11.7

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Problema 11.7. Determinaţi imaginea triunghiului ABC printr-o rotaţie de unghi $-45°$ în jurul vârfului A, urmată de o scalare de factori $(2,1)$ relativ la vârful C. Aplicaţi apoi transformările în ordine inversă.

Forma generală a matricei de rotație:

$$Rot(A,\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & a_1(1-\cos\theta)+a_2\sin\theta\\ \sin\theta & \cos\theta & a_2(1-\cos\theta)-a_1\sin\theta\\ 0 & 0 & 1\end{bmatrix}$$

Pentru problema noastră avem $\left(\sin\left(-\frac{\pi}{4}\right)=-\cos\left(-\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}\right)$:

$$Rot(A,\theta)=\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & a_1-\frac{\sqrt{2}(a_1+a_2)}{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & a_2-\frac{\sqrt{2}(a_2-a_1)}{2}\\ 0 & 0 & 1\end{bmatrix}$$

Forma generală pentru scalare neuniformă:

$$Scale(C,s_x,s_y)=\begin{bmatrix}s_x & 0 & (1-s_x)c_1\\ 0 & s_y & (1-s_y)c_2\\ 0 & 0 & 1\end{bmatrix}$$

Pentru problema noastră avem:

$$Scale(C,s_x,s_y)=\begin{bmatrix}2 & 0 & -c_1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$A(1,1)\Rightarrow Rot(A,\theta)=\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1-\sqrt{2}\\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1\\ 0 & 0 & 1\end{bmatrix}$$

$$C(2,3)\Rightarrow Scale(C,s_x,s_y)=\begin{bmatrix}2 & 0 & -2\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$I_{,,}\Longrightarrow$ " (rotim, și după scalăm)

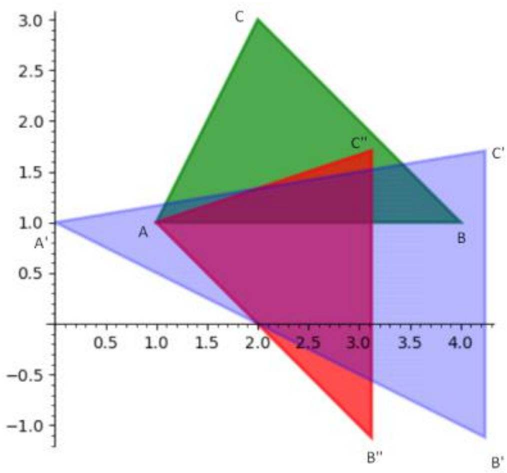
$$T_1=Scale(C,s_x,s_y)\cdot Rot(A,\theta)\Rightarrow$$

$$\Rightarrow T_1 = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1-\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} & -2\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A', B', C'] = T_1 \cdot [A, B, C] = \begin{bmatrix} \sqrt{2} & \sqrt{2} & -2\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \Leftrightarrow$$

$$\Leftrightarrow [A', B', C'] = \begin{bmatrix} 0 & 3\sqrt{2} & 3\sqrt{2} \\ 1 & 1-\frac{3\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} A' = A'(0, 1) \\ B' = B'\left(3\sqrt{2}, 1-\frac{3\sqrt{2}}{2}\right) \\ C' = C'\left(3\sqrt{2}, 1+\frac{\sqrt{2}}{2}\right) \end{cases}$$



$\underline{\text{II}}$ „$\Leftarrow$" (rolăm, după rotim)

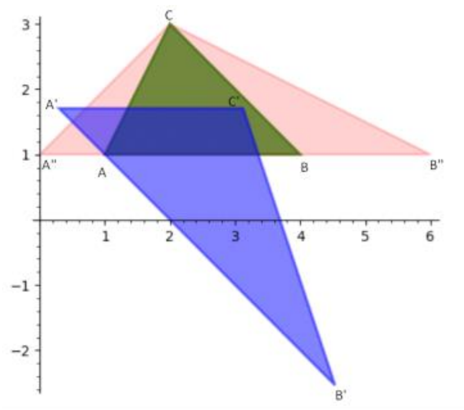
$$T_2 = Rot(A, \theta) \cdot Scale(C, s_x, s_y) \Rightarrow$$

$$\Rightarrow T_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1-\sqrt{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow T_2 = \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} & 1-2\sqrt{2} \\ -\sqrt{2} & \frac{\sqrt{2}}{2} & 1+\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A', B', C'] = T_2 \cdot [A, B, C] = \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} & 1-2\sqrt{2} \\ -\sqrt{2} & \frac{\sqrt{2}}{2} & 1+\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow [A', B', C'] = \begin{bmatrix} 1-\frac{\sqrt{2}}{2} & 1+\frac{5\sqrt{2}}{2} & 1+\frac{3\sqrt{2}}{2} \\ 1+\frac{\sqrt{2}}{2} & 1-\frac{5\sqrt{2}}{2} & 1+\frac{\sqrt{2}}{2} \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$$



$$\Rightarrow \begin{cases} A' = A'\left(1-\frac{\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}\right) \\ B' = B'\left(1+\frac{5\sqrt{2}}{2}, 1-\frac{5\sqrt{2}}{2}\right) \\ C' = C'\left(1+\frac{3\sqrt{2}}{2}, 1+\frac{\sqrt{2}}{2}\right) \end{cases}$$