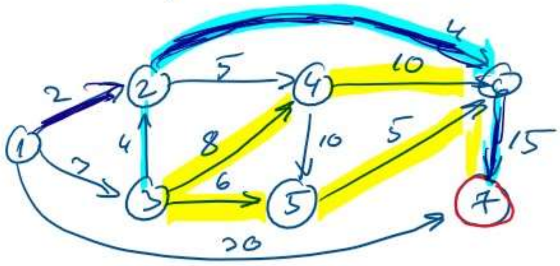
$$a_{ij} = \begin{cases} 0 & , i=j \\ \infty & , \text{dacă } (i,j) \notin E \end{cases}$$

se determină k vectori $k = 1, \dots, n$

$$V_i^{(k)} = \min_{j=\overline{1,n}} \{ a_{ij} + V_j^{(k-1)} \} \quad \text{pt } i = 1, \dots, n$$

până când $V^{(t)} = V^{(t-1)}$ pt un t



$$A = \begin{pmatrix} 0 & 2 & 3 & \infty & \infty & \infty & 30 \\ \infty & 0 & \infty & 5 & \infty & 4 & \infty \\ \infty & 4 & 0 & 8 & 6 & \infty & \infty \\ \infty & \infty & \infty & 0 & 10 & 10 & \infty \\ \infty & \infty & \infty & \infty & 0 & 5 & \infty \\ \infty & \infty & \infty & \infty & \infty & 0 & 15 \\ \infty & \infty & \infty & \infty & \infty & \infty & 0 \end{pmatrix}$$

$$V^{(1)} = \begin{bmatrix} 30 \\ \infty \\ \infty \\ \infty \\ \infty \\ 15 \\ 0 \end{bmatrix} \quad V^{(2)} = \begin{bmatrix} 30 \\ 19 \\ \infty \\ 25 \\ 20 \\ 15 \\ 0 \end{bmatrix} \quad \begin{matrix} \infty + 30 \\ 0 + \infty \\ \infty + \infty \\ 5 + \infty \\ \infty + \infty \\ 4 + 15 \\ \infty + 0 \end{matrix} \quad V^{(3)} = \begin{bmatrix} 21 \\ 19 \\ 23 \\ 25 \\ 20 \\ 15 \\ 0 \end{bmatrix} \quad V^{(4)} = \begin{bmatrix} 21 \\ 19 \\ 23 \\ 25 \\ 20 \\ 15 \\ 0 \end{bmatrix}$$

$$\begin{matrix} 0 + 30 & \infty + 20 \\ 2 + 19 & \infty + 15 \\ 3 + \infty & 30 + 0 \\ \infty + 25 & \end{matrix}$$

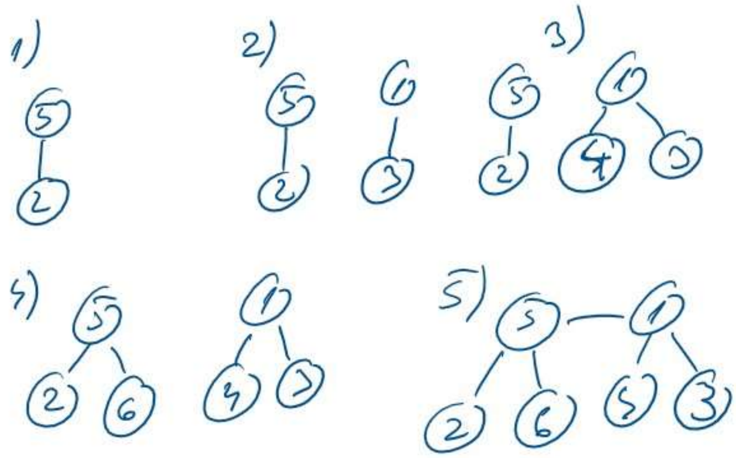
CODARE_PRUFER (T)
 $K = \emptyset$
 while T conține și alte vf. decât rădăcina do
  fie v frunza min din T
  $K \leftarrow$ predecesor(v)
  $T = T \setminus \{v\}$
 return K

