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MT3501 Continuous Assessment

(1) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be def. as:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+y \\ y+2z \end{pmatrix}$$

and let $S: \mathbb{R}^3 \to \mathbb{R}^2$ be def. as:

$$S\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x2 \\ 2z \end{pmatrix}$$

1) Let $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \in \mathbb{R}^3$

for T: $$T\left(\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}\right) = T\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{pmatrix} =$$

$$= \begin{pmatrix} 2(x_1+x_2)+(y_1+y_2) \\ (y_1+y_2)+2(z_1+z_2) \end{pmatrix} = \begin{pmatrix} (2x_1+y_1)+(2x_2+y_2) \\ (y_1+2z_1)+(y_2+2z_2) \end{pmatrix} =$$

$$= \begin{pmatrix} 2x_1+2y_1 \\ y_1+2z_1 \end{pmatrix} + \begin{pmatrix} 2x_2+y_2 \\ y_2+2z_2 \end{pmatrix} = T\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

Let $\alpha \in \mathbb{R}$

$$T\left(\alpha\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\right) = T\begin{pmatrix} \alpha x_1 \\ \alpha y_1 \\ \alpha z_1 \end{pmatrix} = \begin{pmatrix} 2(\alpha x_1)+\alpha y_1 \\ \alpha y_1 + 2\alpha z_1 \end{pmatrix} =$$

$$= \begin{pmatrix} \alpha(2x_1+y_1) \\ \alpha(y_1+2z_1) \end{pmatrix} = \alpha\begin{pmatrix} 2x_1+y_1 \\ y_1+2z_1 \end{pmatrix} =$$

$$= \alpha T\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \Rightarrow T \text{ is a linear transform.}$$

2) Let $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ and $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \in \mathbb{R}^3$

then $S\left(\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}\right) = S\begin{pmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{pmatrix} =$

$= \begin{pmatrix} x_1^2 + 2x_1x_2 + x_2^2 \\ 2(z_1 + z_2) \end{pmatrix} \neq \begin{pmatrix} x_1^2 + x_2^2 \\ 2(z_1 + z_2) \end{pmatrix}$

$\Rightarrow$ S is not a linear transform.

(2) $B_3 = \left\{ \underset{\vec{e}_{31}}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}, \underset{\vec{e}_{32}}{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}, \underset{\vec{e}_{33}}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}} \right\}$ $B_2 = \left\{ \underset{\vec{e}_{21}}{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \underset{\vec{e}_{22}}{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \right\}$

find $Mat_{B_3, B_2}(T)$, $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y \\ y + 2z \end{pmatrix}$

$T(\vec{e}_{31}) = T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} = 2\vec{e}_{21}$

$T(\vec{e}_{32}) = T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \vec{e}_{21} + \vec{e}_{22}$

$T(\vec{e}_{33}) = T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} = 2\vec{e}_{22}$

$\Rightarrow Mat_{B_3, B_2}(T) = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

(3) $M_{2,3}(\mathbb{R}) = \left\{ \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}, \alpha_{11}, \alpha_{22}, \dots \alpha_{23} \in \mathbb{R} \right\}$

$\mathcal{B} = \left\{ \underset{b_{11}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}, \underset{b_{12}}{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}, \underset{b_{13}}{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}, \underset{b_{21}}{\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}, \underset{b_{22}}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}, \underset{b_{23}}{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \right\}$

$M_{B_3 B_2}(T) = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} = 2b_{11} + b_{12} + b_{22} + 2b_{23}$

(4) $T_{kl} : \mathbb{R}^3 \to \mathbb{R}^2$

$$T_{kl}(e_{3,j}) = \begin{cases} e_{2,l} & \text{if } j = k \\ \vec{0} & \text{if } j \neq k \end{cases}$$

where $e_{3,j} \in B_3$ and $e_{2,l} \in B_2$

$$A = \{ T_{kl} \mid 1 \le k \le 3,\ 1 \le l \le 2 \} =$$

$$= \{ T_{11}, T_{12}, T_{21}, T_{22}, T_{31}, T_{32} \}$$

$$\underset{e_{3,1}}{T_{11}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix} e_{2,1} \qquad \underset{e_{3,2}}{T_{21}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y \\ 0 \end{pmatrix} e_{2,1}$$

$$\underset{e_{3,1}}{T_{12}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ x \end{pmatrix} e_{2,2} \qquad \underset{e_{3,2}}{T_{22}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix} e_{2,2}$$

$$\underset{e_{3,3}}{T_{31}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ 0 \end{pmatrix} e_{2,1} \qquad T_{33} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ z \end{pmatrix} e_{22}$$

$$T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x + y \\ y + 2z \end{pmatrix} = 2T_{11} + T_{21} + T_{22} + 2T_{32}$$

(5) B_3' and B_2' for $\mathbb{R}^3$ and $\mathbb{R}^2$, s.t.

$$\mathrm{Mat}_{B_3' B_2'}(T) = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Let $B_2' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$

Then $T(e_{21}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = T\begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix}$

$T(e_{22}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = T\begin{pmatrix} 0 \\ 0 \\ 1/2 \end{pmatrix}$

$T(e_{23}) = \begin{pmatrix} -1 \\ 2 \end{pmatrix} = T\begin{pmatrix} 1 \\ -3 \\ 5/2 \end{pmatrix}$

$\begin{cases} 2x + y = -1 \\ y + 2z = 2 \end{cases}$ let $x = 1$, then $y = -3$, $z = 5/2$

$$\Rightarrow B_3' = \left\{ \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1/2 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 5/2 \end{pmatrix} \right\}$$

(6) let $\mathcal{C}_3 = \{ e_{31}, e_{32}, e_{33} \}$
$\mathcal{C}_2 = \{ e_{21}, e_{22} \}$

let $e_{31} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow T(e_{31}) = T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2x+y \\ y+2z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Rightarrow 2x = -y = 2z$

Let $y = 1 \Rightarrow x = z = -\frac{1}{2}$

$\Rightarrow e_{31} = \begin{pmatrix} -1/2 \\ 1 \\ -1/2 \end{pmatrix}$

$$T\begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 1\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \mathcal{C}_3 = \left\{ \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1/2 \\ 1 \\ -1/2 \end{pmatrix} \right\}$$

$$\mathcal{C}_2 = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$$