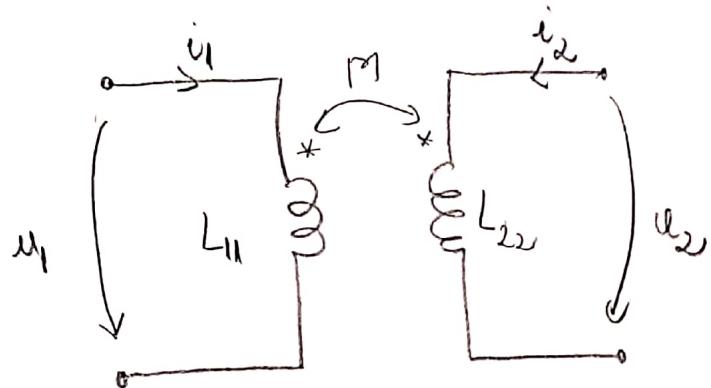


## CURENT ALTERNATIV

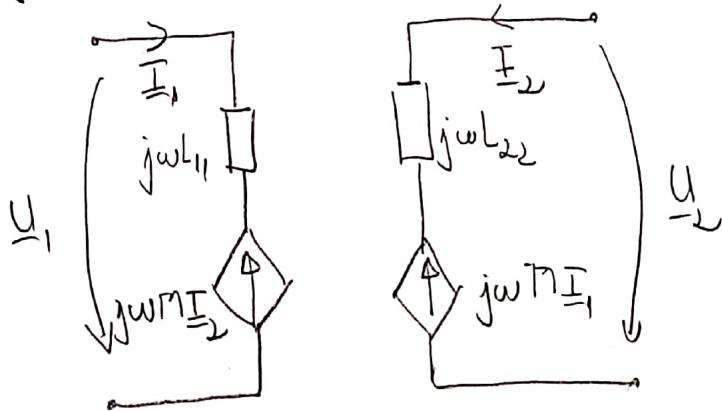
BOBINE IDEALE CU PLATE



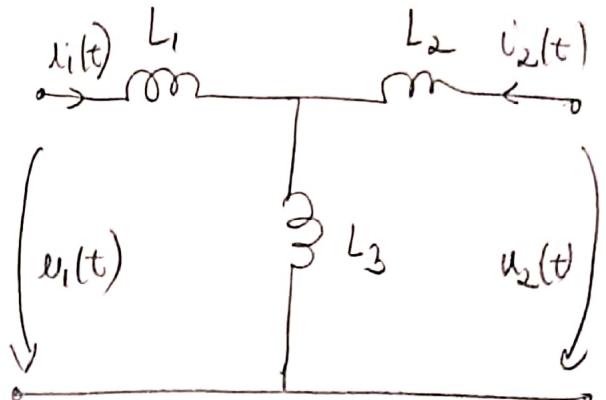
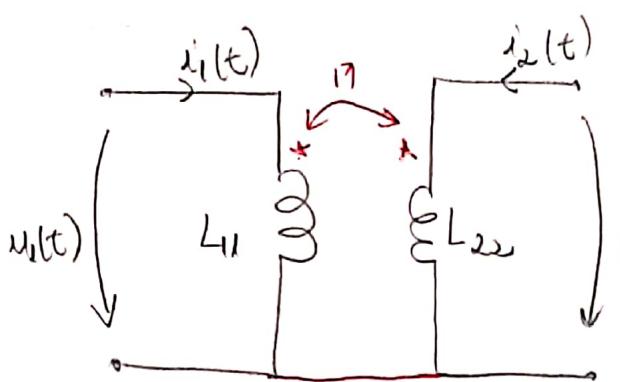
$$\begin{cases} u_1(t) = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} \\ u_2(t) = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt} \end{cases}$$

 $L_{11}$  și  $L_{22} \rightarrow$  inductivitate proprie $L_{12} = L_{21} = M \rightarrow$  inductivitate mutuale

$$\begin{cases} \underline{U}_1 = j\omega L_{11} \underline{I}_1 + j\omega M \underline{I}_2 \\ \underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_{22} \underline{I}_2 \end{cases} \Rightarrow \begin{cases} \underline{U}_1 = Z_{11} \underline{I}_1 + Z_M \underline{I}_2 \\ \underline{U}_2 = Z_M \underline{I}_1 + Z_{22} \underline{I}_2 \end{cases}$$

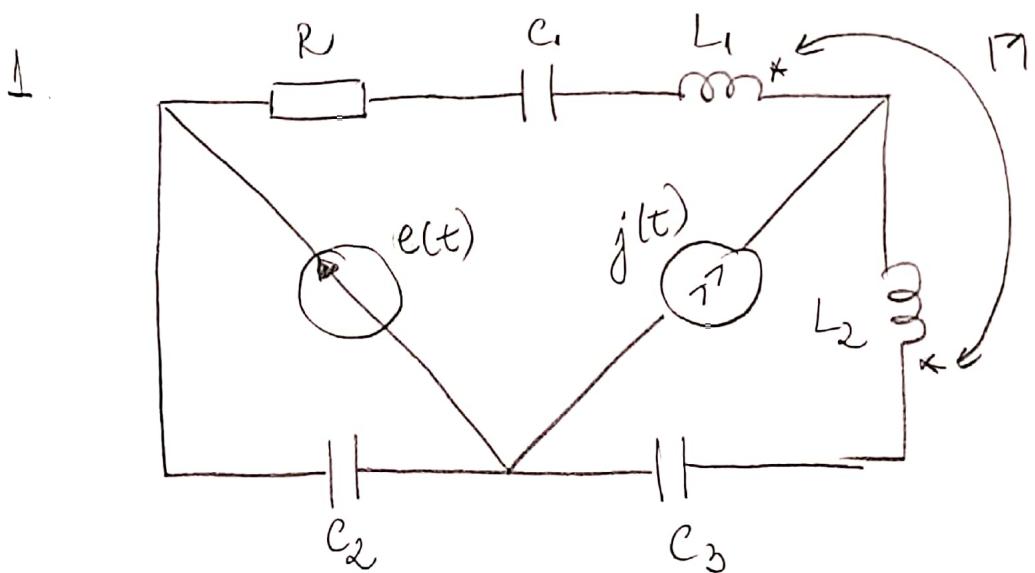


CASO PARTICULAR.



$$L_1 = L_{11} - M; \quad L_2 = L_{22} - M; \quad L_3 = M.$$

PROBLEME RESOLVATE



$$R = 10 \Omega; \quad C_1 = \frac{1000}{\pi} \mu F; \quad C_2 = \frac{2000}{3\pi} \mu F; \quad C_3 = \frac{500}{\pi} \mu F$$

$$L_1 = \frac{100}{\pi} mH; \quad L_2 = \frac{200}{\pi} mH; \quad M = \frac{100}{\pi} mH.$$

$$f = 50 \text{ Hz}.$$

$$e(t) = 30 \sqrt{2} \sin(\omega t + \frac{\pi}{2})$$

$$j(t) = \sqrt{2} \sin(\omega t - \frac{\pi}{2})$$

PAG-2  
/21

PASUL 1 Reprezentare in complex a surselor independente

$$j(t) = \sqrt{2} \sin(\omega t - \frac{\pi}{2}) \rightarrow 1 e^{j(-\frac{\pi}{2})} = \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} = -j$$

$$e(t) = 30\sqrt{2} \sin(\omega t + \frac{\pi}{2}) \rightarrow 30 e^{j\frac{\pi}{2}} = 30 \left( \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} \right) = 30j$$

$$\omega = 2\pi f = 100\pi$$

PASUL 2 Elementele paralele a retelei sunt reprezentabile in complex:

$$\underline{Z}_R = R = 10\Omega$$

$$\underline{Z}_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{j100\pi \cdot \frac{1000}{\pi} 10^{-6}} = -10j$$

$$\underline{Z}_{C_2} = \frac{1}{j\omega C_2} = \frac{1}{j100\pi \cdot \frac{2000}{3\pi} 10^{-6}} = -15j$$

$$\underline{Z}_{C_3} = \frac{1}{j\omega C_3} = \frac{1}{j100\pi \cdot \frac{500}{\pi} 10^{-6}} = -20j$$

