h=I+A+B (case 1)

Computation of the sequences $\eta := (\| h_n \|_2^2)$, $\xi := (\xi_n)$, $\rho := (\eta_n)$, $\zeta := (\zeta_n)$, $m := (m_n)$ *, tables, graphs, and densities for the paper*
"A COMPUTATIONAL APPROACH TO THE THOMPSON GROUP F"
by S. Haagerup, U. Haagerup, M. Ramirez-Solano:

```
η = {0, 0, 0, 0, 0, 0, 0, 400, 800, 1656, 3344, 7032, 14 272, 30 544, 63 120, 137 264, 292 160, 651 960, 1 435 808, 3 310 592, 7 593 024, 18 161 528, 43 488 112, 107 764 880, 268 721 056, 686 850 128,
  1 769 246 208, 4 640 551 024, 12 254 456 800, 32 773 003 720, 88 160 278 544, 239 251 904 104, 652 453 973 392, 1 790 526 123 576, 4 933 923 852 880, 13 660 080 583 776, 37 952 694 315 360}
```

```
η = {3, 6, 12, 24, 48, 96, 192, 400, 800, 1656, 3344, 7032, 14 272, 30 544, 63 120, 137 264, 292 160, 651 960, 1 435 808, 3 310 592, 7 593 024, 18 161 528, 43 488 112, 107 764 880, 268 721 056,
  686 850 128, 1 769 246 208, 4 640 551 024, 12 254 456 800, 32 773 003 720, 88 160 278 544, 239 251 904 104, 652 453 973 392, 1 790 526 123 576, 4 933 923 852 880, 13 660 080 583 776, 37 952 694 315 360}
```

```
{0, 0, 0, 0, 0, 0, 0, 400, 800, 1656, 3344, 7032, 14 272, 30 544, 63 120, 137 264, 292 160, 651 960, 1 435 808, 3 310 592, 7 593 024, 18 161 528, 43 488 112, 107 764 880, 268 721 056, 686 850 128,
 1 769 246 208, 4 640 551 024, 12 254 456 800, 32 773 003 720, 88 160 278 544, 239 251 904 104, 652 453 973 392, 1 790 526 123 576, 4 933 923 852 880, 13 660 080 583 776, 37 952 694 315 360}

{3, 6, 12, 24, 48, 96, 192, 400, 800, 1656, 3344, 7032, 14 272, 30 544, 63 120, 137 264, 292 160, 651 960, 1 435 808, 3 310 592, 7 593 024, 18 161 528, 43 488 112, 107 764 880, 268 721 056,
 686 850 128, 1 769 246 208, 4 640 551 024, 12 254 456 800, 32 773 003 720, 88 160 278 544, 239 251 904 104, 652 453 973 392, 1 790 526 123 576, 4 933 923 852 880, 13 660 080 583 776, 37 952 694 315 360}
```

```
q = 3 - 1;
ξ = η - (q + 1) Table[q^(n-1), {n, 1, Length[η]}]
ρ = ξ - (q - 1) Table[Total[ξ[[Range[n - 1]]]], {n, 1, Length[ξ]}]
ζ = ρ - (q - 1) Table[Total[ρ[[Range[n - 1]]]], {n, 1, Length[ξ]}]
m = Table[Binomial[2 n, n] q^n + Total[Table[Binomial[2 n, n - k] q^(n-k) (ζ[[k]] + 1 - q), {k, 1, n}]], {n, 1, Length[ζ]}]
mq = Table[Binomial[2 n, n] q^n + Total[Table[Binomial[2 n, n - k] q^(n-k) (1 - q), {k, 1, n}]], {n, 1, Length[ζ]}]
Ntuple = Length[m]
```

```
{0, 0, 0, 0, 0, 0, 0, 16, 32, 120, 272, 888, 1984, 5968, 13 968, 38 960, 95 552, 258 744, 649 376, 1 737 728, 4 447 296, 11 870 072, 30 905 200, 82 599 056, 218 389 408, 586 186 832,
 1 567 919 616, 4 237 897 840, 11 449 150 432, 31 162 390 984, 84 939 053 072, 232 809 453 160, 639 569 071 504, 1 764 756 319 800, 4 882 384 245 328, 13 557 001 368 672, 37 746 535 885 152}

{0, 0, 0, 0, 0, 0, 0, 16, 16, 72, 104, 448, 656, 2656, 4688, 15 712, 33 344, 100 984, 232 872, 671 848, 1 643 688, 4 619 168, 11 784 224, 32 572 880, 85 764 176, 235 172 192,
 630 718 144, 1 732 776 752, 4 706 131 504, 12 970 221 624, 35 584 492 728, 98 515 839 744, 272 466 004 928, 758 084 181 720, 2 110 955 787 448, 5 903 188 665 464, 16 535 721 813 272}

{0, 0, 0, 0, 0, 0, 0, 16, 0, 40, 0, 240, 0, 1344, 720, 7056, 8976, 43 272, 74 176, 280 280, 580 272, 1 912 064, 4 457 952, 13 462 384, 34 080 800, 97 724 640,
 258 098 400, 729 438 864, 1 970 016 864, 5 527 975 480, 15 172 024 960, 42 518 879 248, 117 953 204 688, 331 105 376 552, 925 892 800 560, 2 607 169 891 128, 7 336 514 373 472}

{3, 15, 87, 543, 3543, 23 823, 163 719, 1 144 015, 8 100 087, 57 971 735, 418 640 071, 3 046 373 007, 22 314 896 087, 164 407 579 407, 1 217 526 417 687, 9 057 960 864 015, 67 667 981 453 831, 507 425 879 338 551,
 3 818 200 408 513 415, 28 821 799 875 573 303, 218 200 189 786 794 855, 1 656 415 132 760 705 871, 12 606 151 256 856 370 471, 96 166 410 605 134 544 815, 735 237 884 585 469 467 543, 5 632 983 879 577 272 289 359,
 43 241 777 428 163 458 121 799, 332 564 656 181 337 723 832 623, 2 562 203 165 206 920 141 303 479, 19 773 205 160 010 752 987 777 543, 152 837 887 007 013 006 956 440 295, 1 183 157 961 642 417 140 248 556 303,
 9 172 380 845 538 923 831 902 240 519, 71 206 765 648 586 031 626 111 809 367, 553 521 536 480 845 628 126 004 101 879, 4 308 220 957 036 953 495 382 444 267 287, 33 572 939 291 063 083 015 187 615 095 255}

{3, 15, 87, 543, 3543, 23 823, 163 719, 1 143 999, 8 099 511, 57 959 535, 418 441 191, 3 043 608 351, 22 280 372 247, 164 008 329 423, 1 213 166 815 047, 9 012 417 249 663, 67 208 553 680 247, 502 920 171 632 943,
 3 775 020 828 459 687, 28 415 858 155 984 863, 214 444 848 602 732 247, 1 622 146 752 543 427 983, 12 297 086 677 257 812 487, 93 407 024 378 072 517 183, 710 817 216 408 949 234 743, 5 418 515 848 189 548 101 103,
 41 370 969 437 551 748 377 959, 316 342 913 595 655 481 088 159, 2 422 290 412 177 856 021 208 471, 18 572 165 552 209 575 045 630 543, 142 571 626 578 134 353 530 739 911, 1 095 738 430 841 101 539 942 155 007,
 8 430 543 922 804 728 230 924 999 415, 64 931 174 474 685 252 212 909 028 015, 500 582 936 483 899 929 918 138 992 679, 3 862 797 902 645 049 762 611 593 611 615, 29 833 954 602 139 121 152 988 918 731 863}

37
```

```
Grid[Transpose[{Range[Length[η]], η, ξ, ρ, ζ, m}], Alignment → Left]
```

