

**Referee’s report of “A fully labelled proof system for intuitionistic modal logics”,
by Sonia Marin, Marianela Morales and Lutz Straßburger**

The Kripke semantics of intuitionistic modal logic requires—unlike the one of classical modal logic—that there are two accessibility relations rather than just one. This phenomenon carries over to all proof systems based on semantics; in particular, to the labelled proof systems for intuitionistic modal logics, possibly with extensions, that have been studied in many of the papers the authors refer to, and elsewhere (see also below). This circumstance somewhat relativises the originality of the authors’ statement that “The main result of the paper is a ”labelled sequent system for intuitionistic modal logics such that there is not only one, but two relation symbols appearing in sequents: one for the accessibility relation associated with the Kripke semantics for normal modal logics and one for the preorder relation associated with the Kripke semantics for intuitionistic logic.” In fact, a birelational semantics of the type considered by the authors has already been studied in two papers by Božic, and Došen that unfortunately are missing from the paper’s bibliography, for example:

- Božic, M., Došen, K., Models for normal intuitionistic modal logics. *Studia Logica* 43, 217–245, 1984.

By the well-known method of labelled sequent calculi with frame properties that obey the geometric rule scheme, the authors obtain a sequent calculus with admissible structural rules. They then claim that their calculus is inspired by a ”multiple-conclusion nested sequent system a la Maehara”. Such a calculus in fact has appeared in a work which the authors cite as unpublished, but which seems to have been published in the meantime [Archive for Mathematical Logic (2019) 58:359–385], which reference needs to be updated accordingly. This aside, it is quite puzzling that the actual source for the calculus presented in the paper is an entirely different one. In fact, the rules of their calculus are identical (except for the ones that relate the two accessibility relations) to the rules for the intuitionistic-alethic fragment of the calculus for intuitionistic bimodal logic in this publication:

- Maffezioli, P., Naibo, A. & Negri, S., The Church-Fitch knowability paradox in the light of structural proof theory. *Synthese*, vol. 190, 2677–2716, 2013.

The frame conditions on page 1 of the authors’ paper, moreover, have already been treated, albeit in a different context, in yet another publication:

- Maffezioli, P. and A. Naibo, Proof theory of epistemic logic of programs. *Logic and Logical Philosophy*, vol. 23, 301–328, 2014.

Also the proofs of the structural properties closely follow the methodology of the paper by Maffezioli, Naibo & Negri. All this has to be said expressis verbis, and appropriate citations have to be added.

Perhaps even more importantly, the authors of the paper under review neglect an important caveat in the design of rules of contraction-free labelled sequent calculi: to guarantee that contraction be admissible one needs to ensure that a certain closure condition be satisfied, see

- Negri, S. Proof analysis in modal logic. *Journal of Philosophical Logic*, 34, 507–544, 2005.

In Section 7, if the rules G_{klmn} are added without their contracted instances, the calculus will not be complete. For example, in the classical case, Euclidean transitivity $\forall a, b, c(aRb \& aRc \rightarrow bRc)$ has the contracted instance $\forall a, b(aRb \rightarrow bRb)$ that allows for the derivation of $\Box(\Box A \rightarrow A)$. This, however, can hardly be derived without the rule's contracted instance.

On top of this, the authors' claim that labelled calculi "work best for logics with standard Kripke semantics" is utterly wrong: labelled calculi have been developed for several variants of Kripke semantics, such as Lewis's semantics for counterfactuals, neighbourhood semantics, Kripke semantics of impossible worlds, and so on.

In conclusion, the paper does not look highly original. The birelational semantics is not a novelty, nor a calculus internalising that semantics appears an original idea. The novelty seems limited to a detailed presentation for intuitionistic modal logic with a bridge condition between the two accessibility relations, the latter of which already considered in another paper albeit in a different context. All this does not make the paper a strong case for publication in a renowned journal such as JLC.

In any case, the authors need to make quite some effort in contrasting their own contributions with the existing literature, and be explicit and precise about the latter. Also the reason for preferring one semantics for intuitionistic modal logic over another should be discussed in detail, as otherwise the authors' work would rather look a routine exercise.