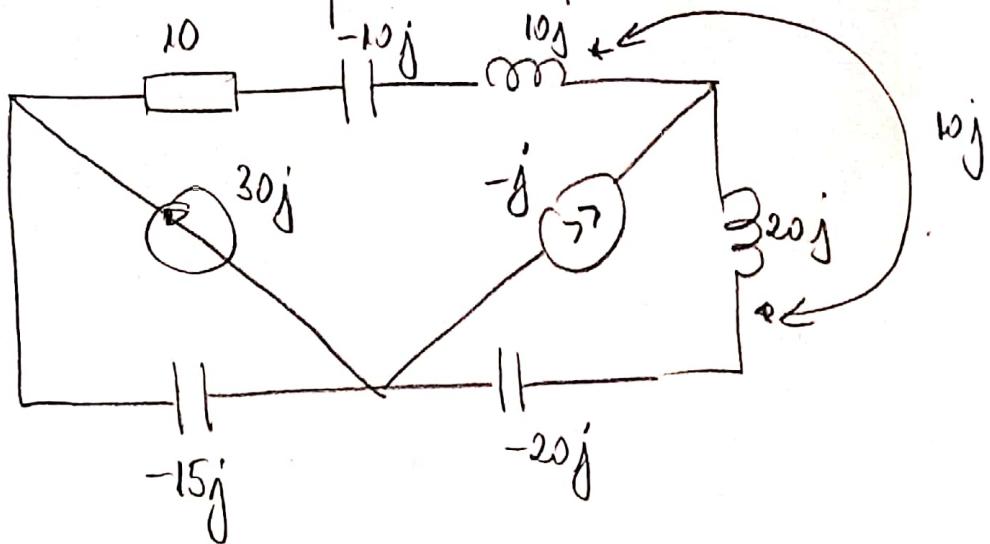
$$\underline{Z}_L = j\omega L_1 = j100\pi \cdot \frac{100}{\pi} 10^{-3} = 10j$$

$$\underline{Z}_L = j\omega L_2 = j100\pi \cdot \frac{200}{\pi} 10^{-3} = 20j$$

$$\underline{Z}_M = j\omega M = j100\pi \cdot \frac{100}{\pi} 10^{-3} = 10j$$

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Reprezentarea în complex a circuitului



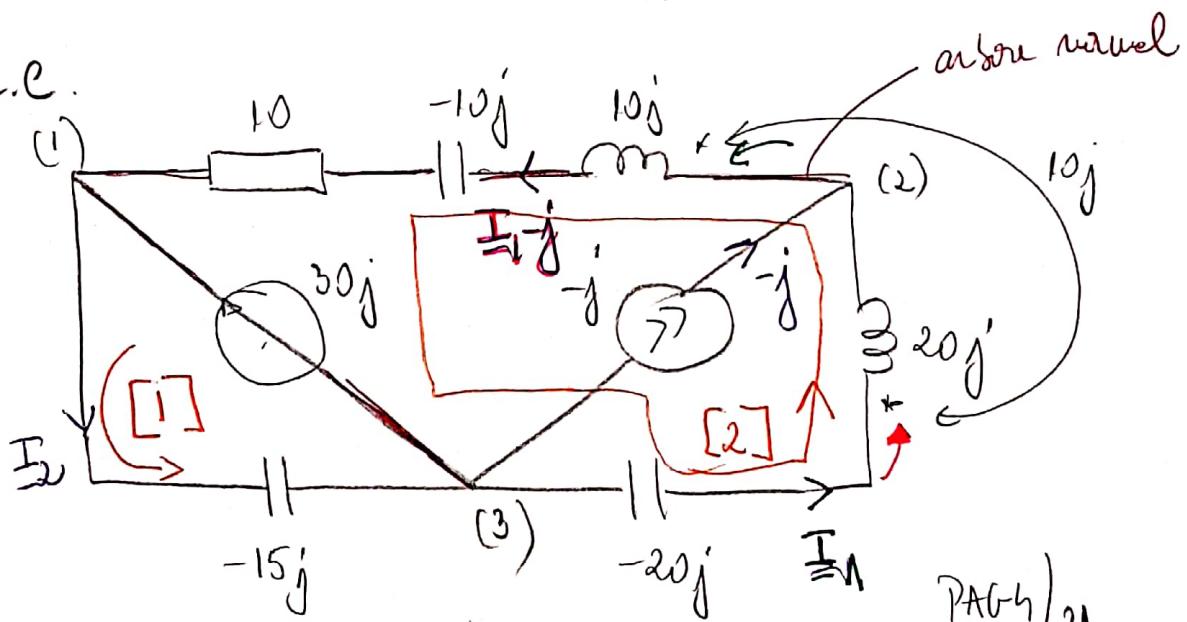
PASUL 3 Se rezolvă circuitul cu metodele de la c.c.

$$N=3; L=5; n_{SIC}=1; n_{SIT}=1$$

- metoda în curenti  $L-N+1-n_{SIT}=5-3+1-1=2$  și

- metoda în tensiuni  $N-1-n_{SIC}=3-1-1=1$  și

M.C.C.



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$$[1] \left\{ -15jI_2 = 30j \right.$$

$$[2] \left\{ (-20j + 20j)I_1 + (10 - 10j + 10j)(I_1 - j) + 10jI_1 + 10j(I_1 - j) = -30j \right.$$

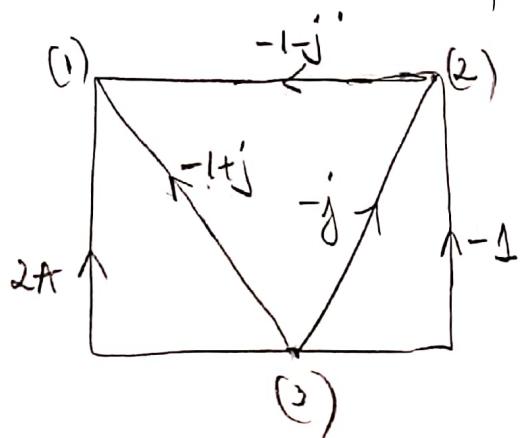
$$I_2 = -2A$$

$$10(I_1 - j) + 10j I_1 + 10j(I_1 - j) = -30j$$

$$I_1(10 + 20j) = -30j + 10j - 10 = -10 - 20j = -(10 + 20j)$$

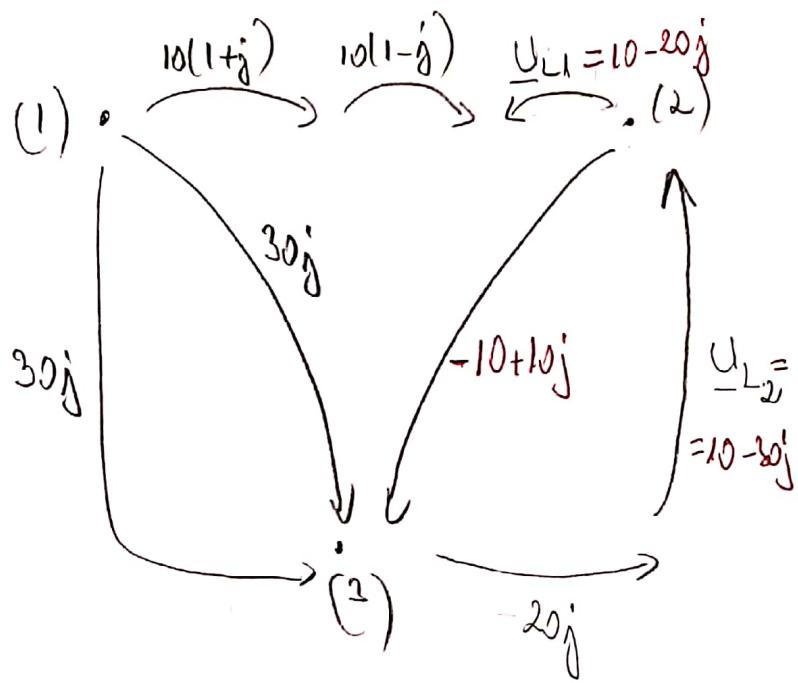
$$\Rightarrow I_1 = -1A$$

GRAFUL DE CURENTI



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GRAFUL DE TENSIUNI



$$\begin{aligned} U_{L1} &= 10j(-1-j) + 10j(-1) = \\ &= -10j + 10 - 10j = 10 - 20j \end{aligned}$$

$$\begin{aligned} U_{L2} &= 20j(-1) + 10j(-1-j) = \\ &= -20j - 10j + 10 = \\ &= 10 - 30j \end{aligned}$$

