Seminar 12

1. Pentru functia $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 + 3xy^2 - 15x - 12y$ si punctul $a = (-2,-1)$, precizati
   a) $\nabla f(a)$, $H(f)(a)$ si $d^2 f(a)$
   b) natura punctului a.

2. Determinati punctele critice si punctele de extrem local (specificand tipul acestora) pentru urmatoarele functii
   a) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x,y,z) = 2x^2 - xy + 2xz - y + y^3 + z^2$
   b) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = x^4 + y^4 - 2x^2$

3. Determinati punctele de extrem conditionat (specificand tipul acestora) si valorile extreme ale functiei f relativ la multimea S indicata (stiind ca aceasta este compacta)

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = (1-x)(1-y), \quad S = \{(x,y) \in \mathbb{R}^2 |\, x^2 + y^2 = 1\}$$

4. Determinati valorile extreme ale urmatoarelor functii relativ la multimea S indicata
   a) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x,y,z) = x + 2y + 3z, \quad S = \{(x,y,z) \in \mathbb{R}^3 |\, x^2 + y^2 + z^2 \leq 1\}$
   b) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = x^2 - 2xy + 2y, \quad S = [0,2] \times [0,4]$

Exercitii suplimentare

1. Determinati constanta $a \in \mathbb{R}$ pentru care functia $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = 4x^2 + 4xy + ay^2$ are puncte de extrem local si determinati aceste puncte.

2. Justificati ca $a = (0, 0)$ este punct critic, dar nu este punct de extrem local al functiei $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x, y) = (x^2 - y)(x^2 - 3y)$.

3. Determinati punctele critice si punctele de extrem local (specificand tipul acestora) pentru urmatoarele functii
   a) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x^3 + y^3 - 3xy$
   b) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x, y, z) = x^3 - x + y^2 + z^2$
   c) $f : (0, \infty) \times \mathbb{R} \to \mathbb{R}, \quad f(x, y) = x(y^2 + \ln^2 x)$
   d) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x, y, z) = z^2(1 + xy) + xy$
   e) $f : (0, \infty)^2 \to \mathbb{R}, \quad f(x, y) = xy + \frac{8}{x} + \frac{8}{y}$
   f) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = (1 + \mathrm{e}^x) \cos y - x\mathrm{e}^x$

4. Determinati punctele de extrem conditionat (specificand tipul acestora) si valorile extreme ale urmatoarelor functii relativ la multimea S indicata (stiind ca aceasta este compacta)
   a) $f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = x + y, \quad S = \{(x, y) \in \mathbb{R}^2 |\, x^2 + xy + y^2 = 1\}$
   b) $f : \mathbb{R}^3 \to \mathbb{R}, \quad f(x, y, z) = xyz, \quad S = \{(x, y, z) \in \mathbb{R}^3 |\, x+y+z = 0,\ x^2+y^2+z^2 = 1\}$

5. Determinati valorile extreme ale functiei f relativ la multimea S indicata

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x, y) = xy, \quad S = \{(x, y) \in \mathbb{R}^2 |\, x^2 + 2y^2 \leq 2\}$$

6. Determinati distanta minima in plan de la punctul $a = (0, -1)$ la hiperbola de ecuatie $xy = \sqrt{2}$, $x > 0$, $y > 0$.