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Signals, filters, and frequency domain

Abstract

The time domain and the frequency domain are the two ways we use to describe signals in the real world. The Fourier transform can be used to represent the relationship between these two domains. It is a formula that equates the energy in the time domain to that in the frequency domain and thus allows us to convert between the two. Oftentimes, an analog to digital converter (ADC) is used to convert analog signals that are measured continuously into a discrete digital signal by first sampling the signal at a particular frequency and quantifying it to determine the resolution of the signal, then setting binary values to the signal to be read as a digital value. Sometimes we need to use a filter (RC circuit) to get rid of any unwanted signals or noise. These filters only keep signals with frequencies within the filter's passband and will reduce the frequencies outside of the pass band. For the purpose of this lab, we used the Arduino Nano 33 Sense in order to generate a square wave signal at 5 different frequencies, as well as an impulse wave. We ran the signals through two separate low pass filters, measuring the output signal in the time domain and plotting it using MATLAB. The signal was then converted to the frequency domain with the use of MATLAB's Fast Fourier transform (FFT) and plotted again. Then, for each filter, gain was plotted as a function of frequency. The results from this lab demonstrate that the frequency response of a filtered signal can be characterized using 3 different methods: by measuring the attenuation of a square wave at different frequencies, by measuring the FFT of the impulse response, and by using the gain formula (see Appendix B).

Results

<u>Characterizing filters with square waves</u>

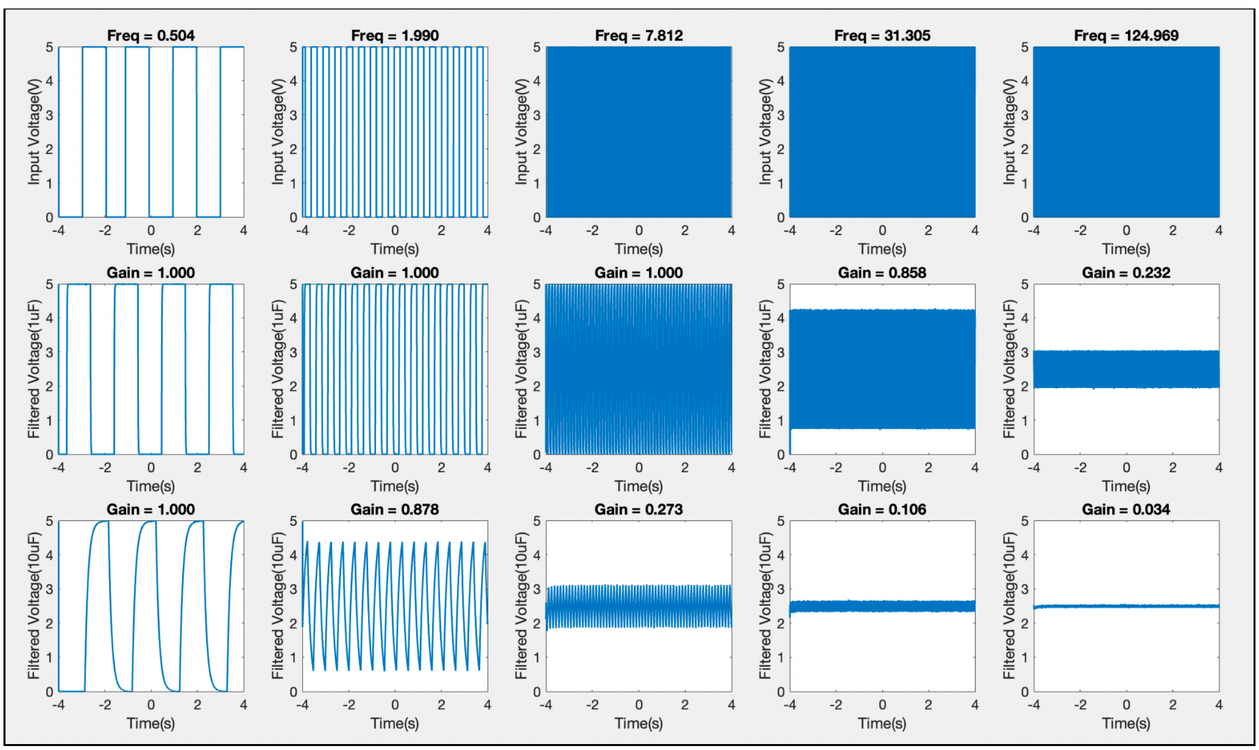

Figure 1: Graph of one unfiltered and two filtered square signals at five different frequencies in the time domain. First row is the unfiltered signal with frequencies as title of each plot; second row is the filtered signal (1μF) with gain as title of each plot; third row is the filtered signal(10μF) with gain as title of each plot.