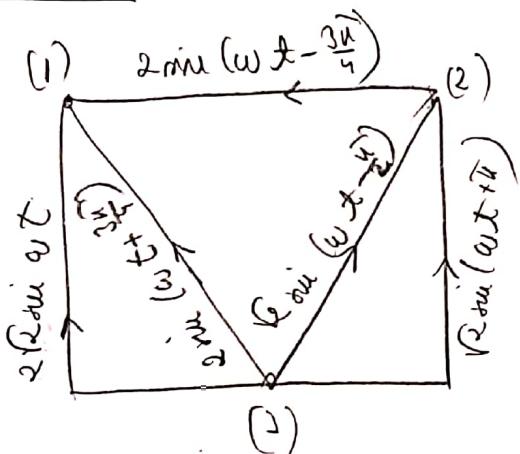
$$\underline{S}_c = (10 - 10j + 10j) (1^2 + 1^2) + (-15j) \cdot 2^2 + (-20j + 20j) \cdot 1^2 + \\ + 10j [(-1-j) \cdot (-1) + (-1+j)(-1)]$$

$$\underline{S}_c = 10 \times 2 - 60j + 10j [1+j + 1-j] = 20 - 60j + 20j = 20 - 40j$$

$$\underline{S}_g = 30j (-1+j)^2 + (-10+10j) (-j)^2 = \\ = 30j (-1-j) + (-10+10j) j = -30j + 30 - 10j - 10 = 20 - 40j$$

$$\Rightarrow \underline{S}_c = \underline{S}_g$$

PASUL h: Zeiger in Stimp.



$$-1-j \Rightarrow | -1-j | = \sqrt{2}$$

$$\left. \begin{array}{l} \cos \alpha = -\frac{\sqrt{2}}{2} \\ \sin \alpha = -\frac{\sqrt{2}}{2} \end{array} \right\} \alpha = -\frac{3\pi}{4}$$

$$-1-j \rightarrow \sqrt{2}, \sqrt{2} \sin \left( \omega t - \frac{3\pi}{4} \right) = \\ = 2 \sin \left( \omega t - \frac{3\pi}{4} \right)$$

$$-1+j \Rightarrow | -1+j | = \sqrt{2}$$

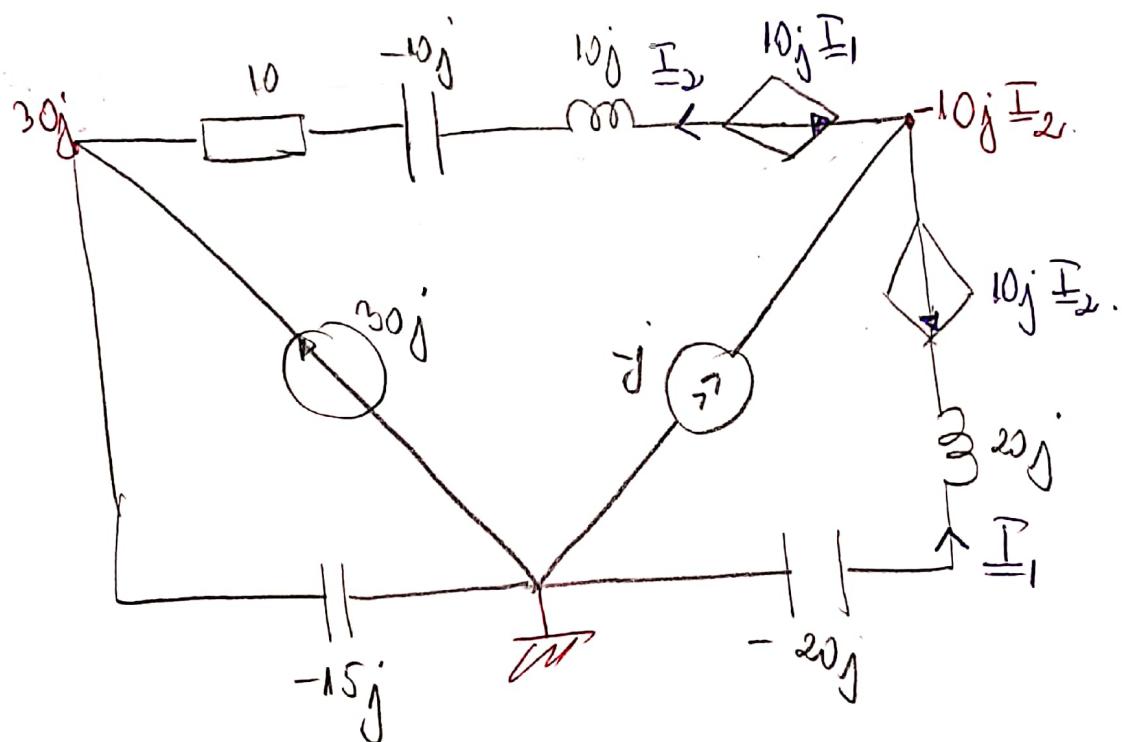
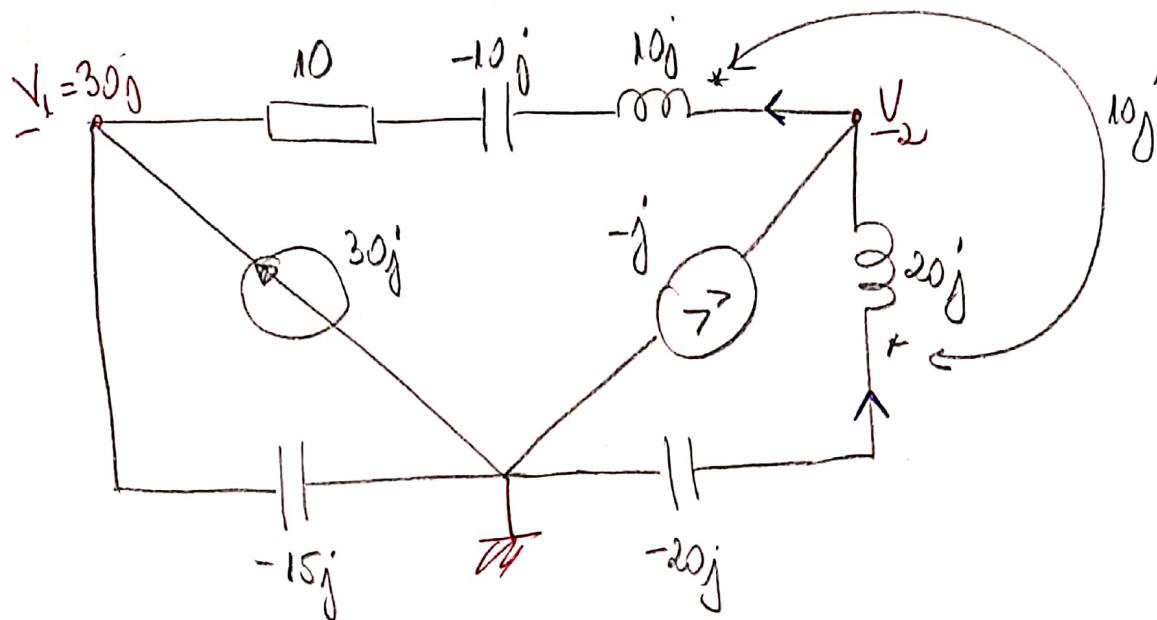
$$\left. \begin{array}{l} \cos \alpha = -\frac{\sqrt{2}}{2} \\ \sin \alpha = \frac{\sqrt{2}}{2} \end{array} \right\} \alpha = \frac{3\pi}{4} \quad \text{PA 6/21}$$

$$\Rightarrow -1+j \rightarrow 2 \sin \left( \omega t + \frac{3\pi}{4} \right)$$

$$-j \Rightarrow | -j | = 1 \quad \left. \begin{array}{l} \cos \alpha = 0 \\ \sin \alpha = -1 \end{array} \right\} \alpha = -\frac{\pi}{2} \Rightarrow -j \rightarrow \sqrt{2} \sin \left( \omega t - \frac{\pi}{2} \right)$$

$$-1 \Rightarrow | -1 | = 1 \quad \left. \begin{array}{l} \cos \alpha = -1 \\ \sin \alpha = 0 \end{array} \right\} \alpha = \pi \Rightarrow -1 \rightarrow \sqrt{2} \sin \left( \omega t + \pi \right)$$

