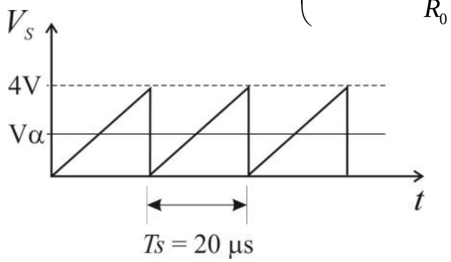
# Sistemas de Control

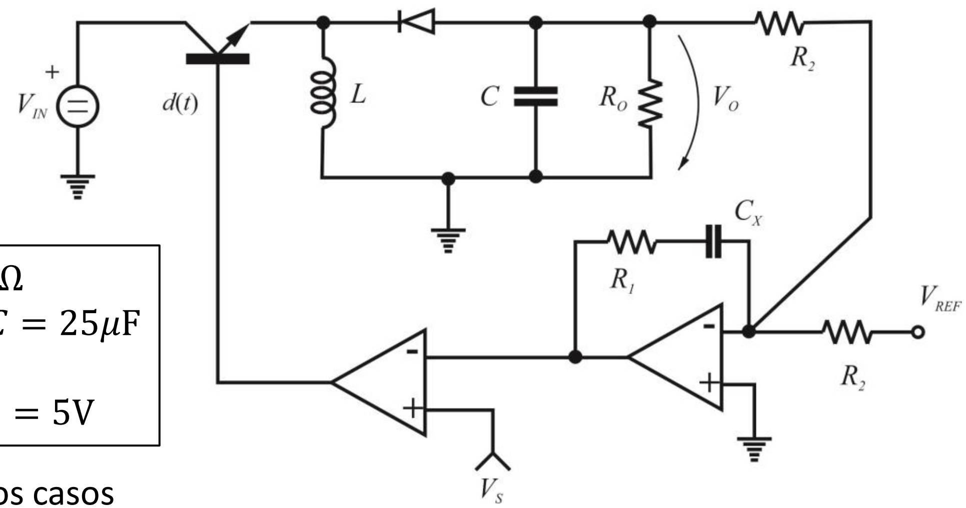
## Guía Nº 7: CONVERTIDORES DC/DC / MODELO PROMEDIADO DE ESTADOS

**Ejercicio N°4:** Considere el convertidor flyback de la figura, cuya función de transferencia respecto de la variable de control esta dada por:

$$G_P(S)=\frac{\tilde{V}_0}{\tilde{d}}=V_{IN}\frac{\left(1-S\frac{L}{R_0}\frac{D}{(1-D)^2}\right)}{\left(S^2LC+S\frac{L}{R_0}+(1-D)^2\right)}$$

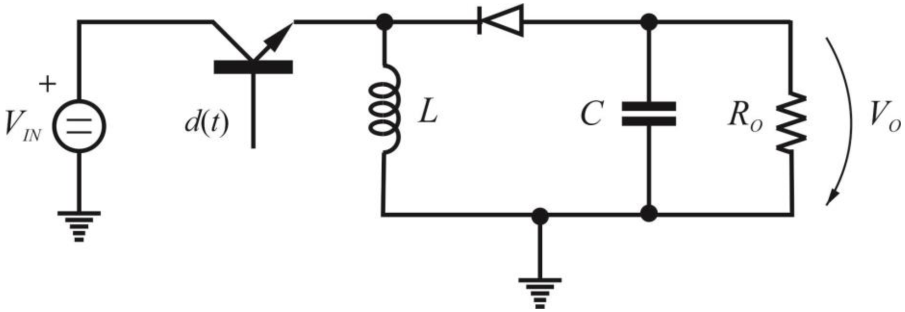


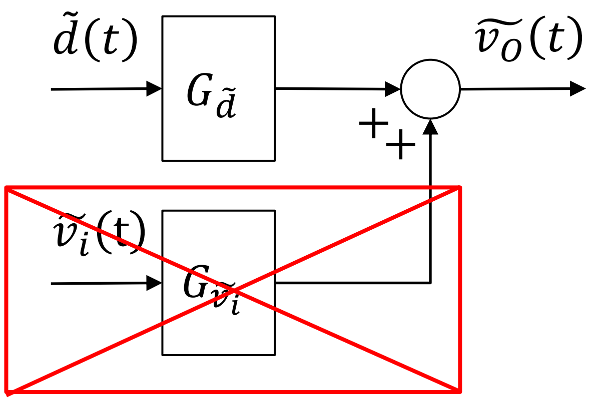

$R_O = 3\Omega \ldots 30\Omega$
$L = 250\mu\text{H} \quad C = 25\mu\text{F}$
$f_s = 50\text{kHz}$
$V_{REF} = 5\text{V}\ V_{IN} = 5\text{V}$




a) Verificar que se cumple CCM para todos los casos
b) Dibujar un diagrama en bloques para el lazo de control, sin considerar la entrada de perturbación de la fuente, identificando claramente Gc, H, GMOD y Gp.
c) Graficar el diagrama de Bode de Gp para los valores extremos de R0, indicando los valores notables de frecuencia y amplitud en cada caso.
d) Compensar el convertidor, calculando R1, R2 y CX, a fin de obtener máximo ancho de banda y rechazo en GH(s), con un margen de fase de aproximadamente 45º. Dibujar el diagrama de Bode. Nota: Se sugiere agregar una red de amortiguación en el convertidor.

# Modelado del convertidor de potencia:



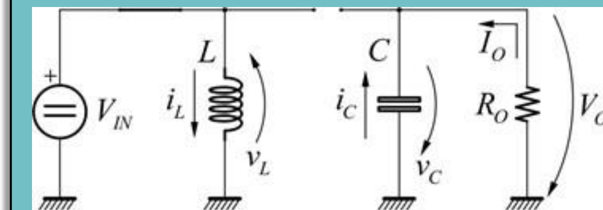




$$v_i(t) = V_{IN}: cte$$

$$G_{\tilde{d}}(S) = \frac{\tilde{v}_0}{\tilde{d}} = \frac{V_{IN}}{(1-D)^2}\,\frac{\left(1 - S\frac{L}{R_0}\frac{D}{(1-D)^2}\right)}{\left(1 + S\frac{L}{(1-D)^2\cdot R_0} + S^2\frac{LC}{(1-D)^2}\right)}$$

