$$h = A + B + A^{-1} + B^{-1}$$

Extrapolation of the ||M_n|| norms described in the paper
"A COMPUTATIONAL APPROACH TO THE THOMPSON GROUP F"
by S. Haagerup, U. Haagerup, M. Ramirez-Solano:

```
norms = {{1, 4.`}, {2, 7.000000000000001`}, {3, 8.605551275463988`},
    {4, 9.541381265149111`}, {5, 10.187816020025469`}, {6, 10.656244451028344`},
    {7, 11.022388732772455`}, {8, 11.308130917701545`}, {9, 11.545716716541902`},
    {10, 11.743086500427827`}, {11, 11.913803073100125`},
    {12, 12.05981096300795`}, {13, 12.189380782293503`}, {14, 12.302920985302379`},
    {15, 12.405655196592425`}, {16, 12.49714624116274`}, {17, 12.580970148269211`},
    {18, 12.656602089719547`}, {19, 12.726679068950057`},
    {20, 12.790534930918907`}, {21, 12.850169509154144`}, {22, 12.904982146260096`},
    {23, 12.956513938208243`}, {24, 13.004187392723907`}};
{xx, yy} = Transpose[norms];
norms = Transpose[{xx, yy^(1/2)}];
norms // MatrixForm
Ntuple = Length[norms]
```

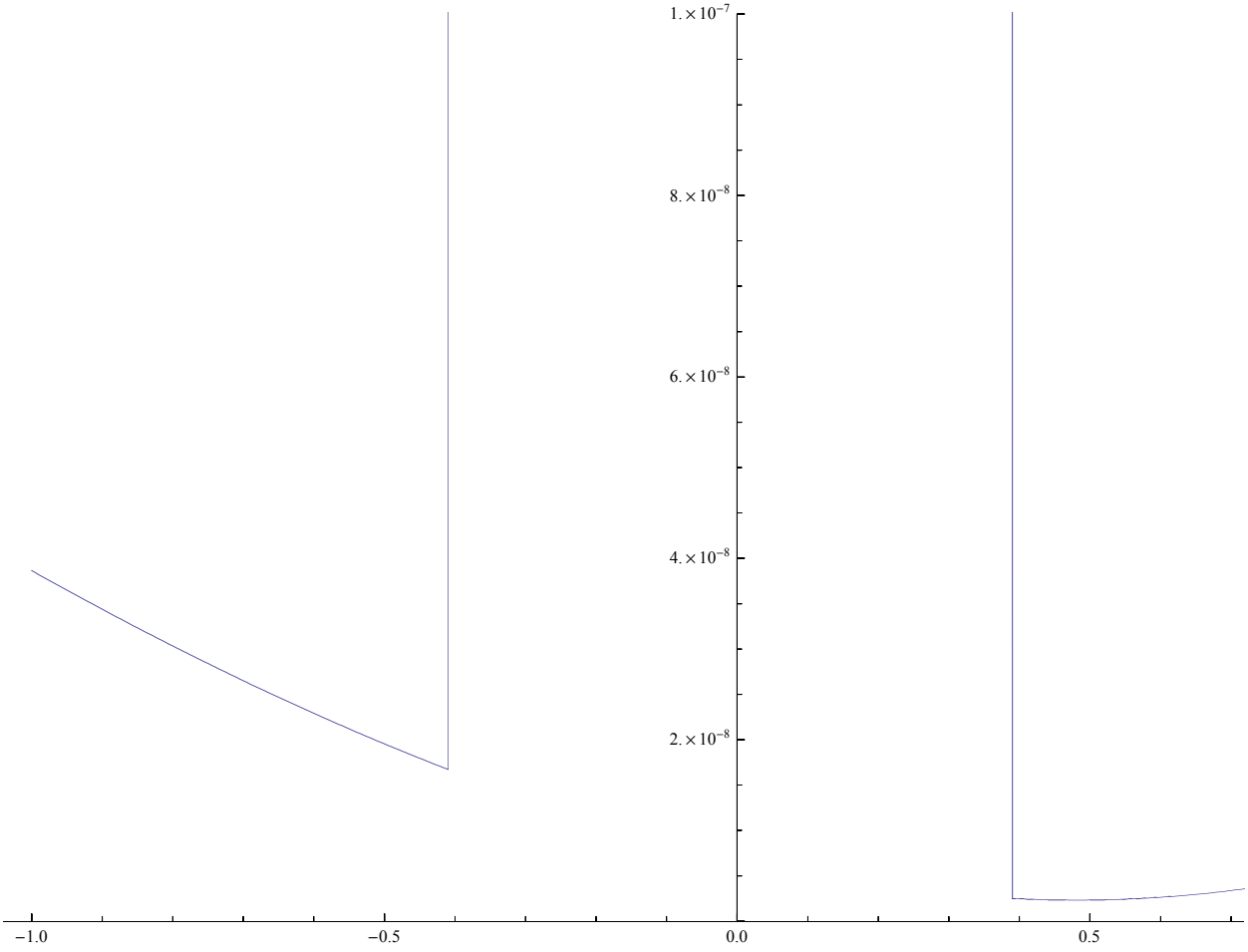
1	2.
2	2.64575
3	2.93352
4	3.08891
5	3.19184
6	3.26439
7	3.32
8	3.36276
9	3.3979
10	3.42682
11	3.45164
12	3.47272
13	3.49133
14	3.50755
15	3.52217
16	3.53513
17	3.54697
18	3.55761
19	3.56745
20	3.57639
21	3.58471
22	3.59235
23	3.59952
24	3.60613

```
24
```

```
variance = Function[d, Module[{nlm, a, b, c, f, g, h},
    nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b ((x - d)^(-c)), {a, b, c}, x];
    {f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}];
    Total[g^2]/(Length[g] - 1) (*variance*)
   ]]
upperlimit = Function[d, Module[{nlm, aa, a, b, c, f, g, h},
    nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b ((x - d)^(-c)), {a, b, c}, x];
    aa = nlm["BestFitParameters"];
    a /. aa
   ]]
```

