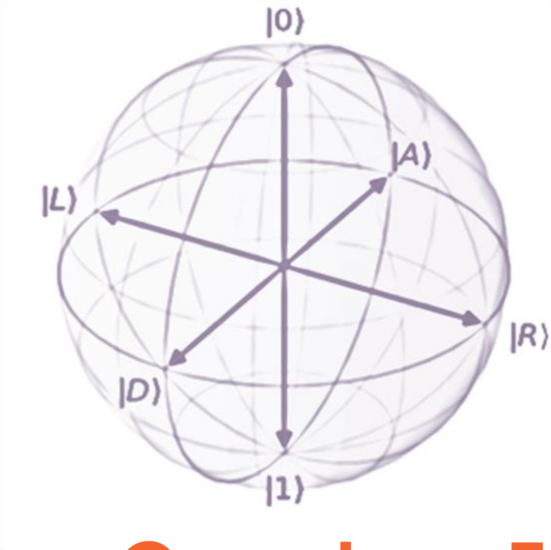

Quantum Error Mitigation (QEM) applied to research in fundamental quantum mechanics

Final project

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Course: Quantum computing and programming (Q511-1)

QEM applied to research in QM

Final project

Introduction

Duality and entanglement are perhaps the most prominent hallmarks of quantum mechanics. Both evoke a sense of weirdness which has been however no obstacle for their instrumentation as powerful tools, useful not only for an accurate description of physical phenomena but for many practical applications as well. It is thus important to elucidate the various possible forms in which duality and entanglement manifest themselves. Recent developments have shown that the defining features of duality and entanglement may arise in both the quantum and the classical domain. An important result in this respect is the established polarization-coherence theorem (PCT).

PCT theorem

$$V^2 + D^2 = P^2$$

Basic concepts

Visibility (V)
Express a degree of wave behaviour by using classical methods like interferometers and this is the definition

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Polarization (P)
When you express a quantum state as a matrix density

$$\rho = \frac{1}{2}(1 + \vec{S} \cdot \hat{\sigma})$$

The polarization es defined as:

$$P^2 = S_x^2 + S_y^2 + S_z^2$$

Distinguishability (D)
To see the particle behaviour, we can use the concept distinguishability which is defined as:

$$D = \frac{1}{2} Tr|\rho_1 - \rho_2|$$

$$D = \frac{1}{2} |\vec{S}_1 - \vec{S}_2|$$

Quantum tomography

$$S_z^{(1)} = \frac{P_{01} - P_{11}}{P_{01} + P_{11}}$$
$$S_z^{(2)} = \frac{P_{00} - P_{10}}{P_{00} + P_{10}}$$

Extended PCT theorem

$$V^2 + D^2 = Cos^2\gamma/2 + P^2 Sin^2\gamma/2$$

$$V^2 = \mathrm{Cos}^2\,\gamma/2 + P^2\,\mathrm{Cos}^2\,\phi\,\mathrm{Sin}^2\,\gamma/2$$

$$D^2 = P^2\,\mathrm{Sin}^2\,\phi\,\mathrm{Sin}^2\,\gamma/2$$

Quantum circuit implementation

$$|\psi_0\rangle = \left(\sqrt{\tfrac{1+P}{2}}\,|0_A, 0_M\rangle + \sqrt{\tfrac{1-P}{2}}\,|1_A, 1_M\rangle\right) \otimes |0_S\rangle$$

$$|\psi_1\rangle = \left(\sqrt{\tfrac{1+P}{2}}\,|0_A, 0_M\rangle + \sqrt{\tfrac{1-P}{2}}\,|1_A, 1_M\rangle\right) \otimes \left(\tfrac{|0_S\rangle + |1_S\rangle}{\sqrt{2}}\right)$$

