$h = A + B + A^{-1} + B^{-1}$ (case 2)

Computation of the sequences $\eta := (\| h_n \|_2^2), \xi := (\xi_n), \rho := (\eta_n), \zeta := (\zeta_n), m := (m_n)$ *, tables, graphs, and densities for the paper*
"A COMPUTATIONAL APPROACH TO THE THOMPSON GROUP F"
by S. Haagerup, U. Haagerup, M. Ramirez-Solano:

```
ηhalf = {2, 6, 18, 54, 172, 538, 1750, 5662, 19354, 67640, 248808, 955226, 3873742, 16469058, 72875074,
   335047398, 1582466740, 7674516890, 37775752458, 189434653576, 959151943910, 4922901950252, 25435065012668, 132837576943418}
idsNotreduced = {0, 0, 0, 0, 0, 0, 0, 0, 0, 20, 0, 124, 0, 528, 0, 2168, 0, 9376, 0, 42340, 0, 191584, 0, 884020}
```

```
{2, 6, 18, 54, 172, 538, 1750, 5662, 19354, 67640, 248808, 955226, 3873742, 16469058, 72875074,
 335047398, 1582466740, 7674516890, 37775752458, 189434653576, 959151943910, 4922901950252, 25435065012668, 132837576943418}
```

```
{0, 0, 0, 0, 0, 0, 0, 0, 20, 0, 124, 0, 528, 0, 2168, 0, 9376, 0, 42340, 0, 191584, 0, 884020}
```

```
Clear[ids]
ids[-1] = 0;
ids[0] = 0;
idsList = Table[ids[n] = idsNotreduced[[n]] - 3 ids[n - 2], {n, 1, Length[idsNotreduced]}];
η = 2 ηhalf - idsList^2
Clear[ids, idsList]
```

```
{4, 12, 36, 108, 344, 1076, 3500, 11324, 38708, 134880, 497616, 1906356, 7747484, 32825220, 145750148,
 668749196, 3164933480, 15314270964, 75551504916, 378261586048, 1918303887820, 9831967554120, 50870130025336, 265393048436340}
```

```
η
```

```
{4, 12, 36, 108, 344, 1076, 3500, 11324, 38708, 134880, 497616, 1906356, 7747484, 32825220, 145750148,
 668749196, 3164933480, 15314270964, 75551504916, 378261586048, 1918303887820, 9831967554120, 50870130025336, 265393048436340}
```

```
q = 4 - 1;
ξ = η - (q + 1) Table[q^(n-1), {n, 1, Length[η]}]
ρ = ξ - (q - 1) Table[Total[ξ[[Range[n - 1]]]], {n, 1, Length[ξ]}]
ζ = ρ - (q - 1) Table[Total[ρ[[Range[n - 1]]]], {n, 1, Length[ξ]}]
m = Table[Binomial[2 n, n] q^n + Total[Table[Binomial[2 n, n - k] q^(n-k) (ξ[[k]] + 1 - q), {k, 1, n}]], {n, 1, Length[ζ]}]
mq = Table[Binomial[2 n, n] q^n + Total[Table[Binomial[2 n, n - k] q^(n-k) (1 - q), {k, 1, n}]], {n, 1, Length[ζ]}]
Ntuple = Length[m]
```

```
{0, 0, 0, 0, 20, 104, 584, 2576, 12464, 56148, 261420, 1197768, 5621720, 26447928, 126618272, 611353568,
 2992746596, 14797710312, 74001822960, 373612540180, 1904356750216, 9790126141308, 50744605786900, 265016475721032}
```

```
{0, 0, 0, 0, 20, 64, 336, 1160, 5896, 24652, 117628, 531136, 2559552, 12142320, 59416808, 290915560,
 1449601452, 7269071976, 36877764000, 188484835300, 972003964976, 5049059855636, 26423287218612, 139205945578944}
```

```
{0, 0, 0, 0, 20, 24, 168, 320, 2736, 9700, 53372, 231624, 1197768, 5661432, 28651280, 141316416,
 718171188, 3638438808, 18708986880, 96560530180, 503109989256, 2636157949964, 13912265601668, 73848349524776}
```

```
{4, 28, 232, 2092, 19884, 196096, 1988452, 20612364, 217561120, 2331456068, 25311956784, 277937245744, 3082543843552, 34493827011868, 389093033592912, 4420986174041164, 50566377945667804,
