h=I+A+B

Extrapolation of the ||M_n|| norms described in the paper
“A COMPUTATIONAL APPROACH TO THE THOMPSON GROUP F”
by S. Haagerup, U. Haagerup, M. Ramirez-Solano:

In[1]:=
```
norms = {{1, 2.9999999999999996`}, {2, 5.000000000000001`},
   {3, 5.999999999999999`}, {4, 6.561552812808831`}, {5, 6.90951596615595`},
   {6, 7.141336115655365`}, {7, 7.304337743007365`}, {8, 7.432163237993422`},
   {9, 7.531993362930429`}, {10, 7.61434713292474`}, {11, 7.681428519345555`},
   {12, 7.73954284784636`}, {13, 7.788108145812049`}, {14, 7.831595176942418`},
   {15, 7.868872357791319`}, {16, 7.902753284389529`}, {17, 7.932409830305334`},
   {18, 7.959849162636051`}, {19, 7.984180489022279`}, {20, 8.006956664241345`},
   {21, 8.027335460419687`}, {22, 8.046597046112455`}, {23, 8.063989225969943`},
   {24, 8.080527409475819`}, {25, 8.095647014527733`}, {26, 8.11006079249735`},
   {27, 8.12330403752793 9`}, {28, 8.136018060209928`}, {29, 8.147759937749639`},
   {30, 8.159067041231319`}, {31, 8.169564285141952`}, {32, 8.179709741060485`},
   {33, 8.189171701328226`}, {34, 8.19833781133207`}, {35, 8.20692641290236`},
   {36, 8.215254352294734`}, {37, 8.22308826173715`}};
{xx, yy} = Transpose[norms];
norms = Transpose[{xx, yy^(1/2)}];
norms // MatrixForm
Ntuple = Length[norms]
```

Out[4]//MatrixForm=

1	1.73205
2	2.23607
3	2.44949
4	2.56155
5	2.6286
6	2.67233
7	2.70265
8	2.7262
9	2.74445
10	2.75941
11	2.77154
12	2.782
13	2.79072
14	2.7985
15	2.80515
16	2.81118
17	2.81645
18	2.82132
19	2.82563
20	2.82966
21	2.83326
22	2.83665
23	2.83972
24	2.84263
25	2.84529
26	2.84782
27	2.85014
28	2.85237
29	2.85443
30	2.85641
31	2.85824
32	2.86002
33	2.86167
34	2.86327
35	2.86477
36	2.86623
37	2.86759

