CURS 11

Stabilitation prim function di ecuitionic

prim function Lyapunov

(1) 
$$\begin{cases} x' = f_1(x_1y) \\ y' = f_2(x_1y) \end{cases}$$
 $\begin{cases} x'' = f_2(x_1y) \\ y'' = f_2(x_1y) \end{cases}$ 

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Pt o functie  $V = V(x,y) \in C^1(D)$  numim derivata

lui V de-2 lungul trajectoriiler sist. (1) si o motaur

 $\bigvee (x_{i}y) = \frac{d}{dt} \bigvee (x(t)_{i}y(t)) = \frac{\partial \bigvee}{\partial x} (x(t)_{i}y(t)) \cdot x'(t) +$ 

+ 34 (x(+),4(+1). 91(t)

7=(2\*13\*)

D⊆R<sup>2</sup> domenin, X\*∈D

Fix 
$$V \in C^{1}(D)$$
 a.i

(i)  $V(X^{*}) = V(x^{*}, y^{*}) = 0$  of  $V(x_{1}y_{1}) > 0$  of  $V(x_{1}y_{1}) \in D \setminus \{(x_{1}y_{1})\} = 0$  of  $V(x_{1}y_{1}) = 0$  of  $V(x_{1}y_{1}) \in D \setminus \{(x_{1}y_{1})\} = 0$  of  $V(x_{1}y_{1}) \in D \setminus \{(x_{1}y_{1})\} = 0$ 

(ii) Data  $V(x_{1}y_{1}) \in 0$  of  $V(x_{1}y_{1}) \in D \setminus \{(x_{1}y_{1})\} = 0$ 

(iv) Data  $V(x_{1}y_{1}) > 0$  of  $V(x_{1}y_{1}) \in D \setminus \{(x_{1}y_{1})\} = 0$ 

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=) X\*(x\*,y\*) est in stabil.

Dbs: Daca V satisf. (i) +(ii) => V fet Lyapunov

Daca V satisf. (i) +(iii) => V fet. Lyapunov stricta

 $\Rightarrow \left| \dot{V}(x,y) = \frac{\partial V}{\partial x}(x,y) \cdot \dot{J}_{2}(x,y) + \frac{\partial V}{\partial y}(x,y) \cdot \dot{J}_{2}(x,y) \right|$ 

Teorema lui Lyapunos

V(x,y) = x4+44

V(0,0)=0

X (0,0) este pet de échilibre

 $\int_{1}^{2} \{(0,0)\} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \implies y^{2} = y^{2} = 0 \quad \text{and brokin.} \implies y^{2} = y^{2} = 0$ 

, D=122

>> nu se poate aplica T. stab. in prima aproximatie

V(x,y)>0, + (x,y) = 122 { (0,0)}

1) 
$$\begin{cases} x' = -y^3 \\ y' = x^3 \end{cases}$$

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$$\begin{cases} y' = x^3 \\ f_1(x,y) = -y^3 \\ f_2(x,y) = x^3 \\ \frac{\partial f_1}{\partial x} \frac{\partial f_2}{\partial y} \\ \frac{\partial f_2}{\partial x} \frac{\partial f_3}{\partial y} \\ \frac{\partial f_2}{\partial x} \frac{\partial f_3}{\partial y} \\ \frac{\partial f_2}{\partial y} \frac{\partial f_3}{\partial y} \\ \frac{\partial f_2}{\partial y} \frac{\partial f_3}{\partial y} \\ \frac{\partial f_3}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_2}{\partial y} \frac{\partial f_3}{\partial y} \\ \frac{\partial f_3}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \\ \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y} \frac{\partial f_4}{\partial y}$$

$$\begin{array}{ll}
\ddot{V}(x,y) \leq 0, & \forall (x,y) \in \mathbb{R}^2 \\
\Rightarrow & \chi^*(0,0) \text{ est local stabil.} \\
2) & \chi' = -x + y^3 \\
& \chi'' = -x - y
\end{array}$$

$$\chi^*(0,0) \text{ pct. de echilibru}$$

 $\sqrt{(x,y)} = \frac{\partial V}{\partial x} \cdot f_1 + \frac{\partial V}{\partial y} \cdot f_2 = 4x^3 \cdot (-y^3) + 4y^3 \cdot x^3 =$ 

 $=-4x^3y^3+4x^3y^3=0$ .

$$f_{1}(x,y) = -x+y^{3}$$

$$f_{2}(x,y) = -x-y$$

$$V(x,y) = \frac{1}{2}x^{2} + \frac{1}{4}y^{4}, D = \mathbb{R}^{2}$$

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$$V(0,0) = 0$$
 of  $V(x,y) > 0$  of  $(x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$  =  $7(i)$  so that  $V(x,y) = \frac{\partial V}{\partial x} \cdot f_1 + \frac{\partial V}{\partial y} \cdot f_2 = X \cdot (-x + y^3) + y^3 \cdot (-x - y) = 0$ 

 $= -x^2 + xy^3 - xy^3 - y^4 = -x^2 - y^4$ 

3) 
$$\int_{1}^{1} x^{1} = x - y^{3}$$
  $\chi^{*}(q_{0})$  pcf de echil.