1	3	0	0	0	3
2	6	0	0	0	15
3	12	0	0	0	87
4	24	0	0	0	543
5	48	0	0	0	3543
6	96	0	0	0	23 823
7	192	0	0	0	163 719
8	400	16	16	16	1 144 015
9	800	32	16	0	8 100 087
10	1656	120	72	40	57 971 735
11	3344	272	104	0	418 640 071
12	7032	888	448	240	3 046 373 007
13	14 272	1984	656	0	22 314 896 087
14	30 544	5968	2656	1344	164 407 579 407
15	63 120	13 968	4688	720	1 217 526 417 687
16	137 264	38 960	15 712	7056	9 057 960 864 015
17	292 160	95 552	33 344	8976	67 667 981 453 831
18	651 960	258 744	100 984	43 272	507 425 879 338 551
19	1 435 808	649 376	232 872	74 176	3 818 200 408 513 415
20	3 310 592	1 737 728	671 848	280 280	28 821 799 875 573 303
21	7 593 024	4 447 296	1 643 688	580 272	218 200 189 786 794 855
22	18 161 528	11 870 072	4 619 168	1 912 064	1 656 415 132 760 705 871
23	43 488 112	30 905 200	11 784 224	4 457 952	12 606 151 256 856 370 471
24	107 764 880	82 599 056	32 572 880	13 462 384	96 166 410 605 134 544 815
25	268 721 056	218 389 408	85 764 176	34 080 800	735 237 884 585 469 467 543
26	686 850 128	586 186 832	235 172 192	97 724 640	5 632 983 879 577 272 289 359
27	1 769 246 208	1 567 919 616	630 718 144	258 098 400	43 241 777 428 163 458 121 799
28	4 640 551 024	4 237 897 840	1 732 776 752	729 438 864	332 564 656 181 337 723 832 623
29	12 254 456 800	11 449 150 432	4 706 131 504	1 970 016 864	2 562 203 165 206 920 141 303 479
30	32 773 003 720	31 162 390 984	12 970 221 624	5 527 975 480	19 773 205 160 010 752 987 777 543
31	88 160 278 544	84 939 053 072	35 584 492 728	15 172 024 960	152 837 887 007 013 006 956 440 295
32	239 251 904 104	232 809 453 160	98 515 839 744	42 518 879 248	1 183 157 961 642 417 140 248 556 303
33	652 453 973 392	639 569 071 504	272 466 004 928	117 953 204 688	9 172 380 845 538 923 831 902 240 519
34	1 790 526 123 576	1 764 756 319 800	758 084 181 720	331 105 376 552	71 206 765 648 586 031 626 111 809 367
35	4 933 923 852 880	4 882 384 245 328	2 110 955 787 448	925 892 800 560	553 521 536 480 845 628 126 004 101 879
36	13 660 080 583 776	13 557 001 368 672	5 903 188 665 464	2 607 169 891 128	4 308 220 957 036 953 495 382 444 267 287
37	37 952 694 315 360	37 746 535 885 152	16 535 721 813 272	7 336 514 373 472	33 572 939 291 063 083 015 187 615 095 255

```
μ = Function[d, If[d == 1, 1, Module[{k, factores, exponentes, isProductOfDistinctPrimes},
    {factores, exponentes} = Transpose[FactorInteger[d]];
    k = Length[factores];
    If[Max[exponentes] == 1 , isProductOfDistinctPrimes = 1, isProductOfDistinctPrimes = 0];
    If[isProductOfDistinctPrimes == 1, (-1)^k, 0]
   ]]]

ζpFunction = Function[n, Module[{divisors, i},
   divisors = Divisors[n];
   Total[Table[μ[n / divisors[[i]]] ζ[[divisors[[i]]]], {i, 1, Length[divisors]}]]
  ]]
```

```
Function[d, If[d == 1, 1, Module[{k, factores, exponentes, isProductOfDistinctPrimes}, {factores, exponentes} = Transpose[FactorInteger[d]];
   k = Length[factores]; If[Max[exponentes] == 1, isProductOfDistinctPrimes = 1, isProductOfDistinctPrimes = 0]; If[isProductOfDistinctPrimes == 1, (-1)^k, 0]]]]

Function[n, Module[{divisors, i}, divisors = Divisors[n]; Total[Table[μ[n / divisors[[i]]] ζ[[divisors[[i]]]], {i, 1, Length[divisors]}]]]]
```

```
Table[{n, ζpFunction[n]}, {n, 1, Length[ζ]}] // ColumnForm
```

```
{1, 0}
{2, 0}
{3, 0}
{4, 0}
{5, 0}
{6, 0}
{7, 0}
{8, 16}
{9, 0}
{10, 40}
{11, 0}
{12, 240}
{13, 0}
{14, 1344}
{15, 720}
{16, 7040}
{17, 8976}
{18, 43 272}
{19, 74 176}
{20, 280 240}
{21, 580 272}
{22, 1 912 064}
{23, 4 457 952}
{24, 13 462 128}
{25, 34 080 800}
{26, 97 724 640}
{27, 258 098 400}
{28, 729 437 520}
{29, 1 970 016 864}
{30, 5 527 974 720}
{31, 15 172 024 960}
{32, 42 518 872 192}
{33, 117 953 204 688}
{34, 331 105 367 576}
{35, 925 892 800 560}
{36, 2 607 169 847 616}
{37, 7 336 514 373 472}
```

$$\text{Table}\left[\left\{n, \frac{\text{ζpFunction}[n]}{2\,n}\right\}, \{n, 1, \text{Length}[ζ]\}\right] \text{ // ColumnForm}$$

```
{1, 0}
{2, 0}
{3, 0}
{4, 0}
{5, 0}
{6, 0}
{7, 0}
{8, 1}
{9, 0}
{10, 2}
{11, 0}
{12, 10}
{13, 0}
{14, 48}
{15, 24}
{16, 220}
{17, 264}
{18, 1202}
{19, 1952}
{20, 7006}
{21, 13 816}
{22, 43 456}
{23, 96 912}
{24, 280 461}
{25, 681 616}
{26, 1 879 320}
{27, 4 779 600}
{28, 13 025 670}
{29, 33 965 808}
{30, 92 132 912}
{31, 244 710 080}
{32, 664 357 378}
{33, 1 787 169 768}
{34, 4 869 196 582}
{35, 13 227 040 008}
{36, 36 210 692 328}
{37, 99 142 086 128}
```

```
mm = Riffle[0 Range[Length[m]], m]
Table[d[n] = Det[Table[If[i + j == 0, 1, mm[[i + j]]], {i, 0, n}, {j, 0, n}]], {n, 0, Length[m]}]
d[-1] = 1;
```

$$\text{Table}\left[k[n] = \left(\frac{d[n-1]}{d[n]}\right)^{1/2}, \{n, 0, \text{Length}[m]\}\right]$$

