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GDP Forecasting Using a Combination of Dynamic Factor Model and Neural Network

Maria Lashina

Urgency


Annual growth rate of Russia's quarterly GDP
growth rate
1,15
1,1
1,05
1
0,95
0,9
0,85
1996Q1
1997Q1
1998Q1
1999Q1
2000Q1
2001Q1
2002Q1
2003Q1
2004Q1
2005Q1
2006Q1
2007Q1
2008Q1
2009Q1
2010Q1
2011Q1
2012Q1
2013Q1
2014Q1
2015Q1
2016Q1
2017Q1
2018Q1
2019Q1
2020Q1
2021Q1
2022Q1

Problem Statement

Aim:

To evaluate the predictive potential of ensemble methods in relation to the task of forecasting the growth rates of Russia's GDP

Ensemble:
- Dynamic Factor Model (DFM)
- Long Short-Term Memory neural network(LSTM)

VS.

- Autoregressive–moving-average with exogenous variables model (ARMAX)
- Vector autoregression (VAR)

- Support vector regression (SVR)
- Boosting (CatBoost)

Steps:

1. Analyze the modeling methods used to predict GDP
2. Collect and process data on selected indicators
3. Build and train DFM, LSTM and ensemble models
4. Build and train competitor models
5. Compare the accuracy of model predictions using root mean square error (RMSE)
6. Интерпретация прогнозов DFM, LSTM

Hypotheses

1. The RMSE of forecasts of an ensemble model comprising DFM and LSTM is lower than that of forecasts of models separately and competing models (Longo, Riccaboni, & Rungi, 2022).

2. The RMSE of forecasts of machine learning methods (SVR, CatBoost, LSTM) is lower than that of forecasts of statistical models (ARMAX, VAR) (Ahmed, Atiya, Gayar, & El-Shishiny, 2010; Richardson, van Florenstein Mulder, & Vehbi, 2020; Teräsvirta, Van Dijk, & Medeiros, 2005).

3. The RMSE of forecasts of models trained on data including indices of economic policy uncertainty and geopolitical risk is lower than that of forecasts of models trained on data without these indicators (Baker, Bloom, Davis, & Terry, 2020).

Literature: Dynamic factor model

Authors	Title	Year	Contribution
Chamberlain & Rothschild	Arbitrage, factor structure and mean-variance analysis in large asset markets	1983	They introduced the concept of an "approximate" factor structure, rejecting the hypothesis that idiosyncratic errors do not correlate with each other
Forni, Hallin, Lippi, & Reichlin	The generalized dynamic-factor model: Identification and estimation	2000	Proposed a new methodology for assessing unobservable factors based on the principal component method (PCA)
Mariano & Murasawa	A new coincident index of business cycles based on monthly and quarterly series	2003	Proposed a DFM specification for simultaneous processing of quarterly and monthly data in a single model
Bai & Ng	Determining the number of factors in approximate factor models	2002	Used DFM to determine the main factors affecting the US economy, such as consumption, investment and exports
Giannone, Reichlin & Small	Nowcasting: the real-time informational content of macroeconomic data	2008	Used DFM, trained on a wide range of monthly indicators, to forecast eurozone GDP immediately after the start of the Great Recession of 2008
Chernis & Sekkel	A dynamic factor model for nowcasting Canadian GDP growth	2017	Have shown that the DFM forecasts are of higher quality than, for example, the mixed data sampling model (MIDAS) and bridge regression
Ponomarev & Pleskachev	Short-term GDP forecasting using a dynamic factor model	2018	Showed that the errors in the forecasts of Russia's GDP made with the help of DFM are lower than those of the reference methods

