Homework 3
Due Wednesday, September 28th

1. Let $R = \mathbb{Z}[x]$ and $M = (2, x)$, the ideal in R generated by 2 and x. Show that $\{2, x\}$ is not a (free) basis for M. Show that the maximal number of R-linearly independent elements in M is 1, but M is not free of rank 1.

2. Show that if R is an integral domain and M is any nonprincipal ideal of R, then M is torsion free and the maximal number of R-linearly independent elements in M is 1, but M is not a free R-module.

3. Read section 8.2 (P.I.D.s), do problem 4 (p. 282) from Dummit and Foote, 3rd edition.

4. Read sections 10.1–10.3 (Module theory), do problem 8 on p. 344 and problems 4, 5, 12, 15, 24 on p. 356–358 from Dummit and Foote, 3rd edition.