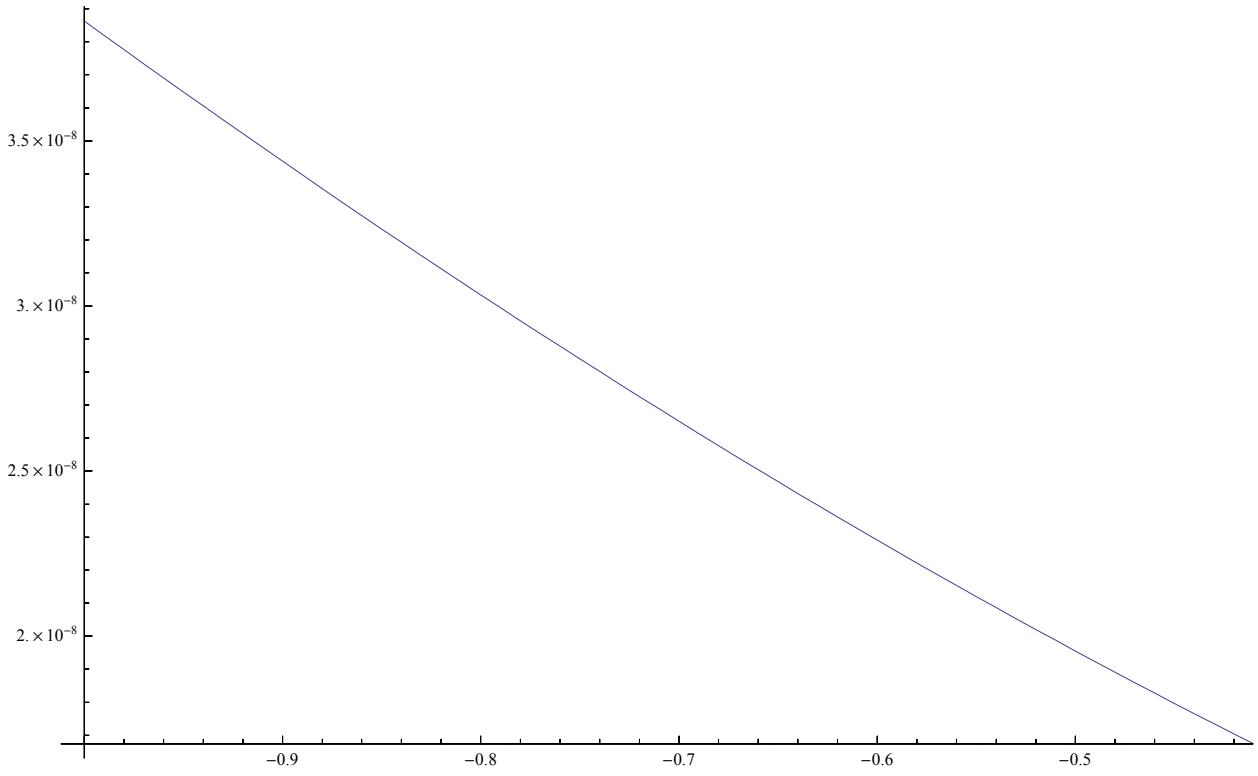
```
Function[d, Module[{nlm, a, b, c, f, g, h},
  nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b (x - d)^-c, {a, b, c}, x];
  {f, g, h} = nlm[{BestFit, FitResiduals, ParameterTable}]; Total[g^2]/(Length[g] - 1)]]

Function[d, Module[{nlm, aa, a, b, c, f, g, h},
  nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b (x - d)^-c, {a, b, c}, x];
  aa = nlm[BestFitParameters]; a /. aa]]
```

```
(*Graph d vs. variance(d)*)
varValues = Table[{d, variance[d]}, {d, -1, 1, .005}];
ListPlot[varValues, Joined → True, PlotRange → {0, 10^-7}]
```