$$\rho_M = \tfrac{1}{2}\begin{pmatrix} 1+P\,\mathrm{Sin}\phi & -i\,P\,\mathrm{Cos}\phi \\ i\,P\,\mathrm{Cos}\phi & 1-P\,\mathrm{Sin}\phi \end{pmatrix} = \tfrac{1}{2}(\mathcal{I} + \vec{\sigma}\cdot\vec{n}\,P) \text{ con } \vec{n} = (0, \mathrm{Cos}\phi, \mathrm{Sen}\phi)$$

$$U = \begin{pmatrix} \mathrm{Cos}[\gamma/2] & \mathrm{Sin}[\gamma/2] \\ -\mathrm{Sin}[\gamma/2] & \mathrm{Cos}[\gamma/2] \end{pmatrix} \quad U = \mathrm{Cos}(\gamma/2)\,\mathcal{I} + i\,\mathrm{Sen}(\gamma/2)\,\vec{\sigma}\cdot\vec{\mu} \text{ con } \vec{\mu} = (0, 1, 0)$$

$$\vec{n}\cdot\vec{\mu} = \mathrm{Cos}\phi$$

$$|\psi_s^{(0)}\rangle = P(\phi)\,H\,|0_s\rangle = \frac{|0_s\rangle + e^{i\phi}\,|1_s\rangle}{\sqrt{2}}$$

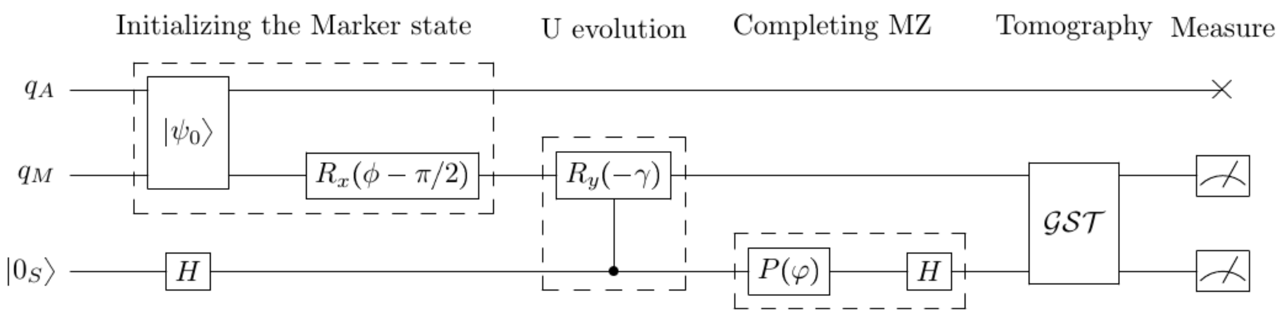
Así $\rho_{SM}^{(0)} = \rho_M^{(0)} \otimes |\psi_s^{(0)}\rangle\langle\psi_s^{(0)}| \;\rightarrow\; U\rho_{SM}^{(0)}\,U^\dagger$