Literature: Machine learning and interpretation of results

Authors	Title	Year	Contribution
Tiffin	Seeing in the Dark: A Machine-learning Approach to Nowcasting in Lebanon	2016	Used elastic net regression and random forest regression for the GDP of Lebanon
Chuku, Simpasa, & Oduor	Intelligent forecasting of economic growth for developing economies	2019	Demonstrated that forecasts of GDP growth rates in African countries of nonparametric regression and artificial neural network models have a lower standard error than the predictions of the "classical" autoregression model-the moving average
Jena, et al.	Impact of COVID-19 on GDP of major economies: Application of the artificial neural network forecaster	2021	Proposed the design of a neural network for predicting financial time series
Shijun, Xiaoli, Yunbin, & Chong	Application of Improved LSTM Algorithm in Macroeconomic Forecasting	2021	Applied LSTM to the data set of the futures index of agricultural products, the forecasts of the neural network are more accurate than those of ARIMA
Kurihara & Fukushima	AR Model or Machine Learning for Forecasting GDP and Consumer Price for G7 Countries	2019	Indicate the preference of the AR model over the LSTM in a situation where the quality of forecasts is at the same level, due to problems with the interpretation of forecasts of neural networks
Shapley	A value for n-person games	1953	Proposed a theory of fair distribution of profits between the coalition members, using the average expected marginal contribution of the player after considering all combinations
Sundararaja, Taly, & Yan	Axiomatic attribution for deep networks	2017	The method of integrated gradients (IG) was proposed to determine the contribution of variables to the prediction of neural networks

Literature: Methods for improving the quality of forecasts

Authors	Title	Year	Contribution
Foroni, Marcellino & Stevanovic	Forecasting the Covid-19 recession and recovery: Lessons from the financial crisis	2022	Used MIDAS to adjust the initial forecasts for information from news releases during the COVID-19 crisis by an amount similar to errors in news releases and forecasts that can be corrected during the Great Recession of 2008-2009
Coulombe, Marcellino & Stevanovic	Can Machine Learning Catch the COVID-19 Recession?	2021	Used the ability of machine learning models to detect nonlinear patterns during the COVID-19 crisis and found an improvement in short-term forecasts
Longo, Riccaboni & Rungi	A neural network ensemble approach for GDP forecasting	2022	Showed that the ensemble consisting of DFM and LSTM surpasses a number of alternative modeling methods, including random walk, vector autoregression, random forest and boosting models
Clark & McCracken	Averaging Forecasts from VARs with Uncertain Instabilities	2010	The structure of the ensemble was proposed: the final forecast is the sum of the weighted average forecasts of the components
Qingwen, Chengming, & Guangxi	A New Multipredictor Ensemble Decision Framework Based on Deep Reinforcement Learning for Regional GDP Prediction	2022	Combined more than two complex algorithms and used an additional model to determine the weights of the ensemble components
Lu	Research on GDP Forecast Analysis Combining BP Neural Network and ARIMA Model	2021	Proposed to assign low weights to ensemble components with large or small forecast errors, and higher weights to models with an average error

Data

Category of variables	Examples of variables in groups	Number of variables	Sources of data
Leading	• Indices of enterprising confidence in the sectors • Actual / expected output price indices • Actual / expected salary indices • Actual / expected employment indices • Actual / expected order portfolio indices	35	• The magazine "Russian economic barometer“ • Rosstat • Investing.com
Real sector	• **Real GDP in 1995 prices** • Export/import volume • Unemployment rate • Production indices by industry	15	• EAESD of The Higher School of Economics • Rosstat
Finance	• CPI • MOEX • The price of the Brent oil futures contract • PMI index in partner countries	23	• Rosstat • EAESD of The Higher School of Economics • Investing.com • The Central Bank of the Russian Federation
Uncertainty	• Economic policy uncertainty index • Geopolitical Risk Index	2	• Economic Policy Uncertainty (Baker, Bloom, Davis, & Terry, 2020)
		76	

- Time horizon of the initial data: 1995Q1 – 2022Q4
- All indicators are monthly, except quarterly GDP, business confidence indices
- Transformations for stationarity: (annual) growth rate, natural logarithm, no transformation
- Training sample: 1996Q1-2016Q4 (84), validation sample: 2013Q1 – 2016Q4 (16), test sample: 2017Q1-2022Q4 (24)
- The data was standardized, null values were replaced by median values

Methodology: Dynamic factor model

1. Observed variables are determined by unobservable dynamic factors (Burns & Mitchell, 1946)
2. The unique characteristics of individual series are captured by idiosyncratic errors
3. Unobservable factors and errors depend on their previous values
4. 1st stage: equation 1 using the principal component method, equations 2 and 3 using OLS. Stage 2: parameters are re-estimated using the Kalman filter. Repeat until convergence
5. Unobservable factors:
   - Global
   - Real
   - Financial
   - Uncertainty

1 Measurement equation:

$$y_{i,t} = \sum_{j=1}^{r} \lambda_{i,j} f_{j,t} + e_{i,t}, \text{где } i = 1, \cdots, n$$

$y_{i,t}$ – observed values of the indicator i;
$f_{j,t}$ – unobservable dynamic factors;
$e_{i,t}$ – idiosyncratic error of the indicator i.

