Course on Proof Theory - Lecture 3

Exercises - Gentzen's sequent calculus

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Exercises for Lecture 3

1. We saw that modus ponens could be simulated in LK. Give cut-free LK proofs for each of the three propositional axioms of HF:

$$\begin{array}{rl}
(wk) & A \to (B \to A) \\
(dist) & (A \to (B \to C)) \to ((A \to B) \to (A \to C)) \\
(neg) & ((A \to \bot) \to \bot) \to A
\end{array}$$

2. What about the quantifier axioms and rule:

$$\text{gen}\,\frac{A}{\forall x A} \qquad \begin{array}{l} \forall x.A \to A[t/x] \\ \forall x.(A \to B) \to (A \to \forall x.B) \quad \text{as long as } x \notin \mathrm{FV}(A) \end{array}$$

   (**NB:** this concludes the proof of completeness of LK).

3. What possible rules might we need in LK if we had $\bot$ in our language?

4. Show that the multiplicative formulation of the rules for $\wedge$ can be derived from the additive formulation of the rules for $\wedge$ using the structural rules, and vice versa (see slide 13).