 581894842848487960, 6733830314028209908, 78331435477025276852, 915607264080561034564, 10750847942401254987096, 126768974481834814357308, 1500753741925909645997904}
```

```
{4, 28, 232, 2092, 19864, 195352, 1970896, 20275660, 211823800, 2240795848, 23951289520, 258255469816, 2805534253552, 30675477376432, 337306474674592, 3727578443380492, 41376874025687032,
 461121792658583272, 5157384457905440752, 57869888433073055272, 651266142688270063312, 7349148747954997832272, 83136542574028253115232, 942624010510370287581112}
```

```
24
```

```
Grid[Transpose[{Range[Length[η]], η, ξ, ρ, ζ, m}], Alignment → Left]
```

1	4	0	0	0	4
2	12	0	0	0	28
3	36	0	0	0	232
4	108	0	0	0	2092
5	344	20	20	20	19884
6	1076	104	64	24	196096
7	3500	584	336	168	1988452
8	11324	2576	1160	320	20612364
9	38708	12464	5896	2736	217561120
10	134880	56148	24652	9700	2331456068
11	497616	261420	117628	53372	25311956784
12	1906356	1197768	531136	231624	277937245744
13	7747484	5621720	2559552	1197768	3082543843552
14	32825220	26447928	12142320	5661432	34493827011868
15	145750148	126618272	59416808	28651280	389093033592912
16	668749196	611353568	290915560	141316416	4420986174041164
17	3164933480	2992746596	1449601452	718171188	50566377945667804
18	15314270964	14797710312	7269071976	3638438808	581894842848487960
19	75551504916	74001822960	36877764000	18708986880	6733830314028209908
20	378261586048	373612540180	188484835300	96560530180	78331435477025276852
21	1918303887820	1904356750216	972003964976	503109989256	915607264080561034564
22	9831967554120	9790126141308	5049059855636	2636157949964	10750847942401254987096
23	50870130025336	50744605786900	26423287218612	13912265601668	126768974481834814357308
24	265393048436340	265016475721032	139205945578944	73848349524776	1500753741925909645997904

```
μ = Function[d, If[d == 1, 1, Module[{k, factores, exponentes, isProductOfDistinctPrimes},
     {factores, exponentes} = Transpose[FactorInteger[d]];
     k = Length[factores];
     If[Max[exponentes] == 1 , isProductOfDistinctPrimes = 1, isProductOfDistinctPrimes = 0];
     If[isProductOfDistinctPrimes == 1, (-1)^k, 0]
    ]]]
ζpFunction = Function[n, Module[{divisors, i},
    divisors = Divisors[n];
    Total[Table[μ[n/divisors[[i]]] ζ[[divisors[[i]]]], {i, 1, Length[divisors]}]]
  ]]
```

```
Function[d, If[d == 1, 1, Module[{k, factores, exponentes, isProductOfDistinctPrimes}, {factores, exponentes} = Transpose[FactorInteger[d]];
   k = Length[factores]; If[Max[exponentes] == 1, isProductOfDistinctPrimes = 1, isProductOfDistinctPrimes = 0]; If[isProductOfDistinctPrimes == 1, (-1)^k, 0]]]]
```

```
Function[n, Module[{divisors, i}, divisors = Divisors[n]; Total[Table[μ[n/divisors[[i]]] ζ[[divisors[[i]]]], {i, 1, Length[divisors]}]]]]
```

```
Table[{n, ζpFunction[n]}, {n, 1, 24}] // ColumnForm
```

```
{1, 0}
{2, 0}
{3, 0}
{4, 0}
{5, 20}
{6, 24}
{7, 168}
{8, 320}
{9, 2736}
{10, 9680}
{11, 53372}
{12, 231600}
{13, 1197768}
{14, 5661264}
{15, 28651260}
{16, 141316096}
{17, 718171188}
{18, 3638436048}
{19, 18708986880}
{20, 96560520480}
{21, 503109989088}
{22, 2636157896592}
{23, 13912265601668}
{24, 73848349292832}
```

$$\text{Table}\left[\left\{n, \frac{\zeta\text{pFunction}[n]}{2\,n}\right\}, \{n, 1, 24\}\right] \text{// ColumnForm}$$

```
{1, 0}
{2, 0}
{3, 0}
{4, 0}
{5, 2}
{6, 2}
{7, 12}
{8, 20}
{9, 152}
{10, 484}
{11, 2426}
{12, 9650}
{13, 46068}
{14, 202188}
{15, 955042}
{16, 4416128}
{17, 21122682}
{18, 101067668}
{19, 492341760}
{20, 2414013012}
{21, 11978809264}
{22, 59912679468}
{23, 302440556558}
{24, 1538507276934}
```

```
mm = Riffle[0 Range[Length[m]], m]
Table[d[n] = Det[Table[If[i + j == 0, 1, mm[[i + j]]], {i, 0, n}, {j, 0, n}]], {n, 0, Length[m]}]
d[-1] = 1;
Table[k[n] = (d[n - 1] / d[n])^(1/2), {n, 0, Length[m]}]
Table[α[n] = k[n - 1] / k[n], {n, 1, Length[m]}]
```

$$\{0,\ 4,\ 0,\ 28,\ 0,\ 232,\ 0,\ 2092,\ 0,\ 19\,884,\ 0,\ 196\,096,\ 0,\ 1\,988\,452,\ 0,\ 20\,612\,364,\ 0,\ 217\,561\,120,\ 0,\ 2\,331\,456\,068,\ 0,\ 25\,311\,956\,784,\ 0,\ 277\,937\,245\,744,\ 0,$$
$$3\,082\,543\,843\,552,\ 0,\ 34\,493\,827\,011\,868,\ 0,\ 389\,093\,033\,592\,912,\ 0,\ 4\,420\,986\,174\,041\,164,\ 0,\ 50\,566\,377\,945\,667\,804,\ 0,\ 581\,894\,842\,848\,487\,960,\ 0,\ 6\,733\,830\,314\,028\,209\,908,$$
$$0,\ 78\,331\,435\,477\,025\,276\,852,\ 0,\ 915\,607\,264\,080\,561\,034\,564,\ 0,\ 10\,750\,847\,942\,401\,254\,987\,096,\ 0,\ 126\,768\,974\,481\,834\,814\,357\,308,\ 0,\ 1\,500\,753\,741\,925\,909\,645\,997\,904\}$$

$$\{1,\ 4,\ 48,\ 1728,\ 186\,624,\ 64\,198\,656,\ 68\,839\,981\,056,\ 238\,010\,587\,938\,816,\ 2\,592\,670\,620\,986\,638\,336,\ 92\,636\,506\,280\,543\,601\,557\,504,\ 10\,630\,707\,614\,753\,855\,994\,907\,852\,800,\ 4\,027\,333\,162\,626\,304\,814\,271\,741\,453\,926\,400,$$
$$4\,943\,652\,405\,070\,075\,593\,186\,874\,013\,951\,262\,195\,712,\ 20\,230\,521\,828\,540\,040\,223\,551\,260\,803\,661\,384\,239\,640\,739\,840,\ 270\,744\,068\,730\,398\,203\,074\,097\,408\,602\,961\,794\,247\,779\,427\,171\,696\,640,$$
$$12\,161\,133\,888\,024\,410\,391\,198\,049\,014\,394\,415\,759\,190\,372\,923\,641\,700\,347\,281\,408,\ 1\,798\,001\,731\,108\,573\,966\,243\,420\,033\,709\,971\,661\,941\,236\,650\,748\,794\,355\,212\,360\,079\,114\,240,$$
$$897\,089\,475\,410\,745\,138\,158\,450\,580\,206\,075\,295\,572\,542\,065\,494\,599\,308\,747\,298\,316\,953\,951\,288\,688\,640,\ 1\,483\,192\,441\,166\,690\,145\,024\,763\,713\,985\,438\,676\,359\,977\,538\,458\,009\,290\,133\,675\,719\,181\,459\,753\,673\,122\,939\,666\,432,$$
$$8\,317\,895\,359\,196\,495\,278\,193\,231\,495\,018\,204\,925\,846\,618\,659\,054\,302\,894\,404\,751\,396\,937\,958\,773\,150\,551\,724\,091\,502\,929\,903\,616,$$
$$155\,328\,115\,254\,591\,067\,890\,780\,863\,990\,027\,741\,172\,240\,362\,980\,181\,239\,058\,952\,668\,502\,602\,612\,382\,220\,939\,427\,596\,490\,065\,611\,405\,584\,760\,832,$$
$$9\,879\,699\,315\,039\,905\,638\,019\,186\,839\,051\,070\,641\,814\,952\,979\,144\,495\,038\,224\,260\,922\,706\,551\,635\,063\,467\,730\,678\,131\,366\,224\,516\,419\,721\,886\,167\,344\,349\,184,$$