2 Transition equations:

$$f_{j,t} = \sum_{l=1}^{p} a_{j,l}\, f_{j,t-l} + u_{j,t}, u_{j,t} \sim i.i.d.N\left(0, \sigma_{u_j}^2\right), j = 1, \cdots r$$

$$e_{i,t} = \sum_{k=1}^{q} \rho_{i,k}\, e_{i,t-k} + \varepsilon_{i,t}, \varepsilon_{i,t} \sim i.i.d.N\left(0, \sigma_{\varepsilon_i}^2\right) \text{для } i = 1, \cdots n$$

3 Equation of quarterly indicators (Mariano & Murasawa, 2003):

$$x_t^Q = \begin{cases} \gamma x_t + 2\gamma x_{t-1} + 3\gamma x_{t-2} + 2\gamma x_{t-3} + \gamma x_{t-4}, t = 3,6,9 \ldots \\ \text{unobservable value, otherwise} \end{cases}$$

x_t^Q – observed quarterly values of the indicator;
x_t – the unobservable monthly value modeled by the equation 1.

Methodology: LSTM neural network and Integrated gradients

1. RNN stores in memory the previous forecast, which is a nonlinear combination of the input data in the previous step
2. LSTM can store information about the previous values of the input data for a long time
3. IG summarizes the gradients on the way from the initial to the actual value of the indicator and calculates the impact of the indicators on the forecast of the neural network

1 Recurrent neural networks (RNN):

$$h^t = \sigma(X^t \cdot W_X + h^{t-1} \cdot W_Y) = \sigma(W \cdot [X^t, h^{t-1}])$$

h^t – neural network prediction at a moment at a time t;
X^t – matrix of input data at a time t;
W – the matrix of weights;
σ – nonlinear activation function, for example, sigmoid.

2 LSTM:

$$h^t = z^t \cdot h^t + (1 - z^t) \cdot h^{t-1}, \ where\ z^t = \sigma(W \cdot [x^t, h^{t-1}])$$

3 The method of integrated gradients (IG):

$$IG_i(x, x') = (x_i - x_i') \times \int_{\alpha=0}^{1} \frac{\delta f\big(x' + \alpha(x - x')\big)}{\delta x_i} d\alpha$$

x_i – the actual value of the indicator i;
x_i' – the initial value of the indicator i, if it was absent in the model.

Methodology: Ensemble model

1. The generally accepted structure of ensemble models is the sum of averaged forecasts of components (Clark & McCracken, 2010)
2. The study proposes an error–correcting structure - the sum of the DFM forecast for the GDP growth rate and the LSTM forecast for the DFM forecast error. LSTM corrects DFM errors to reduce the bias of the final forecast

1 Ensemble with the structure of averaged forecasts:

$$a(x) = {}^{1}\!/_{2}\, a_{DFM}(x) + {}^{1}\!/_{2}\, a_{LSTM}(x)$$

$a(x)$ – prediction of the ensemble model;
$a_{DFM}(x)$ – prediction of DFM;
$a_{LSTM}(x)$ – prediction of LSTM.

2 Ensemble with error correction structure:

$$y_{LSTM} = y - a_{DFM}(x)$$

y – Russia's GDP growth rate;
y_{LSTM} – target variable for training LSTM.

