Circuit Theory and Electronics Fundamentals

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Lab1: Circuit analysis methods

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1 Introduction

The objective of this laboratory assignment is to study a circuit with four elementary meshes and eight nodes. This circuit contains an independent voltage source V_a, an independent current source I_d, a voltage source dependent from a current V_c and a current source dependent from a voltage I_b. Besides this, it has seven resistors, R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.
In Section 2, a theoretical analysis of the circuit is presented, using the mesh method and the node method, in Octave. In Section 5, the circuit is analysed by simulation using ngspice, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 6.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using the mesh method and the node method.
In the first one, we define a current for each elementary mesh, which is a loop containing no other loops, with an arbitrary direction. These currents will be our unknown values and

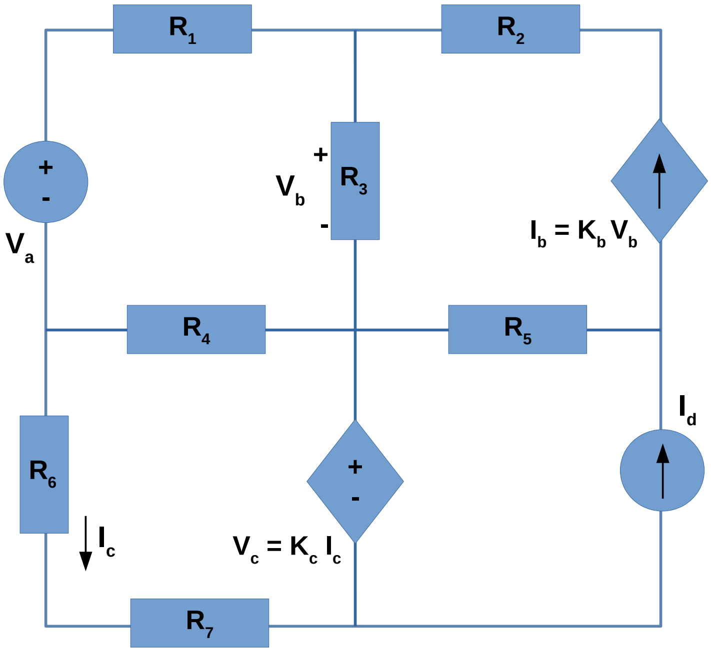

Figure 1: Circuit containing voltage sources, current sources and resistors.

the objetive is to compute them, using the Ohm's Law (V = RI) and Kirchhoff Voltage Law. Therefore, in this circuit we wil have four currents to compute, so we have to find four equations to solve this problem.
In the second one, each circuit node have a defined voltage. We also choose a node to have a null voltage. Then, we use Ohm's Law and Kirchhoff Current Law in nodes that are not connected to a voltage source. In the end, we write additional equations for the nodes related by voltage sources. Consequently, we will have an equal number of nodes and equations. To calculate the unkown voltages and currents, we use the following values (Resistances in kOhm, voltages in Volts, currents in mA, K_b in S and K_c in Ohms):

Table 1: Given values.

R_1	1,00332071212
R_2	2,04460853047
R_3	3,08291730437
R_4	4,16061678649
R_5	3,04022345043
R_6	2,06711403452
R_7	1,03302701196
V_a	5,13988034104
I_d	1,02475824097
K_b	7,0544535009 x 10^{-3}
K_c	8,16113797582 x 10^{3}

