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TEMA #8

- CN -

1) $f(x) = \sin(x)$

$\Delta = \left(-\frac{\pi}{2}, 0, \frac{\pi}{2}\right)$

a) Metoda Neville: Avem nodurile $x_1 = -\frac{\pi}{2}$, $x_2 = 0$, $x_3 = \frac{\pi}{2}$

$P_1(x) = f(x_1) = -1$

$P_2(x) = f(x_2) = 0 \rightarrow P_{12}(x) = \dfrac{(x-x_1)P_2(x) - (x-x_2)P_1(x)}{x_2 - x_1} = (*)$

$P_3(x) = f(x_3) = 1 \rightarrow P_{23}(x) = \dfrac{(x-x_2)P_3(x) - (x-x_3)P_2(x)}{x_3 - x_2} = (**)$

$(*) = \dfrac{(x+\frac{\pi}{2}) \cdot 0 - (x-0)(-1)}{0 + \frac{\pi}{2}} = x \cdot \frac{2}{\pi}$

$(**) = \dfrac{(x-0) \cdot 1 - (x - \frac{\pi}{2}) \cdot 0}{\frac{\pi}{2} - 0} = x \cdot 2/\pi$

$P_{123}(x) = \dfrac{(x-x_1)P_{23}(x) - (x-x_3)P_{12}(x)}{x_3 - x_1} = \dfrac{(x+\frac{\pi}{2}) \cdot \frac{2x}{\pi} - (x - \frac{\pi}{2})\frac{2x}{\pi}}{\pi}$

$= \dfrac{\frac{2x+\pi}{\cancel{2}} \cdot \frac{\cancel{2}x}{\pi} + \frac{-2x+\pi}{\cancel{2}} \cdot \frac{\cancel{2}x}{\pi}}{\pi} = \frac{2}{\pi}x.$

Ex. 2

$f(x) = 3^x$

$\Delta = (-2, -1, 0, 1, 2)$

P_{m_1}

$x_1 = -2 \quad P_1(x) = f(x_1) = \frac{1}{9}$

$x_2 = -1 \quad P_2(x) = \frac{1}{3}$

$x_3 = 0 \quad P_3(x) = 1$

$x_4 = 1 \quad P_4(x) = 3$

$x_5 = 2 \quad P_5(x) = 9$

$P_{m_1 m_2}$

$P_{12}(x) = \frac{(x+2)\cdot\frac{1}{3} - (x+1)\cdot\frac{1}{9}}{-1+2} = \frac{1}{9}(2x+5)$

$P_{23}(x) = \frac{(x+1)\cdot 1 - (x-0)\cdot\frac{1}{3}}{0+1} = \frac{1}{3}(2x+3)$

$P_{34}(x) = \frac{(x-0)\cdot 3 - (x-1)\cdot 1}{1-0} = 2x+1$

$P_{45}(x) = \frac{(x-1)\cdot 9 - (x-2)\cdot 3}{2-1} = 6x-3.$

$P_{m_1 m_2 m_3}$

$2x^2+9x+6 \qquad 2x^2+5x$

$P_{123}(x) = \frac{(x+2)\cdot\frac{1}{3}\cdot(2x+3) - (x+1)\cdot\frac{1}{9}(2x+5)}{2}$

$= \frac{1}{18}(4x^2+16x+18) = 1{,}5$

$2x^2+3x+1 \qquad 2x^2+x+3.$

$P_{234}(x) = \frac{(x+1)\cdot(2x+1) - (x-1)\cdot\frac{1}{3}(2x+3)}{1+1}$

$= \frac{1}{6}(4x^2+10x) \quad 6x^2- = 1{,}8(3)$

$P_{345}(x) = \frac{(x-0)(6x-3) - (x-2)(2x+1)}{2-0} =$

$= \frac{1}{2}(4x^2 \; +2) = 1{,}5.$

$P_{m_1 m_2 m_3 m_4}$

$3(4x^3+10x^2+8x^2+20x) - (4x^3+16x^2+18x-4x^2-16x-18)$

$P_{1234}(x) = \frac{(x+2)\frac{1}{6}(4x^2+10x) - (x-1)\frac{1}{18}(4x^2+16x+18)}{1+2} =$