The Arduino Nano Sense was used to generate square waves at 5 different frequencies which were then passed through two low-pass filters. Both filtered and unfiltered data was graphed (Figure 1). Both low-pass filters were created using an RC circuit with a 10 kOhm resistor. One of the filters was built using a 1μF capacitor and the other, 10μF. The cutoff frequency values for these filters are 15.92Hz and 1.59Hz respectively, which was calculated using the formula (see Appendix A). In the time domain, a low-pass filter will affect a signal by decreasing the peak to peak voltage of the signal if the waveform's frequency passes the cutoff frequency. We measured the peak to peak voltage of the filtered signal and divided the result by the unfiltered peak to peak voltage in order to obtain the gain of the filter. We then analyzed the frequency domain plots to find the point at which the gain begins to decrease. From Figure 1, it is evident that the peak to peak voltage of the filtered signal using a 1μF capacitor starts to decrease at a frequency of 31.305Hz and that the peak to peak voltage of the filtered signal using a 10μF capacitor starts to decrease at a frequency of 1.99Hz. This means that the cutoff frequency for the filters using 1μF and 10μF capacitors must lie in the range 7.812Hz-31.305Hz and 0.504Hz-1.990Hz respectively. As for the frequency domain, the signal will have large peaks that incrementally get lower at factor values of the fundamental frequency of the signal. Here, a low-pass filter will decrease the amplitude of the peaks that are passed the cutoff frequency which will in turn decrease the gain of the filter at said frequencies. According to Figure 2, the amplitudes of the signal before the cutoff frequencies for the filter signals are higher than those for the unfiltered signals, meaning that the signal is being reduced at those frequencies.

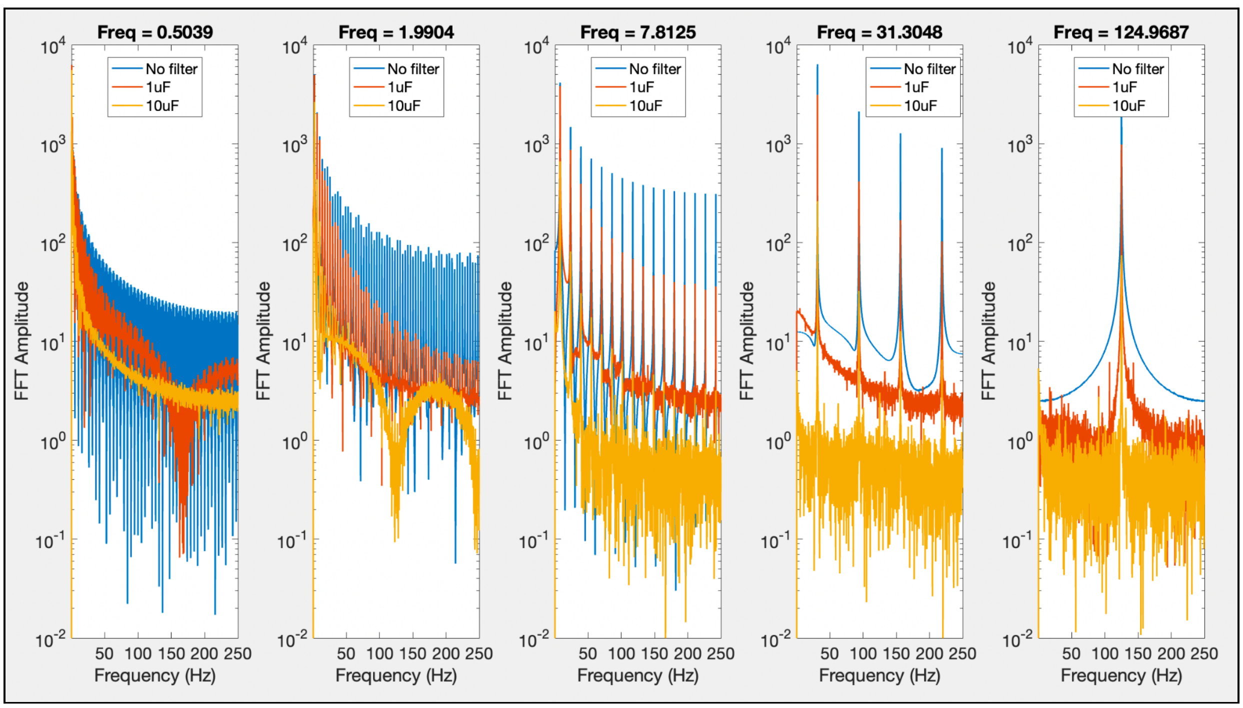

Figure 2: Filtered 1μF (orange), 10μF (yellow) and unfiltered (blue) square signals graphed in the frequency domain. Each subplot represents the Fourier transform of the three signals in Figure 1 at a particular frequency which is labeled as the title of the graph.

