# Package 'AUC comparison'

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Title Statistical test - Between group comparison of AUC

**Version** 0.0.0.9000

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<b>Description</b> The goal of AUCcomparison is to propose a statistical test evaluating the difference of Area under the curve (AUC) of a given outcome between two distinct groups of individuals. To this end, longitudinal data obtained for subjects splitted into G distincts groups are fitted with a Mixed-Effects model whose fixed-effects (marginal dynamics) and random-effects (in dividual dynamics) are respectively modeled by group-structured polynomial or B-splines curves and individual polynomial or B-spline curves.
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Encoding UTF-8
LazyData true
Roxygen list(markdown = TRUE)
RoxygenNote 7.1.1
Imports freeknotsplines, lmec, matlib, splines, ArgumentCheck, dplyr, gtools, plyr, mathjaxr, rmarkdown
RdMacros mathjaxr
Suggests testthat, knitr
Depends R (>= 2.10), mathjaxr
VignetteBuilder knitr
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```

ANOVA\_AUC\_Group\_Specific

One-way analysis of variance of group specific AUC

### **Description**

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This function performs a one-way ANOVA to compare the area under the curves of multiple groups marginal dynamics, modeled by group-structured polynomials or B-spline curve in Mixed-Effects model. Before performing the ANOVA, this function can parform a Bartlett's test to evaluate homoscedasticity. In addition to ANOVA, users can decide to evaluate all the 2 by 2 comparisons.

### Usage

```
ANOVA_AUC_Group_Specific(
   MEM_Pol_group,Groups,Time_groups,
   Nb_id_group,common.interval = TRUE,
   method = "trapezoid",Averaged = FALSE,
   conf_level = 0.95,bartlettTest = FALSE,
   data = NULL,twobytwo.comp = TRUE,
   alternative = "two.sided"
)
```

### Arguments

MEM\_Pol\_group A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).

- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic. type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic. type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

Groups

a vector indicating the names of the groups belonging to the set of groups involved in MEM\_Pol\_group we want to include in the ANOVA (a subset or the entire set of groups involved in the model can be considered).

Time\_groups

a list of numerical vectors indicating the time points to consider in AUC calculation for each group (as much elements than the number of groups in Groups).

Nb\_id\_group

a numerical vector indicating the number of individuals belonging to each group (as much elements than the number of groups in Groups).

common.interval

a logical scalar. If TRUE (default) AUCs of all the compared groups are calculated on the same time interval defined as the intersect of all the time interval defined in Time\_groups. If FALSE, AUC specific to each group is evaluated on its own interval.

method

a character scalar indicating the interpolation method to use to estimate the AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with "not-a-knot" spline boundary conditions.

Averaged

a logical scalar. If TRUE, AUC are evaluated as normalized AUC (nAUC) where nAUC is computed as the AUC divided by the range of time of calculation. If FALSE (default), the classic AUC is calculated (see Group\_specific\_AUC\_estimation for more details).

conf\_level

a numerical value (between 0 and 1) indicating the confidence level of the interval. By default, this variable is fixed at 0.95

bartlettTest

a logial scalar indicating whether a bartettTest must be performed before the ANOVA. Default: FALSE

data

a dataframe gathering data for the groups involved in the ANOVA that have been fitted by the MEM model summarized in MEM\_Pol\_group. This dataframe has to contain at least 4 columns:

- A column labeled 'Group' containing the information of the group for each observation (the same Groups than those defined in Groups).
- A column labeled 'id' containing the information of individual identifier.
- A column labeled 'time' containing information about the time of observations.
- A column labeled 'value' containing the longitudinal observations.

twobytwo.comp

a logical scalar indicating whether all the 2 by 2 comparisons must be evaluated after the ANOVA. Default: TRUE.

alternative

a character scalar specifying the alternative hypothesis for the 2 by 2 comparisons. Options are 'two.sided' (default), 'greater' or 'less'.

#### Value

A list of three elements elements:

- bartlettTest: a list of class "htest" corresponding to the Bartlett's test results (see bartlett.test for more details). If the test is not performed, a character 'Not performed' is returned.
- ANOVA\_F: a list containing:
  - 1. Followup: a vector of two numerical values indicating the time interval when Common.interval = TRUE. If common.interval=FALSE, if variable is not included in the list.
  - 2. Estimated\_AUCs: the vector of AUC estimated for each group involved in the ANOVA.
  - 3. Estimated\_VarAUCs: the vector of the intra group variance estimated by the MEM (equiv. squared Standard error).
  - 4. Between: a vector gathering the between groups sum of squares, degree of freedom and variance with var=SS/df.
  - 5. Within: a vector gathering the within groups sum of squares, degree of dreedom and variance with var=SS/df.
  - 6. ANOVA F: the value of the ANOVA statistic F.
  - 7. Pvalue: the value of the Pvalue.
  - 8. Reject\_H0: a boolean indicating whether the null hypothesis (all AUC are equals) is rejected.
- TwobyTwo\_Comparison: a list of K sublists where each sublist k gathers results of the kth comparison. If the 2 by 2 comparison is nor performed, a character 'Not performed' is returned.
  - 1. Groups: a vector indicating the 2 compared groups.
  - 2. Estimated. AUC: a vector of the 2 estimated values of AUC.
  - 3. Delta\_AUC: the value of the difference of AUC between the 2 compared groups.
  - 4. Tstat: the value of the t-statistic.
  - 5. Pvalue: the P-value (without any adjustment on multiple tests).
  - 6. Conf.int: the confidence interval.
  - 7. Adjusted.Pvalue: the P-value adjusted on multiple testing by "bonferroni" method (see p.adjust for more details).

#### See Also

```
AUC_time_weights_estimation bartlett.test
```

### **Examples**

```
# Download of data
data("HIV_Simu_Dataset_Delta01_cens")
data <- HIV_Simu_Dataset_Delta01_cens</pre>
colnames(data) <- c("id","time","Group","value","cens")</pre>
# Change factors in character vectors
data$id <- as.character(data$id) ; data$Group <- as.character(data$Group)</pre>
{\tt MEM\_estimation} < {\tt MEM\_Polynomial\_Group\_structure} (y = {\tt data\$value}, x = {\tt data\$time}, {\tt Group} = {\tt data\$Group}, x = {\tt data\$time}, x
                                                                                                                                                                              Id=data$id,Cens=data$cens)
Groups <- unique(data$Group)</pre>
Time_groups <- lapply(seq(1,length(Groups)),function(g)</pre>
                                                                                               return(unique(data$time[which(data$Group == Groups[g])])))
Nb_id_group <- sapply(seq(1,length(Groups)),function(g)</pre>
                                                                                     return(length(unique(data$id[which(data$Group == Groups[g])]))))
ANOVA_test <- ANOVA_AUC_Group_Specific(MEM_Pol_group=MEM_estimation,Groups=Groups,
                                                                                                                                          Time_groups=Time_groups,Nb_id_group=Nb_id_group,
                                                                                                                    \verb|common.interval=TRUE, method="trapezoid", Averaged=FALSE, \\
                                                                                                                                          conf_level=0.95,bartlettTest=TRUE,data=data,
                                                                                                                                           twobytwo.comp=TRUE,alternative="two.sided")
```

```
AUC_Lagrange_Cjp_coefficients
```

Time Dependent Coefficients Cjp for AUC Lagrange Interpolation Method

### **Description**

This function calculate the time-dependent coefficients Cjp involved in the calculation of the are under the curve when the Lagrange interpolation method is used.

