# BS2280 - Econometrics 1

## Lecture 10 - Part 2: Nonlinear Models and Transformation of Variables II

Dr. Yichen Zhu

# Structure of today's lecture

# Intended Learning Outcomes

- Interpreting the coefficients of quadratic model

# Review: Log, Semi-log models

| Model | Dependent Variable | Independent Variable | Interpretation |
|---|---|---|---|
| $Y = \beta_1 + \beta_2 X$<br>Level-Level Model | $Y$ | $X$ | 1 unit change in $X$ bring $\beta_2$ units change in $Y$ |
| $logY = \beta_1 + \beta_2 logX$<br>Log-Log Model | $logY$ | $logX$ | 1 % change in $X$ bring $\beta_2$ % change in $Y$ |
| $logY = \beta_1 + \beta_2 X$<br>Log-Level Model | $logY$ | $X$ | 1 unit change in $X$ bring $100 \times \beta_2$ % change in $Y$ |

# Student Task

- We estimate the following model:

$$\widehat{log\ lexp}_i = \hat{\beta}_1 + \hat{\beta}_2 log\ gnppc_i$$

- Where

| | |
|---|---|
| *lexp* | life expectancy at birth (in years) |
| *gnppc* | gross national product per capita (in USD) |

- Write out the regression model and interpret the coefficient $\hat{\beta}_2$

```
Call:
lm(formula = lnlexp ~ lngnppc, data = lifeexp)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.957925   0.035172 112.529  < 2e-16 ***
lngnppc     0.038786   0.004208   9.218 3.65e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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```
Call:
lm(formula = lnlexp ~ gnppck, data = lifeexp)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.2391824  0.0082144 516.069  < 2e-16 ***
gnppck      0.0044648  0.0006014   7.424 4.32e-10 ***
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Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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# Student Task

- We estimate the following model:

$$\widehat{logEARN_i} = \hat{\beta}_1 + \hat{\beta}_2 logS_i + \hat{\beta}_3 EXP_i + \hat{\delta} FEMALE_i$$

**Note**: *FEMALE* is 1 for female and 0 for male

- Interpret the coefficient

```
Call:
lm(formula = LOGEARN ~ LOGS + EXP + FEMALE, data = EAWE21)

Coefficients:
(Intercept)            S          EXP       FEMALE
    -0.9229      1.33918      0.03895     -0.18834
```

$$\widehat{logEARN_i} = -0.922 + 1.339 logS_i + 0.038 EXP_i - 0.188 FEMALE_i$$

# Review: Dummy Variables in Log Regression

$\widehat{logEARN}_i = \hat{\beta}_1 + \hat{\delta} FEMALE_i$; *FEMALE* is 1 for female and 0 for male

| | | |
|---|---|---|
| **Small coefficient** | -0.3 $\leq \hat{\delta} \leq$ 0.3 | For female, hourly earnings increase by $100 \times \hat{\delta}\%$ \$/hour |
| **Large coefficient** | $\hat{\delta}$ < -0.3 and $\hat{\delta}$ > 0.3 | The hourly earnings differences between female and male are $100 \times (e^{\hat{\delta}} - 1)$ \$/hour |

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# Quadratic Model

- We will now consider models with quadratic explanatory variables of the type shown.
- We can use OLS without modifications!!!

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{2i}^2 + u_i$$

- Holding everything else constant cannot be applied anymore
- If you change $X_2$ you will also change $X_2^2$

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- **Question**: If we change $X_2$, what will happen to $Y$?
- To calculate the marginal impact of a change in $X_2$ on $Y$, we need to differentiate with respect to $X_2$

$$\frac{dY}{dX_2} = \beta_2 + 2\beta_3 X_2$$

  (Hint: Make use of the power rule, derivative of $X^2$ is $2X^{2-1} = 2X$)
- The impact of a unit change in $X_2$ on $Y$ is $\beta_2 + 2\beta_3 X_2$
- The interpretations will depend on the value of $\beta_3$

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- $\beta_3$ can be interpreted as the rate of change in the $X_2$ coefficient for every one-unit change in $X_2$

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| If $\beta_3 < 0$ | the marginal effect of $X_2$ decreases with increasing levels of $X_2$ |
| If $\beta_3 > 0$ | the marginal effect of $X_2$ increases with increasing levels of $X_2$ |
| If $\beta_3 = 0$ | the marginal effect of $X_2$ is constant, which is $\beta_2$ |

- $\beta_1$ has a conventional interpretation. It is the value of $Y$ when $X_2 = 0$.

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# Quadratic Model: Example

- We will use again the data on employment growth rate (percentage point), $e$, and GDP growth rate (percentage point), $g$, for 25 OECD countries
- Consider that we use a quadratic function:

$$e_i = \beta_1 + \beta_2 g_i + \beta_3 g_i^2 + u_i$$