## Modelo promediado de estados

**Estado ON**

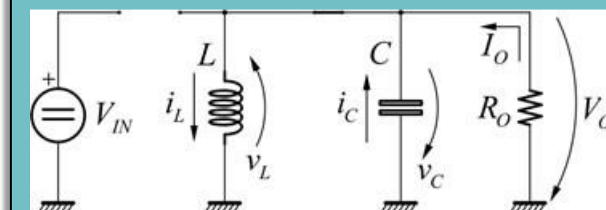



$$\frac{di_L}{dt}, \frac{dv_C}{dt}$$

$$\Downarrow$$

$$A_1, B_1, C_1$$

**Estado OFF**




$$\frac{di_L}{dt}, \frac{dv_C}{dt}$$

$$\Downarrow$$

$$A_2, B_2, C_2$$

$$A = D \cdot A_1 + (1-D) \cdot A_2$$

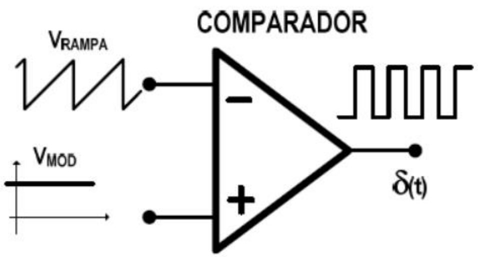
$$B = D \cdot B_1 + (1-D) \cdot B_2$$

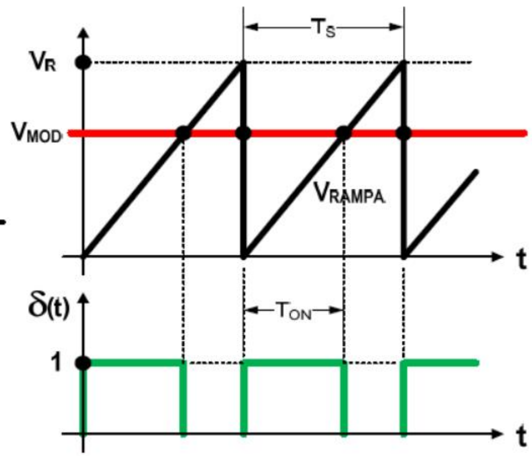
$$C = D \cdot C_1 + (1-D) \cdot C_2$$

$$G_{\tilde{d}} = V_{IN}(C_2 - C_1)(A^{-1}B)$$
$$+ V_{IN}C(sI_d - A)^{-1}[(A_2 - A_1)(A^{-1}B) + (B_1 - B_2)]$$

# Modelado del modulador PWM:

**MODULADOR PWM:**



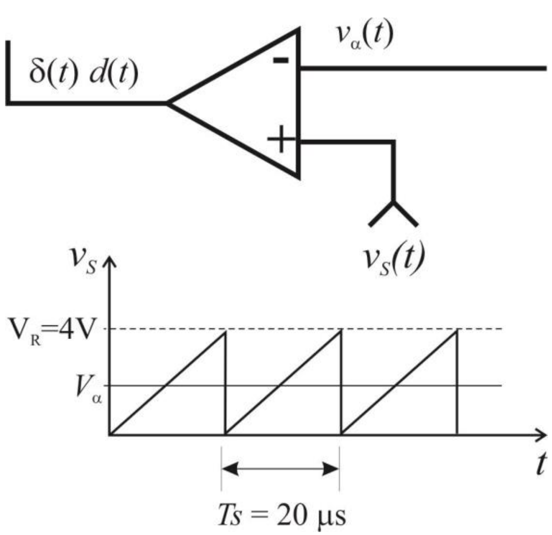



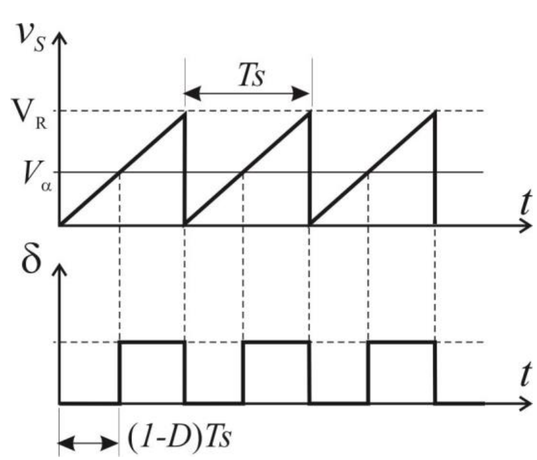

$$0 \le V_{MOD} \le V_R$$

**GANANCIA DEL MODULADOR PWM**

$$G_{MOD} = \frac{d(t)}{V_{MOD}} = \frac{1}{V_R}$$

G.U. Clase 12/11/20



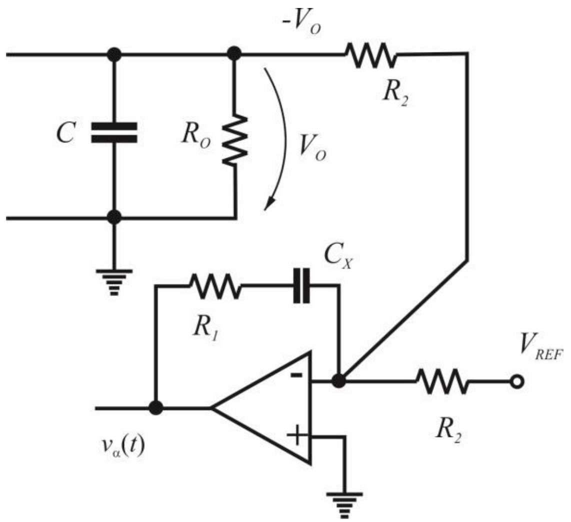



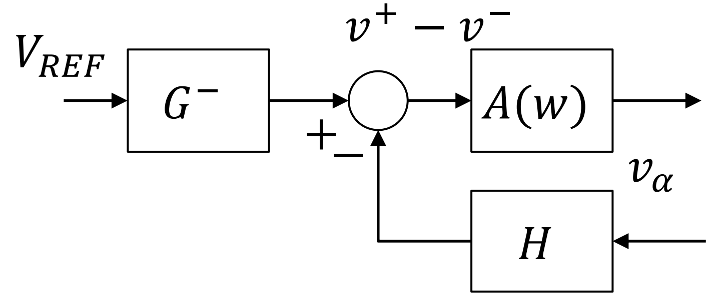

$$(1 - d(t)) = \frac{V_\alpha}{V_R}$$

$$d(t) = 1 - \frac{V_\alpha}{V_R}$$

$$G_{MOD} = \frac{\partial d(t)}{\partial V_\alpha} = -\frac{1}{V_R}$$

## Modelado del compensador:

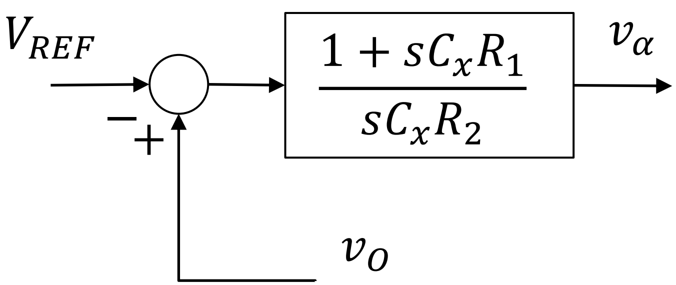


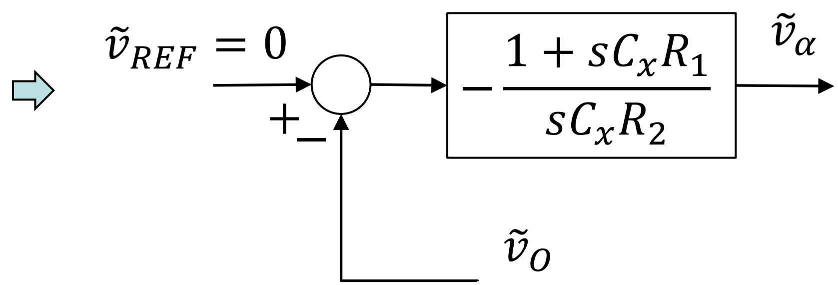

$$G^- = -\frac{v^-}{-v_O} = -\frac{v^-}{V_{REF}} = -\frac{1}{2}\cdot\frac{1+sC_xR_1}{1+sC_x(R_1+R_2/2)}$$

