STA 325 Final Project Code

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```
library(readr)
library(dplyr)
library(tidyverse)
library(gridExtra)
library(mgcv)
library(patchwork)
library(MASS)
library(EnvStats)
library(tree)
library(randomForest)
library(gbm)
library(bst)
library(plyr)
library(caret)
```

Data Load-In and Initial Cleaning

```
# read data
flights <- read_csv("data/flights.csv")

# find unique airlines, destinations, and types of delays
unique(flights$OP_CARRIER)
```

```
## [1] "AA" "DL" "B6" "AS"
```

```
unique(flights$DEST)
```

```
##  [1] "LAX" "SFO" "SJC" "SAN" "PSP" "SMF" "OAK" "LGB" "ONT" "BUR"
```

```
# mutate delays and filter out NA arrival delays
flights <- flights %>%
  mutate(CARRIER_DELAY = case_when(CARRIER_DELAY > 0 ~ 1,
                                   TRUE ~ 0),
         WEATHER_DELAY = case_when(WEATHER_DELAY > 0 ~ 1,
            TRUE ~ 0),
         NAS_DELAY = case_when(NAS_DELAY > 0 ~ 1,
                               TRUE ~ 0),
         SECURITY_DELAY = case_when(SECURITY_DELAY > 0 ~ 1,
                                    TRUE ~ 0),
         LATE_AIRCRAFT_DELAY = case_when(LATE_AIRCRAFT_DELAY > 0 ~ 1,
                                         TRUE ~ 0)) %>%
  filter(!is.na(ARR_DELAY))
```

```r
# glimpse data
flights
```

```
## # A tibble: 2,033 x 34
##     YEAR MONTH DAY_OF_MONTH DAY_OF_WEEK FL_DATE    OP_CARRIER TAIL_NUM
##    <dbl> <dbl>        <dbl>       <dbl> <date>     <chr>      <chr>
##  1  2020     1            1           3 2020-01-01 AA         N110AN
##  2  2020     1            2           4 2020-01-02 AA         N111ZM
##  3  2020     1            3           5 2020-01-03 AA         N108NN
##  4  2020     1            4           6 2020-01-04 AA         N102NN
##  5  2020     1            5           7 2020-01-05 AA         N113AN
##  6  2020     1            6           1 2020-01-06 AA         N103NN
##  7  2020     1            7           2 2020-01-07 AA         N113AN
##  8  2020     1            8           3 2020-01-08 AA         N106NN
##  9  2020     1            9           4 2020-01-09 AA         N102NN
## 10  2020     1           10           5 2020-01-10 AA         N117AN
## # ... with 2,023 more rows, and 27 more variables: OP_CARRIER_FL_NUM <dbl>,
## #   ORIGIN <chr>, ORIGIN_CITY_NAME <chr>, DEST <chr>, DEST_CITY_NAME <chr>,
## #   CRS_DEP_TIME <dbl>, DEP_TIME <dbl>, DEP_DELAY <dbl>, TAXI_OUT <dbl>,
## #   WHEELS_OFF <dbl>, WHEELS_ON <dbl>, TAXI_IN <dbl>, CRS_ARR_TIME <dbl>,
## #   ARR_TIME <dbl>, ARR_DELAY <dbl>, CANCELLED <dbl>, CANCELLATION_CODE <lgl>,
## #   DIVERTED <dbl>, CRS_ELAPSED_TIME <dbl>, ACTUAL_ELAPSED_TIME <dbl>,
## #   AIR_TIME <dbl>, DISTANCE <dbl>, CARRIER_DELAY <dbl>, WEATHER_DELAY <dbl>,
## #   NAS_DELAY <dbl>, SECURITY_DELAY <dbl>, LATE_AIRCRAFT_DELAY <dbl>
```

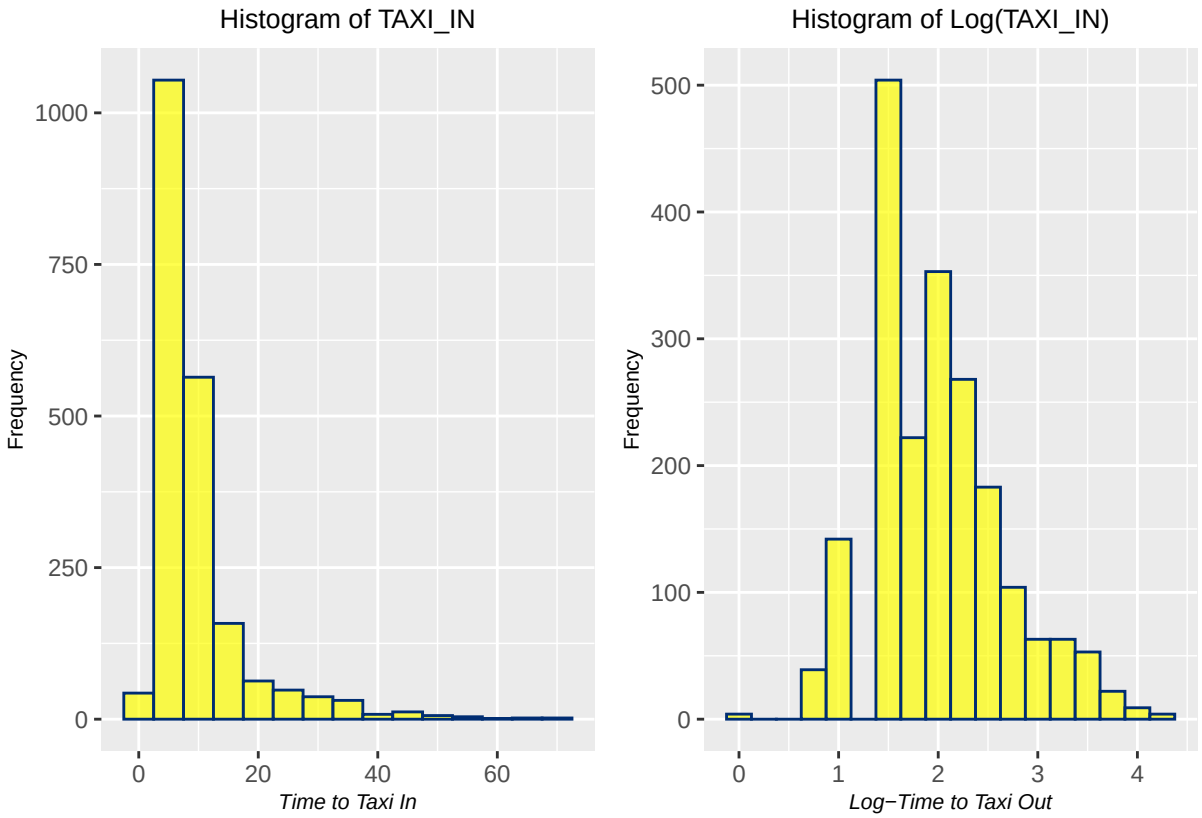
Exploratory Data Analysis

Individual Predictor Variable EDA

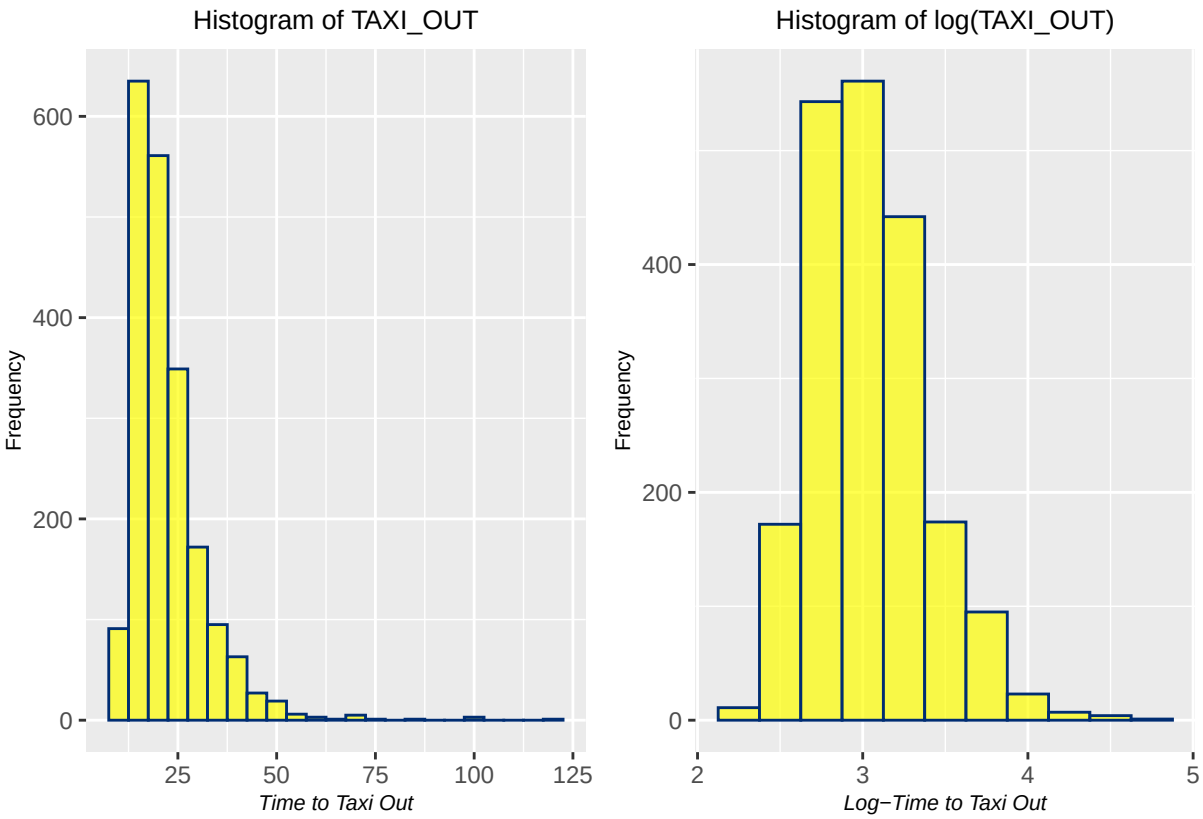
Taxi Histograms

```
# plot untransformed predictor taxi_in
pTAXI_IN <- ggplot(data = flights, aes(x = TAXI_IN)) +
  geom_histogram(binwidth = 5, fill = "#FFFF00", color = "#002D72", alpha = .7) +
  labs(x = "Time to Taxi In",
       y = "Frequency",
       title = "Histogram of TAXI_IN") +
  theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# plot untransformed predictor taxi_out
pTAXI_OUT <- ggplot(data = flights, aes(x = TAXI_OUT)) +
  geom_histogram(binwidth = 5, fill = "#FFFF00", color = "#002D72", alpha = .7) +
  labs(x = "Time to Taxi Out",
       y = "Frequency",
       title = "Histogram of TAXI_OUT") +
  theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# log transform taxi_in and taxi_out
flights$log_TAXI_OUT <- log(flights$TAXI_OUT)
flights$log_TAXI_IN <- log(flights$TAXI_IN)

# plot log transformed taxi_out
plog_TAXI_OUT <- ggplot(data = flights, aes(x = log_TAXI_OUT)) +
  geom_histogram(binwidth = .25, fill = "#FFFF00", color = "#002D72", alpha = .7) +
  labs(x = "Log-Time to Taxi Out",
       y = "Frequency",
       title = "Histogram of log(TAXI_OUT)") +
  theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# plot log transform taxi_in
plog_TAXI_IN <- ggplot(data = flights, aes(x = log_TAXI_IN)) +
  geom_histogram(binwidth = .25, fill = "#FFFF00", color = "#002D72", alpha = .7) +
  labs(x = "Log-Time to Taxi Out",
       y = "Frequency",
       title = "Histogram of Log(TAXI_IN)") +
  theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))
```

```
pTAXI_IN + plog_TAXI_IN
```

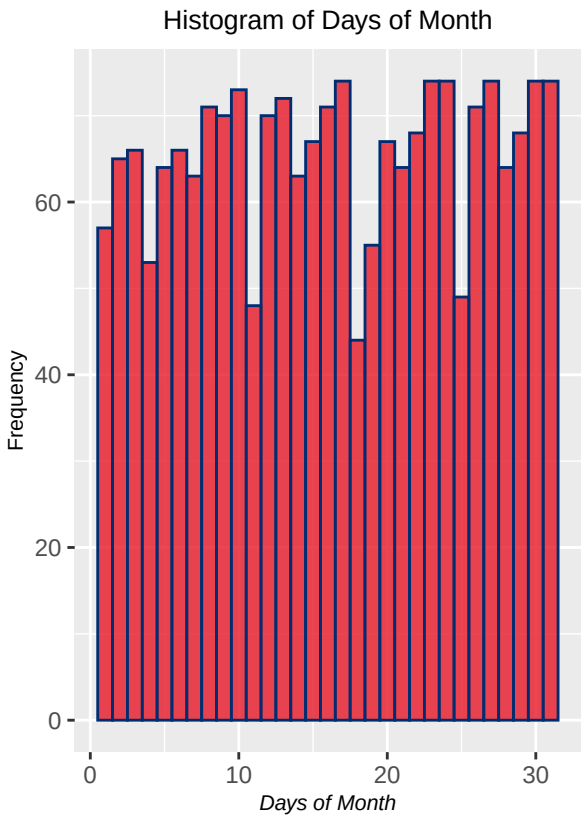


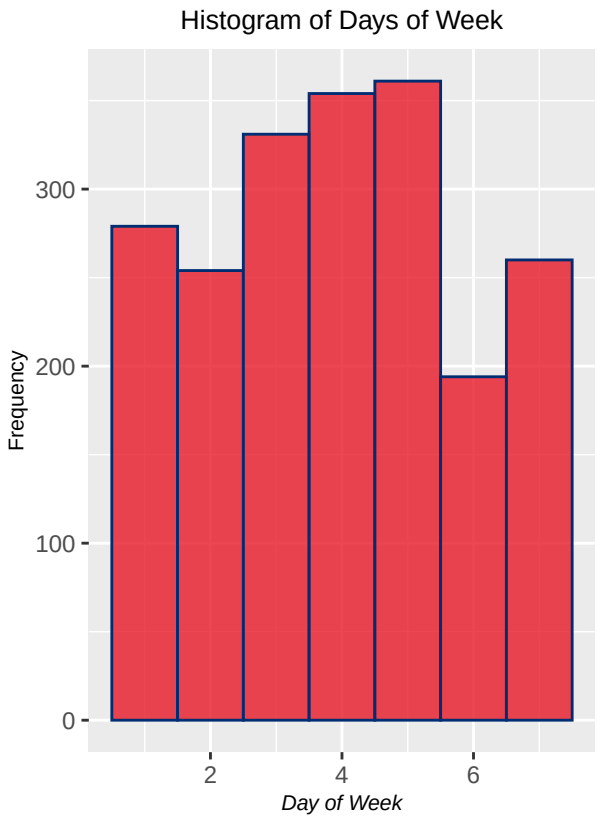

```
pTAXI_OUT + plog_TAXI_OUT
```



Days of Month and Week

```
# plot predictor DAYS_OF_MONTH
pDOM <- ggplot(data = flights, aes(x = DAY_OF_MONTH)) +
  geom_histogram(binwidth = 1, fill = "#E81828", color = "#002D72", alpha = .8) +
  labs(x = "Days of Month",
       y = "Frequency",
       title = "Histogram of Days of Month") +
    theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# plot predictor DAY_OF_WEEK
pDOW <- ggplot(data = flights, aes(x = DAY_OF_WEEK)) +
  geom_histogram(binwidth = 1, fill = "#E81828", color = "#002D72", alpha = .8) +
  labs(x = "Day of Week",
       y = "Frequency",
       title = "Histogram of Days of Week") +
    theme(plot.title = element_text(size = 10,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pDOM + pDOW
```

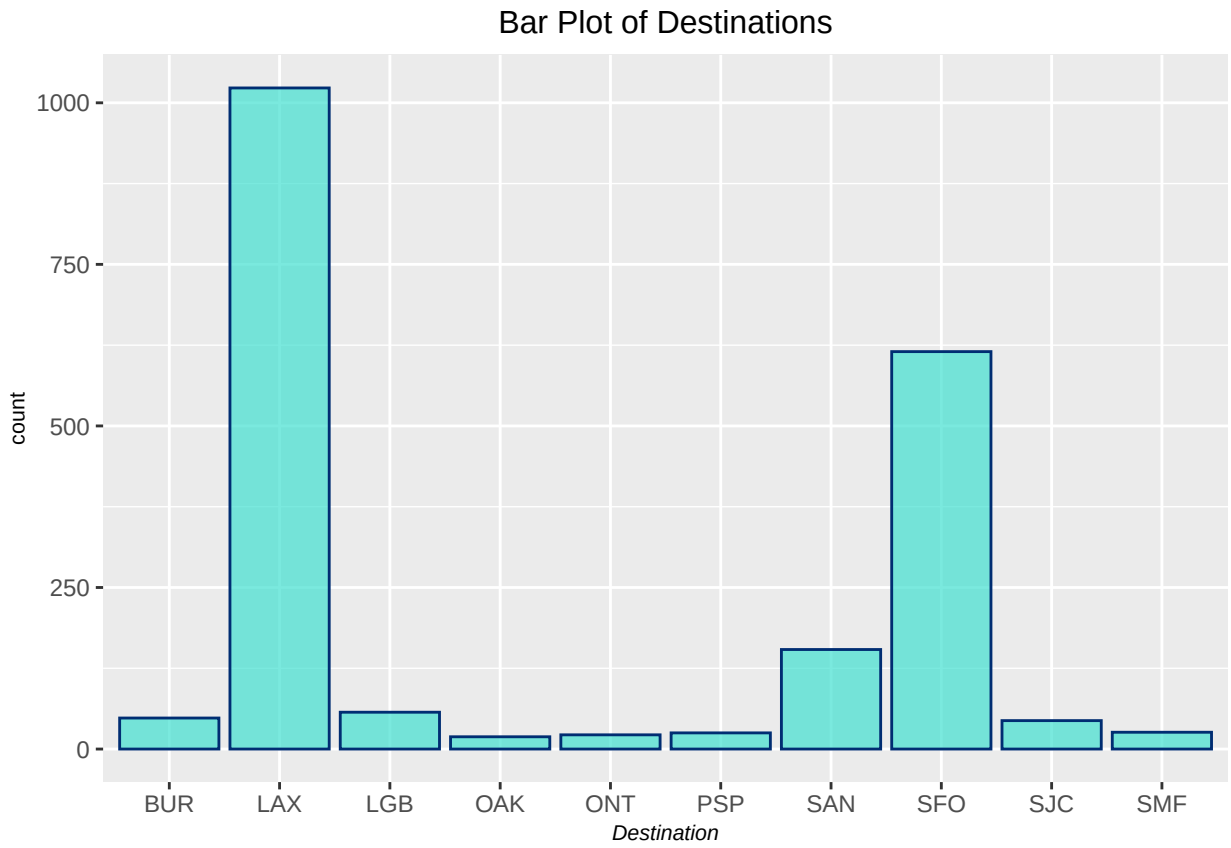




Destination Locations

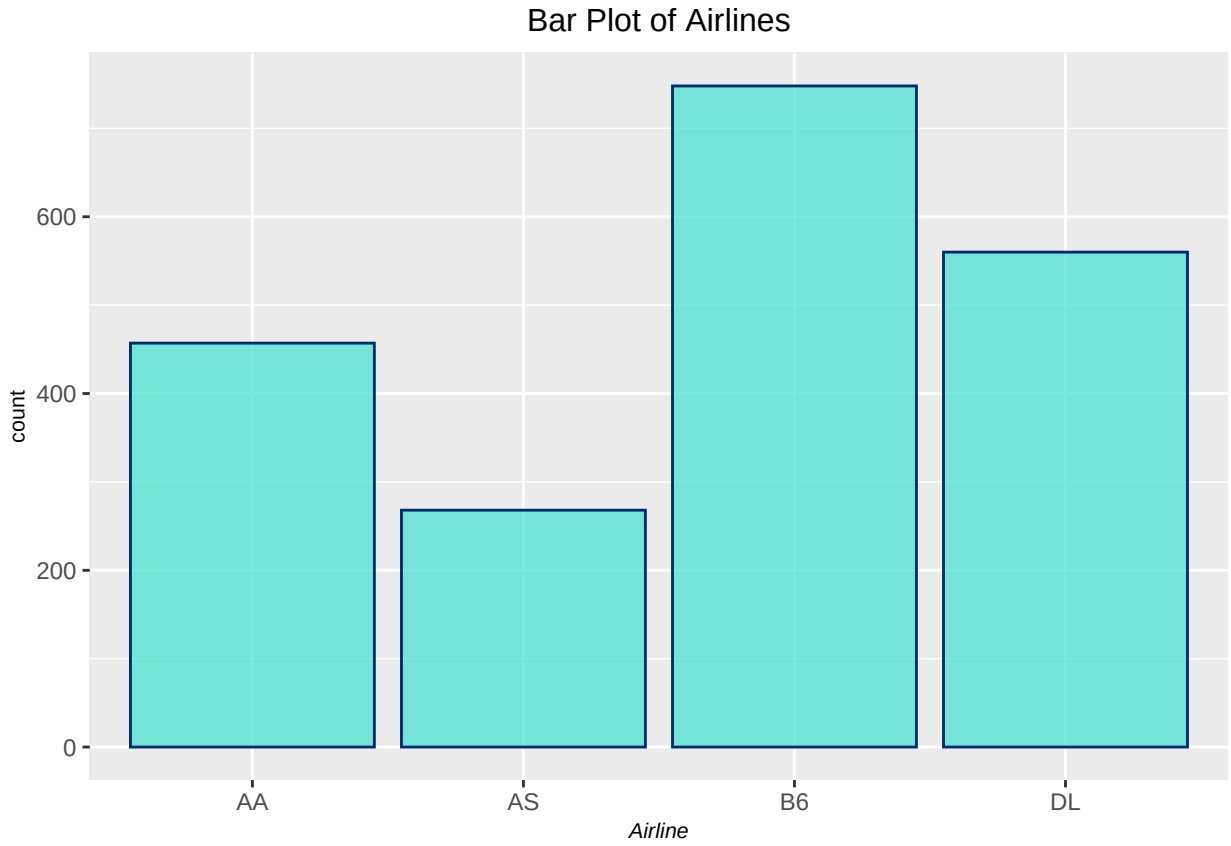
Origin is all JFK, but we could consider the different destination locations.

```
# plot destinations in CA
pDEST <- ggplot(data = flights, aes(x = DEST)) +
  geom_bar(fill = "#40E0D0", color = "#002D72", alpha = .7) +
  labs(x = "Destination",
       title = "Bar Plot of Destinations") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pDEST
```



Airlines

```
# plot airline carriers
pLINE <- ggplot(data = flights, aes(x = OP_CARRIER)) +
  geom_bar(fill = "#40E0D0", color = "#002D72", alpha = .7) +
  labs(x = "Airline",
       title = "Bar Plot of Airlines") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pLINE
```

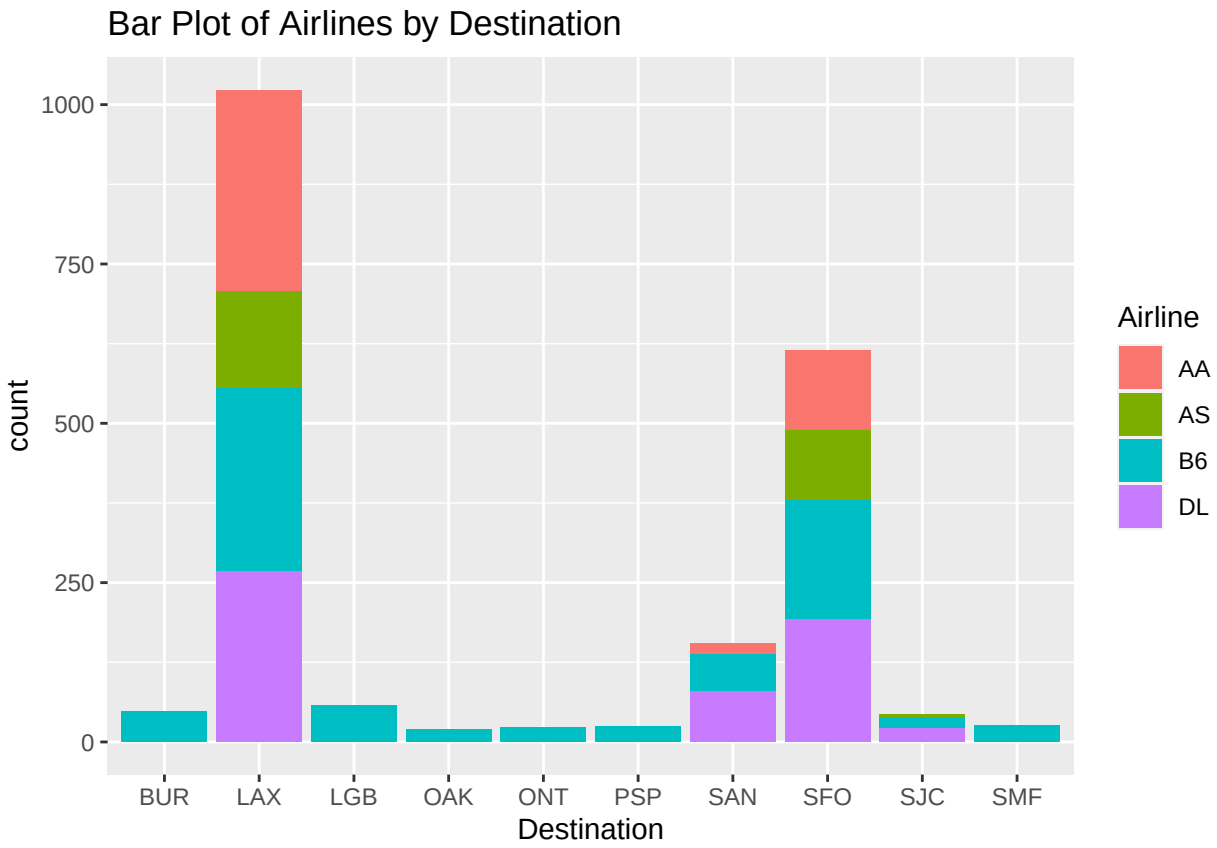



```
# plot airlines by destination
pLINEDEST <- ggplot(data = flights, aes(x = DEST, fill = OP_CARRIER)) +
  geom_bar() +
  labs(x = "Destination",
       title = "Bar Plot of Airlines by Destination",
       fill = "Airline")
    theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))
```

