1.3.55 (doar $|\mathbb{N}| = |\mathbb{Z}|$) aleph_0

$f : \mathbb{N} \to \mathbb{Z}$

$$f(x) = \begin{cases} \frac{x}{2} & , x \in \{2k \mid k \in \mathbb{N}\}, \forall k \\ \frac{-x-1}{2} & , x \in \{2k+1 \mid k \in \mathbb{N}\}, \forall k \end{cases}$$

- trebuie demonstrată bijectivitatea funcției $f$

$f$ bijectivă $\Rightarrow |\mathbb{N}| = |\mathbb{Z}|$

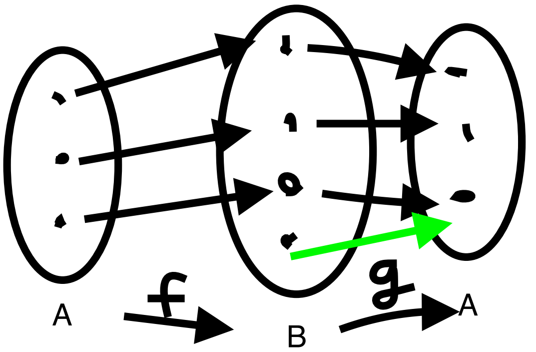
1.3.42

- ne uităm la propoziția 1.3.16 (pentru (1))

$A = \emptyset$, $B$ mulțime oarecare

p -> q <=> ~p v q

$\exists!\ f : A \to B$ funcție injectivă




Obs. (i) $|B^A| = |B|^{|A|} = |B|^{|\emptyset|} = 1$

(ii) Dacă $B \neq \emptyset$, atunci $\nexists\ B \xrightarrow{g} \emptyset$ (deci $f$ nu are inv. la stânga)

(iii) Dacă $B = \emptyset$, atunci $\theta = 1_\emptyset : \emptyset \to \emptyset$ bijectivă

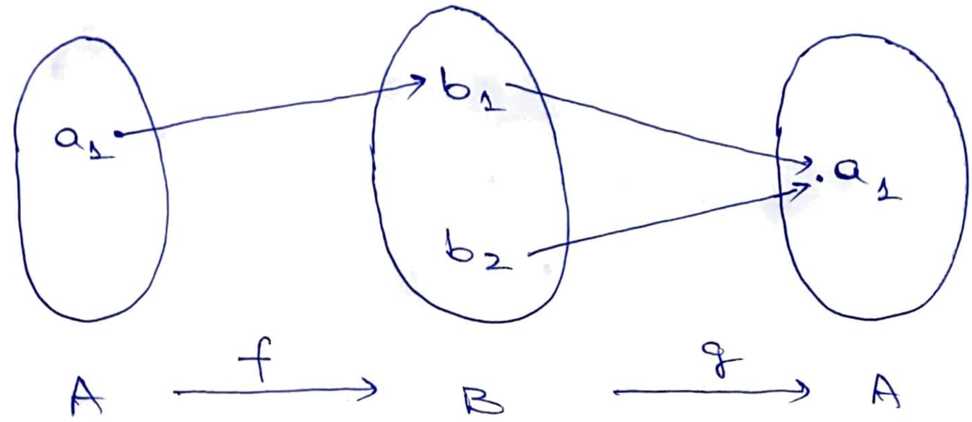
(2) câte inverse la stânga putem găsi?