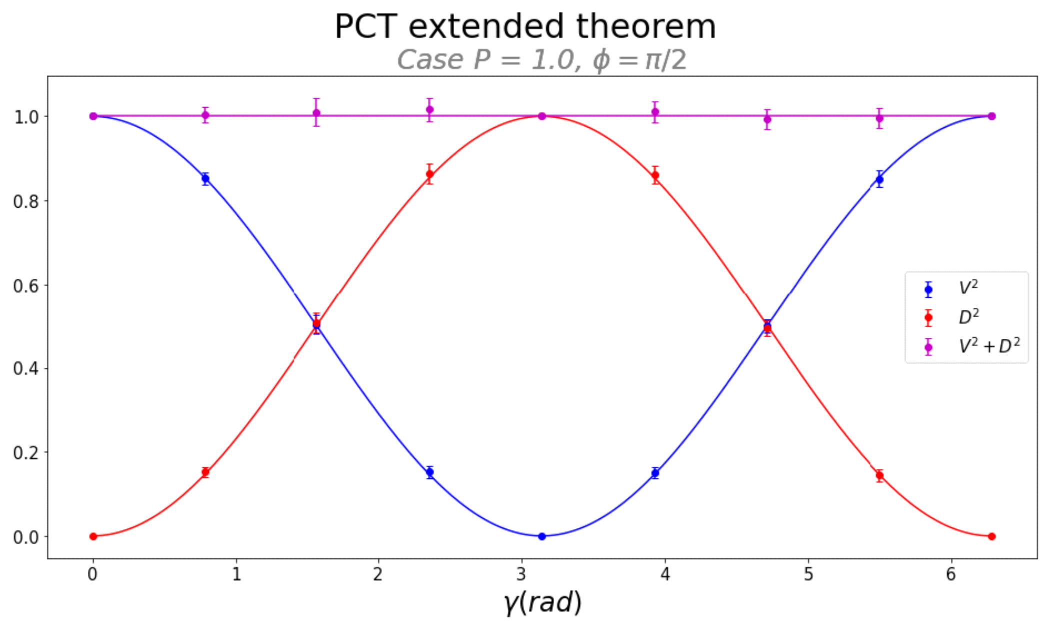
donde $U = |0_s\rangle\langle 0_s| \otimes \mathcal{I}_M + |1_s\rangle\langle 1_s| \otimes \mathrm{Ry}_M(-\gamma)$