3 Mesh method

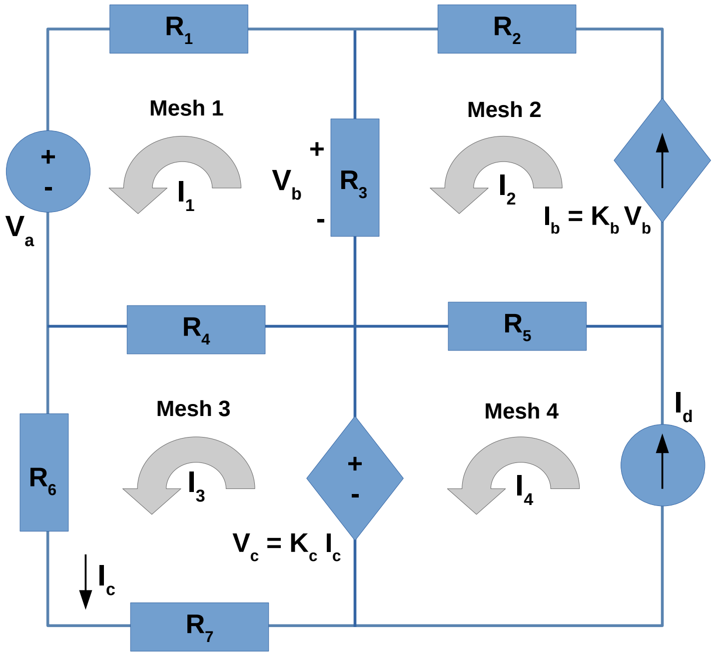

Figure 2: Circuit with current defined in each mesh.

In Figure 2, we defined a current for each circuit essencial loop. As said before, we will have four equations, because we need to compute four currents (I_1, I_2, I_3 and I_4). In this analysis, we consider currents flowing counter clockwise as positive values and currents flowing clockwise as negative values, using KVL.
Starting with mesh 3, we can see by inspection that $I_c = I_3$ and that R_3 depends on I_1 and I_3, which have opposite directions. The other 3 components depend only on I_3. In the voltage source, the current flows to the positive side to the negative, so it is not aligned with the direction we selected. Hence, we must consider it negative. So we have this equation:

$$(I_3 - I_1)R_4 + I_3R_7 + I_3R_6 - K_cI_3 = 0 \quad (1)$$

In mesh 1, the voltage source is aligned with the selected direction, so we consider it positive. The current in R_1 is only dependent from I_1, which we can see by inspection, and R_3 and R_4 depend on two different currents with opposite directions, being I_1 the positive one for this loop. Then we have to switch the signal in R_3 because the negative and opposite sides are not aligned with the current we estimated for this loop. So we have:

$$(I_1 - I_3)R_4 + V_a + R_3(I_2 - I_1) + R_1I_1 = 0 \quad (2)$$

We already have two equations, but we are going to need two additional ones, since we can not use this method in loops with current sources. We can get these two equations by inspection of the two current sources. Starting with I_b, the dependent source, we already know the relation between this source and V_b, so we only have to write this term as a function of resistances and currents, as we have done previously on the equation for mesh 1. So we can write:

$$I_2 = K_bR_3(I_2 - I_1) \quad (3)$$

Finishing with the simplest equation, we can look at the fourth mesh to verify that:

$$I_4 = I_d \tag{4}$$

All these equations can be transformed in a matricial system as it is shown here:

$$\begin{bmatrix} -R_4 & 0 & R_6+R_4+R_7-K_c & 0 \\ R_4-R_3+R_1 & R_3 & -R_4 & 0 \\ K_bR_3 & 1-K_bR_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -V_a \\ 0 \\ I_d \end{bmatrix}$$

Solving this system using Ocatve and the values given, we have obtained this values:

Table 2: Meshes table.

Meshes currents	Values obtained (Ampers)
I_1	2,00
I_2	1,50
I_3	1,50
I_4	3,00

4 Node method

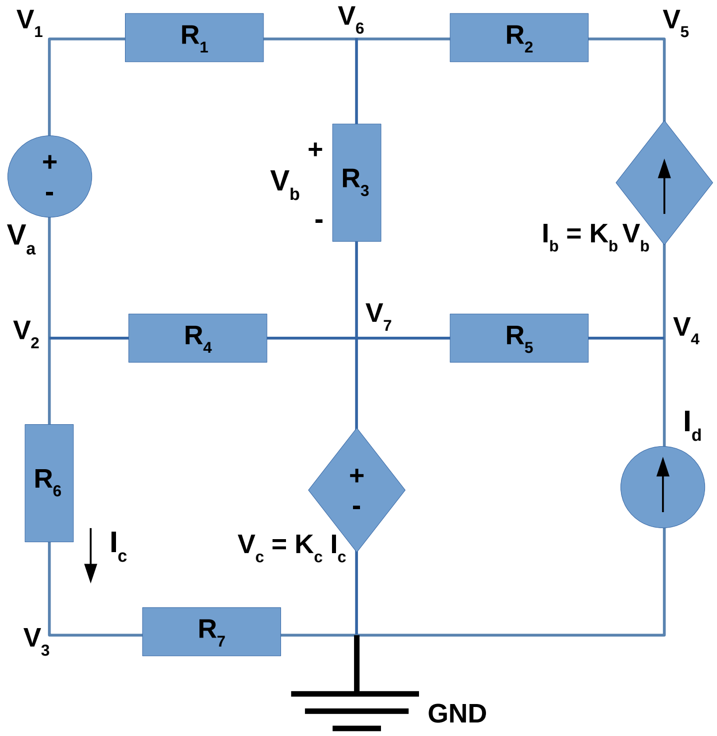

Figure 3: Circuit with voltages defined in each node.

