Optimal Initial Settlement Placement in CATAN®

A Game-Theory Approach

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AGENDA

1. Game Explanation
2. Problem Statement & Objective Function
3. Backward Induction
4. Memoization
5. Pruning Strategies
6. Results and Impact

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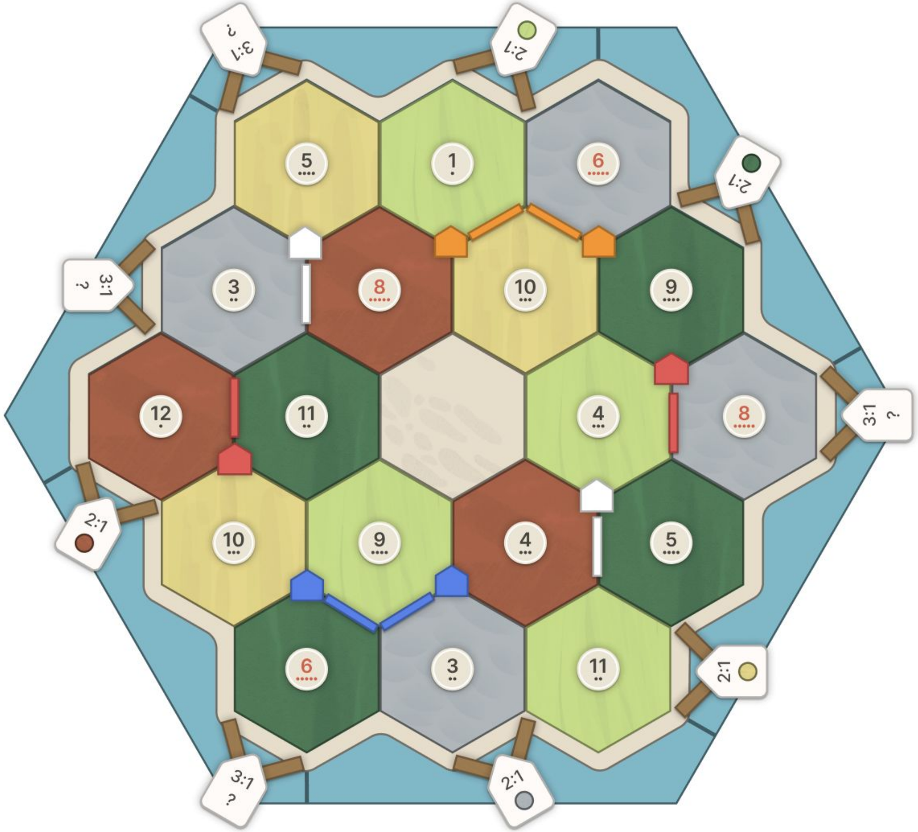
GAME EXPLANATION



BUILDING COSTS

Road

Settlement

+ 1 VP

City

+2 VP

Development Card

+? VP

Problem Statement

Optimize initial placement settlements in Catan

- Players settle in a snake order
- This is a dynamic programming setup.

Goal: Develop a quick solver for this optimal placement.

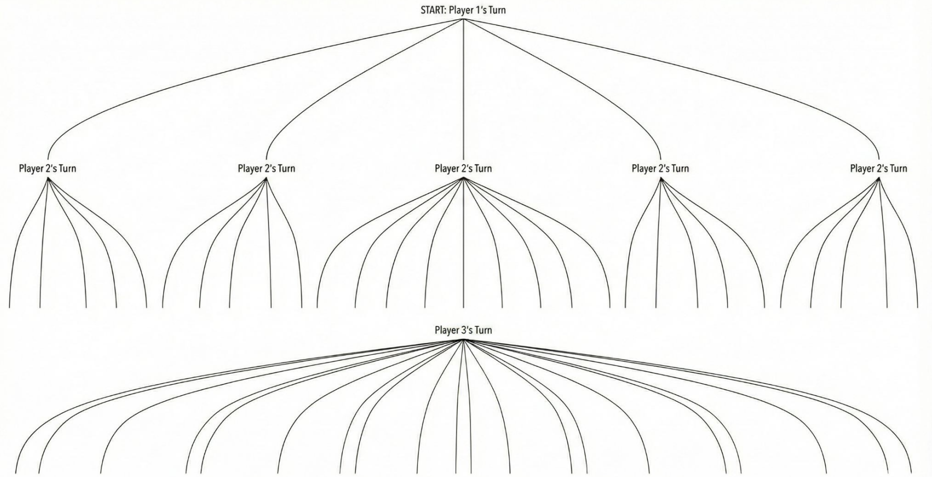
- Implement memoization and branch and bound techniques to speed up our solve time.

Objective Function

Each player want to maximize their reward which is defined by:

- Material Diversity
- Number Probability
- Number Diversity

We assume every player will place optimally in their turn (game-theory)



Backward Induction

We treat settlement placement as a deterministic dynamic-programming problem.

- Each player's optimal move depends on how all later players will respond.

