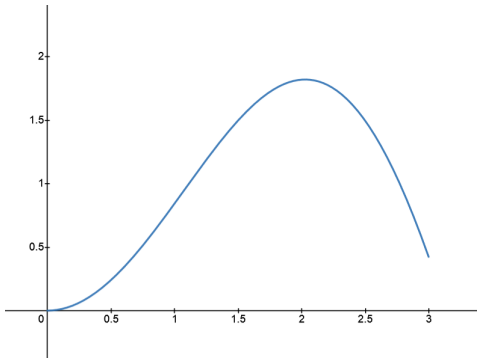
Merit Worksheet 18 Math 220

Problem 1. A moving particle (or car) has position function $f(t)$, where t is the time. The graph of $f(t)$ is given below. We know that its instant speed at time t is $f'(t)$.



(a) Draw on the graph the lines whose slopes give the **instant** speed at $t = 0.5$ and $t = 2.5$

(b) Draw on the graph the line whose slope gives the **average** speed from $t = 0$ to $t = 3$.

(c) Use what you drew in (a) and (b) to decide whether the instant speed at $t = 0.5$ is equal to the average speed. Do the same for $t = 2.5$.

(d) Look at your graph. Is there a time t such that the instant speed at t is equal to the average speed from $t = 0$ to $t = 3$? If so, draw the corresponding line.

(e) Using the terminology from this problem, fill in the blanks:

Mean Value Theorem:

If the motion of a particle is smooth enough,
then there exists a point in time when the _____ speed of the particle will be _____ to the _____ speed of the particle.

(f) Now let's translate the intuitive statement from (e) into math language: Find the precise statement of the MVT in your notes, and fill in the following "conversion table":

Intuition	**Math Language**
the motion of the particle	$f(t)$
the motion is smooth enough	
there exists a point in time	
the instant speed at the time on the previous row	
is equal to	
the average speed from $t = 0$ to $t = 3$	

(g) Read the precise statement of the MVT from your notes one more time to see if it makes more sense now.

(h) Draw an example that will convince me that it is necessary for the motion to be "smooth enough" for the conclusion of the MVT to hold.

Problem 2. (a) Use the MVT to prove that $|\sin(b) - \sin(a)| \leq |b - a|$ for all numbers a and b.

(b) In a similar way, prove that $|\sin(x)| \leq |x|$ for all real numbers x.

(c) Can you give a geometric description of the inequality in (b)?

Problem 3. (Challenge Problem) Two cars start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed.