Coverage Control for Multi-Robot Teams With Heterogeneous Sensing Capabilities

María Santos[1], Yancy Diaz-Mercado[2] and Magnus Egerstedt[1]

Abstract—**This paper investigates how mobile agents with qualitatively different sensing capabilities should be organized in order to effectively cover an area. In particular, by encoding the different capabilities as different density functions in the locational cost, the result is a heterogeneous coverage control problem where the different density functions serve as a way of both abstracting and encapsulating different sensing capabilities. However, different density functions imply that mass is not conserved as the agents move and, as a result, the normal cancellations that occur across boundaries between regions of dominance in the homogeneous case no longer take place when computing the gradient of the locational cost. As a result, new terms are needed if the robots are to execute a descent flow in order to minimize the locational cost, and we show how these additional terms can be formulated as boundary-disagreement terms that are added to the standard Lloyd's algorithm. The results are implemented on real robotic platforms for a number of different use cases.**

Index Terms—**Multi-robot systems, networked robots, distributed robot systems.**

I. Introduction

COVERAGE control concerns itself with the problem of distributing a collection of mobile sensor nodes across a domain in such a way that relevant environmental features and events are detected by at least one sensor node (with sufficiently high probability), e.g., [1], [2]. Different ways of encoding this have been proposed, including the construction of networks with particularly effective topologies, e.g., triangulations [3], [4], deployment according to spatial point processes with desired probability characteristics [5], and the partition of the domain into useful regions of dominance, where each node is in charge of covering its own region [6].

In particular, if a team of N planar robots with positions $p_i \in \mathcal{D} \subset \mathbb{R}^2,\ i = 1, \ldots, N$, are to cover a convex domain $\mathcal{D}$, one natural choice is to let Robot i be in charge of the points in $\mathcal{D}$ that are closest to p_i, i.e., to let Robot i's region of dominance be given by the Voronoi cell

$$\mathcal{V}_i(p) = \{q \in \mathcal{D} \mid \|q - p_i\| \leq \|q - p_j\|,\ \forall j \in \mathcal{N}\},$$

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[1]M. Santos and M. Egerstedt are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA {maria.santos,magnus}@gatech.edu

[2]Y. Diaz-Mercado is with The Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland, USA yancy.diaz@gatech.edu

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where p is the combined positions of all the robots $[p_1^T, \ldots, p_N^T]^T$ and $\mathcal{N}$ is the index set $\{1, \ldots, N\}$.

Given a Voronoi partition of the domain into regions of dominance, one can now ask how well the team is actually covering the area. This question is typically asked relative to an underlying density function $\phi : \mathcal{D} \to [0, \infty)$, which captures the relative importance of points in the domain, with $\phi(q) > \phi(\hat{q})$ meaning that q is more important, has a higher probability of being a place where an event will occur, or contains more relevant features than point $\hat{q}$, as discussed in [6]. If we furthermore assume that the quality of the measurements that Robot i makes is higher for points that are closer to Robot i, the quality of the coverage obtained in region $\mathcal{V}_i(p)$ can be encoded through the cost

$$\int_{\mathcal{V}_i(p)} \|q - p_i\|^2 \phi(q)\, \mathrm{d}q,$$

with a better coverage corresponding to a lower cost. Summing over all agents thus yields the so-called *locational cost*

$$\mathcal{H}_{hom}(p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i(p)} \|q - p_i\|^2 \phi(q)\, \mathrm{d}q, \qquad (1)$$

as described in [2] as a way of capturing the coverage performance, and where the subscript hom refers to the fact that all the robots have the same sensing capabilities, i.e., the team is homogeneous.

The question pursued in this paper is how to introduce heterogeneity into this formulation in a way that reflects the capabilities of the different robots in a natural manner when the sensing modalities are qualitatively different. A number of approaches to the heterogeneous coverage problem have been proposed, focusing on sensor ranges [7], [8], robot footprints [9], and motion performance [10] as the differentiating features among the robots, encoded through weights in the power diagram [11]. Heterogeneity has also been considered in anisotropic sensor networks, where the domain partitions accommodate the specific geometry of the sensor footprint [12], [13]. However, in these cases the sensors still measure the same types of features and, as a result, the density function $\phi(q)$ is common to all the agents.

In this paper, we explicitly try to maintain some of the structural advantages afforded by the formulation of the coverage problem through a locational cost, while capturing qualitatively different sensing capabilities distributed across the robots. To this end, let $\mathcal{S}$ be a set of sensory modalities, with each robot being equipped with a subset of these sensors, denoted by $s(i) \subset \mathcal{S}$. Moreover, for each sensor $j \in \mathcal{S}$, there is a corresponding density of events or features in $\mathcal{D}$

that this particular sensor can detect. For example, a camera can detect color variations associated with wilting crops on a farm field, while chemical gas sensor arrays can be used to measure soil conditions [14], [15]. As a result, we no longer have a single density function, but rather a class of functions $\phi_j : \mathcal{D} \to [0, \infty),\ j \in \mathcal{S}$, with the density associated with point q, as it pertains to Robot i, being given by