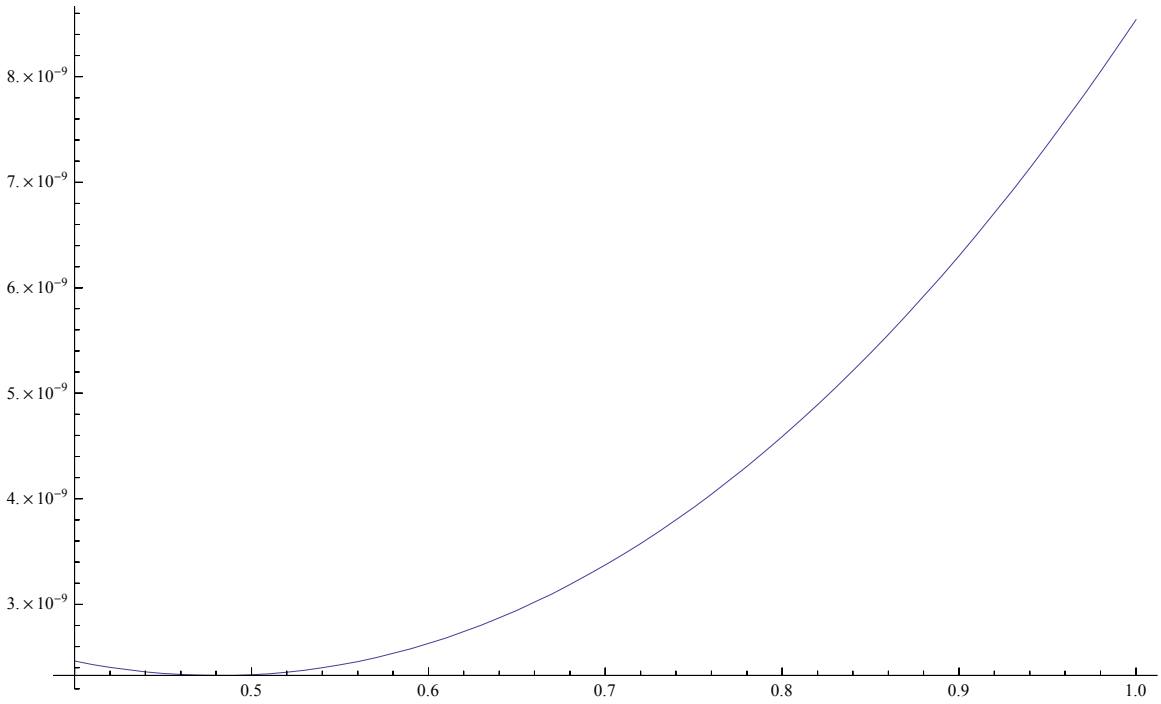
```
(*Graph d vs. variance(d)*)
varValues = Table[{d, variance[d]}, {d, -1, -.40005, .01}];
varValues[[-1]]
ListPlot[varValues, Joined → True]
```

