Circuit Theory and Electronics Fundamentals

Engineering Physics

Lab 1: Circuit Analysis

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1 Introduction

The main goal of this work is to analyze a circuit using 3 methods: mesh (Section 2) and nodal (Section 3) analysis and using an *ngspice* simulation (Section 5).[1]

Our circuit (Figure 1) consists of 7 resistors, 2 voltage sources - 1 independent, and 1 current controlled dependent one, and 2 current sources - 1 independent, and 1 voltage controlled dependent one. There are 4 meshes and 8 nodes. Voltage source V_a is in series with resistor 1; Resistor 2 and current source I_b, and resistors 6 and 7 ares also connected in series.

[1]To get the values of resistance for each resistor and other data, we used the number 96514 (the lowest of the three student Id's).

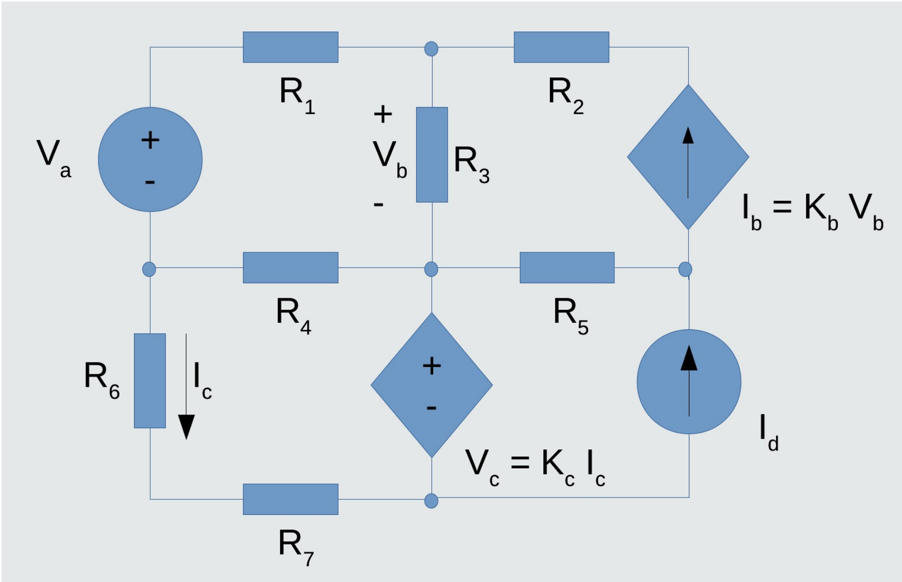

Figure 1: Circuit

We wrote a system of equations for each theoretical method (mesh and nodal), from Kirchhoff and Ohm's Laws. To get the solutions for these analysis, we used *octave*, which solved our systems of equations efficiently and gave us all currents and voltages for each resistor. We then used *ngspice* to get a simulation of this circuit, expecting to obtain the same results in all three methods. We compared the results from different methods in the conclusion (Section 6).

2 Mesh Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically using the Mesh Current Method. This method is based on Kirchhoff's Voltage Law (KVL), which states that the sum of all the potencial differences around the loop must be equal to zero.
The first step is to identify the meshes (in this case, there are four) and assign a current variable to each one (I_1, I_2, I_3, I_4), using a consistent direction (in the four meshes we chose counter-clockwise). This can be seen in Figure 2.

