# BS2280 – Econometrics I
# Homework 2: Introduction to Regression Analysis - Solution

October 11, 2023

## 1

The data below show alcohol expenditure and income (both in £s per week) for sample of 17 families.

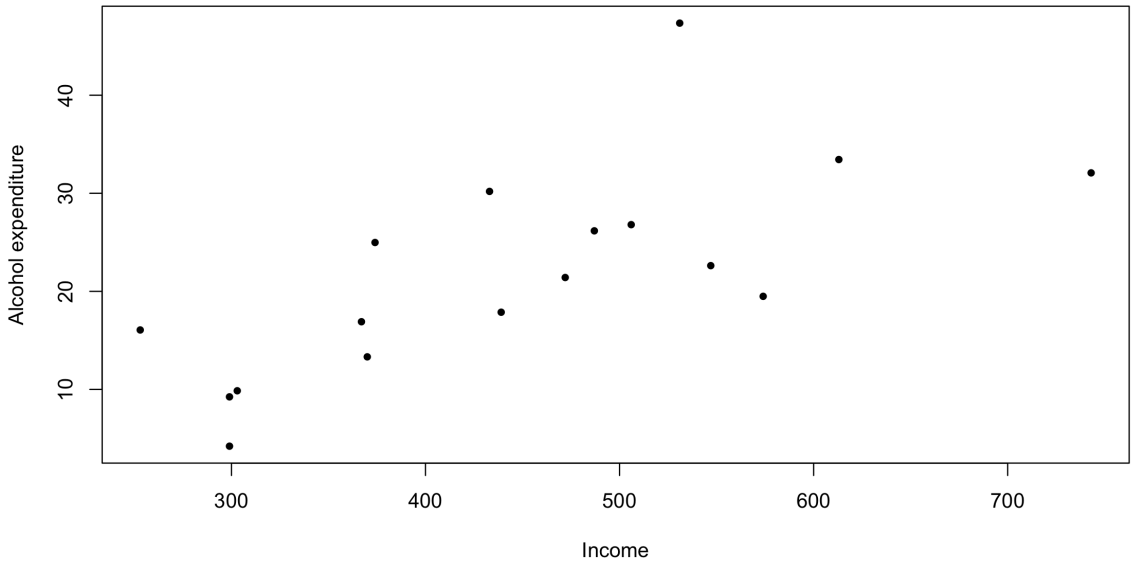
| Family | Alcohol Expenditure | Income |
|---|---|---|
| 1 | 26.17 | 487 |
| 2 | 19.49 | 574 |
| 3 | 17.87 | 439 |
| 4 | 16.90 | 367 |
| 5 | 4.21 | 299 |
| 6 | 32.08 | 743 |
| 7 | 30.19 | 433 |
| 8 | 22.62 | 547 |
| 9 | 9.86 | 303 |
| 10 | 13.32 | 370 |
| 11 | 9.24 | 299 |
| 12 | 47.35 | 531 |
| 13 | 26.80 | 506 |
| 14 | 33.44 | 613 |
| 15 | 21.41 | 472 |
| 16 | 16.06 | 253 |
| 17 | 24.98 | 374 |

Load the data into R by copying the following command:
data <- data.frame(alc = c(26.17, 19.49, 17.87, 16.90, 4.21, 32.08, 30.19, 22.62, 9.86, 13.32, 9.24, 47.35, 26.80, 33.44, 21.41, 16.06, 24.98), inc = c(487, 574, 439, 367, 299, 743, 433, 547, 303, 370, 299, 531, 506, 613, 472, 253, 374))

a. Draw and XY plot of the data and comment.

Use code:

```
plot(data$alc~data$inc, ylab = "Alcohol expenditure", xlab = "Income", pch =20)
```




b. From the chart, would you expect the line of best fit to slope up or down? In theory, which way should it slope?

From the graph there appears to be a positive relationship and the line of best fit should slope upwards. One might expect those with more income to spend more on alcohol.

c. What would you expect the correlation coefficient to be, approximately?

There is a reasonably good fit of the data, so a positive correlation of about 0.6 to 0.7 might be expected.

d. Calculate the Covariance between alcohol expenditure and income.