### Usage

```
AUC_Lagrange_Cjp_coefficients(ind_j,ind_p,t)
```

### **Arguments**

ind_j	a numerical scalar indicating the value of the index j.
ind_p	a numerical scalar indicating the value of the index p
t	a numerical vector of time points (x-axis coordinates) to consider for the AUC calculation.

#### **Details**

The coefficients  $C_{jp}$  involved in the calculation of the AUC are defined as

$$C_{2p} = (t_2 - t_1) \prod_{l=0; l \neq p}^{P=2} t_{1+l} - \frac{(t_2^2 - t_1^2)}{2} \sum_{l=0; l \neq p}^{P=2} t_{1+l} + \frac{(t_2^3 - t_1^3)}{3}$$

$$C_{mp} = (t_m - t_{m-1}) \prod_{l=0; l \neq p}^{P=2} t_{m-2+l} - \frac{(t_m^2 - t_{m-1}^2)}{2} \sum_{l=0; l \neq p}^{P=2} t_{m-2+l} + \frac{(t_m^3 - t_{m-1}^3)}{3}$$

$$C_{jp} = -(t_j - t_{j-1}) \prod_{l=0; l \neq p}^{P=3} t_{j-2+l} + \frac{(t_j^2 - t_{j-1}^2)}{2} \sum_{l=0; l \neq p}^{P-1=2} \sum_{l=2}^{P=3} t_{j-2+l_1} \cdot t_{j-2+l_2} - \frac{(t_j^3 - t_{j-1}^3)}{3} \sum_{l=0; l \neq p}^{P=3} t_{j-2+l} + \frac{(t_j^4 - t_{j-1}^4)}{4}$$

where m is the number of time points in the vector t.

### Value

a numerical scalar corresponding to the coefficient  $C_{jp}$  evaluated for  $j = ind_j$  and  $p = ind_p$ .

AUC\_Spline\_matrix\_A Spline Interpolation Method - Matrix of Second Derivative Coefficients

#### **Description**

In the area under the curve calculation using the spline interpolation method, the vector of the second derivative of the outcome of interest Y is expressed as AY'' = BY + F. This function calculate calculate the matrix A.

#### Usage

AUC\_Spline\_matrix\_A(time)

### Arguments

time

a numerical vector of time points of length m (x-axis coordinates).

#### **Details**

The tridiagonal matrix A is defined as (for the "not-a-knot boundary conditions): The jth line of the matrix,  $A_{[j,:]}$  is given by

$$A_{[j,:]} = \left(\frac{1}{h_2}, -\left[\frac{1}{h_2} + \frac{1}{h_3}\right], \frac{1}{h_3}, 0, \dots, 0\right) if j = 1$$

$$A_{[j,:]} = \left(0, \dots, 0, \frac{1}{h_{m-1}}, -\left[\frac{1}{h_{m-1}} + \frac{1}{h_m}\right], \frac{1}{h_m}\right) if j = m$$

$$A_{[j,:]} = \left(0_1, \dots, 0_{j-2}, \frac{h_j}{6}, \frac{h_j + h_{j+1}}{3}, \frac{h_{j+1}}{6}, 0_{j+2}, \dots, 0_m\right) otherwise$$

#### Value

a tridiagonal matrix corresponding to the weights of the second derivative of the variable of interest in the spline interpolation method. In this version, the matrix is build considering the "not-a-knot" spline boundary conditions.

AUC\_Spline\_matrix\_B Spline Interpolation Method - Matrix of the zero order derivative coefficients

#### **Description**

In the area under the curve calculation using the spline interpolation method, the vector of the second derivative of the outcome of interest Y is expressed as  $AY^{''} = BY + F$ . This function calculate calculate the matrix B.

### Usage

AUC\_Spline\_matrix\_B(time)

### **Arguments**

time

a numerical vector of time points of length m (x-axis coordinates).

### **Details**

The tridiagonal matrix B is defined as (for the "not-a-knot boundary conditions): The jth line of the matrix,  $B_{[j,:]}$  is given by

$$B_{[j,:]} = (0, \cdots, 0) \text{ if } j = 1$$

$$B_{[j,:]} = (0, \cdots, 0) \ if \ j = m$$

$$B_{[j,:]} = \left(0_1, \dots, 0_{j-2}, \frac{1}{h_j}, -\left\lceil \frac{1}{h_j} + \frac{1}{h_{j+1}} \right\rceil, \frac{1}{h_{j+1}}, 0_{j+2}, \dots, 0_m \right) \ otherwise$$

#### Value

a tridiagonal matrix corresponding to the weights of the variable of interest in the spline interpolation method. In this version, the matrix is build considering the "not-a-knot" spline boundary conditions.

AUC\_time\_weights\_estimation

Weights for AUC Matrix Formulation

### **Description**

In matrix formulation, the area under a curve of interest, named Y, can be expressed as matrix product of a vector of weights W and the vector of the values of Y. This function calculates the weights W when AUC is calculated either by the trapezoid, the Lagrange or the Spline interpolation methods.

### Usage

AUC\_time\_weights\_estimation(time, method)

### **Arguments**

time a numerical vector of time points of length m (x-axis coordinates for AUC cal-

culation).

method a character scalar indicating the interpolation method of interest. Options are

'trapezoid', 'lagrange' and 'spline'. In this version the 'spline' interpolation method is implemented with the "not-a-knot" spline boundary conditions.

### **Details**

In matrix formulation, the AUC of the outcome Y can be expressed as  $AUC = W \cdot Y$ , with W defined by the following expressions for the trapezoid, the Lagrange and the spline interpolation methods.

### Trapezoid method:

$$W_j = \frac{t_{j+1} - t_j}{2} \text{ if } j = 1$$
 
$$W_j = \frac{t_j - t_{j-1}}{2} \text{ if } j = m$$
 
$$W_j = \frac{t_{j+1} - t_{j-1}}{2} \text{ otherwise}$$

**Lagrange method:** (see AUC\_Lagrange\_Cjp\_coefficients for the definition of the Cjp coefficients)

$$W_{j} = \frac{C_{[2][j-1]}}{\prod_{l=0;\ l\neq(j-1)}^{P=2}(t_{j}-t_{j+1})} + \sum_{p=0}^{P=3} \frac{C_{[j-1+p][3-p]}}{\prod_{l=0;\ l\neq(3-p)}^{P=3}(t_{j}-t_{j-3+p+l})} \ if \ j=1,2,3$$
 
$$W_{j} = \frac{C_{[m][j-(m-2)]}}{\prod_{l=0;\ l\neq(j-(m-2))}^{P=2}(t_{j}-t_{j-2+l})} + \sum_{p=0}^{m-j} \frac{C_{[j-1+p][3-p]}}{\prod_{l=0;\ l\neq(3-p)}^{P=3}(t_{j}-t_{j-3+p+l})} \ if \ j=m-2,m-1,m$$
 
$$W_{j} = \sum_{p=0}^{m-j} \frac{C_{[j-1+p][3-p]}}{\prod_{l=0;\ l\neq(3-p)}^{P=3}(t_{j}-t_{j-3+p+l})} \ otherwise$$

**Spline method:** (see AUC\_Spline\_matrix\_A and AUC\_Spline\_matrix\_B for the definition of Matrices A and B)

$$W_j = \sum_{p=2}^{m} -\frac{(t_p - t_{p-1})^3}{24} (u_{pj} + u_{p-1j}) + W_j^{trap.}$$

where  $(u_{pj})$  is the element U(p,j) with U a matrix defined as  $U=A^{-1}B$ .

### Value

A numerical scalar with same length than the vector time corresponding to the weights W.

```
Group_specific_AUC_estimation
```

Area Under The Curve of Group-Specific Polynomial Marginal Dynamics

### **Description**

This function estimates the area under the curve of marginal dynamics modeled by group-structured polynomials or B-spline curves.

### Usage

```
Group_specific_AUC_estimation(
   MEM_Pol_group,time,Groups = NULL,
   method = "trapezoid",Averaged = FALSE
)
```

### **Arguments**

MEM\_Pol\_group

A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - 2. the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).
- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic. type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic. type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

time a numerical vector of time points (x-axis coordinates) or a list of numerical vectors (with as much elements than the number of groups in Groups).

a vector indicating the names of the groups belonging to the set of groups involved in MEM\_Pol\_group for which we want to estimate the AUC (a subset or the entire set of groups involved in the model can be considered). If NULL (default), the AUC for all the groups involved the MEM is calculated.

a character scalar indicating the interpolation method to use to estimate the AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with the "not-a-knot" spline boundary conditions.

a logical scalar. If TRUE, the function return the normalized AUC (nAUC) computed as the AUC divided by the range of the time calculation. If FALSE (default), the classic AUC is calculated.

Details

Groups

method

Averaged

The area under the curve for the group g of interest is calculated as an approximation of the integral of the expected value of the estimated outcome Y specific to the group g. Assuming a time interval  $[0, T_g]$ , the AUC is then calculated as

$$AUC_g = \int_0^{T_g} E(\hat{Y_g})(t)dt$$

Similarly, the normalized AUC (nAUC) for this same group is then defined as

$$nAUC_g = \frac{1}{T_g} \int_0^{T_g} E(\hat{Y}_g)(t)dt$$

#### Value

A numerical vector containing the estimation of the AUC (or nAUC) for each group defined in the Groups vector.

### See Also

bs, MEM\_Polynomial\_Group\_structure

#### **Examples**

```
# Download of data
data("HIV_Simu_Dataset_Delta01_cens")
data <- HIV_Simu_Dataset_Delta01_cens</pre>
# Change factors in character vectors
data$id <- as.character(data$id) ; data$Group <- as.character(data$Group)</pre>
# Example 1: We consider the variable \code{MEM_Pol_Group} as the output
# of our function \link[AUCcomparison]{MEM_Polynomial_Group_structure}
MEM_estimation <- MEM_Polynomial_Group_structure(y=data$VL,x=data$time,Group=data$Group,
                                                  Id=data$id,Cens=data$cens)
time_group1 <- unique(data$time[which(data$Group == "Group1")])</pre>
time_group2 <- unique(data$time[which(data$Group == "Group2")])</pre>
# Estimation of the AUC for the two groups defined in the dataset
AUC_estimation <- Group_specific_AUC_estimation(MEM_Pol_group=MEM_estimation,
                     time=list(time_group1,time_group2),
                     Groups=unique(data$Group))
# Estimation of the AUC only for the group "Group1"
AUC_estimation_G1 <- Group_specific_AUC_estimation(MEM_Pol_group=MEM_estimation,
                         time=time_group1,Groups=c("Group1"))
# Example 2: We consider results of MEM estimation from another source.
# We have to give build the variable 'MEM_Pol_group' with the good structure
\# We build the variable 'MEM_Pol_group.1' with the results of MEM estimation obtained
# for two groups (even if only "Group1" is called in AUC estimation function)
MEM_Pol_group.1 <- list(Model_estimation=c(1.077,0.858,-0.061,0.0013,0.887,-0.066,0.0014),
                        Model_features=list(Groups=c("Group1", "Group2"),
                                  Marginal.dyn.feature=list(dynamic.type="polynomial",
                                         intercept=c(global.intercept=TRUE,
                                          group.intercept1=FALSE,group.intercept2=FALSE),
                                          polynomial.degree=c(3,3))))
# We build the variable 'MEM_Pol_group.2' with the results of MEM estimation obtained only for
# the group of interest (extraction)
MEM_Pol_group.2 <- list(Model_estimation=c(1.077,0.858,-0.061,0.0013),</pre>
                        Model_features=list(Groups=c("Group1"),
                                  Marginal.dyn.feature=list(dynamic.type="polynomial",
                              intercept=c(global.intercept=TRUE,group.intercept1=FALSE),
                                          polynomial.degree=c(3))))
# Estimation of the AUC for the group "Group1"
time_group1 <- unique(data$time[which(data$Group == "Group1")])</pre>
AUC_estimation_G1.1 <- Group_specific_AUC_estimation(MEM_Pol_group=MEM_Pol_group.1,
                                  time=time_group1,Groups=c("Group1"))
AUC_estimation_G1.2 <- Group_specific_AUC_estimation(MEM_Pol_group=MEM_Pol_group.2,
                                  time=time_group1)
```

```
Group_specific_Delta_AUC_estimation

*Difference of AUC of Two Group-Specific Polynomial Marginal Dynamics*
```

### **Description**

This function estimates the difference of area under the curve of marginal dynamics from two groups when marginal dynamics are modeled by group-structured polynomials or B-spline curves.

#### **Usage**

```
Group_specific_Delta_AUC_estimation(
   MEM_Pol_group,Group1,Group2,
   time.G1,time.G2,common.interval = TRUE,
   method = "trapezoid",Averaged = FALSE
)
```

#### **Arguments**

MEM\_Pol\_group

A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).
- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic.type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic.type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).

- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

Group1 a character scalar indicating the name of the first group whose marginal dynamics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM\_Pol\_group).

a character scalar indicating the name of the second group whose marginal dynamics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM\_Pol\_group).

time.G1 a numerical vector of time points (x-axis coordinates) to use for the Group1 AUC calculation.

time.G2 a numerical vector of time points (x-axis coordinates) to use for the Group2 AUC calculation.

common.interval

Group2

method

Averaged

a logical scalar. If FALSE, the difference of AUC is calculated as the difference of AUCs where the AUC of each group is calculated on its specific interval of time. If TRUE (default), the difference of AUC is estimated on a common interval of time defined as the intersect of the two group-specific interval (see @details for more details).

a character scalar indicating the interpolation method to use to estimate the AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with "not-a-knot" spline boundary conditions.

a logical scalar. If TRUE, the function return the difference of normalized AUC (nAUC) where nAUC is computated as the AUC divided by the range of time of calculation. If FALSE (default), the classic AUC is calculated.

#### **Details**

The difference of area under the curve between the two groups of interest is calculated as an approximation of the difference of the integrals of the expected value of the estimated outcome Y specific to the two groups g1 and g2. Assuming a time interval  $[0, T_{g1}]$  for the group 1 and  $[0, T_{g2}]$  for the group 2, the difference of AUC is then calculated as

$$\Delta AUC = \int_0^{T_{g_2}} E(\hat{Y}_{g_2})(t)dt - \int_0^{T_{g_1}} E(\hat{Y}_{g_1})(t)dt$$

Similarly, the difference of the normalized AUC ( $\Delta nAUC$ ) for these two same groups is then defined as

$$\Delta nAUC = \frac{1}{T_{g_2}} \int_0^{T_{g_2}} E(\hat{Y_{g_2}})(t)dt - \frac{1}{T_{g_1}} \int_0^{T_{g_1}} E(\hat{Y_{g_1}})(t)dt$$

When we want to estimate the difference of AUC (or nAUC) on the same time interval for the two groups (common.interval = TRUE), we define  $T = \min(T_{g_1}, T_{g_2})$  and we calculate the difference of AUC as

$$\Delta AUC = \int_{0}^{T} E(\hat{Y}_{g_{2}})(t)dt - \int_{0}^{T} E(\hat{Y}_{g_{1}})(t)dt$$
$$\Delta nAUC = \frac{1}{T} \int_{0}^{T} E(\hat{Y}_{g_{2}})(t)dt - \frac{1}{T} \int_{0}^{T} E(\hat{Y}_{g_{1}})(t)dt$$

#### Value

A numerical scalar defined as  $\triangle AUC = AUC_2 - AUC_1$  (or  $\triangle nAUC = nAUC_2 - nAUC_1$ ) with  $AUC_1$  (or  $nAUC_1$ ) and  $AUC_2$  (or  $nAUC_2$ ) being respectively estimated as the AUC (or nAUC) for the Group1 and for the Group2.

#### See Also

MEM\_Polynomial\_Group\_structure, Group\_specific\_AUC\_estimation

### **Examples**

```
# Download of data
data("HIV_Simu_Dataset_Delta01_cens")
data <- HIV_Simu_Dataset_Delta01_cens</pre>
# Change factors in character vectors
data$id <- as.character(data$id) ; data$Group <- as.character(data$Group)</pre>
# Example 1: We consider the variable \code{MEM_Pol_Group} as the output of our function
# \link[AUCcomparison]{MEM_Polynomial_Group_structure}
MEM_estimation_1 <- MEM_Polynomial_Group_structure(y=data$VL,x=data$time,Group=data$Group,
                                                     Id=data$id,Cens=data$cens)
time_group1 <- unique(data$time[which(data$Group=="Group1")])</pre>
time_group2 <- unique(data$time[which(data$Group=="Group2")])</pre>
Delta_AUC_1 <- Group_specific_Delta_AUC_estimation(MEM_Pol_group=MEM_estimation_1,</pre>
                                                     Group1="Group1",Group2="Group2"
                                                  time.G1=time_group1,time.G2=time_group2)
# Example 2: We consider results of MEM estimation from another source.
\mbox{\tt\#} We have to give build the variable 'MEM_Pol_group' with the good structure
# We build the variable 'MEM_estimation_2' with the results of MEM estimation obtained for 2 groups
MEM_estimation_2 < -list(Model_estimation=c(1.077, 0.858, -0.061, 0.0013, 0.887, -0.066, 0.0014),
                         Model_features=list(Groups=c("Group1", "Group2"),
                                     Marginal.dyn.feature=list(dynamic.type="polynomial",
                                                     intercept=c(global.intercept=TRUE,
                                           group.intercept1=FALSE,group.intercept2=FALSE),
                                                     polynomial.degree=c(3,3))))
Delta_AUC_2 <- Group_specific_Delta_AUC_estimation(MEM_Pol_group=MEM_estimation_2,</pre>
                                                     Group1="Group1",Group2="Group2",
                                                  time.G1=time_group1,time.G2=time_group2)
```

```
Group_specific_Var_AUC_estimation
```

Variance of the Area Under The Curve of Group-Specific Polynomial Marginal Dynamics

### **Description**

This function calculates the variance of the area under the curve of marginal dynamics modeled by group-structured polynomials or B-spline curves in Mixed-Effects models

#### **Usage**

```
Group_specific_Var_AUC_estimation(
   MEM_Pol_group,time,Groups = NULL,
   method = "trapezoid",Averaged = FALSE
)
```

### **Arguments**

MEM\_Pol\_group

A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).
- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic. type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic.type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

time

a numerical vector of time points (x-axis coordinates) or a list of numerical vectors (with as much elements than the number of groups in Groups).

Groups

a vector indicating the names of the groups belonging to the set of groups involved in MEM\_Pol\_group for which we want to estimate the AUC (a subset or the entire set of groups involved in the model can be considered). If NULL (default), the AUC for all the groups involved the MEM is calculated.

method a character scalar indicating the interpolation method to use to estimate the

AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with the "not-a-knot" spline boundary

conditions.

Averaged a logical scalar. If TRUE, the function return the normalized AUC (nAUC)

computed as the AUC divided by the range of the time calculation. If FALSE (default), the classic AUC is calculated. (See Group specific AUC estimation

for more details about calculation)

#### Value

A numerical vector containing the estimation of the variance of the AUC (or nAUC) for each group defined in the Groups vector.

#### See Also

bs, MEM\_Polynomial\_Group\_structure

### **Examples**

```
# Download of data
data("HIV_Simu_Dataset_Delta01_cens")
data <- HIV_Simu_Dataset_Delta01_cens</pre>
# Change factors in character vectors
data$id <- as.character(data$id) ; data$Group <- as.character(data$Group)</pre>
# Example 1: We consider the variable \code{MEM_Pol_Group} as the output of our function
# \link[AUCcomparison]{MEM_Polynomial_Group_structure}
MEM_estimation <- MEM_Polynomial_Group_structure(y=data$VL,x=data$time,Group=data$Group,
                                                   Id=data$id,Cens=data$cens)
time_group1 <- unique(data$time[which(data$Group == "Group1")])</pre>
time_group2 <- unique(data$time[which(data$Group == "Group2")])</pre>
Var_AUC_estimation <- Group_specific_Var_AUC_estimation(MEM_Pol_group=MEM_estimation,</pre>
                                                       time=list(time_group1,time_group2))
# Example 2: We consider results of MEM estimation from another source.
# We have to give build the variable 'MEM_Pol_group' with the good structure
# We build the variable 'MEM_Pol_group.1' with the results of MEM estimation obtained for 2 groups
# Generation of random matrix
Covariance_Matrix_1 <- matrix(rnorm(7*7,mean=0,sd=0.01),ncol=7,nrow=7)</pre>
# Transform the matrix into symmetric one
Covariance_Matrix_1 <- Covariance_Matrix_1 %*% t(Covariance_Matrix_1)</pre>
MEM_Pol_group.1 <- list(Model_estimation=Covariance_Matrix_1,</pre>
                        Model_features=list(Groups=c("Group1", "Group2"),
                               Marginal.dyn.feature=list(dynamic.type="polynomial",
                                                 intercept=c(global.intercept=TRUE,
                                           group.intercept1=FALSE,group.intercept2=FALSE),
                                                 polynomial.degree=c(3,3))))\\
Var_AUC_estimation_G1.1 <- Group_specific_Var_AUC_estimation(MEM_Pol_group.1,</pre>
                                                      time=time_group1,Groups=c("Group1"))
# We build the variable 'MEM_Pol_group.2' with the results of MEM estimation obtained only for the
```

```
{\tt Group\_specific\_Var\_Delta\_AUC\_estimation}
```