Mehrde Ausdrücke.



$$\left\{ \begin{array}{l} \underline{I}_2 = \frac{-10j\underline{I}_2 - 10j\underline{I}_1 - 30j}{10 - 10j + 10j} \\ \underline{I}_1 - j = \underline{I}_2 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} 10\underline{I}_2 = -10j\underline{I}_2 - 10j\underline{I}_1 - 30j \\ \underline{I}_1 - j = \underline{I}_2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \underline{I}_2(1+j) + j\underline{I}_1 = -30j \\ \underline{I}_1 - j = \underline{I}_2 \Rightarrow \underline{I}_1(j + \underline{I}_2) \end{array} \right.$$

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$$\underline{I}_2(1+j) + j(j + \underline{I}_2) = -3j$$

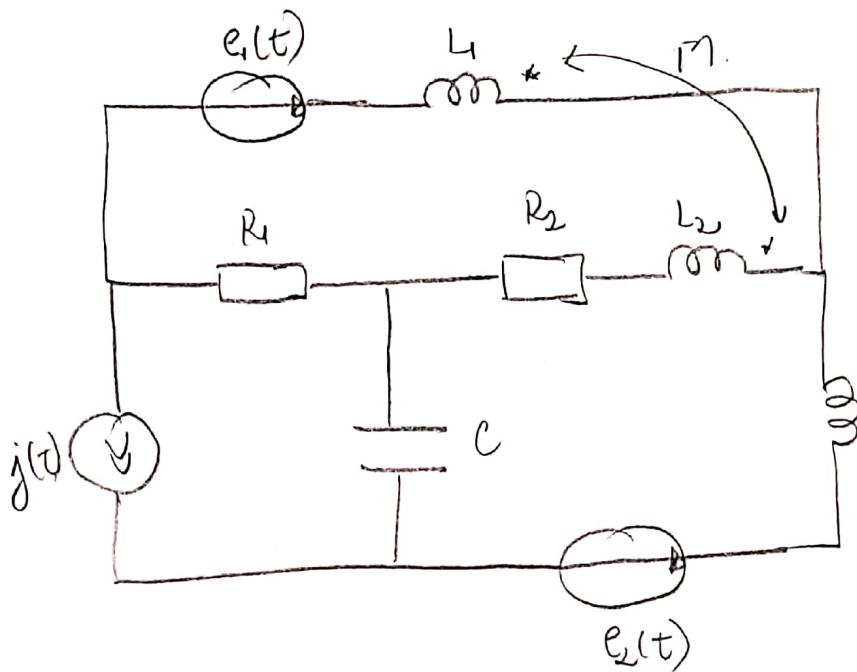
$$\underline{I}_2(1+2j) = -3j + 1 \Rightarrow \underline{I}_2 = \frac{(-3j+1)(1-2j)}{5}$$

$$\boxed{\underline{I}_2 = \frac{-3j-6+1-2j}{5} = -1-j}$$

$$\boxed{\underline{I}_1 = \underline{I}_2 + j = -1-j+1 = -1}$$

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# PROBLEMA REZOLVATĂ 2



$$R_1 = R_2 = 10 \Omega$$

$$L_1 = \frac{150}{\pi} \mu H$$

$$L_2 = \frac{50}{\pi} \mu H$$

$$L_3 = \frac{50}{\pi} \mu H$$

$$R_3 = \frac{50}{\pi} \mu H$$

$$C = \frac{1000}{\pi} \mu F$$

$$e_1(t) = 20\sqrt{2} \sin \omega t$$

$$e_2(t) = 40\sqrt{2} \sin \omega t$$

$$j(t) = 2\sqrt{2} \sin \left( \omega t - \frac{\pi}{2} \right)$$

PASUL 1 Reprezentare în complex a surselor independente

$$e_1(t) = 20\sqrt{2} \sin \omega t \rightarrow \underline{E}_1 = 20$$

$$e_2(t) = 40\sqrt{2} \sin \omega t \rightarrow \underline{E}_2 = 40$$

$$j(t) = 2\sqrt{2} \sin \left( \omega t - \frac{\pi}{2} \right) \rightarrow \underline{j} = 2e^{j\left(-\frac{\pi}{2}\right)} = -2j$$

PASUL 2 Elementele passive se introduc în reprezentările lor în complex

$$\underline{z}_{R_1} = R_1 = 10 \quad ; \quad \underline{z}_{R_2} = R_2 = 10$$

Patru g / 21

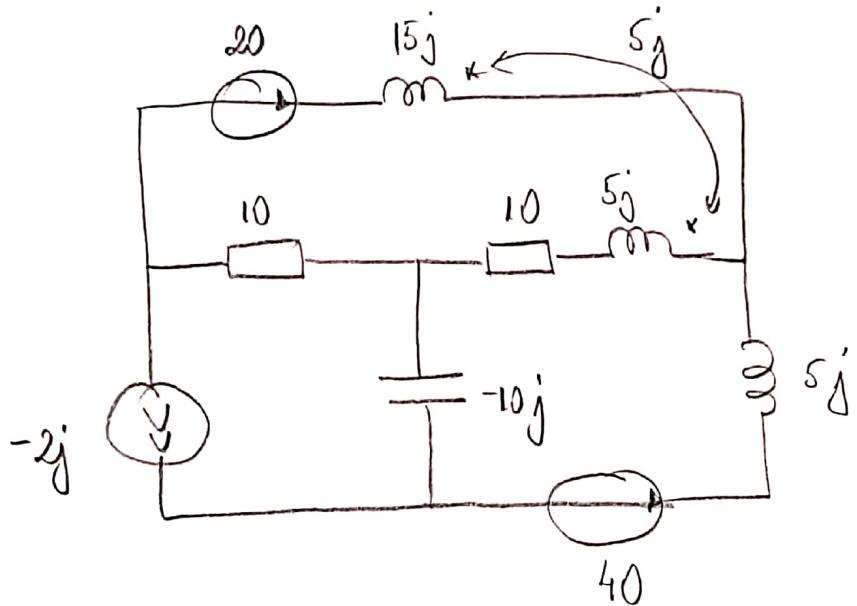
$$Z_L = j\omega L_1 = j 100\pi \cdot \frac{150}{\pi} \cdot 10^3 = 15j$$

$$Z_{L_2} = Z_{L_3} = j\omega L_{2,3} = j \cdot 100\pi \cdot \frac{50}{\pi} 10^3 = 5j$$

$$Z_H = j\omega M = 5j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j 100\pi \cdot \frac{1000}{\pi} 10^{-6}} = -10j$$

Reprezentare în complex a circuitului.