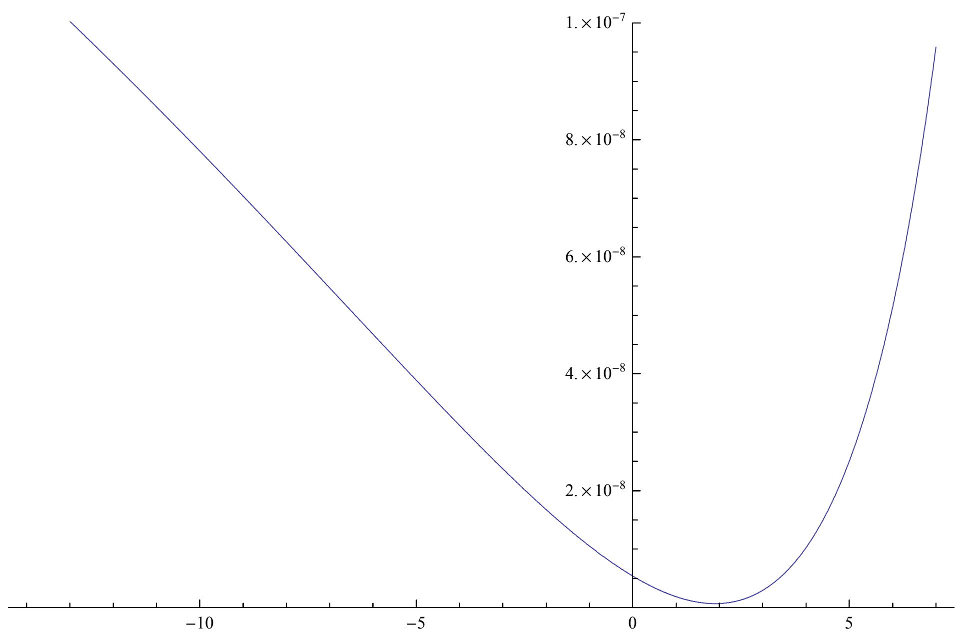
Out[5]= 37

In[6]:=
```
variance = Function[d, Module[{nlm, a, b, c, f, g, h},
    nlm =
     NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b ((x - d) ^ (-c)), {a, b, c}, x];
    {f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}];
    Total[g^2] / (Length[g] - 1) (*variance*)
   ]]
upperlimit = Function[d, Module[{nlm, aa, a, b, c, f, g, h},
    nlm =
     NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b ((x - d) ^ (-c)), {a, b, c}, x];
    aa = nlm["BestFitParameters"];
    a /. aa
   ]]
```

Out[6]=
```
Function[d, Module[{nlm, a, b, c, f, g, h},
  nlm = NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b (x - d)^-c, {a, b, c}, x];
  {f, g, h} = nlm[{BestFit, FitResiduals, ParameterTable}]; Total[g^2] / (Length[g] - 1)]]
```

Out[7]=
```
Function[d, Module[{nlm, aa, a, b, c, f, g, h},
  nlm = NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b (x - d)^-c, {a, b, c}, x];
  aa = nlm[BestFitParameters]; a /. aa]]
```

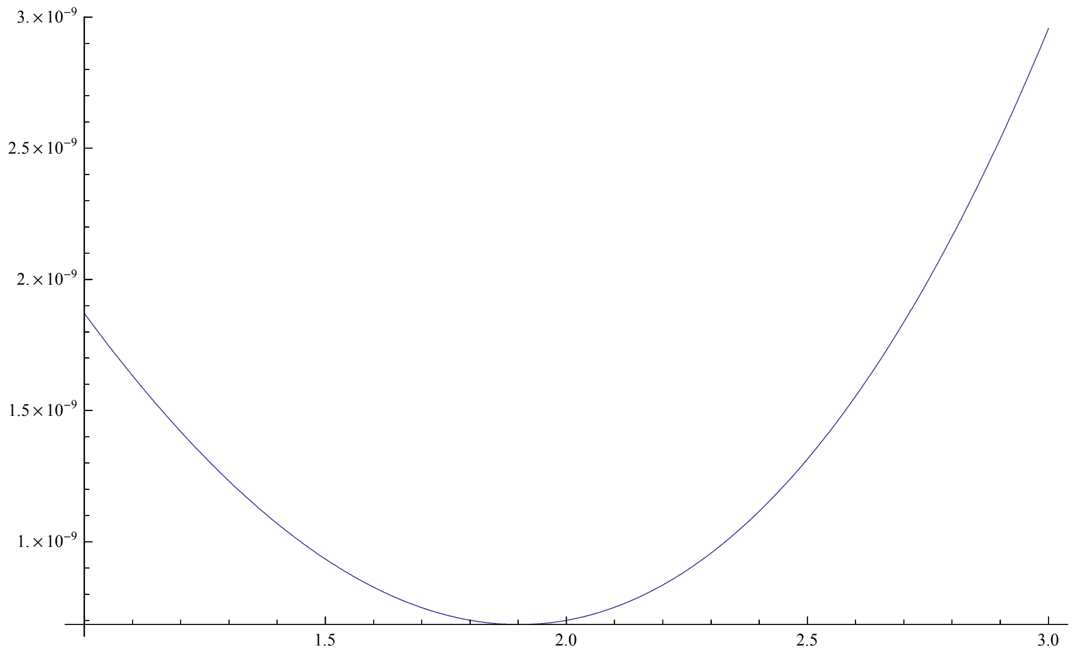
In[8]:=
```
(*Graph d vs. variance(d)*)
varValues = Table[{d, variance[d]}, {d, -14, 7, .01}];
ListPlot[varValues, Joined → True, PlotRange → {0, 10^-7}]
```