$$\text{Table}\left[\alpha[n] = \frac{k[n-1]}{k[n]}, \{n, 1, \text{Length}[m]\}\right]$$

```
{0, 3, 0, 15, 0, 87, 0, 543, 0, 3543, 0, 23 823, 0, 163 719, 0, 1 144 015, 0, 8 100 087, 0, 57 971 735, 0, 418 640 071, 0, 3 046 373 007, 0, 22 314 896 087, 0, 164 407 579 407, 0,
 1 217 526 417 687, 0, 9 057 960 864 015, 0, 67 667 981 453 831, 0, 507 425 879 338 551, 0, 3 818 200 408 513 415, 0, 28 821 799 875 573 303, 0, 218 200 189 786 794 855, 0, 1 656 415 132 760 705 871,
 0, 12 606 151 256 856 370 471, 0, 96 166 410 605 134 544 815, 0, 735 237 884 585 469 467 543, 0, 5 632 983 879 577 272 289 359, 0, 43 241 777 428 163 458 121 799, 0, 332 564 656 181 337 723 832 623,
 0, 2 562 203 165 206 920 141 303 479, 0, 19 773 205 160 010 752 987 777 543, 0, 152 837 887 007 013 006 956 440 295, 0, 1 183 157 961 642 417 140 248 556 303, 0, 9 172 380 845 538 923 831 902 240 519,
 0, 71 206 765 648 586 031 626 111 809 367, 0, 553 521 536 480 845 628 126 004 101 879, 0, 4 308 220 957 036 953 495 382 444 267 287, 0, 33 572 939 291 063 083 015 187 615 095 255}
```

{1, 3, 18, 216, 5184, 248 832, 23 887 872, 4 586 471 424, 1 834 588 569 600, 1 465 224 737 587 200, 2 418 753 924 149 280 768, 8 020 819 523 706 034 323 456, 55 621 455 490 265 529 779 748 864,
771 257 589 297 244 616 685 105 709 056, 22 573 616 473 892 003 259 921 831 624 179 712, 1 323 041 867 885 128 332 176 009 815 756 573 245 440, 163 573 549 074 771 221 819 290 690 208 631 917 044 039 680,
40 768 217 085 427 869 104 745 510 825 374 464 332 475 372 929 024, 21 481 190 983 733 202 819 162 530 507 600 327 688 146 718 150 671 466 496, 22 837 201 998 927 496 678 236 450 191 436 488 430 582 405 591 356 001 161 838 592,
51 449 102 309 117 687 885 760 082 869 028 612 233 504 308 745 305 071 878 330 663 305 216, 234 153 866 021 884 557 163 454 624 665 557 448 380 732 756 615 203 713 543 025 282 144 133 971 968,
2 264 386 863 252 566 562 087 442 955 121 814 099 428 228 945 443 312 703 141 399 167 456 886 015 444 123 648, 44 321 998 812 089 241 420 401 035 177 498 510 846 589 482 285 979 255 997 314 860 566 125 518 097 642 318 035 156 992,
1 844 356 742 848 274 409 001 680 674 450 161 496 069 646 719 285 577 673 387 459 438 469 088 170 293 398 985 808 974 465 466 368,
156 148 813 265 513 459 714 101 313 166 058 667 243 945 354 846 168 950 500 164 843 008 592 551 760 207 484 251 329 502 319 191 238 115 328,
28 024 066 373 624 899 479 750 794 764 818 355 503 420 220 655 479 573 536 260 779 835 020 535 104 561 942 507 349 059 044 269 815 214 899 250 331 648,
10 261 873 501 676 642 143 863 104 639 679 774 521 762 832 082 656 958 686 451 938 306 715 996 079 680 639 182 963 682 312 574 047 158 053 513 500 707 300 311 040,
7 986 372 193 761 184 872 154 254 897 785 610 865 354 327 886 034 818 307 922 160 257 248 139 071 208 863 723 526 960 630 973 434 091 403 690 539 279 056 209 452 162 088 960,
12 689 244 939 837 692 916 561 260 786 571 127 920 189 525 564 695 535 351 525 533 525 614 930 263 093 628 957 190 712 085 673 770 124 727 550 204 930 656 569 626 123 232 294 656 278 528,
42 867 274 401 831 078 879 590 618 247 071 823 317 370 307 120 423 327 195 137 465 383 775 626 371 831 251 049 827 678 053 877 979 129 112 197 534 915 413 075 312 652 637 358 879 301 462 218 768 384,
296 367 122 888 460 807 331 110 729 996 130 493 508 930 521 917 748 019 844 857 239 648 494 844 937 044 275 467 778 144 727 148 524 075 996 782 655 357 501 965 382 796 730 686 184 483 140 580 696 043 267 555 328,
4 358 822 230 321 170 818 306 246 244 632 176 328 859 001 305 629 503 420 436 622 679 778 394 911 190 890 290 024 340 722 320 187 102 914 020 534 501 555 468 441 002 822 982 878 316 657 300 122 119 018 429 752 223 859 736 576,
131 420 589 758 915 372 391 479 480 714 488 244 227 538 539 297 538 167 552 757 211 238 151 744 637 152 210 104 378 918 694 653 306 652 584 236 292 533 687 594 885 233 073 451 657 659 957 993 549 721 608 786 998 383 405 678 933 089 \
189 888,
8 430 074 969 423 268 004 171 739 410 572 180 206 670 240 577 342 208 605 145 853 834 511 294 719 913 585 374 321 371 591 485 259 473 648 831 182 603 452 623 387 925 350 059 802 014 442 331 096 057 335 766 717 531 088 806 736 441 \
268 530 732 110 708 736,
1 111 113 316 417 338 669 984 809 008 714 934 456 399 580 836 701 564 864 200 666 679 175 617 166 503 808 418 619 467 624 679 109 003 794 140 723 273 670 661 931 964 827 904 282 653 911 392 463 905 817 260 166 703 135 268 078 566 \
982 435 274 717 364 347 830 483 812 352,
311 340 582 498 361 610 117 361 139 820 733 982 814 294 106 902 850 258 759 316 406 103 887 681 327 709 718 358 502 932 397 075 115 776 101 858 892 373 442 843 710 235 576 698 852 778 391 592 285 661 389 309 815 437 478 727 838 396 \
067 237 792 983 247 076 107 290 320 400 698 834 944,
179 703 984 806 919 284 308 196 197 113 750 780 327 412 739 797 728 913 168 299 507 560 623 784 116 562 584 347 585 374 940 141 254 855 923 236 496 162 403 052 835 155 975 931 167 218 565 507 147 498 402 238 209 445 788 451 339 399 \
566 622 920 371 179 959 868 243 895 349 967 761 776 449 758 953 472}

$$\Big\{1,\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{6}},\ \frac{1}{2\sqrt{3}},\ \frac{1}{2\sqrt{6}},\ \frac{1}{4\sqrt{3}},\ \frac{1}{4\sqrt{6}},\ \frac{1}{8\sqrt{3}},\ \frac{1}{20},\ \frac{\sqrt{\frac{3}{599}}}{2},\ \frac{5\sqrt{\frac{3}{7738}}}{4},\ \frac{\sqrt{\frac{1797}{372\,439}}}{4},\ \frac{\sqrt{\frac{11\,607}{20\,122\,577}}}{2},\ \sqrt{\frac{1\,117\,317}{15\,492\,928\,193}},\ \sqrt{\frac{120\,735\,462}{3\,533\,755\,844\,749}},\ \frac{\sqrt{\frac{46\,478\,784\,579}{681\,031\,526\,755\,495}}}{2},$$