$$2\,103\,127\,622\,641\,710\,334\,836\,660\,807\,962\,025\,814\,858\,870\,521\,455\,440\,440\,382\,854\,052\,629\,771\,218\,513\,062\,381\,548\,104\,583\,944\,579\,139\,719\,617\,690\,940\,028\,414\,592\,838\,270\,976,$$
$$1\,530\,226\,809\,462\,398\,229\,469\,463\,254\,447\,884\,229\,915\,519\,562\,380\,590\,918\,210\,332\,711\,334\,192\,027\,763\,053\,225\,511\,523\,011\,772\,832\,448\,999\,035\,274\,600\,619\,561\,934\,818\,120\,387\,298\,150\,318\,080,$$
$$3\,741\,303\,042\,285\,262\,109\,902\,211\,197\,312\,530\,335\,878\,170\,973\,670\,669\,725\,759\,853\,287\,101\,872\,920\,720\,063\,735\,142\,114\,733\,759\,452\,114\,771\,551\,497\,041\,743\,077\,816\,987\,992\,518\,769\,882\,825\,976\,757\,186\,723\,840\ \}$$

$$\Big\{1,\ \frac{1}{2},\ \frac{1}{2\sqrt{3}},\ \frac{1}{6},\ \frac{1}{6\sqrt{3}},\ \frac{1}{2\sqrt{86}},\ \frac{3\sqrt{\frac{3}{7238}}}{2},\ \frac{3\sqrt{\frac{43}{83\,627}}}{4},\ \frac{\sqrt{\frac{10\,857}{7\,391\,642}}}{4},\ \frac{\sqrt{\frac{83\,627}{747\,001\,282}}}{2},\ \frac{\sqrt{\frac{3\,695\,821}{16\,964\,885\,238}}}{5},\ \frac{\sqrt{\frac{373\,500\,641}{35\,374\,209\,607\,502}}}{2},\ \frac{5\sqrt{\frac{8\,482\,442\,619}{16\,269\,392\,083\,171\,243}}}{4},\ \sqrt{\frac{17\,687\,104\,803\,751}{72\,379\,554\,729\,412\,245\,070}},$$
$$2\sqrt{\frac{16\,269\,392\,083\,171\,243}{870\,929\,864\,429\,659\,559\,728\,037}},\ \sqrt{\frac{72\,379\,554\,729\,412\,245\,070}{3\,251\,105\,222\,535\,739\,267\,032\,457\,929}},\ \sqrt{\frac{870\,929\,864\,429\,659\,559\,728\,037}{128\,765\,411\,049\,435\,550\,115\,060\,648\,674\,110}},\ \sqrt{\frac{1\,083\,701\,740\,845\,246\,422\,344\,152\,643}{540\,698\,826\,578\,531\,016\,101\,443\,376\,397\,349\,598}},$$
$$\frac{\sqrt{\frac{64\,382\,705\,524\,717\,775\,057\,530\,324\,337\,055}{26\,611\,599\,175\,321\,939\,103\,434\,889\,115\,882\,592\,195\,721}}}{2},\ \frac{\sqrt{\frac{811\,048\,239\,867\,796\,524\,152\,165\,064\,596\,024\,397}{1\,137\,110\,432\,071\,479\,627\,369\,189\,100\,045\,090\,956\,862\,038\,509}}}{2},\ \frac{\sqrt{\frac{26\,611\,599\,175\,321\,939\,103\,434\,889\,115\,882\,592\,195\,721}{124\,236\,040\,648\,288\,575\,985\,580\,719\,863\,131\,115\,560\,609\,275\,227\,323}}}{2},\ \frac{\sqrt{\frac{1\,137\,110\,432\,071\,479\,627\,369\,189\,100\,045\,090\,956\,862\,038\,509}{4\,520\,394\,270\,882\,518\,736\,009\,393\,470\,618\,543\,801\,933\,934\,105\,977\,259\,063}}}{4},$$
$$\frac{\sqrt{\frac{124\,236\,040\,648\,288\,575\,985\,580\,719\,863\,131\,115\,560\,609\,275\,227\,323}{1\,652\,911\,189\,926\,726\,750\,385\,075\,363\,643\,574\,345\,216\,235\,656\,628\,935\,554\,401\,817}}}{4},\ \frac{\sqrt{\frac{4\,520\,394\,270\,882\,518\,736\,009\,393\,470\,618\,543\,801\,933\,934\,105\,977\,259\,063}{822\,254\,963\,057\,831\,796\,189\,798\,685\,572\,834\,210\,858\,415\,320\,211\,630\,709\,219\,797\,792\,510}}}{2},\ \frac{\sqrt{\frac{1\,652\,911\,189\,926\,726\,750\,385\,075\,363\,643\,574\,345\,216\,235\,656\,628\,935\,554\,401\,817}{1\,010\,314\,553\,577\,976\,302\,255\,331\,236\,614\,929\,111\,292\,766\,751\,182\,770\,893\,131\,529\,878\,422\,857\,279}}}{2}\Big\}$$

