Homework 11
Due Wednesday, December 7th

1. (Dimmit and Foote, 2nd edition, p. 634, Exc. 4.) Let p be a prime and let $\{s, t\}$ be algebraically independent over $\mathbb{F}_p$. Let β be a root of $x^2 - sx + t = 0$, and let α be a root of $x^p - \beta = 0$. Denote $K = \mathbb{F}_p(s, t, \alpha)$. Show that $\{\alpha, s\}$ is a *separating transcendence basis* of K over $\mathbb{F}_p$, i.e., $\{\alpha, s\}$ is a transcendence basis of K over $\mathbb{F}_p$ and K is separable over $\mathbb{F}_p(\alpha, s)$.

2. (Dimmit and Foote, 2nd edition, p. 634, Exc. 5.) Let t be transcendental over $\mathbb{F}_p$ and let K be obtained by adjoining to $\mathbb{F}_p(t)$ all p-power roots of t. Prove that $\mathit{tr\,deg}_{\,\mathbb{F}_p} K = 1$ and K has no separating transcendence basis over $\mathbb{F}_p$.

3. (Dimmit and Foote, 2nd edition, p. 634, Exc. 6.) Let t be transcendental over $\mathbb{Q}$. Show that $\mathbb{Q}(t, \sqrt{t^3 - t})$ is not purely transcendental over $\mathbb{Q}$.
Definition: A field extension F of a field K is *purely transcendental* over K if there exists a transcendental basis S of F over K such that $F = K(S)$.