$$a(x) = a_{DFM}(x) + a_{LSTM}(x)$$

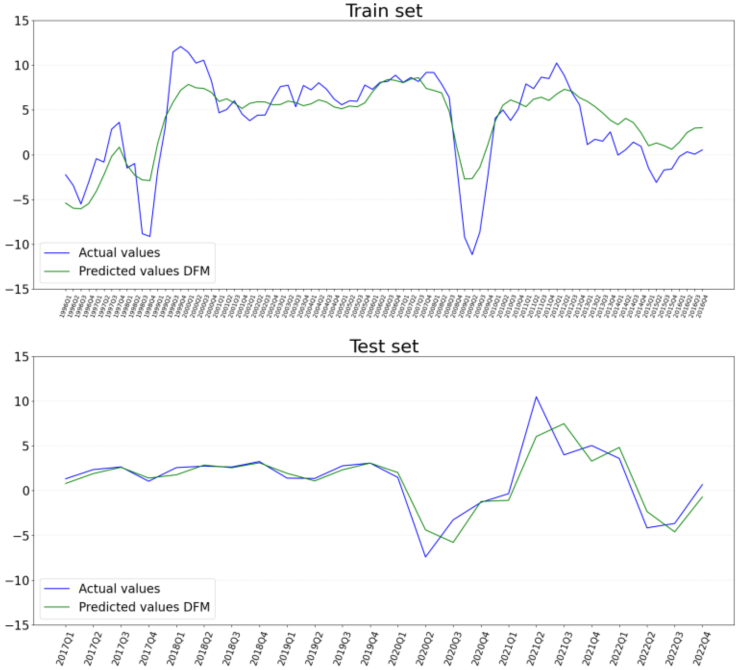
Results: Model accuracy analysis (1/3)

Model	RMSE train set	RMSE test set
DFM	2,76	1,59
LSTM	2,21	3,19
Ensemble DFM, LSTM with weighted average structure	1,32	2,42
Ensemble DFM, LSTM with error correction structure	2,35	1,95
ARMAX	0	8,56
VAR	5,91	3,95
SVR	0,49	3,49
CatBoost	0,27	3,23

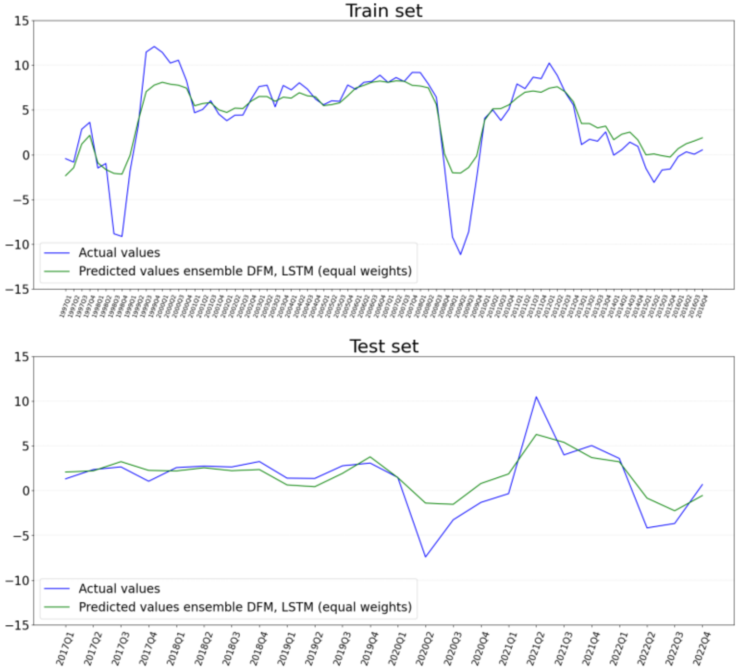
- Hypothesis 1 about the lower RMSE of the ensemble model forecasts was **partially accepted**
- Hypothesis 2 about a lower RMSE of machine learning methods than statistical models was **accepted** (Ahmed et al. (2010); Richardson et al. (2020); Teräsvirta et al. (2005))

Results: Model accuracy analysis (2/3)