$$\frac{\sqrt{\frac{3\,533\,755\,844\,749}{3\,033\,985\,750\,313\,262}}}{12},\ \frac{\sqrt{\frac{681\,031\,526\,755\,495}{2\,652\,136\,577\,268\,463\,069}}}{8},\ \frac{3\sqrt{\frac{1\,516\,992\,875\,156\,631}{449\,616\,981\,095\,932\,586\,851}}}{4},\ \frac{\sqrt{\frac{2\,652\,136\,577\,268\,463\,069}{704\,888\,509\,087\,894\,471\,227\,697}}}{2},\ \frac{\sqrt{\frac{449\,616\,981\,095\,932\,586\,851}{63\,307\,859\,642\,261\,657\,406\,444\,203}}}{4},\ \frac{\sqrt{\frac{704\,888\,509\,087\,894\,471\,227\,697}{200\,504\,433\,155\,195\,459\,612\,351\,514\,791}}}{4},$$

$$\frac{\sqrt{\frac{63\,307\,859\,642\,261\,657\,406\,444\,203}{153\,054\,792\,720\,330\,826\,889\,917\,966\,282\,527}}}{2},\ \sqrt{\frac{200\,504\,433\,155\,195\,459\,612\,351\,514\,791}{3\,924\,575\,518\,583\,541\,317\,160\,840\,308\,241\,161\,839}},\ \sqrt{\frac{153\,054\,792\,720\,330\,826\,889\,917\,966\,282\,527}{6\,369\,018\,693\,759\,599\,584\,770\,112\,292\,838\,940\,884\,883}},\ \frac{\sqrt{\frac{3\,924\,575\,518\,583\,541\,317\,160\,840\,308\,241\,161\,839}{20\,766\,651\,170\,319\,451\,879\,538\,837\,117\,410\,043\,457\,356\,159}}}{4},$$

$$\frac{\sqrt{\frac{6\,369\,018\,693\,759\,599\,584\,770\,112\,292\,838\,940\,884\,883}{285\,762\,342\,466\,992\,982\,072\,949\,973\,422\,544\,850\,055\,668\,650\,882}}}{2},\ \frac{\sqrt{\frac{20\,766\,651\,170\,319\,451\,879\,538\,837\,117\,410\,043\,457\,356\,159}{1\,901\,086\,949\,000\,278\,291\,400\,024\,111\,606\,796\,051\,842\,511\,059\,807\,205}}}{2},\ \frac{\sqrt{\frac{47\,627\,057\,077\,832\,163\,678\,824\,995\,570\,424\,141\,675\,944\,775\,147}{9\,266\,519\,516\,512\,483\,697\,182\,742\,550\,445\,951\,166\,585\,945\,535\,918\,491\,682}}}{2},$$

$$\frac{\sqrt{\frac{633\,695\,649\,666\,759\,430\,466\,674\,703\,868\,932\,017\,280\,837\,019\,935\,735}{62\,928\,441\,732\,066\,008\,679\,742\,534\,768\,806\,506\,376\,328\,038\,154\,050\,661\,867\,303}}}{4},\ \frac{\sqrt{\frac{4\,633\,259\,758\,256\,241\,848\,591\,371\,275\,222\,975\,583\,292\,972\,767\,959\,245\,841}{3\,913\,062\,171\,425\,629\,359\,723\,824\,487\,282\,780\,330\,546\,492\,871\,365\,372\,956\,704\,555\,562}}}{2},\ \frac{\sqrt{\frac{188\,785\,325\,196\,198\,026\,039\,227\,604\,306\,419\,519\,128\,984\,114\,462\,151\,985\,601\,909}{326\,296\,484\,046\,870\,952\,645\,147\,136\,743\,869\,729\,339\,249\,167\,863\,130\,357\,381\,132\,343\,034\,907}}}{2},$$

$$\frac{\sqrt{\frac{5\,869\,593\,257\,138\,444\,039\,585\,736\,730\,924\,170\,495\,819\,739\,307\,048\,059\,435\,056\,833\,343}{21\,581\,774\,424\,576\,737\,649\,468\,270\,267\,854\,331\,895\,904\,975\,102\,875\,833\,439\,702\,192\,078\,693\,802\,114}}}{2},\ \frac{\sqrt{\frac{326\,296\,484\,046\,870\,952\,645\,147\,136\,743\,869\,729\,339\,249\,167\,863\,130\,357\,381\,132\,343\,034\,907}{614\,874\,760\,081\,415\,166\,856\,589\,409\,301\,938\,967\,937\,321\,125\,666\,883\,093\,828\,582\,245\,747\,696\,976\,965\,326}}}{4},$$

$$\frac{\sqrt{\frac{10\,790\,887\,212\,288\,368\,824\,734\,135\,133\,927\,165\,947\,952\,487\,551\,437\,916\,719\,851\,096\,039\,346\,901\,057}{10\,815\,465\,202\,344\,115\,914\,753\,824\,558\,534\,983\,432\,461\,303\,507\,426\,524\,066\,664\,392\,289\,469\,669\,759\,555\,819\,786}}}{8},\ \frac{\sqrt{\frac{307\,437\,380\,040\,707\,583\,428\,294\,704\,650\,969\,483\,968\,660\,562\,833\,441\,546\,914\,291\,122\,873\,848\,488\,482\,663}{2\,532\,582\,510\,881\,411\,734\,028\,508\,979\,056\,820\,168\,589\,440\,906\,603\,567\,459\,495\,422\,358\,880\,420\,880\,940\,532\,905\,348\,601}}}{4},$$

$$\frac{\sqrt{\frac{5\,407\,732\,601\,172\,057\,957\,376\,912\,279\,267\,491\,716\,230\,651\,753\,713\,262\,033\,332\,196\,144\,734\,834\,879\,777\,909\,893}{168\,364\,327\,639\,018\,656\,297\,770\,385\,618\,995\,465\,404\,166\,376\,665\,327\,362\,698\,718\,720\,677\,722\,899\,974\,379\,175\,668\,005\,738\,494}}}{3},\ \frac{\sqrt{\frac{2\,532\,582\,510\,881\,411\,734\,028\,508\,979\,056\,820\,168\,589\,440\,906\,603\,567\,459\,495\,422\,358\,880\,420\,880\,940\,532\,905\,348\,601}{7\,458\,122\,388\,121\,271\,722\,746\,532\,161\,502\,187\,079\,545\,027\,925\,138\,021\,021\,348\,479\,056\,951\,798\,450\,976\,087\,126\,537\,086\,452\,786\,278}}}{14}\Big\}$$