$$\phi_{s(i)}(q) = \bigoplus_{j\in s(i)} \phi_j(q), \tag{2}$$

where $\oplus$ is an appropriately chosen composition operator. The choice of composition operator reflects how the densities from the different sensors on the robot should be combined in order to compute the overall density function. For example, one simple way to combine the density functions is a direct summation,

$$\bigoplus_{j\in s(i)} \phi_j(q) = \sum_{j\in s(i)} \phi_j(q),$$

where the relative importance of a point is reflected by the sum of its importance among different sensors. Another possible composition is to pick the maximum density value among the sensors on Robot i,

$$\bigoplus_{j\in s(i)} \phi_j(q) = \max_{j\in s(i)} \phi_j(q).$$

This choice would ensure that the density associated with a point corresponds to the highest relative importance measured by its sensors.

This paper investigates what the implications are when introducing qualitatively different sensing capabilities for the purpose of coverage control. The outline of the paper is as follows: In Section II, we recall how the standard, homogeneous locational cost formulation lends itself to a very elegant descent algorithm for coverage control, known as Lloyd's algorithm, and formally introduce the heterogeneous locational cost. The gradient to this new cost is derived in Section III together with a gradient-based, distributed controller that minimizes the cost. Section IV presents a series of experiments on real robotic platforms that allows us to make observations about the optimality of the proposed controllers. Lastly, Section V provides conclusions.

II. Locational Costs

Recalling the locational cost for homogeneous coverage in (1), one relevant question is how the robots should move in order to minimize this cost. An approach to this could be to let the individual robots move against the gradient of the cost, i.e., to let

$$\dot{p}_i = -\gamma_i(p)\frac{\partial \mathcal{H}_{hom}(p)}{\partial p_i}^T,\ i \in \mathcal{N},$$

for some positive, possibly state-dependent, gain $\gamma_i(p)$, with the result that

$$\begin{aligned}\frac{d\mathcal{H}_{hom}(p)}{dt} &= -\frac{\partial \mathcal{H}_{hom}(p)}{\partial p}\Gamma(p)\frac{\partial \mathcal{H}_{hom}(p)}{\partial p}^T \\ &= -\left\| \frac{\partial \mathcal{H}_{hom}(p)}{\partial p}^T \right\|^2_{\Gamma(p)},\end{aligned}$$

where $\Gamma(p) = \mathrm{diag}(\gamma_1(p), \ldots, \gamma_N(p))$ is a positive definite diagonal matrix with the individual gains on the diagonal.

This descent formulation has two highly desirable properties, as discussed in [2]. On the one hand, it directly turns $\mathcal{H}_{hom}$ into a Lyapunov function, amenable to the application of LaSalle's invariance principle as a way of showing convergence to a stationary point. On the other hand, the distributed nature of the team is encoded through a Delaunay adjacency relationship [2] – Robots i and j only have to exchange information if they share a boundary in the Voronoi tessellation (as long as $\Gamma(p)$ does not introduce additional dependencies).

Now, in order to compute the gradient to $\mathcal{H}_{hom}(p)$, Leibniz integral rule must be applied, which contains terms involving the derivative of the integrands as well as the domains over which the integrals are defined. However, even though a small change in p_i results in a corresponding change to the boundary of the Voronoi cell $\mathcal{V}_i(p)$, the net contribution from this change to the locational cost is offset by the corresponding changes to the locational cost from the boundaries of the adjacent Voronoi cells, given that the density function, $\phi(q)$, is common to all the agents and the total mass is conserved across $\mathcal{D}$. As a result, the domain terms in Leibniz rule cancel among neighbors and only the integrand terms must be considered when computing the gradient [1], [16], given by

$$\frac{\partial \mathcal{H}_{hom}(p)}{\partial p_i} = 2\int_{\mathcal{V}_i(p)} (p_i - q)^T \phi(q)\, \mathrm{d}q.$$

It is possible to express this gradient in a more compact form by defining the *mass* and *center of mass* associated with Robot i's Voronoi cell as

$$m_i(p) = \int_{\mathcal{V}_i(p)} \phi(q)\, \mathrm{d}q, \qquad c_i(p) = \frac{\int_{\mathcal{V}_i(p)} q\phi(q)\, \mathrm{d}q}{m_i(p)},$$

which yields the gradient

$$\frac{\partial \mathcal{H}_{hom}(p)}{\partial p_i} = 2m_i(p)\left(p_i - c_i(p)\right)^T. \tag{3}$$

Moreover, by letting the gain be

$$\gamma_i(p) = \frac{\kappa}{2m_i(p)},$$

the scaled descent algorithm becomes the well-known Lloyd's algorithm [17],

$$\dot{p}_i = -\kappa(p_i - c_i(p)), \tag{4}$$

where $\kappa > 0$ is a proportional control gain. In fact, using LaSalle's invariance principle, Lloyd's algorithm has been shown to asymptotically achieve a centroidal Voronoi tessellation (CVT), i.e., a configuration where, asymptotically, $p_i = c_i(p)$, which in turn is a necessary condition for optimal coverage, as shown in [1].