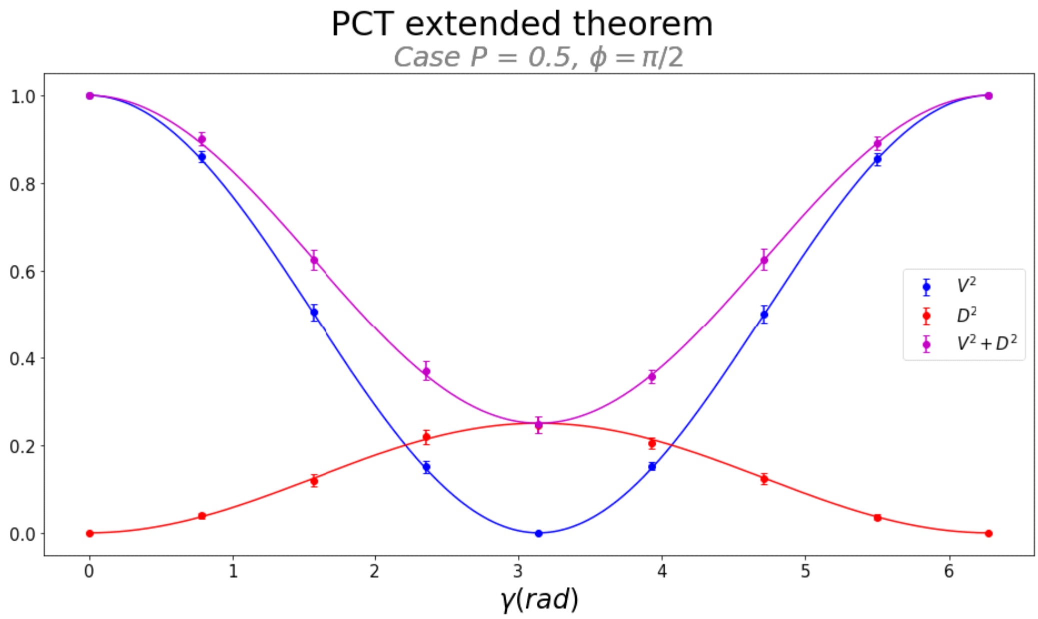
To calculate erros

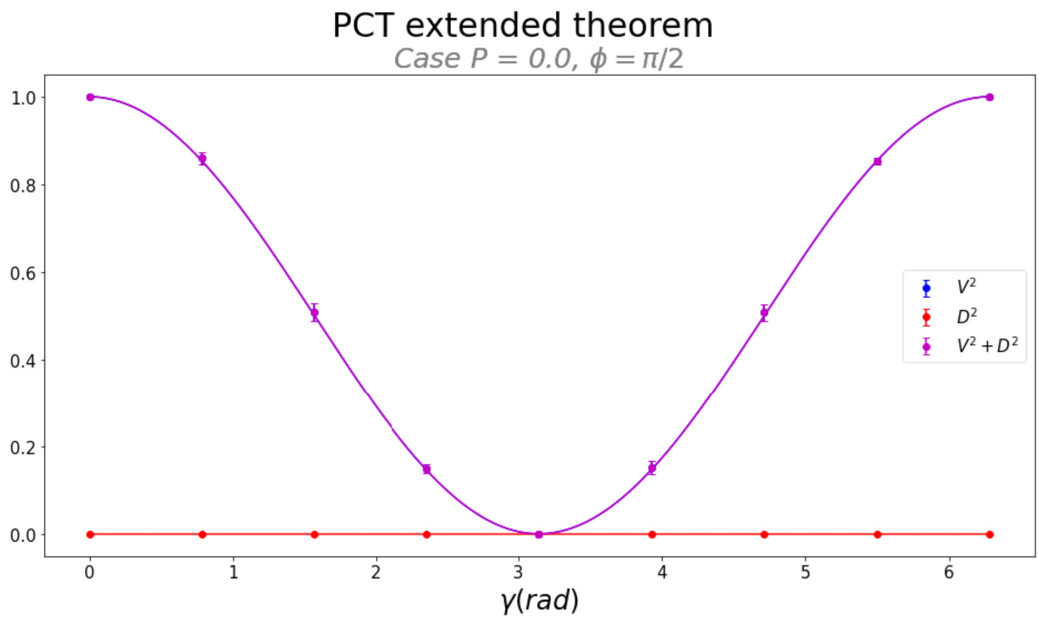
$$(dV^2) = 2V\sqrt{\tfrac{A^2+B^2}{B^4}}\;dA = 2\tfrac{V}{B}\sqrt{1+V^2}\;\sqrt{(dI^{max})^2 + (dI^{min})^2}$$

$$dD^2 = \frac{1}{2}\sqrt{\left(S_x^{(1)} - S_x^{(2)}\right)^2\left(\left(dS_x^{(1)}\right)^2 + \left(dS_x^{(2)}\right)^2\right) + \left(S_y^{(1)} - S_y^{(2)}\right)^2\left(\left(dS_y^{(1)}\right)^2 + \left(dS_y^{(2)}\right)^2\right) + \left(S_z^{(1)} - S_z^{(2)}\right)^2\left(\left(dS_z^{(1)}\right)^2 + \left(dS_z^{(2)}\right)^2\right)}$$






