Exercises for Lecture 3

Proof theory for modal and non-classical logics

June 2023

Exercise 1. All the following formulas are theorems of intuitionistic propositional logic. Provide proofs for them, using the various proof systems for propositional logics presented in Lecture 3: **G3i**, **m-G3i**, **G4ip** and the nested calculus **Nipl**.

You can also try to use the labelled proof system for intuitionistic propositional logic introduced in the following paper:
Dyckhoff, Negri, Proof analysis in intermediate logics. Arch. Math. Logic 51, 7192 (2012). https://link.springer.com/article/10.1007/s00153-011-0254-7

a) $((p \to \bot) \vee q) \to (p \to q)$

b) $p \to ((p \to \bot) \to q)$

c) $(p \vee q) \to (((p \to \bot) \wedge (q \to \bot)) \to \bot)$

d) $(p \to q) \to ((q \to r) \to (p \to r))$

e) $(p \to q) \to ((p \wedge (q \to \bot)) \to \bot)$

Informally compare the various proofs: which proof system produces shorter proofs? Which one you find easier to use?