Signal and Systems in Bioengineering

Master on Biomedical Engineering

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Spectral Analysis and DFT

Lab Session 3

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This work is about the *Discrete Fourier Transform* (DFT) defined as follows,

$$X(k) = X(\omega_k) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}|_{\omega_k = \frac{2\pi}{N}k} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \tag{1}$$

where $\omega_k = \frac{2\pi}{N}k$ and $k = 0, .., M-1$ with $M \geq N$.

Part I

1) Build a MatLab function to compute a M-length DFT of a N-length signal, $x(n)$, $X = matDFT(x, M)$, using matrix notation assuming that $M \geq N$.

2) i) Generate a pure A tone ($f = 440$ Hz) with a sampling frequency, $f_s = 4000$ Hz, and duration of $T = 1$ second.
ii) Compute the DFT of this signal using the function obtained in 1).
iii) Comment the index of the larger coefficient and correlate it with the original A tone frequency.

3) Compute the DFT using the *FFT* algorithm. Use the *tic* and *toc* functions from MatLab to estimate and compare the processing times of the DFT using the definition, in (1), and the FFT algorithm, using the MatLab function *fft()*.
(Take into account that the length of the signal should be as close as possible of $N = 2^\mu$ where μ is an integer).

Part II

1) Generate a chirp signal, $x(n) = sin(2\pi f(n)t(n))$, with $0 \leq t(n) \leq 1$ seconds and $200Hz \leq f(n) \leq 1000Hz$ with a sampling frequency $f_s = 4000$ Hz and compute its DFT using the FFT algorithm. Represent its module and comment.

2) Explain the spectrum obtained in the previous item referring in particular the band limits of the signal. Justify analytically.

3) Compute the spectrogram of $x(n)$ and comment.

4) Compare the spectra of the signals $f(t) = sin(2\pi f_1 t) + sin(2\pi f_2 t)$ and $g(t) = sin(2\pi f_1 t) \times sin(2\pi f_2 t)$ with $f_1 = 1000$ Hz and $f_2 = 1500$ Hz for $0 \leq t \leq 1$ sec and $f_s = 4000$ Hz.
Explain the differences.

Part III

Consider the sine wave $x(t) = sin(2\pi ft)$ where $f = 100$ Hz and a modulated a carrier $c(t) = cos(2\pi f_0)$ with frequency $f_0 = 1000$ Hz. In the next simulations use a sampling frequency of $f_s = 2400$ Hz.

1) Compute the spectrum of $x(t)$, $X(f)$. Display its magnitude, $|X(f)|$. Comment.

2) Compute and display the spectrum of the *Amplitude Modulated* (AM) signal

$$x_{am}(t) = (1 + \alpha x(t))c(t) \tag{2}$$

   for $\alpha = 0.1$ and $\alpha = 1$. Comment. In particular refer the relation between the spectra of $x(t)$ and $x_{am}(t)$

3) Compute and display the spectrum of the *Frequency modulation* (FM) signal $c(t) = cos(2\pi f_0(1 + \alpha x(t))t)$ for $\alpha = 0.1$ and $\alpha = 1$. Comment and compare the results with the ones obtained in the previous item.

4) Comment the main differences between both modulation methods.