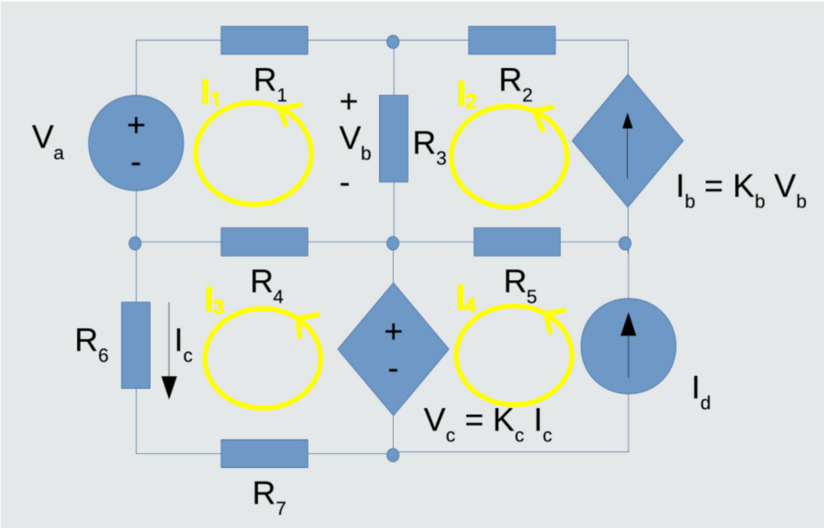

Figure 2: Mesh currents

The second step is to write Kirchhoff's Voltage Law equations around each mesh; however, the value of I_d is already known, so there's no need to write an equation around the fourth mesh. This being said, and knowing by Ohm's Law that

$$V = RI \tag{1}$$

the equation for the first mesh is:

$$V_a + R_4(I_1 - I_3) + R_3(I_1 - I_2) + R_1 I_1 = 0 \Leftrightarrow I_1(R_1 + R_3 + R_4) - I_2 R_3 - I_3 R_4 = -V_a \tag{2}$$

In the second mesh, noticing that $I_b = I_2$, V_b can be written in two ways:

$$V_b = \frac{I_2}{K_b}$$

and

$$V_b = R_3(I_2 - I_1)$$

Therefore:

$$\frac{I_2}{K_b} = R_3(I_2 - I_1) \Leftrightarrow I_2 = K_b R_3(I_2 - I_1) \Leftrightarrow -I_1 K_b R_3 + I_2(K_b R_3 - 1) = 0 \tag{3}$$

And finally, for the third mesh:

$$R_6 I_3 + R_7 I_3 - V_c + R_4(I_3 - I_1) = 0 \tag{4}$$

and since $V_c = K_c I_c$ and $I_c = I_3$:

$$-I_1 R_4 + I_3(R_4 + R_6 + R_7 - K_c) = 0 \tag{5}$$

The next step is to solve the resulting system of equations for the mesh currents:

$$\begin{pmatrix} R_1 + R_3 + R_4 & -R_3 & -R_4 \\ -K_b R_3 & K_b R_3 - 1 & 0 \\ -R_4 & 0 & R_4 + R_6 + R_7 - K_c \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -V_a \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

3 Nodal Analysis

3.1 Systematic procedure

In this circuit, there are 8 nodes. Therefore, to solve the circuit we need to find N (number of nodes) -1 equations, which in this case means we must find 7 independent equations. Firstly, we must choose our ground node (node which, by convention, has voltage 0). Typically the node chosen as the ground node is the one that connects to a voltage source so we chose the node that connects the voltage source V_a and the resistors 4 and 6. Next, we numbered the remaining nodes as shown in Figure 3.

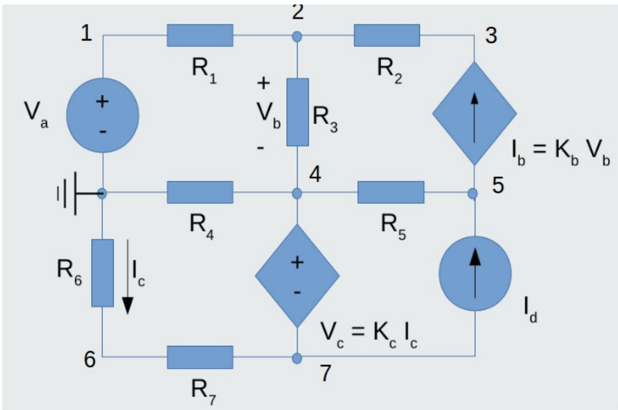

Figure 3: Numbered nodes and ground node

3.2 Discovering the equations

Now to discover the equations for each node we need to use Kirchhoff's Current Law, which states that the sum of all the currents entering and leaving a node must be equal to zero. For simplicity instead of using directly the currents we will also use Ohm's Law.
Starting off at node 1, we will consider that current is leaving node 1 towards the voltage source and that current is entering the node from the resistor 1. That leaves us with the following equation:

$$-i_a + \frac{V_2 - V_1}{R_1} = 0 \Leftrightarrow i_a = \frac{V_2 - V_1}{R_1}, \tag{7}$$

where V_1 and V_2 are the voltages at nodes 1 and 2 and i_a is the current that flows through the voltage source.
Moreover, we know the voltage provided by the voltage source is equal to V_a. Taking into consideration the voltage at the nodes 1 and ground we get:

$$V_a = V_1 - V_{GND} \Leftrightarrow V_a = V_1 \tag{8}$$

where V_{GND} is the voltage at the ground node (by convention = 0).
Now looking at node 2. We consider all currents entering the node. From that we get:

$$\frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} + \frac{V_4 - V_2}{R_3} = 0 \tag{9}$$

