Reinforcement Learning

Policy Gradient Methods

Stefano Albrecht, Pavlos Andreadis

28 February 2020

THE UNIVERSITY of EDINBURGH
informatics

Lecture Outline

- Parameterised policies
- Softmax and Gaussian policies as examples
- Policy gradient theorem
- Policy gradient methods: REINFORCE, baselines, actor-critic family

Value Function Approximation

Previously: approximate value function with parameterised function

$$\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$$
$$\hat{q}(s, a, \mathbf{w}) \approx q_\pi(s, a)$$

Policy was generated *implicitly* from value function (e.g. ϵ-greedy)

- **Compact:** number of parameters in $\mathbf{w}$ can be much smaller than $|\mathcal{S}|$
- **Generalises:** changing one parameter may change value of many states/actions

Policy Function Approximation

Today: approximate *policy* with parameterised function

$$\pi(a|s,\theta) = \Pr\{A_t = a \mid S_t = s, \theta_t = \theta\}$$

$\theta \in \mathbb{R}^{d'}$ is *policy parameter vector*

e.g. linear function, neural network, decision tree, ...

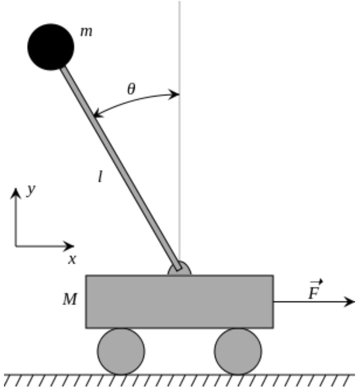
- **Compact:** number of parameters in θ can be much smaller than $|\mathcal{S}|$
- **Generalises:** changing one parameter may change action in many states

Policy vs Value Approximation

Advantages of optimising policy directly:

- Better convergence properties, typically to local optimum
- Effective in high-dimensional and continuous action spaces

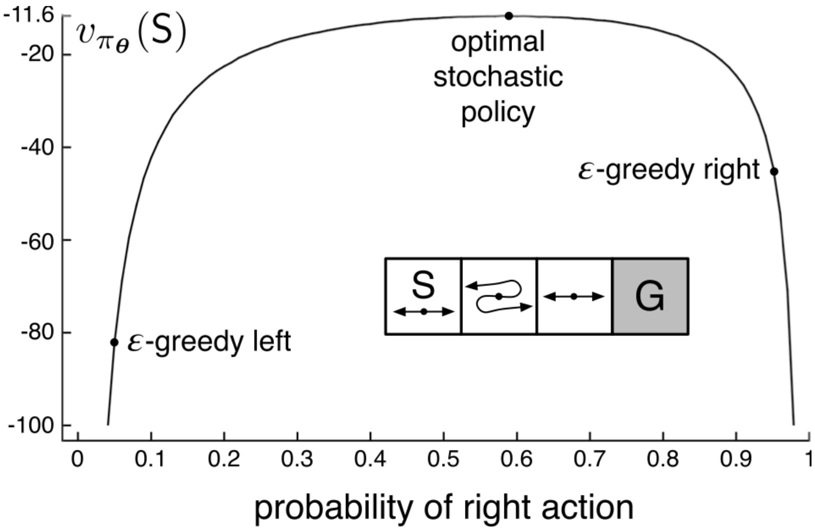
  $\Rightarrow$ Important for robotics applications
- Can learn stochastic policies (assign any probabilities to actions)



Example: Optimal Stochastic Policy in Short Corridor

Reward is −1 until goal state reached

Agent cannot distinguish between states, only between left/right action



Parameterised Policies

How to parameterise policy?

- We focus on gradient-based optimisation $\rightarrow$ need *differentiable* policy
- Examples:
  - Softmax for discrete actions
  - Gaussian for continuous actions
  - Deep neural network (next lectures)

How to optimise policy parameters?

- Policy gradient theorem leads to family of optimisation algorithms
- Monte Carlo, n-step TD, TD(λ), ...

