

# BS2280 - Econometrics 1

## Lecture 4 - Part 2: Hypothesis Testing

Dr. Yichen Zhu

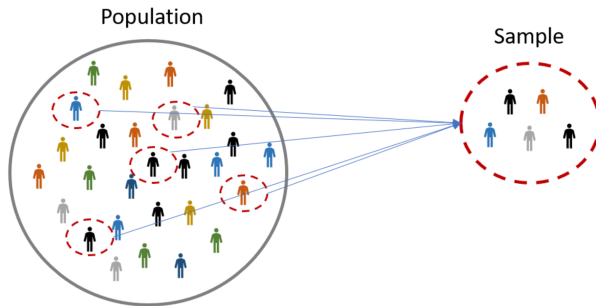
## Structure of today's lecture

- 1 Hypothesis Testing in Econometrics
- 2 Hypothesis Testing in Econometrics: Regression Coefficients
- 3 Hypothesis Testing in Econometrics: Goodness of Fit  $R^2$

## Intended Learning Outcomes

- Undertaking hypothesis tests for regression coefficients  $\beta_1, \beta_2$
- Testing the statistical significance of Goodness of fit  $R^2$

# Review: Hypothesis Testing in Statistics



**Statistics:**

Econometrics:

**Population Parameters**

$$\mu, \sigma^2$$

$$\beta_1, \beta_2$$

**Inference**

←

←

**Sample Coefficients**

$$\bar{X}, s^2$$

$$\hat{\beta}_1, \hat{\beta}_2$$

## Example: Two-Tail Test ( $\sigma$ unknown)

The average cost of a hotel room in New York is said to be \$168 per night. A random sample of 25 hotels resulted in  $\bar{x} = \$172.50$  and  $s = \$15.40$ . Test at the  $\alpha = 0.05$  level. Assume the population distribution is normal.

- $H_0 : \mu = 168$
- $H_1 : \mu \neq 168$

### Steps for hypothesis testing:

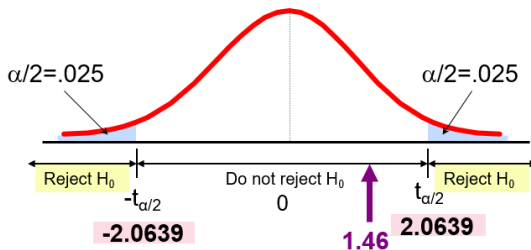
- 1 State the null and alternative hypotheses
- 2 Select the significance level
- 3 Select and calculate the test statistics
- 4 Set the decision rule
- 5 Make statistical decisions

## Example Solution: Two-Tail Test

$$H_0: \mu = 168$$
$$H_A: \mu \neq 168$$

- $\alpha = 0.05$
- $n = 25$
- $\sigma$  is unknown, so use a **t statistic**
- Critical Value:

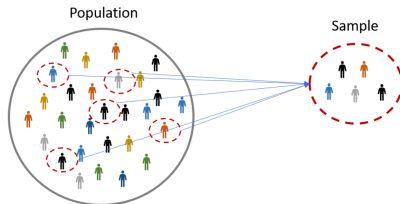
$$t_{24} = \pm 2.0639$$



$$t_{n-1} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{172.50 - 168}{\frac{15.40}{\sqrt{25}}} = 1.46$$

**Do not reject  $H_0$ :** not sufficient evidence that true mean cost is different than \$168

# Estimation Background: Hypothesis Testing in Econometrics



## What is Estimation?

Estimation is the process of using sample data to draw inferences about the population. For example, we use OLS estimator to estimate  $\beta_1$  and  $\beta_2$

	Population Parameters	Inference	Sample Coefficients
Statistics:	$\mu, \sigma^2$	←	$\bar{x}, s^2$
Econometrics:	$\beta_1, \beta_2$	←	$\hat{\beta}_1, \hat{\beta}_2$