Use code:

```
cov(data)
```

```
          alc        inc
alc 111.1661   961.9388
inc 961.9388 17134.6176
```

961.939
Note that we have sample data, therefore we have to divide by $n-1$ rather than n when calculating the covariance.

e. Calculate the correlation coefficient between alcohol spending and income.

Use code:

```
cor(data)
```

```
          alc       inc
alc 1.0000000 0.6969861
inc 0.6969861 1.0000000
```

0.697
Note that we have sample data, therefore we have to divide by $n-1$ rather than n when calculating the variance of income and alcohol expenditure.

## 2

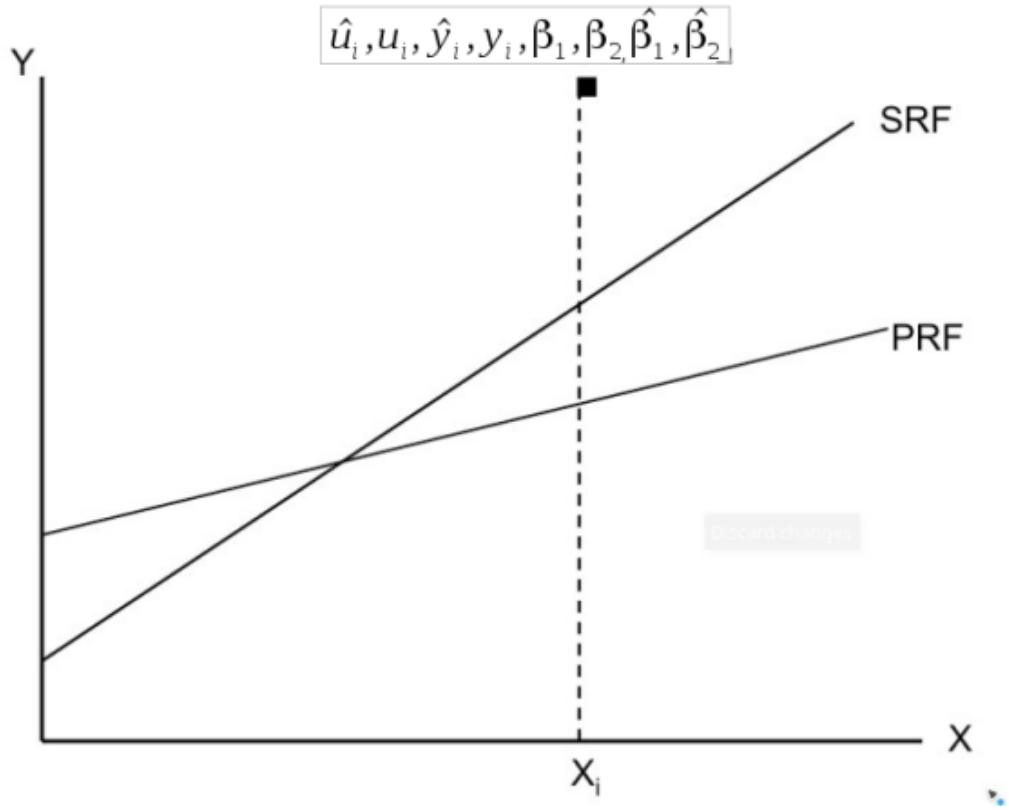
What is the role of the stochastic error term $u_i$ in regression analysis? What is the difference between the stochastic error term and the residual, $\hat{u}_i$?