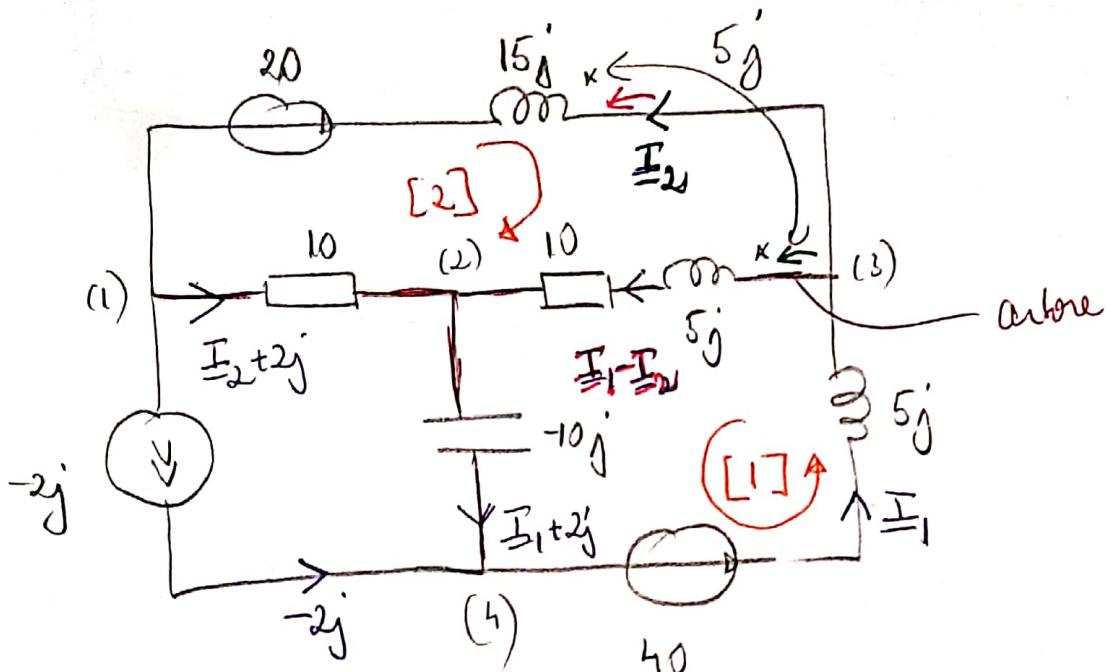
PASUL 3 Se regăsește curentul cu metodele de la c.e.

$$N=4 ; L=6 ; n_{SIC}=1 ; n_{SIT}=0.$$

- metoda în curenti:  $L-N+1-n_{SIT}=6-4+1-0=3$  dc
- metoda în tensiuni:  $N-1-n_{SIC}=4-1-1=2$  dc.

M.c.c.

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$$[1] \left\{ 5j I_1 + (10 + 5j)(I_1 - I_2) - 10j(I_1 + 2j) + 5j I_2 = 40 \right.$$

$$[2] \left\{ (10 + 5j)(I_1 - I_2) - 10(I_2 + 2j) - 15j I_2 - 5j(I_1 - I_2) + 5j I_2 = 20 \right.$$

$$\left\{ I_1(5j + 10 + 5j - 10j) + I_2(-10 - 5j + 5j) = 40 - 20 \right.$$

$$\left\{ I_1(10 + 5j - 5j) - I_2(10 + 5j + 10 + 15j - 5j - 5j) = 20 + 20j \right.$$

$$\left\{ 10 I_1 - 10 I_2 = 20 \right.$$

$$\left. 10 I_1 - I_2 (20 + 10j) = 20 + 20j \right.$$

$$\left\{ I_1 - I_2 = 2 \right.$$

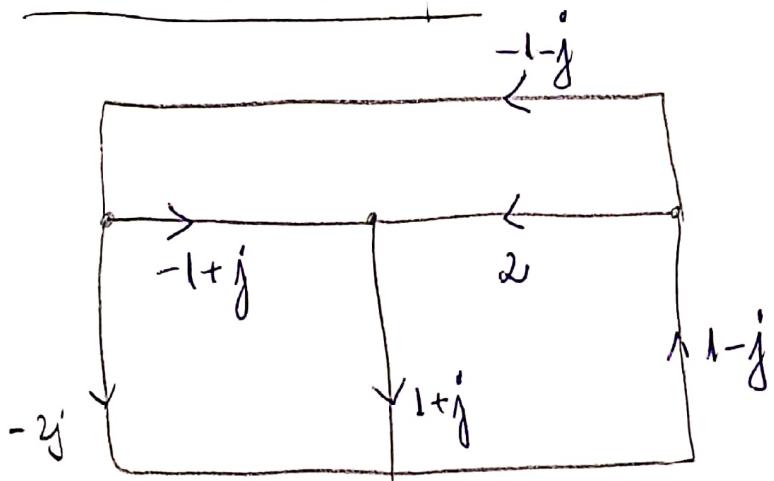
$$\left. I_1 - I_2 (2 + j) = 2 + 2j \right.$$

$$\Leftrightarrow I_2(2 + j - 1) = -2 - 2j + 2$$

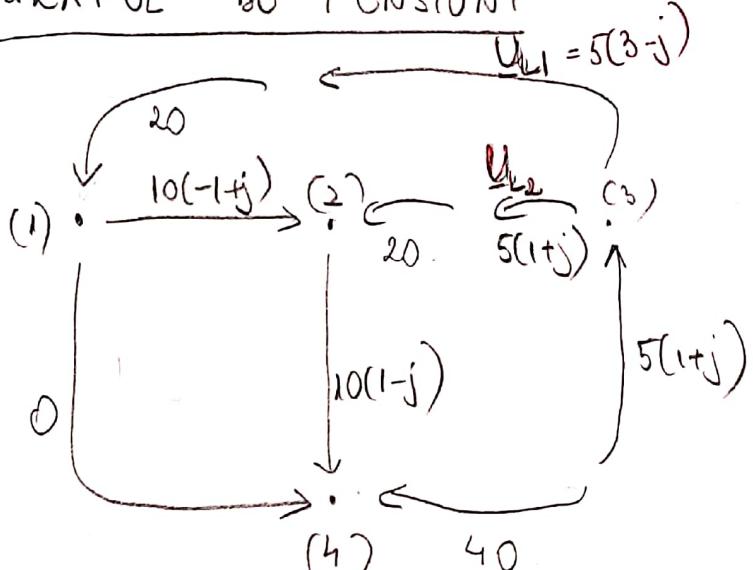
$$I_2(1 + j) = -2j \Rightarrow I_2 = \frac{-2j(1 - j)}{2} = -j - 1$$

$$\boxed{I_1 = 2 + I_2 = 2 - j - 1 = 1 - j}$$

### GRAFUL DE CURENTI



### GRAFUL DE TENSIUNI



$$U_{L_2} = 5j \times 2 + 5j(-1-j) = \\ = 10j - 5j + 5 = 5 + 5j$$

$$U_{L_1} = 15j(-1-j) + 5j \cdot 2 = \\ = -15j + 15 + 10j = \\ = 15 - 5j = 5(3-j)$$