```
## List of 4
##  $ axis.title.x.bottom:List of 11
##   ..$ family       : NULL
##   ..$ face         : chr "italic"
##   ..$ colour       : NULL
##   ..$ size         : num 8
##   ..$ hjust        : NULL
##   ..$ vjust        : NULL
##   ..$ angle        : NULL
##   ..$ lineheight   : NULL
##   ..$ margin       : NULL
##   ..$ debug        : NULL
##   ..$ inherit.blank: logi FALSE
##   ..- attr(*, "class")= chr [1:2] "element_text" "element"
##  $ axis.title.y.left  :List of 11
##   ..$ family       : NULL
```

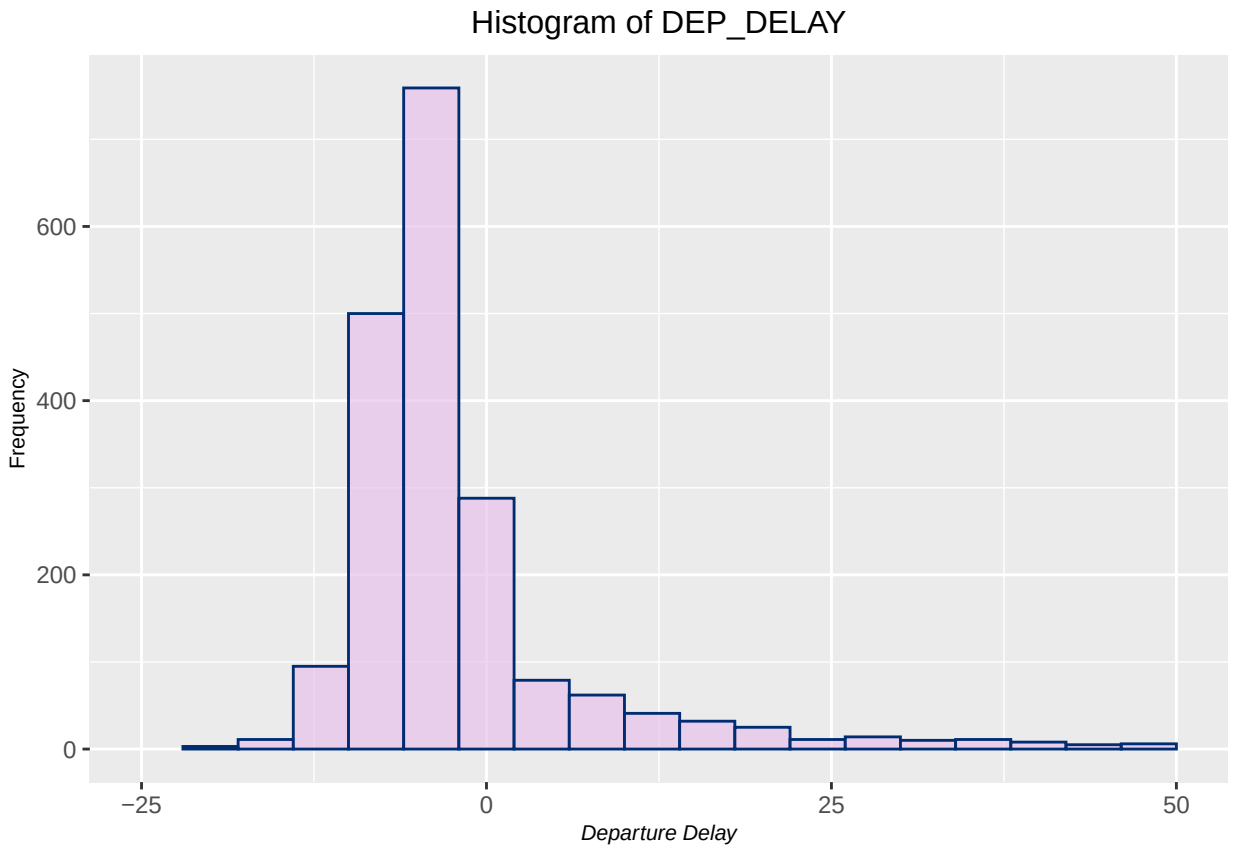
```
##   ..$ face         : NULL
##   ..$ colour       : NULL
##   ..$ size         : num 8
##   ..$ hjust        : NULL
##   ..$ vjust        : NULL
##   ..$ angle        : NULL
##   ..$ lineheight   : NULL
##   ..$ margin       : NULL
##   ..$ debug        : NULL
##   ..$ inherit.blank: logi FALSE
##   ..- attr(*, "class")= chr [1:2] "element_text" "element"
##  $ plot.title         :List of 11
##   ..$ family       : NULL
##   ..$ face         : NULL
##   ..$ colour       : NULL
##   ..$ size         : num 12
##   ..$ hjust        : num 0.5
##   ..$ vjust        : NULL
##   ..$ angle        : NULL
##   ..$ lineheight   : NULL
##   ..$ margin       : NULL
##   ..$ debug        : NULL
##   ..$ inherit.blank: logi FALSE
##   ..- attr(*, "class")= chr [1:2] "element_text" "element"
##  $ plot.subtitle      :List of 11
##   ..$ family       : NULL
##   ..$ face         : NULL
##   ..$ colour       : NULL
##   ..$ size         : NULL
##   ..$ hjust        : num 0.5
##   ..$ vjust        : NULL
##   ..$ angle        : NULL
##   ..$ lineheight   : NULL
##   ..$ margin       : NULL
##   ..$ debug        : NULL
##   ..$ inherit.blank: logi FALSE
##   ..- attr(*, "class")= chr [1:2] "element_text" "element"
##  - attr(*, "class")= chr [1:2] "theme" "gg"
##  - attr(*, "complete")= logi FALSE
##  - attr(*, "validate")= logi TRUE
```

```
pLINEDEST
```

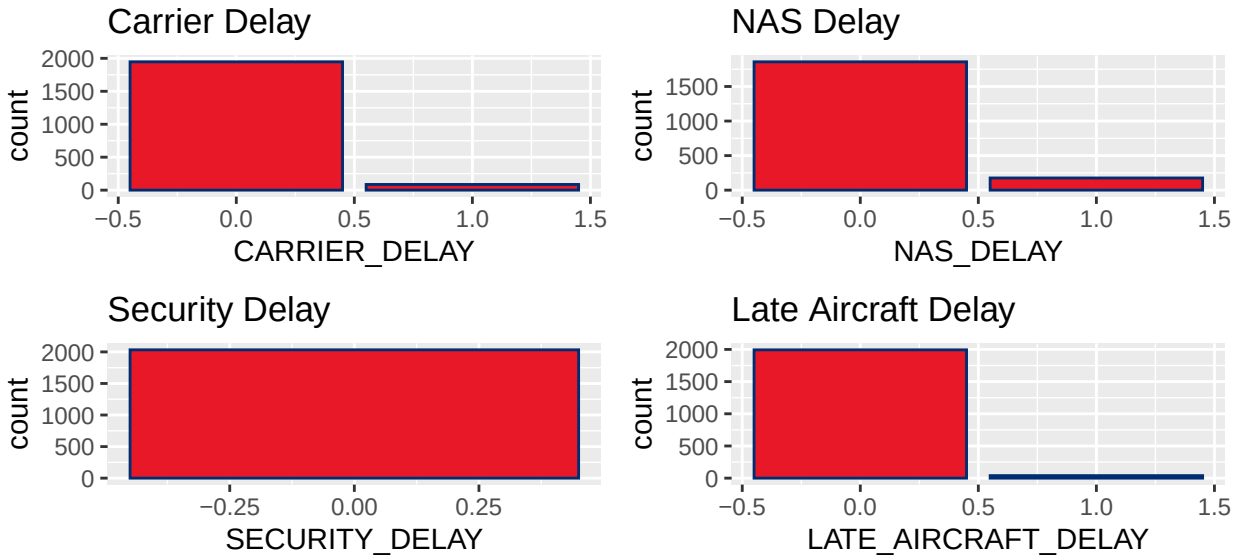


Depart Delay Histogram

```
# plot DEP_DELAY
pDEPDELAY <- ggplot(data = flights, aes(x = DEP_DELAY)) +
  geom_histogram(binwidth = 4, fill = "#e9c2ed", color = "#002D72", alpha = 0.7) +
  xlim(-25, 50) +
  labs(x = "Departure Delay",
       y = "Frequency",
       title = "Histogram of DEP_DELAY") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pDEPDELAY
```




```
# plot types of delays
p1 <- ggplot(data = flights, aes(x = CARRIER_DELAY)) +
  geom_bar(fill = "#E81828", color = "#002D72") +
  labs(title = "Carrier Delay")

#p2 <- ggplot(data = flights, aes(x = WEATHER_DELAY)) +
  #geom_bar(fill = "#E81828", color = "#002D72") +
  #labs(title = "Weather Delay")

p3 <- ggplot(data = flights, aes(x = NAS_DELAY)) +
  geom_bar(fill = "#E81828", color = "#002D72") +
  labs(title = "NAS Delay")

p4 <- ggplot(data = flights, aes(x = SECURITY_DELAY)) +
  geom_bar(fill = "#E81828", color = "#002D72") +
  labs(title = "Security Delay")

p5 <- ggplot(data = flights, aes(x = LATE_AIRCRAFT_DELAY)) +
  geom_bar(fill = "#E81828", color = "#002D72") +
  labs(title = "Late Aircraft Delay")

grid.arrange(p1,p3,p4,p5, nrow = 3)
```



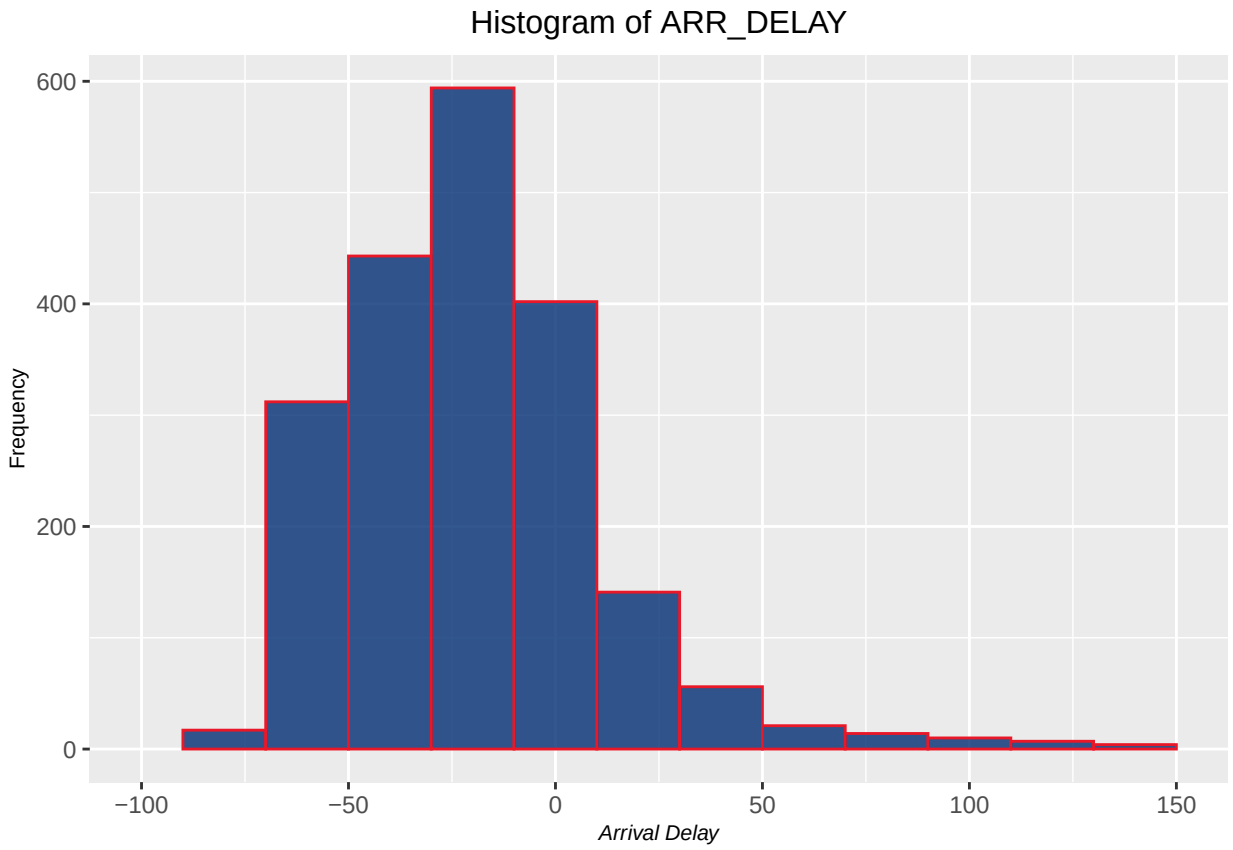
From this EDA of the categorical variables, we probably should not perform analysis with `SECURITY_DELAY` since all of them are classified as 0.

Furthermore, only 9 flights are classified with a weather delay, so it may not be good for our model to include this as a variable for right now.

Overall, the categorical delay predictors I would think we could use are: Carrier Delay, NAS Delay, and Late Aircraft Delay

Response Variable EDA

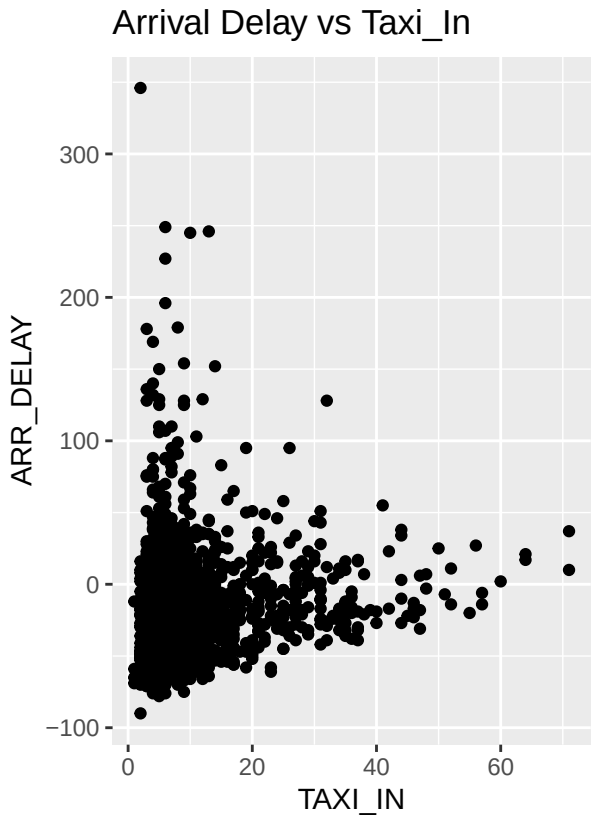
```
# plot ARR_DELAY
pARRDELAY <- ggplot(data = flights, aes(x = ARR_DELAY)) +
  geom_histogram(binwidth = 20, fill = "#002D72", color = "#E81828", alpha = 0.8) +
  xlim(-100, 150) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Histogram of ARR_DELAY") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pARRDELAY
```

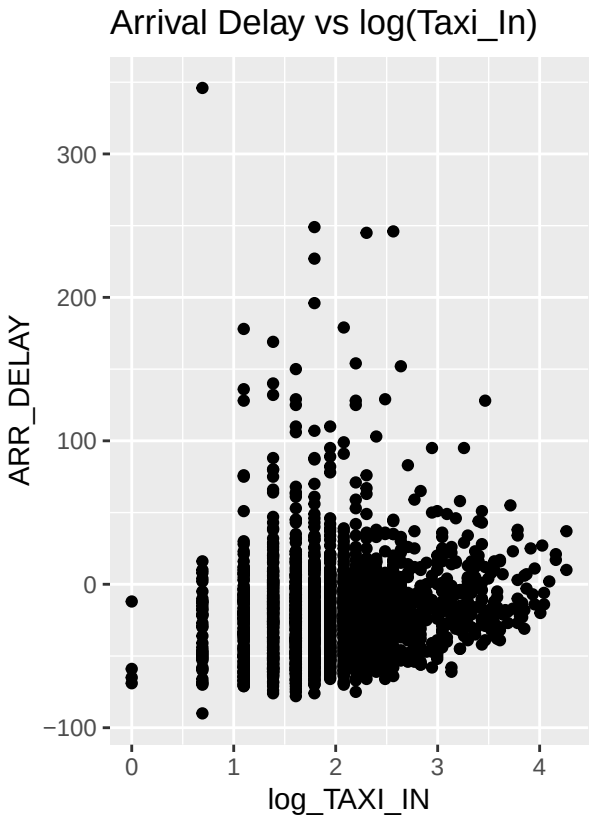


Predictors vs. Response EDA

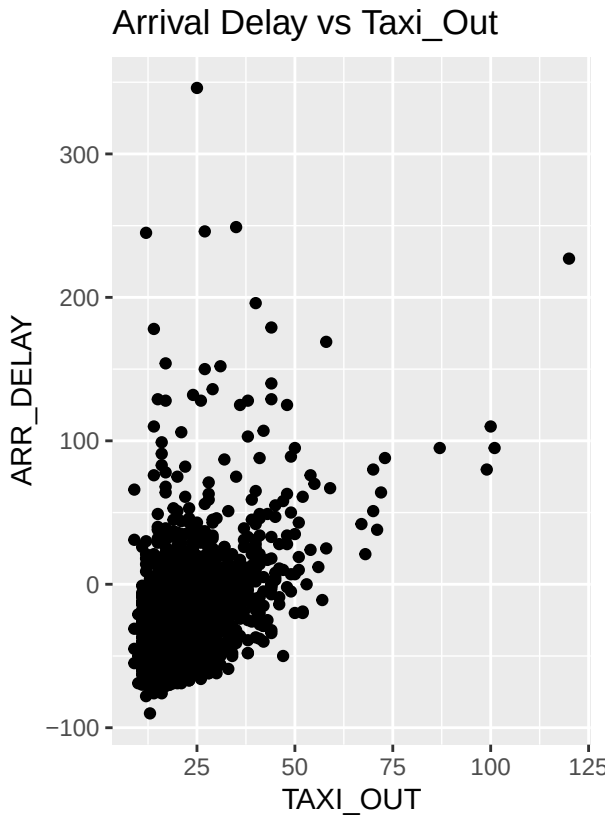
TAXI_IN / TAXI_OUT vs. ARR_DELAY

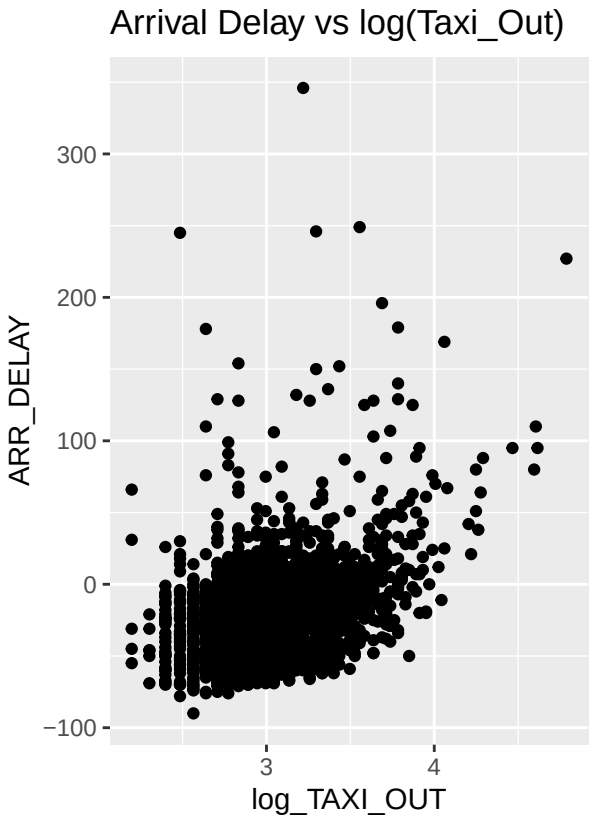
```
p6 <- ggplot(data = flights, aes(y = ARR_DELAY, x = TAXI_IN)) +
  geom_point() +
  labs(title = "Arrival Delay vs Taxi_In")

p7 <- ggplot(data = flights, aes(y = ARR_DELAY, x = TAXI_OUT)) +
  geom_point() +
  labs(title = "Arrival Delay vs Taxi_Out")



plog6 <- ggplot(data = flights, aes(y = ARR_DELAY, x = log_TAXI_IN)) +
  geom_point() +
  labs(title = "Arrival Delay vs log(Taxi_In)")

plog7 <- ggplot(data = flights, aes(y = ARR_DELAY, x = log_TAXI_OUT)) +
  geom_point() +
  labs(title = "Arrival Delay vs log(Taxi_Out)")

p6 + plog6
```


Arrival Delay vs Taxi_In
ARR_DELAY
TAXI_IN
300
200
100
0
−100
0
20
40
60


Arrival Delay vs log(Taxi_In)
ARR_DELAY
log_TAXI_IN
300
200
100
0
−100
0
1
2
3
4

```
p7 + plog7
```

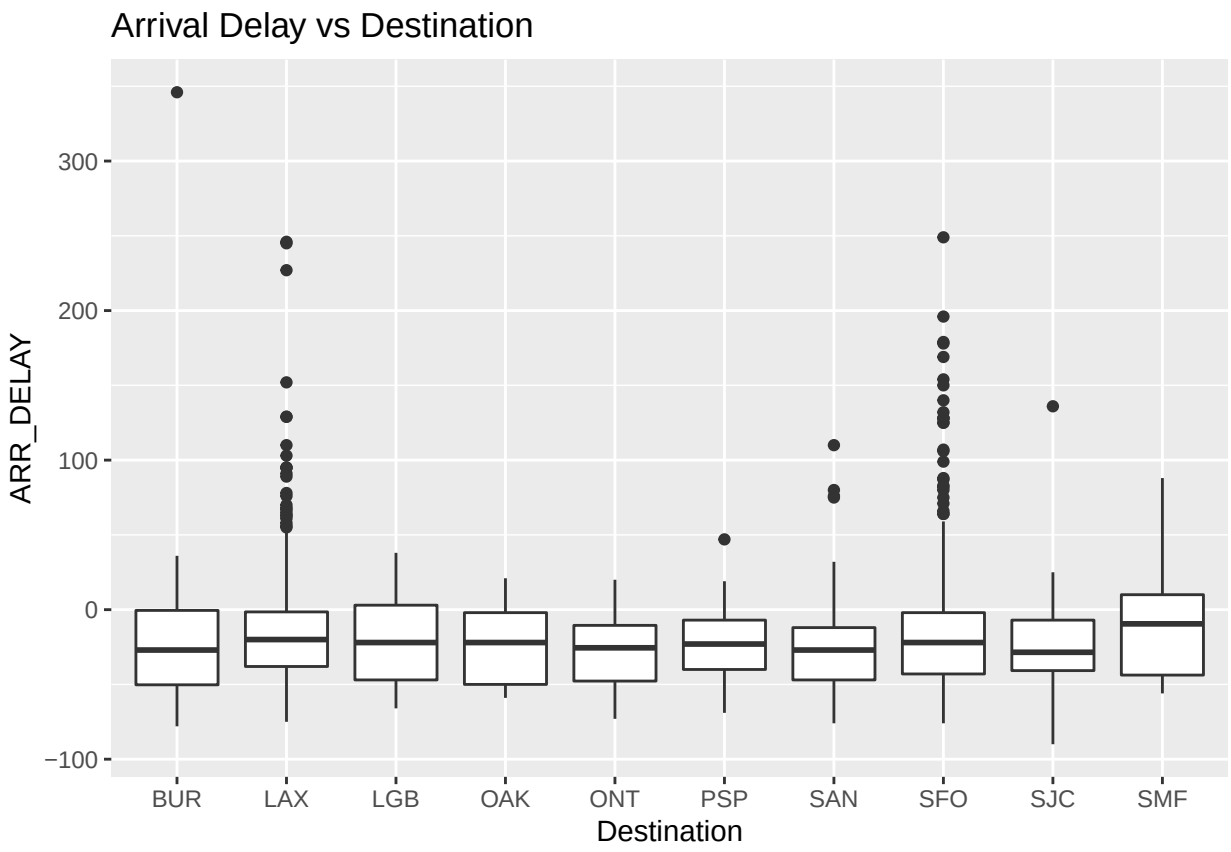

Arrival Delay vs Taxi_Out
ARR_DELAY
TAXI_OUT
300
200
100
0
−100
25
50
75
100
125


Arrival Delay vs log(Taxi_Out)
ARR_DELAY
log_TAXI_OUT
300
200
100
0
−100
3
4

These plots above suggest that we may want to transform the variables at some point.

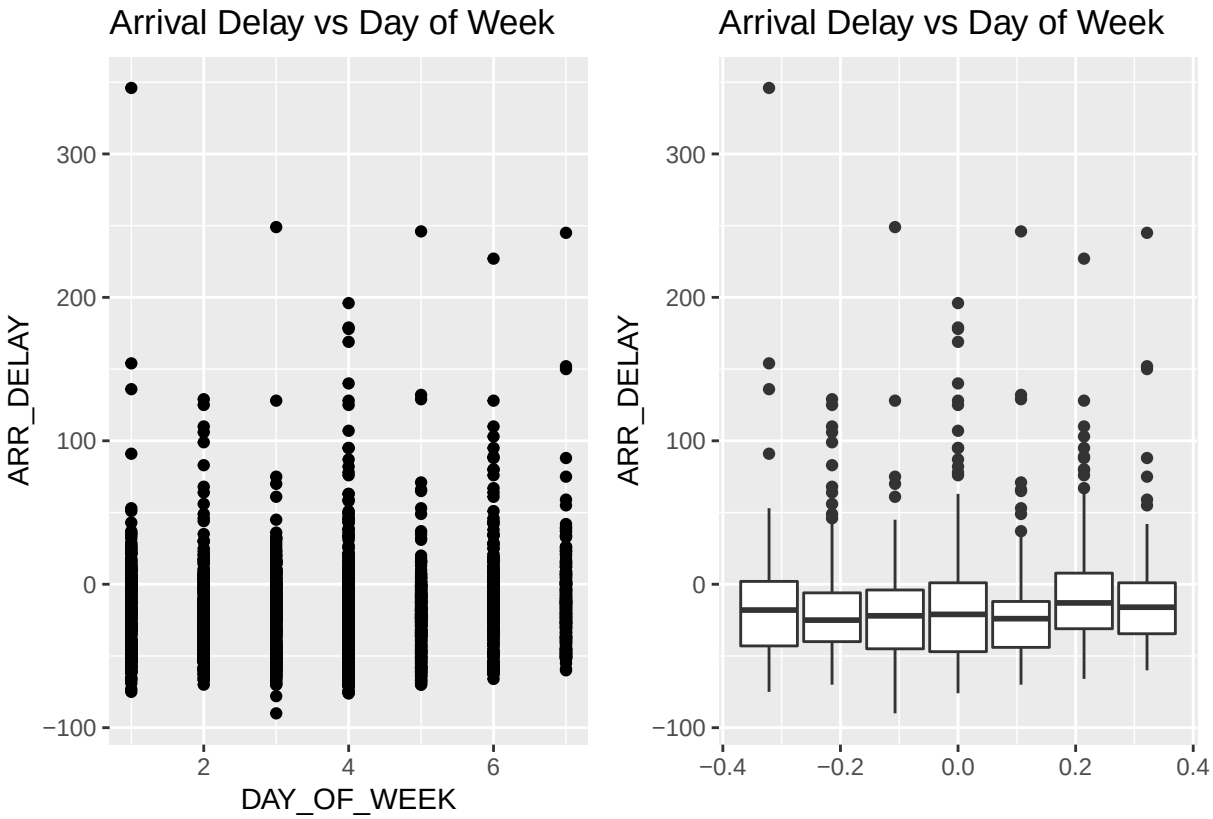
DEST vs. ARR_DELAY