$$\Big\{2,\ \sqrt{3},\ \sqrt{3},\ \sqrt{3},\ \frac{\sqrt{\frac{86}{3}}}{3},\ \frac{\sqrt{\frac{3619}{129}}}{3},\ \sqrt{\frac{501\,762}{155\,617}},\ \sqrt{\frac{953\,521\,818}{302\,646\,113}},\ \frac{\sqrt{\frac{4\,055\,096\,459\,337}{309\,070\,422\,767}}}{2},\ \frac{5\sqrt{\frac{709\,361\,228\,899\,113}{1\,380\,391\,512\,521\,261}}}{2},\ \frac{2\sqrt{\frac{65\,368\,373\,362\,903\,824\,571}{3\,168\,197\,755\,442\,218\,779}}}{5},\ \frac{\sqrt{\frac{12\,153\,256\,743\,489\,569\,146\,533\,526}{150\,029\,851\,594\,057\,113\,463\,869}}}{5},\ \frac{35\sqrt{\frac{6\,264\,851\,222\,255\,198\,371\,451\,435\,085}{575\,516\,885\,736\,733\,161\,982\,483\,464\,986}}}{2},$$
$$\frac{\sqrt{\frac{15\,404\,227\,788\,884\,038\,782\,707\,302\,925\,817\,466\,787}{1\,177\,571\,354\,698\,159\,279\,275\,642\,521\,892\,522\,010}}}{2},\ \sqrt{\frac{105\,787\,011\,138\,159\,277\,254\,465\,563\,066\,815\,230\,600\,271\,494}{31\,518\,757\,893\,983\,065\,508\,550\,196\,956\,256\,613\,397\,013\,795}},\ 10\sqrt{\frac{31\,066\,610\,387\,692\,948\,766\,676\,359\,332\,265\,525\,718\,443\,646\,280\,459}{943\,828\,210\,236\,536\,524\,506\,755\,251\,933\,697\,592\,404\,234\,294\,751\,791}},$$
$$\sqrt{\frac{235\,455\,377\,864\,658\,011\,473\,104\,878\,430\,583\,270\,541\,198\,720\,233\,852\,245\,639\,563}{69\,771\,650\,057\,463\,517\,344\,379\,022\,783\,831\,524\,893\,223\,798\,940\,454\,701\,086\,365}},\ 13\sqrt{\frac{341\,290\,371\,041\,092\,419\,672\,945\,576\,656\,957\,638\,481\,798\,108\,256\,819\,246\,652\,195\,922\,374}{17\,405\,826\,664\,583\,003\,500\,756\,857\,687\,880\,504\,247\,199\,706\,451\,784\,961\,172\,909\,210\,376\,945}},$$
$$\sqrt{\frac{24\,403\,415\,365\,714\,222\,557\,891\,396\,970\,843\,185\,569\,176\,719\,975\,547\,067\,626\,746\,235\,396\,127\,101\,883\,665}{7\,194\,430\,223\,737\,388\,078\,127\,035\,570\,337\,896\,334\,046\,078\,547\,340\,629\,981\,935\,857\,209\,131\,838\,335\,079}},\ \sqrt{\frac{100\,761\,422\,095\,938\,472\,165\,934\,850\,176\,942\,778\,988\,559\,029\,178\,859\,530\,889\,547\,361\,153\,160\,679\,063\,644\,228\,999\,231}{30\,260\,327\,036\,363\,361\,105\,734\,771\,608\,761\,384\,144\,042\,017\,065\,855\,554\,008\,175\,209\,298\,998\,956\,994\,730\,367\,019\,989}},$$
$$2\sqrt{\frac{120\,294\,920\,451\,147\,253\,795\,849\,745\,443\,119\,218\,459\,888\,309\,931\,663\,139\,410\,157\,943\,912\,126\,093\,121\,807\,997\,431\,906\,417\,069\,423}{141\,270\,097\,860\,425\,328\,590\,991\,615\,774\,353\,468\,779\,955\,040\,172\,518\,455\,673\,482\,702\,425\,283\,252\,871\,141\,430\,139\,320\,154\,981\,407}},$$
$$\sqrt{\frac{1\,879\,542\,557\,353\,363\,779\,406\,654\,593\,217\,257\,873\,155\,440\,319\,801\,190\,027\,770\,519\,321\,560\,183\,567\,029\,408\,366\,038\,111\,508\,240\,682\,713\,570\,853}{561\,595\,886\,383\,651\,397\,751\,417\,950\,931\,401\,642\,368\,141\,038\,965\,887\,619\,168\,839\,871\,844\,429\,105\,468\,831\,706\,427\,757\,317\,568\,677\,786\,978\,349}},$$
$$\sqrt{(51\,076\,850\,506\,854\,906\,221\,272\,866\,617\,959\,608\,574\,808\,722\,469\,439\,723\,382\,390\,811\,895\,941\,317\,681\,067\,696\,135\,723\,073\,856\,805\,066\,233\,801\,811\,918\,375\,365\ /$$
