Exercises for Lecture 4

Proof theory for modal and non-classical logics

June 2023

Exercise 1. The following are the axioms of intuitionistic modal logic IK. Provide proofs for them, using the various proof systems for propositional logics presented in Lecture 4: single-conclusion nested sequents NIK_s, multi-conclusion nested sequents NIK_m, and labelled sequents.

a) $\Box(p \supset q) \supset (\Box p \supset \Box q)$

b) $\Box(p \supset q) \supset (\Diamond p \supset \Diamond q)$

c) $\Diamond(p \vee q) \supset (\Diamond p \vee \Diamond q)$

d) $(\Diamond p \supset \Box q) \supset \Box(p \supset q)$

e) $\Diamond\bot \supset \bot$

Exercise 2. Look at the proof of soundness for multi-conclusion nested sequents NIK_m in Section 4 of this paper:
Kuznets, R., Straburger, L. Maehara-style modal nested calculi. Arch. Math. Logic 58, 359385 (2019) https://link.springer.com/article/10.1007/s00153-018-0636-1

Reconstruct the soundness proof of the $\Diamond^\bullet$ and $\Diamond^\circ$ rules, and of the $\supset^\bullet$ and $\supset^\circ$ rules.