```
ggplot(data = flights, aes(y = ARR_DELAY, x = DEST)) +
  geom_boxplot() +
  labs(x = "Destination",
       title = "Arrival Delay vs Destination")
```



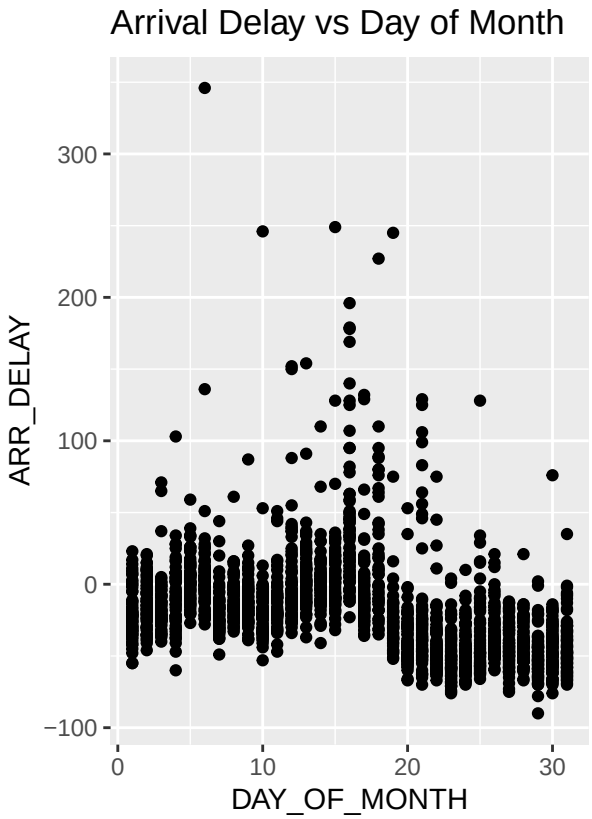
DAY_OF_WEEK vs. ARR_DELAY

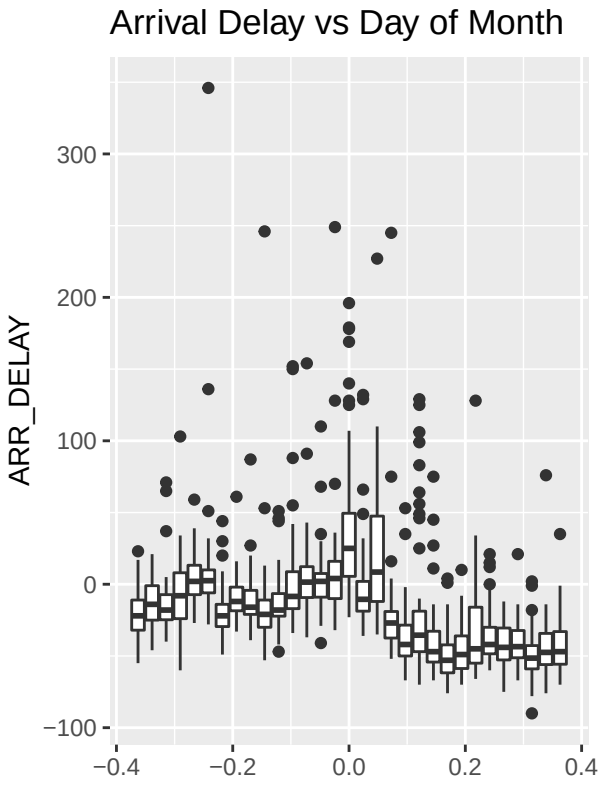
```
p8 <- ggplot(data = flights, aes(y = ARR_DELAY, x = DAY_OF_WEEK)) +
  geom_point() +
  labs(title = "Arrival Delay vs Day of Week")

p9 <- ggplot(data = flights, aes(y = ARR_DELAY, group = DAY_OF_WEEK)) +
  geom_boxplot() +
  labs(title = "Arrival Delay vs Day of Week")

p8 + p9
```



ARR_DELAY and DAY_OF_MONTH

```
p10 <- ggplot(data = flights, aes(y = ARR_DELAY, x = DAY_OF_MONTH)) +
  geom_point() +
  labs(title = "Arrival Delay vs Day of Month")

p11 <- ggplot(data = flights, aes(y = ARR_DELAY, group = DAY_OF_MONTH)) +
  geom_boxplot() +
  labs(title = "Arrival Delay vs Day of Month")

p10 + p11
```


Arrival Delay vs Day of Month
ARR_DELAY
300
200
100
0
−100
0
10
20
30
DAY_OF_MONTH

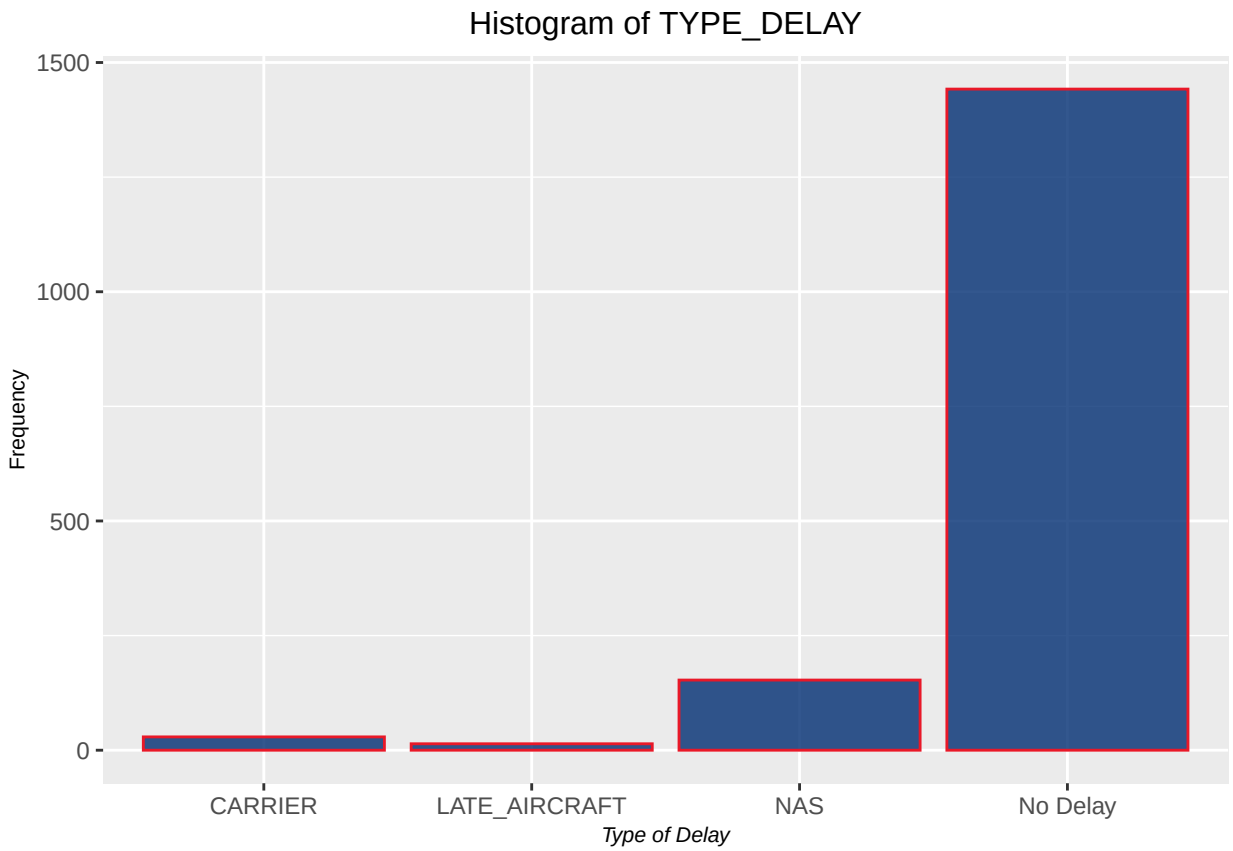

Arrival Delay vs Day of Month
ARR_DELAY
300
200
100
0
−100
−0.4
−0.2
0.0
0.2
0.4

Additional Data Cleaning

New Bounds and Variable

```
# take only SFO/LAX since all 4 carriers fly there
# create TYPE_DELAY as a factor variable for type of delays
flights <- flights %>%
  filter(DEST == "SFO" | DEST == "LAX") %>%
  mutate(TYPE_DELAY = case_when(NAS_DELAY == 1 ~ "NAS",
                                CARRIER_DELAY == 1 ~ "CARRIER",
                                LATE_AIRCRAFT_DELAY == 1 ~ "LATE_AIRCRAFT",
                                WEATHER_DELAY == 1 ~ "WEATHER",
                                TRUE ~ "No Delay"))
```

```
pTYPE <- ggplot(data = flights, aes(x = TYPE_DELAY)) +
  geom_bar(fill = "#002D72", color = "#E81828", alpha = 0.8) +
  labs(x = "Type of Delay",
       y = "Frequency",
       title = "Histogram of TYPE_DELAY") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

pTYPE
```



Test and Training Set Split

We will use a 80-20 split of training and test sets.

```
set.seed(1234)

flights <- flights %>%
  dplyr::mutate(id = row_number())

train <- flights %>%
  sample_frac(0.8)
test <- anti_join(flights, train, by = "id")
```

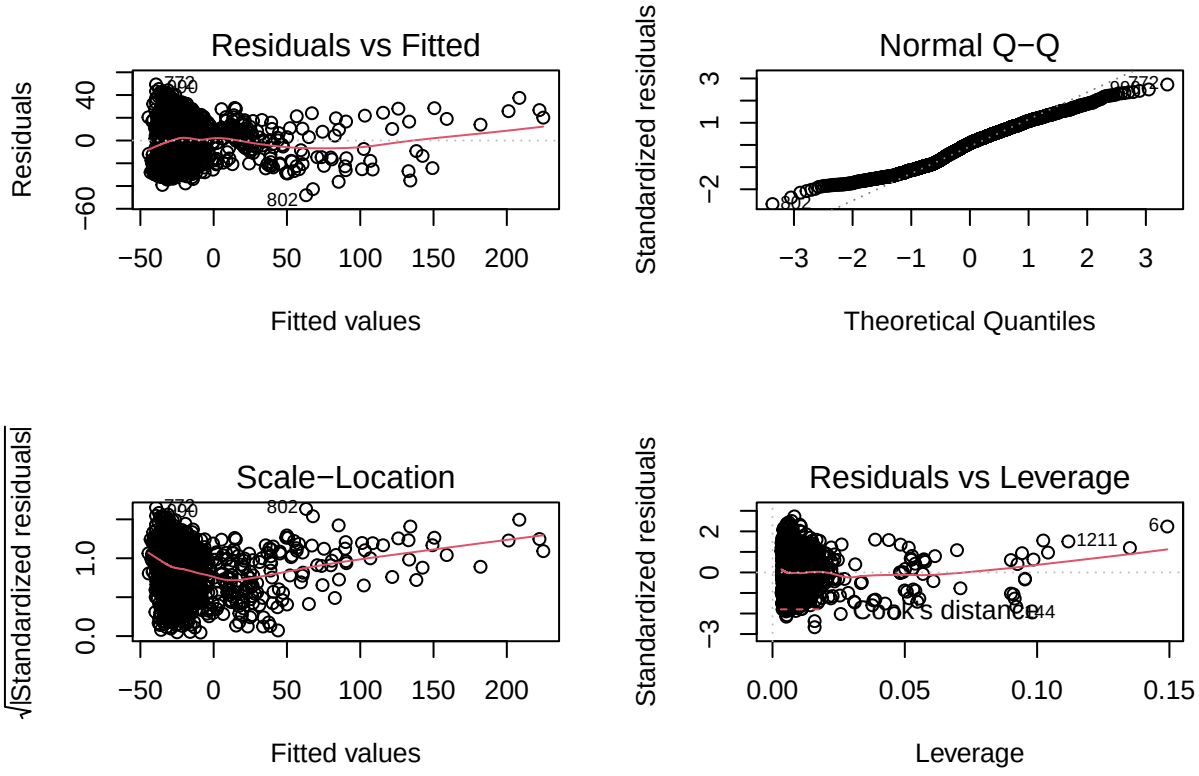
Modeling

(1) Multiple Linear Regression

(a) Baesline Linear Model (with AIC Selection)

```
# create linear model with all revelant variables
full_model <- lm(ARR_DELAY ~
                    DEP_DELAY +
                    DAY_OF_WEEK +
                    OP_CARRIER +
                    DEST +
                    CRS_DEP_TIME +
                    CRS_ARR_TIME +
                    TAXI_OUT +
                    TAXI_IN +
                    TYPE_DELAY, train)

# summary(full_model)
```

```
# use AIC model selection on full model
step_model <- stepAIC(full_model, trace = FALSE)
#summary(step_model)

# update full model
plain_linear_model <- step_model

# plot full model
par(mfrow = c(2,2))
plot(plain_linear_model)
```



(b) Full Log-Transformed Model

```
# fitting models and running ANOVA tests to identify interactions

## first model
lm.01 <- lm(ARR_DELAY ~ DEP_DELAY + DAY_OF_WEEK + OP_CARRIER + DEST + CRS_DEP_TIME + CRS_ARR_TIME + log
#plot(lm.01)
#summary(lm.01)

## second model
step_model <- stepAIC(lm.01, direction = "backward", trace = FALSE)
#summary(step_model)

## third model
lm.02 <- lm(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME  + log_TAXI_OUT + log_TAXI_IN + TY
#summary(lm.02)
#anova(step_model, lm.02)

## fourth model
lm.03 <- lm(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME  + log_TAXI_OUT + log_TAXI_IN + TY
#anova(lm.02, lm.03)

## final log model
log_linear_model <- lm(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME  + log_TAXI_OUT + log_T

anova(lm.03, log_linear_model)
```

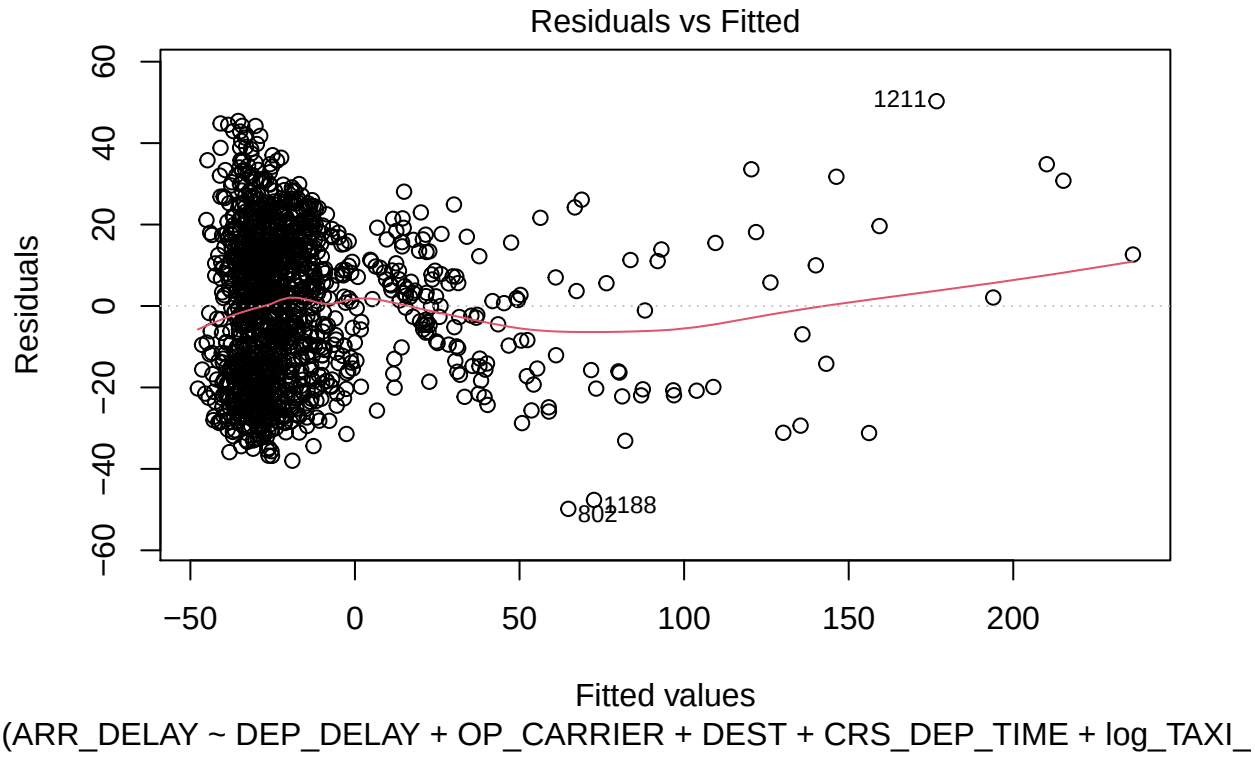
```
## Analysis of Variance Table
##
## Model 1: ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME + log_TAXI_OUT +
##     log_TAXI_IN + TYPE_DELAY + OP_CARRIER:DEST + DEST:log_TAXI_IN
## Model 2: ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME + log_TAXI_OUT +
##     log_TAXI_IN + TYPE_DELAY + OP_CARRIER:DEST + DEST:log_TAXI_IN +
##     log_TAXI_OUT:DEP_DELAY
##   Res.Df    RSS Df Sum of Sq      F  Pr(>F)
## 1   1294 427667
## 2   1293 425449  1      2218 6.7408 0.00953 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(log_linear_model)
```

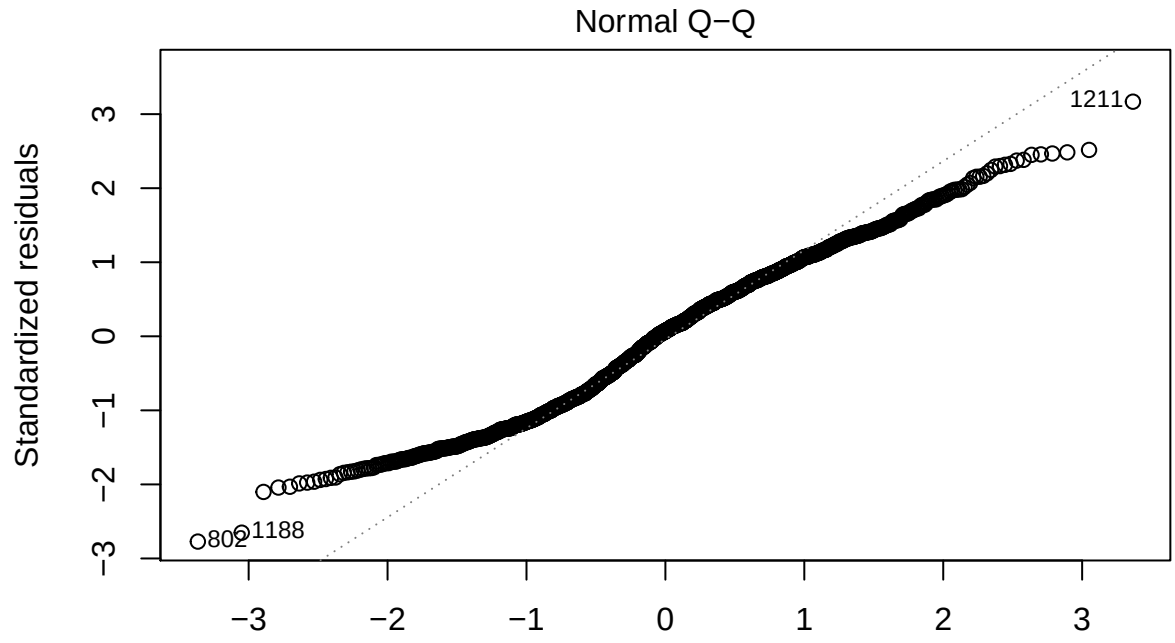
```
##
## Call:
## lm(formula = ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME +
##     log_TAXI_OUT + log_TAXI_IN + TYPE_DELAY + OP_CARRIER:DEST +
##     DEST:log_TAXI_IN + log_TAXI_OUT:DEP_DELAY, data = train)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -49.817 -15.330   1.198  13.897  50.301
##
## Coefficients:
##                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)              -81.901399   7.262197 -11.278  < 2e-16 ***
```

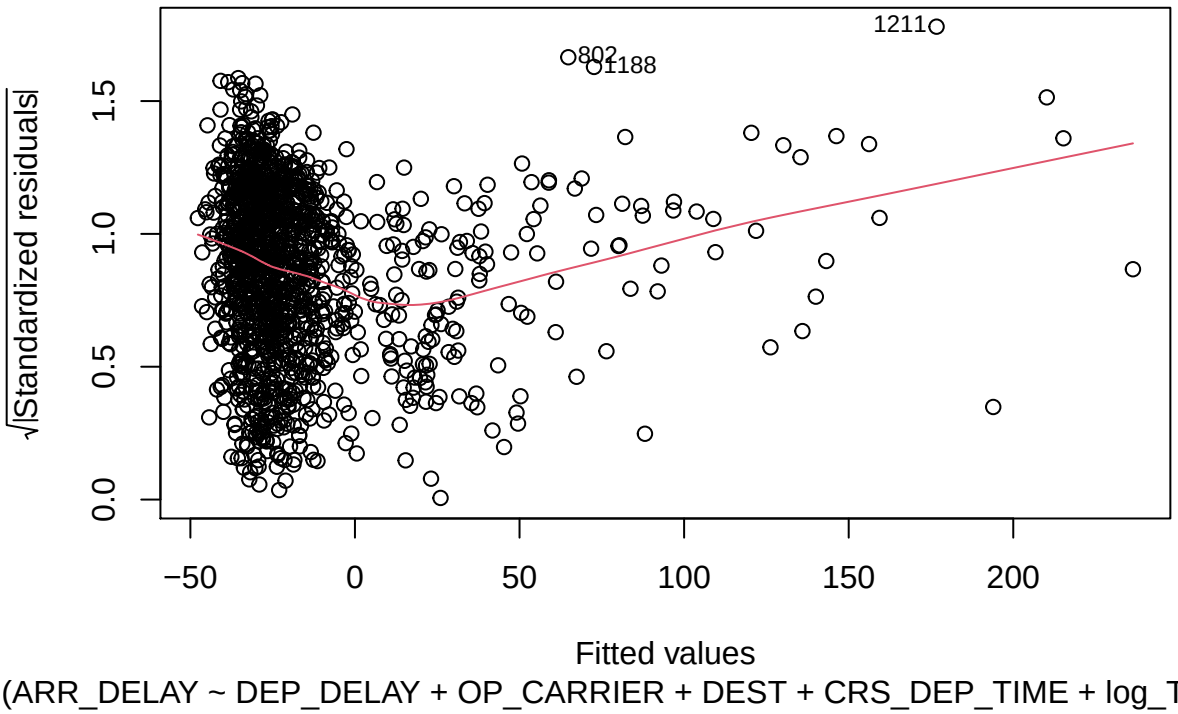
```
## DEP_DELAY                 0.524207   0.141256   3.711 0.000215 ***
## OP_CARRIERAS             -4.458249   2.087733  -2.135 0.032912 *
## OP_CARRIERB6              5.045463   1.669832   3.022 0.002564 **
## OP_CARRIERDL             -1.493672   1.717998  -0.869 0.384775
## DESTSFO                   9.893184   4.366882   2.266 0.023647 *
## CRS_DEP_TIME             -0.004364   0.001070  -4.081 4.77e-05 ***
## log_TAXI_OUT             20.610508   1.617078  12.746  < 2e-16 ***
## log_TAXI_IN               8.433233   1.057392   7.976 3.32e-15 ***
## TYPE_DELAYLATE_AIRCRAFT  -3.973566   6.537317  -0.608 0.543408
## TYPE_DELAYNAS            24.019795   4.598524   5.223 2.05e-07 ***
## TYPE_DELAYNo Delay      -15.676745   4.540377  -3.453 0.000573 ***
## OP_CARRIERAS:DESTSFO      6.630276   3.374581   1.965 0.049655 *
## OP_CARRIERB6:DESTSFO     -4.199151   2.858830  -1.469 0.142121
## OP_CARRIERDL:DESTSFO     -1.424895   2.900122  -0.491 0.623282
## DESTSFO:log_TAXI_IN      -5.261163   1.951509  -2.696 0.007110 **
## DEP_DELAY:log_TAXI_OUT    0.113332   0.043651   2.596 0.009530 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.14 on 1293 degrees of freedom
## Multiple R-squared:  0.7376, Adjusted R-squared:  0.7344
## F-statistic: 227.2 on 16 and 1293 DF,  p-value: < 2.2e-16
```

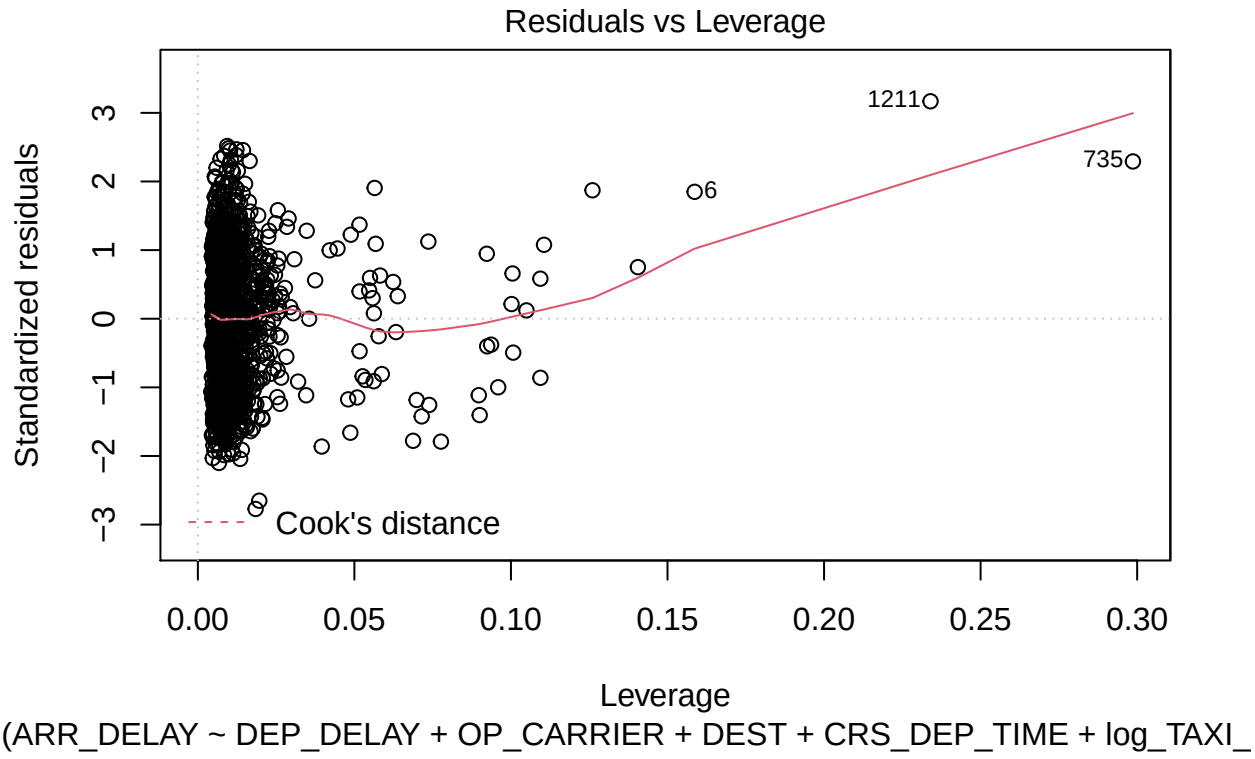
```
plot(log_linear_model)
```



(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME + log_TAXI_


Normal Q−Q
Standardized residuals
Theoretical Quantiles
(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME + log_TAXI_
1211
802
1188
−3
−2
−1
0
1
2
3


Scale−Location
√|Standardized residuals|
Fitted values
(ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST + CRS_DEP_TIME + log_TAXI_
1211
802
1188
0.0
0.5
1.0
1.5
−50
0
50
100
150
200



(c) Box-Cox-Transformed Response (w/o Log-Transformed Predictors)

```
# adjust response with vertical shift to account for negative responses
## min(train$ARR_DELAY) = -74 --> shift up by constant 75

## transform repsonse
train$adj_ARR_DELAY = train$ARR_DELAY + 75

## fit lm for future Box-Cox use
adj_linear_model <- lm(adj_ARR_DELAY ~
                     DEP_DELAY +
                      OP_CARRIER +
                      DEST +
                      CRS_DEP_TIME +
                      CRS_ARR_TIME +
                      TAXI_OUT +
                      TAXI_IN +
                      TYPE_DELAY,
                    data = train)
```

Box-Cox Setup

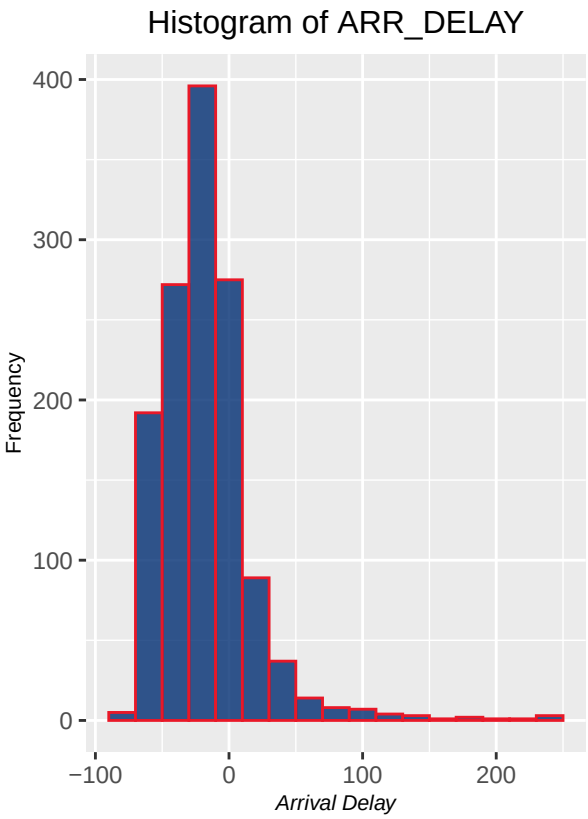
```
# box cox transform (same as case study)
## call EnvStats library
bc_model <- EnvStats::boxcox(adj_linear_model, optimize = TRUE)
# find optimal lambda parameter
bc_lambda <- bc_model$lambda
bc_lambda
```

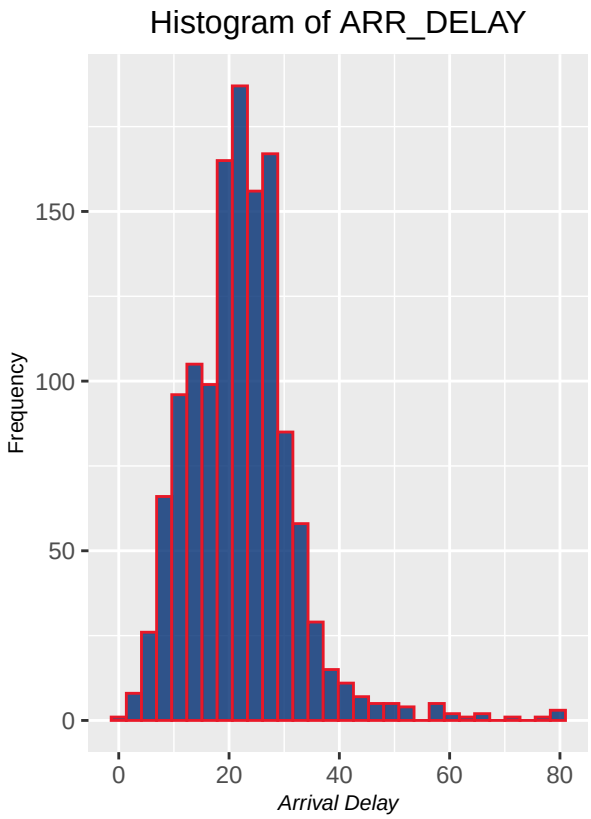
```
## [1] 0.6981479
```