$$H = \frac{v^-}{v_\alpha} = \frac{sC_xR_2/2}{1+sC_x(R_1+R_2/2)}$$

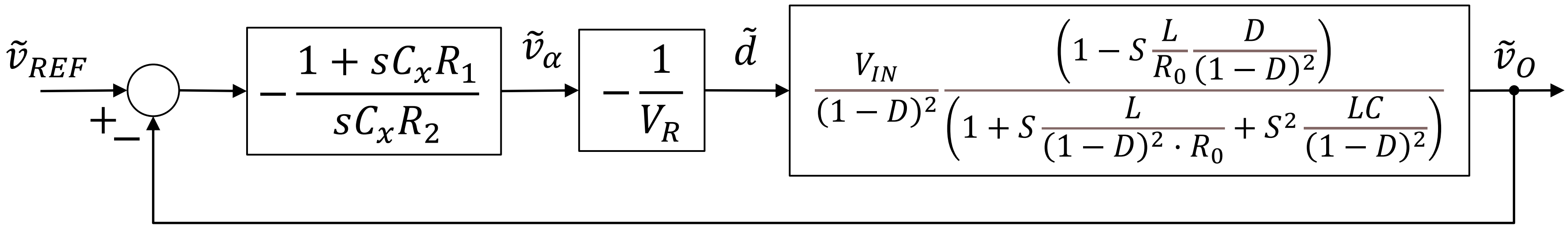
$$\frac{v_\alpha}{-v_O} = \frac{v_\alpha}{V_{REF}} = \frac{G^-}{H} = -\frac{1+sC_xR_1}{sC_xR_2}$$




Compensador tipo PI



## Diagrama en bloques:




Sistema tipo I ⇨ $V_O = V_{REF} = 5\text{V}$

### Verificación CCM

Premisa: CCM ⇨ $i_L > 0\ \forall t$

Condición crítica: $I_L = \frac{\Delta i_L}{2}$

$$L_C = \frac{R_O(1-D)^2 \cdot T_S}{2}$$

$R_O = 3\Omega \ldots 30\Omega$
$L = 250\mu\text{H} \quad C = 25\mu\text{F}$
$f_s = 50\text{kHz}$
$V_{REF} = 5\text{V}\ V_{IN} = 5\text{V}$

$$\frac{V_O}{V_{IN}} = \frac{D}{1-D}$$

$$D = \frac{V_O}{V_O + V_{IN}} = \frac{5V}{5V+5V} = \frac{1}{2}$$

Caso más exigente: $R_O = R_{Omax}$

$$L_C = \frac{R_{Omax}(1-D)^2 \cdot T_S}{2}$$

$$L_C = \frac{30\Omega}{2}\left(1-\frac{1}{2}\right)^2 \frac{1}{50kHz} = 75\mu H$$

## Análisis de $G_{\tilde{d}}$ :

$$G_{\tilde{d}} = \frac{V_{IN}}{(1-D)^2} \frac{\left(1 - s\frac{L}{R_O}\frac{D}{(1-D)^2}\right)}{\left(1 + s\frac{L}{R_O}\frac{1}{(1-D)^2} + s^2\frac{LC}{(1-D)^2}\right)} = \frac{G_0(1 - s/\omega_{Z1})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})} = \frac{G_0(1 - s/\omega_{Z1})}{\left(\frac{s}{\omega_{P12}}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_{P12}}\right) + 1}$$

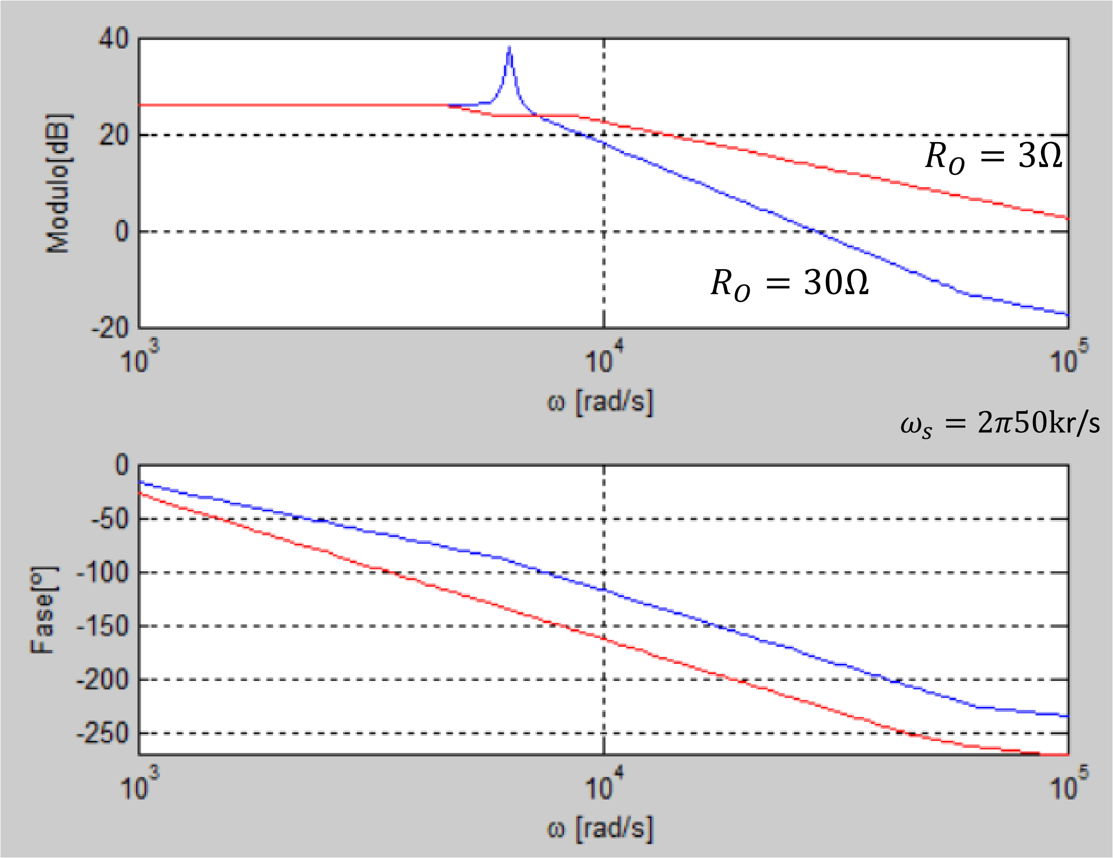
$$R_O = 30\Omega \quad \begin{cases} G_0 = 20\ (26\text{dB}) \\ \omega_Z = 60000\text{rad/s} \\ \omega_{P12} = 666 \pm j6324\text{rad/s} \\ Q = 4{,}73\ (13{,}5\text{dB}) \end{cases}$$