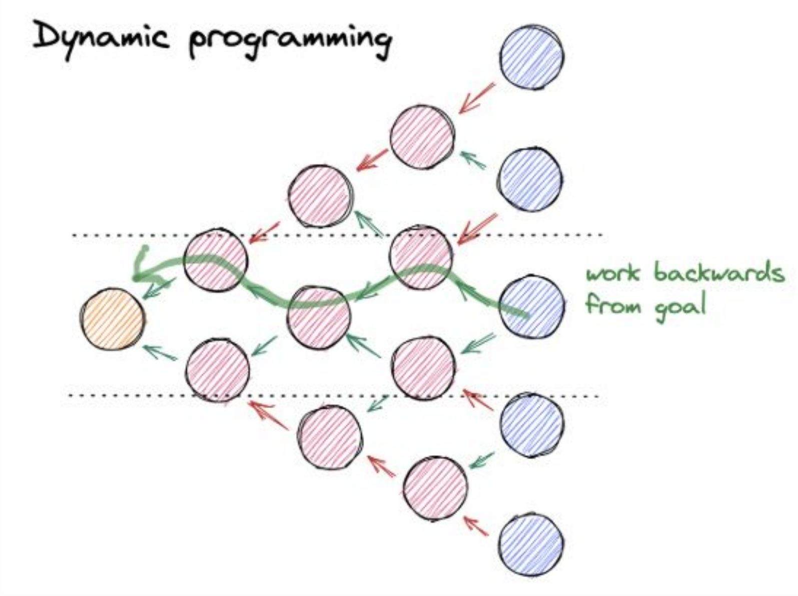
Backward induction:

- Finds the optimal move by solving the last step first
- Then working backward to choose the best action at each earlier step.

Backward Induction Flow

- For a given player, consider all feasible first-settlement vertices.
- Recursively solve for the next player (propagates forward until all 4 players place).

After recursion returns, place the player's second settlement optimally based on all later placements.
Keep the configuration with the best objective value.



Memoization

Many game states share the exact same configuration (occupied vertices and available options)

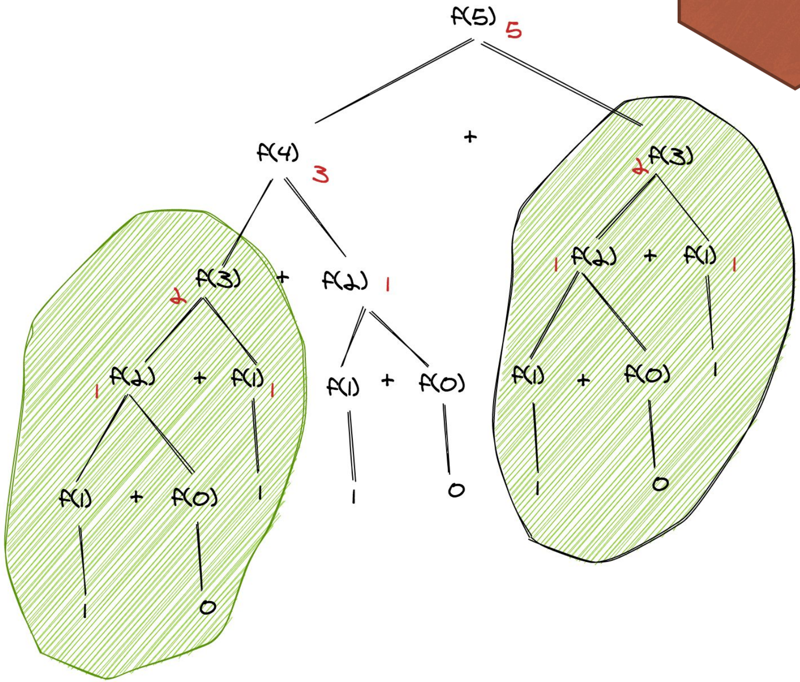
- The optimal resulting board is identical no matter how the state was reached.

Cache Results:
(current player, available vertices) -> best resulting board

How the Solver Uses Memoization

- Check if the current (player, available vertices) is in the memo.
- If yes → return stored best board.
- Otherwise: Try each feasible settlement placement.
- Recursively solve the next player's move.
- Evaluate and track the best resulting board.

Store the best board in the memo before returning it.



Pruning Strategies

Lower Bound

At each recursive node, we create a lower bound by the best complete solution found so far. Any branch that can't exceed this value is pruned

Upper Bound

We compute an upper bound on the max achievable value per branch. This is fully optimistic (relaxed), we say player 1 gets their 2 favorite settlements consecutively. If this UB is below LB, we skip the whole branch.

Move Ordering

We ensure we look at most promising branches first (greedy). This sets a high lower bound early, so we don't waste much time.

Results

Performance Metrics

Modality	Avg. Time	Min Time	Max Time	Avg. Calls	Speedup
Feasibility Pruning Only	444.16 s	441.64 s	454.26 s	5,634,937	1.00×
Feasibility + Memoization	79.20 s	78.61 s	80.28 s	983,839	5.61×
All Prunings (Full Solver)	0.33 s	0.05 s	0.69 s	569	1,361.88×

Key Findings

Model Strategies

Adding memoization reduces solve time by 5.61x
Adding UB pruning speeds up by 1,361x baseline approach

Correctness

All three models found the same optimal solutions across all 30 boards, indicating that neither efficiency technique eliminates optimal solutions

Consistency

Baseline model had the same amount of calls for each board ~5.6million and the full solver showed variable calls from just 65 to 1,110 calls

Future Steps

1

Competition-Aware Objectives

Players make moves not to maximize their own value, but to maximize the difference in value between themselves and all other players.

2

Stochastic Opponents

We make the decision of each player not assuming the other players act optimally each time. We maximize expected utility

3

New Strategies

Players can select different strategies to optimize for, such as a monopoly on certain resources, longest road, or biggest army. (Longest road and biggest army are subgame of Catan we did not touch on)

4

Full Game Features

Incorporate other game features like roads and ports that are also should be considered when we decide placement

THANK YOU!