$$\Big\{\sqrt{3},\ \sqrt{2},\ \sqrt{2},\ \sqrt{2},\ \sqrt{2},\ \sqrt{2},\ \sqrt{2},\ \frac{5}{2\sqrt{3}},\ \frac{\sqrt{\frac{599}{3}}}{10},\ \frac{2\sqrt{\frac{7738}{599}}}{5},\ 5\sqrt{\frac{372\,439}{4\,635\,062}},\ \frac{\sqrt{\frac{12\,053\,423\,623}{1\,440\,966\,491}}}{2},\ \frac{\sqrt{\frac{59\,942\,139\,178\,717}{7\,494\,432\,455\,303}}}{2},\ \sqrt{\frac{1\,316\,108\,493\,062\,472\,811}{623\,515\,281\,038\,226\,722}},\ 2\sqrt{\frac{27\,408\,218\,673\,130\,016\,621\,230}{54\,748\,225\,554\,290\,313\,108\,557}},$$

$$18\sqrt{\frac{15\,668\,441\,122\,729\,531\,807\,198\,522}{2\,406\,599\,138\,130\,565\,428\,902\,645\,755}},\ \frac{\sqrt{\frac{18\,744\,006\,261\,990\,078\,446\,702\,008\,149\,362}{1\,033\,119\,973\,845\,128\,431\,165\,364\,937\,345}}}{3},\ \frac{\sqrt{\frac{306\,203\,339\,090\,959\,503\,149\,335\,654\,328\,996\,245}{4\,023\,272\,291\,658\,552\,242\,107\,770\,013\,960\,539}}}{6},\ \frac{3\sqrt{\frac{1\,069\,310\,846\,066\,116\,053\,671\,254\,850\,581\,740\,408\,807}{1\,192\,445\,641\,325\,545\,914\,133\,295\,394\,710\,128\,505\,719}}}{2},$$

$$2\sqrt{\frac{167\,901\,090\,185\,820\,098\,903\,656\,432\,934\,273\,722\,114\,639\,007}{316\,929\,843\,465\,311\,953\,918\,334\,827\,876\,530\,763\,049\,212\,147}},\ \sqrt{(90\,150\,197\,931\,590\,195\,963\,484\,230\,782\,249\,040\,456\,029\,018\,613\,141\ /\ 44\,624\,982\,796\,779\,503\,924\,911\,108\,582\,650\,024\,820\,176\,738\,690\,491)},$$

$$\frac{\sqrt{\frac{107\,886\,564\,649\,390\,720\,628\,189\,628\,497\,725\,812\,952\,395\,198\,376\,849\,550\,319}{12\,693\,506\,511\,840\,348\,830\,174\,382\,938\,060\,697\,916\,488\,569\,072\,070\,706\,573}}}{2},\ \frac{\sqrt{\frac{248\,456\,476\,085\,943\,090\,609\,713\,266\,607\,330\,983\,153\,960\,021\,214\,660\,578\,446\,369\,317}{30\,688\,164\,456\,075\,868\,920\,089\,090\,725\,338\,656\,660\,449\,163\,503\,724\,471\,525\,356\,857}}}{2},$$

√(1 277 016 482 947 111 936 509 046 538 692 495 733 744 258 906 065 378 978 947 207 002 804 453 / 600 675 092 512 088 779 414 669 374 938 721 822 779 521 282 360 837 525 006 150 204 887 153),
4 √(3 178 435 490 368 659 290 227 519 203 171 882 662 010 966 096 698 503 884 469 842 102 729 057 533 793 / 24 995 694 842 929 849 465 450 239 715 573 974 519 445 743 157 385 369 330 006 919 778 574 671 579 837),
√(560 747 946 689 523 257 119 734 286 383 792 204 224 152 260 107 659 645 149 822 547 323 604 827 938 676 045 999 / 264 526 379 021 098 510 816 674 519 924 867 418 960 183 806 916 842 036 640 493 597 382 144 577 414 300 088 794),
√(4 036 019 438 881 724 985 733 781 914 228 960 801 217 258 167 989 636 174 335 981 828 065 154 230 578 410 232 692 994 005 /
1 978 108 961 208 469 271 649 839 117 136 344 414 625 213 185 180 642 437 888 400 631 421 703 184 833 052 756 701 160 746),
√(64 144 859 454 124 003 655 346 385 132 583 528 746 522 217 966 170 730 205 746 037 611 992 824 666 539 726 856 546 739 584 323 146 /
30 181 058 876 652 685 927 546 414 973 528 156 433 347 608 484 796 503 967 774 616 226 791 887 934 091 010 371 048 950 965 178 045),
√(5 994 192 972 384 286 600 192 311 086 205 934 218 059 587 125 399 161 322 091 616 506 697 497 437 089 803 236 348 362 689 727 306 972 637 082 /
2 936 076 552 583 041 924 163 163 490 593 381 357 267 306 911 812 169 903 411 291 480 229 509 751 484 116 418 653 494 485 500 932 886 028 135),
√(1 239 845 237 453 992 278 572 042 734 291 125 787 525 101 872 758 317 415 179 952 002 051 254 809 819 239 602 186 407 765 774 214 632 696 943 488 404 035 /
583 127 633 453 908 312 907 857 567 804 389 753 786 800 024 459 921 398 719 062 122 624 726 548 828 204 104 951 258 402 358 113 122 873 817 393 273 646),
√(503 938 789 598 288 994 990 306 327 353 059 714 858 691 833 476 532 704 540 209 928 658 683 973 951 082 993 000 087 293 301 285 748 788 868 577 768 162 952 523 929 /
246 242 904 848 509 408 602 495 418 915 733 218 015 083 392 341 303 039 062 117 884 874 117 809 391 286 898 482 812 876 928 559 295 615 110 583 506 057 634 589 286),
√(1 358 107 434 351 569 648 373 482 199 802 657 583 022 680 658 628 762 708 572 076 856 199 438 892 892 307 752 781 918 123 704 930 936 355 279 138 947 646 860 692 268 008 878 542 /
638 409 214 196 498 539 649 731 221 159 437 181 585 455 444 809 685 989 022 771 124 022 340 278 837 659 448 061 484 774 324 679 493 283 467 035 236 895 763 959 213 843 501 367),
6 √(200 503 596 986 582 943 334 663 827 738 312 832 239 849 535 748 484 059 968 051 950 830 751 408 769 308 533 546 675 685 707 730 571 977 465 303 566 335 986 972 751 818 661 263 526 203 601 /
3 521 028 557 116 035 505 132 252 462 257 592 924 849 464 591 601 317 678 900 096 852 004 178 190 669 919 606 052 127 425 489 340 210 290 228 173 335 212 478 706 234 148 089 068 826 196 699),
2 √(1 764 524 134 428 082 369 203 462 215 685 471 964 655 562 249 249 769 649 976 485 404 076 747 418 104 860 598 199 306 075 193 844 893 909 884 256 842 244 625 095 950 279 931 058 799 310 376 799 634 951 /
3 317 522 092 860 710 857 409 488 947 516 964 637 340 997 228 237 009 756 505 241 296 032 877 316 939 924 011 793 616 247 274 689 824 514 394 426 394 427 602 550 827 493 507 083 517 784 345 120 874 791), $\frac{1}{2}$
(√(27 328 812 230 735 394 571 440 080 491 540 541 910 437 573 113 306 695 746 777 875 598 892 555 820 177 430 504 072 252 100 513 579 342 065 515 447 613 257 940 406 047 961 166 377 391 654 617 244 300 149 640 371 257 /
3 325 078 285 730 116 307 288 460 534 917 925 216 105 443 955 681 758 244 590 092 233 992 067 618 158 045 625 570 353 304 602 390 308 159 197 742 271 841 972 133 802 388 355 354 666 631 215 778 753 074 213 370 118)), $\frac{1}{2}$
3 √(25 880 743 890 827 593 188 510 561 582 998 079 267 349 294 534 349 342 288 288 015 756 375 877 294 033 734 165 836 619 918 123 132 885 973 835 814 155 455 646 474 604 158 107 216 067 152 693 670 312 415 445 875 975 415 364 761 /
27 391 058 018 503 196 905 441 312 976 617 730 659 019 074 610 382 855 341 210 774 503 748 421 051 489 049 747 416 086 756 032 533 131 106 999 013 741 753 778 842 739 137 283 150 684 258 481 877 958 634 720 833 976 663 219 386),
$\frac{1}{3}$14
√(20 165 765 790 887 302 770 321 510 109 335 354 983 127 472 845 974 611 225 737 167 188 785 299 273 875 582 409 326 668 103 324 380 712 807 977 009 177 262 128 601 011 419 228 654 725 180 804 104 036 062 451 210 459 095 004 542 \
535 424 127 /
213 198 275 817 443 268 238 143 042 805 483 823 654 043 781 086 177 070 679 340 632 888 485 060 812 476 436 827 733 340 140 471 239 657 015 707 466 293 218 282 515 558 628 969 845 816 207 459 404 215 600 649 220 331 259 737 306 \
757 373 447)}

