

Exercises for Lecture 4

1. Prove that the following are derivable:

- ▷ $(a \rightarrow b) \rightarrow ((a \wedge (b \rightarrow \perp)) \rightarrow \perp)$
- ▷ $((a \wedge (b \rightarrow \perp)) \rightarrow \perp) \rightarrow ((a \rightarrow \perp) \vee b)$

2. Prove the following cases of the Principal Lemma:

- ▷ The cut formula is $B \vee C$ and it is principal in both derivations:

$$\vee_R \frac{\begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \mathcal{D}_1 \end{array} \quad \Gamma \vdash_p^{m-1} \Delta, B, C}{\Gamma \vdash_p^m \Delta, B \vee C}$$

$$\vee_L \frac{\begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \mathcal{D}'_2 \end{array} \quad B, \Gamma \vdash_p^{n1} \Delta \quad \begin{array}{c} \text{---} \\ \diagup \quad \diagdown \\ \mathcal{D}''_2 \end{array} \quad C, \Gamma \vdash_p^{n2} \Delta}{B \vee C, \Gamma \vdash_p^n \Delta}$$

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- ▷ The cut formula is A and it is principal in the leftmost derivation, with $\text{deg}(C) < p$:

$$\text{cut} \frac{\begin{array}{c} \mathcal{D}'_1 \\ \hline \Gamma \vdash_{\substack{\text{orange } m1 \\ \text{green } p}} \Delta, A, C \end{array} \quad \begin{array}{c} \mathcal{D}''_1 \\ \hline C, \Gamma \vdash_{\substack{\text{orange } m2 \\ \text{green } p}} \Delta, A \end{array}}{\Gamma \vdash_{\substack{\text{orange } m \\ \text{green } p}} \Delta, \textcolor{red}{A}}$$

$$\text{R}_1 \frac{\begin{array}{c} \mathcal{D}_2 \\ \hline C, \Gamma' \vdash_{\substack{\text{orange } n-1 \\ \text{green } p}} \Delta' \end{array}}{\textcolor{red}{A}, \Gamma \vdash_{\substack{\text{orange } n \\ \text{green } p}} \Delta}$$

- ▷ The cut formula is $\forall xB$ and it is principal in both R_1 and R_2

$$\forall_R \frac{\begin{array}{c} \mathcal{D}_1 \\ \hline \Gamma \vdash \Delta, B[x/y] \end{array}}{\Gamma \vdash \Delta, \textcolor{red}{\forall x B}}$$

$$\forall_L \frac{\begin{array}{c} \mathcal{D}_2 \\ \hline B[x/t], \forall x.B, \Gamma \vdash \Delta \end{array}}{\textcolor{red}{\forall x.B}, \Gamma \vdash \Delta}$$