As discussed in Section I, the objective behind this work is to introduce heterogeneity in the sensing capabilities through heterogeneous density functions constructed as in (2),

$$\phi_{s(i)}(q) = \bigoplus_{j\in s(i)} \phi_j(q),$$

where $s(i) \in \mathcal{S}$ is the set of sensing modalities associated with Robot i, $\mathcal{S}$ is the set of all such sensing modalities, and ϕ_j is the density associated with (and detectable by) sensor $j \in \mathcal{S}$. A direct usage of this formulation in the locational cost gives

$$\mathcal{H}_{\mathcal{C}}(p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i(p)} \|q - p_i\|^2 \phi_{s(i)}(q)\,\mathrm{d}q. \tag{5}$$

Note that, under this formulation, the original Voronoi partition is employed, giving each individual robot the sole responsibility for its region of dominance. The reason for this is twofold, namely (a) a desire to recover as much as possible from the homogeneous coverage control case in terms of the structure of the derivations, and (b) the fact that *coordination* emerges explicitly from the regions of dominance – hence the subscript $\mathcal{C}$.

However, in the heterogeneous case, it is no longer true that whichever area Robot i does not cover outside of $\mathcal{V}_i(p)$ is automatically covered by the adjacent robots. Since the robots may be equipped with different sensor suites, it may be necessary to let coverage responsibilities encroach on other robots' cells, i.e., we no longer have a strict partition of the domain into regions of dominance. In the extreme case, if Robot i is the only robot equipped with a particular sensor, and that sensor is needed to cover the whole domain (as well as possible), it is necessary to define an additional cost over the whole domain. As such, in order to let the agents embrace their *domain objectives*, denoted by the subscript $\mathcal{O}$, a different locational cost is needed,

$$\mathcal{H}_{\mathcal{O}}(p) = \sum_{i \in \mathcal{N}} \int_{\mathcal{D}} \|q - p_i\|^2 \phi_{s(i)}(q)\,\mathrm{d}q, \tag{6}$$

where each integral measures how well Robot i is covering the entire domain $\mathcal{D}$ with respect to its particular sensor configuration.

Armed with these two different locational costs, we let the *heterogeneous locational cost* be given by a convex combination of the costs in (5) and (6),

$$\begin{aligned}\mathcal{H}_{het}(p) &= \sigma \mathcal{H}_{\mathcal{C}}(p) + (1-\sigma)\mathcal{H}_{\mathcal{O}}(p) \\ &= \sigma \sum_{i \in \mathcal{N}} \int_{\mathcal{V}_i(p)} \|q - p_i\|^2 \phi_{s(i)}(q)\,\mathrm{d}q \\ &\quad + (1-\sigma) \sum_{i \in \mathcal{N}} \int_{\mathcal{D}} \|q - p_i\|^2 \phi_{s(i)}(q)\,\mathrm{d}q, \end{aligned} \tag{7}$$

where $\sigma \in (0, 1]$ acts as a regularizer between the two competing objectives. We do not let $\sigma = 0$ since, with this choice, no coordination among agents is present. The effect of selecting different values of σ is further discussed in subsequent sections.

These changes in the locational cost, as compared to the homogeneous case, have significant implications for how the gradient should be computed. In the following sections, we untangle these implications and present a controller that achieves convergence to the critical points of the heterogeneous locational cost in (7), which constitutes a necessary condition for optimal, heterogeneous coverage.

III. Heterogeneous Gradient Descent

If we were to obtain the gradient to the heterogeneous locational cost in (7), a descent direction that achieves a local minimizer could be computed for the robots. To this end, we compute the gradient to $\mathcal{H}_{het}$ by considering the two locational costs $\mathcal{H}_{\mathcal{C}}$ and $\mathcal{H}_{\mathcal{O}}$ separately, starting with the former of the two.

Let $\mathcal{N}_i$ encode the Delaunay neighborhood of Robot i, i.e., the set of agents whose Voronoi cells share a face with agent i's Voronoi cell, as was done in [18]. We can now break $\partial \mathcal{H}_{\mathcal{C}} / \partial p_i$ down into three terms, namely Robot i's contribution, the contributions from robots in $\mathcal{N}_i$, and the contributions from the remaining robots,

$$\begin{aligned}\frac{\partial \mathcal{H}_{\mathcal{C}}}{\partial p_i}(p) &= \frac{\partial}{\partial p_i}\left(\int_{\mathcal{V}_i(p)} \|q - p_i\|^2 \phi_{s(i)}(q)\mathrm{d}q\right) \\ &\quad + \frac{\partial}{\partial p_i}\left(\sum_{j \in \mathcal{N}_i} \int_{\mathcal{V}_j(p)} \|q - p_j\|^2 \phi_{s(j)}(q)\mathrm{d}q\right) \\ &\quad + \frac{\partial}{\partial p_i}\left(\sum_{j \notin \mathcal{N}_i \cup \{i\}} \int_{\mathcal{V}_j(p)} \|q - p_j\|^2 \phi_{s(j)}(q)\mathrm{d}q\right). \end{aligned} \tag{8}$$