<u>Characterizing filters with impulses</u>

The Arduino Nano sense was used in this part of the lab in order to output impulse waves, which were graphed in both the time domain and the frequency domain. While in the time domain, the impulse wave is characterized by a sharp peak, in the frequency domain it is more

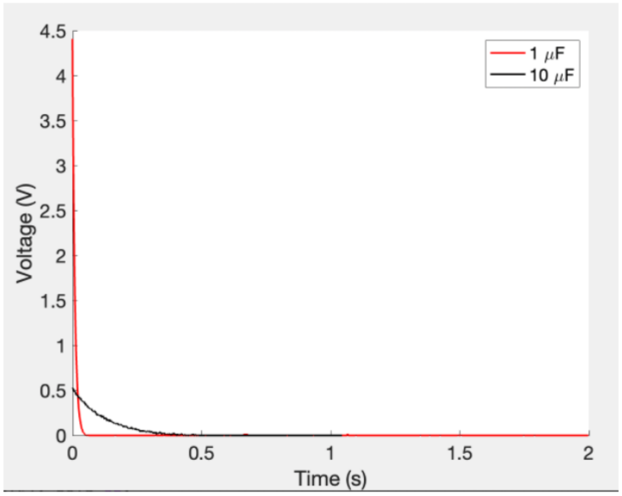

Figure 3: Time domain graph of the first filtered impulse wave Data cropped at 2s after the start of the impulse wave.

like a unit function, where all frequencies within a particular range are of equal magnitude. When an impulse signal goes through a low-pass filter, the magnitude of the impulse wave decreases based on the cutoff frequency in the time domain and the amplitudes of the signal past the cutoff frequency in the frequency domain also decrease. As demonstrated in the time domain response of a filtered signal in Figure 3, higher capacitor values mean lower cutoff frequency, which ultimately lowers the magnitude of the impulse signal. On the other hand, in the frequency domain, this will lead to more of the signal frequencies being reduced. The frequency domain of a signal can be used in order to characterize a filter and find the cutoff frequency by using impulse waves by analyzing where the gains of the frequencies decrease most rapidly.

<u>Comparing estimates of the filter frequency response</u>

There are three methods for characterizing filters in the frequency domain. In the first method, the gain of the low pass filter was measured from the attenuation of square waves of different frequencies and was plotted using large, filled dots connected by lines (Figure 4). For the second method, the gain was measured from the fast Fourier transform (FFT) of the impulse response and is plotted using solid lines (Figure 4). Finally in the last method, the values were calculated using the formula for $Vout/Vin$ (see Appendix B). The data calculated via the third method can be found on Figure 4 and is plotted using dotted lines. We can use the gain vs frequency plots to find the cutoff frequency of the low-pass filter. To do this, we need to locate the point at which the slope is most negative, since that is the point of significant attenuation. From Figure 4, we can see that the cutoff frequency estimated using the first method (attenuation of square wave) is an overestimate. The other two estimates, calculating the gain using the formula and using the FFT response of the impulse wave have near identical gain vs frequency curves and also a closer estimate to the expected value.

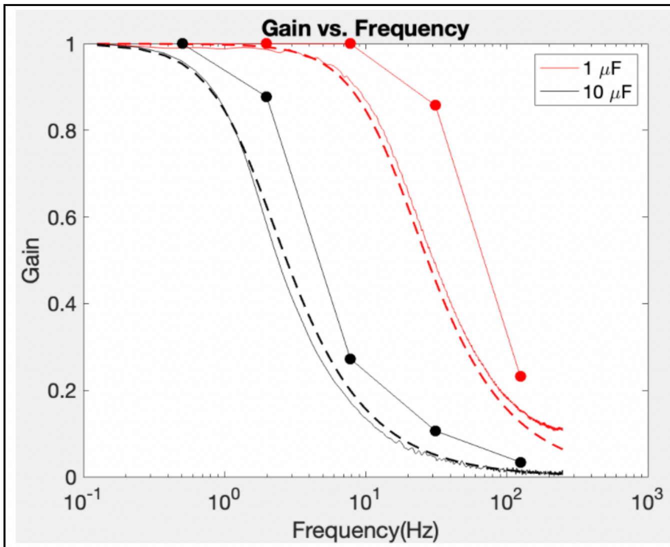

Figure 4: Calculated Gain vs frequency for signal going through both low-pass filters independently via 3 different methods: 1) By attenuation of square waves (solid lines with markers), 2) by the FFT of the impulse response (solid lines), 3) by calculating gain using the formula (dashed lines

Discussion

Each of the three methods used to characterize the low-pass filters has its own advantages and disadvantages. For the first method, when square waves were used, one of the advantages was that we were able to see how the filters changed the shape of the square wave as the frequency went past the fundamental frequency. A disadvantage of this method, however, was that it results in an overestimate of the cutoff frequency. As for the fast Fourier transform method, its advantage lies in that the impulse wave consists of many frequencies of the same magnitude, which allows us to view the effects of said filter over a large range of frequencies. On the other hand, this method requires intensive computational analysis, which is certainly a disadvantage. The opposite can be said for the last method - using the Vout/Vin formula, which requires no intensive analysis but something that could be seen as a disadvantage to this method is the disregard for the capacitor and resistor wear over time, which could eventually alter the RC circuit and ultimately affect the cutoff frequency.

Appendix

(A) $$\mathrm{f_{cutoff}} = \frac{1}{2\pi \mathrm{RC}}$$

(B) $$\frac{Vout}{Vin} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$