```
#plot(bc_model)
```

```
# add Box-Cox response as variable to train set
train <- train %>%
  mutate(bc_adj_ARR_DELAY = ((adj_ARR_DELAY^bc_lambda) - 1)/bc_lambda)

# plot training ARR_DELAY
ptrain_ARRDELAY <- ggplot(data = train, aes(x = ARR_DELAY)) +
  geom_histogram(binwidth = 20, fill = "#002D72", color = "#E81828", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Histogram of ARR_DELAY") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# plot Box-Cox training ARR_DELAY
ptrain_bcARRDELAY <- ggplot(data = train, aes(x = bc_adj_ARR_DELAY)) +
  geom_histogram(fill = "#002D72", color = "#E81828", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Histogram of ARR_DELAY") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

ptrain_ARRDELAY + ptrain_bcARRDELAY
```

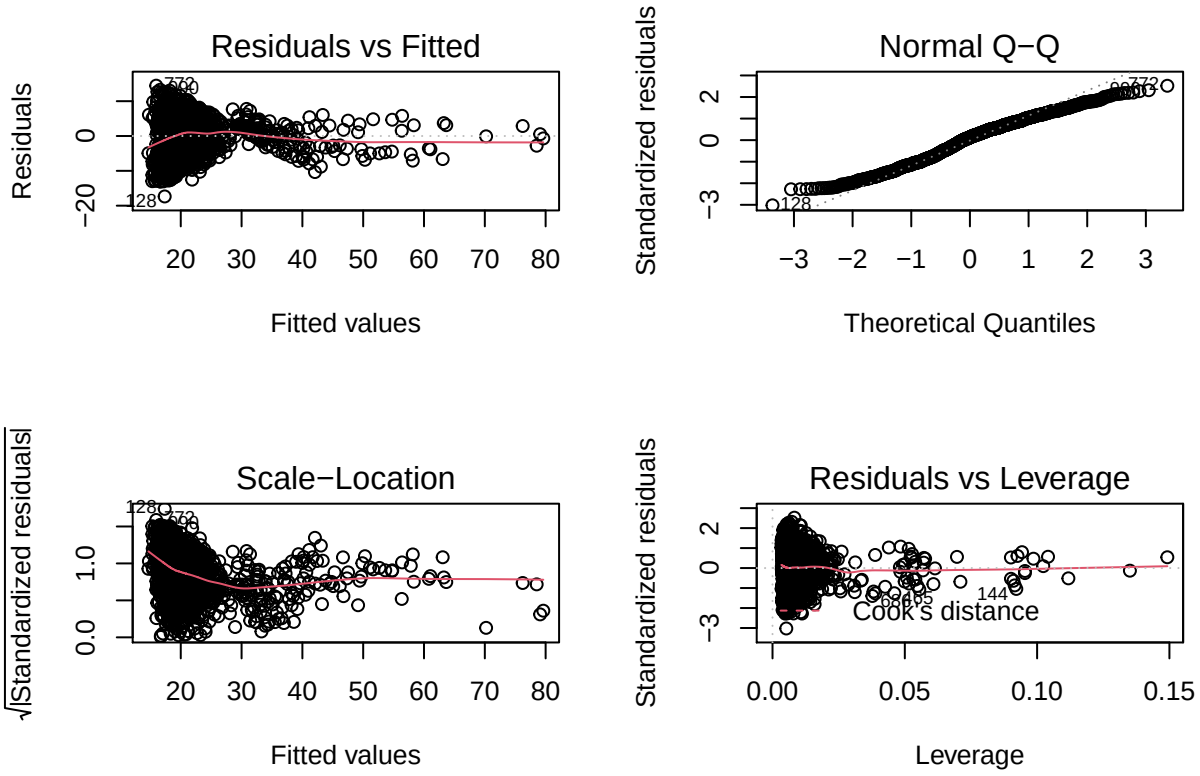
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```





Final Box-Cox Linear Model

```
bc_adj_linear_model <- lm(bc_adj_ARR_DELAY ~
                     DEP_DELAY +
                      OP_CARRIER +
                      DEST +
                      CRS_DEP_TIME +
                      CRS_ARR_TIME +
                      TAXI_OUT +
                      TAXI_IN +
                      TYPE_DELAY,
                    data = train)
par(mfrow = c(2,2))
plot(bc_adj_linear_model)
```




```
summary(bc_adj_linear_model)
```

```
##
## Call:
## lm(formula = bc_adj_ARR_DELAY ~ DEP_DELAY + OP_CARRIER + DEST +
##     CRS_DEP_TIME + CRS_ARR_TIME + TAXI_OUT + TAXI_IN + TYPE_DELAY,
##     data = train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.3556  -4.6043   0.7628   4.3534  14.4508
##
## Coefficients:
##                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)                21.9481352  1.5512203  14.149  < 2e-16 ***
## DEP_DELAY                   0.2019305  0.0085688  23.566  < 2e-16 ***
## OP_CARRIERAS               -0.3865272  0.5255474  -0.735 0.462183
## OP_CARRIERB6                0.4704172  0.4324948   1.088 0.276937
## OP_CARRIERDL               -0.3935191  0.4384069  -0.898 0.369559
## DESTSFO                    -0.6404205  0.3427150  -1.869 0.061894 .
## CRS_DEP_TIME               -0.0012308  0.0003474  -3.543 0.000410 ***
## CRS_ARR_TIME               -0.0005200  0.0002776  -1.873 0.061326 .
## TAXI_OUT                    0.2468270  0.0192971  12.791  < 2e-16 ***
## TAXI_IN                     0.1471062  0.0194314   7.571 7.02e-14 ***
## TYPE_DELAYLATE_AIRCRAFT    -0.9730991  2.0595772  -0.472 0.636667
## TYPE_DELAYNAS               5.1599555  1.4212105   3.631 0.000294 ***
## TYPE_DELAYNo Delay         -5.5982679  1.4072908  -3.978 7.33e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Residual standard error: 5.755 on 1297 degrees of freedom
## Multiple R-squared:  0.6445, Adjusted R-squared:  0.6412
## F-statistic: 195.9 on 12 and 1297 DF,  p-value: < 2.2e-16
```

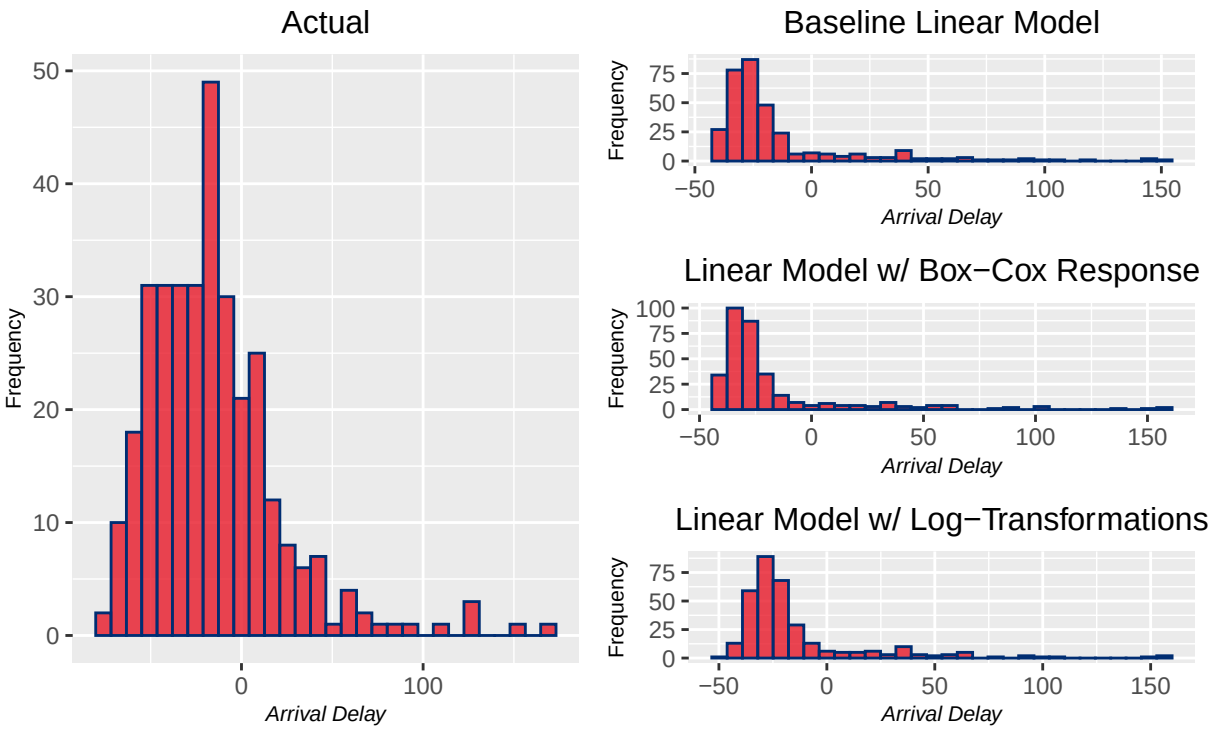
MLR Test Error Metrics & Predictions

LOOKING AT: - model without any interactions or transformations: ENDED UP PERFORMING THE BEST!!! - log transformed predictors (taxi_in and taxi_out), interactions, no boxcox transformed response - box-cox transformed response, no interactions or predicted transformations

```
# predictions
## baseline
test$plain_mlr_pred <- predict(plain_linear_model, test)
## log
test$log_linear_preds <- predict(log_linear_model, test)
## bc
test$adj_ARR_DELAY = test$ARR_DELAY + 77
test$bc_adj_linear_preds <- predict(bc_adj_linear_model, test)
test <- test %>%
 mutate(adj_linear_preds = ((bc_adj_linear_preds*(bc_lambda) + 1)^(1/bc_lambda)))
test$bc_mlr_pred = test$adj_linear_preds - 77

## histogram of predictions vs. actual
# actual ARR_DELAY in test set
#hist(test$ARR_DELAY)
ptest_actual <- ggplot(data = test, aes(x = ARR_DELAY)) +
  geom_histogram(fill = "#E81828", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Actual") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- baseline lm
ptest_baseline_preds <- ggplot(data = test, aes(x = plain_mlr_pred)) +
  geom_histogram(fill = "#E81828", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Baseline Linear Model") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- lm with log-trans
# hist(test$log_linear_preds)
ptest_lm_log_preds <- ggplot(data = test, aes(x = log_linear_preds)) +
  geom_histogram(fill = "#E81828", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Linear Model w/ Log-Transformations") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
```

```
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- lm with Box-Cox model
#hist(test$log_linear_preds)
ptest_bc_preds <- ggplot(data = test, aes(x = bc_mlr_pred)) +
  geom_histogram(fill = "#E81828", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Linear Model w/ Box-Cox Response") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

mlr_patchwork <- ptest_actual + (ptest_baseline_preds / ptest_bc_preds / ptest_lm_log_preds)

mlr_patchwork + plot_annotation(
  title = 'Comparing Distributions of ARR_DELAY',
  subtitle = 'Histograms of actual test values and MLR-predicted values'
)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



```r
# test MSE calculations
plain_linear_model_MSE <- sum((test$ARR_DELAY - test$plain_mlr_pred)^2, na.rm=T)/length(test$ARR_DELAY)
plain_linear_model_MSE
```

```
## [1] 322.4588
```

```r
log_linear_MSE <- sum((test$log_linear_preds-test$ARR_DELAY)^2, na.rm=T)/length(test$ARR_DELAY)
log_linear_MSE
```

```
## [1] 333.8962
```

```r
bc_adj_linear_model_MSE <- sum((test$ARR_DELAY - test$bc_mlr_pred)^2, na.rm=T)/length(test$ARR_DELAY)
bc_adj_linear_model_MSE
```

```
## [1] 334.9217
```

(2) Generalized Additive Models

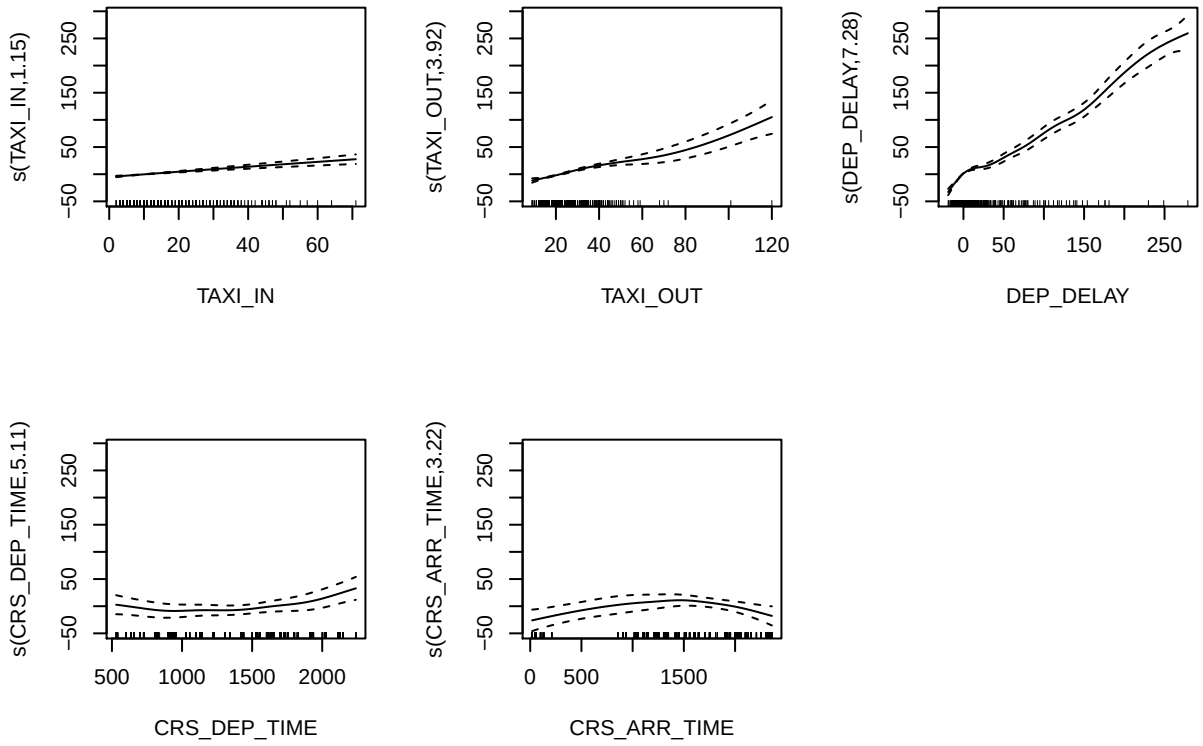
(a) Initial GAM: No Box-Cox on Response

```r
gam00 <- gam(ARR_DELAY ~ DAY_OF_WEEK +
                   OP_CARRIER +
                   s(TAXI_IN) +
                   s(TAXI_OUT) +
                   DEST +
                   s(DEP_DELAY) +
                   s(CRS_DEP_TIME) +
                   s(CRS_ARR_TIME) +
                   TYPE_DELAY, data = train)

summary(gam00)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## ARR_DELAY ~ DAY_OF_WEEK + OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) +
##     DEST + s(DEP_DELAY) + s(CRS_DEP_TIME) + s(CRS_ARR_TIME) +
##     TYPE_DELAY
##
## Parametric coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 0.6248     4.6488   0.134   0.8931
## DAY_OF_WEEK                 0.2465     0.2625   0.939   0.3479
## OP_CARRIERAS               -1.4083     1.6722  -0.842   0.3999
## OP_CARRIERB6                2.8700     1.3616   2.108   0.0352 *
## OP_CARRIERDL               -2.7519     1.3905  -1.979   0.0480 *
## DESTSFO                    -0.5607     1.1267  -0.498   0.6188
## TYPE_DELAYLATE_AIRCRAFT    -3.3786     6.5848  -0.513   0.6080
## TYPE_DELAYNAS              18.9194     4.5466   4.161 3.38e-05 ***
## TYPE_DELAYNo Delay        -22.2130     4.5742  -4.856 1.34e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
```

```
##                   edf Ref.df       F  p-value
## s(TAXI_IN)      1.148  1.283  44.974 6.24e-13 ***
## s(TAXI_OUT)     3.922  4.851  46.982  < 2e-16 ***
## s(DEP_DELAY)    7.279  8.272 136.783  < 2e-16 ***
## s(CRS_DEP_TIME) 5.114  6.050   2.121   0.0415 *
## s(CRS_ARR_TIME) 3.221  3.897   1.845   0.1277
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.749   Deviance explained = 75.5%
## GCV = 317.86  Scale est. = 310.66    n = 1310
```

```
par(mfrow = c(2,3))
plot.gam(gam00, se=TRUE)
```



Checking Lineartiy TAXI_IN may be linear

```
gam01 <- gam(ARR_DELAY ~ DAY_OF_WEEK +
                    OP_CARRIER +
                    TAXI_IN +
                    s(TAXI_OUT) +
                    DEST +
                    s(DEP_DELAY) +
                    s(CRS_DEP_TIME) +
                    s(CRS_ARR_TIME) +
                    TYPE_DELAY, data = train)

anova(gam00, gam01, test = "F")
```

```
## Analysis of Deviance Table
##
```

```
## Model 1: ARR_DELAY ~ DAY_OF_WEEK + OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) +
##     DEST + s(DEP_DELAY) + s(CRS_DEP_TIME) + s(CRS_ARR_TIME) +
##     TYPE_DELAY
## Model 2: ARR_DELAY ~ DAY_OF_WEEK + OP_CARRIER + TAXI_IN + s(TAXI_OUT) +
##     DEST + s(DEP_DELAY) + s(CRS_DEP_TIME) + s(CRS_ARR_TIME) +
##     TYPE_DELAY
##   Resid. Df Resid. Dev       Df Deviance      F Pr(>F)
## 1    1276.6     397738
## 2    1276.9     397845 -0.29646  -106.39 1.1552 0.1929
```

based on anova test, the model with a smoothing spline on TAXI_IN is a better fit

More ANOVA Tests DAY_OF_WEEK, DEST, and CRS_ARR_TIME have very high p-values, so let's try an anova test without including them

```
gam02 <- gam(ARR_DELAY ~ OP_CARRIER +
                    s(TAXI_IN) +
                    s(TAXI_OUT) +
                    s(DEP_DELAY) +
                    s(CRS_DEP_TIME) +
                    TYPE_DELAY, data = train)

anova(gam00, gam02, test = "F")
```

```
## Analysis of Deviance Table
##
## Model 1: ARR_DELAY ~ DAY_OF_WEEK + OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) +
##     DEST + s(DEP_DELAY) + s(CRS_DEP_TIME) + s(CRS_ARR_TIME) +
##     TYPE_DELAY
## Model 2: ARR_DELAY ~ OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) + s(DEP_DELAY) +
##     s(CRS_DEP_TIME) + TYPE_DELAY
##   Resid. Df Resid. Dev      Df Deviance      F  Pr(>F)
## 1    1276.6     397738
## 2    1280.0     400550 -3.3459  -2811.2 2.7045 0.03822 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

based on the anova test, the model excluding these variables is a better fit

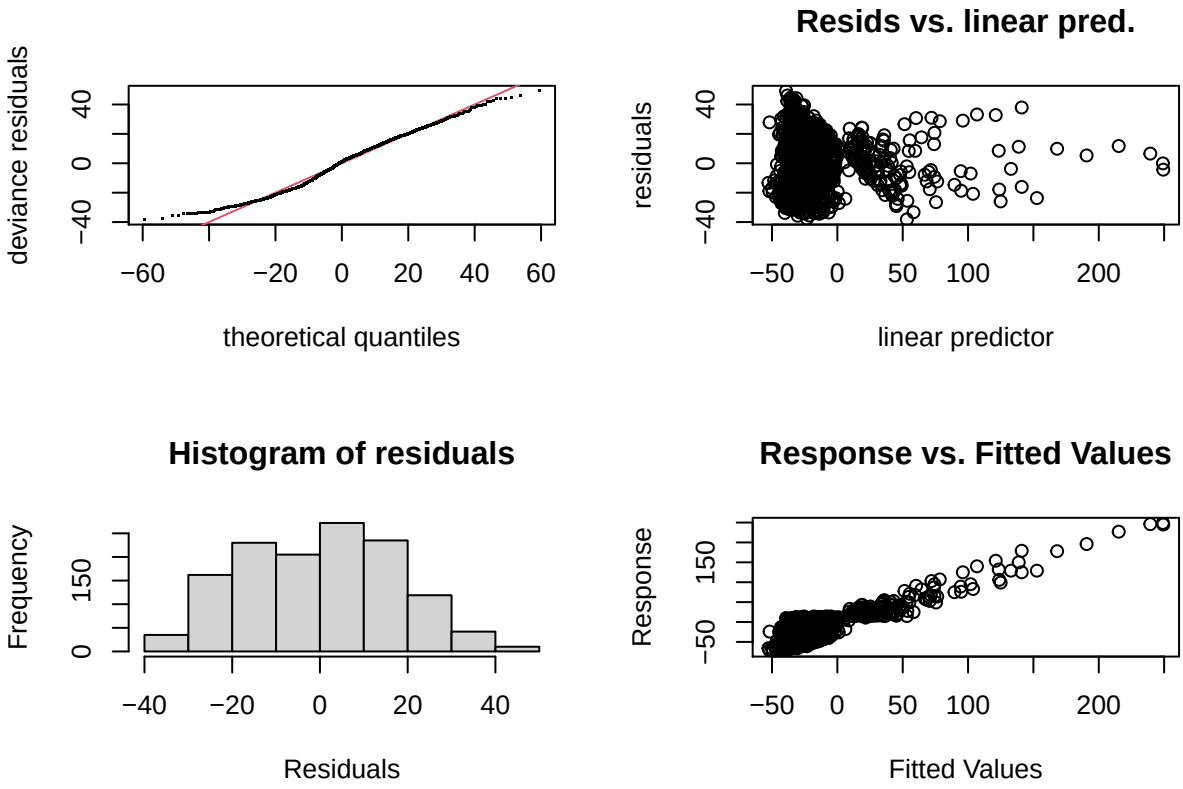
Tuned Initial GAM

```
# final fit
summary(gam02)
```

```
##
## Family: gaussian
## Link function: identity
##
## Formula:
## ARR_DELAY ~ OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) + s(DEP_DELAY) +
##     s(CRS_DEP_TIME) + TYPE_DELAY
##
## Parametric coefficients:
##                          Estimate Std. Error t value Pr(>|t|)
## (Intercept)                 1.828      4.530   0.404   0.6865
## OP_CARRIERAS               -1.676      1.664  -1.007   0.3140
```

```
## OP_CARRIERB6                2.486      1.354   1.836   0.0666 .
## OP_CARRIERDL               -3.137      1.381  -2.272   0.0233 *
## TYPE_DELAYLATE_AIRCRAFT    -3.199      6.601  -0.485   0.6281
## TYPE_DELAYNAS              18.795      4.556   4.126 3.93e-05 ***
## TYPE_DELAYNo Delay        -22.413      4.588  -4.885 1.16e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                    edf Ref.df       F  p-value
## s(TAXI_IN)       1.260  1.478  43.326 7.39e-14 ***
## s(TAXI_OUT)      4.308  5.298  44.490  < 2e-16 ***
## s(DEP_DELAY)     7.384  8.348 134.478  < 2e-16 ***
## s(CRS_DEP_TIME) 6.781  7.883   5.229 2.36e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.748   Deviance explained = 75.3%
## GCV = 318.64  Scale est. = 312.13    n = 1310
```

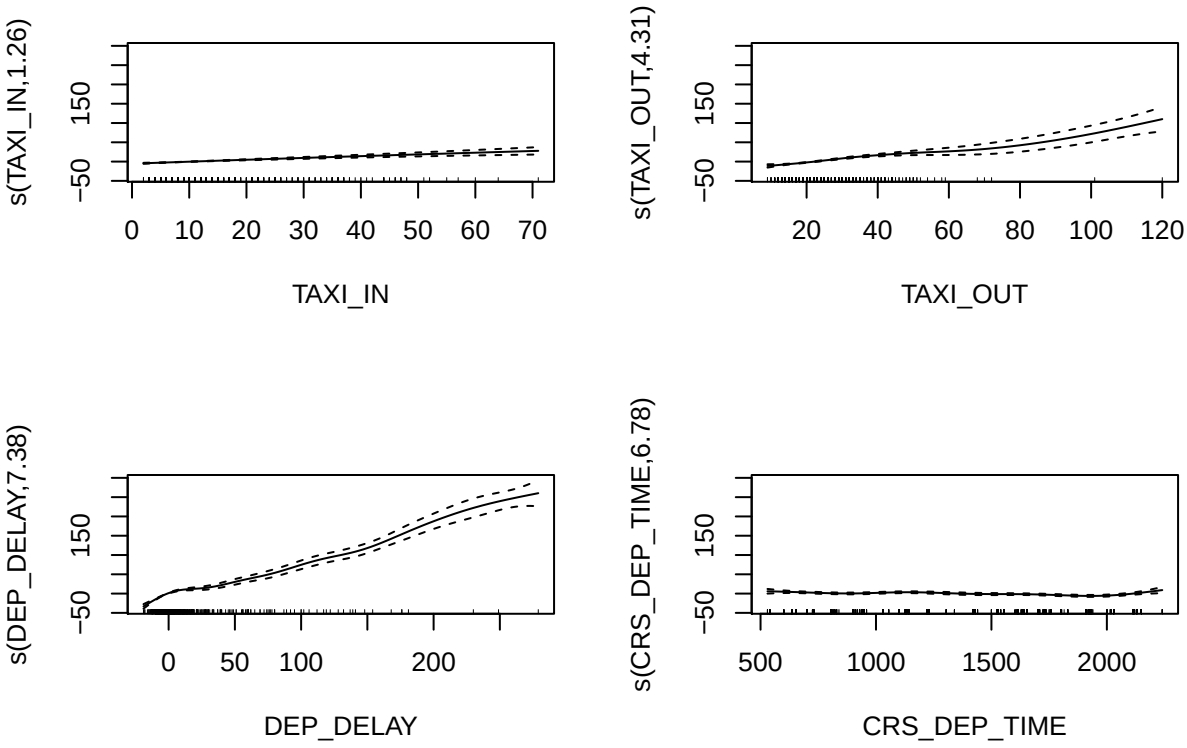
```
# diagnostic plots
par(mfrow = c(2,2))
gam.check(gam02)
```




```
##
## Method: GCV   Optimizer: magic
## Smoothing parameter selection converged after 12 iterations.
## The RMS GCV score gradient at convergence was 0.0007156955 .
## The Hessian was positive definite.
## Model rank =  43 / 43
```

```
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                   k'  edf k-index p-value
## s(TAXI_IN)      9.00 1.26    0.99    0.32
## s(TAXI_OUT)     9.00 4.31    1.07    0.99
## s(DEP_DELAY)    9.00 7.38    0.99    0.31
## s(CRS_DEP_TIME) 9.00 6.78    0.97    0.14
```