**Peor condición en amplitud!**

$$R_O = 3\Omega \quad \begin{cases} G_0 = 20\ (26\text{dB}) \\ \omega_Z = 6000\text{rad/s} \\ \omega_{P1} = 4560\text{rad/s} \\ \omega_{P2} = 8768\text{rad/s} \end{cases}$$

**Peor condición en fase!**

**Bode de $G_{\tilde{d}}/1V$**



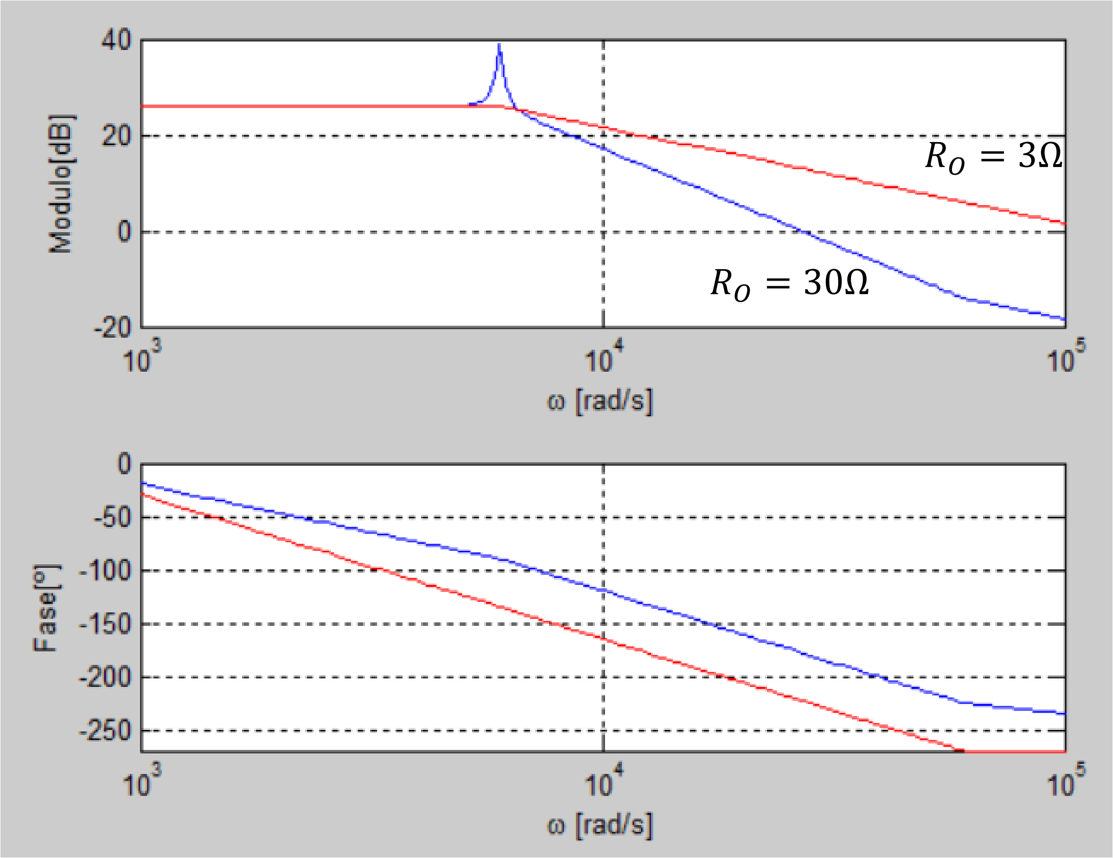
## Análisis de estabilidad (sin amortiguamiento):

$$G_{\tilde{d}} = \frac{V_{IN}}{(1-D)^2}\frac{\left(1 - s\frac{L}{R_O}\frac{D}{(1-D)^2}\right)}{\left(1 + s\frac{L}{R_O}\frac{1}{(1-D)^2} + s^2\frac{LC}{(1-D)^2}\right)} = \frac{G_0(1 - s/\omega_{Z1})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})} = \frac{G_0(1 - s/\omega_{Z1})}{\left(\frac{s}{\omega_{P12}}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_{P12}}\right) + 1}$$

$$R_O = 30\Omega \left[\begin{array}{l} G_0 = 20\ (26\text{dB}) \\ \omega_Z = 60000\text{rad/s} \\ \omega_{P12} = 666 \pm \text{j}6324\text{rad/s} \\ Q = 4{,}73\ (13{,}5\text{dB}) \end{array}\right.$$

$$R_O = 3\Omega \left[\begin{array}{l} G_0 = 20\ (26\text{dB}) \\ \omega_Z = 6000\text{rad/s} \\ \omega_{P1} = 4560\text{rad/s} \\ \omega_{P2} = 8768\text{rad/s} \end{array}\right.$$

**Bode de $G_{\tilde{d}}/1V$ simplificado**



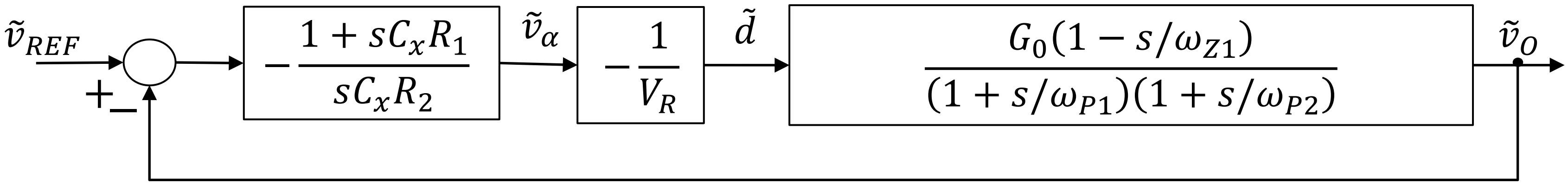

**Simplificación transitoria del problema**

$$R_O = 30\Omega \left[\begin{array}{l} G_0 = 26\text{dB} \\ \omega_Z = 60000\text{rad/s} \\ |\omega_{P12}| \approx 6000\text{rad/s} \\ Q \approx 13\text{dB} \end{array}\right.$$

$$R_O = 3\Omega \left[\begin{array}{l} G_0 = 26\text{dB} \\ \omega_Z \approx 6000\text{rad/s} \\ \omega_{P1} \approx 6000\text{rad/s} \\ \omega_{P2} \approx 6000\text{rad/s} \end{array}\right.$$

