Practice 2. The Classes P and NP

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Summary

In this Computational Complexity practice, it is explained what the chromatic number and the clique number of a graph are. These concepts are later used in the exam scheduling. I have develope some functions to generate exam scheduling instances, translate them into DIMACS format and solve them using the backtracking algorithm. Finally, I am going to conclude telling that the performance of the algorithm is significantly affected by the exponential nature of the problem.

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1 Questions

1.1 What does the chromatic number mean in this context?

Given a graph, the chromatic number is the smallest number of colours needed to colour each vertex differently from its adjacent vertices. In this case, as the colours represents the different days to do an exam, the chromatic number represents the minimum number of days on which there will be exams, so that all students can take their exams without any coincidence.

1.2 What does the clique number mean in this context?

The clique number of a graph is the size of the largest complete subgraph. In this context, it means that we are going to need the clique number o more days to date all the exams, so that each student can take all their exams without any of them coinciding.

1.3 What is the relation between both numbers?

The relation between the chromatic number and the clique number is that the chromatic number is always greater than or equal to the clique number by definition, due to if we have a complete subgraph of the size of the clique number, each vertex of that subgraph must be coloured with a different colour.

1.4 What is their relation to the maximum vertex degree?

The chromatic number of a graph is at most the maximum degree plus one (Brooks' theorem). In the other hand, we have that the clique number is less than or equal to the minimum degree plus one.

2 Research Tasks

In the files generator.hpp, translator.hpp and solver.hpp I have developed the different functions to generate random examen scheduling instances, translate them to the DIMACS format and solve them with the backGCP algorithm. As the algorithm is exponential, I have limited the execution time to 20 minutes for each file and I have increased the number of total students to 5000 in order to reduce the files that will get the best solution in more than the limit time. Also, each subject should enrols a sample of [250 · P(k)] students, but I have reduced that 250 to 5 with the same target said before.

The generator function creates six new files, each one with a different number of subjects, with the format asked:

- Lines start with a subject followed by its students.
- Each subject is represented by its code.
- Each student is represented by its identifier.

The translator reads all the files generated before, and create six new files with the DIMACS format:

- One problem line: p edge nVertices nEdges
- One edge line per edge: e vertex1 vertex2

Finally, the solver reads all the files that have the DIMACs format, creating the corresponding graphs, and calling the bactracking function that returns the chromatic number and the colour of each vertex.

Computing an optimal exam schedule for each instance, I have gotten the following execution times:

n	5	10	15	20	25	30
t	0.000169	0.000069	0.000194	0.000511	20	20

Table 1: Execution times

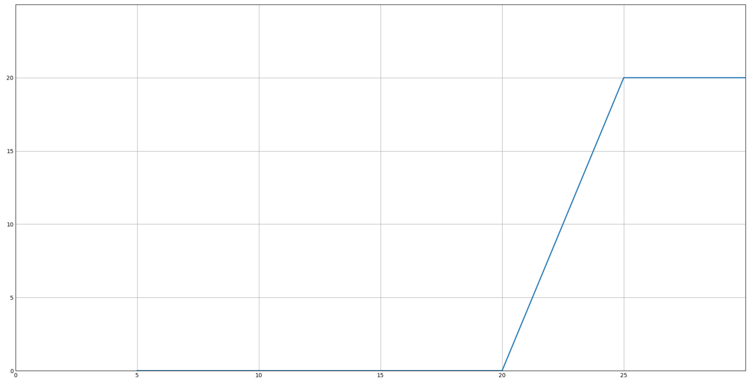

Figure 1: Execution times

As we can see, the algorithm is so exponential that by limiting the execution time to 20 minutes, it goes from taking less than a second to not being able to finish executing the algorithm.

Repeating the experiments with the same instances, but with the vertex selection upgraded, we are not going to see a lot of difference because the instances have almost no edges, so the times will be very similar. If the generator produced a complete graph, the times would be better with respect the first experiment, but we can't see the difference due to the limit time.

Bibliography

[1] Wolfram MathWorld. *Clique Number*. 2002.
URL: https://mathworld.wolfram.com/CliqueNumber.html.