BS2280 - Econometrics 1

Lecture 2 - Part 1: Analysing Economic Relationships

by Dr Yichen Zhu

Outline

- Data Structure
- Covariance and Correlation
- Simple Regression Analysis

- Data sets for econometric analysis generally come in three forms:
 - 1) Cross-section data
 - 2) Time series data
 - 3) Panel data
- Each gives rise to particular econometric challenges
- Large literature in econometrics devoted to each
- We will focus on cross-sectional data, but useful to be able to understand and recognise each of these different data structure

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- Observations on different individuals at a single point in time
- Individuals can be people, companies, countries, households, etc.

Person	Year	Income (GBP)	Gender
1	2010	22,000	Male
2	2010	23,265	Male
3	2010	58,000	Female
4	2010	55,998	Male
5	2010	15,000	Male
6	2010	12,350	Female

- Note: only the person changes the year is the same for everyone, i.e. each person is observed in year 2010 only.
- Each person is observed only once in the period under consideration.

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- Conventional to refer to variables sourced from cross-sectional data with a subscript i
- *Income*ⁱ refers to the income for each individual *i*, hence:

$$(i = 1) \rightarrow Income_1 = £22,000$$

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and so on

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- Observations on one individual at several points in time
- Time can be (mili-)seconds, minutes, days, weeks, months, quarters, years, five-year periods, etc.

Person	Year	Income (GBP)	Gender
3	2010	58,000	Female
3	2011	59,740	Female
3	2012	61,533	Female
3	2013	63,378	Female
3	2014	65,280	Female
3	2015	67,433	Female

- Note: the same person, but time (year) changes.
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Time Series Data II

- Conventional to refer to variables sourced from time series data with a subscript t.
- $Income_t$ refers to the income for the individual at time t, hence: $(t=1) \rightarrow Income_{2010} = £58,000$ $(t=2) \rightarrow Income_{2011} = £59,740$
- Notations X_t and Y_t in econometrics refer to time series data. In other words, values on variables X and Y are observed over time t

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Panel Data I

- Observations on many individuals at different points in time
- A combination of cross-sectional and time series data

Person	Year	Income (GBP)	Gender
1	2010	22,000	Female
1	2011	22,240	Female
1	2012	22,442	Female
2	2010	23,265	Male
2	2011	23,963	Male
2	2012	24,682	Male
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 Examples: GDP data on all countries in the world since 1975, daily admissions in every hospital in England in 2016

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Panel Data II

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```
(i=1,t=1) 
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Data Structure Summary

Cross-sectional Data	Time Series Data	Panel Data
Different individuals, single time point	One individual, several points in time	Many individuals, different points in time
Notations X_i and Y_i	Notations X_t and Y_t	Notations X_{it} and Y_{it}
$(i=1) \rightarrow \textit{Income}_1 = £22,000$	$(t=1) \rightarrow \textit{Income}_{2010} = £58,000$	$(i = 1, t = 1) \rightarrow Income_{1, 2010} = $ £22,000
$(i=2) \rightarrow \textit{Income}_2 = £23,265$	$(t=2) \rightarrow \textit{Income}_{2011} = £59,740$	$(i = 1, t = 2) \rightarrow Income_{1, 2011} = $ £22,240

Covariance and Correlation

- We often want to analyse the relationship between two variables
- We know already know two numerical measures: Covariance and correlation

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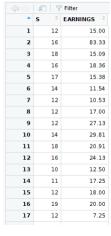
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 - Shall I attend classes? Attendance and exam marks.
 - Shall the government increase numbers of police officers? Police officers and crime rate
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Example: Earnings and education

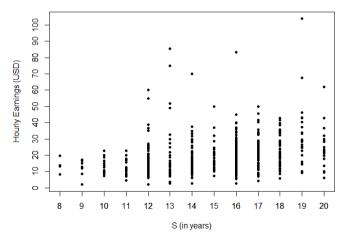


- Snapshot of data on the two variables of interest
- Useful to know summary statistics of these
- Hourly earning range from a minimum of \$2 to a maximum of approx. \$104
- School level ranges from the 8th grade (middle school) to the 20th grade (PhD)

```
> summary(EAWE21.simple)
             EARNINGS
Min.
       : 8.00
                Min.
                          2.00
1st Ou.:12.00 1st Ou.: 11.98
Median: 15.00 Median: 17.00
Mean
       :14.87
                Mean
                       : 19.58
                3rd Ou.: 23.93
3rd Qu.:17.00
       :20.00
                       :103.85
Max.
                Max.
```

Relationship between hourly wages and education

Hourly Earnings vs Schooling



Covariance

 Estimating the covariance: this measures how closely two variables are move together.

$$s_{XY} = \frac{\sum_{t=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

- Note: We usually use the sample measure rather than the population measure, therefore we divide by n-1
- Result (using R):

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> cov(EAWE21.simple$EARNINGS, EAWE21.simple$S)
[1] 9.522066
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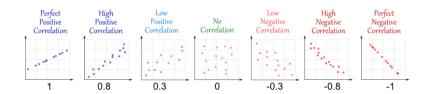
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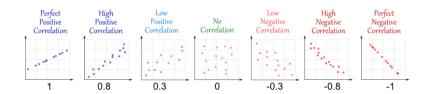
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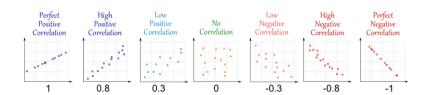
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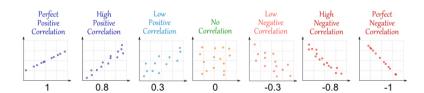
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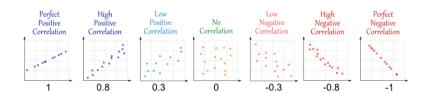
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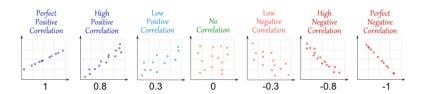
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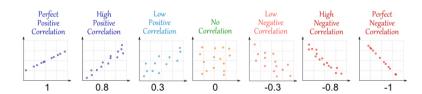
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Formula:

$$r_{XY} = \frac{s_{XY}}{s_X s_Y} - 1 \le s_{XY} \le 1$$

• Result (using R):

```
> cor(EAWE21.simple$EARNINGS, EAWE21.simple$S
[1] 0.2924262
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- This shows that education and earnings are positively but rather weakly correlated.
- We will require further statistical tools to analyse this relationship!

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- ρ_{XY} is independent of units of measurement. If earnings were measured in GDP, USD or EUR, correlation still would be 0.29
- The correlation between Y and X is the same as between X and Y, i.e. it does not matter which variable is labelled X and which Y.
- Finding an association between variables might be suggestive, but is rarely sufficient
- Establishing a casual relationship is more desirable
- Ceteris paribus plays an important role in causal analysis
- Need to control for sufficient number of variables

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- How much would your grade increase by 1 extra hour of study?
- How much effect does education have on wages?
- What is the price elasticity of demand for smart phones?
- How much would the price of a stock change with a change in the previous year's profit?
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$$Y = \beta_1 + \beta_2 X + u$$

- The regression model hypothesizes a mathematical relationship between variables
- We start with only two variables of interest (Y and X)
- We hypothesise that X affects Y (and not the other way round)
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Dougherty & some textbooks	Other textbooks
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Student task

- **1** Assume that $\beta_1 = 1$ and $\beta_2 = 2$. Draw the regression line.
- ② Show how the regression line will change when $\beta_1 = 0$
- **3** Show how the regression line will change when $\beta_2 = -1$

