Homework 5
Due Wednesday, October 19th

1. Read sections 8.1, 8.2, do problems 1(a,b), 2(a,b), 3, 4, 8, 11 on p. 277–279 from Dummit and Foote, 3rd edition.

2. Let F be a field and let $R = F[x_1, x_2, x_3, x_4]$. Show that $R/(x_1x_2 - x_3x_4)$ is not a UFD. Here $(x_1x_2 - x_3x_4)$ is an ideal in R generated by $x_1x_2 - x_3x_4$.

3. Let R be a PID. For $x, y \in R$ with $gcd(x, y) = 1$ show that the map $R/(xy) \to R/(x) \times R/(y)$ induced by the map $R \to R \times R$, $r \to (r, r)$ is a ring isomorphism.

4. Show that in a PID an ideal is prime if and only if it is maximal.

5. (a) Read sections 9.1–9.5 from Dummit and Foote, 3rd edition.

(b) Let F be a field and let $R = F[x_1, x_2, \ldots, x_n]$. Define $N : R \to \mathbb{Z}_{\geq 0}$ via $N(p(x_1, x_2, \ldots, x_n))$ is the degree of $p(x_1, x_2, \ldots, x_n) \in R$. Show that R together with N is not a Euclidean domain.