PCT extended theorem
Case P = 0.5, ϕ = π/2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
γ(rad)
V^2
D^2
V^2+D^2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D^2
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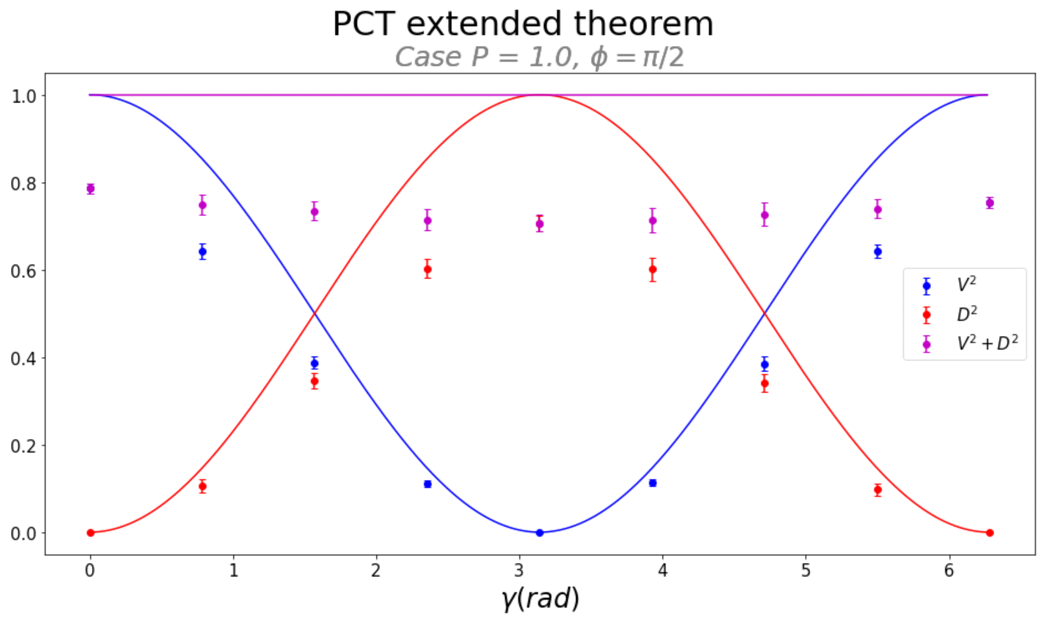
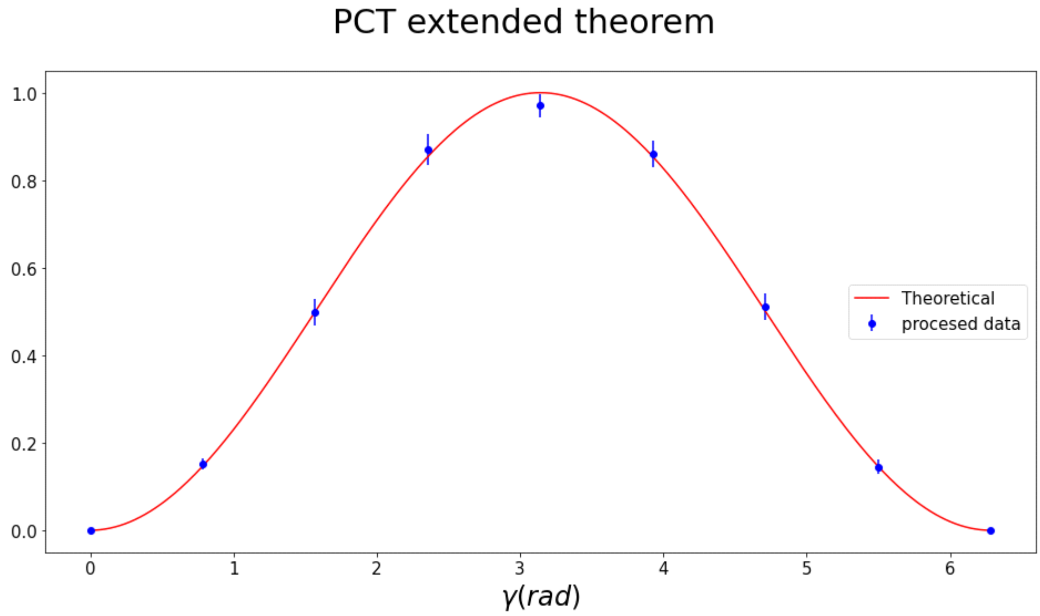
Noisy quantum computer



Quantum error mitigation



Conclusion

- By using QEM I improved the results when I had used a noisy model of a quantum computer

References

[1]. Hidden coherences and two-state systems, F. De Zela, Optica, https://doi.org/10.1364/OPTICA.5.000243

[2]. Experimental display of the extended polarization coherence theorem,P. Sánches, J. Gonzales, V. Avalos, F.Auccapuclla, E. Suarez and F. De Zela, https://doi.org/10.1364/OL.44.001052