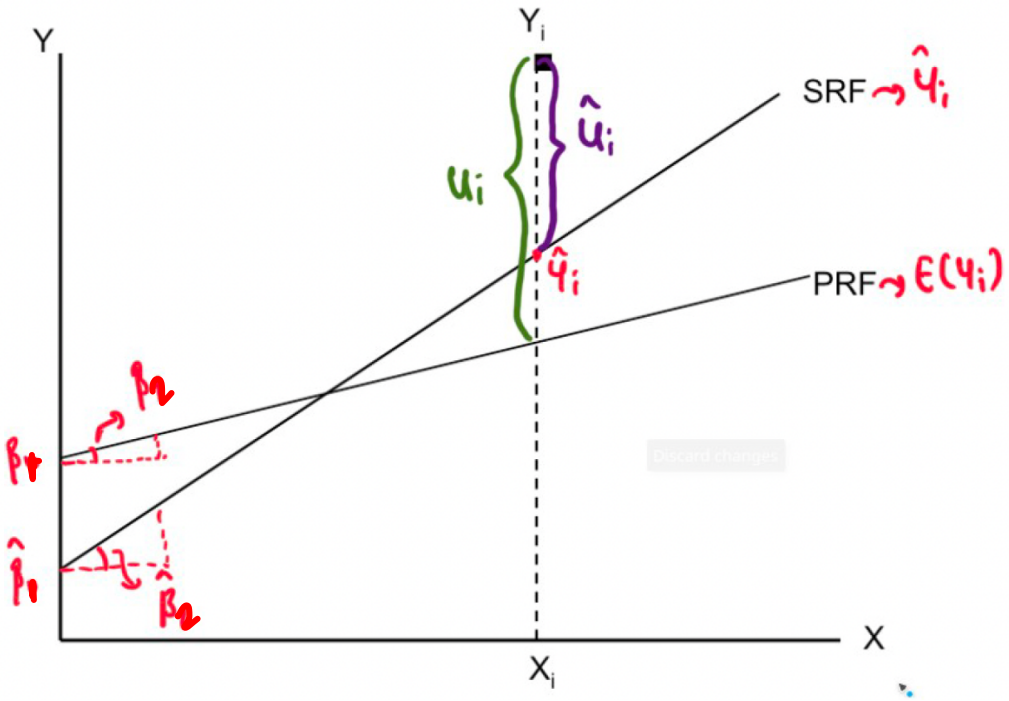
The stochastic error term is the effect of many small (so small we are unable to effectively model them) effects on the dependent variable, or the result of pure random variation in the dependent variable.

The residual is the estimated stochastic error term. This will include any over/underestimate because of differences between the population and estimated parameters – and can be smaller or larger than the stochastic error term.

## 3

The figure below shows the regression line based on variable X ans Y. SRF represents the sample regression function and PRF the population regression function. Label the diagram with the following:


$\hat{u}_i, u_i, \hat{y}_i, y_i, \beta_1, \beta_2, \hat{\beta}_1, \hat{\beta}_2$
Y
SRF
PRF
X
$X_i$


Y
$Y_i$
SRF → $\hat{Y}_i$
$\hat{u}_i$
$u_i$
$\hat{Y}_i$
PRF → $E(Y_i)$
$\beta_2$
$\beta_1$
$\hat{\beta}_1$
$\hat{\beta}_2$
X
$X_i$

# 4

The table below shows the average annual percentage rates of growth of employment, e, and real GDP, g, for 31 OECD countries for the period 2002–2007. The regression output shows the result of regressing e on g. Provide an interpretation of the coefficients.

Average annual percentage rates of growth of employment and real GDP, 2002–2007

| | *Employment* | *GDP* | | *Employment* | *GDP* |
|---|---|---|---|---|---|
| Australia | 2.57 | 3.52 | Korea | 1.11 | 4.48 |
| Austria | 1.64 | 2.66 | Luxembourg | 1.34 | 4.55 |
| Belgium | 1.06 | 2.27 | Mexico | 1.88 | 3.36 |
| Canada | 1.90 | 2.57 | Netherlands | 0.51 | 2.37 |
| Czech Republic | 0.79 | 5.62 | New Zealand | 2.67 | 3.41 |
| Denmark | 0.58 | 2.02 | Norway | 1.36 | 2.49 |
| Estonia | 2.28 | 8.10 | Poland | 2.05 | 5.16 |
| Finland | 0.98 | 3.75 | Portugal | 0.13 | 1.04 |
| France | 0.69 | 2.00 | Slovak Republic | 2.08 | 7.04 |
| Germany | 0.84 | 1.67 | Slovenia | 1.60 | 4.82 |
| Greece | 1.55 | 4.32 | Sweden | 0.83 | 3.47 |
| Hungary | 0.28 | 3.31 | Switzerland | 0.90 | 2.54 |
| Iceland | 2.49 | 5.62 | Turkey | 1.30 | 6.90 |
| Israel | 3.29 | 4.79 | United Kingdom | 0.92 | 3.31 |
| Italy | 0.89 | 1.29 | United States | 1.36 | 2.88 |
| Japan | 0.31 | 1.85 | | | |

Regression output:

```
Call:
lm(formula = e ~ g, data = oecd_exercises)

Residuals:
     Min       1Q   Median       3Q      Max
-1.03915 -0.42605 -0.08701  0.30295  1.65834

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.49195    0.28325   1.737  0.09303 .
g            0.23794    0.07025   3.387  0.00205 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6692 on 29 degrees of freedom
Multiple R-squared:  0.2834,    Adjusted R-squared:  0.2587
F-statistic: 11.47 on 1 and 29 DF,  p-value: 0.002049
```

Literally the regression implies that a 1 percent increase in the growth of GDP generates a 0.24 percent increase in the rate of growth of employment. The intercept suggests that, if GDP is static, employment will still grow at a rate of 0.49 percent per year.