# What about hypothesis testing for regressions?

	Statistics	Econometrics
<b>Model</b>	$X$ , unknown $\mu, \sigma^2$	$Y = \beta_1 + \beta_2 X + u$
<b>Estimator</b>	$\bar{X}$ and $s^2$	$\hat{\beta}_1$ and $\hat{\beta}_2$
<b>Null Hypothesis</b>	$H_0 : \mu = \mu_0$	$H_0 : \beta_1 = \beta_1^0 \quad H_0 : \beta_2 = \beta_2^0$
<b>Alternative Hypothesis</b>	$H_1 : \mu \neq \mu_0$	$H_1 : \beta_1 \neq \beta_1^0 \quad H_1 : \beta_2 \neq \beta_2^0$
<b>Test statistic</b>	$t = \frac{\bar{X} - \mu_0}{s.e.(\bar{X})}$	$t = \frac{\hat{\beta}_1 - \beta_1^0}{s.e.(\hat{\beta}_1)} \quad t = \frac{\hat{\beta}_2 - \beta_2^0}{s.e.(\hat{\beta}_2)}$
<b>Reject <math>H_0</math> if</b>	$ t  > t_{crit}$	$ t  > t_{crit}$
<b>Degrees of Freedom</b>	$n - 1$	$n - k = n - 2$



# Hypothesis Testing for $\beta_2$

## Example

Impact of wage inflation ( $w$ ) on price inflation ( $p$ ); both measured in % annual growth rate. Sample size  $n = 20$

Population regression model:

$$p = \beta_1 + \beta_2 w + u$$

Sample estimated regression model:

$$\hat{p} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82} w \quad (1)$$

**Note:** Standard Errors (s.e.) in brackets

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<b>Null Hypothesis</b>	$H_0 : \beta_2 = 1.0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 1.0$

---

# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (2)$$

**Note:** Standard Errors (s.e.) in brackets

- 1 State the null and alternative hypotheses

<b>Null Hypothesis</b>	$H_0 : \beta_2 = 1.0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 1.0$

- 2 Select the significance level. Significance level  $\alpha = 5\%$

- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use  $t$  statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.0}{0.10} = -1.80$

- 4 Set the decision rule.  $n = 20$ , degree of freedom  $= n - k = 20 - 2 = 18$ ,  $t_{crit,5\%} = 2.101$

- 5 Make statistical decisions.  $|t| = 1.80 < t_{crit,5\%} = 2.101$ , cannot reject the null  $H_0 : \beta_2 = 1.0$

# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (2)$$

**Note:** Standard Errors (s.e.) in brackets

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<b>Null Hypothesis</b>	$H_0 : \beta_2 = 1.0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 1.0$

- 2 Select the significance level. **Significance level  $\alpha = 5\%$**

- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use  $t$  statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.0}{0.10} = -1.80$

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# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (2)$$

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- 2 Select the significance level. Significance level  $\alpha = 5\%$
- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use  $t$  statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.0}{0.10} = -1.80$

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- 5 Make statistical decisions.  $|t| = 1.80 < t_{crit,5\%} = 2.101$ , cannot reject the null  $H_0 : \beta_2 = 1.0$

# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (2)$$

**Note:** Standard Errors (s.e.) in brackets

- 1 State the null and alternative hypotheses

<b>Null Hypothesis</b>	$H_0 : \beta_2 = 1.0$
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# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (2)$$

**Note:** Standard Errors (s.e.) in brackets

- 1 State the null and alternative hypotheses

<b>Null Hypothesis</b>	$H_0 : \beta_2 = 1.0$
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- 2 Select the significance level. Significance level  $\alpha = 5\%$
- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use  $t$  statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = \frac{0.82 - 1.0}{0.10} = -1.80$

- 4 Set the decision rule.  $n = 20$ , degree of freedom  $= n - k = 20 - 2 = 18$ ,  $t_{crit,5\%} = 2.101$
- 5 Make statistical decisions.  $|t| = 1.80 < t_{crit,5\%} = 2.101$ , cannot reject the null  $H_0 : \beta_2 = 1.0$

## Hypothesis Testing for $\beta_2$

- Usually, we do not have a feeling for the actual value of the coefficients.
- We do not have any prior notion on how big the actual relationship is!!!!
- **Often or Always**, we just want to demonstrate that  $Y$  is influenced by  $X$ , therefore:

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<b>Null Hypothesis</b>	$H_0 : \beta_2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 0$

---

Population regression model:

$$p = \beta_1 + \beta_2 w + u$$

Sample estimated regression model:

$$\hat{p} = \underset{(0.05)}{1.21} + \underset{(0.10)}{0.82} w \quad (3)$$

**Note:** Standard Errors (s.e.) in brackets

# Hypothesis Testing for $\beta_2$

$$\hat{p} = \begin{matrix} 1.21 \\ (0.05) \end{matrix} + \begin{matrix} 0.82 \\ (0.10) \end{matrix} w \quad (4)$$

**Note:** Standard Errors (s.e.) in brackets

- 1 State the null and alternative hypotheses

<b>Null Hypothesis</b>	$H_0 : \beta_2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 0$

- 2 Select the significance level. Significance level  $\alpha = 5\%$
- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use  $t$  statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{s.e.(\hat{\beta}_2)} = \frac{\hat{\beta}_2}{s.e.(\hat{\beta}_2)} = \frac{0.82 - 0}{0.10} = 8.2$

- 4 Set the decision rule.  $n = 20$ , degree of freedom  $= n - k = 20 - 2 = 18$ ,  $t_{crit,5\%} = 2.101$
- 5 Make statistical decisions.  $|t| = 8.2 > t_{crit,5\%} = 2.101$ , reject the null  $H_0 : \beta_2 = 0$ .  
Wage inflation ( $w$ ) will affect price inflation ( $p$ ).



# Student Task

We estimated in previous lectures the relationship between hourly earnings (\$) and years of schooling. Sample size  $n = 500$

$$\widehat{EARNINGS} = \begin{matrix} 0.765 \\ (2.804) \end{matrix} + \begin{matrix} 1.266 \\ (0.185) \end{matrix} S \quad (5)$$

**Note:** Standard Errors (s.e.) in brackets

- 1 State the null and alternative hypotheses

---

<b>Null Hypothesis</b>	$H_0 : \beta_2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : \beta_2 \neq 0$

---

- 2 Select the significance level. Significance level  $\alpha = 5\%$
- 3 Select and calculate the test statistics

Do not know the population variance  $\sigma^2$ , so use t statistic:  $t = \frac{\hat{\beta}_2 - \beta_2^0}{\text{s.e.}(\hat{\beta}_2)} = ?$

- 4 Set the decision rule.  $n = 50$ , degree of freedom  $= n - k = ?$ ,  $t_{crit,5\%} = 1.96$
- 5 Make statistical decisions. ?

# Hypothesis Testing for $\beta_1$ and $\beta_2$ with R

$$\widehat{EARNINGS} = 0.765 + 1.266 S \quad (6)$$

(2.804)      (0.185)

```
> summary(earnfit)
```

Call:

```
lm(formula = EARNINGS ~ S, data = EAWE21.simple)
```

Residuals:

Min	1Q	Median	3Q	Max
-20.079	-6.726	-2.203	3.451	79.037

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.7647	2.8038		
S	1.2657	0.1855		

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.36 on 498 degrees of freedom

Multiple R-squared: 0.08551, Adjusted R-squared: 0.08368

F-statistic: 46.57 on 1 and 498 DF, p-value: 2.579e-11

# Hypothesis Testing for $\beta_1$ and $\beta_2$ with R

$$\widehat{EARNINGS} = \begin{matrix} 0.765 \\ (2.804) \end{matrix} + \begin{matrix} 1.266 \\ (0.185) \end{matrix} S \quad (7)$$