```
# predictor plots
par(mfrow = c(2,2))
plot(gam02)
```



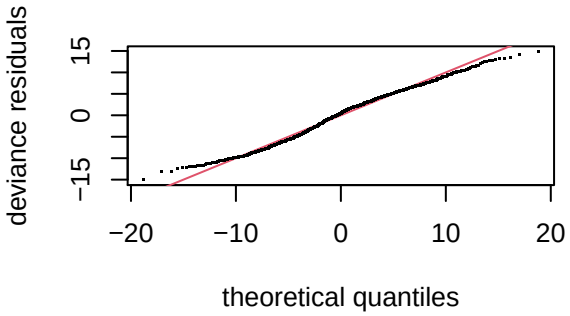
(b) Secondary GAM: Box-Cox on Response

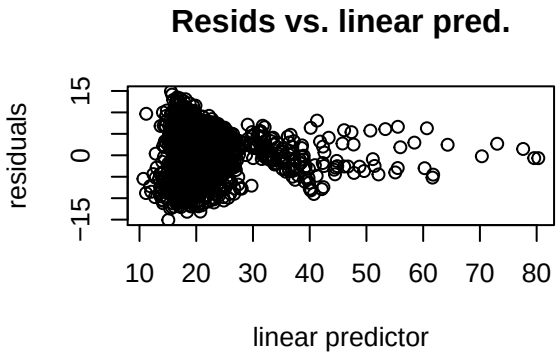
```
gambc <- gam(bc_adj_ARR_DELAY ~ OP_CARRIER +
                    s(TAXI_IN) +
                    s(TAXI_OUT) +
                    s(DEP_DELAY) +
                    s(CRS_DEP_TIME) +
                    TYPE_DELAY, data = train)

# final BC GAM fit
summary(gambc)
```

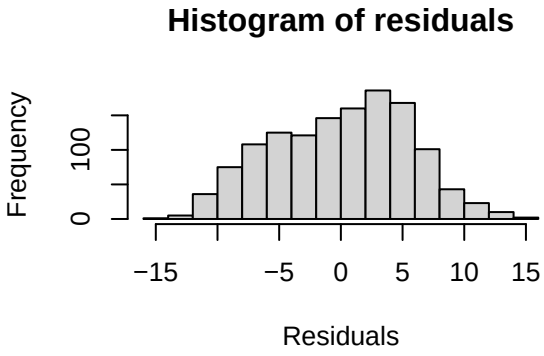
```
##
## Family: gaussian
## Link function: identity
##
```

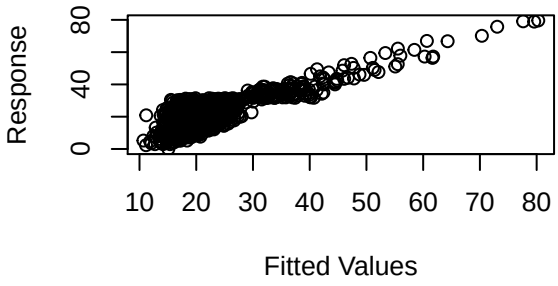
```
## Formula:
## bc_adj_ARR_DELAY ~ OP_CARRIER + s(TAXI_IN) + s(TAXI_OUT) + s(DEP_DELAY) +
##     s(CRS_DEP_TIME) + TYPE_DELAY
##
## Parametric coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)                28.2688     1.4316  19.746  < 2e-16 ***
## OP_CARRIERAS               -0.5081     0.5258  -0.966  0.33407
## OP_CARRIERB6                0.7942     0.4275   1.858  0.06340 .
## OP_CARRIERDL               -0.9165     0.4366  -2.099  0.03598 *
## TYPE_DELAYLATE_AIRCRAFT    -1.0709     2.0885  -0.513  0.60822
## TYPE_DELAYNAS               3.9308     1.4403   2.729  0.00644 **
## TYPE_DELAYNo Delay         -7.1102     1.4503  -4.903 1.07e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Approximate significance of smooth terms:
##                    edf Ref.df      F  p-value
## s(TAXI_IN)       1.409  1.721 38.501 2.66e-14 ***
## s(TAXI_OUT)      3.241  4.048 46.659  < 2e-16 ***
## s(DEP_DELAY)     7.435  8.383 76.966  < 2e-16 ***
## s(CRS_DEP_TIME) 6.825  7.920  5.521 8.48e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## R-sq.(adj) =  0.662   Deviance explained = 66.8%
## GCV = 31.859  Scale est. = 31.229    n = 1310
```

```r
# diagnostic plots
par(mfrow = c(2,2))
gam.check(gambc)
```



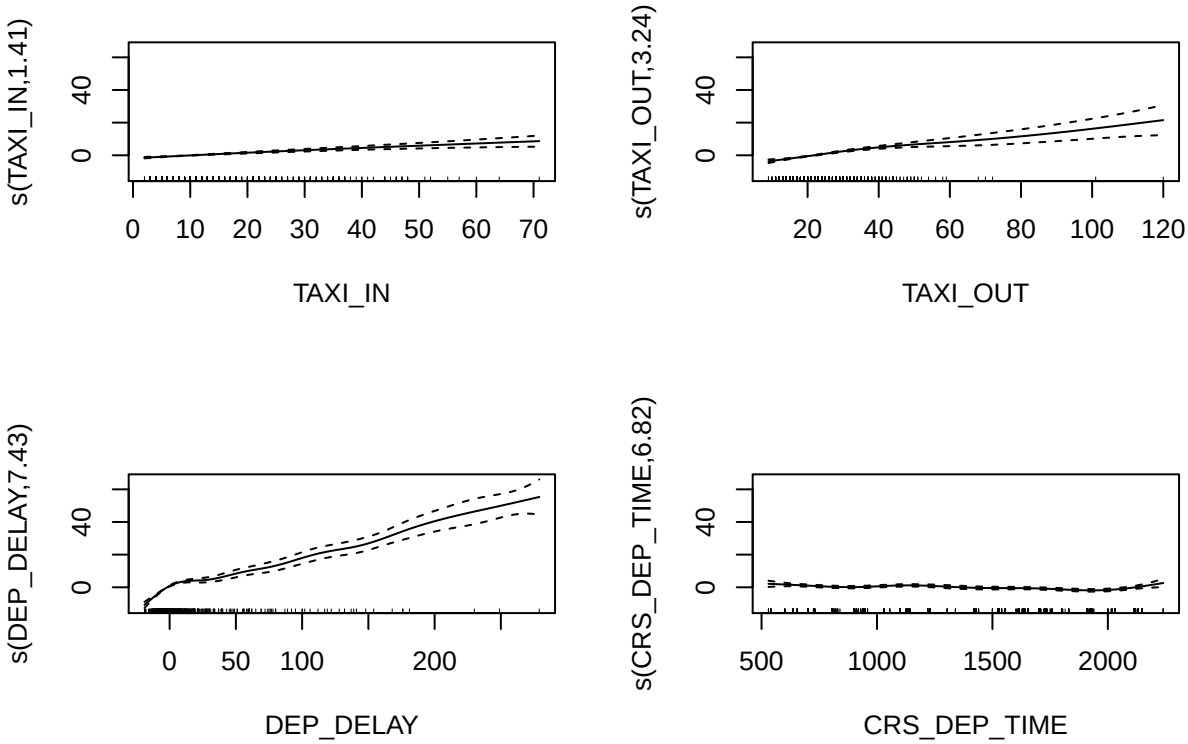






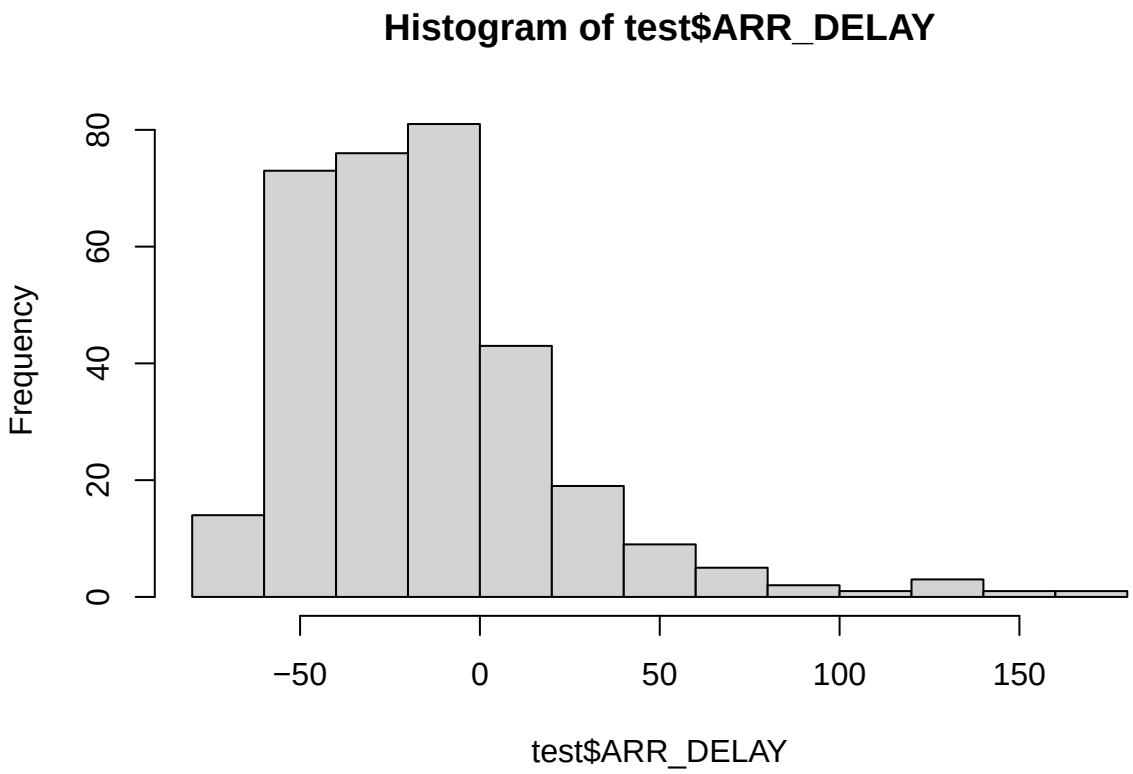

```
##
## Method: GCV   Optimizer: magic
## Smoothing parameter selection converged after 11 iterations.
## The RMS GCV score gradient at convergence was 0.0002369716 .
## The Hessian was positive definite.
## Model rank =  43 / 43
##
## Basis dimension (k) checking results. Low p-value (k-index<1) may
## indicate that k is too low, especially if edf is close to k'.
##
##                   k'  edf k-index p-value
## s(TAXI_IN)      9.00 1.41    0.99    0.36
## s(TAXI_OUT)     9.00 3.24    1.06    0.98
## s(DEP_DELAY)    9.00 7.43    0.98    0.23
## s(CRS_DEP_TIME) 9.00 6.82    0.96    0.03 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# predictor plots
par(mfrow = c(2,2))
plot(gambc)
```



GAM Test Error Metrics & Predictions

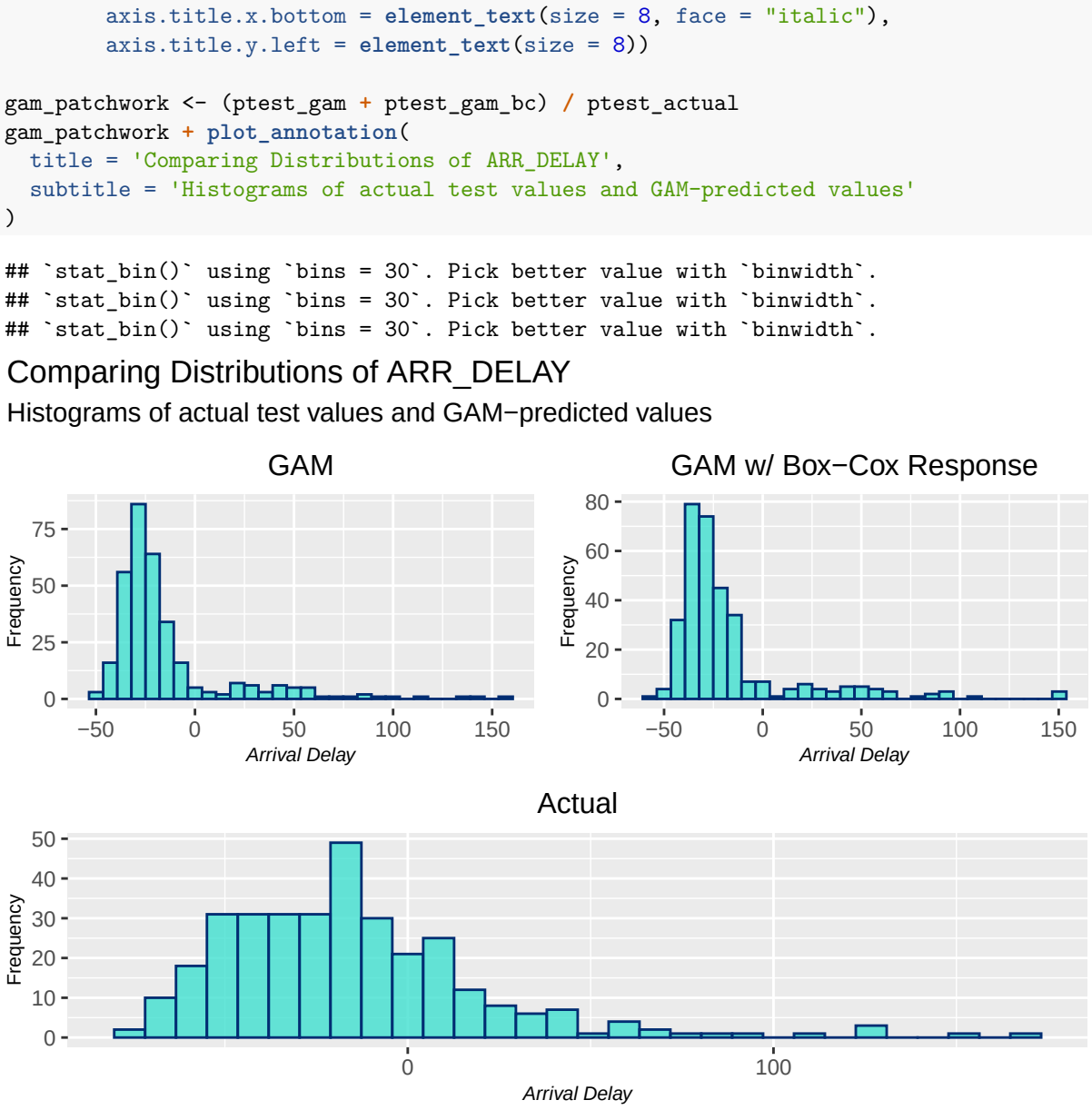
```
# GAM predictions
## no BC
gam_preds <- predict.gam(gam02, newdata = test)
## BC
gambc_preds <- predict.gam(gambc, newdata = test)
adjgam_preds <- ((gambc_preds*(bc_lambda) + 1)^(1/bc_lambda))
bc_gam_pred = adjgam_preds - 77


## histogram of predictions vs. actual
# actual ARR_DELAY in test set
hist(test$ARR_DELAY)
```




```
ptest_actual <- ggplot(data = test, aes(x = ARR_DELAY)) +
  geom_histogram(fill = "#40E0D0", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Actual") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- baseline lm
ptest_gam <- ggplot(data = test, aes(x = gam_preds)) +
  geom_histogram(fill = "#40E0D0", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "GAM") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- lm with log-trans
ptest_gam_bc <- ggplot(data = test, aes(x = bc_gam_pred)) +
  geom_histogram(fill = "#40E0D0", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "GAM w/ Box-Cox Response") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
```

```
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

gam_patchwork <- (ptest_gam + ptest_gam_bc) / ptest_actual
gam_patchwork + plot_annotation(
  title = 'Comparing Distributions of ARR_DELAY',
  subtitle = 'Histograms of actual test values and GAM-predicted values'
)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
# test MSE calculations
gam_MSE <- sum((test$ARR_DELAY - gam_preds)^2, na.rm=T)/length(test$ARR_DELAY)
gam_MSE
```

```
## [1] 312.2953
```

```
gambc_MSE <- sum((test$ARR_DELAY - bc_gam_pred)^2, na.rm=T)/length(test$ARR_DELAY)
gambc_MSE
```

```
## [1] 317.4533
```

(3) Tree-Based Models

(a) Random Forests

By default, `randomForest()` uses $p/3$ variables when building a random forest of regression trees.

```r
set.seed(1)

# optimal number of predictors (param = mtry) used = 2 based on CV
rf.delay <- randomForest(ARR_DELAY ~ DAY_OF_MONTH +
                    TAXI_IN +
                    TAXI_OUT +
                    DEST +
                    DEP_DELAY +
                    CARRIER_DELAY +
                    NAS_DELAY,
                    data = train, na.action = na.omit, importance = TRUE,
                    ntree=10000, mtry=2)
```

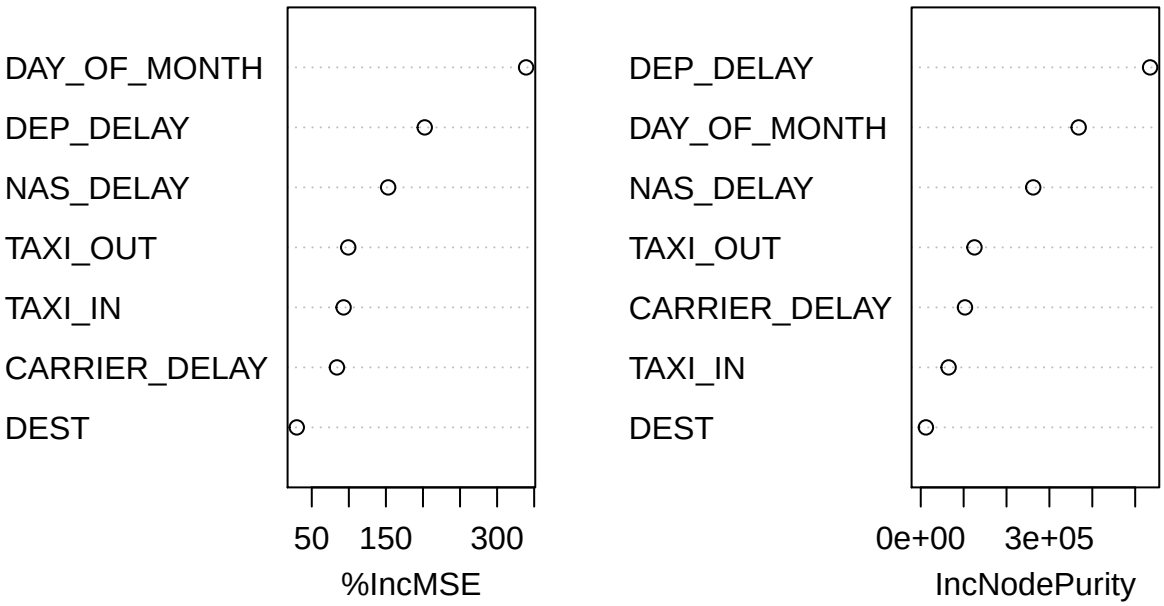
Using the `importance()` function, we can view the importance of each variable.

```r
importance(rf.delay)
```

```
##                  %IncMSE IncNodePurity
## DAY_OF_MONTH  339.08627     368261.46
## TAXI_IN        92.93643      64905.19
## TAXI_OUT       99.21626     125258.87
## DEST           29.76223      12009.51
## DEP_DELAY     202.38571     534776.35
## CARRIER_DELAY  83.95478     103105.51
## NAS_DELAY     153.05251     262232.63
```

Two measures of variable importance are reported. The former is based on the mean decrease in accuracy in predictions on the out of bag samples when a given variable is excluded from the model. The latter is a measure of the total decrease in node impurity that results from splits over that variable, averaged over all trees (this was plotted in Figure 8.9 in the text). In the case of regression trees, the node impurity is measured by the training RSS and for classification trees by the deviance. Plots of these importance measures can be produced using the `varImpPlot()` function.

```r
varImpPlot(rf.delay)
```



(b) Boosting

Here we use the `gbm()` package, and within it the `gbm()` function, to fit boosted regression trees to the `train` data set. We run `gbm()` with the option `distribution = "gaussian"` since this is a regression problem. The argument `n.trees = 150` indicates that we want 150 trees, and the option `interaction.depth = 3` limits the depth of each tree.

```
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1150.5648             nan     0.1000  108.0894
##      2     1062.7120             nan     0.1000   98.0251
##      3      999.8411             nan     0.1000   54.3611
##      4      937.5774             nan     0.1000   53.2725
##      5      877.3090             nan     0.1000   60.5384
##      6      834.8623             nan     0.1000   42.6809
##      7      782.6249             nan     0.1000   41.6720
##      8      736.9349             nan     0.1000   47.9831
##      9      695.5806             nan     0.1000   40.5227
##     10      657.6592             nan     0.1000   36.4710
##     20      421.6438             nan     0.1000   15.4628
##     40      244.7935             nan     0.1000    1.2456
##     60      184.0732             nan     0.1000    1.4904
##     80      155.0727             nan     0.1000    0.9043
##    100      140.8694             nan     0.1000    0.3294
##    120      133.2812             nan     0.1000    0.2873
##    140      127.0999             nan     0.1000    0.1086
##    150      124.7120             nan     0.1000   -1.0101
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1091.1076             nan     0.1000  130.0192
```

```
##      2      961.6851             nan     0.1000  157.4429
##      3      858.5946             nan     0.1000  101.8085
##      4      790.1614             nan     0.1000   64.1696
##      5      710.8182             nan     0.1000   78.4191
##      6      649.3869             nan     0.1000   63.5996
##      7      598.3666             nan     0.1000   39.1273
##      8      545.5722             nan     0.1000   54.2270
##      9      500.1991             nan     0.1000   34.5864
##     10      467.1205             nan     0.1000   34.3254
##     20      287.3160             nan     0.1000    5.7970
##     40      166.8623             nan     0.1000    3.5853
##     60      132.0353             nan     0.1000    0.5102
##     80      118.0411             nan     0.1000    0.0546
##    100      110.1089             nan     0.1000    0.7461
##    120      103.2643             nan     0.1000   -0.5363
##    140      100.1627             nan     0.1000   -0.0438
##    150       98.7409             nan     0.1000   -0.4050
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1066.5079             nan     0.1000  173.6475
##      2      923.2312             nan     0.1000  140.7612
##      3      801.1209             nan     0.1000   93.9866
##      4      714.0700             nan     0.1000   88.0321
##      5      640.2316             nan     0.1000   68.1676
##      6      569.5613             nan     0.1000   55.4898
##      7      505.9868             nan     0.1000   58.5620
##      8      456.4054             nan     0.1000   49.7937
##      9      419.3526             nan     0.1000   27.1763
##     10      388.8173             nan     0.1000   26.1372
##     20      233.6893             nan     0.1000    7.8144
##     40      135.9065             nan     0.1000    1.1181
##     60      110.4129             nan     0.1000    0.0033
##     80      101.0143             nan     0.1000   -1.0713
##    100       93.8919             nan     0.1000    0.3906
##    120       90.0736             nan     0.1000   -0.2786
##    140       86.1896             nan     0.1000    0.0213
##    150       84.9967             nan     0.1000   -0.3078
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1038.4961             nan     0.1000   79.2910
##      2      964.6743             nan     0.1000   63.7220
##      3      904.4326             nan     0.1000   63.8304
##      4      846.0564             nan     0.1000   35.8399
##      5      796.9160             nan     0.1000   33.8357
##      6      741.3574             nan     0.1000   53.0141
##      7      697.1677             nan     0.1000   42.4855
##      8      658.3158             nan     0.1000   31.6678
##      9      620.2354             nan     0.1000   34.9740
##     10      583.9651             nan     0.1000   37.1194
##     20      365.3850             nan     0.1000   16.3536
##     40      211.8031             nan     0.1000    3.8751
##     60      162.5473             nan     0.1000    1.5584
##     80      139.3494             nan     0.1000    0.7306
##    100      126.2946             nan     0.1000    0.2218
```

```
##    120      117.2882             nan     0.1000    0.1613
##    140      110.9744             nan     0.1000    0.3039
##    150      108.7748             nan     0.1000    0.2844
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      977.1061             nan     0.1000  113.4577
##      2      866.4147             nan     0.1000  113.7672
##      3      768.6013             nan     0.1000   79.1478
##      4      692.0857             nan     0.1000   73.6998
##      5      626.7474             nan     0.1000   64.8212
##      6      575.0267             nan     0.1000   52.8815
##      7      521.4352             nan     0.1000   40.5435
##      8      473.4024             nan     0.1000   32.9714
##      9      435.6518             nan     0.1000   33.5803
##     10      404.0142             nan     0.1000   30.8877
##     20      237.3946             nan     0.1000    9.0847
##     40      148.7742             nan     0.1000    0.6326
##     60      116.9303             nan     0.1000   -0.0815
##     80      103.5219             nan     0.1000    0.3205
##    100       97.1446             nan     0.1000    0.0267
##    120       93.7601             nan     0.1000   -0.2853
##    140       90.1817             nan     0.1000   -0.2491
##    150       88.4311             nan     0.1000    0.0747
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      979.2186             nan     0.1000  157.4003
##      2      843.8291             nan     0.1000  140.3897
##      3      728.9358             nan     0.1000  107.9369
##      4      657.4075             nan     0.1000   73.6883
##      5      583.9841             nan     0.1000   74.7565
##      6      525.9856             nan     0.1000   47.9538
##      7      471.0073             nan     0.1000   45.7894
##      8      421.1825             nan     0.1000   45.8401
##      9      381.2552             nan     0.1000   28.7010
##     10      349.2513             nan     0.1000   25.4642
##     20      197.4749             nan     0.1000    7.5401
##     40      118.1161             nan     0.1000    0.2761
##     60       96.2397             nan     0.1000    0.0293
##     80       89.0182             nan     0.1000    0.3721
##    100       83.6028             nan     0.1000   -0.2090
##    120       79.7778             nan     0.1000   -0.2337
##    140       76.0114             nan     0.1000    0.0092
##    150       74.8119             nan     0.1000   -0.2349
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1043.1800             nan     0.1000   65.1818
##      2      959.4253             nan     0.1000   70.1816
##      3      896.6926             nan     0.1000   61.6784
##      4      843.1070             nan     0.1000   50.6219
##      5      796.5353             nan     0.1000   40.8339
##      6      746.4385             nan     0.1000   46.4100
##      7      702.8579             nan     0.1000   41.7416
##      8      663.6213             nan     0.1000   38.0004
##      9      625.0179             nan     0.1000   39.5442
```