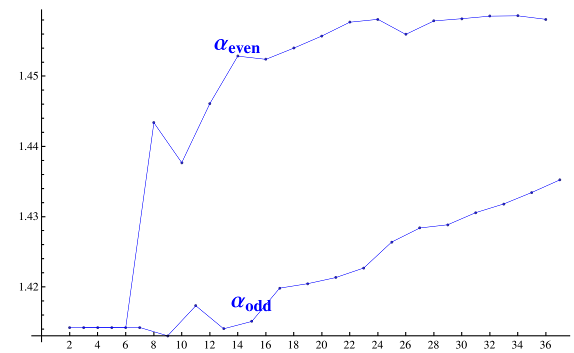
```
MnormFunctiontest = Function[n, Module[{i, j, eigenvalues, A},
    A = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
    Table[A[[i, i + 1]] = α[i]^2, {i, 1, n}];
    Table[A[[i + 1, i]] = 1, {i, 1, n}];
    A
   ]];
Table[(MnormFunctiontest[i] // MatrixForm), {i, 0, 7}]
MnormFunction = Function[n, Module[{i, j, eigenvalues, A},
    A = Table[0, {i, 1, n + 1}, {j, 1, n + 1}];
    Table[A[[i, i + 1]] = α[i]^2, {i, 1, n}];
    Table[A[[i + 1, i]] = 1, {i, 1, n}];
    eigenvalues = Eigenvalues[A];
    eigenvalues = eigenvalues // N;
    Max[eigenvalues]
   ]];
Table[Mnorm[i] = MnormFunction[i], {i, 0, Length[m]}]
```

$$\left\{\begin{pmatrix}0\end{pmatrix}, \begin{pmatrix}0&3\\1&0\end{pmatrix}, \begin{pmatrix}0&3&0\\1&0&2\\0&1&0\end{pmatrix}, \begin{pmatrix}0&3&0&0\\1&0&2&0\\0&1&0&2\\0&0&1&0\end{pmatrix}, \begin{pmatrix}0&3&0&0&0\\1&0&2&0&0\\0&1&0&2&0\\0&0&1&0&2\\0&0&0&1&0\end{pmatrix}, \begin{pmatrix}0&3&0&0&0&0\\1&0&2&0&0&0\\0&1&0&2&0&0\\0&0&1&0&2&0\\0&0&0&1&0&2\\0&0&0&0&1&0\end{pmatrix}, \begin{pmatrix}0&3&0&0&0&0&0\\1&0&2&0&0&0&0\\0&1&0&2&0&0&0\\0&0&1&0&2&0&0\\0&0&0&1&0&2&0\\0&0&0&0&1&0&2\\0&0&0&0&0&1&0\end{pmatrix}, \begin{pmatrix}0&3&0&0&0&0&0&0\\1&0&2&0&0&0&0&0\\0&1&0&2&0&0&0&0\\0&0&1&0&2&0&0&0\\0&0&0&1&0&2&0&0\\0&0&0&0&1&0&2&0\\0&0&0&0&0&1&0&2\\0&0&0&0&0&0&1&0\end{pmatrix}\right\}$$

```
{0., 1.73205, 2.23607, 2.44949, 2.56155, 2.6286, 2.67233, 2.70265, 2.7262, 2.74445, 2.75941, 2.77154, 2.782, 2.79072, 2.7985, 2.80515, 2.81118, 2.81645, 2.82132,
 2.82563, 2.82966, 2.83326, 2.83665, 2.83972, 2.84263, 2.84529, 2.84782, 2.85014, 2.85237, 2.85443, 2.85641, 2.85824, 2.86002, 2.86167, 2.86327, 2.86477, 2.86623, 2.86759}
```