Variance of the Difference of AUC of Two Group-Specific Polynomial Marginal Dynamics

### **Description**

This function calculates the variance of the difference of area under the curve of two marginal dynamics modeled by group-structured polynomials or B-spline curve in Mixed-Effects models.

### Usage

```
Group_specific_Var_Delta_AUC_estimation(
   MEM_Pol_group,Group1,Group2,
   time.G1,time.G2,common.interval = TRUE,
   method = "trapezoid",Group.dependence = TRUE,
   Averaged = FALSE
)
```

### **Arguments**

MEM\_Pol\_group

A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).
- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:

- dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
- intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic. type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic.type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

Group1

a character scalar indicating the name of the first group whose marginal dynamics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM\_Pol\_group).

Group2

a character scalar indicating the name of the second group whose marginal dynamics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM\_Pol\_group).

time.G1

a numerical vector of time points (x-axis coordinates) to use for the variance of the Group1 AUC calculation.

time.G2

a numerical vector of time points (x-axis coordinates) to use for the variance of the Group2 AUC calculation.

common.interval

a logical scalar. If FALSE, the variance of difference of AUC is calculated as the variance of the difference of AUCs where the AUC of each group is calculated on its specific interval of time. If TRUE (default), the variance is estimated on a common interval of time defined as the intersect of the two group-specific interval (see Group\_specific\_Delta\_AUC\_estimation for more details about calculation.).

method

a character scalar indicating the interpolation method to use to estimate the AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with "not-a-knot" spline boundary conditions.

Group.dependence

a logical scalar indicating whether the two groups, whose the difference of AUC  $(\Delta AUC)$  is studied, are considered as dependent. By default, this variable is defined as TRUE.

Averaged

a logical scalar. If TRUE, the function return the difference of normalized AUC (nAUC) where nAUC is computed as the AUC divided by the range of time of calculation. If FALSE (default), the classic AUC is calculated.

#### Value

A numerical scalar corresponding to the variance of the difference of AUC ( $\Delta AUC$ ) between the Group1 and the Group2. If the two groups are considered as dependent (Group. dependence=TRUE), the variance of  $\Delta AUC$  is calculated as  $Var(AUC_1) + Var(AUC_2) - 2Cov(AUC_1, AUC_2)$ . Otherwise, only the sum of the two variance is used.

#### See Also

bs, Group\_specific\_Var\_AUC\_estimation, MEM\_Polynomial\_Group\_structure

#### **Examples**

```
# Download of data
data("HIV_Simu_Dataset_Delta01_cens")
data <- HIV_Simu_Dataset_Delta01_cens</pre>
# Change factors in character vectors
data$id <- as.character(data$id) ; data$Group <- as.character(data$Group)</pre>
# Example 1: We consider the variable \code{MEM_Pol_Group} as the output of our function
# \link[AUCcomparison]{MEM_Polynomial_Group_structure}
MEM_estimation <- MEM_Polynomial_Group_structure(y=data$VL,x=data$time,Group=data$Group,</pre>
                                                   Id=data$id,Cens=data$cens)
time_group1 <- unique(data$time[which(data$Group=="Group1")])</pre>
time_group2 <- unique(data$time[which(data$Group=="Group2")])</pre>
Var_Delta_AUC_estimation <- Group_specific_Var_Delta_AUC_estimation(</pre>
                                                MEM_Pol_group=MEM_estimation,
                                                Group1="Group1",Group2="Group2",
                                                time.G1=time_group1,time.G2=time_group2)
# Example 2: We consider results of MEM estimation from another source.
# We have to give build the variable 'MEM_Pol_group' with the good structure
# We build the variable 'MEM_Pol_group.1' with the results of MEM estimation obtained for 2 groups
# Generation of random matrix
Covariance_Matrix_1 <- matrix(rnorm(7*7, mean=0, sd=0.01), ncol=7, nrow=7)</pre>
# Transform the matrix into symmetric one
Covariance_Matrix_1 <- Covariance_Matrix_1 %*% t(Covariance_Matrix_1)</pre>
MEM_Pol_group.1 <- list(Model_estimation=Covariance_Matrix_1,</pre>
                        Model_features=list(Groups=c("Group1", "Group2"),
                               Marginal.dyn.feature=list(dynamic.type="polynomial",
                                                intercept=c(global.intercept=TRUE,
                                           group.intercept1=FALSE,group.intercept2=FALSE),
                                                polynomial.degree=c(3,3))))
Var_Delta_AUC_estimation_2 <- Group_specific_Var_Delta_AUC_estimation(</pre>
                                                MEM_Pol_group=MEM_Pol_group.1,
                                                Group1="Group1",Group2="Group2",
                                                time.G1=time_group1,time.G2=time_group2)
```

HIV\_Simu\_Dataset\_Delta0

HIV RNA Load Simulated Data for Delta AUC = 0

### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. This dataset was simulated to obtain a similar area under the mean HIV RNA load curves, despite the difference of the mean dynamics shape.

### Usage

HIV\_Simu\_Dataset\_Delta0

#### **Format**

A data frame with 4 columns and 1000 rows:

id The subject id.

time Time of HIV RNA load measurements.

Group Treatment group.

VL The HIV RNA load measures.

### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

### **Examples**

HIV\_Simu\_Dataset\_Delta0

 ${\sf HIV\_Simu\_Dataset\_Delta01}$ 

HIV RNA Load Simulated Data for Delta AUC = -0.1

### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. This dataset was simulated to obtain a difference of area under the mean HIV RNA load curves between the two groups (AUC2 - AUC1) equals to -0.1 log10 cp/ml.

### Usage

```
HIV_Simu_Dataset_Delta01
```

#### **Format**

A data frame with 4 columns and 1000 rows:

id The subject id.

time Time of HIV RNA load measurements.

Group Treatment group.

VL The HIV RNA load measures.

#### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

#### **Examples**

```
HIV_Simu_Dataset_Delta01
```

```
HIV_Simu_Dataset_Delta01_cens
```

HIV RNA Load Simulated Data for Delta AUC = -0.1 With Leftcensored data

### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. HIV RNA load values are left-censored due to a limit of detection (LOD), considering log10(50) as threshold value. This dataset was simulated to obtain a difference of area under the mean HIV RNA load curves between the two groups (AUC2 - AUC1) equals to -0.1 log10 cp/ml.