$f : A \to B$

$g : B \to A$

~~$|\mathrm{Im} f|$~~ ~~$|A|$~~

$|A|^{|B \setminus \mathrm{Im} f|}$

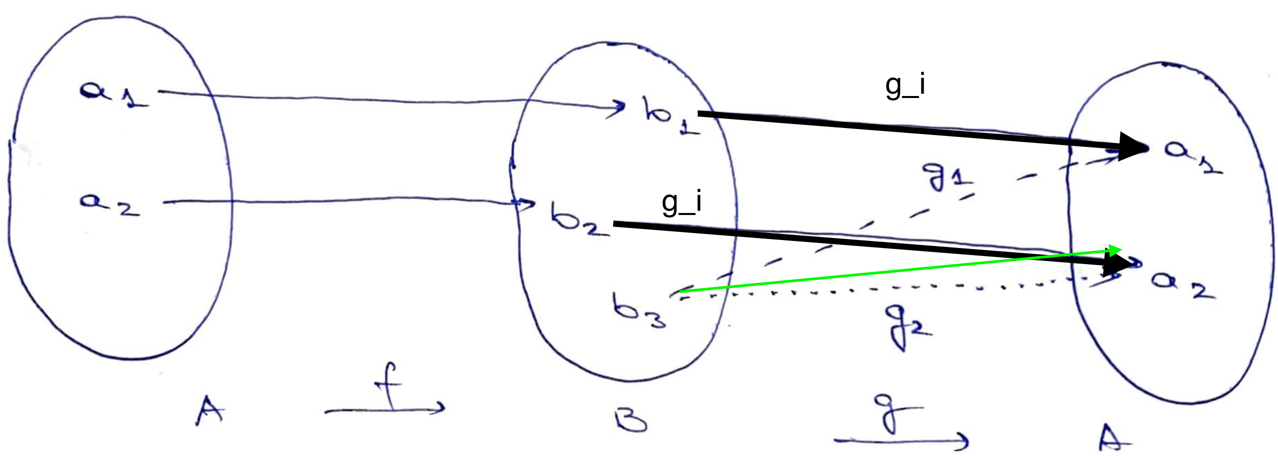



$\exists!\ g : B \to A$

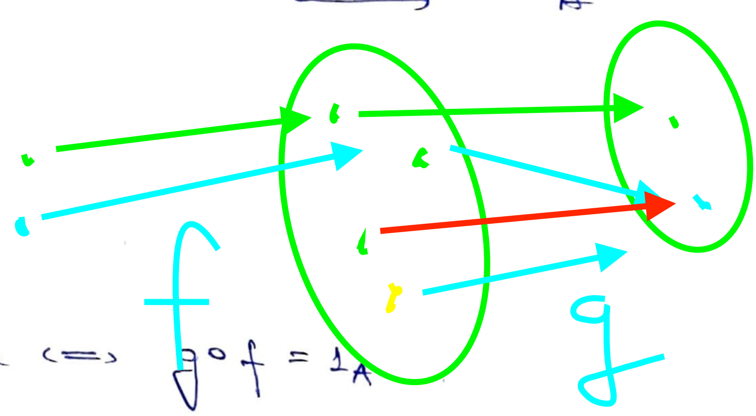
1_A = functia identica pe multimea A: 1_A(x) = x

$g(x) = a_1,\ \forall x \in B$ a.î. $g \circ f = 1_A$

(3)




