2.2585509	9.1989758
309	-40.3987381	-8.6196806
310	-38.9604090	-5.4488565
330	23.6906196	-4.8143503
331	22.2603126	-3.0699116
332	9.0395679	-0.2721770
333	16.8405086	-0.2236361
334	-7.2326151	1.0745816
335	-0.3336684	-10.7521652
337	34.8904868	8.6282652
349	-25.2102286	1.1734322
350	-13.0700342	6.6142178
351	4.5778642	-3.0152621
352	20.8636782	3.5360011
369	3.2754656	0.8722149
370	-25.6129993	4.8224850
371	0.8070461	-0.9881562
372	12.3145921	1.2840221

Subject number 309 had a better reaction time at the start and was less affected by sleep deprivation

Subject number 337 was slower to start and was more affected by sleep deprivation.

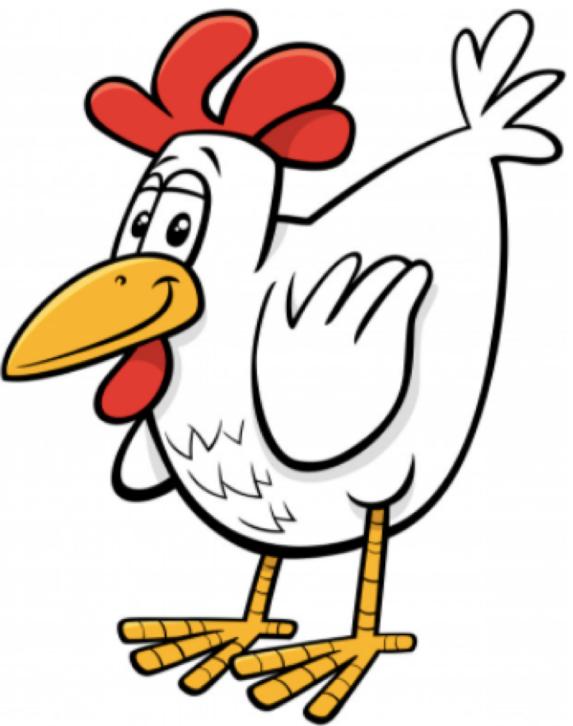
Subject number 370 was faster at the start and was more affected by sleep deprivation.

# Chick Weight



- ▶ 578 rows and 4 columns from a study on the effect of diet on chicken growth.
- ▶ The chicken weight (weight) at baseline (Time = 0) and several days later (Chick) was recorded.
- ▶ The type of diet (Diet) was also recorded.