### Usage

```
HIV_Simu_Dataset_Delta01_cens
```

### **Format**

A data frame with 4 columns and 1000 rows:

id The subject id.

time Time of HIV RNA load measurements.

Group Treatment group.

VL The HIV RNA load measures. Measures are either equals to their true value (if VL > LOD) or to the LOD value

Cens The indicator of left-censoring: 1 if VLtrue <= LOD and 0 otherwise.

#### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

#### **Examples**

```
HIV_Simu_Dataset_Delta01_cens
```

```
HIV_Simu_Dataset_Delta025
```

HIV RNA Load Simulated Data for Delta AUC = -0.25

### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. This dataset was simulated to obtain a difference of area under the mean HIV RNA load curves between the two groups (AUC2 - AUC1) equals to -0.25 log10 cp/ml.

### Usage

```
HIV_Simu_Dataset_Delta025
```

### **Format**

A data frame with 4 columns and 1000 rows:

id The subject id.

time Time of HIV RNA load measurements.

Group Treatment group.

VL The HIV RNA load measures.

### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

### **Examples**

```
HIV_Simu_Dataset_Delta025
```

```
HIV_Simu_Dataset_Delta025_cens
```

 $\it HIV\ RNA\ Load\ Simulated\ Data\ for\ Delta\ AUC = -0.25\ With\ Left-censored\ data$ 

#### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. HIV RNA load values are left-censored due to a limit of detection (LOD), considering log10(50) as threshold value. This dataset was simulated to obtain a difference of area under the mean HIV RNA load curves between the two groups (AUC2 - AUC1) equals to -0.25 log10 cp/ml.

### Usage

HIV\_Simu\_Dataset\_Delta025\_cens

#### **Format**

A data frame with 4 columns and 1000 rows:

id The subject id.

time Time of HIV RNA load measurements.

Group Treatment group.

VL The HIV RNA load measures. Measures are either equals to their true value (if VL > LOD) or to the LOD value

Cens The indicator of left-censoring: 1 if VLtrue <= LOD and 0 otherwise.

#### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

### **Examples**

```
{\tt HIV\_Simu\_Dataset\_Delta025\_cens}
```

```
HIV_Simu_Dataset_Delta0_cens
```

HIV RNA Load Simulated Data for Delta AUC = 0 With Left-censored data

### **Description**

This dataset gives simulated longitudinal log-transformed HIV RNA load dynamics for two distinct groups of patients who interrupted their antiretroviral treatment at baseline (time=0). The group 'Group1' was simulated to mimic a control group while the group 'Group2' gathers patients assigned to a treatment group. Each group accounts for 20 patients and each of them were followed for 24 weeks with one measure every week. HIV RNA load values are left-censored due to a limit of detection (LOD), considering log10(50) as threshold value. This dataset was simulated to obtain a similar area under the mean HIV RNA load curves, despite the difference of the mean dynamics shape (without considering the left-censorship).

### Usage

```
HIV_Simu_Dataset_Delta0_cens
```

#### **Format**

A data frame with 4 columns and 1000 rows:

```
id The subject id.time Time of HIV RNA load measurements.Group Treatment group.
```

VL The HIV RNA load measures. Measures are either equals to their true value (if VL > LOD) or to the LOD value Cens The indicator of left-censoring: 1 if VLtrue <= LOD and 0 otherwise.

#### **Details**

A dataset containing simulated data used in the accompanying paper to this package.

#### **Examples**

```
HIV_Simu_Dataset_Delta0_cens
```

```
MEM_Polynomial_Group_structure

Polynomial Mixed-Effects Models with Censored and Group-
Structured Responses
```

### **Description**

This function fits a mixed-effects model (MEM) to potentially censored data structured by group when marginal and individual dynamics are described either by polynomials or B-spline curves.

#### Usage

```
MEM_Polynomial_Group_structure(
   y,x = NULL,Group = NULL,Id = NULL,Cens = NULL,
   marginal_dyn_type = "polynomial",ind_dyn_type = "polynomial",
   global_intercept = TRUE,group_intercept = FALSE,
   degree_group = 3,Adaptive = "none",knotnumcrit = "AIC",
   min_knots_group = 2,max_knots_group = 2,
   knots_group = NULL,df_group = NULL,Boundary.knots_group = NULL,
   ind_intercept = FALSE,degree_ind = 3,
   min_knots_ind = 2,max_knots_ind = 2,
   same_base_group_ind = FALSE,knots_ind = NULL,
   df_ind = NULL,Boundary.knots_ind = NULL,
   ...
)
```

### **Arguments**

y observed responses described either as a data frame containing at least a column named 'y' and possibly the columns 'x', 'Group', 'Id' and 'Cens' (among others), or as a vector of numerical values.

x a numerical vector representing the x-axis coordinates corresponding to the observed responses (e.g. time of observations) which can be defined is y is a vector

or a data frame without x column. By default, this variable is defined as NULL.

Group a vector of group indicator for each observed responses which can be defined if

y is a vector or a data frame without Group column. If this variable is defined as NULL (default) and y does not contain group information, all observed data are

assumed to belong to the same group.

Id a vector of individual ID for each observed responses which can be defined if

y is a vector or a data frame without 'Id' column. By default, this variable is

defined as NULL.

Cens a vector of censoring indicator (if  $y \ge y$ true, then Cens == 1). If this variable is

defined as NULL (default) and y does not contain 'Cens' column, observed data

are assumed as uncensored.

marginal\_dyn\_type

a character variable indicating the type of marginal dynamics. Options are 'poly-

nomial' (default) and 'spline'.

ind\_dyn\_type a character variable indicating the type of individual dynamics (random effects).

Options are 'polynomial' (default) or 'spline'.

global\_intercept

a logical scalar. If TRUE (default) a global intercept (no group-specific) is in-

cluded in the marginal dynamics.

group\_intercept

a logical scalar (same option for all groups) or vector. For each group, if TRUE,

a group-specific intercept is included in the marginal dynamics. By default, this

variable is defined as FALSE

degree\_group an integer scalar (same option for all groups) or vector. The variable indicates for

each group either the degree of polynomial functions or spline curves describing

marginal dynamics. By default, the variable is fixed at 3.

Adaptive an optional character variable whether B-spline curves are built with internal

knot positions optimally estimated according to data (see Optimal\_knot\_research and @details for more details). Options are 'none' (default), 'group , 'individ-

ual', and 'both'.

knotnumcrit an optional character variable indicating the criterion to be used for determining

the number of internal knots in B-splines. See @details for more details.

min\_knots\_group

an optional integer scalar indicating the minimum number of internal knots to

consider in the research of optimal knots for marginal dynamics. By default,

this variable fixed at 2. See @details for more details.

max\_knots\_group

an optional integer scalar indicating the maximum number of internal knots to

consider in the research of optimal knots for marginal dynamics. By default,

this variable fixed at 2. See @details for more details.

knots\_group a numerical vector (same option for all groups) or a list of either numerical

vectors or NULL (one for each group) indicating the internal knots for group-specific B-spline curves. By default, this variable is defined as NULL (see bs

and @details for more details).

df\_group an integer scalar (same option for all groups) or vector indicating the degrees

of freedom to consider to build marginal B-spline curves. One can specify df\_group rather than knots\_group (see bs for more details). By default, this

variable is defined as NULL. See @details for more details.