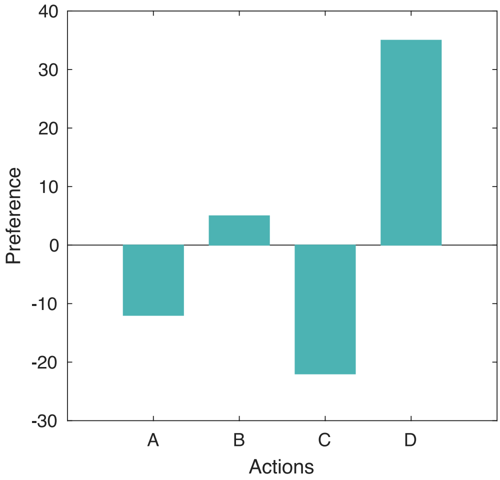
Softmax Policy for Discrete Actions

For discrete actions, can use softmax policy:

$$\pi(a|s,\theta) \doteq \frac{e^{h(s,a,\theta)}}{\sum_b e^{h(s,b,\theta)}}$$

- Action preference $h(s,a,\theta)$ can be parameterised arbitrarily, e.g. linear in features

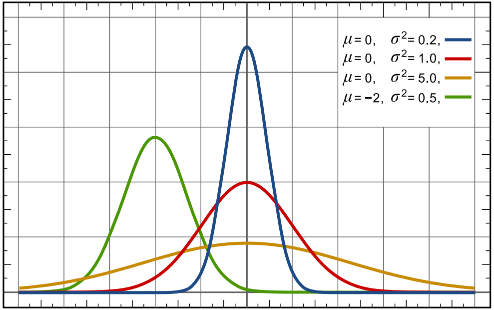
$$h(s,a,\theta) = \theta^\top \mathbf{x}(s,a)$$



Gaussian Policy for Continuous Actions

For continuous actions, can use Gaussian policy:

$$a \sim \mathcal{N}\left(\mu(s, \theta), \sigma^2\right)$$



- Mean μ can be parameterised arbitrarily, e.g. linear in features

$$\mu(s, \theta) \doteq \theta^\top \mathbf{x}(s)$$

- Variance σ^2 can be fixed or also parameterised (see book)

Policy Optimisation Objective

Goal: given policy representation $\pi(a|s, \theta)$, find optimal parameters θ

How to measure quality of θ?

- In episodic tasks, can use value of start state s_0:

$$J(\theta) \doteq v_{\pi_\theta}(s_0)$$

- In continuing tasks, can use average reward:

$$J(\theta) \doteq \sum_s P_\pi(s) \sum_a \pi(a|s, \theta) \sum_{s', r} p(s', r|s, a)\, r$$

$P_\pi(s)$ is steady-state distribution under π

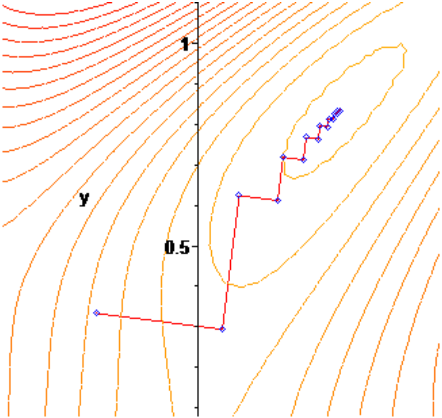
Policy Gradient

- Policy gradient algorithms search for a *local* maximum in $J(\theta)$ by ascending the gradient of π wrt θ

$$\theta_{t+1} = \theta_t + \alpha \nabla J(\theta_t)$$

- $\nabla J(\theta)$ is the policy gradient

$$\nabla J(\theta) = \left(\frac{\partial J(\theta)}{\partial \theta_1}, \cdots, \frac{\partial J(\theta)}{\partial \theta_{d'}} \right)$$



Policy Gradient Theorem:

For any *differentiable* policy π, the policy gradient is

$$\nabla J(\theta) \;=\; \sum_s d_\pi(s) \sum_a q_\pi(s,a)\,\nabla\pi(a|s,\theta)$$

$d_\pi(s)$ is the on-policy distribution under π:

- For start-state value: $d_\pi(s) = \sum_{t=0}^{\infty} \gamma^t \Pr\{S_t = s \mid s_0, \pi\}$
- For average reward: $d_\pi(s) = \lim_{t\to\infty} \Pr\{S_t = s \mid \pi\}$ (steady-state dist.)

Note: does not require derivative of environment dynamics $p(s', r|s, a)$!