**Verificaremos al finalizar!**

# Análisis de estabilidad (sin amortiguamiento):

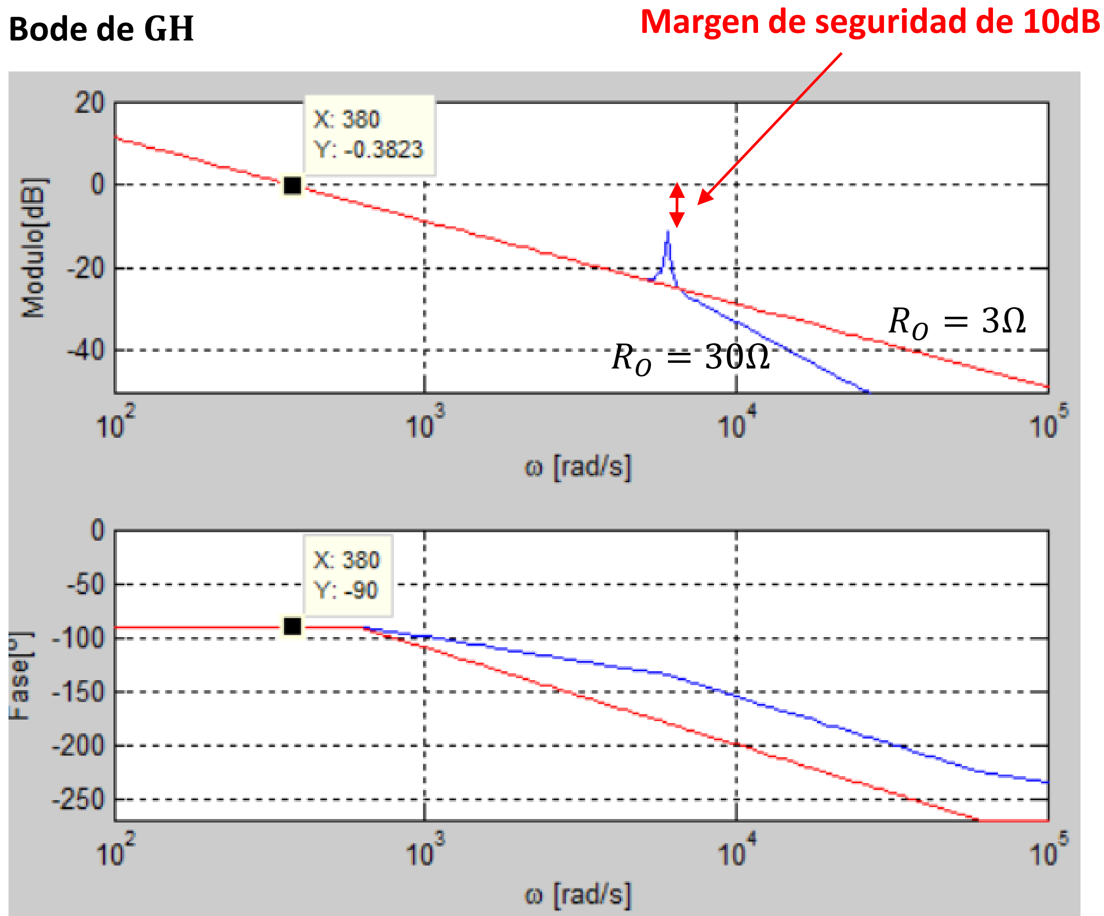



Ajuste del PI:
Polo para garantizar sobrepico debajo de 0dB
Cero para compensar uno de los polos.

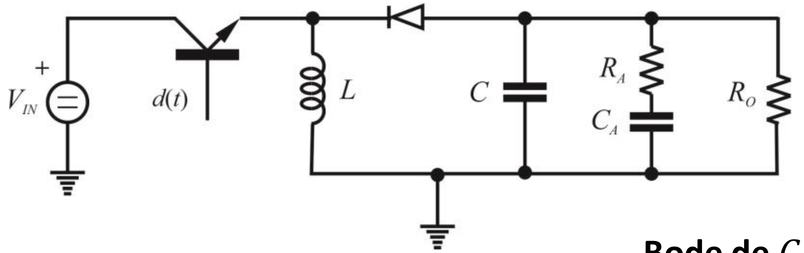
$$\frac{1+sC_xR_1}{sC_xR_2} = \underbrace{\frac{V_R/G_0}{5{,}5\cdot 3}}_{(13dB)\quad(10dB)} \cdot \frac{1+\frac{s}{6\text{krad}}}{\frac{s}{6\text{krad}}}$$

Margen de fase = 90º en ambos casos.
El caso de $R_{Omax}$ resulta más restrictivo!!