$= \frac{2{,}5\cdot 1{,}8(3) + 0{,}5\cdot 1{,}5}{3} = 1{,}(7)$

$P_{2345}(x) = \frac{(x+1)\cdot 1{,}5 - (x-2)\cdot 1{,}8(3)}{2+1} = 1{,}(6)$

$P_{m_1 m_2 m_3 m_4 m_5}$

$P_{12345}(x) = \frac{(x+2)\cdot 1{,}(6) - (x-2)\cdot 1{,}(7)}{4}$

$= 1{,}7083.$

3). $x_j = j \ \forall j = \overline{1,4}$

$P_{12}(x) = x+1$

$P_{23}(x) = 3x-1$

$P_{234}(3/2) = 4.$

$P_{1234}(\frac{3}{2}) = ?$

$$P_{1234}\left(\frac{3}{2}\right) = \frac{\left(\frac{3}{2}-x_1\right)P_{234}\left(\frac{3}{2}\right) - \left(\frac{3}{2}-x_4\right)\cdot P_{123}\left(\frac{3}{2}\right)}{x_4-x_1} =$$

$$= \frac{0{,}5\cdot 4 + 2{,}5\cdot P_{123}\left(\frac{3}{2}\right)}{3} = \frac{2+2{,}5\cdot 2{,}75}{3} = 2{,}958(3)$$

$$P_{123}\left(\frac{3}{2}\right) = \frac{\left(\frac{3}{2}-x_1\right)\cdot P_{23}\left(\frac{3}{2}\right) - \left(\frac{3}{2}-x_3\right)\cdot P_{12}\left(\frac{3}{2}\right)}{x_3-x_1} =$$

$$= \frac{0{,}5\cdot\left(3\cdot\frac{3}{2}-1\right) + 1{,}5\cdot\left(\frac{3}{2}+1\right)}{2} = \frac{1{,}75+3{,}75}{2} = 2{,}75.$$

4). $P_2(x) = f[x_1] + f[x_1,x_2](x-x_1) + \cancel{g} \, (x-x_1)(x-x_2)$

$P_2(x_3)$.

$a_3 = f[x_1, x_2, x_3]$.

$P(x_i) = f(x_i) \quad \forall i = \overline{1,3}$.

Presupunem, fara restrangerea generalitatii ca

$x_1 \neq x_2 \neq x_3 \neq x_1$.

Vrem $a_3 = \frac{f[x_2,x_3] - f[x_1,x_2]}{x_3 - x_1}$

Fie $x = x_3$ avem ca $(x_3 - x_1)(x_3 - x_2) \neq 0$

$$a_3 = \frac{P_2(x_3) - f[x_1] - f[x_1 x_2](x_3 - x_1)}{(x_3 - x_1)(x_3 - x_2)}$$

$$a_3 = \frac{f(x_3) - f[x_1] - f[x_1 x_2](x_3 - x_1)}{(x_3 - x_1)(x_3 - x_2)}$$

$$a_3 = \frac{\frac{f[x_3] - f[x_1]}{x_3 - x_2} - f[x_1 x_2]\frac{x_3 - x_1}{x_3 - x_2}}{(x_3 - x_1)}$$

$$a_3 = \frac{\frac{f[x_3] - f[x_1]}{x_3 - x_2} - \frac{f[x_2] - f[x_1]}{x_2 - x_1} \cdot \frac{x_3 - x_1}{x_3 - x_2}}{x_3 - x_1}$$

$$= \frac{\frac{x_2 f[x_3] - x_2 f[x_1] - x_1 f[x_3] + \cancel{x_1 f[x_1]} - x_3 f[x_2] + x_3 f[x_1] + x_1 f[x_2] - \cancel{x_1 f[x_1]}}{(x_3 - x_2)(x_2 - x_1)}}{(x_3 - x_1)}$$

$$= \frac{\frac{f[x_1](x_3 - x_2)}{(x_3 - x_2)} + \frac{f[x_2](x_1 - x_3) + f[x_3](x_2 - x_1)}{(x_2 - x_1)}}{(x_3 - x_1)}$$

$$= \frac{x_2 f[x_3] - x_2 f[x_2] - x_1 f[x_3] + x_1 f[x_2] - x_3 f[x_2] + x_3 f[x_1] + x_2 f[x_2] - x_2 f[x_1]}{\frac{(x_3-x_2)(x_2-x_1)}{(x_3-x_1)}}$$

au fost introdusi.

$$= \frac{\frac{(x_2-x_1)\left(f[x_3]-f[x_2]\right) - (x_3-x_2)\left(f[x_2]-f[x_1]\right)}{(x_3-x_2)(x_2-x_1)}}{(x_3-x_1)}$$

$$= \frac{\frac{f[x_3]-f[x_2]}{x_3-x_2} - \frac{f[x_2]-f[x_1]}{x_2-x_1}}{x_3-x_1}$$

$$= \frac{f[x_2,x_3] - f[x_1,x_2]}{x_3-x_1}$$

$$= f[x_1,x_2,x_3]$$