In Figure 3, we defined a voltage for each circuit node and a node with null voltage. As said before, we will have seven equations, because we need to compute seven voltages (V_1, V_2, V_3, V_4, V_5, V_6 and V_7). In this analysis, we consider currents diverging from the node as positive values and currents converging as negative values, using KCL.
Starting with node 3, for this node, we considered all currents diverging, so we have this equation:

$$\frac{V_3 - V_2}{R_6} + \frac{V_3}{R_7} = 0 \tag{5}$$

In node 4, only I_d is converging. V_B is equal to V_6 - V_7, due to the direction represented in the circuit for R_3 voltage:

$$\frac{V_4 - V_7}{R_5} - I_d + K_b(V_6 - V_7) = 0 \tag{6}$$

Moving on to node 5, in which only I_b is converging, having this equation:

$$\frac{V_5 - V_6}{R_2} - K_b(V_6 - V_7) = 0 \tag{7}$$

On the other side, in node 6, we considered all currents diverging:

$$\frac{V_6 - V_7}{R_3} + \frac{V_6 - V_5}{R_2} + \frac{V_6 - V_1}{R_1} = 0 \tag{8}$$

The next equation establish the relation between nodes 1 and 2 voltages:

$$V_1 - V_2 = V_a \tag{9}$$

The relation between GND and node 7 is showed in the next equation:

$$V_7 = \frac{K_c(V_2 - V_3)}{R_6} \tag{10}$$

Since there is still an equation missing, we considered a super node containing V_a and R_6 and all currents diverging:

$$\frac{V_1 - V_6}{R_1} + \frac{V_2 - V_7}{R_4} + \frac{V_3}{R_7} = 0 \tag{11}$$

All this equations can be transformed in a matricial system as it is showed here:

$$\begin{bmatrix} 0 & -\frac{1}{R_6} & \frac{1}{R_6} + \frac{1}{R_7} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_5} & 0 & K_b & -K_b - \frac{1}{R_5} \\ 0 & 0 & 0 & 0 & \frac{1}{R_2} & -K_b - \frac{1}{R_2} & K_b \\ -\frac{1}{R_1} & 0 & 0 & 0 & -\frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_c}{R_6} & -\frac{K_c}{R_6} & 0 & 0 & 0 & -1 \\ \frac{1}{R_1} & \frac{1}{R_4} & \frac{1}{R_7} & 0 & 0 & -\frac{1}{R_1} & -\frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} 0 \\ I_d \\ 0 \\ 0 \\ V_a \\ 0 \\ 0 \end{bmatrix}$$

Solving this system using Ocatve and the values given, we have obtained this values:

Table 3: Nodes table.

Nodes Voltages	Values obtained (Volts)
V_1	2,00
V_2	1,50
V_3	1,50
V_4	3,00
V_5	2,00
V_6	10,00
V_7	7,00

5 Simulation Analysis

5.1 Currents and Voltages Analysis

The values obtained, by ngspice, for currents flowing in each resistance (Ampers) and nodes voltages (Volts) are showed in the following table:

Table 4: Values obtained by ngspice.

I_{R1} = I_1	-2,12029 x 10^{-4}
I_{R2}	-2,22248 x 10^{-4}
I_{R3}	-1,02191 x 10^{-5}
I_{R4}	-1,19181 x 10^{-3}
I_{R5}	1,247007 x 10^{-3}
I_{R6} = I_3	9,797777 x 10^{-4}
I_{R7}	9,797777 x 10^{-4}
I_d = I_4	1,024758 x 10^{-3}
I_b = I_2	-2,22248 x 10^{-4}
I_{Va}	-2,12029 x 10^{-4}
I_{Vc}	4,498058 x 10^{-5}
V_1	8,177329
V_2	3,037449
V_3	1,012137
V_4	11,78728
V_5	7,510185
V_6	7,964596
V_7	7,996101

6 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.

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