| $x$ | $a_1$ | $a_2$ |
|---|---|---|
| $f(x)$ | $b_1$ | $b_2$ |



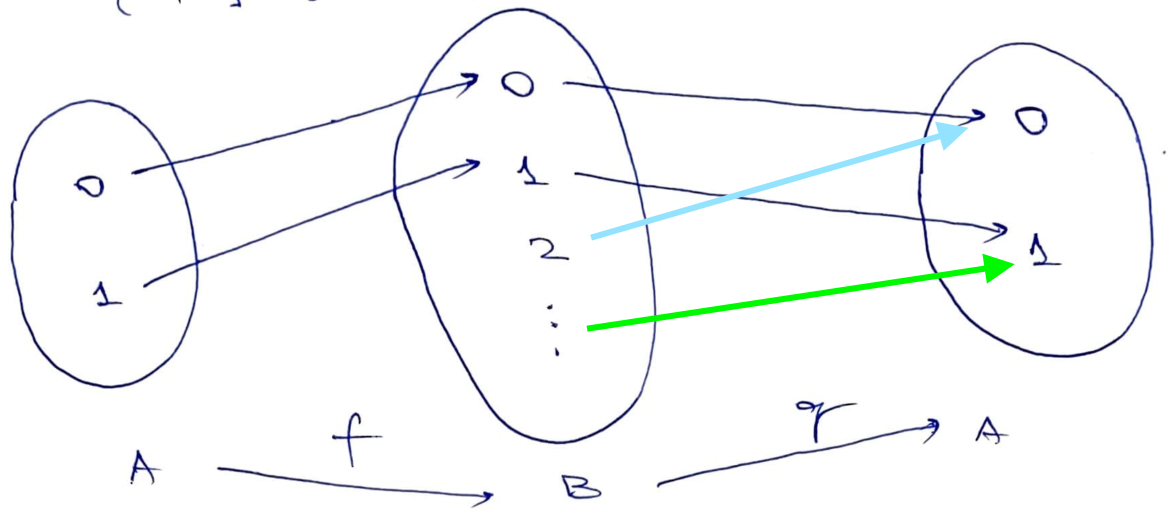

$g : B \to A$ inv. la stânga $\Longleftrightarrow g \circ f = 1_A$

$$\left.\begin{array}{l} \Rightarrow g(b_1) = a_1 \\ \quad g(b_2) = a_2 \\ \text{și } g(b_3) \in \{a_1, a_2\} \ - \ 2 \text{ posib.} \end{array}\right\} \Longrightarrow$$

$\Rightarrow$ f are exact două inverse la stânga

(4) $A = \{0,1\}$, $B = \mathbb{N}$

N = {0, 1, 2, …}



Există $\infty$ fcţii. $g: \mathbb{N} \to \{0,1\}$

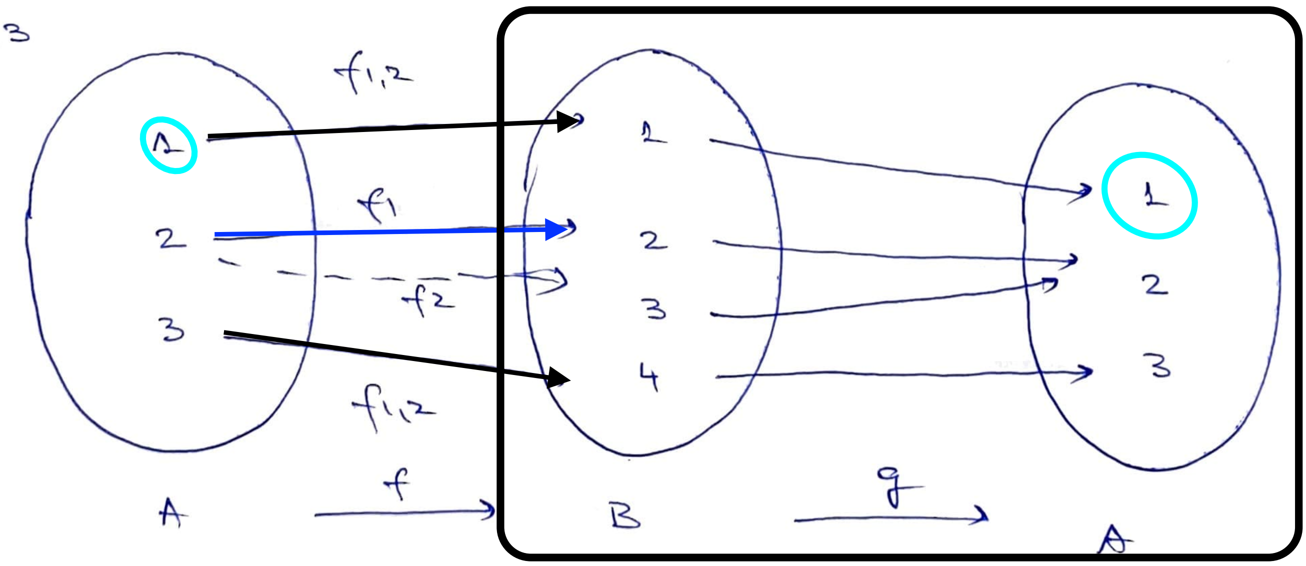
astfel încât $g(0) = 0$, $g(1) = 1$

şi $g(n) \in \{0,1\}$, când $n \geq 3$.

