

Seminar 1 - Exercițiul 1. Sa se rezolve ecuatiile diferentiale:

4.  $xy + (2x-1)y' = 0$

Solutie singulara:  $y(x) = 0$

Solutie generala in forma implicita:  $\ln|y| = -\frac{1}{2}x - \frac{1}{4}\ln|2x-1| + c$

Solutie in forma explicita: dacă înlocuim  $c$  cu  $\ln c$  obținem soluția în formă explicită

$$y(x) = c \cdot e^{-\frac{1}{2}x} \cdot |2x-1|^{-\frac{1}{4}}, c \in \mathbb{R}.$$

5.  $y' = k \cdot \frac{y}{x}, k \in \mathbb{R}^*$

Solutie singulara:  $y(x) = 0$

Solutie in forma explicita:  $y(x) = cx^k, c \in \mathbb{R}$

6.  $y - xy' = a(1+x^2y')$ ,  $a \in \mathbb{R}^*$

Solutie singulara:  $y(x) = a$

Solutie in forma explicita:  $y(x) = a + \frac{cx}{1+ax}, c \in \mathbb{R}$

Exercițiul 2. Sa se rezolve:

4.  $xy' = \sqrt{x^2 - y^2} + y$

Solutii singulare:  $x = \pm 1$

Solutia generala:  $y = x \sin(\ln|x| + c), c \in \mathbb{R}$

5.  $y' = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$

Solutia generala:  $y(x) = x \arcsin(cx), c \in \mathbb{R}$

6.  $x - y \cos\left(\frac{y}{x}\right) + x \cos\left(\frac{y}{x}\right)y' = 0$

Solutii singulare:  $y(x) = k\pi x, k \in \mathbb{Z}$

Solutia generala:  $y(x) = x \arcsin(\ln|x| + c)$

### Exercitiul 3. Sa se rezolve:

3.  $y' + 2x \cdot y = 2xe^{-x^2};$

4.  $xy' - y + x = 0;$

5.  $y' - y = \sin(x)$

6.  $y' + \frac{x}{1-x^2}y = x + \arcsin(x)$

### Solutii:

3.  $y(x) = (c + x^2)e^{-x^2};$

4.  $y(x) = cx - x \ln|x|;$

5.  $y(x) = xe^x - \frac{1}{2}(\cos(x) + \sin(x));$

6.  $y(x) = c\sqrt{1-x^2} - 1 + x^2 + \frac{1}{2}\sqrt{1-x^2} \cdot \arcsin^2(x).$

### Seminar 2 - Exercitiul 1. Sa se rezolve ecuatiile diferentiale:

3.  $y'' = \frac{1}{x}$

Solutia generala:  $y(x) = x \ln x - x + c_1x + c_2, \quad c_1, c_2 \in \mathbb{R}.$

4.  $y'' = \ln x$

Solutia generala:  $y(x) = \frac{1}{2}x^2 \cdot \ln x - \frac{3}{4}x^2 + c_1x + c_2, \quad c_1, c_2 \in \mathbb{R}.$

5.  $y'' = xe^x$

Solutia generala:  $y(x) = (x-2) \cdot e^x + c_1x + c_2, \quad c_1, c_2 \in \mathbb{R}.$

6.  $y'' = \frac{2x^2}{(1+x^2)^2}$

Solutia generala:  $y(x) = x \cdot \arctg x - \ln(1+x^2) + c_1x + c_2, \quad c_1, c_2 \in \mathbb{R}.$

### Exercitiul 2. Sa se rezolve:

3.  $y'' - 2y' = -x^2$

4.  $(1+x^2)y'' + (y')^2 + 1 = 0$

5.  $(1+x^2)y'' = 2xy'$

6.  $y'(1+(y')^2) = ay'', \quad a \in \mathbb{R}^*$

### Solutii:

$y(x) = \frac{1}{2}c_1e^{2x} + \frac{1}{6}x^3 + \frac{1}{4}x^2 + \frac{1}{4}x + \frac{1}{2} + c_2, \quad c_1, c_2 \in \mathbb{R}.$

Soluția generală:  $y(x) = -\frac{x}{c_1} + \left(1 + \frac{1}{c_1^2}\right) \ln|1 + c_1x| + c_2, \quad c_1 \in \mathbb{R}^*, \quad c_2 \in \mathbb{R},$  și  $y(x) = -\frac{x^2}{2} + c, \quad c \in \mathbb{R}, \quad (c_1 = 0).$

$y(x) = c_1 \left(\frac{x^3}{3} + x\right) + c_2, \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = \arcsin(c_1e^{\frac{x}{a}}) + c_2, \quad c_1, c_2 \in \mathbb{R}.$

### Exercitiul 3. Sa se determine solutiile generale corespunzatoare urmatoarelor ecuatii diferentiale:

### Solutii:

$y(x) = c_1e^{2x} + c_2e^{3x} + x^2, \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = c_1e^{-x} + c_2e^{-2x} + \frac{1}{6}e^x, \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = c_1e^{-x} \cos x + c_2e^{-x} \sin x + xe^{-x} + x + 2, \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = c_1e^x + c_2e^{-2x} - \frac{1}{2} \cos(2x) - \frac{3}{2} \sin(2x), \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = c_1e^x + c_2e^{2x} - 3xe^x + \sin x + 3 \cos x, \quad c_1, c_2 \in \mathbb{R}.$

$y(x) = c_1 \cos x + c_2 \sin x + (2x+2)e^{-x} + x \sin x, \quad c_1, c_2 \in \mathbb{R}.$

3.  $y'' + 2y' + y = 0$

4.  $y'' + 3y' + 2y = e^x$

5.  $y'' + 2y' + 2y = xe^{-x} + 2x + 4$

6.  $y'' + y' - 2y = 10 \sin 2x$

7.  $y'' - 3y' + 2y = 3e^x + 10 \sin x$

8.  $y'' + y = 4xe^{-x} + 2 \cos x$