9.4 ecuatia planului tangent la paraboloidul eliptic $\frac{x^2}{5} + \frac{y^2}{3} = z$ paralel cu planul

$P_1: x - 3y + 2z - 1 = 0$

$$\frac{x^2}{p} + \frac{y^2}{q} = 2z$$

$$\frac{x^2}{5} + \frac{y^2}{3} = z \quad | \cdot 2$$

$$\frac{x^2}{\frac{5}{2}} + \frac{y^2}{\frac{3}{2}} = 2z \Rightarrow p = \frac{5}{2},\ q = \frac{3}{2}$$

Ecuatia planului tangent la paraboloidul eliptic in punctul $M(x_0, y_0, z_0)$

$$\frac{xx_0}{p} + \frac{yy_0}{q} = z + z_0$$

$P_2: \frac{2x_0}{5}x + \frac{2y_0}{3}y - z - z_0 = 0$ paralel cu P_1

$\Rightarrow$ coeficienti proportionali $\Rightarrow$

$$\frac{2x_0}{5} = -\frac{2y_0}{9} = -\frac{1}{2}$$

$$\left.\begin{aligned} x_0 &= -\frac{5}{4} \\ y_0 &= \frac{9}{4} \end{aligned}\right\} \Rightarrow M\left(-\frac{5}{4}, \frac{9}{4}, z_0\right)$$

$M \in$ paraboloidului eliptic $\Rightarrow$

$$\frac{\left(-\frac{5}{4}\right)^2}{5} + \frac{\left(\frac{9}{4}\right)^2}{3} = z \Rightarrow z = \frac{5}{16} + \frac{27}{16} \Rightarrow z = 2$$

$$\Rightarrow M\left(-\frac{5}{4}, \frac{9}{4}, 2\right)$$

$P_2: \frac{2\cdot(-\frac{5}{4})}{5}x + \frac{2\cdot(\frac{9}{4})}{3}y - z - 2 = 0$

$\boxed{P_2: -\frac{1}{2}x + \frac{3}{2}y - z - 2 = 0}$ $|\cdot 2$

$P_2: -x + 3y - 2z - 4 = 0$

b) ecuatia planului tangent la paraboloidul hiperbolic $x^2 - \frac{y^2}{4} = z$ paralel cu planul

$P_1: x - 3y + 2z - 1 = 0$

$\frac{x^2}{p} - \frac{y^2}{q} = 2z$

$x^2 - \frac{y^2}{4} = z \quad |\cdot 2$

$2x^2 - \frac{y^2}{2} = 2z$

$\frac{x^2}{\frac{1}{2}} - \frac{y^2}{2} = 2z \Rightarrow p = \frac{1}{2},\ q = 2$

Ecuatia planului tangent la paraboloidul hiperbolic in punctul $M(x_0, y_0, z_0)$

$\frac{xx_0}{p} + \frac{yy_0}{q} = z + z_0$

$P_2: 2x_0 x - \frac{y_0}{2}y - z - z_0 = 0$ paralel cu P_1

$\Rightarrow$ coeficienti proportionali $\Rightarrow$

$2x_0 = \frac{y_0}{6} = -\frac{1}{2}$

$x_0 = -\frac{1}{4}\ ;\ y_0 = -3 \Rightarrow M(-\frac{1}{4}, -3, z_0)$

$M \in$ paraboloidului hiperbolic $\Rightarrow$

$\frac{1}{16} - \frac{9}{4} = z \Rightarrow z = -\frac{35}{16} \Rightarrow M\left(-\frac{1}{4}, -3, -\frac{35}{16}\right)$

$$P_2: -\frac{1}{2}x + \frac{3}{2}y - z + \frac{35}{16} = 0 \quad 1.2$$

$$P_2: -x + 3y - 2z + \frac{35}{8} = 0$$