- **p-values:** probability of obtaining the corresponding t statistic as a matter of chance, if the null hypothesis  $H_0 : \beta = 0$  is true.
- $p\text{-values} < \alpha$ , reject the null hypothesis  $H_0 : \beta = 0$

```
> summary(earnfit)

Call:
lm(formula = EARNINGS ~ S, data = EAWE21.simple)

Residuals:
    Min       1Q   Median       3Q      Max
-20.079  -6.726  -2.203   3.451   79.037

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.7647     2.8038    0.785  0.785
S             1.2657     0.1855   2.58e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## Review: Goodness of Fit $R^2$

- How good is our regression model?
- Good: variations in  $Y$  can be explained by variations in  $X$

$Y_i$	$= \beta_1 + \beta_2$	$X_i$	+	$u_i$
Total variations, variations in $Y$		Explained variations, variations in $X$		Residual variations, variations in $u_i$
Total Sum of Squares, TSS		Explained sum of squares, ESS		Residual sum of squares, RSS
$TSS = \sum (Y_i - \bar{Y})^2$		$ESS = \sum (\hat{Y}_i - \bar{Y})^2$		$RSS = \sum (Y_i - \hat{Y}_i)^2 = \sum u_i^2$
				(8)

$$TSS = ESS + RSS \quad (9)$$

- Total variation in  $Y$  can be decomposed into variations arising from the  $X$  variable and variations arising from the residuals  $u_i$

$$R^2 = \frac{\text{Explained Variations in } Y}{\text{Total Variations in } Y} = \frac{ESS}{TSS} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

## Review: Goodness of Fit $R^2$

- Create a measure of the proportion of ESS in TSS: the  $R^2$  - the coefficient of determination

$$R^2 = \frac{\text{Explained Variations in } X}{\text{Total Variations in } Y} = \frac{ESS}{TSS} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

- $0 \leq R^2 \leq 1$ : The  $R^2$  is bound between 0 and 1
- $R^2 = 0$ : variations in X cannot explain any variations in Y
- $R^2 = 1$ : variations in X can fully explain variations in Y
- A high  $R^2$  means that the independent variable X is good at predicting Y
- **Example:**  $R^2 = 0.75$ , it means that 75% of the variations in Y can be explained by variations in the X variable.
- Remaining 25% comes from unobservables through the residual term

```
> summary(earnfit)
Call:
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Residuals:
    Min       1Q   Median       3Q      Max
-20.079  -6.726  -2.203   3.451  79.037

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)    0.7647     2.8038   0.273   0.785
S              1.2657     0.1855   6.824 2.58e-11 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.36 on 498 degrees of freedom
Multiple R-squared:  0.08551, Adjusted R-squared:  0.08368
F-statistic: 46.57 on 1 and 498 DF,  p-value: 2.579e-11
```

$R^2 = 0.0855$ , only 8.55% of the **variations in earnings** can be explained by **variations in schooling**. 91.45% of the variations in earnings is left unexplained.

# Hypothesis Testing for Goodness of Fit $R^2$ : $F$ statistic

$$Y_i = \beta_1 + \beta_2 X_i + u_i \quad (10)$$

$$R^2 = \frac{\text{Explained Variations in } Y}{\text{Total Variations in } Y} = \frac{ESS}{TSS} = \frac{\sum_{i=1}^n (\hat{Y}_i - \bar{Y})^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2}$$

---

<b>Null Hypothesis</b>	$H_0 : R^2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : R^2 \neq 0$

---

Test the entire regression model, cannot use  $t$  statistic, we need  $F$  statistic.

$$F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)} \quad (11)$$

Make use of  $TSS = ESS + RSS$ ,  $F$  statistic will change to

$$F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} \quad (12)$$

# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \frac{0.765}{(2.804)} + \frac{1.266}{(0.185)} S \quad (13)$$

- 1 State the null and alternative hypotheses

<b>Null Hypothesis</b>	$H_0 : R^2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : R^2 \neq 0$

- 2 Select the significance level. Significance level  $\alpha = 5\%$

- 3 Select and calculate the test statistics

Test the entire regression model, so use  $F$  statistic. Two ways:

$$\text{Option 1. } F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)}$$

Using ESS and RSS we get by using `anova()` command in R

$$\text{Option 2. } F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

Using  $R^2$  from when using `summary()` command in R



# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \frac{0.765}{(2.804)} + \frac{1.266}{(0.185)} S \quad (13)$$

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Using  $R^2$  from when using `summary()` command in R

# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \frac{0.765}{(2.804)} + \frac{1.266}{(0.185)} S \quad (13)$$

- 1 State the null and alternative hypotheses

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Using  $R^2$  from when using `summary()` command in R

# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \frac{0.765}{(2.804)} + \frac{1.266}{(0.185)} S \quad (14)$$

Select and calculate the test statistics

Test the entire regression model, so use  $F$  statistic. Two ways:

$$\text{Option 1. } F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)}$$

Using ESS and RSS we get by using anova() command in R

