

Lists

Lists

- The list is a fundamental data structure in functional programming
- A list having x_1, \dots, x_n as elements is written $\text{List}(x_1, \dots, x_n)$
- Examples:
 - * **`val fruit: List[String] = List("apples", "oranges", "pears")`**
 - * **`val nums: List[Int] = List(1, 2, 3, 4)`**
 - * **`val id3: List[List[Int]] = List(List(1, 0, 0), List(0, 1, 0), List(0, 0, 1))`**
 - * **`val empty: List[Nothing] = List()`**
- There are two important differences between lists and arrays:
 - * Lists are **immutable** – the elements of a list cannot be changed
 - * Lists are **recursive**, while arrays are flat
- Like arrays, lists are homogeneous: the elements of a list must all have the same type
- The type of a list with elements of type T is written **`scala.List[T]`** or shorter just **`List[T]`**

Constructors of Lists

All lists are constructed from:

- The empty list **`Nil`**, and
- The construction operation **`:: (cons)`**: $x :: xs$ gives a new list with the first element x followed by the elements of xs
- **Convention**: Operators ending in `:` associate to the right

Operations on Lists

- All operations on lists can be expressed in terms of the following three:
 - * **`head`** – the first element of the list
 - * **`tail`** – the list composed of all the elements except the first
 - * **`isEmpty`** – ‘true’ if the list is empty, ‘false’ otherwise
- These operations are defined as methods of objects of type `List`
- It is also possible to decompose lists with pattern matching:
 - * **`Nil`** – the `Nil` constant
 - * **`p :: ps`** – a pattern that matches a list with a head matching p and tail matching ps
 - * **`List(p1, ..., pn)`** – same as $p_1 :: \dots :: p_n :: \text{Nil}$

Exercise: Consider the pattern $x :: y :: \text{List}(xs, ys) :: zs$. What is the condition that describes most accurately the length L of the list it matches?

$L \geq 3$ (because zs is a tail and can have none or more elements)

Sorting Lists

- Suppose we want to sort a list of numbers in ascending order:
 - * One way to sort the list `List(7, 3, 9, 2)` is to sort the tail `List(3, 9, 2)` to obtain `List(2, 3, 9)`
 - * The next step is to insert the head 7 in the right place to obtain the result list `List(2, 3, 7, 9)`
- This idea describes Insertion Sort:
def isort(xs: List[Int]): List[Int] = xs match
case List() => List()
case y :: ys => insert(y, isort(ys))

```
def insert(x: Int, xs: List[Int]): List[Int] = xs match  
case List() => List(x)  
case y :: ys =>  
  if x < y then x :: xs  
  else y :: insert(x, ys)
```

Exercise: What is the worst-case complexity of insertion sort relative to the length of the input list N ? – **proportional to $N * N$**