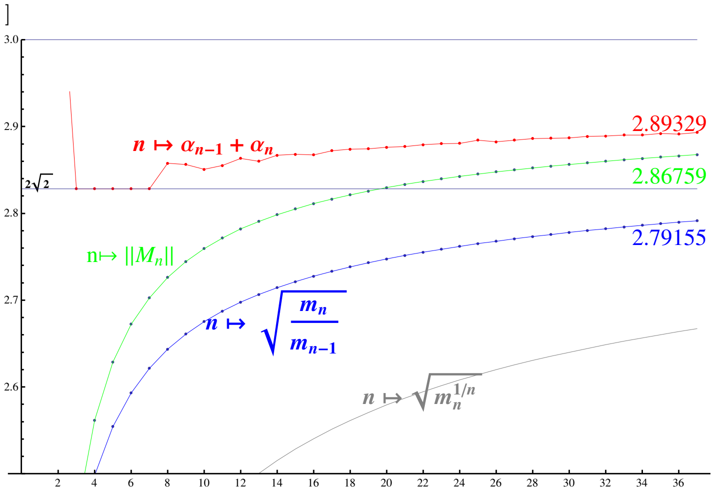
```
Table[α[n], {n, 1, Ntuple}] // N
Show[
 ListPlot[Table[{i, α[i]}, {i, 2, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, α[i]}, {i, 2, Ntuple, 2}], Joined → True, PlotStyle → {Blue}], ListPlot[Table[{i, α[i]}, {i, 3, Ntuple, 2}], Joined → True, PlotStyle → {Blue}],
 Graphics[{Blue, Text[Style[HoldForm[α_even], Large, Bold], {14, .002 + α[14]}]}],
 Graphics[{Blue, Text[Style[HoldForm[α_odd], Large, Bold], {15, .003 + α[15]}]}]
]
```

```
{1.73205, 1.41421, 1.41421, 1.41421, 1.41421, 1.41421, 1.41421, 1.44338, 1.41303, 1.43768, 1.41733, 1.4461, 1.41406, 1.45286, 1.41509, 1.45239, 1.41982, 1.454,
 1.42044, 1.45571, 1.42133, 1.45768, 1.42269, 1.45807, 1.42638, 1.45596, 1.42841, 1.45785, 1.42883, 1.45815, 1.43056, 1.45854, 1.43178, 1.4586, 1.43344, 1.45806, 1.43523}
```



```
ma[0] = 1;
Table[ma[i] = m[[i]], {i, 1, Length[m]}];
Show[
 ListPlot[Table[{i, Mnorm[i]}, {i, 0, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}, PlotRange → {2.5, 3}],
 ListPlot[Table[{i, Mnorm[i]}, {i, 0, Ntuple}], Joined → True, PlotStyle → {Green}],
 Graphics[{Green, Text[Style[HoldForm["n↦ ||Mₙ||"], Large], {6, .08 + Mnorm[6]}]}],
 Graphics[{Green, Text[Style[N[Mnorm[Ntuple], 6], Large], {Ntuple - 1.5, -.025 + Mnorm[Ntuple]}]}],
 Graphics[{Red, Text[Style[N[α[Ntuple - 1] + α[Ntuple], 6], Large], {Ntuple - 1.5, .01 + α[Ntuple - 1] + α[Ntuple]}]}],
 Graphics[{Blue, Text[Style[N[(ma[Ntuple]/ma[Ntuple - 1])^(1/2), 6], Large], {Ntuple - 1.5, -.02 + (ma[Ntuple]/ma[Ntuple - 1])^(1/2)}]}],
 Plot[8, {x, 0, Ntuple}],
 ListPlot[Table[{i, (ma[i]/ma[i - 1])^(1/2)}, {i, 1, Ntuple}], Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, (ma[i]/ma[i - 1])^(1/2)}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Blue}],
 Graphics[{Blue, Text[Style[HoldForm[n ↦ (mₙ/mₙ₋₁)^(1/2)], Large, Bold], {14, -.04 + (ma[14]/ma[13])^(1/2)}]}],
 ListPlot[Table[{i, α[i - 1] + α[i]}, {i, 1, Ntuple}], PlotStyle → {Red}, Ticks → {Range[2, Ntuple, 2]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[Table[{i, α[i - 1] + α[i]}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Red}],
 Graphics[{Red, Text[Style[HoldForm[n ↦ αₙ₋₁ + αₙ], Large, Bold], {10, .03 + α[9] + α[10]}]}],
 ListPlot[Table[{i, (ma[i]^(1/i))^(1/2)}, {i, 1, Ntuple, 1}], Joined → True, PlotStyle → {Gray}],
 Graphics[{Gray, Text[Style[HoldForm[n ↦ Sqrt[(mₙ)^(1/n)]], Large, Bold], {22, -.00 + (ma[22]^(1/(2·22)))}]}],
 Graphics[{Black, Text[Style[HoldForm["2√2 "], Medium, Bold], {1, 2 Sqrt[2] + .007}]}],
 Plot[Sqrt[8], {x, 0, Ntuple}],
 Plot[3, {x, 0, Ntuple}]

]
```




```
Grid[Transpose[
  {Range[Length[η]],
   N[Table[(ma[i]^(1/i))^(1/2), {i, 1, Ntuple, 1}], 6],
   N[Table[(ma[i]/ma[i - 1])^(1/2), {i, 1, Ntuple}], 6],
   N[Table[α[n], {n, 1, Ntuple}], 6],
   N[Table[Mnorm[i], {i, 1, Ntuple}], 6],
   N[Table[α[i - 1] + α[i], {i, 1, Ntuple}], 6]}
 ], Alignment → Left]
```

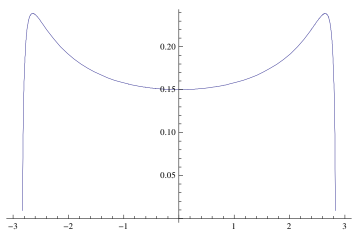
1	1.73205	1.73205	1.73205	1.73205	1.73205 + α[0]
2	1.96799	2.23607	1.41421	2.23607	3.14626
3	2.10501	2.40832	1.41421	2.44949	2.82843
4	2.19710	2.49828	1.41421	2.56155	2.82843
5	2.26432	2.55438	1.41421	2.6286	2.82843
6	2.31606	2.59306	1.41421	2.67233	2.82843
7	2.35741	2.62151	1.41421	2.70265	2.82843
8	2.39140	2.64342	1.44338	2.7262	2.85759
9	2.41994	2.66090	1.41303	2.74445	2.85641
10	2.44434	2.67524	1.43768	2.75941	2.85071
11	2.46548	2.68728	1.41733	2.77154	2.85500
12	2.48403	2.69756	1.44610	2.782	2.86343
13	2.50048	2.70649	1.41406	2.79072	2.86016
14	2.51518	2.71434	1.45286	2.7985	2.86691
15	2.52842	2.72131	1.41509	2.80515	2.86795
16	2.54043	2.72757	1.45239	2.81118	2.86749
17	2.55138	2.73323	1.41982	2.81645	2.87222
18	2.56143	2.73839	1.45400	2.82132	2.87382
19	2.57069	2.74311	1.42044	2.82563	2.87444
20	2.57925	2.74746	1.45571	2.82966	2.87615
21	2.58720	2.75148	1.42133	2.83326	2.87704
22	2.59461	2.75522	1.45768	2.83665	2.87901
23	2.60154	2.75871	1.42269	2.83972	2.88037
24	2.60803	2.76198	1.45807	2.84263	2.88076
25	2.61414	2.76505	1.42638	2.84529	2.88445
26	2.61989	2.76793	1.45596	2.84782	2.88234
27	2.62533	2.77066	1.42841	2.85014	2.88437
28	2.63047	2.77323	1.45785	2.85237	2.88626
29	2.63535	2.77568	1.42883	2.85443	2.88669
30	2.63998	2.77800	1.45815	2.85641	2.88698
31	2.64439	2.78021	1.43056	2.85824	2.88871
32	2.64860	2.78231	1.45854	2.86002	2.88910
33	2.65261	2.78432	1.43178	2.86167	2.89032
34	2.65645	2.78625	1.45860	2.86327	2.89039
35	2.66012	2.78809	1.43344	2.86477	2.89204
36	2.66365	2.78986	1.45806	2.86623	2.89150
37	2.66702	2.79155	1.43523	2.86759	2.89329

```
ρFree = Function[x, 3/(2 π) Sqrt[8 - x^2]/(9 - x^2)]
(*Integrate[ρFree[x], {x, -Sqrt[8], Sqrt[8]}]==1*)
Plot[ρFree[x], {x, -(q+1), (q+1)}]
ρLebesgueCoords = Function[n, Module[{coeffs, mlist},
   If[Mod[n, 2] == 0 && n ≤ 2 Ntuple,
    coeffs = CoefficientList[LegendreP[n, x/(q+1)], x];
    coeffs = coeffs[[Range[1, Length[coeffs], 2]]];
    mlist = Table[ma[i], {i, 0, n/2}];
    Sqrt[(2 n + 1)/(2 (q + 1))] coeffs.mlist, 0]]]

p = Function[n, Sqrt[(2 n + 1)/(2 (q + 1))] LegendreP[n, x/(q+1)]]

ρLebesgue = Total[Table[ρLebesgueCoords[i] p[i], {i, 0, 2 Ntuple}]];
ρLebesgueNminus1 = Total[Table[ρLebesgueCoords[i] p[i], {i, 0, 2 (Ntuple - 1)}]];
ρLebesgueGraph = Table[{x, ρLebesgue}, {x, 0, q + 1, 1/100}];
ρLebesgueGraphTail = Table[{x, ρLebesgue}, {x, 27/10, q + 1, 1/1000}];
ρLebesgueGraphTailNminus1 = Table[{x, ρLebesgueNminus1}, {x, 27/10, q + 1, 1/1000}];
```