Moving along to node 3. Here we have a current that is already named, I_b. We consider the current moving as is indicated by the symbol, so entering node 3 and then we also consider that current is leaving node 3 toward the resistor 2. This leaves us with:

$$I_b - \left(\frac{V_3 - V_2}{R_2}\right) = 0 \Leftrightarrow I_b = \frac{V_3 - V_2}{R_2} \tag{10}$$

Next on the list is node 4. Here we consider that current is leaving the node towards resistor 5 and the current dependant voltage source V_c and entering the node from resistors 3 and 4. The KCL equation becomes:

$$-i_c - \left(\frac{V_4 - V_5}{R_5}\right) + \frac{V_2 - V_4}{R_3} + \frac{V_{GND} - V_4}{R_4} = 0 \Leftrightarrow i_c + \frac{V_4 - V_5}{R_5} + \frac{V_4 - V_2}{R_3} + \frac{V_4}{R_4} = 0 \tag{11}$$

where i_c is the current through the current dependant voltage source.
From nodes 2, 3 and 4 we can also extract one more equation. Because of the voltage dependant current source and the resistor 3 we know 2 things about V_b:

$$V_b = V_2 - V_4 \qquad V_b = \frac{I_b}{K_b}$$

Using equation (4) we can further develop this relation between the two equations.

$$V_b = \frac{I_b}{K_b} \Leftrightarrow V_b = \frac{V_3 - V_2}{R_2 K_b} \Leftrightarrow V_2 - V_4 = \frac{V_3 - V_2}{R_2 K_b} \Leftrightarrow V_2 - V_4 + \frac{V_2 - V_3}{R_2 K_b} = 0 \tag{12}$$

Now onto node 5. We again use the already named voltage dependant current, I_b (that is defined in equation 4) and current source I_d. We define the current direction in accordance with their symbols (I_b is leaving the node and I_d is entering). We also have a resistor, resistor 5, and we assume current is leaving the resistor and entering the node.

$$-I_b + I_d + \frac{V_4 - V_5}{R_5} = 0 \Leftrightarrow \frac{V_3 - V_2}{R_2} + \frac{V_5 - V_4}{R_5} = I_d \tag{13}$$

The next node is node 6. Here we have a named current entering the node, from resistor 6 called I_c, and we assume that current is leaving the node towards resistor 7.

$$I_c - \left(\frac{V_6 - V_7}{R_7}\right) = 0 \Leftrightarrow \frac{V_6 - V_7}{R_7} = I_c \tag{14}$$

We also know that

$$I_c = \frac{V_{GND} - V_6}{R_6} = \frac{-V_6}{R_6} \tag{15}$$

Combining this with equation (8) we get:

$$\frac{V_7 - V_6}{R_7} - \frac{V_6}{R_6} = 0 \tag{16}$$

The final node is node 7. Here we again come across named current I_d and current dependant voltage source V_c. Current I_d is leaving the node and current is entering from the current dependant voltage source and resistor 7. The KCL equation becomes:

$$i_c + \frac{V_6 - V_7}{R_7} - I_d = 0 \Leftrightarrow i_c + \frac{V_6 - V_7}{R_7} = I_d \tag{17}$$

Since we have common terms we can combine equations (5) and (11) and get:

$$I_d + \frac{V_7 - V_6}{R_7} + \frac{V_4 - V_5}{R_5} + \frac{V_4 - V_2}{R_3} + \frac{V_4}{R_4} = 0 \tag{18}$$

