Signal and Systems in Bioengineering

Master on Biomedical Engineering

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Space of Signals

Lab Session 1

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In this lab some basic concepts on vector spaces of signals are addressed, such as norms, inner product and linear combination.

1) **Synthetic data**
   a) Implement a function to generate a column vector containing a sine wave, $sin(2\pi f(t) t)$, with a growing frequency, $f(t)$ from $f(0) = f_1$ to $f(T) = f_2$. The inputs of the function are the duration, T in seconds, the frequencies, f_1 and f_2, in Hz and the sampling rate, f_s, in samples per second

$$x = chirpTone(T, f_1, f_2, f_s) \tag{1}$$

tk=kTs, Ts=1/fs

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   b) Listen the sound produced in the previous item with $f_1 = 100$, $f_2 = 2000$ and $f_s = 4000$ Hz using the MatLab function *soundsc*.
   c) Save the sound vector in an audio file to be read in normal audio players (use the MatLab function *audiowrite*).
   d) Generate a stereo audio signal simulating the siren sound of an ambulance passing at high speed.

2) **Real Data**
   a) Read the audio file "Let It Be.mp3" from the Data section of the webpage of the discipline, using the MatLab function *audioread*. Read carefully the help documentaion of this function.
   b) Compute the length of the file in seconds. Explain the approach you used.
   c) Reproduce the music backwards.
   d) Simulate a movement of the music from the left to the right ear.

3) **Vectors**
   a) Compute a vector of time points, $\mathbf{t} = \{t_k\}$ corresponding to a sampling rate of $f_s = 4000$ Hz in the interval $[0, T]$, with $T = 2$ sec.
   b) Generate three signals/vectors $\mathbf{p}_1 = \{p_1(t_k)\}$, $\mathbf{p}_2 = \{p_2(t_k)\}$ and $\mathbf{p}_2 = \{p_3(t_k)\}$ where
      i) $p_1(t_k) = \sin(2\pi f t_k)$
      ii) $p_2(t_k) = \sin(2\pi f t_k + \pi/4)$
      iii) $p_3(t_k) = \sin(2\pi f t_k) + t_k$

      where $f = 440$ Hz.
   c) Compute the norm of each vector and the angles between them. Are these vectors orthogonal? Why?

d) Compute the signal $\mathbf{z} = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2 + \gamma \mathbf{p}_3$ where $\alpha = 0.5$, $\beta = 1$ and $\gamma = 1.5$ and reproduce it with the MatLab function *soundsc* with the appropriated sampling frequency.

e) Compute the Graminian of the basis $[\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$.
What is the Graminian of a orthonormated basis ($\|\mathbf{p}_k\| = 1$ and $< \mathbf{p}_i, \mathbf{p}_j >= \delta(i-j)$)?

4) **Approximation and representation of vectors**
Consider a linear combination of the vectors $\mathbf{p}_k$

$$\hat{\mathbf{z}}(\mathbf{a}) = \sum_{k=1}^{3} a_k \mathbf{p}_k \tag{2}$$

and the following error vector

$$\mathbf{e}(\mathbf{a}) = \mathbf{z} - \hat{\mathbf{z}}(\mathbf{a}) \tag{3}$$

where $\mathbf{a} = [a_1, a_2, a_3]^T$ is the vector of coefficients and $\mathbf{z} = \{z_k\}$ is the column vector generated in 3.d). The goal is to solve the following optimization problem

$$\mathbf{a}^* = \arg\min_{\mathbf{a}} \|\mathbf{e}\|^2, \tag{4}$$

where $\mathbf{a}^*$ is the optimum vector of coefficients that minimizes the norm of the error or equivalently that maximizes the similarity between $\mathbf{z}$ and $\hat{\mathbf{z}}(\mathbf{a}^*)$. This means that $\hat{\mathbf{z}}(\mathbf{a}^*)$ is the closest representation of $\mathbf{z}$ using the basis vectors $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$.

a) Estimate $\mathbf{a}^*$
   i) using the orthogonality principle.

   ii) using the gradient method where the error defined in (3) may be re-written as follows

$$\mathbf{e}(\mathbf{a}) = \mathbf{z} - \Phi\mathbf{a}. \tag{5}$$

   Derive the matrix Φ, the expression of the square norm of error and the close form solution for $\mathbf{a}^*$.

   iii) Compare the results obtained using both methods.

b) Compute the signal $\mathbf{g} = \alpha \mathbf{p}_1 + \beta \mathbf{p}_2$ (α and β defined in 3.d)) and display the error surface, $\|\mathbf{e}(x,y)\|^2 = \|\mathbf{g} - x\mathbf{p}_1 - y\mathbf{p}_2\|_2^2$ for 100 points in the intervals $x \in [0,1]$ and $y \in [0.5, 1.5]$ with the MatLab function *mesh*. Display in the same graph the point $(x^*, y^*, \|\mathbf{e}(x^*, y^*)\|)$ where

$$(x^*, y^*)^T = \arg\min_{(x,y)^T} \|\mathbf{e}(x,y)\|^2 \tag{6}$$

Comment.

c) Repeat 4.b) with the following vectors
   i) $p_1(t_k) = \sin(2\pi f t_k)$
   ii) $p_2(t_k) = \sin(2\pi (2f) t_k)$
   What are the main differences to the results obtained in 4.b)? Comment.