```
Analysis of Variance Table

Response: EARNINGS
          Df Sum Sq Mean Sq F value    Pr(>F)
S           1    6014   6014.0    46.568 2.579e-11 ***
Residuals 498   64315    129.1
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$F(1, n-2) = \frac{ESS}{RSS/(n-2)} = \frac{6014}{64315/(500-2)} = \frac{6014}{129.15} = 46.57$$

# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \begin{matrix} 0.765 \\ (2.804) \end{matrix} + \begin{matrix} 1.266 \\ (0.185) \end{matrix} S \quad (15)$$

Select and calculate the test statistics

$$\text{Option 2. } F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = \frac{\frac{ESS}{TSS}/(k-1)}{\frac{RSS}{TSS}/(n-k)} = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

Using  $R^2$  from when using `summary()` command in R

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.7647      2.8038   0.273   0.785
S              1.2657      0.1855   6.824 2.58e-11 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.36 on 498 degrees of freedom
Multiple R-squared:  0.08551,    Adjusted R-squared:  0.08368
F-statistic: 46.57 on 1 and 498 DF,  p-value: 2.579e-11
```

$$F(1, n-2) = \frac{R^2}{(1-R^2)/(n-2)} = \frac{0.0855}{(1-0.0855)/(500-2)} = 46.56$$

# Hypothesis Testing for Goodness of Fit $R^2$

$$\widehat{EARNINGS} = \begin{matrix} 0.765 \\ (2.804) \end{matrix} + \begin{matrix} 1.266 \\ (0.185) \end{matrix} S \quad (16)$$

- 1 State the null and alternative hypotheses

---

<b>Null Hypothesis</b>	$H_0 : R^2 = 0$
<b>Alternative Hypothesis</b>	$H_1 : R^2 \neq 0$

---

- 2 Select the significance level. Significance level  $\alpha = 5\%$
- 3 Select and calculate the test statistics  
Test the entire regression model, so use  $F$  statistic. Calculate in two ways and  $F$  statistics is 46.57.
- 4 Set the decision rule.  $k = 2, n = 500$ .  $F_{crit,5\%}(k-1, n-k) = F_{crit,5\%}(1, 498) = 3.84$
- 5 Make statistical decisions.  $F = 46.57 > F_{crit,5\%}(1, 498) = 3.84$ , can reject the null  $H_0 : R^2 = 0$ .  
Our model is statistically significant at 5% level.

# Student Task

Use the F-test to check if the model is statistically significant

$$F_{crit,5\%}(1, 24) = 4.26$$

```
> anova(efit)
```

Analysis of Variance Table

Response: EMPLOY

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
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GDP	1	14.060			
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Residuals	24	23.994			
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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

$$F(k-1, n-k) = \frac{ESS/(k-1)}{RSS/(n-k)} = ?$$

## What to do next:

- Attempt homework 4
- Revise basic R commands from R Workshop 1
- Read chapter 2.8 - 2.11 of Dougherty