## 5 *

In the lecture we introduced the first order conditions of the RSS minimisation problem:

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = 2n\hat{\beta}_1 - 2\sum_{i=1}^{N} Y + 2\hat{\beta}_2 \sum_{i=1}^{N} X = 0$$

$$\frac{\partial RSS}{\partial \hat{\beta}_2} = 2\hat{\beta}_2 \sum_{i=1}^{N} X^2 - 2\sum_{i=1}^{N} XY + 2\hat{\beta}_1 \sum_{i=1}^{N} X = 0$$

Derive $\hat{\beta}_1$ and $\hat{\beta}_2$ mathematically using the first order conditions. (Hint: $\sum_{i=1}^{N} X = n\bar{X}$)

First, look at the hint and we know

$$\sum_{i=1}^{N} X = n\bar{X}$$
$$\sum_{i=1}^{N} Y = n\bar{Y}$$

Make use of these hints for $\frac{\partial RSS}{\partial \hat{\beta}_1}$, we get

$$\begin{aligned} 2n\hat{\beta}_1 - 2\textstyle\sum_{i=1}^{N} Y + 2\hat{\beta}_2 \sum_{i=1}^{N} X &= 0 \\ 2n\hat{\beta}_1 - 2n\bar{Y} + 2\hat{\beta}_2 n\bar{X} &= 0 \\ \hat{\beta}_1 - \bar{Y} + \hat{\beta}_2\bar{X} &= 0 \\ \hat{\beta}_1 &= \bar{Y} - \hat{\beta}_2\bar{X} \end{aligned}$$

*apply $\sum_{i=1}^{N} X = n\bar{X}$ and $\sum_{i=1}^{N} Y = n\bar{Y}$*
*divide by 2n*
*rearrange*

Now we need to make use of this expression and substitute into $\frac{\partial RSS}{\partial \hat{\beta}_2}$, then use hints again we get

$$\begin{aligned} 2\hat{\beta}_2 \textstyle\sum_{i=1}^{N} X^2 - 2\sum_{i=1}^{N} XY + 2\hat{\beta}_1 \sum_{i=1}^{N} X &= 0 \\ 2\hat{\beta}_2 \textstyle\sum_{i=1}^{N} X^2 - 2\sum_{i=1}^{N} XY + 2(\bar{Y} - \hat{\beta}_2\bar{X})n\bar{X} &= 0 \\ 2\hat{\beta}_2 \textstyle\sum_{i=1}^{N} X^2 - 2\sum_{i=1}^{N} XY + 2\bar{Y}n\bar{X} - 2\hat{\beta}_2\bar{X}n\bar{X} &= 0 \\ \hat{\beta}_2 \textstyle\sum_{i=1}^{N} X^2 - \sum_{i=1}^{N} XY + \bar{Y}n\bar{X} - \hat{\beta}_2\bar{X}n\bar{X} &= 0 \\ \hat{\beta}_2(\textstyle\sum_{i=1}^{N} X^2 - n\bar{X}^2) &= \sum_{i=1}^{N} XY - n\bar{Y}\bar{X} \\ \hat{\beta}_2 &= \frac{\sum_{i=1}^{N} XY - n\bar{Y}\bar{X}}{\sum_{i=1}^{N} X^2 - n\bar{X}^2} \\ \hat{\beta}_2 &= \frac{Cov(X,Y)}{Var(X)} \end{aligned}$$

*apply $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$ and $\sum_{i=1}^{N} X = n\bar{X}$*
*simplify expression*
*simplify expression again, divide by 2*
*rearrange*

The expression here is a bit different from what we see in the lecture slides. However, if you are interested in how we get the same expression as lecture slides, please read Chapter 1 of Dougherty's book, just some math work.