```
Call:
lm(formula = e ~ g + gsq, data = oecd_exercises)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.25273    0.58503  -0.432    0.669
g            0.65916    0.29921   2.203    0.036 *
gsq         -0.04879    0.03373  -1.447    0.159
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1

Residual standard error: 0.6569 on 28 degrees of freedom
Multiple R-squared:  0.3333,     Adjusted R-squared:
0.2857
F-statistic: 6.998 on 2 and 28 DF,  p-value: 0.003429
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- $g$: GDP growth rate (percentage point)
  $e$: employment growth rate (percentage point)
- **Question**: If $g$ changes, what will happen to $e$?
- The marginal effect of $g$ on $e$ is:

$$\frac{d\hat{e}_i}{dg} = \beta_2 + 2\beta_3 g = 0.659 - 2 \times 0.048 g$$

- Marginal effect will depend on $g$!
- If $g = 0$, $\frac{d\hat{e}_i}{dg} = 0.659 - 2 \times 0.048 \times 0 = 0.659$
  A 1 percentage point increase in GDP growth would lead to a 0.66 percentage point increase in employment growth
- If $g = 2$, $\frac{d\hat{e}_i}{dg} = 0.659 - 2 \times 0.048 \times 2 = 0.467$
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- $g$: GDP growth rate (percentage point)
  $e$: employment growth rate (percentage point)
- The interpretation of the intercept is straight forward
- If the GDP growth rate $g$ is 0, the employment growth rate $e$ is on average -0.252 percent

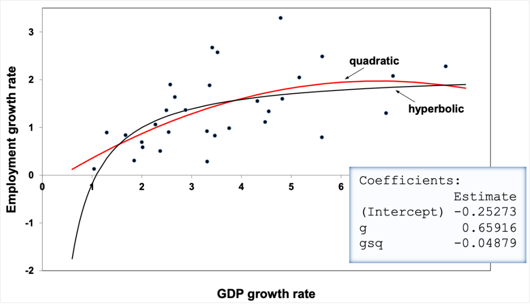
# Quadratic Model: Example

- Hyperbolic model:

$$\hat{e}_i = 2.17 - \frac{2.35}{g_i}$$

- Quadratic model:

$$\hat{e}_i = -0.252 + 0.659 g_i - 0.048 g_i^2$$




```
Coefficients:
             Estimate
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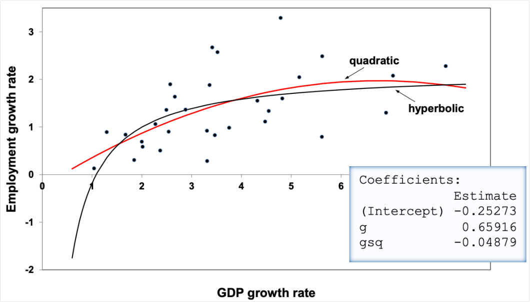

# Student Task

- We estimated the following regression:

$$EARNINGS_i = \beta_1 + \beta_2 S_i + \beta_3 S_i^2 + u_i$$

- Interpret the impact of one more year of education when a person has already 6 years, 12 year and 18 years of education
- What shape will the regression line have?