At this point, we have 7 variables (the voltages at the 7 nodes) and only 6 equations. However, we know that:

$$V_c = V_4 - V_7 \qquad V_c = K_c I_c$$

Combining this fact with equation (9) we get:

$$V_4 - V_7 + \frac{K_c V_6}{R_6} = 0 \tag{19}$$

The next step is to solve the system of equations resulting from equatios 2, 3, 6, 7, 10, 12 and 13 for the node voltages:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{R_1} & -\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} & \frac{1}{R_2} & \frac{1}{R_3} & 0 & 0 & 0 \\ 0 & 1 + \frac{1}{R_2 K_b} & -\frac{1}{R_2 K_b} & -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_3} & 0 & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_5} & -\frac{1}{R_7} & \frac{1}{R_7} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & -\frac{1}{R_5} & \frac{1}{R_5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_6} - \frac{1}{R_7} & \frac{1}{R_7} \\ 0 & 0 & 0 & 1 & 0 & \frac{K_c}{R_6} & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{pmatrix} = \begin{pmatrix} V_a \\ 0 \\ 0 \\ -I_d \\ I_d \\ 0 \\ 0 \end{pmatrix} \tag{20}$$

4 *Octave* Results

Putting both systems of equations in *Octave*, we get the solutions for mesh currents in the first case, and node voltages in the second. From this and Ohm's law we get the following tables for resistors' voltages and currents. The last two lines of both tables pertain to the dependent sources, where V_c refers to the current dependent voltage source and I_b to the voltage dependent current source.

4.1 Mesh Analysis

Solution (mA): $\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -0.233449 \\ -0.244673 \\ 0.992188 \end{pmatrix}$

Table 1: Mesh Analysis results

	Voltage (V)	Current (mA)
R_1	-0.239529	-0.233449
R_2	-0.498337	-0.244673
R_3	-0.034777	-0.011224
R_4	-5.028594	-1.225637
R_5	-4.050308	-1.290879
R_6	2.004013	0.992188
R_7	1.030260	0.992188
V_c	8.062868	0.992188
I_b	-0.034777	-0.244673

4.2 Nodal Analysis

$$\text{Solution (V):} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{pmatrix} = \begin{pmatrix} 5.233347 \\ 4.993818 \\ 4.495481 \\ 5.028594 \\ 9.078902 \\ -2.004013 \\ -3.034274 \end{pmatrix}$$

Table 2: Nodal Analysis results

	Voltage (V)	Current (mA)
R_1	-0.239529	-0.233449
R_2	-0.498337	-0.244673
R_3	-0.034777	-0.011224
R_4	-5.028594	-1.225637
R_5	-4.050308	-1.290879
R_6	2.004013	0.992188
R_7	1.030260	0.992188
V_c	8.062868	0.992188
I_b	-0.034777	-0.244673

5 Simulation

To make this simulation, we listed all components that make up the circuit. However, when declaring a current controlled voltage source (V_c, dependent on I_c), we need to find a way to access this current. For this, we need a voltage source where there isn't one which would sense the current I_c. Therefore, we created a fictitious one. This new source has voltage 0V and its sole purpose is to sense I_c, so it is in series with both resistors 6 and 7, between them (so their current should be the same). This makes node 6 connect R_6 to the positive terminal of this new source and create a new node that connects its negative terminal to R_7 (node 7). This makes the previous node 7 (that connects R_7 to I_d and V_c) be called node 8. In Table 3, we can see the operating point analysis results from the *ngspice* simulation. g1[i] corresponds to the current I_b; i1 is I_d and the voltages correspond to each node. As we can see the values obtained through this analysis are exactly the same as the ones derived from theoretical analysis.

Name	Value [A or V]
@g1[i]	-2.44673e-04
@i1[current]	1.046207e-03
@r1[i]	2.334490e-04
@r2[i]	2.446726e-04
@r3[i]	-1.12237e-05
@r4[i]	-1.22564e-03
@r5[i]	-1.29088e-03
@r6[i]	9.921880e-04
@r7[i]	9.921880e-04
v(1)	5.233347e+00
v(2)	4.993818e+00
v(3)	4.495481e+00
v(4)	5.028594e+00
v(5)	9.078902e+00
v(6)	-2.00401e+00
v(7)	-2.00401e+00
v(8)	-3.03427e+00

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

6 Conclusion

In this laboratory assignment the objective of analysing the circuit has been achieved.
The results obtained by *octave* in both the nodal and mesh theoretical analysis matched each other. The simulation results matched the theoretical results. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. Of these three methods, nodal analysis was the most prone to error for its high number of equations (7) compared with mesh analysis (3). The simulation was simple and quite effective. With very few commands we got the same results as the lengthier theoretical analysis.