$$14\,943\,620\,546\,444\,764\,832\,732\,664\,728\,794\,885\,994\,452\,602\,925\,658\,200\,947\,956\,217\,793\,189\,432\,672\,655\,402\,580\,716\,842\,805\,547\,480\,137\,701\,780\,413\,834\,942\ \},$$
$$\sqrt{(4\,567\,020\,119\,783\,113\,597\,706\,317\,682\,724\,990\,541\,517\,992\,490\,068\,612\,700\,522\,998\,062\,404\,079\,144\,298\,474\,019\,141\,320\,164\,544\,584\,985\,684\,179\,528\,149\,625\,717\,258\,269\,577\ /$$
$$1\,359\,114\,429\,411\,077\,499\,904\,670\,466\,477\,693\,428\,966\,517\,397\,054\,572\,773\,927\,046\,035\,544\,023\,563\,590\,923\,609\,389\,025\,560\,798\,842\,605\,868\,794\,985\,646\,825\,390\,132\,990\,670\ )\Big\}$$

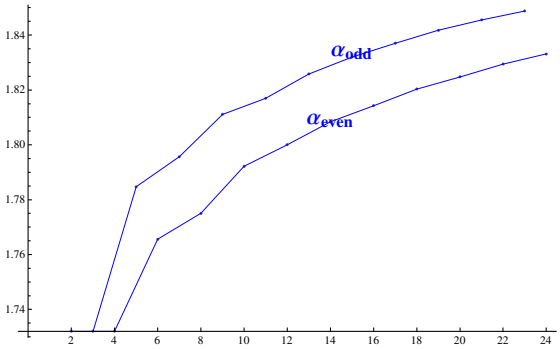
```
MnormFunctiontest = Function[n, Module[{i, j, eigenvalues, A},
    A = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
    Table[A[[i, i + 1]] = α[i]^2, {i, 1, n}];
    Table[A[[i + 1, i]] = 1, {i, 1, n}];
    A
   ]];
Table[(MnormFunctiontest[i] // MatrixForm), {i, 0, 7}]
MnormFunction = Function[n, Module[{i, j, eigenvalues, A},
    A = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
    Table[A[[i, i + 1]] = α[i]^2, {i, 1, n}];
    Table[A[[i + 1, i]] = 1, {i, 1, n}];
    eigenvalues = Eigenvalues[A];
    eigenvalues = eigenvalues // N;
    Max[eigenvalues]
   ]];
Table[Mnorm[i] = MnormFunction[i], {i, 0, Length[m]}]
```

$$\Big\{\begin{pmatrix}0\end{pmatrix},\ \begin{pmatrix}0&4\\1&0\end{pmatrix},\ \begin{pmatrix}0&4&0\\1&0&3\\0&1&0\end{pmatrix},\ \begin{pmatrix}0&4&0&0\\1&0&3&0\\0&1&0&3\\0&0&1&0\end{pmatrix},\ \begin{pmatrix}0&4&0&0&0\\1&0&3&0&0\\0&1&0&3&0\\0&0&1&0&3\\0&0&0&1&0\end{pmatrix},\ \begin{pmatrix}0&4&0&0&0&0\\1&0&3&0&0&0\\0&1&0&3&0&0\\0&0&1&0&3&0\\0&0&0&1&0&\frac{86}{27}\\0&0&0&0&1&0\end{pmatrix},\ \begin{pmatrix}0&4&0&0&0&0&0\\1&0&3&0&0&0&0\\0&1&0&3&0&0&0\\0&0&1&0&3&0&0\\0&0&0&1&0&\frac{86}{27}&0\\0&0&0&0&1&0&\frac{3619}{1161}\\0&0&0&0&0&1&0\end{pmatrix},\ \begin{pmatrix}0&4&0&0&0&0&0&0\\1&0&3&0&0&0&0&0\\0&1&0&3&0&0&0&0\\0&0&1&0&3&0&0&0\\0&0&0&1&0&\frac{86}{27}&0&0\\0&0&0&0&1&0&\frac{3619}{1161}&0\\0&0&0&0&0&1&0&\frac{501\,762}{155\,617}\\0&0&0&0&0&0&1&0\end{pmatrix}\Big\}$$

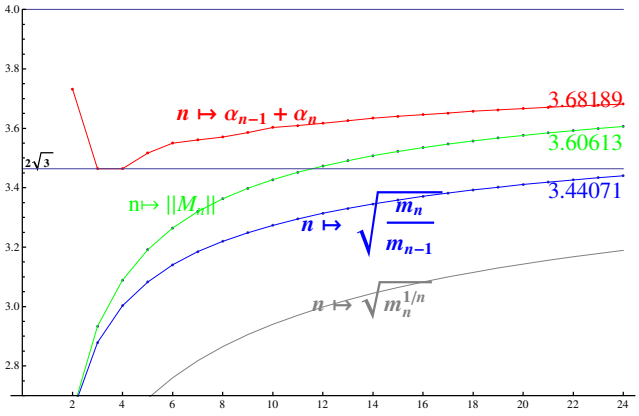
$$\{0.,\ 2.,\ 2.64575,\ 2.93352,\ 3.08891,\ 3.19184,\ 3.26439,\ 3.32,\ 3.36276,\ 3.3979,\ 3.42682,\ 3.45164,$$
$$3.47272,\ 3.49133,\ 3.50755,\ 3.52217,\ 3.53513,\ 3.54697,\ 3.55761,\ 3.56745,\ 3.57639,\ 3.58471,\ 3.59235,\ 3.59952,\ 3.60613\}$$

```
Table[α[n], {n, 1, Ntuple}] // N
Show[
 ListPlot[Table[{i, α[i]}, {i, 2, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, α[i]}, {i, 2, Ntuple, 2}], Joined → True, PlotStyle → {Blue}], ListPlot[Table[{i, α[i]}, {i, 3, Ntuple, 2}], Joined → True, PlotStyle → {Blue}],
 Graphics[{Blue, Text[Style[HoldForm[α_even], Large, Bold], {14, .002 + α[14]}]}],
 Graphics[{Blue, Text[Style[HoldForm[α_odd], Large, Bold], {15, .003 + α[15]}]}]
]
```

```
{2., 1.73205, 1.73205, 1.73205, 1.78471, 1.76554, 1.79564, 1.775, 1.8111, 1.79214, 1.81693,