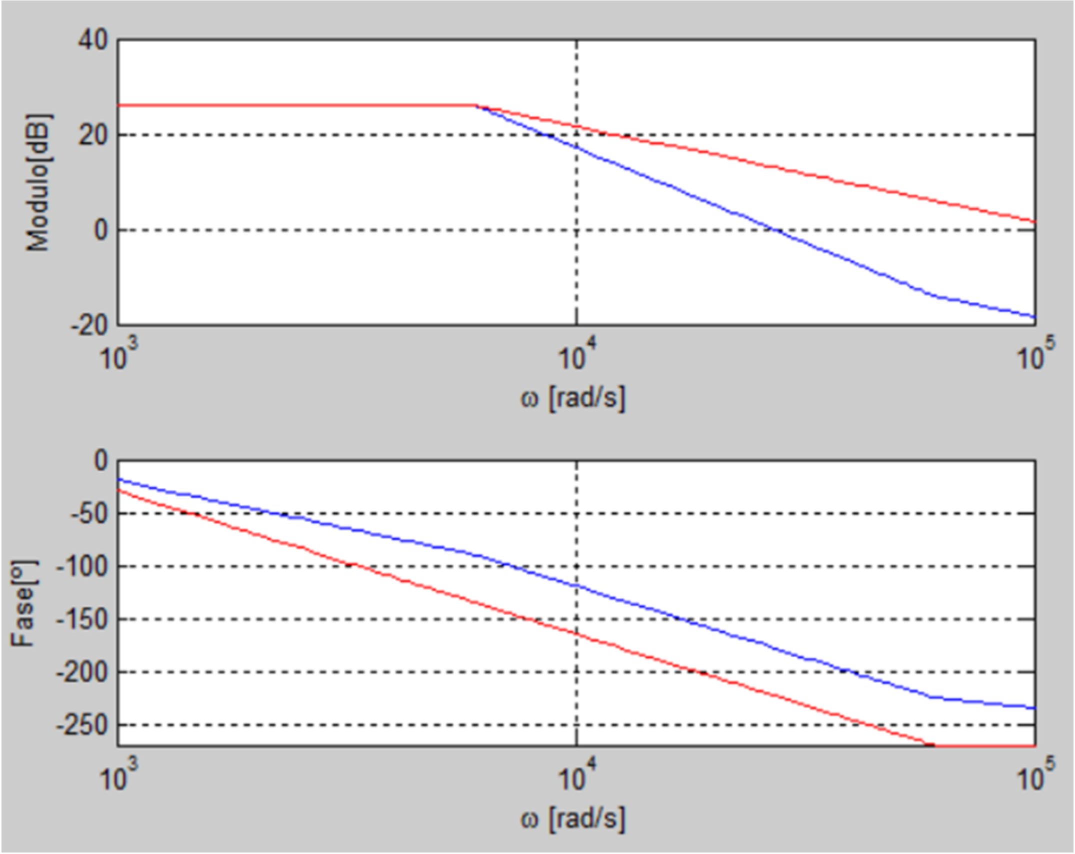
Bode de GH



## Análisis de estabilidad (con amortiguamiento):

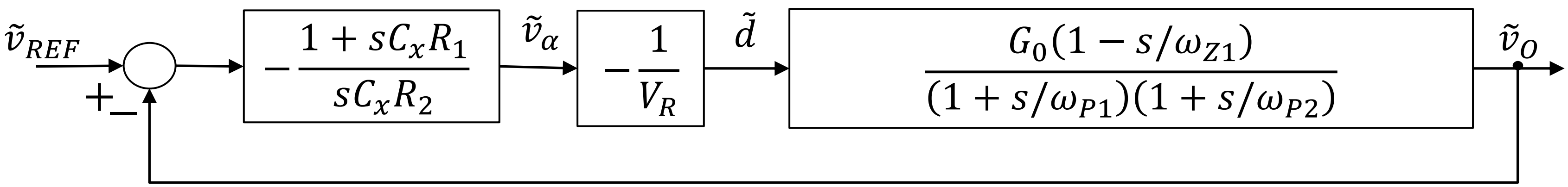



Bode de $G_{\tilde{d}}/1V$ amortiguado




Asumimos por el momento que podemos eliminar el sobrepico sin modificar el resto de las transferencias!

## Análisis de estabilidad (sin amortiguamiento):




**PI con ganancia ajustada para $R_O = 30\Omega$**

$$\frac{1+sC_xR_1}{sC_xR_2} = \frac{V_R}{G_0} \cdot \frac{1+\frac{s}{6\text{krad}}}{\frac{s}{6\text{krad}}}$$

$R_O = 30\Omega$

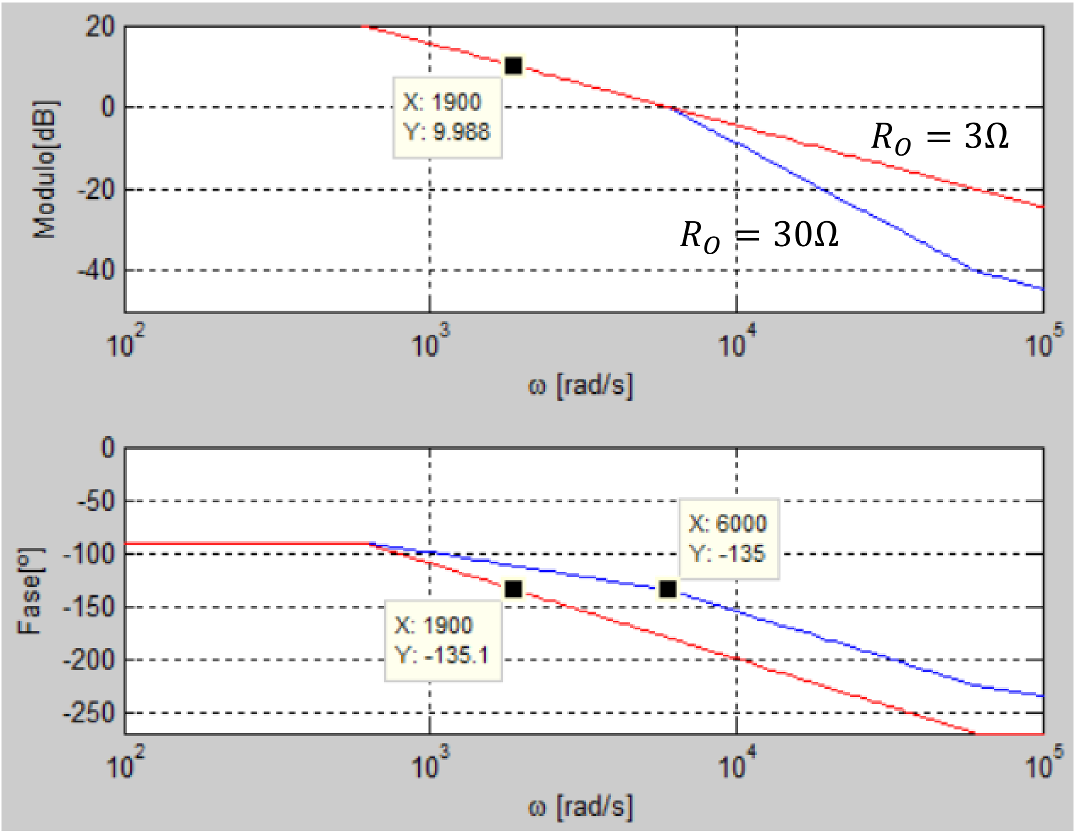
$$\varphi_{GH} = -90 - \text{argtg}\left(\frac{\omega_X}{6\text{krad}}\right)$$

$R_O = 3\Omega$

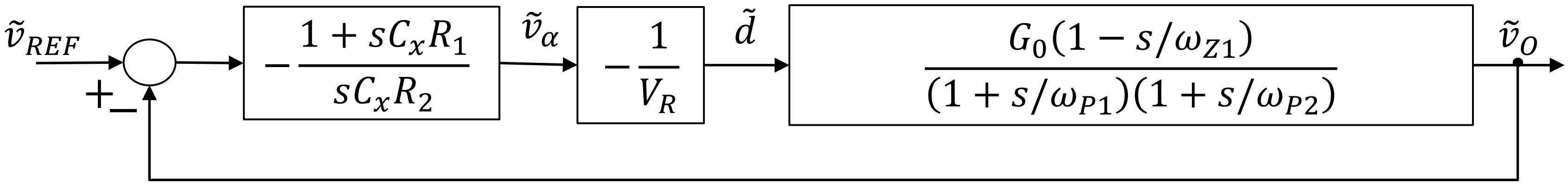
$$\varphi_{GH} = -90 - 2\cdot\text{argtg}\left(\frac{\omega_X}{6\text{krad}}\right)$$

El caso de $R_{Omin}$ resulta más restrictivo!!

**Bode de GH con $G_{\tilde{d}}/1V$ amortiguado**



# Análisis de estabilidad (sin amortiguamiento):




**PI con ganancia ajustada para $R_O = 3\Omega$**

$$\frac{1+sC_xR_1}{sC_xR_2} = \underbrace{\frac{V_R}{3\cdot G_0}}_{(10dB)} \cdot \frac{1+\frac{s}{6\text{krad}}}{\frac{s}{6\text{krad}}}$$