K : 5 1 1 5 1

DECODARE_PRUFER (K, n)
 $T = \emptyset$
 for $i = 1$ to $n-1$ do
  x primul elem din K (stg)
  y cel mai mic nr. natural care nu este în K

y cel mai mic nr. natural care nu este în K
$(x,y) \in E(T)$, x părintele lui y în T
șterg pe x din K și adaug pe y în coada secv K
return T

x	y	
5 1 1 5 1	2	$\to (5,2) \in E$
1 1 5 1 2	3	$\to (1,3) \in E$
1 5 1 2 3	4	$\to (1,4) \in E$
5 1 2 3 4	6	$\to (5,6) \in E$
1 2 3 4 6	5	$\to (1,5) \in E$



Huffman

aaaa b cccdd   80 biți

C

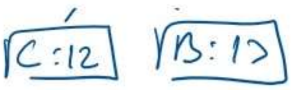
a = 0   ~~b=0~~
b = 01
a/b
0 0 01
0 a

Huffman(C)
  n = |C|
  Q = C
  for $1 \le i \le n-1$ do
    aloca un nou vf. z
    z.stg = x = EXTRACT_MIN(Q)
    z.dr = y = EXTRACT_MIN(Q)
    z.fr = x.fr + y.fr
    INSERT(Q, z)
  return EXTRACT_MIN(Q)

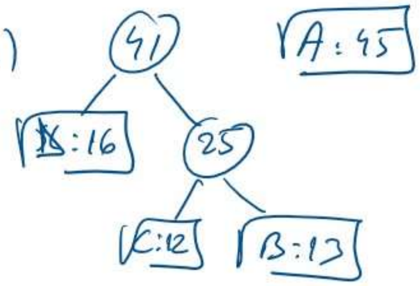
	a	b	c	d
fr	45	13	12	16

Q: [C:12] (x), [B:13] (y); [D:16], [A:45]

1) Q: [D:16] (x), 25 (y) [C:12] [B:13], [A:45]

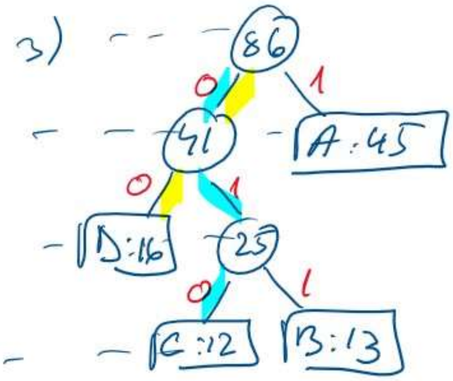


2)



aaaa bcccdd 80biți

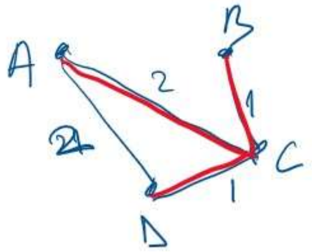
3)



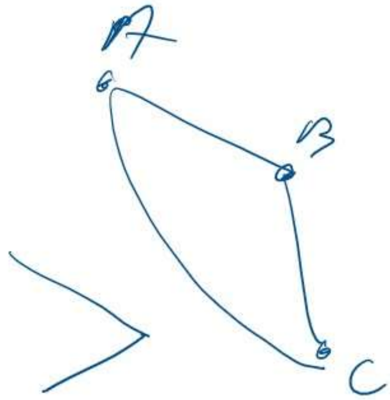
00 = D
010 = C
011 = B
1 = A

1111 011 010 010 010 00 00 20biți

1 1 1 1 011 . . . . .
a a a a b



(A, D)

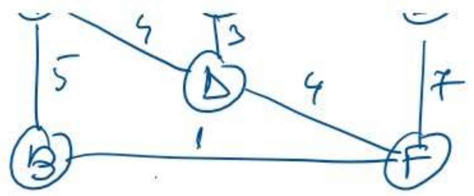


kruskal (G, w) disjoint set

```
kruskal(G, w)
  T = ∅
  for v ∈ V do
      MAKE_SET(v)
  sortează crescător muchiile după ponderea w
  for (u, v) ∈ E luate crescător după w do
      if FIND_SET(u) ≠ FIND_SET(v) then
          T = T ∪ {(u, v)}
          UNION(u, v)
  return T
```