Out[9]=


In[10]:=
```
(*Graph d vs. variance(d)*)
varValues = Table[{d, variance[d]}, {d, 1, 3, .001}];
varValues[[-1]]
ListPlot[varValues, Joined → True]
```

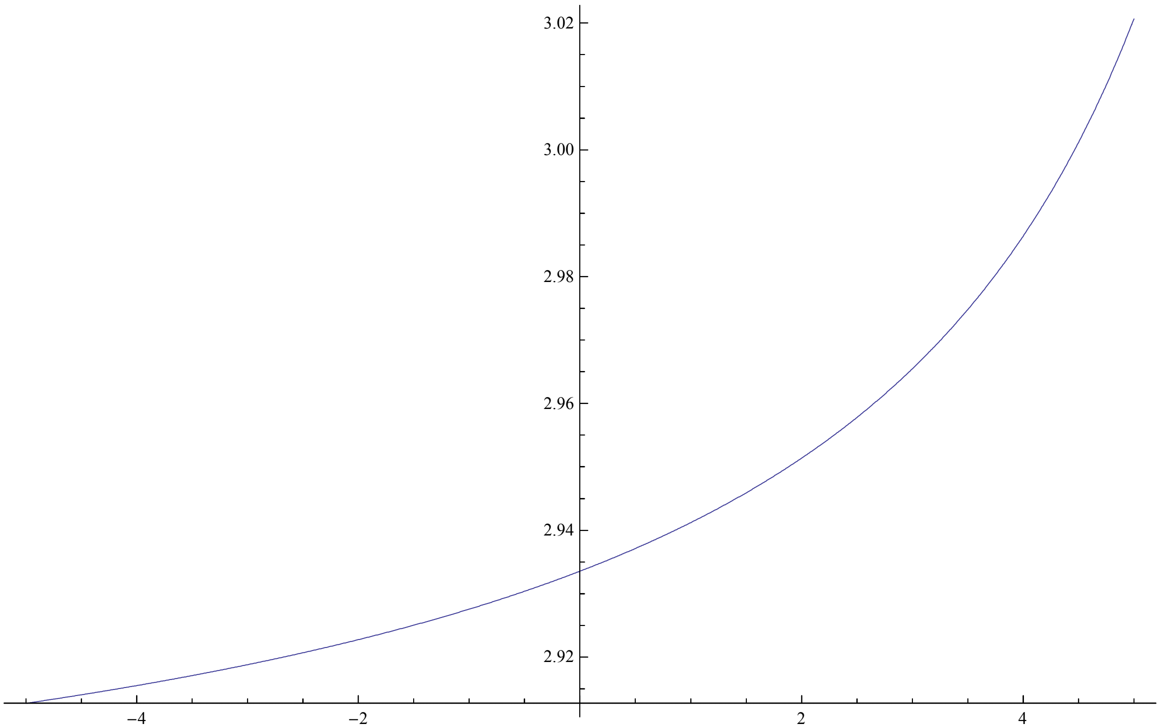
Out[11]= $\{3., 2.95643\times10^{-9}\}$

Out[12]=



In[13]:=
```
(*Graph d vs. a=upperbound*)
upperlimitValues = Table[{d, upperlimit[d]}, {d, -5, 5, .01}];
ListPlot[upperlimitValues, Joined → True]
```

Out[14]=



In[15]:=
```
(*minimum variance*)
d = 1.900
nlm = NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b ((x - d) ^ (-c)), {a, b, c}, x]
nlm["BestFitParameters"]
{f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}]
Total[g^2] / (Length[g] - 1)
Show[ListPlot[norms], Plot[f, {x, 0, 37}]]
```

Out[15]= 1.9

Out[16]= FittedModel[$2.9502 - \frac{0.629977}{(-1.9 + x)^{0.570942}}$]

