Temă - seminar 12

12.5.

Determinați imaginea triunghiului ABC prin reflexia relativ la dreapta AB

A(1,1)

B(4,1)

C(2,3)

Determinăm ecuația generală a dreptei AB:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{1-1}{4-1} = 0$$

AB: $y - y_A = m_{AB}(x - x_A)$

AB: $y - 1 = 0$

AB: $y = 1$

vectorul director al dreptei: $\vec{w} = \vec{i} = (1,0)$ (dreapta e paralelă cu ~~axa~~ axa Ox, al cărei vector director este $\vec{i}$)

Matricea omogenă a reflexiei:

$$Mirror(Q, w) = \begin{pmatrix} I_2 - 2(w^{\perp} \otimes w^{\perp}) & 2(i^{\perp} \otimes i^{\perp}) \cdot Q \\ 0 & 1 \end{pmatrix}$$

$$Mirror(A, \vec{i}) = \begin{pmatrix} I_2 - 2(i^{\perp} \otimes i^{\perp}) & 2(i^{\perp} \otimes i^{\perp}) A \\ 0 & 1 \end{pmatrix}$$

$i^{\perp} = (0, 1)$

$$I_2 - 2(i^{\perp} \otimes i^{\perp}) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 0 \\ 1 \end{vmatrix} \cdot (0, 1)$$

$$= \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

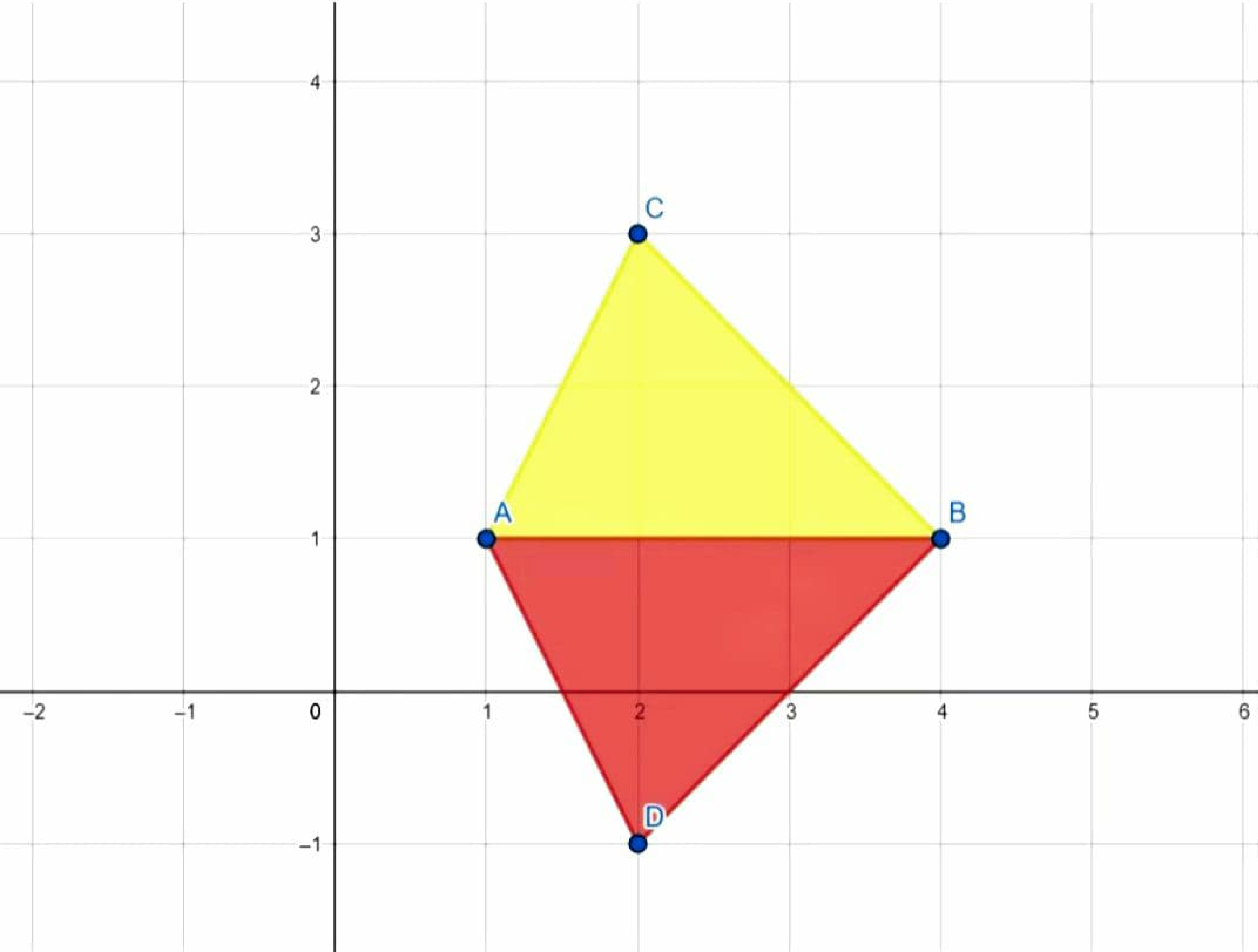
$$2 \cdot (i^{\perp} \otimes i^{\perp}) \cdot A = \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 2 \end{vmatrix}$$

$$\text{Mirror}(A, i) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$[A', B', C'] = \text{Mirror}(A, i) \cdot [A, B, C]$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 2 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow \begin{cases} A'(1, 1) \\ B'(4, 1) \\ C'(2, -1) \end{cases}$$



C
A
B
D
4
3
2
1
0
−1
−2
−1
1
2
3
4
5
6