We immediately note that the last term in the above expression does not depend on p_i, and as such, will be zero. For the remaining terms, we need to recall Leibniz integral rule:

Lemma 1 (Leibniz Integral Rule [16])**.** *Let $\Omega(p)$ be a region that depends smoothly on p such that the unit outward normal vector $n(p)$ is uniquely defined almost everywhere on the boundary $\partial\Omega(p)$. Let*

$$F = \int_{\Omega(p)} f(q)\,\mathrm{d}q.$$

Then,

$$\frac{\partial F}{\partial p} = \int_{\partial\Omega(p)} f(q) n(q)^T \frac{\partial q}{\partial p}\,\mathrm{d}q,$$

where $\int_{\partial\Omega(p)}$ denotes the line integral over the boundary of $\Omega(p)$.

This expression needs to be connected to the coordination locational cost in (5). Assuming that $\mathcal{V}_i$ and $\mathcal{V}_j$ share a boundary, this boundary will be orthogonal to the line connecting the Voronoi cell generators, as is observed in [19]. In other words, for any point q on this boundary,

$$\left(q - \frac{p_i + p_j}{2}\right)^T (p_i - p_j) = 0.$$

Differentiating this with respect to p_i yields

$$(p_j - p_i)^T \frac{\partial q}{\partial p_i} = (q - p_i)^T. \tag{9}$$

As $(p_j - p_i)/\|p_j - p_i\|$ is the unit outward normal from $\mathcal{V}_i$ on this shared boundary, by dividing (9) by $\|p_j - p_i\|$ the term $n(q)^T \frac{\partial q}{\partial p}$ in the integrand of Lemma 1 is obtained.

Considering coverage control when mass conservation no longer holds is not new. For example, [8] considers coverage control with visibility constraints and, analogously to what was done in [8], we can calculate the gradient to $\mathcal{H}_\mathcal{C}$ by applying the Leibniz integral rule to (8),

$$\begin{aligned}\frac{\partial \mathcal{H}_\mathcal{C}}{\partial p_i} &= 2m_i\,(p_i - c_i)^T \\ &\quad + \sum_{j\in\mathcal{N}_i} \int_{\partial\mathcal{V}_{ij}} \|q-p_i\|^2 \frac{(q-p_i)^T}{\|p_j-p_i\|}\phi_{s(i)}(q)\,\mathrm{d}q \\ &\quad - \sum_{j\in\mathcal{N}_i} \int_{\partial\mathcal{V}_{ij}} \|q-p_j\|^2 \frac{(q-p_i)^T}{\|p_j-p_i\|}\phi_{s(j)}(q)\,\mathrm{d}q, \end{aligned} \tag{10}$$

where we, for notational convenience, have suppressed the explicit dependence of p on $\partial\mathcal{V}_{ij}$ – the boundary between Voronoi cells $\mathcal{V}_i$ and $\mathcal{V}_j$ – and where $\int_{\partial\mathcal{V}_{ij}}$ refers to the line integral evaluated along this boundary. Moreover, m_i and c_i are the *heterogeneous mass* and *center of mass* in Robot i's Voronoi cell, given by

$$m_i = \int_{\mathcal{V}_i} \phi_{s(i)}(q)\mathrm{d}q, \qquad c_i = \frac{\int_{\mathcal{V}_i} q\phi_{s(i)}(q)\mathrm{d}q}{m_i}. \tag{11}$$

From the definition of the Voronoi tessellation, all points on a boundary between cells are equidistant from the seeds for those cells, i.e., for all $q\in\partial\mathcal{V}_{ij}$ we have that $\|q-p_i\| = \|q-p_j\|$. Substituting $\|q-p_j\|$ by $\|q-p_i\|$ in (10) yields

$$\begin{aligned}\frac{\partial \mathcal{H}_\mathcal{C}}{\partial p_i} \;&=\; 2m_i\,(p_i-c_i)^T \\ &+\; \sum_{j\in\mathcal{N}_i}\left(\int_{\partial\mathcal{V}_{ij}} (q-p_i)^T\,\frac{\|q-p_i\|^2}{\|p_j-p_i\|}\phi_{s(i)}(q)\,\mathrm{d}q \right.\\ &\qquad \left. - \int_{\partial\mathcal{V}_{ij}} (q-p_i)^T\,\frac{\|q-p_i\|^2}{\|p_j-p_i\|}\phi_{s(j)}(q)\,\mathrm{d}q\right),\end{aligned}$$

where the integral terms simplify to

$$\sum_{j\in\mathcal{N}_i}\int_{\partial\mathcal{V}_{ij}} (q-p_i)^T\,\frac{\|q-p_i\|^2}{\|p_j-p_i\|}\left(\phi_{s(i)}(q)-\phi_{s(j)}(q)\right)\,\mathrm{d}q. \tag{12}$$

From this, we directly see that the gradient of the coordination term differs from the one obtained in the homogeneous case. Since the densities are no longer the same in adjacent cells, the net increase over $\mathcal{V}_i(p)$ caused by a small movement in p_i is not offset by the changes in adjacent Voronoi cells. Note though that if the density functions are identical for all robots, $\phi_{s(i)} = \phi_{s(j)}, i,j\in\mathcal{N}$, then the additional term cancels out and the homogeneous gradient (3) from Section II is immediately recovered.