Boundary.knots\_group

a numerical vector (same option for all groups) or a list of either numerical vectors or NULL (one for each group) indicating the boundary knots for group-specific B-spline curves. By default, this variable is defined as NULL (see bs and @details for more details).

ind\_intercept

a logical scalar. If TRUE, an intercept is included in the individual dynamics (random effects). By default, this variable is defined as FALSE.

degree\_ind

an integer scalar indicating either the degree of the polynomial functions or the B-spline curves describing individual dynamics. By default, this variable is fixed at 2.

min\_knots\_ind

an optional interger scalar indicating the minimum number of internal knots to consider in the research of optimal knots for individual dynamics. By default, this variable is fixed at 2. See @details for more details.

max\_knots\_ind

an optional interger scalar indicating the maximum number of internal knots to consider in the research of optimal knots for individual dynamics. By default, this variable is fixed at 2. See @details for more details.

same\_base\_group\_ind

an optional logical scalar indicating whether or not the same B-spline basis must be considered in group-specific and individual dynamics. If TRUE, each individual B-spline basis will be build as the corresponding group-specific B-spline basis evaluated at the individual predictor variable. By default, this variable is defined as FALSE. See @details for more details.

knots\_ind

a numerical vector (same option for all individuals) or a list of numerical values or NULL indicating the internal knots for individual-specific B-spline curves. If this variable is defined as a list, internal knots can either be defined individually (one vector or NULL for each Id value), equivalent for each individual belong to the same group (one vector or NULL for each Group), or equivalent for each individual (one vector or NULL). By default, this variable is defined as NULL (see bs and @details for more details).

df\_ind

an integer scalar (same option for all individuals) or vector indicating the degrees of freedom to consier to build individual B-splines curves. This variable can be choosen different for each individual or equivalent for each individual belonging to the same group (one value for each group). By default, this variable is defined as NULL(see bs and @details for more details).

 $Boundary.knots\_ind$ 

a numerical vector indicating the boundary knots for individual-specific B-spline curves. By default, this variable is defined as NULL (see bs and @details for more details).

Further arguments to be passed (see lmec for more details).

#### **Details**

The variable adaptive can take 4 differents values:

- 'none' (default): internal knot positions defining B-spline curves in the model are not optimally estimated according to data, whether for fixed or random effects. Knots are then defined by users.
- 'group': only internal knot positions defining B-spline curves in fixed effects are optimally estimated, even if random effects involve B-splines.
- 'individual': only internal knot positions defining B-spline curves in random effects are optimally estimated, even if fixed effects involve B-splines.

• 'both': internal knot positions defining B-spline curves for both fixed and random effects are optimally estimated.

At group level (fixed effects),

- the variables Adaptive, knotnumcrit, min\_knots\_group, max\_knots\_group, knots\_group, df\_group and Boundary.knots\_group will be used only if the variable marginal\_dyn\_type has been chosen as 'spline'.
  - among them, the variables knotnumcrit, min\_knots\_group and max\_knots\_group will be used only if adaptive knots are requested (the variable Adaptive chosen as 'group' or 'both').
  - the variables knots\_group, df\_group and Boundary.knots\_group will be used without adaptive knots.

Similarly, at individual level (random effects),

- the variables Adaptive, knotnumcrit, min\_knots\_ind, max\_knots\_ind, knots\_ind, df\_ind
  and Boundary.knots\_ind will be used only if the variable ind\_dyn\_type has been chosen
  as 'spline'.
  - among them, the variables knotnumcrit, min\_knots\_ind and max\_knots\_ind will be used only if adaptive knots are requested (the variable Adaptive chosen as 'individual' or 'both').
  - the variables knots\_ind, df\_ind and Boundary.knots\_ind will be used without adaptive knots.
- the variable same\_base\_group\_ind can be used only if both variables marginal\_dyn\_type and ind\_dyn\_type are chosen as 'spline'.

The Mixed-Effects model describing the outcome of interest  $Y_{ij,g_i}$  of the subject i in the group  $g_i$  at the jth time point (x-axis) is given by

$$Y_{ij,g_i} = \gamma_0 + \sum_{g=1}^{G} 1_{g_i=g} \times F_g(t_{ij,g}) + h_i(t_{ij,g_i}) + \varepsilon_{ij}$$

where  $\gamma_0$  is the global (non group-specific) intercept and the functions  $F_g$  and  $h_i$  are the non-linear smooth functions describing respectively the marginal group-specific dynamics and the individual dynamics (random effects). Through this function, the group-specific functions are defined as following, where  $\beta_0^g$  is the optional group\_intercept:

$$F_g(t_{ij,g}) = \beta_0^g + \sum_{k=1}^{K_g} \beta_k^g f_g^k(t_{ij,g})$$

If marginal\_dyn\_type is defined as 'polynomial',  $K_g = \text{degree\_group}$  and  $f_g^k(t_{ij,g}) = t_{ij,g}^k$  and if marginal\_dyn\_type is defined as 'spline',  $K_g = \text{df\_group}$  (see bs for more details) and  $f_g^k(t_{ij,g})$  is the kth group-specific spline basis. Similarly, the individual dynamics are described by the following functions, with  $b_{0i}$  as optional ind\_intercept:

$$h_i(t_{ij,g}) = b_{0i} + \sum_{k=1}^{K_i} b_{ki} \Psi_k^i(t_{ij,g})$$

If ind\_dyn\_type is defined as 'polynomial',  $K_i = \text{degree\_ind}$  and  $\Psi_k^i(t_{ij,g}) = t_{ij,g}^k$  and if ind\_dyn\_type is defined as 'spline',  $K_i = \text{df\_ind}$  (see bs for more details) and  $\Psi_k^i(t_{ij,g})$  is the kth individual spline basis.

#### Value

### A list containing:

- Model\_estimation: a list containing the results of the model estimation provided by the function lmec. In this list, the vector of fixed parameters called beta, the parameters are returned in the following order:  $\beta = (\gamma_0, \beta_0^1, \cdots, \beta_{K_1}^1, \cdots, \beta_0^g, \cdots, \beta_{K_G}^g, \cdots, \beta_{K_G}^G, \cdots, \beta_{K_G}^G)$ .
- Model\_features: a list of 3 elements:
  - 1. Groups: a vector indicating the names of the groups involved in the model.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model (through input arguments):
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1 values).

For 'polynomial' marginal dynamics:

 polynomial.degree: an integer vector indicating the degree of polynomial functions.

For 'spline' marginal dynamics:

- spline.degree: an integer vector indicating the degree of B-spline curves.
- adaptive.splines: a logical scalar indicating whether or not adaptive internal knots have been considered.
- knots: a list of group-specific internal knots used to build B-spline basis. If the degrees of freedom were equals to the spline degrees, then NULL.
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis.
- 3. Individual.dyn.feature: a list summarizing the features of the individual dynamics defined in the model (through input arguments)
  - dynamic.type: a character scalar indicating the chosen type of individual dynamics.
  - intercept: a logical scalar indicating whether a random intercept was included in the model.