$\{-0.41, 1.67381 \times 10^{-8}\}$

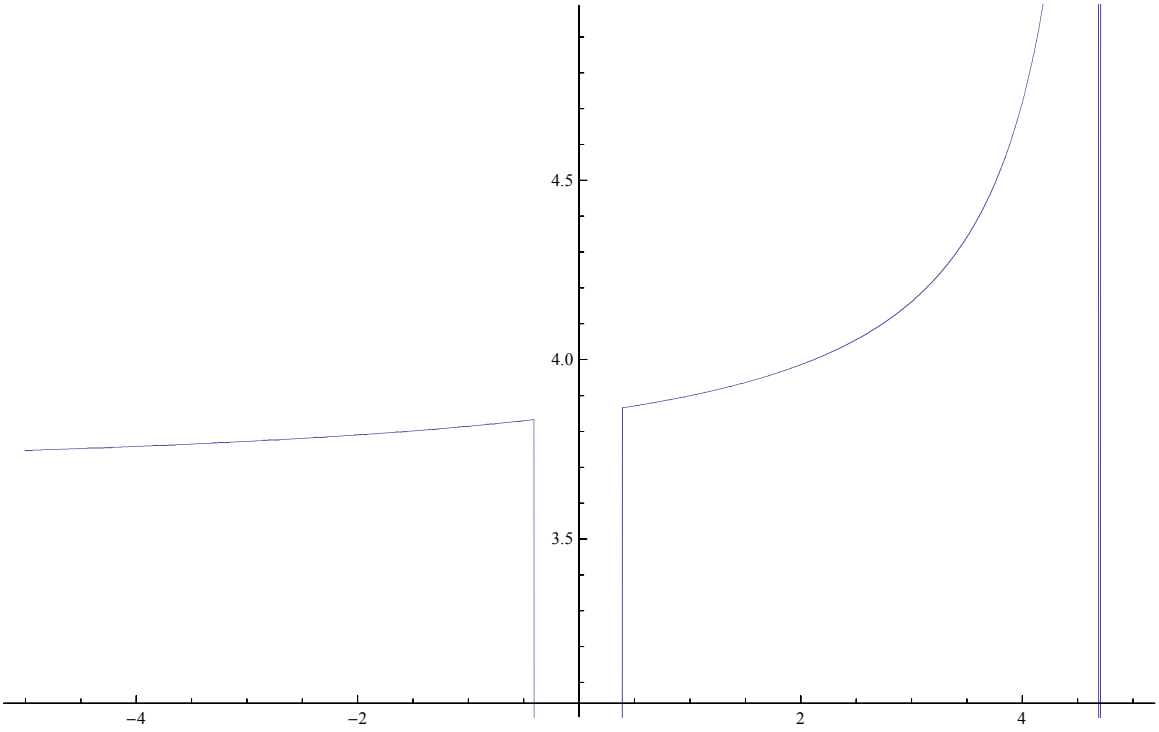


```
(*Graph d vs. variance(d)*)
varValues = Table[{d, variance[d]}, {d, .4, 1, .01}];
varValues[[-1]]
ListPlot[varValues, Joined → True]
```