Sampling Policy Gradient

Since $d_\pi(s)$ is on-policy, we can *sample* approximate gradient:

$$
\begin{aligned}
\nabla J(\theta) &= \sum_s d_\pi(s) \sum_a q_\pi(s,a)\, \nabla \pi(a|s,\theta) \\
&= \mathbb{E}_\pi \left[\sum_a q_\pi(S_t,a)\, \nabla \pi(a|S_t,\theta) \right] \\
&= \mathbb{E}_\pi \left[\sum_a \pi(a|S_t,\theta)\, q_\pi(S_t,a)\, \frac{\nabla \pi(a|S_t,\theta)}{\pi(a|S_t,\theta)} \right] \\
&= \mathbb{E}_\pi \left[q_\pi(S_t,A_t)\, \frac{\nabla \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)} \right] \\
&= \mathbb{E}_\pi [q_\pi(S_t,A_t)\, \nabla \ln \pi(A_t|S_t,\theta)]
\end{aligned}
$$

General Gradient Update

General gradient update: $\theta_{t+1} = \theta_t + \alpha\big(q_\pi(S_t, A_t)\, \nabla \ln \pi(A_t|S_t, \theta_t)\big)$

Policy gradient method needs to:

- Compute/approximate $\nabla \ln \pi(A_t|S_t, \theta_t)$
  - Softmax policy: $\nabla \ln \pi(a|s, \theta) = \mathbf{x}(s, a) - \sum_b \pi(b|s, \theta)\, \mathbf{x}(s, b)$
  - Gaussian policy: $\nabla \ln \pi(a|s, \theta) = (a - \mu(s, \theta))\, \mathbf{x}(s) \;/\; \sigma^2$
- Approximate $q_\pi(S_t, A_t)$

  e.g. Monte Carlo: use G_t, since $\mathbb{E}_\pi[G_t|S_t, A_t] = q_\pi(S_t, A_t)$

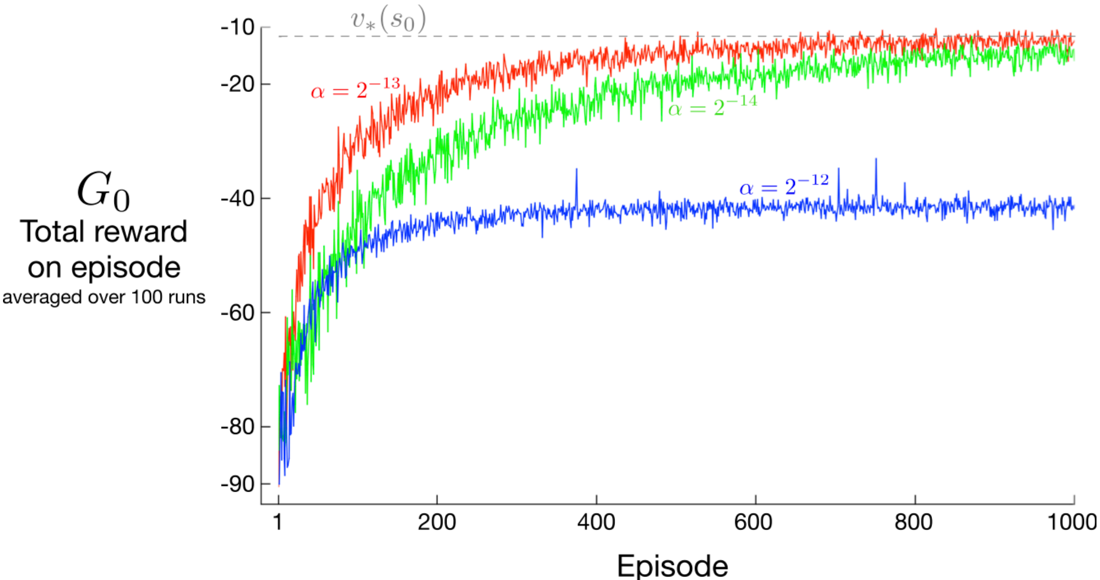
  $\Rightarrow$ REINFORCE algorithm

REINFORCE: Monte Carlo Policy Gradient

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
    Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
    Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
        $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

REINFORCE in Corridor Example



Baseline to Reduce Variance in Updates

Can generalise policy update to include baseline:

$$\big(q_\pi(S_t, A_t) - b(S_t)\big)\nabla \ln \pi(A_t|S_t, \theta)$$

Does not change expectation:

$$\begin{aligned}
\mathbb{E}_\pi[\nabla \ln \pi(A_t|S_t, \theta)\, b(S_t)] &= \sum_s d_\pi(s) \sum_a \nabla \pi(a|s, \theta)\, b(s) \\
&= \sum_s d_\pi(s)\, b(s)\, \nabla \sum_a \pi(a|s, \theta) \\
&= \sum_s d_\pi(s)\, b(s)\, \nabla 1 \quad = \quad 0
\end{aligned}$$