For 'polynomial' individual dynamics:

 polynomial.degree: an integer scalar indicating the degree of polynomial functions.

For 'spline' marginal dynamics:

- spline.degree: an integer scalar indicating the degree of B-spline curves
- adaptive.splines: a logical scalar indicating whether or not adaptive internal knots have been considered.
- knots: a data frame of individually estimated internal knots (if Adaptive chosen as 'individual' or 'both'), or a list of chosen individual internal knots.
- df: a numerical vector of individual degrees of freedom.
- boundary.knots: a numerical vector of individual boundary knots.

### See Also

### **Examples**

### **Description**

This function uses AIC criterion to estimate optimal position and number of knots by fitting free-knot splines to data with one independent and one dependent variable. It is assumed that knots are estimated for least-squares splines with no penalty using genetic algorithm.

### Usage

```
Optimal_knot_research(
  data,degree = 3,minknot = 2,maxknot = 2,
  criteria = "AIC",...
)
```

### **Arguments**

data	a data frame containing the independent variable $(y)$ and the dependent one $(x)$ .
degree	an integer scalar indicating the degree of the spline it. By default, this value is fixed at $\bf 3$
minknot	an integer scalar indicating the minimum number of knots to consider. By default, this variable is fixed at $2$
maxknot	an integer scalar indicating the maximum number of knots to consider. By default, this variable is fixed at 2.
criteria	a character varaiable indicating the criterion to be used for determining the number and the positions of knots. Choices are "AIC" for Akaike information criterion (by default), "AICc" for corrected AIC, "BIC" for Bayesian information criterion, "adjAIC" for an adjusted version of AIC, "GCV" for generalized cross-validation and "adjGCV" for an adjusted version of GCV.
	Further arguments to be passed (see freeknotfit for more details).

#### Value

A numerical vector of optimal knots whose number can varied from minknot to maxknot

#### See Also

freeknotfit for more details about the method.

```
\begin{tabular}{ll} Stat\_test\_Delta\_AUC\_Group\_Specific \\ T-statistic of the Difference of AUC of Two Group-Specific Polynomial \\ Marginal Dynamics \end{tabular}
```

### **Description**

This function performs the t-test evaluating whether the difference of area under the curve of two marginal dynamics, modeled by group-structured polynomials or B-spline curve in Mixed-Effects model, is null.

#### **Usage**

```
Stat_test_Delta_AUC_Group_Specific(
   MEM_Pol_group,Group1,Group2,
   time.G1,time.G2,common.interval = TRUE,
   method = "trapezoid",Group.dependence = TRUE,
   Averaged = FALSE,conf_level = 0.95,
   alternative = "two.sided"
)
```

### **Arguments**

MEM\_Pol\_group

A list with similar structure than the output provided by the function MEM\_Polynomial\_Group\_struct A list containing:

- Model\_estimation: a list containing at least 2 elements:
  - 1. the vector of the marginal (fixed) parameters estimates (at least for the groups whose AUC is to estimate), labeled 'beta'.
  - the variance-covariance matrix of these parameters, labeled 'varFix' (see MEM\_Polynomial\_Group\_structure for details about the parameter order).
- Model\_features: a list of at least 2 elements:
  - 1. Groups: a vector indicating the names of the groups whose fixed parameters are given.
  - 2. Marginal.dyn.feature: a list summarizing the features of the marginal dynamics defined in the model:
    - dynamic.type: a character scalar indicating the chosen type of marginal dynamics. Options are 'polynomial' or 'spline'.
    - intercept: a logical vector summarizing choices about global and group-specific intercepts (Number of groups + 1) elements whose elements are named as ('global.intercept', 'group.intercept1', ..., 'group.interceptG') if G Groups are defined in MEM\_Pol\_group. For each element of the vector, if TRUE, the considered intercept is considered as included in the model (see *Examples*).

If dynamic. type is defined as 'polynomial':

polynomial.degree: an integer vector indicating the degree of polynomial functions, one value for each group.

If dynamic. type is defined as 'spline':

- spline.degree: an integer vector indicating the degree of B-spline curves, one for each group.
- knots: a list of group-specific internal knots used to build B-spline basis (one numerical vector for each group) (see bs for more details).
- df: a numerical vector of group-specific degrees of freedom used to build B-spline basis, (one for each group).
- boundary.knots: a list of group-specific boundary knots used to build B-spline basis (one vector for each group) (see bs for more details).

Group1 a character scalar indicating the name of the first group whose marginal dy-

namics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM Pol. group)

involved in the MEM (see Groups vector in MEM\_Pol\_group).

Group2 a character scalar indicating the name of the second group whose marginal dy-

namics must be considered. This group name must belong to the set of groups involved in the MEM (see Groups vector in MEM\_Pol\_group).

time.G1 a numerical vector of time points (x-axis coordinates) to use for the Group1

AUC calculation.

time. G2 a numerical vector of time points (x-axis coordinates) to use for the Group2

AUC calculation.

common.interval

a logical scalar. If FALSE, the difference of AUC is calculated as the difference of AUCs where the AUC of each group is calculated on its specific interval of time. If TRUE (default), the difference of AUC is estimated on a common interval of time defined as the intersect of the two group-specific interval (see Group\_specific\_Delta\_AUC\_estimation for more details about calculation).

method a character scalar indicating the interpolation method to use to estimate the

AUC. Options are 'trapezoid' (default), 'lagrange' and 'spline'. In this version, the 'spline' interpolation is implemented with "not-a-knot" spline boundary con-

ditions.

Group.dependence

a logical scalar indicating whether the two groups, whose the difference of AUC ( $\Delta$ AUC) is studied, are considered as dependent. By default, this variable is

defined as TRUE.

Averaged a logical scalar. If TRUE, the function return the difference of normalized AUC

 $(\mbox{nAUC})$  where nAUC is computated as the AUC divided by the range of time of

calculation. If FALSE (default), the classic AUC is calculated.

conf\_level a numerical value (between 0 and 1) indicating the confidence level of the inter-

val. By default, this variable is fixed at 0.95

alternative a character scalar specifying the alternative hypothesis. Options are 'two.sided'

(default), 'greater' or 'less'.

#### Value

### A list containing:

• Tstat: the value of the t-statistic.

- Pvalue: the P-value.
- Conf. int: the confidence interval.
- Delta\_AUC: the estimated value of the difference of AUC between the two groups (nAUC2 nAUC1) (see Group\_specific\_Delta\_AUC\_estimation for more details).
- AUCs: the estimated values of the Group-specific AUC (AUC1 and AUC 2) (see Group\_specific\_AUC\_estimation for more details).

#### See Also

```
\label{lem:mem_polynomial_Group_structure} MEM\_Polynomial\_Group\_structure, Group\_specific\_Delta\_AUC\_estimation, Group\_specific\_AUC\_estimation
```

### **Examples**

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