Bilouțul puternic

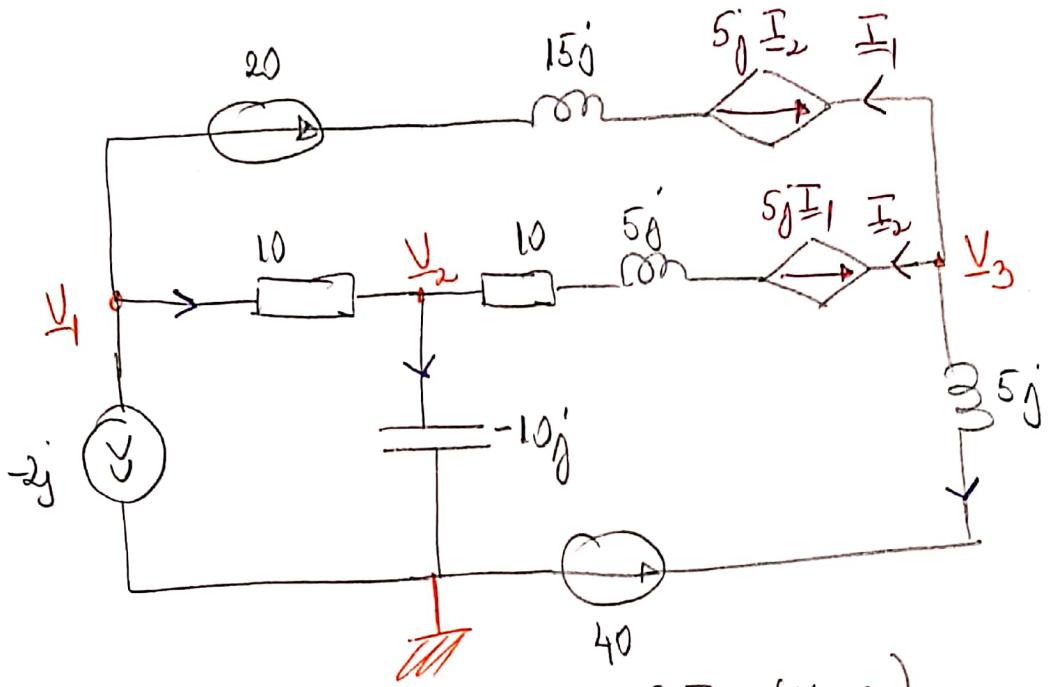
$$S_C = 10(1+j) - 10j(1+j) + (10 + 5j) + 2^2 + 5j(1+j) + 5j((-1+j) \cdot 2 + \\ + 15j(1+j))$$

$$S_C = 20 - 20j + 40 + 20j + 10j + 5j(-2 + 2j - 2 - 2j) + 30j \quad \text{PAG-12/21}$$

$$S_C = 60 + 10j - 20j + 30j = 60 + 20j$$

$$S_g = 0 \cdot (-2j)^* + 40 \cdot (1-j)^* + 20(1+j)^* = 40(1+j) + 20(1-j) = \\ = 40 + 40j + 20 - 20j = 60 + 20j$$

## METODA MODALĂ

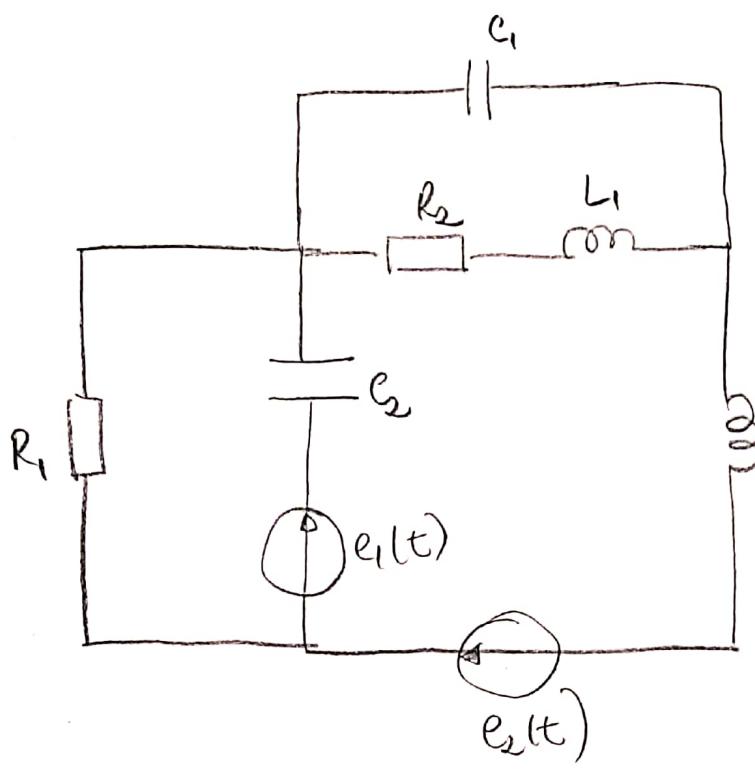


$$\left\{ \begin{array}{l} \text{TKI}_{V_1} : -2j + \frac{V_1 - V_2}{10} - \frac{V_3 - 5jI_2 - (V_1 + 20)}{15j} = 0 \\ \text{TKI}_{V_2} : - \frac{V_1 - V_2}{10} + \frac{V_2 - 0}{-10j} - \frac{V_3 - 5jI_1 - V_2}{10 + 5j} = 0 \\ \text{TKI}_{V_3} : + \frac{V_3 - 40}{5j} + \frac{V_3 - 5jI_1 - V_2}{10 + 5j} + \frac{V_3 - 5jI_2 - (V_1 + 20)}{15j} = 0 \end{array} \right.$$

$$I_1 = \frac{V_3 - 5jI_2 - (V_1 + 20)}{15j}$$

$$I_2 = \frac{V_3 - 5jI_1 - V_2}{10 + 5j}$$

PROBLEMA REZOLVATĂ 3



$$R_1 = 5\Omega$$

$$R_2 = 10\Omega$$

$$C_1 = \frac{500}{\pi} \mu F$$

$$L_1 = \frac{100}{\pi} \mu H$$

$$L_2 = \frac{300}{\pi} \mu H$$

$$C_2 = \frac{1000}{\pi} \mu F$$

$$e_1(t) = 20 \sin(\omega t - \frac{\pi}{4})$$

$$e_2(t) = 60 \sin(\omega t - \frac{3\pi}{4})$$

PASUL 1 - reprezentarea în complex a sursei independente

$$e_1(t) = 20 \sin(\omega t - \frac{\pi}{4}) \rightarrow E_1 = \frac{20}{\sqrt{2}} e^{j(-\frac{\pi}{4})} = \frac{20}{\sqrt{2}} \left( \cos \frac{\pi}{4} - j \sin \frac{\pi}{4} \right) = \frac{20}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} (1-j) = 10(1-j)$$

$$e_2(t) = 60 \sin(\omega t - \frac{3\pi}{4}) \rightarrow E_2 = \frac{60}{\sqrt{2}} e^{j(-\frac{3\pi}{4})} = \frac{60}{\sqrt{2}} \left( \cos \frac{3\pi}{4} - j \sin \frac{3\pi}{4} \right) = \frac{60}{\sqrt{2}} \left( -\frac{\sqrt{2}}{2} (1+j) \right) = -30(1+j)$$

$$\omega = 2\pi f = 100\pi$$

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PASUL 2 Elementele pozitive se înlocuiesc cu reprezentările lor în complex

$$Z_{R_1} = R_1 = 5 \Omega; \quad Z_{R_2} = R_2 = 10 \Omega$$

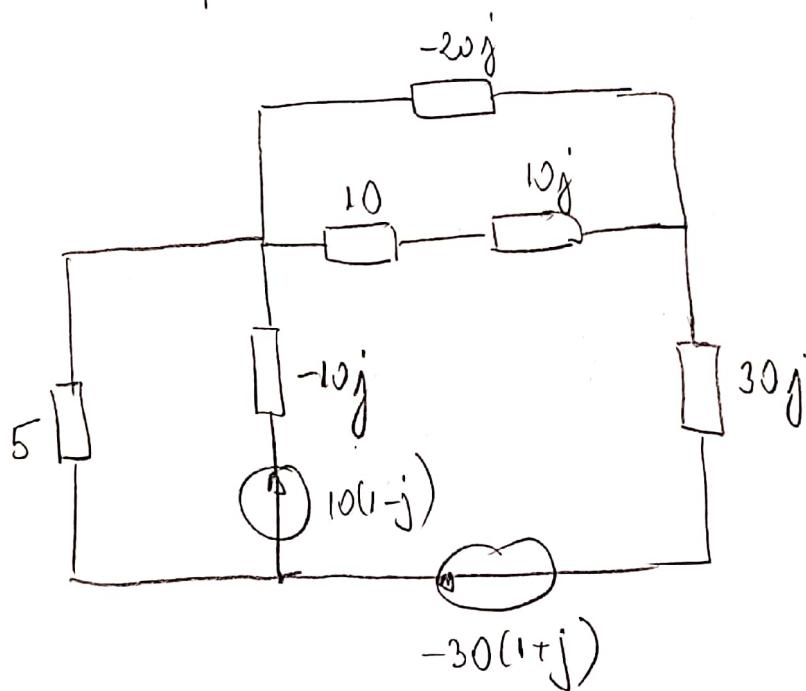
$$Z_{L_1} = j\omega L_1 = j 100\pi \cdot \frac{100}{\pi} 10^{-3} = 10j$$

$$Z_{L_2} = j\omega L_2 = j 100\pi \cdot \frac{300}{\pi} 10^{-3} = 30j$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j 100\pi \cdot \frac{500}{\pi} 10^{-6}} = -20j$$

$$Z_{C_2} = \frac{1}{j\omega C_2} = \frac{1}{j 100\pi \cdot \frac{1000}{\pi} 10^{-6}} = -10j$$