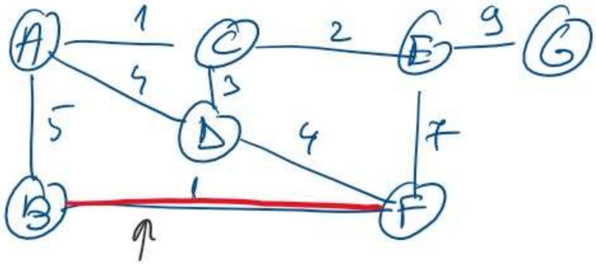
T

(A) (B) (C) (D)



(A) (B) (C) (D)

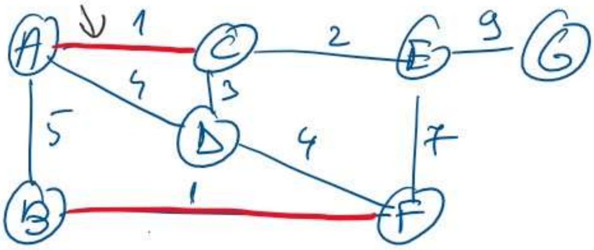
(E) (F) (G)



T = {(B,F)

(A) (B F) (C) (D)

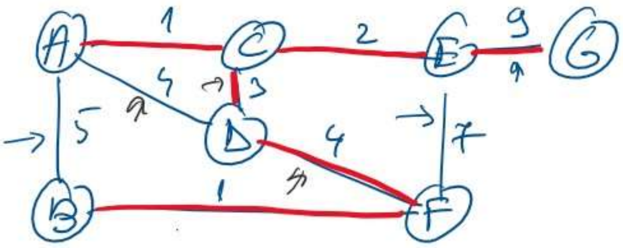
(E) ~~(F)~~ (G)



T = {(B,F), (A,C)

(A,C) (B,F) ~~(C)~~ (D)

(E) (G)



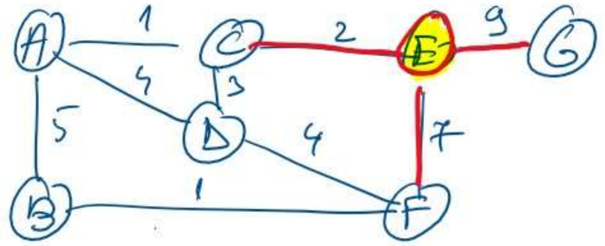
T = {(B,F), (A,C), (C,E), (C,D), (D,F), (E,G) }

(A,C,E,D , B,F,G) ~~(D)~~

~~(G)~~

```
PRIM(G, w, r)                key, π
  for v ∈ V do
      v.key = ∞
      v.π = NIL
  r.key = 0
  Q = V
  while Q ≠ ∅ do
      u = EXTRACT_MIN(Q)
      for v ∈ Adj[u]
          if v ∈ Q şi w(u,v) < v.key then
              v.π = u
              v.key = w(u,v)
```

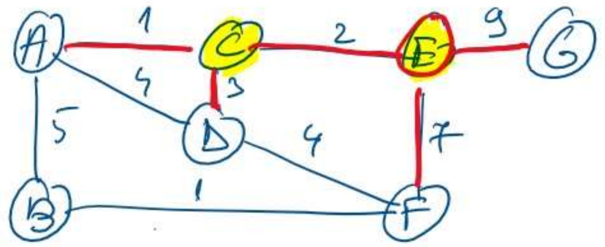
| A B C D E F G



	A	B	C	D	E	F	G
key	∞	∞	$\cancel{\infty}$ 2	∞	0	$\cancel{\infty}$ 7	$\cancel{\infty}$ 9
π	NIL	NIL	~~NIL~~ E	NIL	NIL	~~NIL~~ E	~~NIL~~ E

Q: A B C D E F G

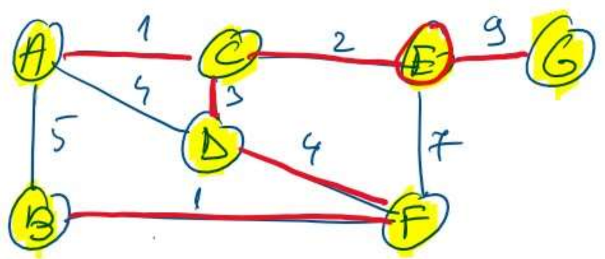
A	B	C	D	E	F	G
∞	∞	$\cancel{\infty}$ 2	∞	0	$\cancel{\infty}$ 7	$\cancel{\infty}$ 9