```
##     10      591.8549             nan     0.1000   27.8403
##     20      378.0878             nan     0.1000   12.1186
##     40      231.7994             nan     0.1000    3.0440
##     60      180.3416             nan     0.1000    1.4298
##     80      156.3828             nan     0.1000    0.6180
##    100      144.4843             nan     0.1000    0.4221
##    120      136.6853             nan     0.1000   -0.0499
##    140      130.7566             nan     0.1000    0.0179
##    150      128.6565             nan     0.1000   -0.3876
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      988.9693             nan     0.1000  135.6243
##      2      878.1054             nan     0.1000  115.2549
##      3      782.7530             nan     0.1000   68.8872
##      4      706.5929             nan     0.1000   65.4505
##      5      647.2079             nan     0.1000   53.1487
##      6      584.1888             nan     0.1000   65.6355
##      7      536.8114             nan     0.1000   43.1166
##      8      496.6526             nan     0.1000   42.0299
##      9      460.4912             nan     0.1000   35.7958
##     10      431.7768             nan     0.1000   27.7186
##     20      263.1444             nan     0.1000    9.9121
##     40      160.8285             nan     0.1000    1.3644
##     60      131.1793             nan     0.1000    0.8514
##     80      117.1487             nan     0.1000    0.2811
##    100      109.9067             nan     0.1000   -0.3520
##    120      103.3028             nan     0.1000   -0.0456
##    140       99.4621             nan     0.1000   -1.4216
##    150       97.8404             nan     0.1000   -0.1779
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      963.7173             nan     0.1000  145.6485
##      2      830.0235             nan     0.1000  140.3019
##      3      729.2431             nan     0.1000  112.9741
##      4      642.8786             nan     0.1000   85.5526
##      5      579.7839             nan     0.1000   67.1794
##      6      525.0721             nan     0.1000   54.9593
##      7      482.4734             nan     0.1000   44.9557
##      8      440.6059             nan     0.1000   36.2972
##      9      405.1308             nan     0.1000   30.6701
##     10      377.1782             nan     0.1000   32.2158
##     20      223.1937             nan     0.1000    8.3633
##     40      137.5210             nan     0.1000    2.7908
##     60      115.6683             nan     0.1000   -0.1178
##     80      103.5987             nan     0.1000    0.1486
##    100       97.4186             nan     0.1000    0.0468
##    120       92.1760             nan     0.1000   -0.6028
##    140       87.8751             nan     0.1000   -0.0790
##    150       86.0851             nan     0.1000   -0.2352
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1114.9069             nan     0.1000  111.0734
##      2     1026.6882             nan     0.1000   60.7724
##      3      960.8236             nan     0.1000   65.4554
```

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##      4      905.6734             nan     0.1000   56.6590
##      5      844.7358             nan     0.1000   56.0554
##      6      790.0563             nan     0.1000   37.1147
##      7      747.9614             nan     0.1000   43.9995
##      8      703.2455             nan     0.1000   37.1534
##      9      660.5436             nan     0.1000   37.6661
##     10      620.8812             nan     0.1000   37.8569
##     20      394.0971             nan     0.1000   13.1611
##     40      227.3770             nan     0.1000    3.0090
##     60      175.9880             nan     0.1000    0.6697
##     80      153.5385             nan     0.1000    0.4214
##    100      141.7942             nan     0.1000    0.0400
##    120      133.9781             nan     0.1000    0.2538
##    140      128.5789             nan     0.1000   -0.6216
##    150      127.0912             nan     0.1000    0.2749
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1058.4375             nan     0.1000  179.6786
##      2      934.4213             nan     0.1000  123.0235
##      3      826.0630             nan     0.1000  119.9622
##      4      741.0864             nan     0.1000   76.9391
##      5      673.9855             nan     0.1000   59.9329
##      6      614.3471             nan     0.1000   63.7129
##      7      565.6583             nan     0.1000   51.6028
##      8      515.4718             nan     0.1000   48.5894
##      9      473.3732             nan     0.1000   39.6733
##     10      437.3717             nan     0.1000   25.2694
##     20      255.5560             nan     0.1000    9.7985
##     40      158.6039             nan     0.1000    0.8082
##     60      130.2847             nan     0.1000    1.6124
##     80      119.4850             nan     0.1000    0.1706
##    100      113.3957             nan     0.1000   -0.4501
##    120      107.7894             nan     0.1000    0.0992
##    140      103.8321             nan     0.1000   -0.2636
##    150      102.7621             nan     0.1000   -0.2770
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1050.1920             nan     0.1000  157.4568
##      2      908.9149             nan     0.1000  134.2770
##      3      813.2572             nan     0.1000   88.2433
##      4      713.7046             nan     0.1000   97.3873
##      5      630.2140             nan     0.1000   73.9902
##      6      555.7110             nan     0.1000   72.0744
##      7      505.4664             nan     0.1000   57.3740
##      8      459.7692             nan     0.1000   50.6590
##      9      418.2648             nan     0.1000   38.7313
##     10      381.5394             nan     0.1000   28.0108
##     20      218.5273             nan     0.1000    8.5399
##     40      139.6013             nan     0.1000    1.0679
##     60      115.4817             nan     0.1000    0.4317
##     80      105.5851             nan     0.1000   -0.4976
##    100       98.7866             nan     0.1000   -0.0657
##    120       94.8003             nan     0.1000   -0.2332
##    140       92.0789             nan     0.1000   -0.4928
```

```
##    150       89.9621             nan     0.1000   -0.4581
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1363.0377             nan     0.1000  115.9742
##      2     1261.0838             nan     0.1000   95.4869
##      3     1169.9134             nan     0.1000   89.4782
##      4     1098.3658             nan     0.1000   58.2659
##      5     1033.3767             nan     0.1000   65.9647
##      6      975.8048             nan     0.1000   57.3396
##      7      916.9678             nan     0.1000   51.0980
##      8      866.2389             nan     0.1000   48.7142
##      9      822.0977             nan     0.1000   45.7419
##     10      780.3579             nan     0.1000   40.0106
##     20      473.4706             nan     0.1000   21.2260
##     40      261.5182             nan     0.1000    4.9859
##     60      188.8009             nan     0.1000    2.0729
##     80      156.3942             nan     0.1000    0.2772
##    100      139.4622             nan     0.1000    0.4014
##    120      130.4820             nan     0.1000    0.2730
##    140      123.5861             nan     0.1000   -0.1129
##    150      120.8073             nan     0.1000    0.0414
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1313.9267             nan     0.1000  178.6950
##      2     1154.3750             nan     0.1000  158.1254
##      3     1021.4058             nan     0.1000   96.9070
##      4      907.3896             nan     0.1000   93.8171
##      5      821.2174             nan     0.1000   79.4315
##      6      735.9728             nan     0.1000   65.2562
##      7      672.5909             nan     0.1000   59.5681
##      8      625.1451             nan     0.1000   44.8287
##      9      569.7428             nan     0.1000   37.7258
##     10      521.7478             nan     0.1000   38.0977
##     20      296.2888             nan     0.1000   12.0787
##     40      167.2275             nan     0.1000    1.9542
##     60      133.7161             nan     0.1000    0.8959
##     80      115.3132             nan     0.1000   -0.3287
##    100      107.6759             nan     0.1000    0.3238
##    120      102.2019             nan     0.1000   -0.2904
##    140       96.2270             nan     0.1000    0.0778
##    150       94.6262             nan     0.1000   -0.3259
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1284.6787             nan     0.1000  216.3611
##      2     1110.8225             nan     0.1000  156.8866
##      3      957.2629             nan     0.1000  157.3410
##      4      837.9827             nan     0.1000  107.2450
##      5      739.2871             nan     0.1000   94.1477
##      6      657.4487             nan     0.1000   70.9209
##      7      592.1601             nan     0.1000   65.7733
##      8      533.5499             nan     0.1000   54.9894
##      9      481.1973             nan     0.1000   54.6397
##     10      450.3253             nan     0.1000   32.3668
##     20      242.1455             nan     0.1000   16.5360
```

```
##     40      134.1906             nan     0.1000    1.2931
##     60      107.5346             nan     0.1000    0.6984
##     80       96.4049             nan     0.1000    0.4291
##    100       88.7862             nan     0.1000    0.3195
##    120       84.0447             nan     0.1000   -0.3354
##    140       79.9662             nan     0.1000   -0.0714
##    150       78.3658             nan     0.1000   -0.0550
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1119.6872             nan     0.1000   64.0268
##      2     1028.6291             nan     0.1000   89.2680
##      3      955.2203             nan     0.1000   51.7304
##      4      886.0096             nan     0.1000   76.9672
##      5      833.6934             nan     0.1000   52.1605
##      6      792.9316             nan     0.1000   36.7743
##      7      744.9836             nan     0.1000   38.6176
##      8      697.4934             nan     0.1000   37.4781
##      9      656.1072             nan     0.1000   39.4088
##     10      622.1726             nan     0.1000   33.2767
##     20      396.7593             nan     0.1000   17.3952
##     40      235.1882             nan     0.1000    1.0045
##     60      179.7141             nan     0.1000    1.2229
##     80      154.9271             nan     0.1000    0.6423
##    100      143.1362             nan     0.1000    0.2508
##    120      135.1987             nan     0.1000    0.0048
##    140      129.8053             nan     0.1000   -0.5869
##    150      127.0031             nan     0.1000    0.1124
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1030.7566             nan     0.1000  138.8045
##      2      931.4097             nan     0.1000  108.5589
##      3      823.2101             nan     0.1000  119.9196
##      4      737.1424             nan     0.1000   83.8840
##      5      682.1829             nan     0.1000   52.8919
##      6      617.9732             nan     0.1000   55.2543
##      7      564.1432             nan     0.1000   44.3180
##      8      518.3703             nan     0.1000   41.4856
##      9      485.0086             nan     0.1000   35.0296
##     10      459.1011             nan     0.1000   24.7205
##     20      265.7766             nan     0.1000    6.5308
##     40      163.6818             nan     0.1000    1.6940
##     60      134.7479             nan     0.1000    0.6209
##     80      121.5619             nan     0.1000    0.4184
##    100      111.6734             nan     0.1000   -0.3092
##    120      106.5058             nan     0.1000   -0.0169
##    140      101.4909             nan     0.1000    0.1760
##    150       98.6352             nan     0.1000    0.0204
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1030.4886             nan     0.1000  169.4195
##      2      894.3337             nan     0.1000  147.4280
##      3      781.2207             nan     0.1000   99.2478
##      4      692.5225             nan     0.1000   86.5989
##      5      614.2082             nan     0.1000   74.9347
```

```
##      6      556.0481             nan     0.1000   53.3782
##      7      506.4840             nan     0.1000   45.6967
##      8      456.4495             nan     0.1000   44.0783
##      9      425.2643             nan     0.1000   34.2144
##     10      394.9264             nan     0.1000   30.4285
##     20      233.0086             nan     0.1000    7.0539
##     40      142.0020             nan     0.1000    1.1186
##     60      116.1049             nan     0.1000    0.5247
##     80      102.4701             nan     0.1000   -0.5115
##    100       94.1522             nan     0.1000    0.1074
##    120       88.0607             nan     0.1000    0.2569
##    140       84.4240             nan     0.1000    0.1709
##    150       82.9162             nan     0.1000   -0.0851
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1151.1140             nan     0.1000   74.8637
##      2     1073.9338             nan     0.1000   73.2535
##      3     1014.7446             nan     0.1000   60.9543
##      4      947.6675             nan     0.1000   59.7251
##      5      902.7024             nan     0.1000   48.6272
##      6      840.8978             nan     0.1000   61.2264
##      7      795.5811             nan     0.1000   39.0083
##      8      748.1772             nan     0.1000   48.2803
##      9      709.2787             nan     0.1000   36.1646
##     10      675.0291             nan     0.1000   35.6680
##     20      425.3133             nan     0.1000   16.5287
##     40      244.8662             nan     0.1000    1.7448
##     60      187.7142             nan     0.1000    1.3188
##     80      158.9160             nan     0.1000    0.8889
##    100      145.4590             nan     0.1000    0.4266
##    120      136.4194             nan     0.1000    0.2414
##    140      130.0089             nan     0.1000    0.3067
##    150      128.3017             nan     0.1000   -1.5994
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1089.0163             nan     0.1000  147.1273
##      2      969.9221             nan     0.1000  116.2787
##      3      880.5462             nan     0.1000   78.6197
##      4      812.5230             nan     0.1000   59.9873
##      5      726.9211             nan     0.1000   84.5285
##      6      677.4496             nan     0.1000   52.2021
##      7      612.9353             nan     0.1000   50.6274
##      8      561.4688             nan     0.1000   48.4978
##      9      512.3283             nan     0.1000   45.8716
##     10      483.8197             nan     0.1000   29.7787
##     20      277.0257             nan     0.1000   10.2003
##     40      170.0154             nan     0.1000    0.9824
##     60      138.0782             nan     0.1000    0.1468
##     80      121.8979             nan     0.1000    1.0083
##    100      113.0314             nan     0.1000    0.7136
##    120      106.1635             nan     0.1000    0.4506
##    140      101.5379             nan     0.1000   -0.1965
##    150       99.3361             nan     0.1000    0.0973
##
```

```
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1076.3660             nan     0.1000  166.0326
##      2      934.8358             nan     0.1000  127.5496
##      3      821.5449             nan     0.1000  106.6992
##      4      746.0042             nan     0.1000   77.6186
##      5      662.0705             nan     0.1000   88.5528
##      6      587.9474             nan     0.1000   68.8832
##      7      524.7118             nan     0.1000   57.8505
##      8      479.7467             nan     0.1000   47.0668
##      9      436.0507             nan     0.1000   39.3705
##     10      399.8243             nan     0.1000   31.9315
##     20      220.4503             nan     0.1000    6.6917
##     40      137.8457             nan     0.1000    0.0340
##     60      111.5257             nan     0.1000    0.4765
##     80      103.2541             nan     0.1000    0.3290
##    100       95.8835             nan     0.1000    0.0615
##    120       90.3473             nan     0.1000   -1.0049
##    140       86.4337             nan     0.1000   -0.4626
##    150       84.6927             nan     0.1000   -0.2075
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1339.4377             nan     0.1000  113.2613
##      2     1216.2615             nan     0.1000   99.2026
##      3     1126.4295             nan     0.1000   68.4633
##      4     1046.2734             nan     0.1000   82.6301
##      5      983.9641             nan     0.1000   62.1021
##      6      914.1832             nan     0.1000   58.4017
##      7      852.3048             nan     0.1000   55.8210
##      8      808.5896             nan     0.1000   38.0359
##      9      761.6549             nan     0.1000   46.5411
##     10      717.0981             nan     0.1000   45.1370
##     20      436.3161             nan     0.1000    5.0169
##     40      247.8442             nan     0.1000    4.0908
##     60      189.6155             nan     0.1000    1.2188
##     80      163.7345             nan     0.1000   -0.4299
##    100      151.1381             nan     0.1000    0.3984
##    120      143.2722             nan     0.1000    0.3737
##    140      137.0679             nan     0.1000    0.2390
##    150      134.8423             nan     0.1000   -0.2984
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1293.9024             nan     0.1000  152.8322
##      2     1141.1291             nan     0.1000  163.8377
##      3     1004.9357             nan     0.1000  125.6723
##      4      904.6677             nan     0.1000  101.5808
##      5      806.7888             nan     0.1000   95.9764
##      6      727.2010             nan     0.1000   56.4767
##      7      671.4384             nan     0.1000   51.6276
##      8      611.6293             nan     0.1000   47.2025
##      9      561.1445             nan     0.1000   48.9747
##     10      523.9407             nan     0.1000   41.5284
##     20      290.1424             nan     0.1000    8.3678
##     40      169.4780             nan     0.1000    2.2472
##     60      139.4834             nan     0.1000    0.5103
```

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##     80      124.4535             nan     0.1000    0.4551
##    100      116.3915             nan     0.1000    0.0335
##    120      109.1921             nan     0.1000   -0.3405
##    140      105.2343             nan     0.1000   -0.4163
##    150      103.8512             nan     0.1000   -0.3317
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1267.3354             nan     0.1000  185.0019
##      2     1107.6952             nan     0.1000  184.8420
##      3      963.0019             nan     0.1000  143.9640
##      4      842.8009             nan     0.1000  109.3526
##      5      751.5434             nan     0.1000  106.4052
##      6      668.2674             nan     0.1000   99.3912
##      7      596.6896             nan     0.1000   53.4083
##      8      542.7084             nan     0.1000   57.2947
##      9      501.0338             nan     0.1000   42.9532
##     10      455.1269             nan     0.1000   46.6512
##     20      255.6455             nan     0.1000    8.0047
##     40      152.1644             nan     0.1000    0.5435
##     60      122.5953             nan     0.1000    0.5605
##     80      109.4411             nan     0.1000    0.0806
##    100      102.2738             nan     0.1000    0.0991
##    120       96.9834             nan     0.1000   -0.6580
##    140       92.7473             nan     0.1000   -0.6350
##    150       90.8796             nan     0.1000   -0.2192
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1309.6891             nan     0.1000  119.7429
##      2     1192.8718             nan     0.1000  131.6083
##      3     1106.9567             nan     0.1000   79.0799
##      4     1030.6708             nan     0.1000   83.9500
##      5      960.5429             nan     0.1000   60.8600
##      6      892.6716             nan     0.1000   56.2380
##      7      833.7406             nan     0.1000   62.1241
##      8      785.0503             nan     0.1000   50.2280
##      9      731.9059             nan     0.1000   58.2727
##     10      686.1060             nan     0.1000   47.9452
##     20      400.7869             nan     0.1000   13.7796
##     40      224.7410             nan     0.1000    2.2285
##     60      170.6971             nan     0.1000    1.0392
##     80      145.5117             nan     0.1000    0.8516
##    100      132.2849             nan     0.1000    0.1497
##    120      123.5844             nan     0.1000   -0.1524
##    140      117.6299             nan     0.1000    0.0698
##    150      115.2245             nan     0.1000   -0.0096
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1215.0085             nan     0.1000  178.3146
##      2     1083.5683             nan     0.1000  136.7143
##      3      980.2517             nan     0.1000  107.0384
##      4      875.5395             nan     0.1000   91.5845
##      5      777.3021             nan     0.1000   89.5319
##      6      697.5710             nan     0.1000   81.7018
##      7      629.7203             nan     0.1000   48.0097
```

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##      8      574.1415             nan     0.1000   61.9125
##      9      524.1105             nan     0.1000   43.0415
##     10      487.4983             nan     0.1000   38.4445
##     20      262.9601             nan     0.1000    5.6366
##     40      159.6440             nan     0.1000    1.8309
##     60      129.1143             nan     0.1000    0.1300
##     80      114.9550             nan     0.1000    0.0602
##    100      106.5549             nan     0.1000    0.1337
##    120      101.1085             nan     0.1000   -0.0446
##    140       96.6816             nan     0.1000   -0.2191
##    150       94.8257             nan     0.1000   -0.4427
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1202.8157             nan     0.1000  211.6199
##      2     1043.5734             nan     0.1000  158.7406
##      3      894.0216             nan     0.1000  117.4613
##      4      778.1679             nan     0.1000  113.2346
##      5      696.3307             nan     0.1000   97.7456
##      6      617.0662             nan     0.1000   76.7895
##      7      548.1352             nan     0.1000   55.9634
##      8      497.3939             nan     0.1000   48.5479
##      9      452.4059             nan     0.1000   43.9317
##     10      409.5711             nan     0.1000   38.9295
##     20      226.4281             nan     0.1000    9.2007
##     40      131.8420             nan     0.1000    1.4361
##     60      107.8706             nan     0.1000    0.4472
##     80       97.8616             nan     0.1000    0.1277
##    100       90.9571             nan     0.1000   -0.1795
##    120       86.1733             nan     0.1000   -0.4309
##    140       81.2866             nan     0.1000   -0.3627
##    150       79.7317             nan     0.1000   -0.1314
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1197.8704             nan     0.1000  105.9670
##      2     1116.4273             nan     0.1000   79.7892
##      3     1039.6990             nan     0.1000   79.0002
##      4      967.4720             nan     0.1000   69.3039
##      5      914.7650             nan     0.1000   54.4291
##      6      861.5939             nan     0.1000   60.7256
##      7      806.6787             nan     0.1000   52.6622
##      8      764.8375             nan     0.1000   42.8954
##      9      732.0972             nan     0.1000   34.8340
##     10      682.1197             nan     0.1000   22.3085
##     20      422.4240             nan     0.1000   15.8138
##     40      244.9176             nan     0.1000    2.6851
##     60      186.9602             nan     0.1000    1.5573
##     80      160.2296             nan     0.1000    0.5846
##    100      147.3465             nan     0.1000   -0.0290
##    120      140.7966             nan     0.1000   -0.7363
##    140      135.9467             nan     0.1000    0.0404
##    150      133.1098             nan     0.1000    0.0520
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1138.3148             nan     0.1000  161.1864
```

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##      2     1003.7702             nan     0.1000  111.5627
##      3      885.9315             nan     0.1000   88.6904
##      4      787.9267             nan     0.1000   80.9427
##      5      718.3539             nan     0.1000   73.0769
##      6      654.9425             nan     0.1000   66.9388
##      7      593.8499             nan     0.1000   57.3728
##      8      552.5312             nan     0.1000   42.2236
##      9      506.1835             nan     0.1000   32.1548
##     10      470.7422             nan     0.1000   20.3838
##     20      280.8118             nan     0.1000   10.4112
##     40      168.5224             nan     0.1000    1.9261
##     60      138.6952             nan     0.1000    0.5653
##     80      125.8292             nan     0.1000    0.5666
##    100      116.6583             nan     0.1000   -0.0397
##    120      110.5151             nan     0.1000   -0.0843
##    140      104.5755             nan     0.1000   -0.5364
##    150      102.4542             nan     0.1000   -0.0456
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1122.9821             nan     0.1000  178.7064
##      2      977.8391             nan     0.1000  140.7097
##      3      857.8848             nan     0.1000  101.6812
##      4      750.4677             nan     0.1000   97.1855
##      5      662.0855             nan     0.1000   56.7001
##      6      585.8205             nan     0.1000   66.1089
##      7      536.4547             nan     0.1000   49.5552
##      8      486.1739             nan     0.1000   47.3448
##      9      446.9455             nan     0.1000   40.6060
##     10      411.8919             nan     0.1000   34.5107
##     20      234.3615             nan     0.1000    8.2405
##     40      140.8457             nan     0.1000    0.6081
##     60      115.9656             nan     0.1000   -0.3337
##     80      105.0992             nan     0.1000   -0.0829
##    100       99.1904             nan     0.1000   -0.3598
##    120       93.7467             nan     0.1000   -0.6929
##    140       89.7672             nan     0.1000   -0.1951
##    150       88.6174             nan     0.1000   -0.6462
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1023.4582             nan     0.1000   68.3441
##      2      967.9584             nan     0.1000   59.5204
##      3      909.0685             nan     0.1000   63.8819
##      4      854.1288             nan     0.1000   43.4207
##      5      810.2001             nan     0.1000   44.7913
##      6      765.1487             nan     0.1000   46.8162
##      7      725.9762             nan     0.1000   37.0026
##      8      675.6022             nan     0.1000   31.4953
##      9      644.0094             nan     0.1000   28.9816
##     10      616.3479             nan     0.1000   29.9801
##     20      393.2961             nan     0.1000    8.9774
##     40      235.7948             nan     0.1000    3.5970
##     60      179.8041             nan     0.1000    1.5017
##     80      150.9343             nan     0.1000    0.7221
##    100      136.3889             nan     0.1000   -1.4436
```

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##    120      129.0320             nan     0.1000    0.1599
##    140      123.3141             nan     0.1000    0.2362
##    150      121.3434             nan     0.1000   -0.7116
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      982.6173             nan     0.1000  147.4154
##      2      873.1261             nan     0.1000   93.7166
##      3      780.8316             nan     0.1000   83.4734
##      4      705.9900             nan     0.1000   79.7419
##      5      650.7736             nan     0.1000   59.2770
##      6      603.7625             nan     0.1000   46.4562
##      7      557.2159             nan     0.1000   44.0364
##      8      507.2621             nan     0.1000   38.9514
##      9      458.6382             nan     0.1000   36.5356
##     10      425.9856             nan     0.1000   32.0136
##     20      268.7626             nan     0.1000   10.8314
##     40      163.7822             nan     0.1000    1.5170
##     60      129.1078             nan     0.1000   -0.2815
##     80      115.6694             nan     0.1000   -0.5982
##    100      108.3858             nan     0.1000    0.0414
##    120      102.1804             nan     0.1000    0.2199
##    140       97.5815             nan     0.1000   -1.0489
##    150       95.6573             nan     0.1000   -0.4461
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      960.5786             nan     0.1000  154.4063
##      2      843.3030             nan     0.1000  117.0651
##      3      741.9885             nan     0.1000  110.7751
##      4      647.6381             nan     0.1000   82.1691
##      5      577.1404             nan     0.1000   57.7678
##      6      518.5489             nan     0.1000   49.1300
##      7      471.1645             nan     0.1000   41.3249
##      8      436.8741             nan     0.1000   25.9718
##      9      404.6363             nan     0.1000   25.0365
##     10      374.0788             nan     0.1000   25.2631
##     20      221.2741             nan     0.1000    8.9664
##     40      130.5958             nan     0.1000    1.2295
##     60      104.8111             nan     0.1000   -0.9402
##     80       96.0620             nan     0.1000   -0.1090
##    100       89.7521             nan     0.1000   -0.1655
##    120       85.2111             nan     0.1000   -0.5734
##    140       81.8484             nan     0.1000   -0.4011
##    150       79.9122             nan     0.1000    0.0527
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1173.3057             nan     0.1000   53.0337
##      2     1104.1929             nan     0.1000   75.5040
##      3     1034.8149             nan     0.1000   73.8075
##      4      976.3153             nan     0.1000   57.9518
##      5      916.5336             nan     0.1000   55.0927
##      6      871.8483             nan     0.1000   46.3780
##      7      820.0678             nan     0.1000   41.5526
##      8      777.0558             nan     0.1000   52.3215
##      9      733.4139             nan     0.1000   29.6760
```

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##     10      698.0279             nan     0.1000   33.0200
##     20      441.0445             nan     0.1000   16.1338
##     40      264.6720             nan     0.1000    2.5522
##     60      199.0969             nan     0.1000    1.4790
##     80      170.6772             nan     0.1000    0.8804
##    100      156.7461             nan     0.1000    0.2365
##    120      148.7043             nan     0.1000   -0.8564
##    140      142.8518             nan     0.1000    0.1958
##    150      140.7197             nan     0.1000   -0.0893
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1172.7678             nan     0.1000  100.4373
##      2     1038.9700             nan     0.1000  143.6029
##      3      956.5260             nan     0.1000   68.2986
##      4      860.1925             nan     0.1000   92.9443
##      5      776.9312             nan     0.1000   62.7130
##      6      701.8254             nan     0.1000   71.9763
##      7      639.9996             nan     0.1000   59.3219
##      8      586.9620             nan     0.1000   39.7598
##      9      540.6001             nan     0.1000   41.4294
##     10      509.9882             nan     0.1000   31.6544
##     20      310.5632             nan     0.1000   11.7259
##     40      187.8151             nan     0.1000    2.0228
##     60      150.1506             nan     0.1000   -0.0571
##     80      133.2271             nan     0.1000    0.3945
##    100      122.1332             nan     0.1000    0.6666
##    120      117.1130             nan     0.1000   -0.1163
##    140      112.4926             nan     0.1000   -0.3512
##    150      111.4169             nan     0.1000   -0.2453
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1098.0963             nan     0.1000  136.6717
##      2     1001.6572             nan     0.1000  110.3699
##      3      876.4455             nan     0.1000  116.3827
##      4      768.7673             nan     0.1000   89.0480
##      5      694.6607             nan     0.1000   69.1399
##      6      628.2615             nan     0.1000   65.2577
##      7      566.9362             nan     0.1000   58.2404
##      8      523.7986             nan     0.1000   44.3024
##      9      476.6739             nan     0.1000   34.2358
##     10      441.0850             nan     0.1000   36.1024
##     20      252.1497             nan     0.1000    9.0238
##     40      151.3068             nan     0.1000    1.0013
##     60      126.2079             nan     0.1000    0.3360
##     80      115.3667             nan     0.1000   -0.8247
##    100      106.5691             nan     0.1000    0.0017
##    120      101.4904             nan     0.1000    0.1990
##    140       97.1520             nan     0.1000   -0.1910
##    150       95.2360             nan     0.1000   -0.2821
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1201.4410             nan     0.1000  106.1190
##      2     1112.0699             nan     0.1000   89.6509
##      3     1043.7033             nan     0.1000   68.3250
```

```
##      4      987.0400             nan     0.1000   55.9010
##      5      924.1874             nan     0.1000   53.4938
##      6      877.9404             nan     0.1000   48.3207
##      7      823.8676             nan     0.1000   46.5479
##      8      773.5133             nan     0.1000   50.9513
##      9      726.6435             nan     0.1000   41.1811
##     10      687.2881             nan     0.1000   32.1867
##     20      428.2893             nan     0.1000   16.4960
##     40      247.9174             nan     0.1000    3.8608
##     60      183.2643             nan     0.1000    1.8177
##     80      153.9239             nan     0.1000    0.7093
##    100      139.5839             nan     0.1000   -0.0992
##    120      130.6775             nan     0.1000   -0.0728
##    140      124.8761             nan     0.1000   -0.5797
##    150      122.2978             nan     0.1000    0.0473
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1142.7927             nan     0.1000  151.0889
##      2     1014.7561             nan     0.1000  124.4026
##      3      908.0770             nan     0.1000   88.4686
##      4      812.2367             nan     0.1000   92.8222
##      5      754.5227             nan     0.1000   55.9591
##      6      686.4518             nan     0.1000   64.5867
##      7      626.9016             nan     0.1000   60.8306
##      8      568.7257             nan     0.1000   39.6262
##      9      533.7047             nan     0.1000   38.9119
##     10      490.3844             nan     0.1000   25.6961
##     20      280.8341             nan     0.1000   11.5409
##     40      162.4504             nan     0.1000    2.3763
##     60      125.9654             nan     0.1000   -0.4606
##     80      113.8271             nan     0.1000    0.2730
##    100      107.0324             nan     0.1000   -0.2279
##    120      100.7149             nan     0.1000   -0.0411
##    140       97.1166             nan     0.1000   -0.1578
##    150       95.0111             nan     0.1000   -0.1561
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1156.1045             nan     0.1000  168.0392
##      2      991.6610             nan     0.1000  178.6425
##      3      873.8349             nan     0.1000  122.0701
##      4      775.5410             nan     0.1000   89.8189
##      5      685.0005             nan     0.1000   82.8261
##      6      624.5754             nan     0.1000   57.5314
##      7      558.5024             nan     0.1000   43.5408
##      8      503.7201             nan     0.1000   45.5906
##      9      461.8048             nan     0.1000   49.5252
##     10      429.3099             nan     0.1000   32.1101
##     20      226.6858             nan     0.1000   10.5130
##     40      129.7483             nan     0.1000    1.1179
##     60      106.9612             nan     0.1000    0.1356
##     80       96.7065             nan     0.1000    0.0488
##    100       91.2641             nan     0.1000    0.0641
##    120       87.7454             nan     0.1000   -0.2613
##    140       84.6140             nan     0.1000    0.0046
```