In order to get the gradient expression in a more compact form, we introduce the total *mass* and *center of mass* (both interpreted in terms of line integrals) *on the boundaries* between Voronoi cells using the following notation,

$$\mu_{ij} = \int_{\partial\mathcal{V}_{ij}} \frac{\|q-p_i\|^2}{\|p_j-p_i\|}\phi_{s(i)}(q)\,\mathrm{d}q,$$

$$\rho_{ij} = \frac{\int_{\partial\mathcal{V}_{ij}} q\dfrac{\|q-p_i\|^2}{\|p_j-p_i\|}\phi_{s(i)}(q)\,\mathrm{d}q}{\mu_{ij}}.$$

Plugging these into (12) yields the derivative of (5) with respect to Robot i's position

$$\begin{aligned}\frac{\partial \mathcal{H}_\mathcal{C}}{\partial p_i}^T &= 2m_i(p_i-c_i) \\ &\quad + \sum_{j\in\mathcal{N}_i} \mu_{ij}\,(\rho_{ij}-p_i) - \mu_{ji}\,(\rho_{ji}-p_i)\,.\end{aligned} \tag{13}$$

The computation of $\partial\mathcal{H}_\mathcal{O}/\partial p_i$ is less involved as the area of integration is the entire domain $\mathcal{D}$, which does not depend on the position of the agents,

$$\begin{aligned}\frac{\partial \mathcal{H}_\mathcal{O}}{\partial p_i} &= \frac{\partial}{\partial p_i}\left(\int_\mathcal{D} \|q-p_i\|^2\phi_{s(i)}(q)\,\mathrm{d}q\right) \\ &= 2M_i(p_i-C_i)^T.\end{aligned} \tag{14}$$

Here M_i and C_i denote the *mass* and *center of mass of the domain* according to the density function of agent i, i.e.,

$$M_i = \int_\mathcal{D} \phi_{s(i)}(q)\,\mathrm{d}q, \qquad C_i = \frac{\int_\mathcal{D} q\phi_{s(i)}(q)\,\mathrm{d}q}{M_i}.$$

The gradient of the heterogeneous locational cost thus becomes

$$\begin{aligned}\frac{\partial \mathcal{H}_{het}}{\partial p_i}^T &= \sigma\frac{\partial \mathcal{H}_\mathcal{C}}{\partial p_i}^T + (1-\sigma)\frac{\partial \mathcal{H}_\mathcal{O}}{\partial p_i}^T \\ &= 2\sigma m_i(p_i-c_i) \\ &\quad + \sigma\sum_{j\in\mathcal{N}_i}\mu_{ij}\,(\rho_{ij}-p_i) - \mu_{ji}\,(\rho_{ji}-p_i) \\ &\quad + 2(1-\sigma)M_i(p_i-C_i).\end{aligned} \tag{15}$$

Letting Robot i follow a negative gradient flow establishes the following heterogeneous gradient descent theorem.

Theorem 1 (Heterogeneous Gradient Descent)**.** *Let Robot i, with planar position p_i, evolve according to the control law $\dot p_i = u_i$, where*

$$\begin{aligned}u_i &= -2\kappa\left(\sigma m_i(p_i-c_i) + (1-\sigma)M_i(p_i-C_i)\right) \\ &\quad - \sigma\kappa\sum_{j\in\mathcal{N}_i}\left(\mu_{ij}\,(\rho_{ij}-p_i) - \mu_{ji}\,(\rho_{ji}-p_i)\right).\end{aligned} \tag{16}$$

Then, as $t\to\infty$, the robots will converge to a critical point of the heterogeneous location cost in (7) *under positive gain $\kappa > 0$.*

Proof. From (15), we already know the form for the gradient. What remains to be shown is that convergence to a critical point is indeed achieved.