Reprezentare în complex a circuitului



$$H = 3$$

$$L = 5$$

$$n_{SIC} = 0$$

$$n_{SIT} = 0.$$

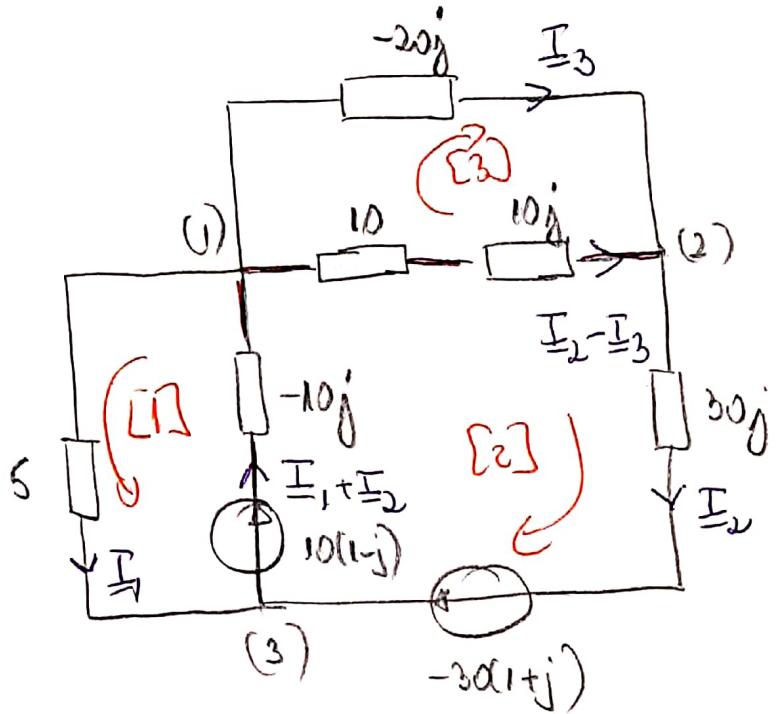
PAG 15/21

PASUL 3: să rezolvă circuitul cu neteriole de c.c.

- neteriole în treapta  $H-1 - n_{SIT} = 2$  ec.

- neteriole în curent  $L-N+1 - n_{SIC} = 5-3+1-0 = 3$  ec.

M.C.C.



$$\left\{ \begin{array}{l} [1] \quad 5I_1 - 10j(I_1 + I_2) = 10(1-j) \\ [2] \quad 30jI_2 - 10j(I_1 + I_2) + (10 + 10j)(I_2 - I_3) = -30(1+j) + 10(1-j) \\ [3] \quad -20jI_3 - (10 + 10j)(I_2 - I_3) = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} I_1 - 2jI_1 - 2jI_2 = 2(1-j) \\ 3jI_2 - j(I_1 + I_2) + (1+j)(I_2 - I_3) = -3(1+j) + (1-j) \\ -2jI_3 - (1+j)(I_2 - I_3) = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} I_1(1-2j) - 2jI_2 = 2(1-j) \quad (\alpha\alpha) \\ -jI_1 + I_2(3j - j + 1 + j) - (1+j)I_3 = -2 - 4j \quad (***) \\ -(1+j)I_2 + I_3(1+j - 2j) = 0 \Rightarrow (1+j)I_2 = I_3(1-j) \end{array} \right.$$

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$$\boxed{I_3 = I_2 \frac{1+j}{1-j} = I_2 \frac{(1+j)^2}{2} = I_2 \frac{2j}{2} = jI_2. \quad (**)}$$

(+)  $\rightarrow (*+)$

$$-jI_1 + I_2 (3j+1) - (1+j) j I_2 = -2 - 4j$$

$$-jI_1 + I_2 (3j+1-j+1) = -2 - 4j$$

$$-jI_1 + I_2 (2j+2) = -2 - 4j$$

$$\left\{ \begin{array}{l} -jI_1 + 2(1+j)I_2 = -2 - 4j \\ I_1(1-2j) - 2jI_2 = 2(1-j) \end{array} \right| \cdot j$$