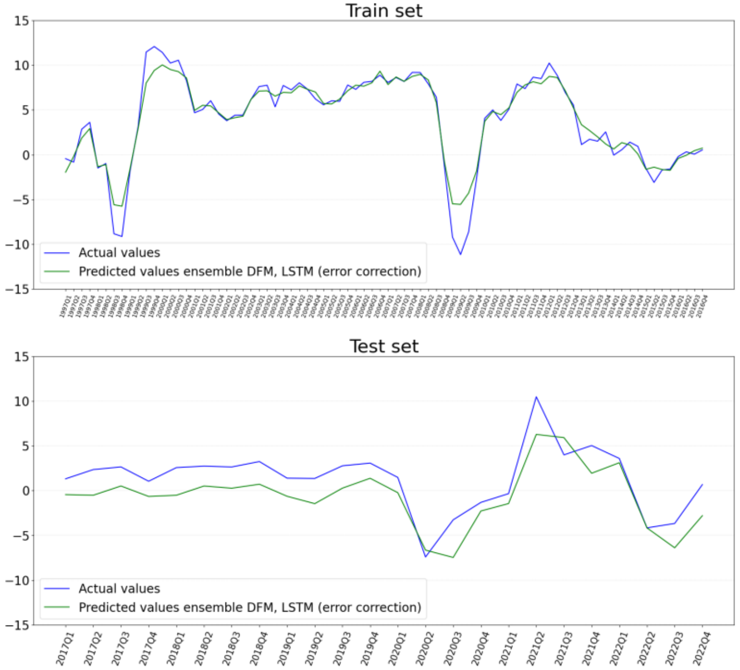
DFM Forecasts for Russia's GDP growth rate



Ensemble forecasts with average weights structure for the growth rate of Russia's GDP



Ensemble forecasts with correction of errorsfor the growth rate of Russia's GDP



- The RMSE of DFM forecasts is lower than statistical methods (Chernis & Sekkel, 2017; Ponomarev & Pleskachev, 2018).
- The superiority of DFM over machine learning methods was not noted earlier (Lovermann & Maas, 2019). Chu & Qureshi (2022): the principal component method is superior to machine learning with frequency indicators with high predictive power. DFM is able to work with data of different frequencies without disturbing their structure, unlike other models.
- The error-correcting structure is prone to overfitting and involves the use of simpler algorithms.
- The ensemble approach does not improve the DFM predictions when the information in the sample is not enough to train the neural network.

Results: Model accuracy analysis (3/3)

Model	RMSE sample with uncertainty indices	RMSE sample without uncertainty indices
DFM	1,59	1,59
LSTM	3,19	3,32
Ensemble DFM, LSTM with weighted average structure	2,42	2,37
Ensemble DFM, LSTM with error correction structure	1,95	2,11
ARMAX	8,56	7,45
VAR	3,95	3,96
SVR	3,49	3,51
CatBoost	3,23	3,31

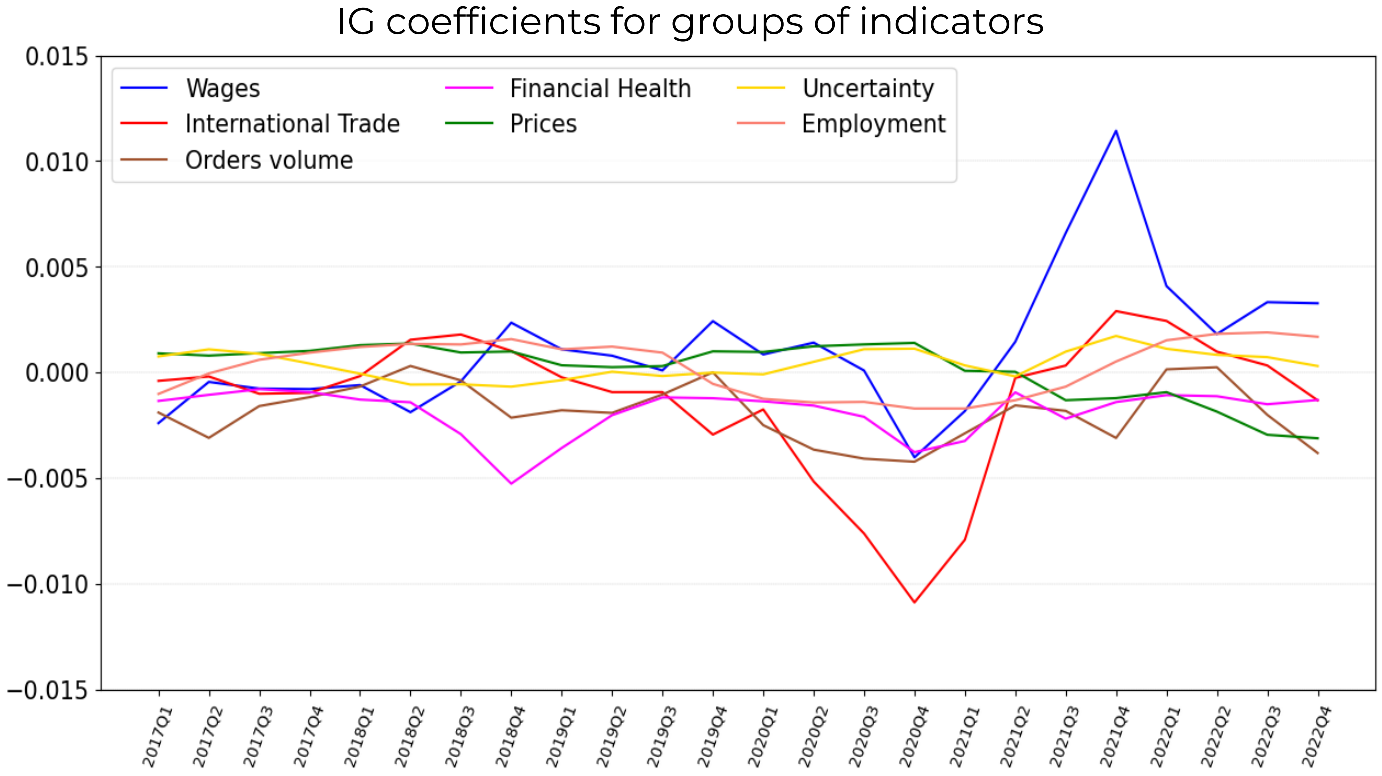
- Hypothesis 3 about a lower RMSE of model forecasts based on data with indices of economic policy uncertainty and geopolitical risk than models without these indicators was **partially accepted** (Baker, Bloom, Davis, & Terry, 2020)