Consider the total derivative of the locational cost

$$\frac{d\mathcal{H}_{het}(p)}{dt} = \sum_{i\in\mathcal{N}}\frac{\partial\mathcal{H}_{het}}{\partial p_i}\dot p_i = -\kappa\left\|\frac{\partial\mathcal{H}_{het}}{\partial p}^T\right\|^2 \le 0. \tag{17}$$

For (17) to be zero, we need $\partial\mathcal{H}_{het}/\partial p = 0$, in which case the control law becomes $\dot p_i = 0$. By LaSalle's invariance principle, the multi-robot system converges to the largest invariant set contained in the set of all points such that $d\mathcal{H}_{het}(p)/dt = 0$, which are the critical points to the heterogeneous locational cost in (7). □

Note that, unlike the homogeneous case, CVTs are no longer the only critical points to the locational cost. Indeed,

TABLE I
SENSOR MODALITIES FOR THE DIFFERENT EXPERIMENTS

	Sensor modalities: $\mathcal{S}$	Robot sensors
Exp. 1	$\{1\}$	$s(i) = 1 \ \forall i \in \mathcal{N}$
Exp. 2	$\{1, \ldots, 6\}$	$s(i) = i \ \forall i \in \mathcal{N}$
Exp. 3	$\{1, \ldots, 6\}$	$s(i) = i \ \forall i \in \mathcal{N}$
Exp. 4	$\{1, \ldots, 4\}$	$s(1) = s(2) = 1$ $s(3) = 2, \ s(4) = 3$ $s(5) = s(6) = 4$

TABLE II
DENSITY PARAMETERS FOR THE DIFFERENT EXPERIMENTS

α_i		ν_i (cm)					
Agent	i	1	2	3	4	5	6
Exp. 1	$\beta_i = 1$			0			
				0			
Exp. 2	$\beta_i = i$			0			
				0			
Exp. 3	$\beta_i = 1$	-40	-20	0	0	20	40
		0	0	20	-20	0	0
Exp. 4	$\beta_i = 1$	-30	-30	0	0	30	30
		0	0	20	-20	0	0

as it will be observed in Section IV, in some situations, placing the agents in a CVT may yield higher costs than non-CVT configurations. Determining whether the achieved critical point is a local minimizer to the locational cost is difficult to establish – this remains an open issue even in the homogeneous case [16].

IV. Experimental Results

The proposed heterogeneous coverage algorithm is implemented on the Robotarium [20], a remotely accessible swarm robotics testbed at the Georgia Institute of Technology, whose arena serves as the region to be covered by the robot team. The team is composed of six GRITSBots [21], which are miniature, differential-drive robots. A webcam-based tracking system provides information about the position and orientation of the different robots in the team. This information is fed to the control algorithm, which produces velocity commands for the robots.

As the descent algorithm ultimately produces desired velocities $\dot{p}_i, \ i \in \mathcal{N}$, an implicit assumption behind this construction is that the robot dynamics can be expressed as (or at least can execute) single integrator dynamics. But the differential-drive configuration does not directly support single integrator dynamics and, as such, the control commands resulting from Theorem 1 must be converted into suitable, low level inputs for the GRITSBots. To this end, let $p_i = (x_i, y_i)^T$ be the position of Robot i, and θ_i its orientation. Then, the differential-drive configuration can be modeled using unicycle dynamics,

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i,$$

where v_i and ω_i are the translational and rotational velocities to be commanded to the robot, respectively. Using a model similar to the one in [2], we can approximately convert the single integrator dynamics into unicycle dynamics as follows,

$$v_i = k_v[\cos\theta_i \quad \sin\theta_i]\dot{p}_i,$$
$$\omega_i = k_\omega \arctan\left(\frac{[-\sin(\theta_i) \quad \cos(\theta_i)]\dot{p}_i}{[\cos(\theta_i) \quad \sin(\theta_i)]\dot{p}_i}\right),$$

with k_v and k_ω positive gains.

To evaluate the control law in Theorem 1, its performance is compared to a baseline controller. To this end, we compare it to a heterogeneous version of Lloyd's algorithm, whereby $\dot{p}_i = -\kappa(p_i - c_i(p))$, where c_i is evaluated using the heterogeneous densities as in (11). Given that the locational cost is an instantaneous measure, we moreover add a temporal component by evaluating the total cost of the controllers

$$\int_0^{t_f} \mathcal{H}_{het}(p(t))\mathrm{d}t$$

under identical initial conditions.

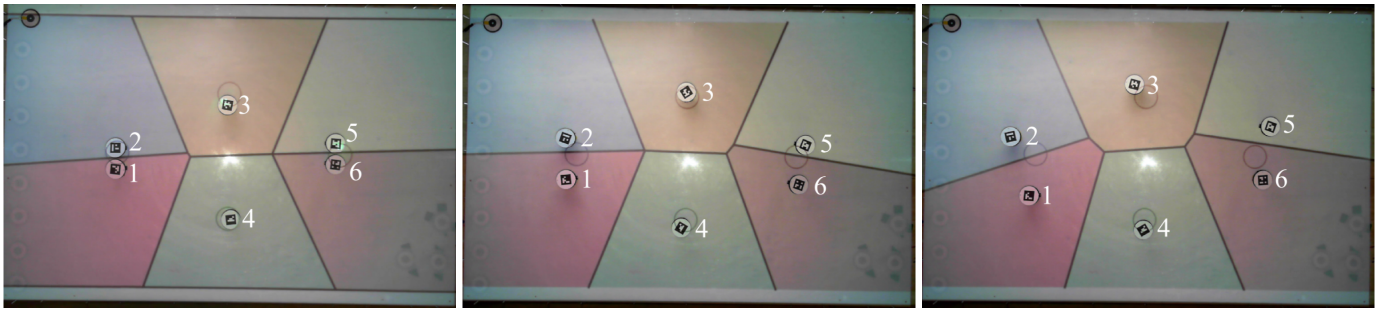
The experiment consists of four different configurations both in terms of the sensor suites assigned to the robots, $s(i)$, $i \in \mathcal{N}$, and the density functions associated with each sensor type, $\phi_j, \ j \in \mathcal{S}$. The sensory capabilities of each robot are simulated using the overhead camera, which provides each robot with the information that its sensors would measure according to the corresponding density functions. Table I shows the sensor modalities for each experiment. In the first experiment, all the robots have the same sensor, therefore being in an equivalent configuration to the homogeneous case. Experiments 2 and 3 reflect situations where each robot has a unique sensor configuration, while in Experiment 4 some robots share sensor configurations.