$2^{\infty}$ astfel de funcţii

1.3.43

(1)



$f_i(1) = 1$, $i=1,2$

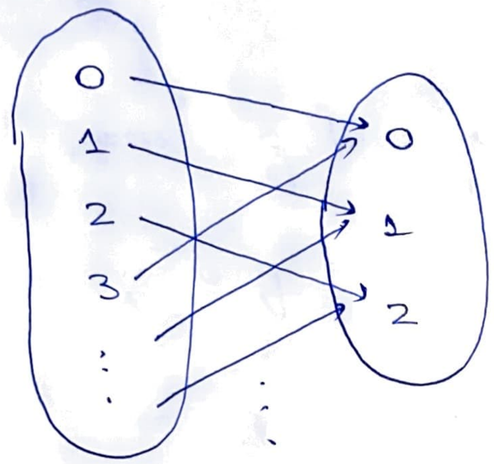
$f_1(2) = 2$, $f_2(2) = 3$

$f_i(3) = 4$, $i=1,2$

2 3 2

(2) $B = \mathbb{N}$, $A = \{0,1,2\}$

$g : \mathbb{N} \to A$



$B \longrightarrow A$

Căutăm $f : \{0,1,2\} \to \mathbb{N}$ a.î.

$g \circ f = 1_A \iff$

$\forall x \in A,\ g(f(x)) = x \iff$

$\forall x \in A,\ \underline{f(x) \equiv x \pmod{3}}$

$f(0) = 3k_0,\ k_0 \in \mathbb{N}$

$f(1) = 3k_1 + 1,\ k_1 \in \mathbb{N}$

$f(2) = 3k_2 + 2,\ k_2 \in \mathbb{N}$

$g: B \to A$ bij. $\overset{?}{\Longleftrightarrow}$ $\exists!\, f: A \to B$ a.î. $g \circ f = 1_A$

dem.

"$\Rightarrow$" $g$ bij. $\Rightarrow \exists\, g^{-1}: A \to B$ a.î.

$g^{-1} \circ g = 1_B$ și $g \circ g^{-1} = 1_A$

$\Rightarrow g^{-1}$ este o inv. la dreapta pt. $g$

Presupunem $f: A \to B$ altă inv. la dr.,

$g \circ f = 1_A$ | , atunci

compunem la stanga cu g^(-1)

$(g^{-1} \circ g) \circ f = g^{-1} \circ 1_A \Leftrightarrow g^{-1} = f$

"$\Leftarrow$" $\exists!\, f: A \to B$ a.î. $g \circ f = 1_A \Rightarrow g$ surj.

trebuie arătat că $g$ este injectivă

Temă: MRA

1.3.57

$|A| = n$

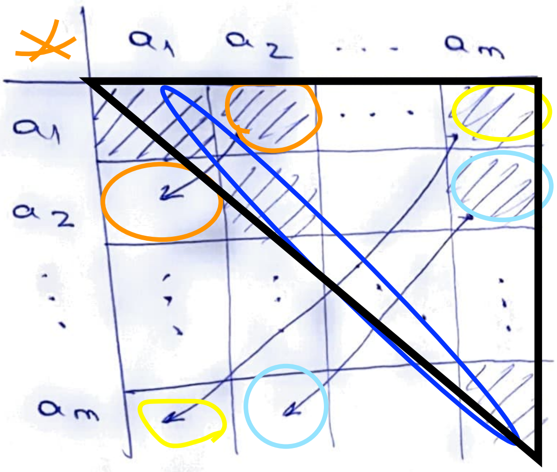
ex:
+ : A x A -> A
+(a, b) = a + b;

$* : A \times A \longrightarrow A$ (operație)

(1) $|A^{A\times A}| = n^{(n^2)}$ $\left(\neq (n^n)^2 = n^{2n}\right)$

IA^(A x A)I = IAI^(IA x AI) = n ^ (n^2)

(2) $A = \{a_1, \dots, a_n\}$




$\boxed{a * b = b * a}$

IB ^ AI = IBI ^ IAI =
n ^ [n + (n - 1) … + 2 + 1] = n ^ [n(n + 1)/2]

$n^{(1+2+\dots+n)} = n^{\left(\frac{n(n+1)}{2}\right)}$, operații comutative

(3) Temă

e * x = x * e = x