```
Call:
lm(formula = EARNINGS ~ S + SSQ, data = EAWE21)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   8.35840   12.86047   0.650    0.516
S             0.19107    1.78582   0.107    0.915
SSQ           0.03668    0.06063   0.605    0.545

Residual standard error: 11.37 on 497 degrees of freedom
Multiple R-squared:  0.08619,    Adjusted R-squared:
0.08251
F-statistic: 23.44 on 2 and 497 DF,  p-value: 1.876e-10
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Multiple R-squared:  0.08619,    Adjusted R-squared:  
0.08251
F-statistic: 23.44 on 2 and 497 DF,  p-value: 1.876e-10
```

# Student Task

- We estimated the following regression:

$$EARNINGS_i = \beta_1 + \beta_2 S_i + \beta_3 S_i^2 + u_i$$

- Interpret the impact of one more year of education when a person has already 6 years, 12 year and 18 years of education
- What shape will the regression line have?

```
Call:
lm(formula = EARNINGS ~ S + SSQ, data = EAWE21)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   8.35840   12.86047   0.650    0.516
S             0.19107    1.78582   0.107    0.915
SSQ           0.03668    0.06063   0.605    0.545

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# Why Stop At A Quadratic Form?

- Why not consider a cubic, or quartic, or a polynomial of even higher order?
- There are several good reasons for not doing so:
  1. Diminishing marginal effects are standard in economic theory, justifying quadratic specifications.
  2. There will be an improvement in fit as higher-order terms are added, but because these terms are not theoretically justified, the improvement will be sample-specific.
  3. Unless the sample is very small, the fits of higher-order polynomials are unlikely to be very different from those of a quadratic over the main part of the data range

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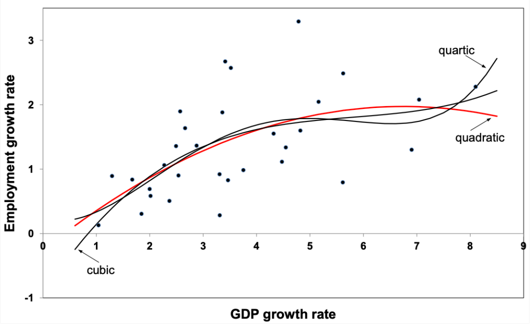


- These points are illustrated by the figure
- Over the main data range, from $g = 1.5$ to $g = 5$, the fits of the cubic and quartic are very similar to that of the quadratic.
- $R^2$ for the quadratic specification is 0.334. For the cubic and quartic it is 0.345 and 0.355, relatively small improvements.
- Cubic and quartic curves both exhibit implausible characteristics.

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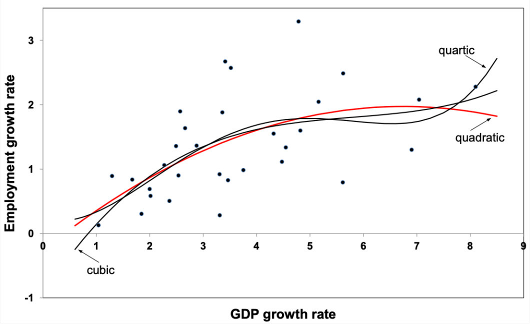


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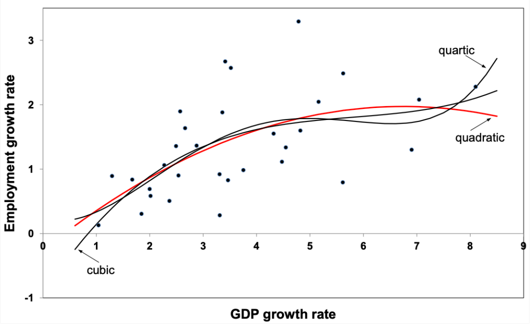


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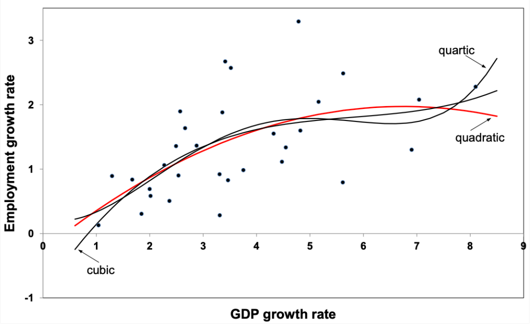


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