Tema 7 - CN

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3) $f(x) = \sin(x)$
$\Delta = \left(-\frac{\pi}{2}, 0, \frac{\pi}{2}\right)$
$\left|P_2\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{6}\right)\right|$

Avem ca $x_1 = -\frac{\pi}{2}$ $\quad y_1 = f(x_1) = -1$
$x_2 = 0$ $\quad y_2 = f(x_2) = 0$
$x_3 = \frac{\pi}{2}$ $\quad y_3 = f(x_3) = 1$

I Metoda directă

$P_2(x) = a_1 + a_2 x + a_3 x^2$

$$\begin{cases} P_2\left(-\frac{\pi}{2}\right) = -1 \\ P_2(0) = 0 \\ P_2\left(\frac{\pi}{2}\right) = 1 \end{cases} \Rightarrow \begin{cases} a_1 - \frac{\pi}{2} a_2 + \frac{\pi^2}{4} a_3 = -1 \\ a_1 = 0 \\ a_1 + \frac{\pi}{2} a_2 + \frac{\pi^2}{4} a_3 = 1 \end{cases} \Rightarrow$$

$$\begin{cases} a_1 = 0 \\ a_2 = 2/\pi \\ a_3 = 0 \end{cases} \Rightarrow P_2(x) = \frac{2}{\pi} x$$

II Metoda Lagrange

$P_2(x) = L_{21}(x) y_1 + L_{22}(x) y_2 + L_{23}(x) y_3$ unde

$$L_{21}(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_2-x_3)} = \frac{x\left(x-\frac{\pi}{2}\right)}{-\frac{\pi}{2}\cdot\left(-\frac{\pi}{2}\right)} = \frac{x^2 - x\frac{\pi}{2}}{\frac{\pi^2}{4}} =$$

$$= \frac{\frac{2x^2 - \pi x}{2}}{\frac{\pi^2}{4}} = \frac{2x^2 - \pi x}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{\pi^2} = \frac{4x^2 - 2\pi x}{\pi^2}$$

$$L_{22}(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} = \frac{\left(x+\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)}{\frac{\pi}{2}\cdot\left(-\frac{\pi}{2}\right)} = \frac{x^2-\frac{\pi^2}{4}}{-\frac{\pi^2}{4}} =$$

$$= \frac{-4x^2+\pi^2}{\pi^2}$$

$$L_{23}(x) = \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)} = \frac{\left(x+\frac{\pi}{2}\right)x}{\pi\cdot\frac{\pi}{2}} = \frac{\frac{2x^2}{2}+\frac{\pi}{2}x}{\frac{\pi^2}{2}} = \frac{2x^2+\pi x}{\pi^2}$$

$$P_2(x) = \frac{-4x^2+2\pi x}{\pi^2} + 0 + \frac{+\cancel{2}x^2+\pi x}{\pi^2} = \frac{-2x^2+3\pi x}{\pi^2}.$$

III. Metodo Newton.

$$P_2(x) = C_1 + C_2(x-x_1) + C_3(x-x_1)(x-x_2)$$

$$\begin{cases} P_2\left(-\frac{\pi}{2}\right) = -1 \\ P_2(0) = 0 \\ P_2\left(\frac{\pi}{2}\right) = 1 \end{cases} \Rightarrow \begin{cases} C_1 = -1 \\ C_1 + C_2\frac{\pi}{2} = 0 \\ C_1 + C_2\pi + C_3\cdot\pi\cdot\frac{\pi}{2} = 1. \end{cases} \Rightarrow \begin{cases} C_1 = -1 \\ C_2 = 2/\pi \\ C_3 = 0 \end{cases}$$

$$P_2(x) = -1 + \frac{2}{\pi}\left(x+\frac{\pi}{2}\right) = -1 + \frac{2}{\pi}x + 1 = \frac{2}{\pi}x$$

IV. Metodo Newton cu diferente divizate.

$$f[x_1] = y_1 = -1 \qquad f[x_2] = 0 \qquad f[x_3] = 1.$$

$$f[x_1x_2] = \frac{f[x_2]-f[x_1]}{x_2-x_1} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$f[x_2x_3] = \frac{f[x_3]-f[x_2]}{x_3-x_2} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$f[x_1x_2x_3] = \frac{f[x_2x_3]-f[x_1x_2]}{x_3-x_1} = 0$$

$$P_2(x) = f[x_1] + f[x_1 x_2](x - x_1) + f[x_1 x_2 x_3](x - x_1)(x - x_2)$$

$$= -1 + \frac{2}{\pi}\left(x + \frac{\pi}{2}\right) = -1 + \frac{2}{\pi}x + 1 = \frac{2x}{\pi}$$

Deci pentru toate cazurile ~~în toate metodele~~ mai puțin Lagrange. $P_2(x) = \frac{2}{\pi}x$. $\underline{\underline{\text{Not}}}$ $P_1(x)$

Și pentru metoda Lagrange $P_2(x) = \frac{-2x^2 + 3\pi x}{\pi^2}$ $\underline{\underline{\text{Not}}}$ $Q(x)$

Deci avem de estimat $\underbrace{\left|P\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{6}\right)\right|}_{n_1}$ și $\underbrace{\left|Q\left(\frac{\pi}{6}\right) - f\left(\frac{\pi}{6}\right)\right|}_{n_2}$

$$n_1 = \left|\frac{2}{\pi} \cdot \frac{\pi}{6} - \frac{1}{2}\right| = \left|\frac{1}{3} - \frac{1}{2}\right| = \frac{1}{6} = 0{,}166 = 16{,}66\%$$

$$n_2 = \left|\frac{-2 \cdot \frac{\pi^2}{36} + 3\pi \frac{\pi}{6}}{\pi^2} - \frac{1}{2}\right| = \left|-\frac{1}{18} + \frac{1}{2} - \frac{1}{2}\right| = \frac{1}{18} = 0{,}055 = 5{,}55\%.$$