

Lab 4

Medical Imaging

IST 2020-2021

Consider the study of a sample with $T_1/T_2 = 700 \text{ ms} / 70 \text{ ms}$ by $^1\text{H-NMR}$, using the rotating reference frame and assuming on-resonance spins. Consider the equilibrium magnetisation vector $M_0 = [0; 0; 1]$ and simulate the evolution of the magnetisation vector (displaying the time course of each of its components) using a time step of 0.1 ms in the following conditions:

1. Excitation (ignoring relaxation) by a 90° flip angle B_1 pulse along $+x$ with duration 10 ms.
2. Relaxation following the 90° excitation simulated in 2., for an observation period of 490 ms.
3. Repeat 1. and 2. for a 60° flip angle.
4. Now repeat 3. over 10 consecutive cycles of excitation and relaxation (repetition time (TR) = 500 ms).

Bloch equations for excitation by B_1 along x in matrix form:

$$\begin{bmatrix} M_x(t_{n+1}) \\ M_y(t_{n+1}) \\ M_z(t_{n+1}) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma B_1 \Delta t) & \sin(\gamma B_1 \Delta t) \\ 0 & -\sin(\gamma B_1 \Delta t) & \cos(\gamma B_1 \Delta t) \end{bmatrix} \begin{bmatrix} M_x(t_n) \\ M_y(t_n) \\ M_z(t_n) \end{bmatrix}$$

Bloch equations for magnetisation relaxation in matrix form:

$$\begin{bmatrix} M_x(t_{n+1}) \\ M_y(t_{n+1}) \\ M_z(t_{n+1}) \end{bmatrix} = \begin{bmatrix} \exp\left\{-\frac{\Delta t}{T_2}\right\} & 0 & 0 \\ 0 & \exp\left\{-\frac{\Delta t}{T_2}\right\} & 0 \\ 0 & 0 & \exp\left\{-\frac{\Delta t}{T_1}\right\} \end{bmatrix} \begin{bmatrix} M_x(t_n) \\ M_y(t_n) \\ M_z(t_n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ M_0 \left(1 - \exp\left\{-\frac{\Delta t}{T_1}\right\}\right) \end{bmatrix}$$