$R_O = 30\Omega$ ⇨ $MF = 66{,}5º$

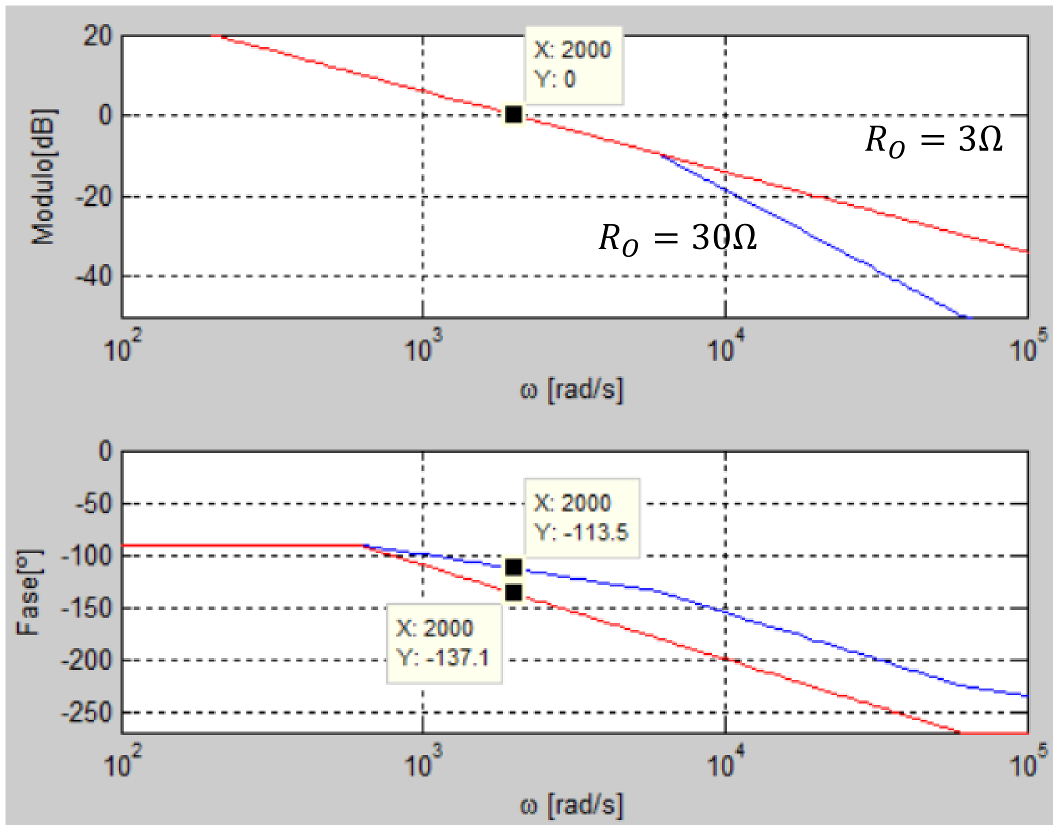
$R_O = 3\Omega$ ⇨ $MF = 42{,}9º$

$$\frac{1+sC_xR_1}{sC_xR_2} = \frac{1+\frac{s}{6\text{krad}}}{\frac{s}{400\text{rad}}}$$

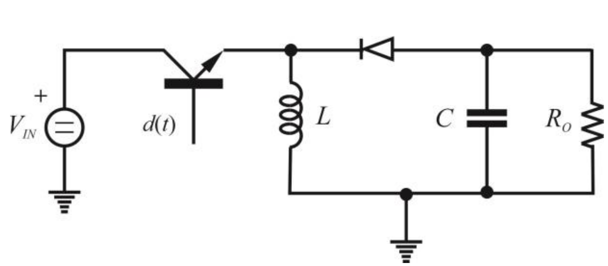
$C_x = 2\mu F$ ⇨ $R_1 = 83{,}3\Omega$ $R_2 = 1{,}25\text{k}\Omega$

Adopto $R_1 = 100\Omega$ y $R_2 = 1{,}5\text{k}\Omega$

Bode de GH con $G_{\tilde{d}}/1V$ amortiguado



## Cálculo de la red de amortiguamiento:



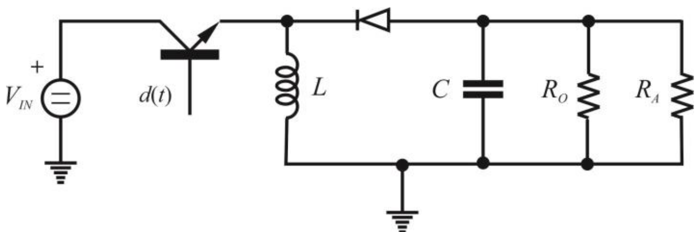

$$\left(1 + s\frac{L}{R_O}\frac{1}{(1-D)^2} + s^2\frac{LC}{(1-D)^2}\right) = \left(\frac{s}{\omega_{P12}}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_{P12}}\right) + 1$$

$$\omega_{P12} = \sqrt{\frac{(1-D)^2}{LC}} \qquad Q = \frac{(1-D)^2}{\omega_{P12}} \cdot \frac{R_O}{L} = R_O(1-D)\sqrt{\frac{C}{L}}$$

Cálculo de $R_O$ para transferencia monótonamente decreciente:

$$Q = \frac{1}{\sqrt{2}} \Rightarrow R_{Omd} = \frac{1}{\sqrt{2}} \cdot \frac{1}{1-D}\sqrt{\frac{L}{C}} = 4{,}47\Omega$$

Amortiguamiento sólo con resistencia $R_A$:

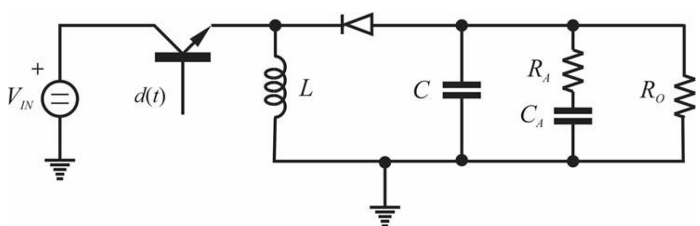



$$R_A // R_{Omax} = R_{Omd}$$

$$\Downarrow$$

$$R_A = 5{,}25\Omega$$

Amortiguamiento con $R_A$ y $C_A$ :

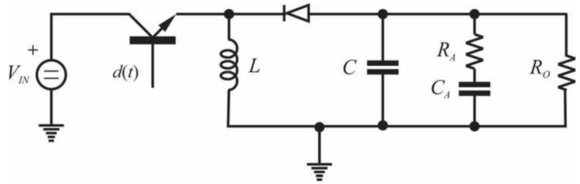



$R_A = 2{,}7\Omega$
$C_A = 68\mu\mathrm{F}$

$$Z_A(\omega_{P12}) = 5{,}25\Omega \Rightarrow \begin{cases} R_A = 5{,}25\Omega/2 \\ \dfrac{1}{\omega_{P12} C_A} = 5{,}25\Omega/2 \end{cases}$$