 1.80006, 1.82585, 1.80841, 1.83203, 1.81426, 1.83702, 1.82036, 1.84173, 1.82478, 1.84556, 1.82942, 1.84878, 1.83311}
```




```
ma[0] = 1;
Table[ma[i] = m[[i]], {i, 1, Length[m]}];
Show[
 ListPlot[Table[{i, Mnorm[i]}, {i, 0, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}, PlotRange → {2.7, 4}],
 ListPlot[Table[{i, Mnorm[i]}, {i, 0, Ntuple}], Joined → True, PlotStyle → {Green}],
 Graphics[{Green, Text[Style[HoldForm["n↦ ||M_n||"], Large], {6, .08 + Mnorm[6]}]}],
 Graphics[{Green, Text[Style[Mnorm[Ntuple], Large], {Ntuple - 1.5, -.055 + Mnorm[Ntuple]}]}],
 Graphics[{Red, Text[Style[N[α[Ntuple - 1] + α[Ntuple]], Large], {Ntuple - 1.5, .015 + α[Ntuple - 1] + α[Ntuple]}]}],
 Graphics[{Blue, Text[Style[N[(ma[Ntuple]/ma[Ntuple - 1])^(1/2)], Large], {Ntuple - 1.5, -.05 + (ma[Ntuple]/ma[Ntuple - 1])^(1/2)}]}],
 Plot[8, {x, 0, Ntuple}],
 ListPlot[Table[{i, (ma[i]/ma[i - 1])^(1/2)}, {i, 1, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, (ma[i]/ma[i - 1])^(1/2)}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Blue}],
 Graphics[{Blue, Text[Style[HoldForm[n ↦ (m_n/m_(n-1))^(1/2)], Large, Bold], {14, -.065 + (ma[14]/ma[13])^(1/2)}]}],
 ListPlot[Table[{i, α[i - 1] + α[i]}, {i, 1, Ntuple}], PlotStyle → {Red}, Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, α[i - 1] + α[i]}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Red}],
 Graphics[{Red, Text[Style[HoldForm[n ↦ α_(n-1) + α_n], Large, Bold], {10 - 1, .05 + α[9] + α[10]}]}],
 ListPlot[Table[{i, (ma[i]^(1/i))^(1 / 2)}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Gray}],
 Graphics[{Gray, Text[Style[HoldForm[n ↦ Sqrt[(m_n)^(1/n)]], Large, Bold], {14, -.02 + (ma[14]^(1/(2 14)))}]}],
 Graphics[{Black, Text[Style[HoldForm["2√3 "], Medium, Bold], {.7, 2 Sqrt[3] + .025}]}],
 Plot[Sqrt[12], {x, 0, Ntuple}],
 Plot[4, {x, 0, Ntuple}]
]
```

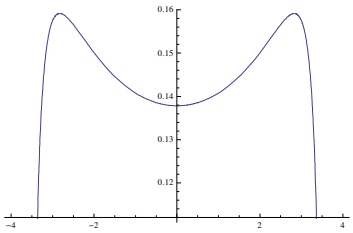


```
Grid[Transpose[
  {Range[Length[η]],
   N[Table[(ma[i]^(1/1))^(1/2), {i, 1, Ntuple, 1}], 6],
   N[Table[(ma[i]/ma[i-1])^(1/2), {i, 1, Ntuple}], 6],
   N[Table[α[n], {n, 1, Ntuple}], 6],
   N[Table[Mnorm[i], {i, 1, Ntuple}], 6],
   N[Table[α[i-1]+α[i], {i, 1, Ntuple}], 6]}
 ], Alignment→Left]
```

1	2.00000	2.00000	2.00000	2.	2.00000 + α[0]
2	2.30033	2.64575	1.73205	2.64575	3.73205
3	2.47884	2.87849	1.73205	2.93352	3.46410
4	2.60058	3.00287	1.73205	3.08891	3.46410
5	2.69061	3.08298	1.78471	3.19184	3.51676
6	2.76083	3.14038	1.76554	3.26439	3.55025
7	2.81769	3.18437	1.79564	3.32	3.56119
8	2.86506	3.21963	1.77500	3.36276	3.57064
9	2.90535	3.24883	1.81110	3.3979	3.58610
10	2.94023	3.27358	1.79214	3.42682	3.60324
11	2.97083	3.29495	1.81693	3.45164	3.60907
12	2.99800	3.31368	1.80006	3.47272	3.61699
13	3.02234	3.33028	1.82585	3.49133	3.62591
14	3.04432	3.34515	1.80841	3.50755	3.63425
15	3.06433	3.35858	1.83203	3.52217	3.64043
16	3.08264	3.37080	1.81426	3.53513	3.64629
17	3.09949	3.38198	1.83702	3.54697	3.65129
18	3.11507	3.39228	1.82036	3.55761	3.65739
19	3.12954	3.40180	1.84173	3.56745	3.66210
20	3.14303	3.41065	1.82478	3.57639	3.66651
21	3.15565	3.41890	1.84556	3.58471	3.67034
22	3.16749	3.42663	1.82942	3.59235	3.67498