```
##    150       83.3537             nan     0.1000   -0.1391
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1035.7901             nan     0.1000   58.7296
##      2      965.8432             nan     0.1000   50.9052
##      3      906.2053             nan     0.1000   65.9404
##      4      842.6515             nan     0.1000   62.7205
##      5      795.6941             nan     0.1000   43.9271
##      6      751.0854             nan     0.1000   44.0143
##      7      705.3684             nan     0.1000   43.3203
##      8      662.9367             nan     0.1000   29.9252
##      9      628.5286             nan     0.1000   23.8324
##     10      591.7874             nan     0.1000   36.1055
##     20      380.0726             nan     0.1000   13.3722
##     40      215.3355             nan     0.1000    3.7943
##     60      164.8433             nan     0.1000    1.1424
##     80      140.1305             nan     0.1000    0.1823
##    100      127.3285             nan     0.1000    0.3753
##    120      118.5926             nan     0.1000    0.1254
##    140      112.7833             nan     0.1000    0.2753
##    150      110.4796             nan     0.1000    0.1521
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      984.4417             nan     0.1000  104.8746
##      2      876.2773             nan     0.1000  106.6737
##      3      801.2080             nan     0.1000   77.5352
##      4      712.2964             nan     0.1000   79.4696
##      5      643.4845             nan     0.1000   42.7919
##      6      579.3777             nan     0.1000   58.4736
##      7      525.9397             nan     0.1000   40.6531
##      8      478.4460             nan     0.1000   44.2109
##      9      448.4120             nan     0.1000   31.1903
##     10      418.4793             nan     0.1000   30.5114
##     20      248.7991             nan     0.1000    6.4246
##     40      150.1354             nan     0.1000    0.9824
##     60      119.8076             nan     0.1000    0.0967
##     80      107.4000             nan     0.1000   -0.7326
##    100       99.6906             nan     0.1000   -0.2750
##    120       94.1053             nan     0.1000    0.2156
##    140       91.0177             nan     0.1000   -0.1580
##    150       89.8213             nan     0.1000    0.1896
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      951.9572             nan     0.1000  128.8859
##      2      842.2726             nan     0.1000  117.5267
##      3      744.2457             nan     0.1000   88.7939
##      4      659.7421             nan     0.1000   79.6048
##      5      595.3616             nan     0.1000   65.9342
##      6      547.8268             nan     0.1000   49.5625
##      7      501.8491             nan     0.1000   45.1066
##      8      453.5627             nan     0.1000   53.1859
##      9      408.1017             nan     0.1000   38.3713
##     10      376.2601             nan     0.1000   26.7730
##     20      209.4982             nan     0.1000    8.3822
```

```
##     40      123.2374             nan     0.1000    2.6032
##     60       97.5851             nan     0.1000    0.9637
##     80       87.1513             nan     0.1000    0.1307
##    100       80.4227             nan     0.1000   -0.0476
##    120       75.7581             nan     0.1000   -0.0517
##    140       72.9355             nan     0.1000   -0.2326
##    150       71.9447             nan     0.1000   -0.1492
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1201.6287             nan     0.1000   90.9115
##      2     1119.7142             nan     0.1000   83.7937
##      3     1040.6954             nan     0.1000   87.8484
##      4      985.5103             nan     0.1000   58.0410
##      5      923.7542             nan     0.1000   56.8231
##      6      870.1986             nan     0.1000   47.7431
##      7      813.9796             nan     0.1000   59.0307
##      8      771.3084             nan     0.1000   45.1861
##      9      734.6957             nan     0.1000   38.0202
##     10      691.2948             nan     0.1000   45.7223
##     20      426.1964             nan     0.1000   16.5375
##     40      247.8649             nan     0.1000    3.8290
##     60      185.7757             nan     0.1000    1.6255
##     80      156.0575             nan     0.1000    0.7319
##    100      142.2254             nan     0.1000   -0.8207
##    120      134.5285             nan     0.1000    0.2423
##    140      128.5601             nan     0.1000    0.1166
##    150      126.1168             nan     0.1000    0.2170
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1157.7912             nan     0.1000  143.5886
##      2     1022.0738             nan     0.1000  138.6245
##      3      905.0947             nan     0.1000   97.2034
##      4      809.8742             nan     0.1000   90.6408
##      5      728.5079             nan     0.1000   73.5228
##      6      655.3187             nan     0.1000   60.1245
##      7      604.2588             nan     0.1000   35.3958
##      8      556.4355             nan     0.1000   45.5072
##      9      511.7440             nan     0.1000   45.0474
##     10      476.4448             nan     0.1000   27.4272
##     20      272.3418             nan     0.1000   10.5469
##     40      165.1427             nan     0.1000    3.5678
##     60      133.0993             nan     0.1000   -0.6801
##     80      121.5918             nan     0.1000   -0.0320
##    100      114.5075             nan     0.1000    0.0147
##    120      109.9636             nan     0.1000   -0.1104
##    140      104.0401             nan     0.1000    0.0509
##    150      102.6555             nan     0.1000    0.0085
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1133.5197             nan     0.1000  172.7048
##      2      987.1972             nan     0.1000  143.5850
##      3      872.7645             nan     0.1000  118.7208
##      4      764.0503             nan     0.1000  112.4616
##      5      665.5178             nan     0.1000   92.4002
```

```
##      6      592.2673             nan     0.1000   80.0497
##      7      536.5960             nan     0.1000   64.6191
##      8      485.7798             nan     0.1000   47.8213
##      9      445.0319             nan     0.1000   41.1958
##     10      411.4633             nan     0.1000   31.3130
##     20      232.8683             nan     0.1000    9.9203
##     40      137.9395             nan     0.1000    1.1614
##     60      115.5033             nan     0.1000   -0.2852
##     80      103.5464             nan     0.1000   -0.1662
##    100       96.6148             nan     0.1000    0.3704
##    120       92.0743             nan     0.1000   -0.4650
##    140       87.9470             nan     0.1000   -0.5296
##    150       86.5364             nan     0.1000    0.1443
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1072.0599             nan     0.1000   83.9007
##      2      995.1452             nan     0.1000   75.6876
##      3      930.6467             nan     0.1000   60.1760
##      4      882.8934             nan     0.1000   36.4684
##      5      826.8277             nan     0.1000   57.7570
##      6      777.8295             nan     0.1000   45.0836
##      7      738.5092             nan     0.1000   38.4305
##      8      694.2602             nan     0.1000   35.9653
##      9      657.8653             nan     0.1000   14.7778
##     10      614.2662             nan     0.1000   45.3666
##     20      381.4929             nan     0.1000   13.7471
##     40      225.4717             nan     0.1000    2.9268
##     60      169.7447             nan     0.1000    1.1790
##     80      143.4027             nan     0.1000    0.1550
##    100      130.2860             nan     0.1000    0.3389
##    120      121.7222             nan     0.1000    0.2099
##    140      115.8403             nan     0.1000   -0.0558
##    150      113.2317             nan     0.1000    0.2352
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1019.1468             nan     0.1000   93.3516
##      2      923.9510             nan     0.1000   96.2785
##      3      817.5182             nan     0.1000   95.1558
##      4      746.5360             nan     0.1000   67.5560
##      5      669.3585             nan     0.1000   64.8072
##      6      605.2731             nan     0.1000   61.6085
##      7      552.7118             nan     0.1000   31.8123
##      8      509.3889             nan     0.1000   34.3121
##      9      473.4643             nan     0.1000   37.5760
##     10      434.2131             nan     0.1000   32.7532
##     20      250.3380             nan     0.1000    9.0373
##     40      149.8441             nan     0.1000    2.3425
##     60      121.5830             nan     0.1000    0.4301
##     80      107.6926             nan     0.1000   -0.6998
##    100      100.4119             nan     0.1000    0.0089
##    120       96.2707             nan     0.1000   -0.3374
##    140       90.8735             nan     0.1000   -0.0021
##    150       90.0521             nan     0.1000   -0.2075
##
```

```
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      997.9016             nan     0.1000  156.2244
##      2      874.7602             nan     0.1000  136.0710
##      3      766.1950             nan     0.1000   88.8837
##      4      673.5437             nan     0.1000   86.0353
##      5      608.7514             nan     0.1000   68.2482
##      6      548.6146             nan     0.1000   57.5847
##      7      501.5420             nan     0.1000   49.3042
##      8      456.5943             nan     0.1000   40.2833
##      9      417.2413             nan     0.1000   37.7478
##     10      387.6704             nan     0.1000   27.5050
##     20      216.7646             nan     0.1000    9.6328
##     40      132.6365             nan     0.1000    1.6443
##     60      107.6826             nan     0.1000    0.0679
##     80       94.8867             nan     0.1000    0.1861
##    100       86.7963             nan     0.1000    0.4310
##    120       80.0393             nan     0.1000    0.0909
##    140       76.8072             nan     0.1000   -0.1429
##    150       75.2394             nan     0.1000   -0.0434
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1080.8189             nan     0.1000   76.6571
##      2     1003.9710             nan     0.1000   66.6505
##      3      942.4747             nan     0.1000   58.5403
##      4      886.9356             nan     0.1000   55.1046
##      5      839.6075             nan     0.1000   47.9903
##      6      781.6543             nan     0.1000   58.4151
##      7      735.4703             nan     0.1000   39.6462
##      8      691.4888             nan     0.1000   42.5628
##      9      654.6201             nan     0.1000   36.2675
##     10      622.4645             nan     0.1000   28.5020
##     20      401.3246             nan     0.1000    8.4799
##     40      234.4538             nan     0.1000    3.2475
##     60      175.0800             nan     0.1000    0.7847
##     80      150.1060             nan     0.1000    0.2590
##    100      137.6066             nan     0.1000   -0.6271
##    120      128.8004             nan     0.1000    0.3464
##    140      123.6679             nan     0.1000   -0.3460
##    150      121.3646             nan     0.1000    0.1333
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1032.4696             nan     0.1000  155.1064
##      2      945.9092             nan     0.1000  107.4908
##      3      833.1954             nan     0.1000   98.7479
##      4      748.2650             nan     0.1000   94.3671
##      5      680.7824             nan     0.1000   70.5340
##      6      622.6394             nan     0.1000   63.3733
##      7      568.8359             nan     0.1000   61.2356
##      8      521.6860             nan     0.1000   39.0593
##      9      482.9395             nan     0.1000   44.5384
##     10      444.5591             nan     0.1000   37.3012
##     20      257.2536             nan     0.1000    9.2497
##     40      156.4679             nan     0.1000    1.0572
##     60      124.0190             nan     0.1000    0.7324
```

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##     80      110.4133             nan     0.1000    0.6957
##    100      101.9957             nan     0.1000    0.7617
##    120       95.8166             nan     0.1000    0.0013
##    140       93.3269             nan     0.1000   -0.2322
##    150       91.7920             nan     0.1000   -0.2663
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1      996.8675             nan     0.1000  155.6846
##      2      868.4262             nan     0.1000  125.6263
##      3      766.1887             nan     0.1000  104.5108
##      4      675.4711             nan     0.1000   58.7944
##      5      600.3957             nan     0.1000   68.9915
##      6      533.0195             nan     0.1000   66.1509
##      7      477.9536             nan     0.1000   37.5246
##      8      440.1525             nan     0.1000   41.8343
##      9      402.6834             nan     0.1000   37.8598
##     10      374.8409             nan     0.1000   31.5991
##     20      210.2067             nan     0.1000    8.9055
##     40      128.6086             nan     0.1000   -1.2271
##     60      109.0801             nan     0.1000   -0.2201
##     80       98.2993             nan     0.1000   -0.1650
##    100       92.4266             nan     0.1000    0.0777
##    120       86.3903             nan     0.1000   -0.2130
##    140       82.5415             nan     0.1000   -0.2469
##    150       81.0329             nan     0.1000   -0.4306
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1222.9937             nan     0.1000   95.5819
##      2     1145.0232             nan     0.1000   77.1941
##      3     1077.9556             nan     0.1000   67.2316
##      4     1009.5096             nan     0.1000   67.9725
##      5      945.3913             nan     0.1000   58.7099
##      6      894.1958             nan     0.1000   54.7122
##      7      841.2722             nan     0.1000   47.8715
##      8      798.8638             nan     0.1000   30.1145
##      9      756.3627             nan     0.1000   37.6571
##     10      714.7252             nan     0.1000   42.2972
##     20      434.9873             nan     0.1000   17.1626
##     40      248.3850             nan     0.1000    4.5311
##     60      189.8807             nan     0.1000    1.6512
##     80      163.0919             nan     0.1000    0.6497
##    100      150.7166             nan     0.1000   -0.4042
##    120      142.7196             nan     0.1000    0.2574
##    140      136.4626             nan     0.1000   -0.1399
##    150      133.4146             nan     0.1000   -0.1580
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1160.9798             nan     0.1000  137.7709
##      2     1025.0834             nan     0.1000  130.3225
##      3      926.5422             nan     0.1000  101.9054
##      4      835.0766             nan     0.1000   95.1952
##      5      755.7430             nan     0.1000   67.7841
##      6      696.4705             nan     0.1000   59.9653
##      7      639.3003             nan     0.1000   52.7958
```

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##      8      578.5829             nan     0.1000   60.5109
##      9      525.8460             nan     0.1000   45.8725
##     10      483.9510             nan     0.1000   44.4311
##     20      277.9536             nan     0.1000    9.0038
##     40      169.2679             nan     0.1000    0.9201
##     60      139.9209             nan     0.1000   -0.2076
##     80      126.1292             nan     0.1000   -0.7480
##    100      117.6557             nan     0.1000    0.2180
##    120      110.9438             nan     0.1000   -0.6432
##    140      106.7862             nan     0.1000    0.0718
##    150      104.1988             nan     0.1000   -0.0185
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1122.3496             nan     0.1000  214.4010
##      2      977.3170             nan     0.1000  141.5716
##      3      881.4311             nan     0.1000   89.5670
##      4      802.2643             nan     0.1000   73.8740
##      5      706.5898             nan     0.1000  103.1031
##      6      638.2308             nan     0.1000   74.1872
##      7      570.3551             nan     0.1000   51.2075
##      8      521.9023             nan     0.1000   47.8098
##      9      471.4959             nan     0.1000   40.5367
##     10      430.4478             nan     0.1000   38.1211
##     20      234.7109             nan     0.1000    9.8324
##     40      139.1237             nan     0.1000    1.2925
##     60      115.0282             nan     0.1000    0.5032
##     80      103.9629             nan     0.1000   -0.3077
##    100       96.6829             nan     0.1000   -0.2997
##    120       91.8252             nan     0.1000    0.0046
##    140       88.5128             nan     0.1000   -0.2414
##    150       86.9650             nan     0.1000   -0.2051
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1087.8570             nan     0.1000   76.6095
##      2     1013.8603             nan     0.1000   62.7495
##      3      957.5153             nan     0.1000   55.9620
##      4      897.3468             nan     0.1000   61.6687
##      5      836.9038             nan     0.1000   49.7647
##      6      787.3263             nan     0.1000   47.1981
##      7      744.5487             nan     0.1000   36.7272
##      8      699.7224             nan     0.1000   44.5974
##      9      664.5324             nan     0.1000   34.5989
##     10      632.6207             nan     0.1000   32.1634
##     20      404.4873             nan     0.1000   14.4809
##     40      243.6616             nan     0.1000    4.6382
##     60      187.3818             nan     0.1000    1.8495
##     80      161.7907             nan     0.1000    0.4776
##    100      149.1679             nan     0.1000    0.4633
##    120      141.2548             nan     0.1000   -0.4979
##    140      135.7785             nan     0.1000   -0.1640
##    150      133.6173             nan     0.1000   -0.5843
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1050.9876             nan     0.1000  112.8077
```

```
##      2      918.7068             nan     0.1000  129.8786
##      3      808.9025             nan     0.1000   98.3689
##      4      728.8537             nan     0.1000   78.2674
##      5      676.8000             nan     0.1000   55.3235
##      6      612.2399             nan     0.1000   54.5006
##      7      559.6181             nan     0.1000   50.6218
##      8      515.0434             nan     0.1000   40.8312
##      9      473.0174             nan     0.1000   36.9899
##     10      433.1366             nan     0.1000   33.8391
##     20      272.3469             nan     0.1000    9.4617
##     40      170.5693             nan     0.1000    2.2894
##     60      143.1123             nan     0.1000    0.3532
##     80      126.9598             nan     0.1000    0.1493
##    100      118.7461             nan     0.1000   -0.8097
##    120      113.7615             nan     0.1000   -0.1565
##    140      109.6232             nan     0.1000   -0.4357
##    150      107.6110             nan     0.1000    0.4073
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1002.9726             nan     0.1000  130.6249
##      2      874.5304             nan     0.1000  106.8671
##      3      784.4297             nan     0.1000   92.7181
##      4      697.1941             nan     0.1000   88.3228
##      5      626.8668             nan     0.1000   74.1835
##      6      573.3715             nan     0.1000   54.5405
##      7      514.4491             nan     0.1000   67.7620
##      8      464.2703             nan     0.1000   44.0149
##      9      427.8894             nan     0.1000   36.7926
##     10      396.0215             nan     0.1000   30.5870
##     20      228.4352             nan     0.1000    7.3448
##     40      143.6600             nan     0.1000    0.7433
##     60      121.9373             nan     0.1000    0.2120
##     80      111.5679             nan     0.1000   -0.0402
##    100      103.4978             nan     0.1000    0.1086
##    120       97.0031             nan     0.1000   -0.1387
##    140       92.7187             nan     0.1000   -0.0460
##    150       90.3598             nan     0.1000   -0.0037
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1090.7159             nan     0.1000   84.5760
##      2     1007.2289             nan     0.1000   74.1523
##      3      946.8949             nan     0.1000   60.3607
##      4      887.4432             nan     0.1000   59.7682
##      5      819.2801             nan     0.1000   70.1452
##      6      771.3496             nan     0.1000   47.5261
##      7      720.1783             nan     0.1000   53.3296
##      8      681.0785             nan     0.1000   40.7998
##      9      643.3239             nan     0.1000   30.1461
##     10      606.5380             nan     0.1000   36.8915
##     20      377.9292             nan     0.1000   12.3079
##     40      217.9400             nan     0.1000    3.2746
##     60      166.1905             nan     0.1000    1.2989
##     80      141.4818             nan     0.1000    0.7027
##    100      128.4313             nan     0.1000    0.1308
```

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##    120      119.5264             nan     0.1000   -0.1771
##    140      113.3529             nan     0.1000   -0.0105
##    150      110.9493             nan     0.1000    0.0478
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1033.9575             nan     0.1000  132.8711
##      2      923.8711             nan     0.1000   96.0836
##      3      819.2553             nan     0.1000   97.9891
##      4      725.4915             nan     0.1000   69.5400
##      5      664.0824             nan     0.1000   62.4558
##      6      606.7370             nan     0.1000   58.9446
##      7      551.1123             nan     0.1000   55.5651
##      8      507.4330             nan     0.1000   44.5145
##      9      467.8108             nan     0.1000   36.8782
##     10      434.9003             nan     0.1000   32.6813
##     20      248.0589             nan     0.1000   12.5799
##     40      146.4711             nan     0.1000    1.8295
##     60      114.5459             nan     0.1000    0.7619
##     80      102.0801             nan     0.1000    0.0312
##    100       94.9411             nan     0.1000    0.5516
##    120       89.8323             nan     0.1000   -0.5575
##    140       86.1568             nan     0.1000    0.0435
##    150       84.8165             nan     0.1000   -0.1532
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1026.8621             nan     0.1000  142.8041
##      2      921.2654             nan     0.1000  108.3263
##      3      795.7200             nan     0.1000  106.8504
##      4      696.4259             nan     0.1000  106.4723
##      5      617.6322             nan     0.1000   72.1743
##      6      552.1506             nan     0.1000   65.2443
##      7      494.2715             nan     0.1000   50.5568
##      8      445.6477             nan     0.1000   46.1631
##      9      408.5658             nan     0.1000   39.8022
##     10      374.8992             nan     0.1000   30.8978
##     20      208.5083             nan     0.1000    6.9347
##     40      122.2220             nan     0.1000    2.0757
##     60       99.9701             nan     0.1000    0.5794
##     80       89.6386             nan     0.1000    0.2813
##    100       83.1020             nan     0.1000    0.1541
##    120       78.2733             nan     0.1000   -0.2972
##    140       75.3554             nan     0.1000   -0.1857
##    150       73.6533             nan     0.1000   -0.1366
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1118.8115             nan     0.1000   73.4980
##      2     1050.4907             nan     0.1000   66.5460
##      3      978.8147             nan     0.1000   67.2210
##      4      917.3993             nan     0.1000   54.8446
##      5      857.4446             nan     0.1000   58.4640
##      6      810.0807             nan     0.1000   42.8145
##      7      764.8915             nan     0.1000   45.2697
##      8      717.6212             nan     0.1000   48.6299
##      9      678.6333             nan     0.1000   38.5026
```

```
##     10      642.8427             nan     0.1000   36.2583
##     20      400.0696             nan     0.1000   15.5698
##     40      232.2462             nan     0.1000    3.8881
##     60      174.7613             nan     0.1000    1.5945
##     80      148.1350             nan     0.1000    0.5106
##    100      135.2250             nan     0.1000   -0.9972
##    120      126.5791             nan     0.1000    0.3248
##    140      119.9147             nan     0.1000    0.1349
##    150      117.7438             nan     0.1000    0.0888
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1062.1767             nan     0.1000  153.5348
##      2      936.4652             nan     0.1000  115.1786
##      3      829.3559             nan     0.1000   85.4330
##      4      757.2288             nan     0.1000   79.2473
##      5      700.1942             nan     0.1000   61.7006
##      6      636.2802             nan     0.1000   66.7213
##      7      580.7349             nan     0.1000   56.5523
##      8      534.5268             nan     0.1000   46.2899
##      9      498.0534             nan     0.1000   35.5052
##     10      458.5808             nan     0.1000   36.8635
##     20      272.1561             nan     0.1000   10.5912
##     40      163.0717             nan     0.1000    1.0363
##     60      126.9545             nan     0.1000    1.4551
##     80      113.7320             nan     0.1000   -0.0505
##    100      106.1635             nan     0.1000   -0.7638
##    120      100.3572             nan     0.1000    0.5649
##    140       95.7085             nan     0.1000   -0.1186
##    150       93.0152             nan     0.1000    0.3598
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1041.6123             nan     0.1000  154.3690
##      2      914.0338             nan     0.1000  142.5481
##      3      801.1364             nan     0.1000  110.1066
##      4      733.4377             nan     0.1000   74.4354
##      5      652.7770             nan     0.1000   91.6788
##      6      578.6338             nan     0.1000   59.0238
##      7      526.6763             nan     0.1000   49.7396
##      8      479.9661             nan     0.1000   42.8199
##      9      438.8906             nan     0.1000   41.4579
##     10      404.4224             nan     0.1000   35.9367
##     20      229.6770             nan     0.1000    9.7288
##     40      135.5758             nan     0.1000    0.8887
##     60      109.8398             nan     0.1000   -0.2751
##     80       97.5181             nan     0.1000   -0.1177
##    100       91.6485             nan     0.1000   -0.3932
##    120       85.1088             nan     0.1000   -0.1882
##    140       80.8995             nan     0.1000   -0.4833
##    150       78.4903             nan     0.1000   -0.2523
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1119.8773             nan     0.1000   94.3423
##      2     1036.5403             nan     0.1000   84.1755
##      3      967.3030             nan     0.1000   68.0679
```

```
##      4      900.4328             nan     0.1000   51.4887
##      5      840.7201             nan     0.1000   57.2646
##      6      789.3604             nan     0.1000   53.1029
##      7      737.3305             nan     0.1000   48.8495
##      8      694.6946             nan     0.1000   46.8030
##      9      651.7255             nan     0.1000   37.3301
##     10      615.7655             nan     0.1000   37.5008
##     20      383.5992             nan     0.1000   11.8737
##     40      228.2604             nan     0.1000    2.4331
##     60      178.3682             nan     0.1000    1.1799
##     80      152.9885             nan     0.1000    0.4506
##    100      139.4953             nan     0.1000    0.1046
##    120      131.4240             nan     0.1000    0.2497
##    140      125.9992             nan     0.1000   -0.6988
##    150      123.9954             nan     0.1000    0.1049
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1056.6943             nan     0.1000  152.1538
##      2      922.9485             nan     0.1000  127.0554
##      3      812.9224             nan     0.1000  112.0540
##      4      735.7370             nan     0.1000   77.1514
##      5      660.4394             nan     0.1000   77.7973
##      6      594.5295             nan     0.1000   54.8795
##      7      538.1276             nan     0.1000   50.0000
##      8      496.2987             nan     0.1000   41.3740
##      9      463.3502             nan     0.1000   29.0453
##     10      423.1220             nan     0.1000   32.9575
##     20      254.4723             nan     0.1000   10.8771
##     40      157.7539             nan     0.1000    3.1032
##     60      129.3769             nan     0.1000    0.6312
##     80      115.9494             nan     0.1000    0.5027
##    100      109.1067             nan     0.1000   -0.1916
##    120      103.6767             nan     0.1000   -0.1559
##    140      100.4952             nan     0.1000   -0.0890
##    150       98.7492             nan     0.1000   -0.2977
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1048.6867             nan     0.1000  169.7189
##      2      898.1875             nan     0.1000  132.8233
##      3      779.0176             nan     0.1000  115.3169
##      4      688.4945             nan     0.1000   76.3549
##      5      601.5160             nan     0.1000   76.0706
##      6      535.5268             nan     0.1000   49.9857
##      7      491.4052             nan     0.1000   44.9608
##      8      456.8617             nan     0.1000   33.7188
##      9      427.1252             nan     0.1000   27.1048
##     10      396.9068             nan     0.1000   34.3282
##     20      227.6345             nan     0.1000    7.7327
##     40      139.1342             nan     0.1000    2.4652
##     60      116.2420             nan     0.1000    0.1765
##     80      104.1271             nan     0.1000   -0.5082
##    100       96.7440             nan     0.1000   -0.1467
##    120       92.1058             nan     0.1000   -0.1483
##    140       87.7991             nan     0.1000    0.0628
```

```
##    150       86.2169             nan     0.1000   -0.0998
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1189.0101             nan     0.1000   86.6900
##      2     1114.6414             nan     0.1000   71.3947
##      3     1047.4181             nan     0.1000   57.8647
##      4      977.1987             nan     0.1000   66.0326
##      5      907.3896             nan     0.1000   67.8968
##      6      851.5658             nan     0.1000   54.0091
##      7      800.4883             nan     0.1000   43.4781
##      8      753.3136             nan     0.1000   40.4555
##      9      709.8473             nan     0.1000   39.5049
##     10      668.1243             nan     0.1000   37.6030
##     20      423.7775             nan     0.1000   18.7096
##     40      248.4562             nan     0.1000    3.1495
##     60      192.5803             nan     0.1000    1.5092
##     80      169.4225             nan     0.1000    0.4061
##    100      156.9656             nan     0.1000    0.2653
##    120      149.3569             nan     0.1000    0.0663
##    140      145.4897             nan     0.1000   -0.1711
##    150      143.3884             nan     0.1000   -0.4252
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1129.1701             nan     0.1000  153.7598
##      2      996.7433             nan     0.1000   97.6723
##      3      909.4208             nan     0.1000   88.9440
##      4      821.4093             nan     0.1000   82.8959
##      5      755.1422             nan     0.1000   67.1834
##      6      678.6369             nan     0.1000   67.5101
##      7      618.9197             nan     0.1000   56.0494
##      8      569.8699             nan     0.1000   52.9390
##      9      533.0902             nan     0.1000   36.0775
##     10      490.8521             nan     0.1000   41.1104
##     20      279.8185             nan     0.1000    8.8339
##     40      181.4204             nan     0.1000    1.5549
##     60      150.0737             nan     0.1000    1.6910
##     80      137.2608             nan     0.1000    0.7824
##    100      129.9877             nan     0.1000   -0.4570
##    120      123.8775             nan     0.1000   -0.3129
##    140      118.0144             nan     0.1000   -0.7412
##    150      115.0476             nan     0.1000   -0.3169
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1106.8309             nan     0.1000  137.4255
##      2      983.8373             nan     0.1000  111.8664
##      3      876.5559             nan     0.1000  110.1780
##      4      766.9048             nan     0.1000  109.8097
##      5      682.1685             nan     0.1000   80.4737
##      6      622.5092             nan     0.1000   66.1048
##      7      550.4590             nan     0.1000   62.3545
##      8      498.6682             nan     0.1000   46.0724
##      9      457.5230             nan     0.1000   46.5795
##     10      421.0962             nan     0.1000   37.5097
##     20      236.7550             nan     0.1000    9.0588
```