## Verificación del cálculo de la red de amortiguamiento:

El sobrepico no fue completamente eliminado




⇩

Modelo de estados promediado de 3er orden

⇩

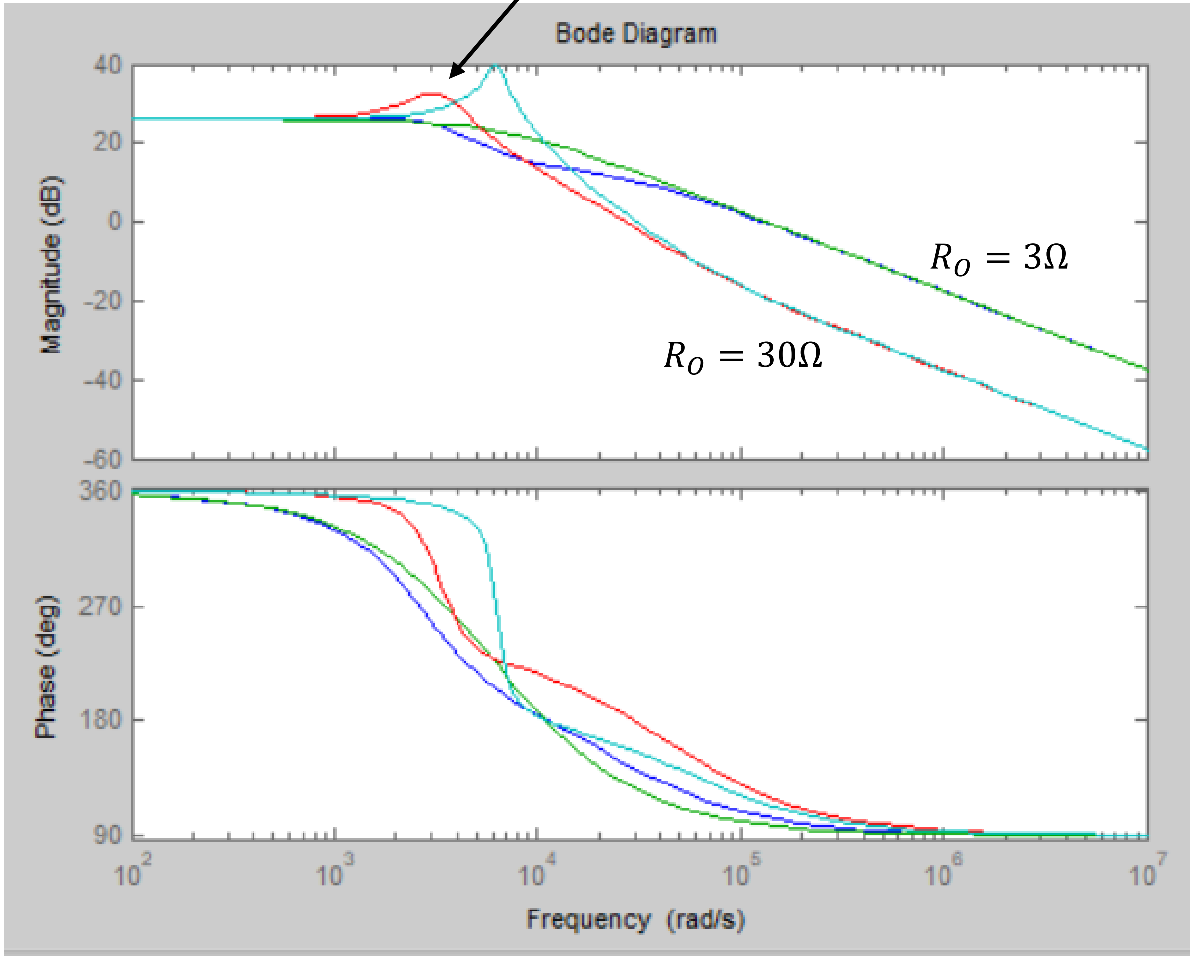
Cálculo de ecuación de $G_{\tilde{d}}$

⇩

Evaluación para diferentes $R_O$ y casos con y sin amortiguamiento.

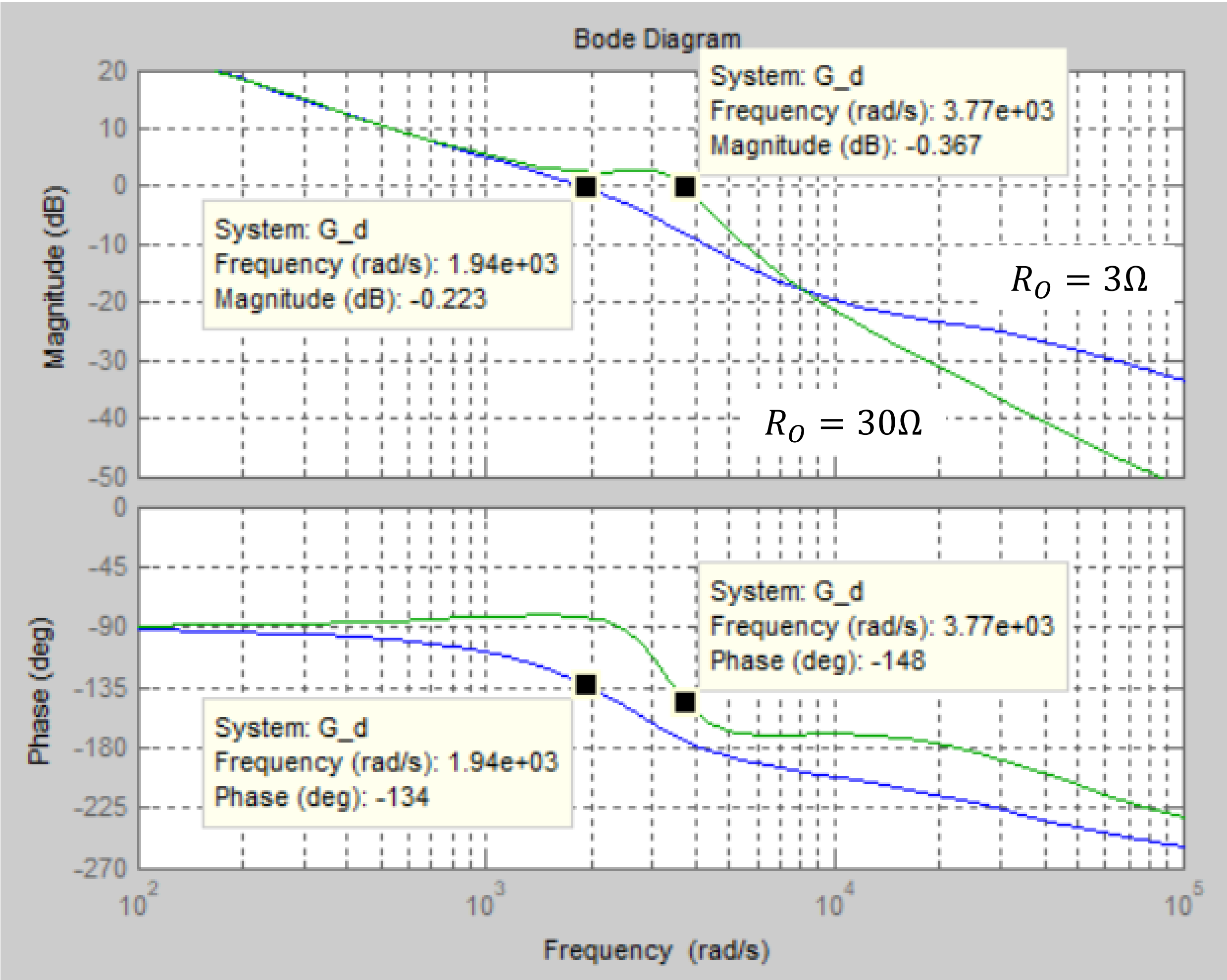
⇨

**Bode de** $G_{\tilde{d}}/1V$



## Verificación del análisis de estabilidad (con amortiguamiento):

### Bode de GH




A causa del residuo del sobrepico, la frecuencia de corte aumentó para el caso de $R_{Omax}$.

⇩

El caso de $R_{Omax}$ resulta Nuevamente más restrictivo!!