$\{1., 8.5409 \times 10^{-9}\}$



```
(*Graph d vs. a=upperbound*)
upperlimitValues = Table[{d, upperlimit[d]}, {d, -5, 5, .01}];
ListPlot[upperlimitValues, Joined → True]
```



```
(*minimum variance*)
d = .480
nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b ((x - d)^(-c)), {a, b, c}, x]
nlm["BestFitParameters"]
{f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}]
 Total[g^2]
Length[g] - 1
Show[ListPlot[norms], Plot[f, {x, 0, 37}]]
```

0.48

$$\text{FittedModel}\left[\boxed{3.87007 - \frac{1.61201}{(-0.48 + x)^{0.573052}}}\right]$$

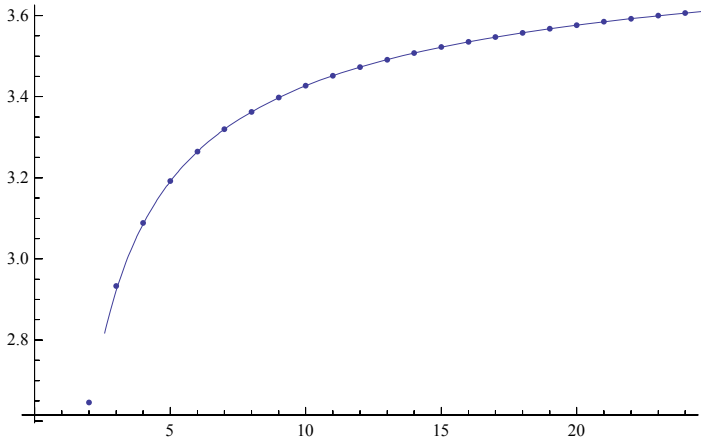
{a → 3.87007, b → 1.61201, c → 0.573052}

$$\left\{3.87007 - \frac{1.61201}{(-0.48 + x)^{0.573052}},\right.$$

{-0.0000290401, 0.000087408, -0.0000915188, 0.000074146, -0.0000604803,
0.0000385427, -0.0000563515, 0.0000354226, -0.0000299461, 0.0000340506,
-0.0000221157, 0.0000313229, -0.0000147162, 0.0000243775, -0.0000148438,
0.0000154702, -0.0000217279},

	Estimate	Standard Error	t-Statistic	P-Value
a	3.87007	0.000703032	5504.82	9.41056×10^{-46}
b	1.61201	0.00147217	1094.99	6.19768×10^{-36}
c	0.573052	0.00110118	520.399	2.06616×10^{-31}

2.32497×10^{-9}



```
(*intersection with x axis for F to be amenable*)
d = 2.118
nlm = NonlinearModelFit[norms[[Range[8, 24]]], a - b ((x - d)^(-c)), {a, b, c}, x]
nlm["BestFitParameters"]
{f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}]
 Total[g^2]
Length[g] - 1
Show[ListPlot[norms], Plot[f, {x, 0, 37}]]
```

```
2.118
```

$$\text{FittedModel}\left[\boxed{4.00001 - \frac{1.22339}{(-2.118 + x)^{0.36765}}}\right]$$

```
{a → 4.00001, b → 1.22339, c → 0.36765}
```

$$\left\{4.00001 - \frac{1.22339}{(-2.118 + x)^{0.36765}},\right.$$

```
 {0.000499332, -0.000130328, -0.00050052, -0.00028628, -0.000281945, -0.0000260868,
  0.0000188674, 0.000217707, 0.000220655, 0.000313324, 0.000247841, 0.000256826,
  0.00013451, 0.0000688909, -0.000100224, -0.00022196, -0.000430607},
```

	Estimate	Standard Error	t-Statistic	P-Value
a	4.00001	0.00750912	532.686	1.49034×10^{-31}
b	1.22339	0.00240121	509.489	2.77958×10^{-31}
c	0.36765	0.00553748	66.3931	6.68757×10^{-19}

$$7.92836\times10^{-8}$$