$$I_1 [1 + (1-2j)(1+j)] = -2j + 4 + 2(1-j)(1+j)$$

$$I_1 [1 + 1 + j - 2j + 2] = -2j + 4 + 4 \Rightarrow 8 - 2j$$

$$I_1 [4 - j] = 2 [4 - j] \Rightarrow \boxed{I_1 = 2A}$$

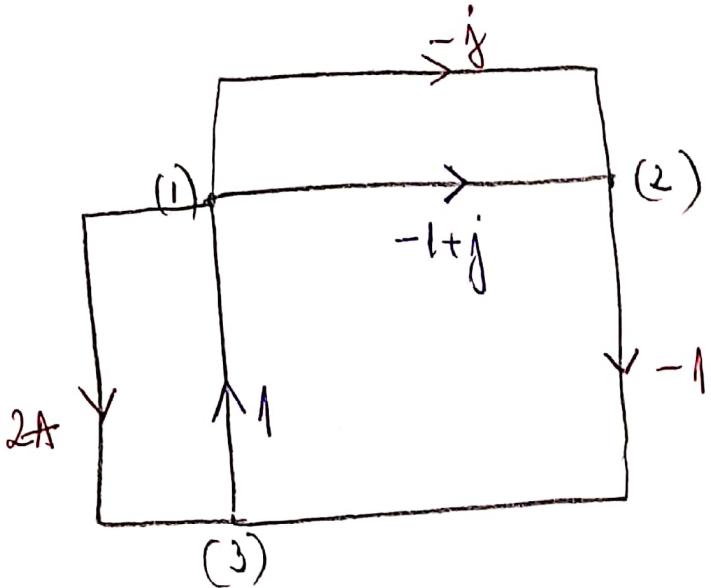
$$2(1-2j) - jI_2 = 2(1-j)$$

$$-2j - j + j = jI_2 \Rightarrow -j = jI_2 \Rightarrow \boxed{I_2 = -1A}$$

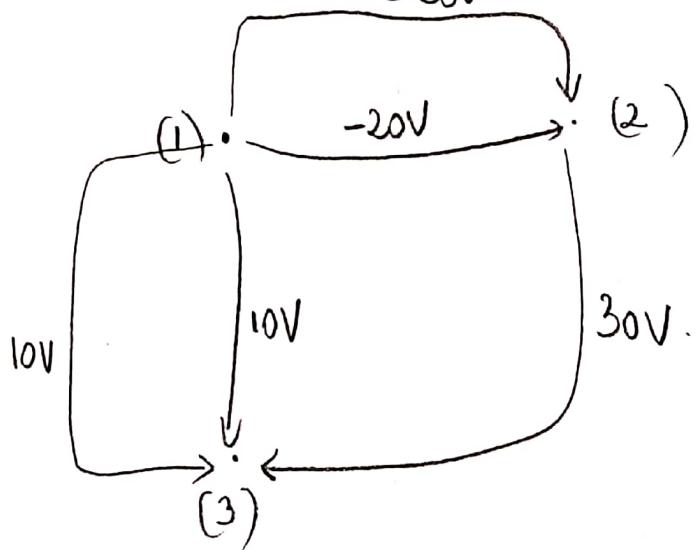
$$\boxed{I_3 = jI_2 = -j}$$

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## GRAFUL DE CIRCUIT



## GRAFUL DE TENSIUNI



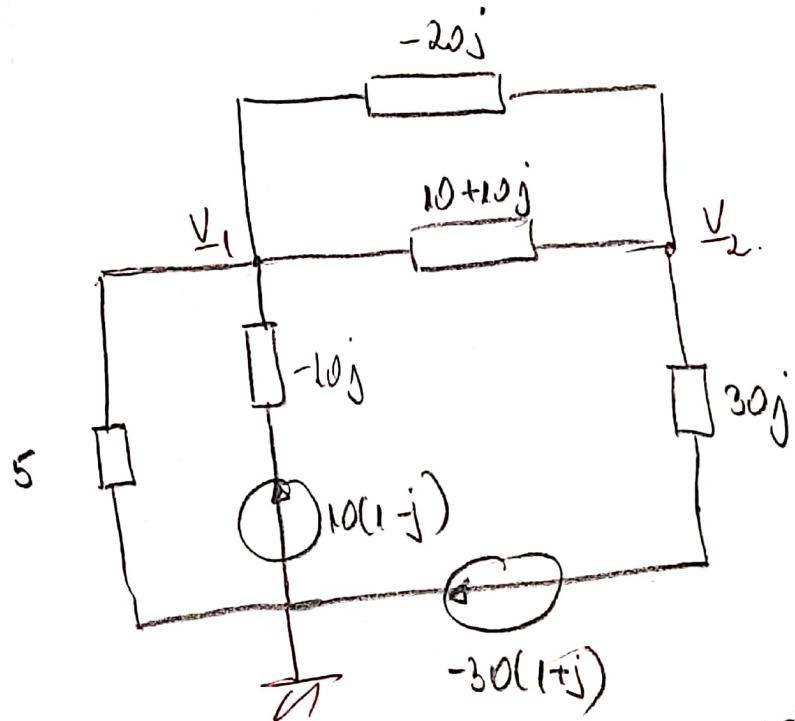
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Bilanțul puterilor

$$\begin{aligned}
 S_C &= 5 \times 2^2 + 10(1+j)(1+j) - 20j \cdot 1^2 + 30j \cdot 1^2 - 10j \cdot 1^2 = \\
 &= 20 + 20(1+j) - 20j + 30j - 10j = 20(2+j) = 40 + 20j
 \end{aligned}$$

$$\begin{aligned}
 S_g &= 10(1-j) \cdot 1^2 - 30(1+j) \cdot (-1)^2 = \\
 &= 10(1-j) + 30(1+j) = 10[1-j + 3 + 3j] = 10(4+2j) = \\
 &= 40 + 20j
 \end{aligned}$$

# Metoda nodalna



$$\begin{aligned}V_1: & \left\{ \frac{V_1}{5} + \frac{V_1 - 10(1-j)}{-10j} + \frac{V_1 - V_2}{-20j} + \frac{V_1 - V_2}{10(1+j)} = 0\right. \\V_2: & \left. \frac{V_2 - 30(1+j)}{30j} + \frac{V_2 - V_1}{-20j} + \frac{V_2 - V_1}{10(1+j)} = 0\right.\end{aligned}$$

$$\left\{ \begin{array}{l} \frac{V_1}{5} + \frac{jV_1 - 10j(1-j)}{10} + \frac{jV_1 - jV_2}{20} + \frac{(V_1 - V_2)(1-j)}{20} = 0 \\ \frac{-jV_2 + 30j(1+j)}{30} + \frac{jV_2 - jV_1}{20} + \frac{(V_2 - V_1)(1-j)}{20} = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} 4V_1 + 2jV_1 - 20j(1-j) + jV_1 - jV_2 + V_1(1-j) - V_2(1-j) = 0 \\ -2jV_2 + 60j(1+j) + 3jV_2 - 3jV_1 + 3(1-j)V_2 - 3(1-j)V_1 = 0 \end{array} \right.$$

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$$\begin{cases} \underline{V}_1(4+2j+j+1-j) + \underline{V}_2(-j-1+j) = 20j(1-j) \\ \underline{V}_1(-3j-3+3j) + \underline{V}_2(-2j+2j+3-3j) = -60j(1+j) \end{cases}$$

$$\begin{cases} \underline{V}_1(5+2j) - \underline{V}_2 = 20(1+j) \\ -3\underline{V}_1 + \underline{V}_2(3-2j) = 60(1-j) \end{cases} \quad | \cdot 3-2j$$

$$\underline{V}_1[(5+2j)(3-2j) - 3] = 20(1+j)(3-2j) + 60(1-j)$$

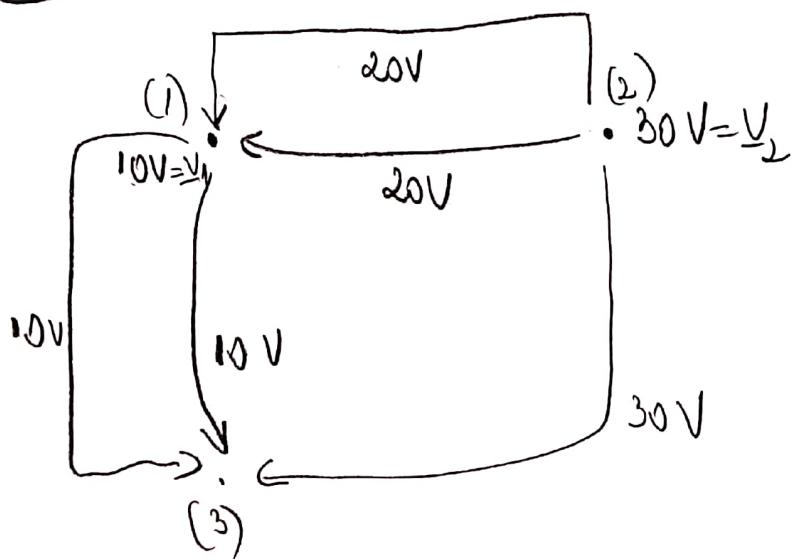
$$\underline{V}_1[15 - 10j + 6j + 4 - 3] = 20[3 - 2j + 3j + 2 + 3 - 3j]$$

$$\underline{V}_1[16 - 4j] = 20[8 - 2j] \Rightarrow \underline{V}_1 \cancel{4(1+j)} = \cancel{20} \times 2 \cancel{(4-j)}$$

$$\boxed{\underline{V}_1 = 10V}$$

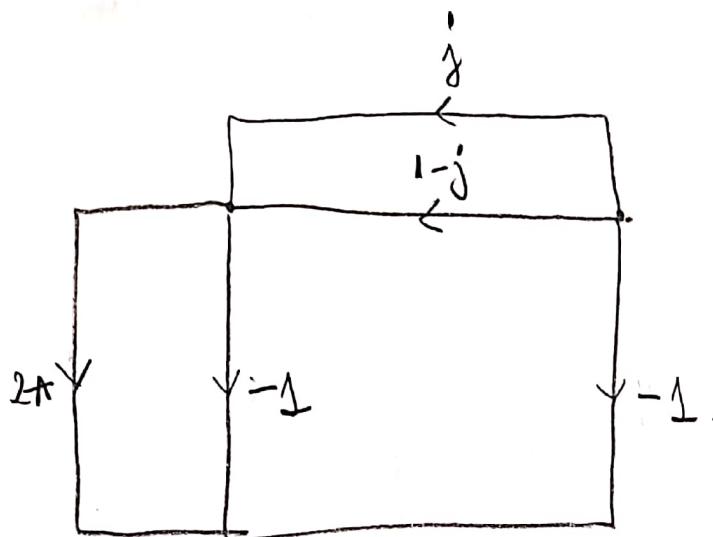
$$10(5+2j) - 20(1+j) = \underline{V}_2$$

$$\boxed{\underline{V}_2 = 10[5+2j-2-2j] = 30V}$$



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GRAFUL DE CURENTI

PASUL 4 : Se rezolvă în stîrpe

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