Gaussian radial basis functions have been used in robotic networks to model sensors whose noisy signals represent physical quantities, such as magnetic forces, heat, radio signal, or chemical concentrations [22]. Following along these lines, for each sensor $j \in \mathcal{S}$, the corresponding density function is modeled as a bivariate normal distribution,

$$\phi_j(q) = \frac{\beta_j}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2}(q - \nu_j)^T \Sigma^{-1} (q - \nu_j)\right),$$

where ν_j is the mean of the density and Σ is the covariance matrix, which is kept constant for all the sensors. β_j serves as a scale factor that models the strength of the density function. Table II indicates the density parameters used for each of the experiments, corresponding to the sensor modalities in Table I. Note that the values for ν_j are measured with respect to the center of the Robotarium arena, a 120×70 cm rectangle.

A value of $\sigma = 0.9$ is used in all four experiments. The value given to the regularizer σ is selected to favor the coordination component, $\mathcal{H}_C$, over the domain objectives. A comparison of the effect of different regularizer values on the behavior of the robot team for the sensor configuration of Experiment 4 is presented in Fig. 1, where we can observe that lower values of σ tend to excessively favor the domain objectives term, $\mathcal{H}_O$, concentrating the robots around



(a) $\sigma = 0.50$ (b) $\sigma = 0.70$ (c) $\sigma = 0.90$

Fig. 1. Effect of the regularizer term, σ, on the final configuration of the robot team for the sensor configuration of Experiment 4. As specified in Table II, Robots 1 and 2 share the same density function, as do Robots 5 and 6. We can observe how, as the value of the regularizer decreases, the coordination between agents vanishes, making the robots that share the same objectives crowd together.

their individual density functions and therefore reducing the coordinated nature of the coverage algorithm.

Table III presents a comparison of the total cost observed for the four sensor configurations, where both the heterogeneous version of Lloyd's algorithm and the descent law in Theorem 1 are executed for a total time of 2 minutes. Except for the first experiment, which corresponds to the homogeneous case, the total cost for the proposed algorithm is consistently smaller than the total cost attained by the heterogeneous Lloyd's algorithm, which confirms that the control law in Theorem 1 is better suited for teams with heterogeneous sensing capabilities. The differences in performance between the two algorithms are also depicted in Fig. 2, where the absence of the boundary terms makes the heterogeneous version of Lloyd's algorithm converge to a configuration with a higher final cost, showing that, for a heterogeneous cost, a CVT is not necessarily on its own a minimizer for the cost function.

TABLE III
COMPARISON OF THE OBSERVED TOTAL COST

	Heterogeneous Lloyd's	Gradient Descent
Exp. 1	0.14	0.14
Exp. 2	1.68	1.08
Exp. 3	0.70	0.61
Exp. 4	0.67	0.53

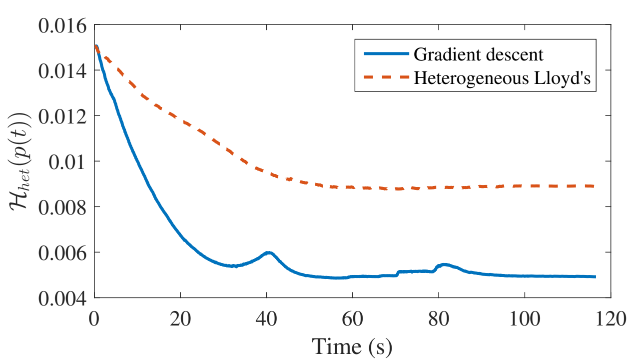


Fig. 2. Evolution of the cost $\mathcal{H}_{het}(p(t))$ with respect to time in Experiment 4. The difference between the cost for heterogeneous Lloyd's and the proposed gradient descent results from ignoring the boundary terms in (13) necessary to minimize the heterogeneous cost. Note that the increase in cost around $t = 40$ is due to the fact that the algorithm assumes single integrator dynamics while the actual robots are subject to nonholomic constraints.

