Temă - seminar 9

9.6. Să se scrie ecuațiile generatoarelor rectilinii ale paraboloidului hiperbolic $\frac{x^2}{16} - \frac{y^2}{4} = z$ care sunt paralele cu planul $\pi: 3x + 2y - 4z = 0$.

ecuația unui paraboloid hiperbolic:

$$\frac{x^2}{p} - \frac{y^2}{q} = 2z, \quad p, q \in \mathbb{R}_+^*$$

$$\frac{x^2}{16} - \frac{y^2}{4} = z \quad \Big| \cdot 2$$

$$\frac{x^2}{8} - \frac{y^2}{2} = 2z.$$

<u>Generatoarele rectilinii ale paraboloidului hiperbolic</u>

$$D_{\lambda,\mu}: \begin{cases} \lambda\left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right) = 2\mu z \\ \mu\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = \lambda \end{cases} \qquad \lambda^2 + \mu^2 \neq 0$$

$$D_{\alpha,\beta}: \begin{cases} \alpha\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = 2\beta z \\ \beta\left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right) = \alpha \end{cases} \qquad \alpha^2 + \beta^2 \neq 0$$

$$D_{\lambda,\mu}: \begin{cases} \lambda\left(\frac{x}{2\sqrt{2}} - \frac{y}{\sqrt{2}}\right) = 2\mu z \\ \mu\left(\frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}}\right) = \lambda \end{cases}$$

$$D_{\lambda,\mu}: \begin{cases} \lambda x - 2\lambda y - 4\sqrt{2}\mu z = 0 \\ \mu x + 2\mu y - 2\sqrt{2}\lambda = 0 \end{cases} \quad \begin{aligned} &\Rightarrow \vec{n_1} = (\lambda, -2\lambda, -4\sqrt{2}\mu) \\ &\Rightarrow \vec{n_2} = (\mu, 2\mu, 0) \end{aligned}$$

vectorul director al dreptei : $\vec{n} = \vec{n_1} \times \vec{n_2}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \lambda & -2\lambda & -4\sqrt{2}\mu \\ \mu & 2\mu & 0 \end{vmatrix} = \vec{i} \begin{vmatrix} -2\lambda & -4\sqrt{2}\mu \\ 2\mu & 0 \end{vmatrix} - \vec{j} \begin{vmatrix} \lambda & -4\sqrt{2}\mu \\ \mu & 0 \end{vmatrix}$$

$$+ \vec{k} \begin{vmatrix} \lambda & -2\lambda \\ \mu & 2\mu \end{vmatrix} = 8\sqrt{2}\mu^2 \vec{i} - 4\sqrt{2}\mu^2 \vec{j} + 4\lambda\mu \vec{k}$$

$$= (8\sqrt{2}\mu^2, -4\sqrt{2}\mu^2, 4\lambda\mu)$$

$D_{\lambda\mu} \parallel \pi : 3x + 2y - 4z = 0 \qquad \vec{n_\pi} = (3, 2, -4)$

$\Rightarrow \vec{n} \perp \vec{n_\pi} \Rightarrow 24\sqrt{2}\mu^2 - 8\sqrt{2}\mu^2 - 16\lambda\mu = 0$

$16\sqrt{2}\mu^2 - 16\lambda\mu = 0 \Rightarrow \sqrt{2}\mu^2 - \lambda\mu = 0$

Alegem $\lambda = 1$

$\sqrt{2}\mu^2 - \mu = 0$

$\mu(\mu\sqrt{2} - 1) = 0$

μ - nu poate să fie 0 deoarece în a 2-a ec a

generatoarei am obține $0 = 1$

$$\sqrt{2}\mu - 1 = 0 \Rightarrow \sqrt{2}\mu = 1 \Rightarrow \mu = \frac{1}{\sqrt{2}}$$

$$D_{\lambda,\mu} : \begin{cases} \frac{x}{2\sqrt{2}} - \frac{y}{\sqrt{2}} = 2 \cdot \frac{1}{\sqrt{2}} z \\ \frac{1}{\sqrt{2}}\left(\frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}}\right) = 1 \end{cases} \Rightarrow D_{\lambda,\mu} \begin{cases} x - 2y - 4z = 0 \\ x + 2y - 4 = 0 \end{cases}$$

$$D_{\alpha,\beta}: \begin{cases} \alpha\left(\frac{x}{\sqrt{8}} + \frac{y}{\sqrt{2}}\right) = 2\beta z \\ \beta\left(\frac{x}{\sqrt{8}} - \frac{y}{\sqrt{2}}\right) = \alpha \end{cases}$$

$$D_{\alpha,\beta}: \begin{cases} \alpha\left(\frac{x}{2\sqrt{2}} + \frac{y}{\sqrt{2}}\right) = 2\beta z \quad {}^{2\sqrt{2}} \\ \beta\left(\frac{x}{2\sqrt{2}} - \frac{y}{\sqrt{2}}\right) = \alpha \quad {}^{2\sqrt{2}} \end{cases}$$

$$D_{\alpha,\beta}: \begin{cases} \alpha x + 2\alpha y - 4\sqrt{2}\beta z = 0 & \Rightarrow \vec{n_1} = (\alpha, 2\alpha, -4\sqrt{2}\beta) \\ \beta x - 2\beta y - 2\sqrt{2}\alpha = 0 & \Rightarrow \vec{n_2} = (\beta, -2\beta, 0) \end{cases}$$

vectorul director al dreptei.

$$\vec{n} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \alpha & 2\alpha & -4\sqrt{2}\beta \\ \beta & -2\beta & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 2\alpha & -4\sqrt{2}\beta \\ -2\beta & 0 \end{vmatrix} -$$

$$-\vec{j}\begin{vmatrix} \alpha & -4\sqrt{2}\beta \\ \beta & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} \alpha & 2\alpha \\ \beta & -2\beta \end{vmatrix}$$

$$= -8\sqrt{2}\beta^2\vec{i} - 4\sqrt{2}\beta^2\vec{j} - 4\alpha\beta\vec{k} = (-8\sqrt{2}\beta^2, -4\sqrt{2}\beta^2, -4\alpha\beta)$$

$$D_{\alpha,\beta} \parallel \pi \Rightarrow \vec{n} \perp \vec{n_\pi} \Rightarrow -24\sqrt{2}\beta^2 - 8\sqrt{2}\beta^2 + 16\alpha\beta = 0$$

$$-32\sqrt{2}\beta^2 + 16\alpha\beta = 0 \Rightarrow -2\sqrt{2}\beta^2 + \alpha\beta = 0$$

Alegem $\alpha = 1$

$$-2\sqrt{2}\,\beta^2 + \beta = 0$$

$$\beta(-2\sqrt{2}\,\beta + 1) = 0$$

β - nu poate fi 0 deoarece în a 2-a ec. a generatoarei am obține $0 = 1$

$$-2\sqrt{2}\,\beta + 1 = 0 \Rightarrow \beta = \frac{1}{2\sqrt{2}}$$

$$D_{\alpha,\beta} : \begin{cases} \dfrac{x}{2\sqrt{2}} + \dfrac{y}{\sqrt{2}} = 2 \cdot \dfrac{1}{2\sqrt{2}}^2 \\[2ex] \dfrac{1}{2\sqrt{2}}\left(\dfrac{x}{2\sqrt{2}} - \dfrac{y}{\sqrt{2}}\right) = 1 \end{cases}$$

$$D_{\alpha,\beta} : \begin{cases} x + 2y - 2z = 0 \\ x - y\,2 - 8 = 0 \end{cases}$$