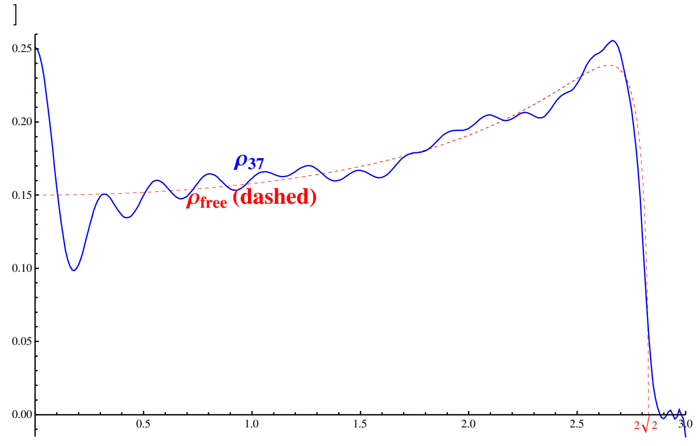
```
Function[x, (3 Sqrt[8 - x^2])/((2 π) (9 - x^2))]
```



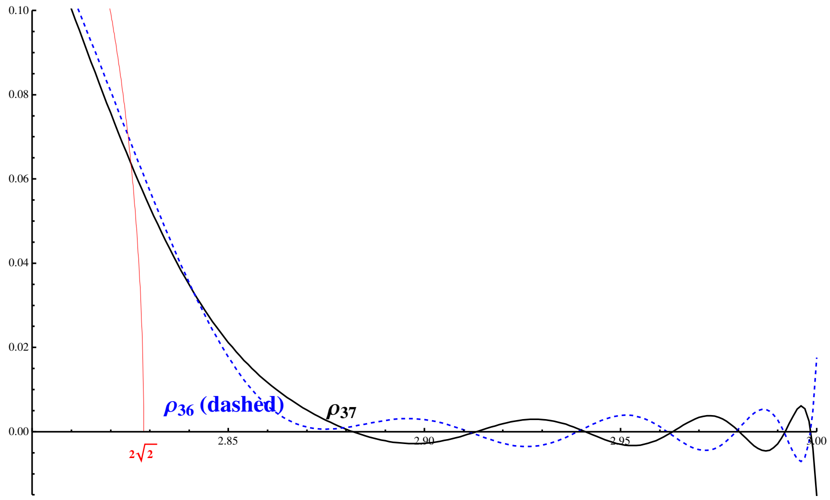
```
Function[n, Module[{coeffs, mlist},
  If[Mod[n, 2] == 0 && n ≤ 2 Ntuple, coeffs = CoefficientList[LegendreP[n, x/(q+1)], x]; coeffs = coeffs[[Range[1, Length[coeffs], 2]]]; mlist = Table[ma[i], {i, 0, n/2}]; Sqrt[(2 n + 1)/(2 (q + 1))] coeffs.mlist, 0]]]

Function[n, Sqrt[(2 n + 1)/(2 (q + 1))] LegendreP[n, x/(q+1)]]
```

```
Show[
 ListPlot[Join[Table[{x, ρFree[x]}, {x, 0, 2 Sqrt[2], 1/10000}], {{2 Sqrt[2], 0}}], Joined → True, PlotStyle → {Dashed, Red},
  PlotRange → {{0, q + 1}, {-.015, .26}}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}],
 ListPlot[ρLebesgueGraph, Joined → True, PlotStyle → {Directive[Blue, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Blue, Text[Style[HoldForm[ρ37], Large, Bold], Evaluate[ρLebesgueGraph[[100]] + {0, .015}]]}],
 Graphics[{Red, Text[Style[HoldForm["2√2 "], Medium], {2 Sqrt[2], -.006}]}],
 Graphics[{Red, Text[Style[HoldForm[ρfree "(dashed)"], Large, Bold], Evaluate[{1, ρFree[0.7] - .005}]]}]
]
```



```
Show[ListPlot[Table[{x, ρFree[x]}, {x, 3.43, 3.5, 1/10000}], Joined → True, PlotStyle → {Red}, PlotRange → {{2.8, q + 1}, {-.015, .10}},
  AxesOrigin → {2.8, 0}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}, PlotRange → {2.8, q + 1}],
 ListPlot[ρLebesgueGraphTail, Joined → True, PlotStyle → {Directive[Black, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Black, Text[Style[HoldForm[ρ37], Large, Bold], Evaluate[ρLebesgueGraphTail[[170]] + {.01, .00}]]}],
 Graphics[{Red, Text[Style[HoldForm["2√2 "], Medium, Bold], {2 Sqrt[2], -.005}]}],
 ListPlot[ρLebesgueGraphTailNminus1, Joined → True, PlotStyle → {Dashed, Directive[Blue, Thickness[.002]]}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, Thickness[.002]]}],
 Graphics[{Blue, Text[Style[HoldForm[ρ36 "(dashed)"], Large, Bold], Evaluate[ρLebesgueGraphTailNminus1[[170]] + {-.02, 0.005}]]}], ListPlot[
  Join[Table[{x, ρFree[x]}, {x, 28/10, Sqrt[8], 1/10000}], {{Sqrt[8], 0}}], Joined → True, PlotStyle → {Red}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}]]
```




```
(ρLebesgueGraphTail + ρLebesgueGraphTailNminus1) / 2 // N // ColumnForm
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```

```
Show[ListPlot[Table[{x, ρFree[x]}, {x, 3.43, 3.5, 1/10000}], Joined → True, PlotStyle → {Red}, PlotRange → {{2.8, q + 1}, {-.005, .10}},
  AxesOrigin → {2.8, 0}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}, PlotRange → {2.8, q + 1}],
 ListPlot[(ρLebesgueGraphTail + ρLebesgueGraphTailNminus1) / 2 // N, AxesOrigin → {27/10, 0}, Joined → True, PlotStyle → {Thickness[.002]},
  AxesStyle → {Directive[Black, 12, Thickness[.02]], Directive[Black, Thickness[.02]]}],
 Graphics[{Blue, Text[Style[HoldForm[1/2 (ρ36 + ρ37)], Large, Bold], Evaluate[ρLebesgueGraphTail[[170]] + {.01, .01}]]}],
 Graphics[{Red, Text[Style[HoldForm["2√2 "], Medium, Bold], {2 Sqrt[2], -.003}]}],
 ListPlot[Join[Table[{x, ρFree[x]}, {x, 28/10, Sqrt[8], 1/1000}], {{2 Sqrt[2], 0}}],
  Joined → True, PlotStyle → {Red}, AxesStyle → {Directive[Black, 12, Thickness[.002]], Directive[Black, 12, Thickness[.002]]}]]
```