# Chick Weight

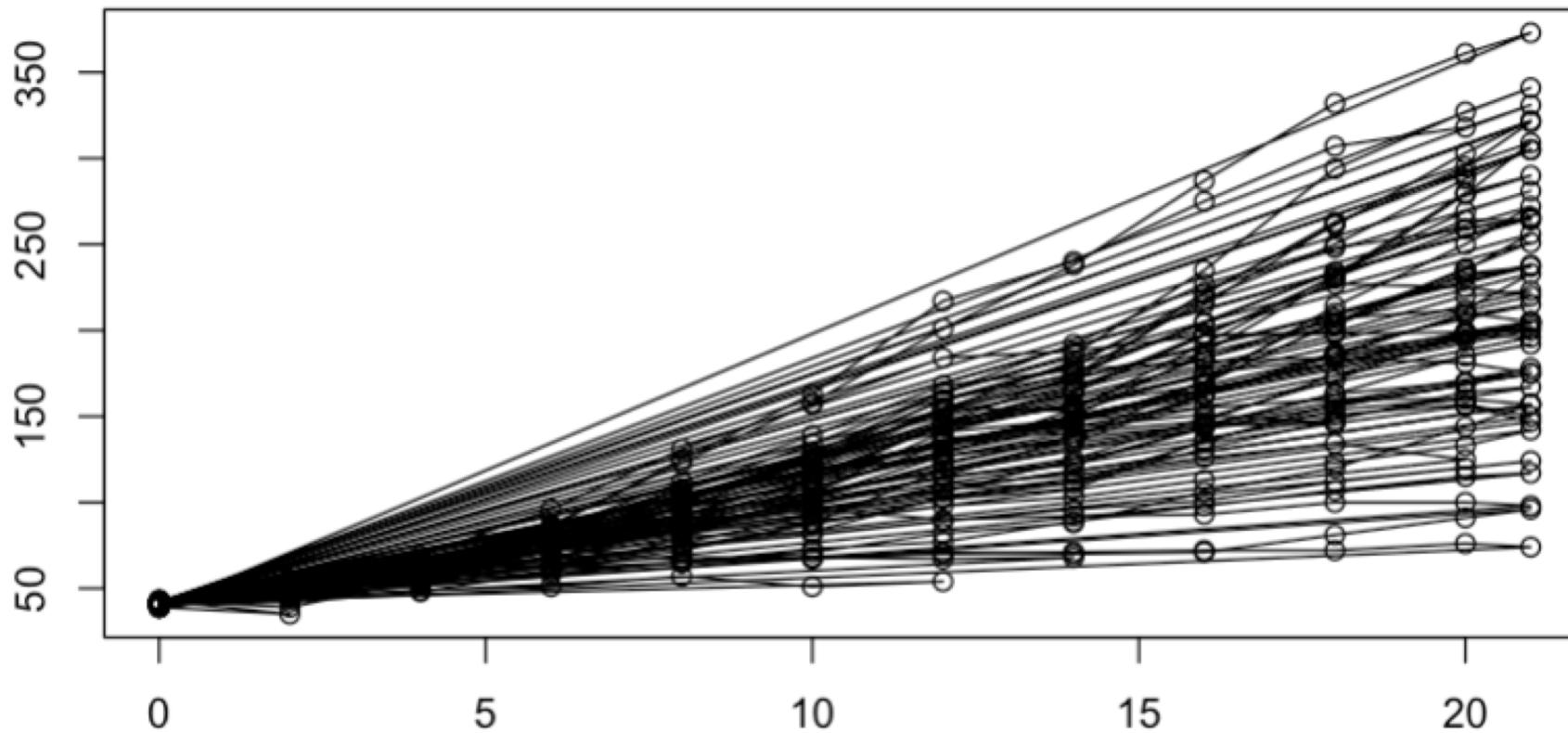


- ▶ 578 rows and 4 columns from a study on the effect of diet on chicken growth.
- ▶ The weight of the hens (weight) at baseline (Time = 0) and several days later for each hen (Chick) was recorded. The type of diet (Diet) was also recorded.

# Chick Weight

	weight	Time	Chick	Diet
1	42	0	1	1
2	51	2	1	1
3	59	4	1	1
4	64	6	1	1
5	76	8	1	1
6	93	10	1	1
7	106	12	1	1
8	125	14	1	1
9	149	16	1	1
10	171	18	1	1
11	199	20	1	1
12	205	21	1	1
13	40	0	2	1
14	49	2	2	1
15	58	4	2	1
16	72	6	2	1
17	84	8	2	1
18	103	10	2	1
19	122	12	2	1
20	138	14	2	1
21	162	16	2	1
22	187	18	2	1
23	209	20	2	1
24	215	21	2	1
25	43	0	3	1

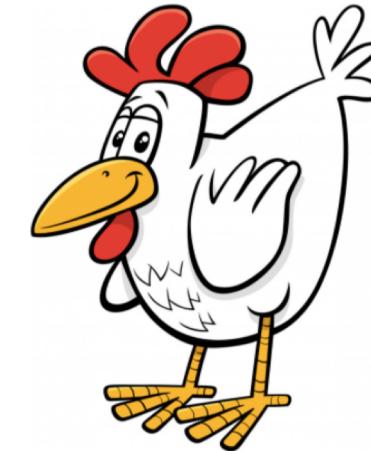
# Chick Weight



# Chick Weight

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	29.1780	1.9573	14.91
Time	8.4531	0.5408	15.63



average weight of hens at the beginning = 29.178

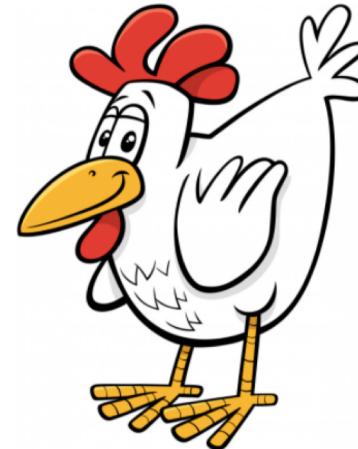
average weight gain 8.4531 / day

But... what about the chicken diet?

# Chick Weight

Fixed effects:

	Estimate	Std. Error	t value
(Intercept)	33.6613	2.9192	11.531
Diet2	-5.0277	5.0108	-1.003
Diet3	-15.4110	5.0108	-3.076
Diet4	-1.7505	5.0179	-0.349
Time	6.2770	0.7614	8.245
Diet2:Time	2.3321	1.3044	1.788
Diet3:Time	5.1459	1.3044	3.945
Diet4:Time	3.2550	1.3051	2.494



But... what about the chicken diet?

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