23	3.17863	3.43388	1.84878	3.59952	3.67820
24	3.18914	3.44071	1.83311	3.60613	3.68189

```
ρFree = Function[x, 2/π Sqrt[12 - x^2]/(16 - x^2)]
(*Integrate[ρFree[x], {x, -Sqrt[12], Sqrt[12]}]==1*)
Plot[ρFree[x], {x, -(q+1), (q+1)}]
ρLebesgueCoords = Function[n, Module[{coeffs, mlist},
   If[Mod[n, 2] == 0 && n ≤ 2 Ntuple,
    coeffs = CoefficientList[LegendreP[n, x/(q+1)], x];
    coeffs = coeffs[[Range[1, Length[coeffs], 2]]];
    mlist = Table[ma[i], {i, 0, n/2}];
    Sqrt[(2 n+1)/(2 (q+1))] coeffs.mlist, 0]]]
p = Function[n, Sqrt[(2 n+1)/(2 (q+1))] LegendreP[n, x/(q+1)]]
ρLebesgue = Total[Table[ρLebesgueCoords[i] p[i], {i, 0, 2 Ntuple}]];
ρLebesgueNminus1 = Total[Table[ρLebesgueCoords[i] p[i], {i, 0, 2 (Ntuple-1)}]];
ρLebesgueGraph = Table[{x, ρLebesgue}, {x, 0, q+1, 1/100}];
ρLebesgueGraphTail = Table[{x, ρLebesgue}, {x, 34/10, q+1, 1/1000}];
ρLebesgueGraphTailNminus1 = Table[{x, ρLebesgueNminus1}, {x, 34/10, q+1, 1/1000}];
```

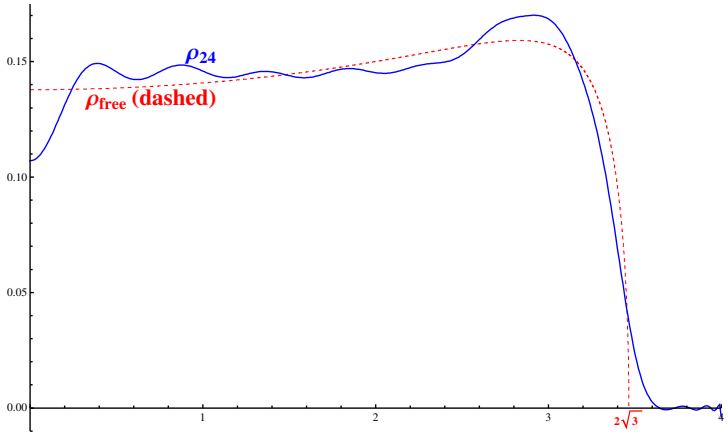
```
Function[x, 2 Sqrt[12 - x^2]/(π (16 - x^2))]
```



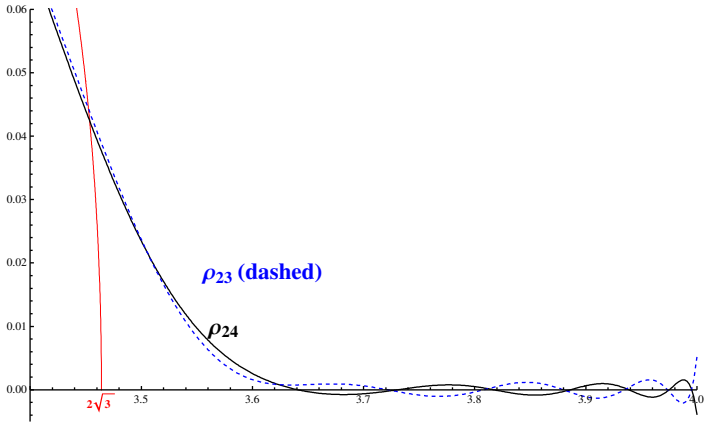

```
Function[n, Module[{coeffs, mlist},
  If[Mod[n, 2] == 0 && n ≤ 2 Ntuple, coeffs = CoefficientList[LegendreP[n, x/(q+1)], x]; coeffs = coeffs[[Range[1, Length[coeffs], 2]]]; mlist = Table[ma[i], {i, 0, n/2}]; Sqrt[(2 n+1)/(2 (q+1))] coeffs.mlist, 0]]]
```

```
Function[n, Sqrt[(2 n+1)/(2 (q+1))] LegendreP[n, x/(q+1)]]
```

```
Show[
 ListPlot[Table[{x, ρFree[x]}, {x, 0, q + 1 - 1/100, 1/10 000}]], Joined → True, PlotStyle → {Dashed, Red},
  PlotRange → {{0, q + 1}, {-.01, .175}}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[ρLebesgueGraph, Joined → True, PlotStyle → {Directive[Blue, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Blue, Text[Style[HoldForm[ρ24], Large, Bold], Evaluate[ρLebesgueGraph[[100]] + {0, .008}]]}],
 Graphics[{Red, Text[Style[HoldForm[ρfree "(dashed)"], Large, Bold], Evaluate[{0.7, ρFree[0.7] - .005}]]}], Graphics[{{Red, Text[Style[HoldForm["2√3 "], Medium, Bold], {2 Sqrt[3], -0.005}]}}]
]
```




```
Show[ListPlot[Join[Table[{x, ρFree[x]}, {x, 3.4, 2 Sqrt[3], 1/10 000}], {{2 Sqrt[3], 0}}], Joined → True, PlotStyle → {Red}, PlotRange → {{3.4, q + 1}, {-.005, .06}},
  AxesOrigin → {3.4, 0}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}, PlotRange → {2.7, 4}],
 ListPlot[ρLebesgueGraphTail, Joined → True, PlotStyle → {Directive[Black, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Black, Text[Style[HoldForm[ρ24], Large, Bold], Evaluate[ρLebesgueGraphTail[[150]] + {.025, .00}]]}],
 ListPlot[ρLebesgueGraphTailNminus1, Joined → True, PlotStyle → {Directive[Dashed, Blue, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Blue, Text[Style[HoldForm[ρ23 "(dashed)"], Large, Bold], Evaluate[ρLebesgueGraphTailNminus1[[150]] + {0.06, 0.01}]]}],
 Graphics[{{Red, Text[Style[HoldForm["2√3 "], Medium, Bold], {2 Sqrt[3], -0.002}]}}]]
```




```
(ρLebesgueGraphTail + ρLebesgueGraphTailNminus1) / 2 // N // ColumnForm
```

```
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