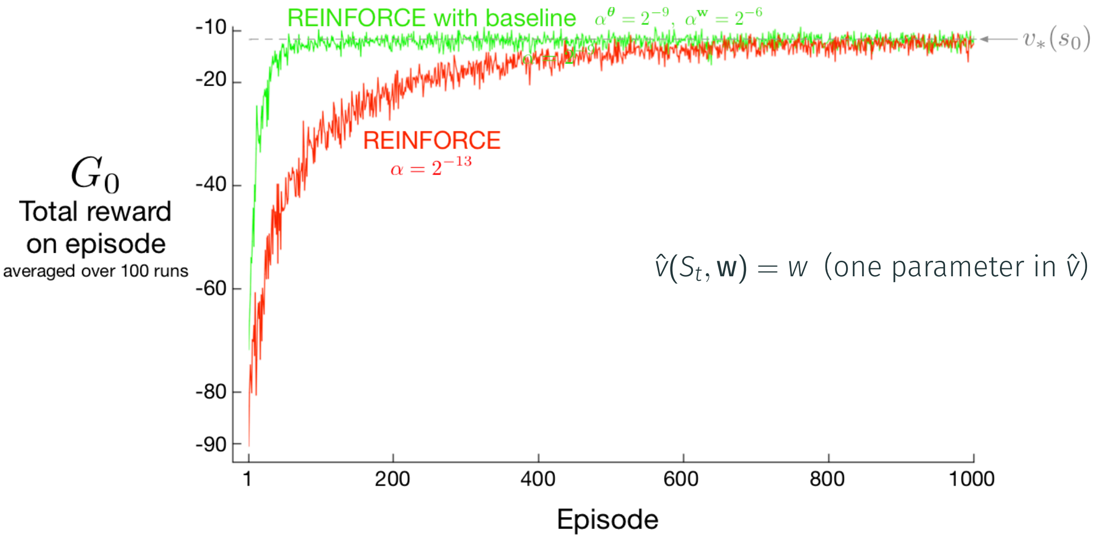
But can reduce *variance* of updates, e.g. use $b(s) = \hat{v}(S_t, \mathbf{w})$

REINFORCE with Baseline

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

REINFORCE with Baseline in Corridor Example



$\hat{v}(S_t, \mathbf{w}) = w$ (one parameter in $\hat{v}$)

Actor-Critic Methods

REINFORCE uses MC updates:

- large variance in updates (as any MC method)
- has to wait until end of episode (as any MC method)

Policy gradient can also use TD methods $\rightarrow$ then called **Actor-Critic method**

e.g. semi-gradient TD(0):

$$\theta_{t+1} = \theta_t + \alpha \left(R_{t+1} + \gamma\, \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \right) \nabla \ln \pi(A_t | S_t, \theta)$$

- *Critic* updates value function parameters $\mathbf{w}$
- *Actor* updates policy function parameters θ

Actor-Critic with Semi-Gradient TD(0)

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
    Initialize S (first state of episode)
    $I \leftarrow 1$
    Loop while S is not terminal (for each time step):
        $A \sim \pi(\cdot|S,\boldsymbol{\theta})$
        Take action A, observe S', R
        $\delta \leftarrow R + \gamma\hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if S' is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
        $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}}\delta\nabla\hat{v}(S,\mathbf{w})$
        $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I\delta\nabla\ln\pi(A|S,\boldsymbol{\theta})$
        $I \leftarrow \gamma I$
        $S \leftarrow S'$

Advanced Policy Gradient Methods

More advanced policy gradient methods:

- Natural Policy Gradient
- Trust Region Policy Optimisation
- Proximal Policy Optimisation
- Deterministic Policy Gradient

(Search on Google Scholar)

Reading

Required:

- RL book, chapter 13 (13.1–13.5, 13.7)

Optional:

- S. Bhatnagar, R. Sutton, M. Ghavamzadeh, M. Lee (2009). Natural Actor–Critic Algorithms. Automatica, 45(11)
- Marc P. Deisenroth, G. Neumann, J. Peters (2013). A Survey on Policy Search for Robotics. Foundations and Trends in Robotics, Vol. 2: No. 1–2, pp 1-142