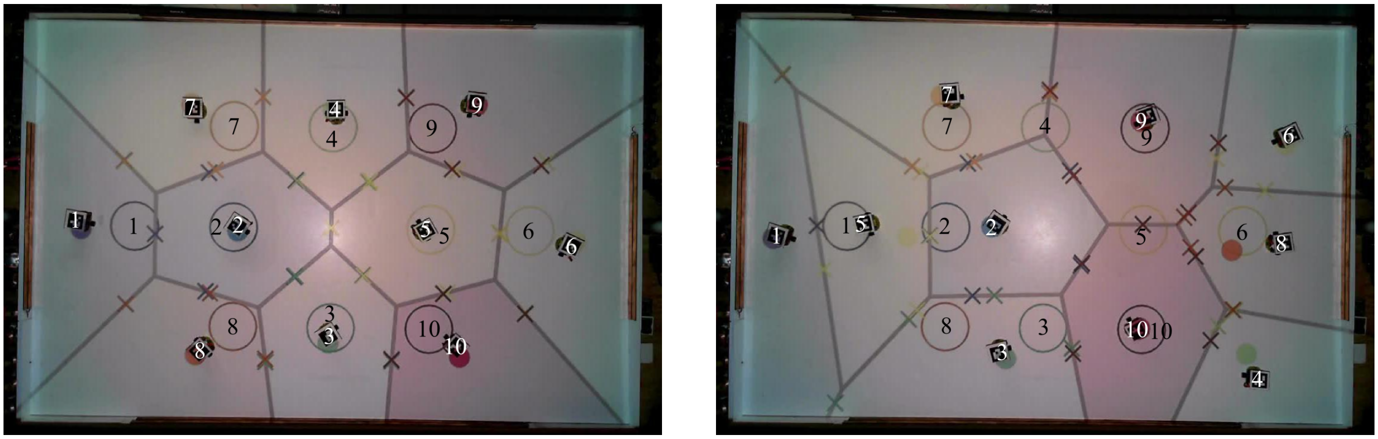
A group of ten robots is used to illustrate the team behavior when $\sigma = 1$ in (16), that is, when the control law is solely determined by the gradient of the coordination cost, $\mathcal{H}_{\mathcal{C}}$. In this case, the movement of a robot only depends on the values of its density function within its Voronoi cell and boundaries. Consequently, the team may be deterred from adequately covering an area associated with a particular density function, ϕ_j, if the robots equipped with the associated sensor, j, are located in areas with low values of the density ϕ_j, and are unable to move to higher density areas due to the position of their Delaunay neighbors, as shown in Fig. 3b.

In Section II, $\mathcal{H}_O$ was introduced as an additional locational cost to palliate the lack of coverage of areas outside each robot's region of dominance when the team is equipped with disjointed sets of sensors[1]. The results from the convex combination of both locational costs, $\mathcal{H}_O$ and $\mathcal{H}_C$, are shown in Fig. 3a. This situation illustrates how the proposed controller, thanks to the introduction of the domain objectives term, achieves a better spatial configuration of the agents in the domain while each robot still coordinates with the other members of the team.

V. CONCLUSIONS

In this paper, we presented a new locational cost function that encodes qualitatively different sensing capabilities through heterogeneous density functions. In order to cover the areas of interest, we adopted a distributed gradient descent approach which drives the agents in the team in a direction that decreases the cost. A series of experiments were performed on a team of differential-drive robots to assess the performance of the proposed gradient descent method as compared to a heterogeneous version of Lloyd's algorithm. The experiments suggest that the additional terms obtained due to the heterogeneous nature of the performance metric resulted in

[1]This paper has supplementary downloadable material available at http://ieeexplore.ieee.org, provided by the authors. This includes a multimedia AVI format movie clip, which shows the effect of the domain objectives term on the behavior of the multi-robot team. This material is 46.8 MB in size.



(a) (b)

Fig. 3. A group of ten GRITSBots executing the control law in Theorem 1 with $\sigma = 0.975$, (a), versus a pure coordination algorithm, with $\sigma = 1$, (b). An overhead projector is used to visualize relevant information in the robot arena. For Robot i, the filled circle represents the center of mass of its Voronoi cell, c_i, while the centers of mass on the boundary, $\rho_{ij},\ j \in \mathcal{N}_i$, are depicted using crosses at the boundaries of the cells. For this experiment, each robot has a unique sensor configuration with only one sensor. The location of the mean of the associated density function, $\phi_{s(i)} = \phi_i$, corresponds to the empty circle labeled with the robot's numerical identifier. Making $\sigma = 1$ implies the sole consideration of the coordination term in the control law, which may result in some robots staying in areas with low information density, as in (b). This situation is alleviated by making $\sigma < 1$ in the control law and therefore involving the term $\mathcal{H}_{\mathcal{O}}$, which allows the robot team to attain a better spatial configuration in the domain, (a).

overall better coverage than a heterogeneous version of Lloyd's algorithm for a number of different density configurations.

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