=> X\*(0,0) este instabil

= x2-xg3+xg3+y4 = x2+ y4

V(x,y)>0, + (x,y) ER? (10,0) => (iv) satisf.

$$f_2(x,y) = x+y$$
  
 $V(x,y) = \frac{1}{2}x^2 + \frac{1}{4}y^4 \rightarrow D = 1R^2$ 

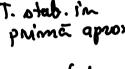
$$\sqrt[4]{(x_1y_1)} = \frac{9x}{9x_1} \cdot \frac{1}{4} + \frac{9x}{9x_1} \cdot \frac{1}{4} = x \cdot (x - x_3) + x_3(x + x_1) = x$$

4) Siotemul prada- pradator.

$$\begin{aligned}
x' &= ax - bxy & a_1b_1x_1, d > 0 \\
y' &= -cyt d \cdot xy & \\
f_1(x_1y) &= ax - bxy \\
f_2(x_1y) &= -c + d \cdot xy
\end{aligned}$$

$$\begin{cases} y' = -cy \cdot d \cdot xy \\ f_{1}(x_{1}y) = ax - bxy \\ f_{2}(x_{1}y) = -c + d \cdot xy \end{cases}$$

$$\chi_{100}^{*}$$
  $|\chi_{2}^{*}| \left(\frac{c}{d}, \frac{a}{b}\right)$   $\chi_{12}^{*} = \pm i \sqrt{ac}$ 



prima aprox.) ). Atab. In prima 
$$V(x,y) = (d \cdot x - c \ln x - c + c \ln \frac{c}{d}) + c \ln \frac{c}{d}$$

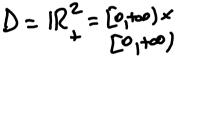
$$= d \left( x - \frac{1}{\alpha} - \frac{1}{\alpha} \ln \frac{xd}{x} \right) + b \left( y - \frac{2}{3} - \frac{2}{5} \ln \frac{xb}{a} \right)$$

$$V(\frac{1}{3}, \frac{2}{5}) = 0$$

$$\lambda_{1/2} = \pm i \sqrt{ac}$$

$$\rho_0 \lambda_{1/2} = 0$$

Re 21,2=0





oe stie ca 
$$\frac{ht+1 \le t}{egalitatea}$$
 au loc  $pt t=1$ .

$$V(x_iy) = d\left[x - \frac{c}{d}\left(1 + \frac{hx_id}{c}\right)\right] + b\left[y - \frac{a}{b}\left(1 + \frac{h\frac{by}{a}}{a}\right)\right]$$

t= yb = 1 hh by < 54

J # 96

$$V(x_{i}y) = d\left[x - \frac{1}{d}(1 + hx_{i}) + b\left[y - \frac{1}{6}(1 + hx_{i})\right] + b\left[y - \frac{1}{6}(1 + hx_{i})\right] + a\left[x - \frac{1}{d}(1 + hx_{i})\right] + a\left[x - \frac{1}{d}(1 + hx_{i})\right]$$

en xa +1 <x a

=> V(x14)>0 , x(x14) ER \{X\_2\*}

(11) satisf.

$$\dot{V}(x,y) = \frac{\partial x}{\partial x} \cdot \dot{f}_1 + \frac{\partial y}{\partial y} \cdot \dot{f}_2 =$$

$$= (d - \frac{c}{x})(ax + bxy) + (b - \frac{a}{5})(-xy + d.xy) =$$

$$= adx - bdxy - ac + cby - bcy + bdxy + ac - adx$$

$$= adx - bdxy - gc + cby - bcy + bdxy + gx - baxy = 0$$

- =) V(x1y)=0 <0, f(x1y) <1P2 => (ii) pahof.
  - → X\* ( f, 2) est local stabil

$$\int_{y}^{x} \frac{dx}{dy} = \frac{ax - bxy}{-cy + dxy} = c. dif. a = bih br$$

$$\frac{dx}{dy} = \frac{a - by}{y} \cdot \frac{x}{-c + d \cdot x}$$

$$\frac{a - by}{y} \cdot dy = \frac{-c + d \cdot x}{x} \cdot dx$$

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