	A	B	C	D	E	F	G
key	$\cancel{\infty}$ 1	∞	$\cancel{\infty}$ 2	$\cancel{\infty}$ 3	0	$\cancel{\infty}$ 7	$\cancel{\infty}$ 9
π	~~NIL~~ C	NIL	~~NIL~~ E	~~NIL~~ C	NIL	~~NIL~~ E	~~NIL~~ E

Q: A B C D F G

A	B	C	D	F	G
$\cancel{\infty}$ 1	∞	$\cancel{\infty}$ 2	$\cancel{\infty}$ 3	$\cancel{\infty}$ 7	$\cancel{\infty}$ 9



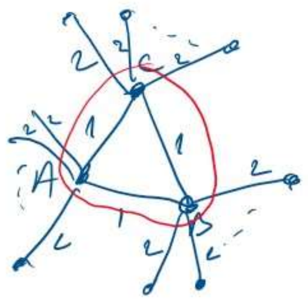
	A	B	C	D	E	F	G
key	$\cancel{\infty}$ 1	$\cancel{\infty}$ $\cancel{5}$ 1	$\cancel{\infty}$ 2	$\cancel{\infty}$ 3	0	$\cancel{\infty}$ $\cancel{7}$ 4	$\cancel{\infty}$ 9
π	~~NIL~~ C	~~NIL~~ ~~A~~ F	~~NIL~~ E	~~NIL~~ C	NIL	~~NIL~~ ~~E~~ D	~~NIL~~ E

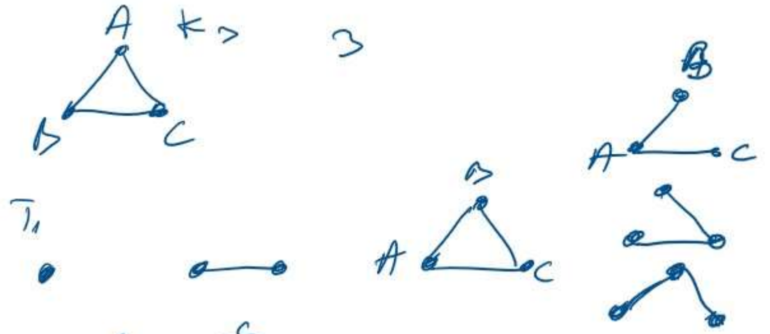
Q:

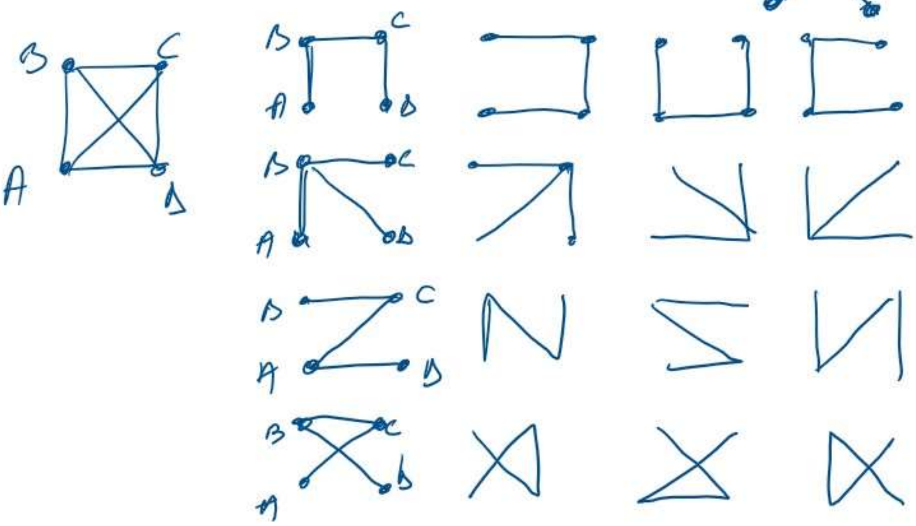
	A	B	C	D	E	F	G
key	1	1	2	3	0	4	9
π	C	F	E	C	NIL	D	E

K_{1000} ponderat și cu vf. etichetate. În graf există un K_3 pt care ponderea muchiilor este 1, restul muchiilor au ponderea 2.
Câți arbori minimi de acoperire există pt. graful dat?

