

Monads

Monads

- Data structures with `map` and `flatMap` seem to be quite common
- In fact, there's a name that describes this class of data structures together with some algebraic laws that they should have. They are called monads
- A monad `M` is a parametric type `M[T]` with two operations: ***flatMap*** and ***unit***, that have to satisfy some laws

```
trait M[T]{  
  def flatMap[U](f: T => M[U]): M[U]  
}  
def unit[T](x: T): M[T]
```

- In the literature, `flatMap` is more commonly called `bind`

Examples of Monads

- ***List*** is a monad with ***unit(x) = List(x)***
- ***Set*** is a monad with ***unit(x) = Set(x)***
- ***Option*** is a monad with ***unit(x) = Some(x)***
- ***Generator*** is a monad with ***unit(x) = single(x)***
- ***flatMap*** is an operation on each of these types, whereas ***unit*** in Scala is different for each monad

Monads and map

- `map` can be defined for every monad as a combination of `flatMap` and `unit`:

```
m map f == m flatMap (x => unit(f(x))  
                == m flatMap (f andThen unit)
```

- `map` is a Scala primitive function that is also defined on every monad

Monad Laws

- To qualify as a monad, a type has to satisfy three laws:
 1. **Associativity**: `m flatMap f flatMap g == m flatMap (x => f(x) flatMap g)`
 2. **Left unit**: `unit(x) flatMap f == f(x)`
 3. **Right unit**: `m flatMap unit == m`
- Let's check the monad laws for `Option`:

```
abstract class Option[+T]{  
  def flatMap[U](f: T => Option[U]): Option[U] = this match{  
    case Some(x) => f(x)  
    case None => None  
  }  
}
```

1. **Checking the Left Unit Law**: `Some(x) flatMap f == f(x)`

```
Some(x) flatMap f == Some(x) match{  
  case Some(x) => f(x)  
  case None => None  
}
```

2. **Checking the Right Unit Law:** *opt flatMap Some == opt*

$$\begin{aligned} & \text{opt flatMap Some} == \text{opt match}\{ \\ & \quad \text{case Some}(x) \Rightarrow \text{Some}(x) \\ & \quad \text{case None} \Rightarrow \text{None} \\ & \} \end{aligned} == \text{opt}$$
3. **Checking Associativity:** *opt flatMap f flatMap g == opt flatMap (x => f(x) flatMap g)*

$$\begin{aligned} & \text{opt flatMap f flatMap g} \\ & == \text{opt match}\{ \text{case Some}(x) \Rightarrow f(x) \text{ case None} \Rightarrow \text{None} \} \\ & \quad \text{opt match}\{ \text{case Some}(y) \Rightarrow g(y) \text{ case None} \Rightarrow \text{None} \} \\ & == \text{opt match}\{ \\ & \quad \text{case Some}(x) \Rightarrow \\ & \quad \quad f(x) \text{ match}\{ \text{case Some}(y) \Rightarrow g(y) \text{ case None} \Rightarrow \text{None} \} \\ & \quad \text{case None} \Rightarrow \\ & \quad \quad \text{None match}\{ \text{case Some}(y) \Rightarrow g(y) \text{ case None} \Rightarrow \text{None} \} \\ & \} \\ & == \text{opt match}\{ \\ & \quad \text{case Some}(x) \Rightarrow \\ & \quad \quad f(x) \text{ match}\{ \text{case Some}(y) \Rightarrow g(y) \text{ case None} \Rightarrow \text{None} \} \\ & \quad \text{case None} \Rightarrow \text{None} \\ & \} \\ & == \text{opt match}\{ \\ & \quad \text{case Some}(x) \Rightarrow f(x) \text{ flatMap g} \\ & \quad \text{case None} \Rightarrow \text{None} \\ & \} \\ & == \text{opt flatMap (x => f(x) flatMap g)} \end{aligned}$$

Significance of the Laws for For-Expressions

- We have seen that monad-type expressions are typically written as for-expressions
 - The monad laws give a justification for certain refactorings of for expressions that are quite intuitive
1. Associativity says essentially that one can “inline” nested for expressions:

$$\begin{aligned} & \text{for (y <- for (x <- m; y <- f(x)) yield y} \\ & \quad \text{z <- g(y)) yield z} \\ & == \text{for (x <- m; y <- f(x); z <- g(y)) yield z} \end{aligned}$$
 2. Right unit says:

$$\text{for (x <- m) yield x} == m$$
 3. Left unit does not have an analogue for for-expressions.

Try type

- Try resembles Option, but instead of Some/None there is a Success case with a value and a Failure case that contains an exception:

$$\begin{aligned} & \text{abstract class Try}[+T] \\ & \text{case class Success}[T](x: T) \text{ extends Try}[T] \\ & \text{case class Failure}(ex: Exception) \text{ extends Try}[Nothing] \end{aligned}$$
- Try is used to pass results of computations that can fail with an exception between threads and computers, bottling up the exception into a value that can be freely passed around

- Nothing is the bottom type; it doesn't have any value and it typically refers to something that is either a missing value in the Exception or a computation that does not really return normally because maybe the computation throws an exception or it loops infinitely

Creating a Try

- You can wrap up an arbitrary computation in a Try (anything that has an apply method can be used as a function):

Try(expr) // gives *Success(someValue)* or *Failure(someException)*

- Here's an implementation of Try:

```
object Try{
  def apply[T](expr: => T): Try[T] =
    try Success(expr)
    catch{
      case NonFatal(ex) => Failure(ex)
    }
}
```

- It uses the Java try, and if a computation here throws an exception, that will be caught in the catch block and then the exception will be wrapped in a failure value and returned as a result; if all goes well a Success type wrapping the computation result is returned
- That happens for any exception that is not fatal(it doesn't make sense to export this beyond a single thread)

What is important here is that the expression is parsed as a by name parameter, because otherwise we would already have a value here, so there would not be a computation that will throw an exception

Composing Try

- Just like with Option, Try-valued computations can be composed in for expressions:

```
for {
  x <- computeX
  y <- computeY
} yield f(x, y)
```

- If computeX and computeY succeed with results ***Success(x)*** and ***Success(y)***, this will return ***Success(f(x, y))***
- If either computation fails with an exception ex, this will return ***Failure(ex)***

Definition of flatMap and map on Try

```
abstract class Try[T]{
  def flatMap[U](f: T => Try[U]): Try[U] = this match {
    case Success(x) => try f(x) catch { case NonFatal(ex) => Failure(ex) }
    case fail: Failure => fail
  }
```

```

def map[U](f: T => U): Try[U] = this match {
  case Success(x) => Try(f(x))
  case fail: Failure => fail
}

```

- So, for a Try value *t*, ***t map f == t flatMap (x => Try(f(x)) == f flatMap (f andThen Try)***

Exercise: *It looks like Try might be a monad, with unit == Try. Is it?*

No, the left unit law fails.

Try(expr) flatMap f != f(expr)

Indeed, the left-hand side will never raise a non-fatal exception whereas the right-hand side will raise any exception thrown by expr or f. Hence, Try trades one monad law for another law, which is more useful in this context:

An expression composed from “Try”, “map” and “flatMap” will never throw a non-fatal exception.

Call this the “bullet-proof” principle.

Conclusion

- We have seen that for-expressions are useful not only for collections
- Many other types also define map, flatMap and withFilter operations and with them for-expressions(ex: Generator, Option, Try)
- Many of the types defining flatMap are monads
- If they also define withFilter, they are called “monads with zero”
- The three monad laws give useful guidance in the design of library APIs