Out[17]= {a → 2.9502, b → 0.629977, c → 0.570942}

Out[18]= $\left\{2.9502 - \frac{0.629977}{(-1.9 + x)^{0.570942}},\right.$

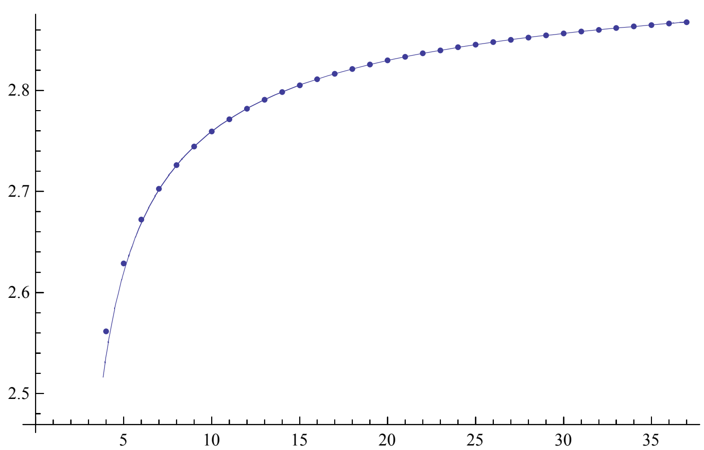
{0.000036066, -0.0000773036, 0.0000426917, -0.0000310409, 0.000036832, -0.0000282977, 0.0000316027, -0.0000194067, 0.0000312903, -0.0000159012, 0.000023514, -0.0000176541, 0.000010598, -0.000015521, 8.53386×10^{-6}, -0.000016669, 6.06873×10^{-6}, -0.0000142542, 4.90762×10^{-6}, -0.0000123199, 5.24568×10^{-6}, -8.3502×10^{-6}, 7.58877×10^{-6}, -2.43102×10^{-6}, 0.0000112766, 2.93358×10^{-6}},

	Estimate	Standard Error	t-Statistic	P-Value
a	2.9502	0.000265289	11120.7	6.53379×10^{-79}
b	0.629977	0.000952053	661.704	1.00164×10^{-50}
c	0.570942	0.0012903	442.486	1.04704×10^{-46}

}

Out[19]= 6.84329×10^{-10}

Out[20]=



In[21]:=
```
(*intersection with x axis for F to be amenable*)
d = 4.466
nlm = NonlinearModelFit[norms[[Range[12, Ntuple]]], a - b ((x - d)^(-c)), {a, b, c}, x]
nlm["BestFitParameters"]
{f, g, h} = nlm[{"BestFit", "FitResiduals", "ParameterTable"}]
Total[g^2]
----------
Length[g] - 1
Show[ListPlot[norms], Plot[f, {x, 0, 37}]]
```

Out[21]= 4.466

Out[22]= FittedModel$\left[3.00002 - \frac{0.436373}{(-4.466 + x)^{0.342838}} \right]$

Out[23]= {a → 3.00002, b → 0.436373, c → 0.342838}

Out[24]= $\left\{3.00002 - \frac{0.436373}{(-4.466 + x)^{0.342838}},\right.$

{0.000343008, -0.0000760081, -0.0000947412, -0.000214101, -0.000140805, -0.000174036, -0.000070344, -0.0000743658, 0.0000213668, 0.0000142878, 0.0000872708, 0.0000723469, 0.000119265, 0.000104301, 0.000132264, 0.000104094, 0.000117424, 0.0000817055, 0.0000799437, 0.0000367104, 0.0000236154, -0.0000248926, -0.0000477403, -0.000100071, -0.000131873, -0.000188627},

	Estimate	Standard Error	t-Statistic	P-Value
a	3.00002	0.00258749	1159.43	2.50319×10^{-56}
b	0.436373	0.000600807	726.311	1.17543×10^{-51}
c	0.342838	0.00526936	65.0626	1.39195×10^{-27}

}

Out[25]= 1.60171×10^{-8}

Out[26]=