Câți arbori minimi de acoperire există pt. graful dat !







$|T| = n^{n-2}$

Cayley

$|V| = 100 - 2 = 998$ $\qquad$ 998^{996}

K_{998} $\qquad$ $|T| = 998^{996}$

$3 \cdot 998^{996}$

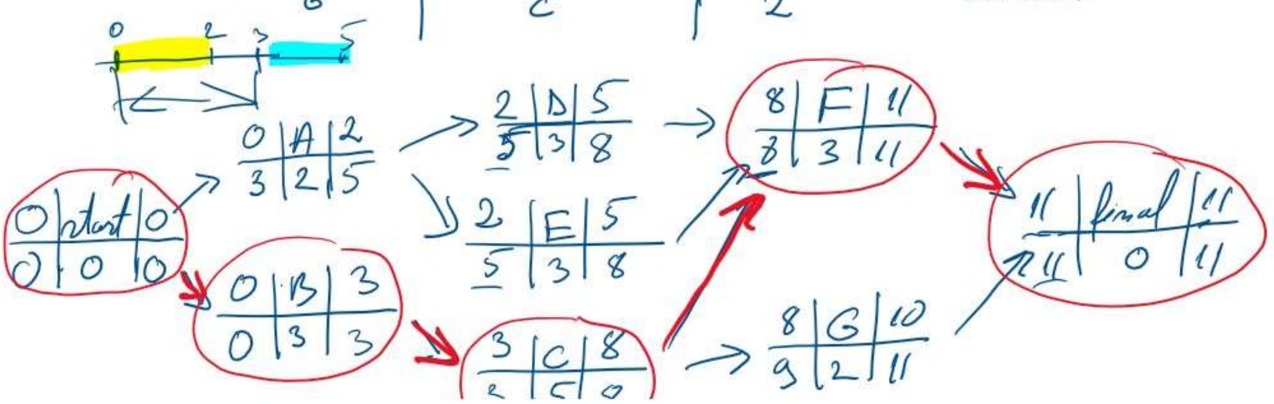
K_3 $\qquad$ $|T| = 3$

Activitate	dependență	durata
A	—	2
B	—	3
C	B	5
D	A	3
E	A	3
F	C, D, E	3
G	C	2

early start $\qquad$ early finish

ES	activitate	EF
LS	durata	LF

late start $\qquad$ late finish



0 | 3 | 3 → 3 | 6 | 8 / 3 | 5 | 8 → 8 | 6 | 10 / 9 | 2 | 11

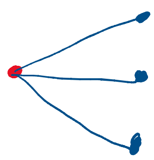
LF = EF
LS = ES

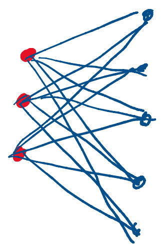
S4 Grafuri Euleriene, Hamiltoniene

G este Euler
G - toate vf. au grad par
muchiile din G pot fi partiționate în cicluri disjuncte

K_n $n = ?$
n - impar $\Rightarrow d(x_i) = n-1$
pt $n = 2$

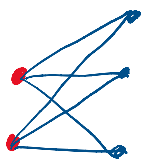
$K_{n,m}$





n, m - impar ×
n, m - par ✓ ciclu eulerian

$\left.\begin{matrix} n = 2 \\ m - \text{par} \end{matrix}\right\}$ ciclu eulerian $\left.\begin{matrix} n = 2 \\ m - \text{impar} \end{matrix}\right\}$ lanț eulerian



Corolar: G conex conține un lanț eulerian dacă G are 2 vf. de grad impar și restul vf. de grad par.

G, $n = 100$, $1, \ldots, 100$, $\{i, j\} \in E$ dacă $|i-j| \leq 2$
conține G un ciclu eulerian? lanț eulerian?
× ✓