Results: DFM interpretation

6 of the 10 most important indicators in DFM are Leading (Porshakov, Ponomarenko, & Sinyakov, 2016)

№	Indicator	The value of the eigenvector in the Global factor	Category
1	Diffuse employment index, expected changes	0,27	Leading
2	Diffuse index of equipment purchases, expected changes	0,26	Leading
3	Diffuse output index, expected changes	0,25	Leading
4	Overdue accounts receivable	-0,24	Finance
5	Overdue accounts payable	-0,24	Finance
6	Unemployment rate	-0,23	Real sector
7	Diffuse index of the order portfolio, actual changes	0,22	Leading
8	Commercial cargo turnover of transport	0,22	Real sector
9	Diffuse wage index, actual changes	0,21	Leading
10	Diffuse output index, actual changes	0,20	Leading

Results: LSTM interpretation



Group	Показатели
Wages	• Actual salary index • Expected. salary index
International Trade	• Export growth • Import growth
Orders volume	• Actual order portfolio index • Expected. order portfolio index • Volume of orders
Financial Health	• Actual financial state of enterprises • Expected financial state of enterprises • % of enterprises without debts to banks
Prices	• Actual output price index • Expected output price index • Actual purchase price index • Expected purchase price index.
Uncertainty	• Economic policy uncertainty index • Geopolitical risk index
Employment	• Actual employment index • Expected employment index

- **Wages** has a positive impact in 2021 during the economic recovery period, in 2022 the impact is decreasing
- **International Trade** has a negative impact in 2020 due to the disruption of supply chains and drop of oil prices
- **Orders volume** has a negative impact in 2020 and 2022 due to a decrease in economic activity
- **Financial Health** has a negative impact in 2018 Q4 due to increasing sanctions pressure
- **Prices** are negatively affected in 2022 due to rising inflation due to a reduction in supply in the economy
- **Employment** has a negative impact in 2019Q2 – 2021Q2 due to lockdown and job cuts due to falling demand

Conclusion

- DFM has shown the most accurate forecasts for Russia's GDP growth rates for the quarter ahead
- The ensemble model is superior to all competitor models except DFM
- The error of machine learning methods is lower than that of "classical" statistical models
- Inclusion of uncertainty indices improves model predictions
- The most important indicators in DFM are expected employment, equipment purchases and output
- Indicators of foreign trade, indices of actual and expected employment were more important for predicting the crisis of the COVID-19 pandemic
- The development of the crisis in 2022 is well predicted in terms of the volume of orders in industry, as well as in terms of price indices for manufactured and purchased products