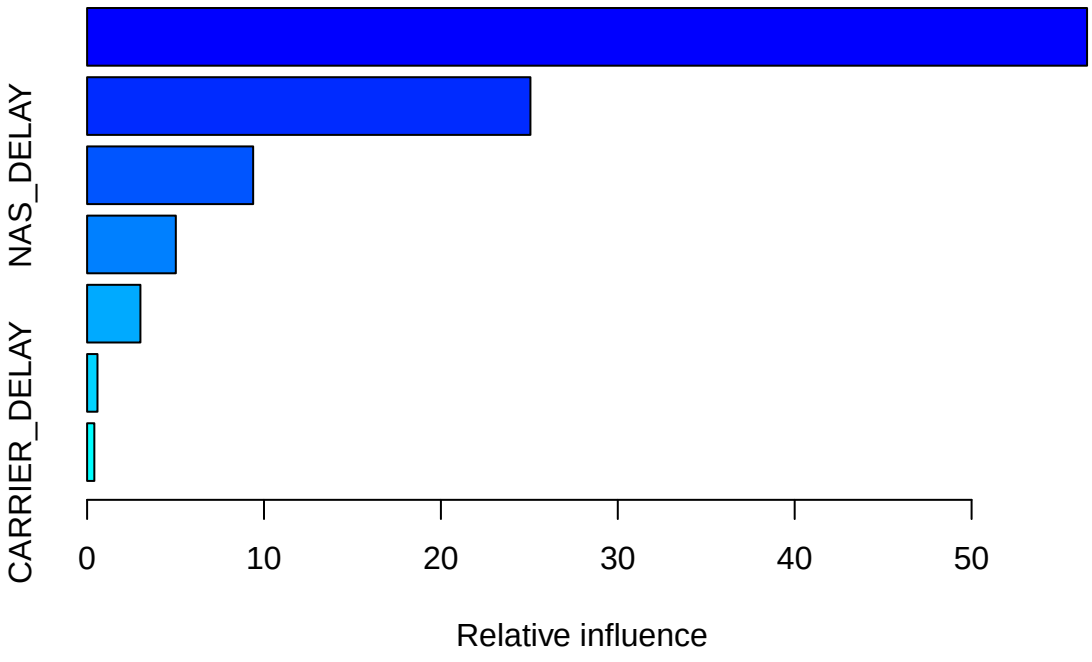
```
##     40      151.0315             nan     0.1000    0.7579
##     60      127.0778             nan     0.1000   -0.1092
##     80      116.4214             nan     0.1000   -0.0738
##    100      107.3310             nan     0.1000   -0.5775
##    120      102.4719             nan     0.1000   -0.3353
##    140       96.2524             nan     0.1000   -0.0157
##    150       94.1987             nan     0.1000    0.1255
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1283.6954             nan     0.1000   73.1715
##      2     1182.6265             nan     0.1000   84.5721
##      3     1092.3751             nan     0.1000   87.0078
##      4     1020.7277             nan     0.1000   66.1521
##      5      954.5623             nan     0.1000   65.1395
##      6      892.8784             nan     0.1000   38.6726
##      7      839.0687             nan     0.1000   52.9767
##      8      785.7557             nan     0.1000   57.1875
##      9      743.5452             nan     0.1000   39.0069
##     10      695.9698             nan     0.1000   48.5670
##     20      422.0979             nan     0.1000   15.6932
##     40      243.9402             nan     0.1000    4.0555
##     60      186.8147             nan     0.1000    0.8337
##     80      161.0412             nan     0.1000    0.2119
##    100      147.5159             nan     0.1000    0.2091
##    120      138.3987             nan     0.1000    0.0832
##    140      131.6827             nan     0.1000    0.1823
##    150      128.7122             nan     0.1000   -0.0164
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1207.7910             nan     0.1000  180.1243
##      2     1063.2926             nan     0.1000  153.4980
##      3      965.0687             nan     0.1000   85.7630
##      4      864.6020             nan     0.1000   91.2689
##      5      773.1352             nan     0.1000   82.4426
##      6      700.6816             nan     0.1000   69.2203
##      7      631.2889             nan     0.1000   68.3590
##      8      573.0636             nan     0.1000   51.0187
##      9      528.0273             nan     0.1000   45.9456
##     10      476.1582             nan     0.1000   44.3333
##     20      278.1279             nan     0.1000    9.1232
##     40      169.9533             nan     0.1000    1.3212
##     60      137.5625             nan     0.1000    0.8528
##     80      123.0038             nan     0.1000   -0.2596
##    100      112.5488             nan     0.1000    0.2349
##    120      105.2482             nan     0.1000   -0.0700
##    140      101.6530             nan     0.1000   -0.3782
##    150      100.3467             nan     0.1000   -0.7668
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1187.7043             nan     0.1000  214.3587
##      2     1020.9794             nan     0.1000  181.5930
##      3      893.9061             nan     0.1000  127.2004
##      4      778.8480             nan     0.1000  120.6036
##      5      695.0268             nan     0.1000   74.5797
```

```
##      6      629.3644             nan     0.1000   77.4514
##      7      561.2605             nan     0.1000   67.4979
##      8      510.4674             nan     0.1000   53.8901
##      9      459.3352             nan     0.1000   37.7979
##     10      416.3479             nan     0.1000   41.0710
##     20      227.3097             nan     0.1000    6.8951
##     40      136.2936             nan     0.1000    1.5643
##     60      115.3820             nan     0.1000    0.0036
##     80      103.7335             nan     0.1000   -0.0722
##    100       96.6125             nan     0.1000    0.0211
##    120       92.4467             nan     0.1000   -0.1759
##    140       88.4003             nan     0.1000   -0.2170
##    150       86.1169             nan     0.1000   -0.0322
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1210.0056             nan     0.1000   77.0098
##      2     1131.9875             nan     0.1000   77.4601
##      3     1050.1754             nan     0.1000   78.9251
##      4      985.7537             nan     0.1000   63.0267
##      5      929.2235             nan     0.1000   51.8651
##      6      868.4312             nan     0.1000   66.4035
##      7      813.4592             nan     0.1000   52.3478
##      8      772.3243             nan     0.1000   43.1009
##      9      738.1129             nan     0.1000   33.0682
##     10      699.9654             nan     0.1000   40.1154
##     20      434.8572             nan     0.1000   14.7241
##     40      252.9659             nan     0.1000    4.0820
##     60      190.3498             nan     0.1000    2.0213
##     80      162.8234             nan     0.1000    0.6628
##    100      150.7828             nan     0.1000    0.5502
##    120      142.9481             nan     0.1000    0.0537
##    140      137.5727             nan     0.1000   -0.5494
##    150      135.5533             nan     0.1000    0.0043
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1144.5054             nan     0.1000  171.3193
##      2     1026.1129             nan     0.1000  122.7750
##      3      929.5790             nan     0.1000   92.9228
##      4      835.4790             nan     0.1000   75.4115
##      5      752.1271             nan     0.1000   78.2724
##      6      683.9247             nan     0.1000   71.9112
##      7      620.0264             nan     0.1000   54.5971
##      8      565.8734             nan     0.1000   40.3096
##      9      516.9392             nan     0.1000   38.0251
##     10      484.2238             nan     0.1000   36.4699
##     20      292.2535             nan     0.1000   10.8410
##     40      165.6017             nan     0.1000    2.9954
##     60      134.2776             nan     0.1000    0.5575
##     80      120.1004             nan     0.1000    0.0713
##    100      111.6263             nan     0.1000   -1.0180
##    120      105.9832             nan     0.1000   -0.2969
##    140      102.5158             nan     0.1000    0.0533
##    150      100.7794             nan     0.1000   -0.2743
##
```

```
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1163.8655             nan     0.1000  156.2903
##      2     1016.5769             nan     0.1000  143.8181
##      3      882.9052             nan     0.1000  133.6532
##      4      772.9999             nan     0.1000   83.3790
##      5      688.3052             nan     0.1000   60.3592
##      6      613.6813             nan     0.1000   68.7803
##      7      550.7211             nan     0.1000   64.4275
##      8      497.8653             nan     0.1000   47.6486
##      9      458.5626             nan     0.1000   40.8449
##     10      420.3436             nan     0.1000   30.5602
##     20      229.2189             nan     0.1000   15.5717
##     40      134.3119             nan     0.1000    0.8417
##     60      112.1139             nan     0.1000    1.0587
##     80       99.2406             nan     0.1000    0.1316
##    100       92.5674             nan     0.1000   -0.1656
##    120       87.5723             nan     0.1000    0.1431
##    140       83.9197             nan     0.1000   -0.3731
##    150       81.6334             nan     0.1000    0.2738
##
## Iter   TrainDeviance   ValidDeviance   StepSize   Improve
##      1     1066.3687             nan     0.1000  169.8292
##      2      927.1047             nan     0.1000  114.6841
##      3      807.6555             nan     0.1000  121.2462
##      4      718.3953             nan     0.1000   99.4117
##      5      634.5350             nan     0.1000   79.5987
##      6      573.8356             nan     0.1000   69.3062
##      7      515.3659             nan     0.1000   47.3729
##      8      470.2866             nan     0.1000   36.9176
##      9      429.8505             nan     0.1000   39.8279
##     10      400.3352             nan     0.1000   30.4731
##     20      229.6228             nan     0.1000    8.5911
##     40      144.8718             nan     0.1000    1.4341
##     60      123.6587             nan     0.1000    1.1849
##     80      111.3196             nan     0.1000    0.1940
##    100      105.9478             nan     0.1000   -0.0948
##    120      101.1540             nan     0.1000   -0.2309
##    140       97.2260             nan     0.1000   -0.1688
##    150       95.4646             nan     0.1000   -0.2985
```

The `summary()` function also provides a relative influence plot and also outputs the relative influence statistics.

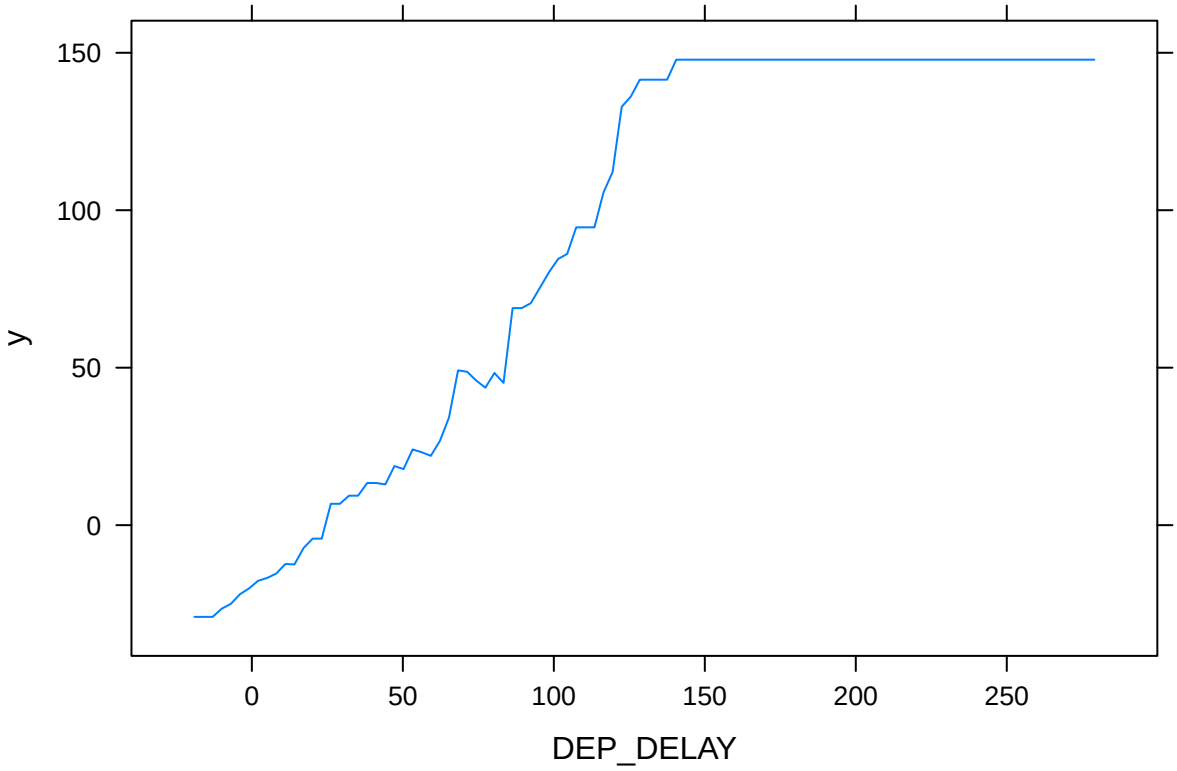
```
summary(boost.delay)
```



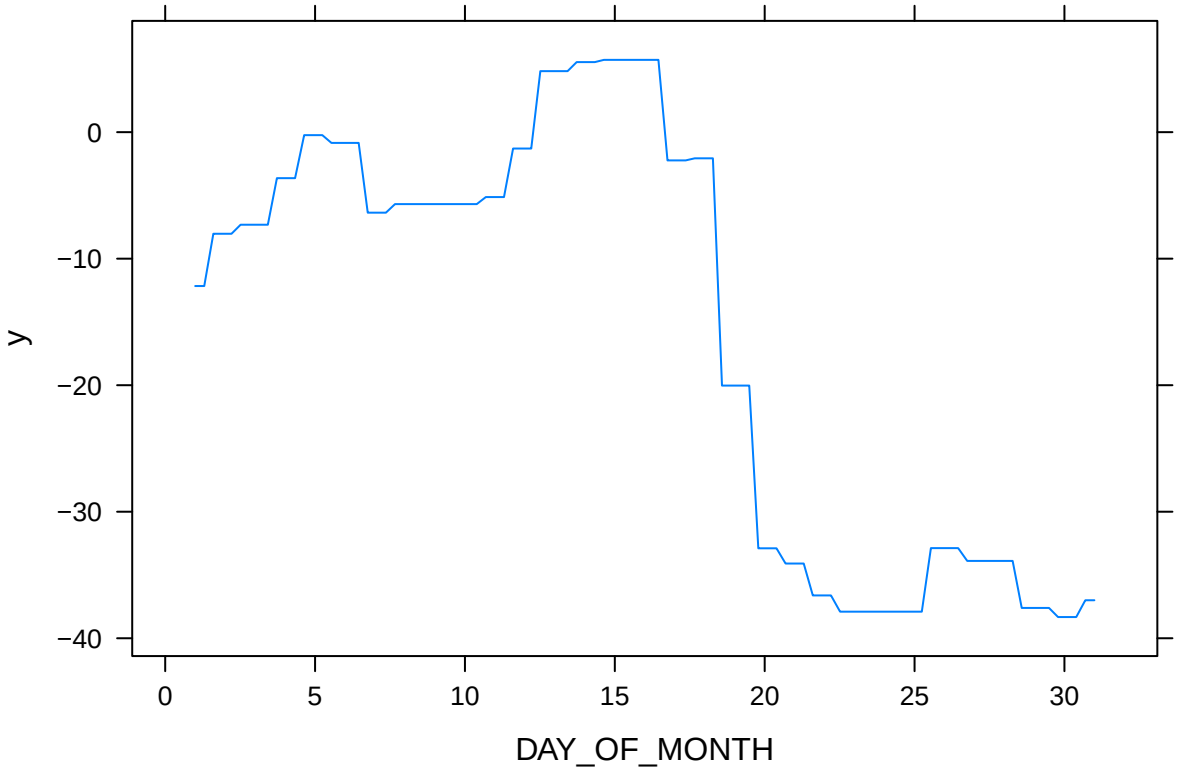

```
##                                    var    rel.inf
## DEP_DELAY                    DEP_DELAY 56.5279213
## DAY_OF_MONTH              DAY_OF_MONTH 25.0608132
## NAS_DELAY                    NAS_DELAY  9.3867001
## TAXI_OUT                      TAXI_OUT  5.0157480
## TAXI_IN                        TAXI_IN  3.0135950
## LATE_AIRCRAFT_DELAY LATE_AIRCRAFT_DELAY  0.5862760
## CARRIER_DELAY            CARRIER_DELAY  0.4089464
```

We see that `DEP_DELAY` and `DAY_OF_MONTH` are by far the most important variables. We can also produce *partial dependence plots* for these two variables. These plots illustrate the marginal effect of the selected variables on the response after `integrating` out the other variables.

```
par(mfrow = c(1,2))
plot(boost.delay, i = "DEP_DELAY")
```




```
plot(boost.delay, i = "DAY_OF_MONTH")
```



Trees Test Error Metrics & Predictions

```
# predictions
## random forest
yhat.rf <- predict(rf.delay, newdata = test)
## boosting
yhat.boost <- predict(boost.delay, newdata =test,
                      n.trees = 150)

## histogram of predictions vs. actual
# actual ARR_DELAY in test set
ptest_actual <- ggplot(data = test, aes(x = ARR_DELAY)) +
  geom_histogram(fill = "#FFFF00", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Actual") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- baseline lm
ptest_rf <- ggplot(data = test, aes(x = yhat.rf)) +
  geom_histogram(fill = "#FFFF00", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Random Forest") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

# predicted ARR_DELAY in test set -- lm with log-trans
ptest_boost <- ggplot(data = test, aes(x = yhat.boost)) +
  geom_histogram(fill = "#FFFF00", color = "#002D72", alpha = 0.8) +
  labs(x = "Arrival Delay",
       y = "Frequency",
       title = "Boosting") +
  theme(plot.title = element_text(size = 12,hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5),
        axis.title.x.bottom = element_text(size = 8, face = "italic"),
        axis.title.y.left = element_text(size = 8))

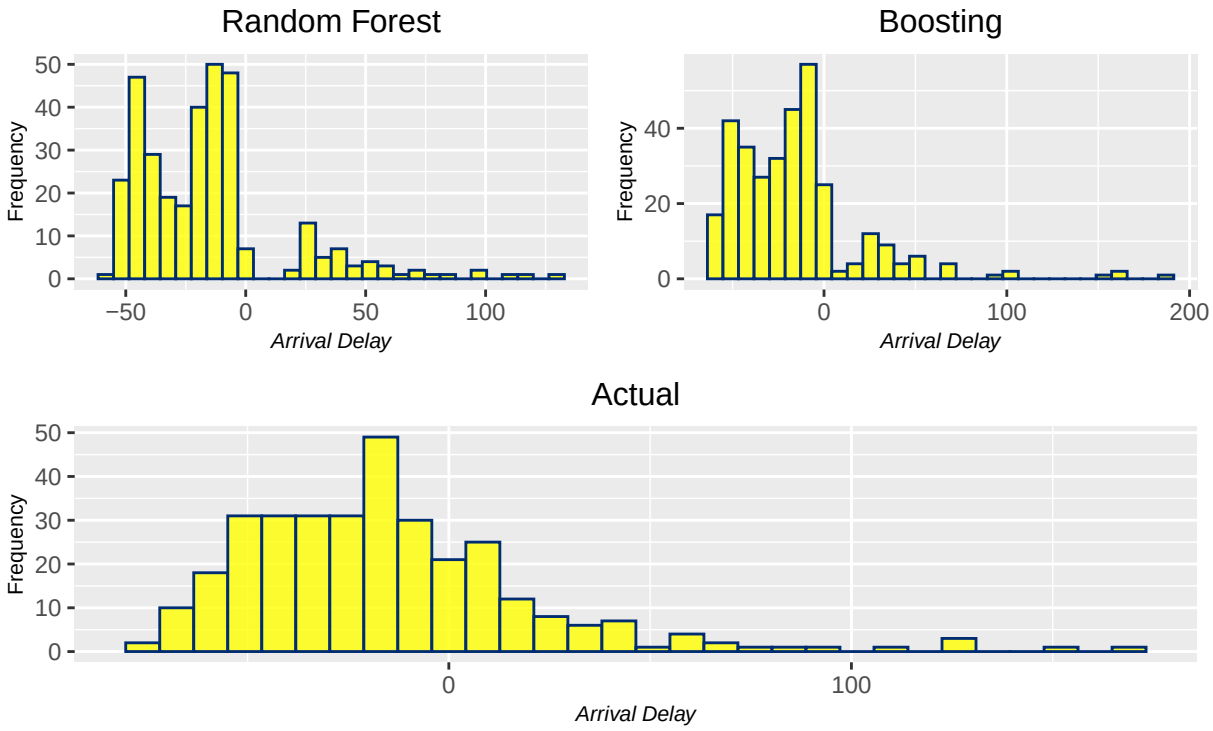

# grid.arrange(ptest_actual, ptest_rf, ptest_boost, ncol = 2,
#              width = c(2,1,1),
#              layout_matrix = rbind(c(1,2),
#                                    c(1, 3)))

trees_patchwork <- (ptest_rf | ptest_boost) / ptest_actual
trees_patchwork + plot_annotation(
  title = 'Comparing Distributions of ARR_DELAY',
  subtitle = 'Histograms of actual test values and tree-based predicted values'
)
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

Comparing Distributions of ARR_DELAY

Histograms of actual test values and tree−based predicted values




```
# test MSE calculations
rf.MSE <- sum((test$ARR_DELAY - yhat.rf)^2, na.rm=T)/length(test$ARR_DELAY)
rf.MSE
```

```
## [1] 155.0148
```

```
boost_MSE <- sum((test$ARR_DELAY-yhat.boost)^2, na.rm = T)/length(test$ARR_DELAY)
boost_MSE
```

```
## [1] 129.7965
```

Test Error Tables

```
customGreen = "#71CA97"
customRed = "#ff7f7f"

library(formattable)
```

```
##
## Attaching package: 'formattable'
```

```
## The following object is masked from 'package:bst':
##
##     gradient
```

```
## The following object is masked from 'package:MASS':
##
##     area
```

```
## The following object is masked from 'package:patchwork':
##
##     area
```

```
options(scipen = 5, digits = 4)
model.names <- c("Baseline Linear", "Selected Linear w/ Log-Transformed Predictors", "Selected Linear w/
model.types <- c("Multiple Linear Regression", "Multiple Linear Regression", "Multiple Linear Regressio

#model4.ints <- c("FALSE", "FALSE", "FALSE", "FALSE")

model.mse <- c(plain_linear_model_MSE, log_linear_MSE,
                   bc_adj_linear_model_MSE, gam_MSE,
                   gambc_MSE, rf.MSE, boost_MSE)
model.mse.char <- c("322.46", "333.90", "334.92", "312.30", "317.45", "155.01", "129.80")

#model4.cvmse_var <- c(mlr4_1_cv, mlr4_4_bc_cv, ridge.mom4.cvmse, gam4_bc_gcv)
#model4.cvmse <- c(2.284e+20, "25.66", 25.62, 25.79)


pctchange_1 <- round(-((log_linear_MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, digits = 4
pctchange_2 <- round(-((bc_adj_linear_model_MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, d
pctchange_3 <- round(-((gam_MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, digits = 4)
pctchange_4 <- round(-((gambc_MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, digits = 4)
pctchange_5 <- round(-((rf.MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, digits = 4)
pctchange_6 <- round(-((boost_MSE - plain_linear_model_MSE)/plain_linear_model_MSE)*100, digits = 4)


model.pctchange <- c("---", pctchange_1, pctchange_2, pctchange_3, pctchange_4, pctchange_5, pctchange_

errors.df <- data.frame(model.names,
                        model.types,
                        model.mse.char,
                       model.pctchange
                        )

 #model4.ints,
#model4.cvmse,
                        #model4.pctchange
```

```
#colnames(errors.df4) <- c("Model Name", "Model Type", "Interactions?", "Model MSE", "Model CV MSE", "M
errors.df
```

```
##                                    model.names                model.types
## 1                              Baseline Linear Multiple Linear Regression
## 2 Selected Linear w/ Log-Transformed Predictors Multiple Linear Regression
## 3                     Selected Linear w/ Box-Cox Multiple Linear Regression
## 4                                          GAM Generalized Additive Model
## 5                                GAM w/ Box-Cox Generalized Additive Model
## 6                                 Random Forest      Tree-Based Regression
## 7                                      Boosting     Tree-Based Regression
##   model.mse.char model.pctchange
## 1         322.46             ---
## 2         333.90         -3.5469
## 3         334.92          -3.865
## 4         312.30          3.1519
## 5         317.45          1.5523
## 6         155.01         51.9272
## 7         129.80         59.7479
```

```
mlr.model.names <- c("Baseline Linear", "Selected Linear w/ Log-Transformed Predictors", "Selected Linea
mlr.mse.char <- c("322.46", "333.90", "334.92")

mlr.error.df <- data.frame(mlr.model.names, mlr.mse.char)

formattable(mlr.error.df,
            col.names = c("Model Name", "Model MSE"),
            list(
  mlr.model.names = formatter("span", style = x ~ ifelse(x == "Baseline Linear",
    style(color = "purple", font.weight = "bold"), NA)),
  mlr.mse.char = formatter("span", style = x ~ ifelse(x == "322.46",
    style(color = "purple", font.weight = "bold"), NA))
            ))
```

Model Name

Model MSE

Baseline Linear

322.46

Selected Linear w/ Log-Transformed Predictors

333.90

Selected Linear w/ Box-Cox

334.92

```
gam.model.names <- c("GAM", "GAM w/ Box-Cox")
gam.mse.char <- c("312.30", "317.45")

gam.error.df <- data.frame(gam.model.names, gam.mse.char)

formattable(gam.error.df,
            col.names = c("Model Name", "Model MSE"),
```

```
            list(
  gam.model.names = formatter("span", style = x ~ ifelse(x == "GAM",
    style(color = "purple", font.weight = "bold"), NA)),
  gam.mse.char = formatter("span", style = x ~ ifelse(x == "312.30",
    style(color = "purple", font.weight = "bold"), NA))
            ))
```

Model Name

Model MSE

GAM

312.30

GAM w/ Box-Cox

317.45

```
tree.model.names <- c("Random Forest", "Boosting")
tree.mse.char <- c("155.01", "129.80")

tree.error.df <- data.frame(tree.model.names, tree.mse.char)

formattable(tree.error.df,
            col.names = c("Model Name", "Model MSE"),
            list(
  tree.model.names = formatter("span", style = x ~ ifelse(x == "Boosting",
    style(color = "purple", font.weight = "bold"), NA)),
  tree.mse.char = formatter("span", style = x ~ ifelse(x == "129.80",
    style(color = "purple", font.weight = "bold"), NA))
            ))
```

Model Name

Model MSE

Random Forest

155.01

Boosting

129.80

```
formattable(errors.df,
            col.names = c("Model Name", "Model Type", "Model MSE", "Model Percent Improvement"),
            list(
  model.names = formatter("span", style = x ~ ifelse(x == "Boosting",
    style(color = "purple", font.weight = "bold"), NA)),
  model.mse.char = formatter("span", style = x ~ ifelse(x == "129.80",
    style(color = "purple", font.weight = "bold"), NA)),
  model.types = formatter("span", style = x ~ ifelse(x == "Tree-Based Regression ",
    style(color = "purple", font.weight = "bold"), NA)),
  model.pctchange = formatter("span",
                                  style = x ~ style(font.weight = "bold",
                                                    color = ifelse(x == "---", "black",
                                                                   ifelse(x > 0, customGreen, ifelse(x < 0
                                  x ~ icontext(ifelse(x>0, "arrow-up", "arrow-down"), x)
```

```
                                        )
))
```

Model Name

Model Type

Model MSE

Model Percent Improvement

Baseline Linear

Multiple Linear Regression

322.46

—

Selected Linear w/ Log-Transformed Predictors

Multiple Linear Regression

333.90

-3.5469

Selected Linear w/ Box-Cox

Multiple Linear Regression

334.92

-3.865

GAM

Generalized Additive Model

312.30

3.1519

GAM w/ Box-Cox

Generalized Additive Model

317.45

1.5523

Random Forest

Tree-Based Regression

155.01

51.9272

Boosting

Tree-Based Regression

129.80

59.7479