BS2280 - Econometrics 1

Lecture 9 - Part 1: Nonlinear Models and Transformation of Variables I

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Structure of today's lecture

1. Background

2. Linear and Nonlinear in Variables

3. Linear and Nonlinear in Parameters

Intended Learning Outcomes

- Understanding methods how models that are non-linear in variables and parameters can be estimated with OLS

Background

- Until now we assumed that our regression model is linear in variables and parameters
- You will come across many situations where a nonlinear regression model is superior!
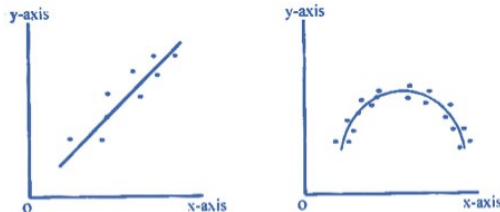
- **Example.** Engel curve showing the relationship between consumption of single good and income

Linear and Nonlinear in Variables

- **Linear in variables**: The variables included on the right side of the regression are exactly as defined, rather than as functions

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + ... + \beta_k X_{ki} + u_i$$

- **Example.**
- Linear in variables:

$$EARNINGS_i = \beta_1 + \beta_2 S_i + \beta_3 EXP_i + u_i$$

- Nonlinear in variables:

$$EARNINGS_i = \beta_1 + \beta_2 S_i^2 + \beta_3 \sqrt{EXP_i} + u_i$$

- The variables included on the right side of the regression are defined using functions

Linear and Nonlinear in Variables

- Nonlinear in variables:

$$EARNINGS_i = \beta_1 + \beta_2 S_i^2 + \beta_3 \sqrt{EXP_i} + u_i$$

- Good news: We can easily estimate this equation with some adjustments. See below.

$$let\ Z_{2i} = S_i^2\ and\ Z_{3i} = \sqrt{EXP_i}$$

- Then the regression model changes to

$$EARNINGS_i = \beta_1 + \beta_2 Z_{2i} + \beta_3 Z_{3i} + u_i$$

Linear and Nonlinear in Parameters

Linear in parameters: The parameters $\beta_2, \beta_3, ..., \beta_k$ are multiplied with the *X* variables.

Assumption 1. Model is linear in parameters and correctly specified:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + ... + \beta_k X_{ki} + u_i$$

Example.

Linear in parameters (this model is also linear in variables):

$$EARNINGS_i = \beta_1 + \beta_2 S_i + \beta_3 EXP_i + u_i$$

Nonlinear in parameters:

$$EARNINGS_i = \beta_1 + S_i^{\beta_2} + \beta_2 \beta_3 EXP_i + u_i$$

The parameter is not just a simple multiplication with S_i or EXP_i anymore!!!

Linear and Nonlinear in Parameters

Nonlinear in parameters:

$$EARNINGS_i = \beta_1 + S_i^{\beta_2} + \beta_2\beta_3 EXP_i + u_i$$

The parameter is not just a simple multiplication with S_i or EXP_i anymore!!!

Good news: Some models which are nonlinear in parameters can be linearised!

Bad news: This one is not one of those that can be linearised.