Code Documentation

The expected photon fluence from ALP decays after SN1987A

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1 Introduction

The authors of Ref. [1] compute limits on axion-like particles (ALPs) from the type-II supernova SN1987A measurements by the Gamma-Ray Spectrometer onboard the Solar Maximum Mission (SMM) [2]. We replicate the results of their Monte Carlo simulations extending the parameter space considered. We summarise our implementation of their algorithm and describe the Python code that we used to calculate our results. The code is publicly available on Github at `https://github.com/marie-lecroq/ALP-fluence-calculation`. It can be used to explore the constraints on the ALP properties in supernovae even beyond the parameter space considered here. We use these results to study heavy ALPs in cosmology using GAMBIT [3–5].

2 Quick start guide

Our code is fully contained in the Python script `code/fluence_calc_mc.py`. The function `expected_photon_fluence` returns the photon fluence (in units of cm^{-2}) given an ALP mass `m` (in units of eV) and ALP-photon coupling `g` (in units of GeV^{-1}). To call this function in parallel, we provide a corresponding MPI implementation (using `mpi4py`) in the script `run_analysis_mpi.py`.

3 Fluence calculation

The ALP spectrum from SN1987A, i.e. the number of ALPs with a given energy E_a is given by fitting the results of Ref. [6] to the following ansatz:

$$\frac{\mathrm{d}N_a}{\mathrm{d}E_a} = C\,\frac{E_a^2}{\exp(E_a/T)-1}\,\sigma(m_a, g_{a\gamma\gamma}, E_a)\,, \tag{1}$$

where $C = 2.54\times10^{77}\,\mathrm{MeV}^{-1}$ and $T = 30.6\,\mathrm{MeV}$ being the results of the fit [1]. The Primakoff cross section $\sigma(m_a, g_{a\gamma\gamma}, E_a)$ can be found in Eq. (A.3) of Ref. [7] and is implemented as `sigma` in our code,

$$\begin{aligned}\sigma(m_a, g_{a\gamma\gamma}, E_a) = \frac{\alpha\, g_{a\gamma\gamma}^2}{8}\Bigg[&\left(1+\frac{k_\mathrm{s}^2}{4E_a^2}-\frac{m_a^2}{2E_a^2}\right)\log\left(\frac{2E_a^2(1+\beta)+k_\mathrm{s}^2-m_a^2}{2E_a^2(1-\beta)+k_\mathrm{s}^2-m_a^2}\right)-\beta\\ &-\frac{m_a^4}{4k_\mathrm{s}^2E_a^2}\log\left(\frac{m_a^4+k_\mathrm{s}^2(2E_a^2(1+\beta)-m_a^2)}{m_a^4+k_\mathrm{s}^2(2E_a^2(1-\beta)-m_a^2)}\right)\Bigg]\,,\end{aligned} \tag{2}$$

where $k_\mathrm{s} = 16.8\,\mathrm{MeV}$ is the Debye screening scale [1]. We calculate the axion fluence resulting from this spectrum using the `scipy.integrate` package in Python to integrate Eq. (1) over the energies, divided by the surface of the sphere centred on the supernova with the distance to the detector as the radius. The result is the `axion_fluence` variable in our code. For practical reasons, we define a minimum and a maximum value for the energy (`min_erg` and `max_erg`) and only integrate over this range.

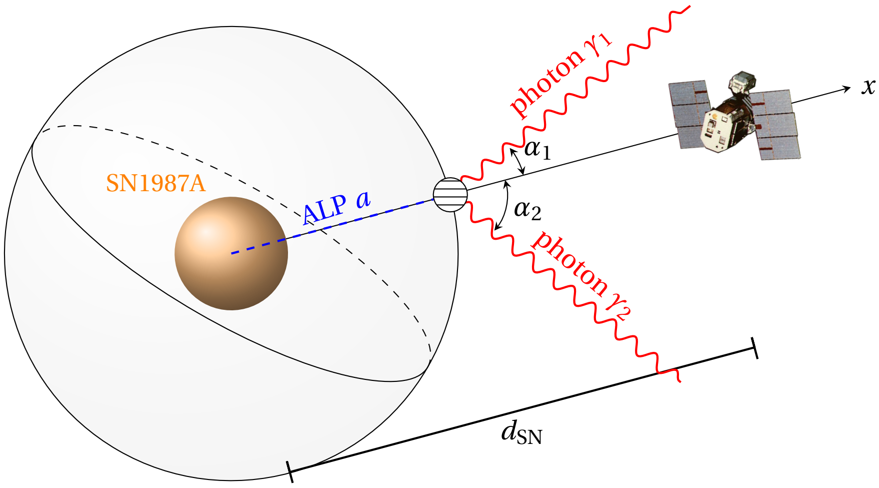

Figure 1: Geometry for SN1987A ALPs decaying into two photons in the laboratory frame i.e. the reference frame of the SMM satellite. Image credit for the SSM satellite to G. Nelson/NASA (JSC image library; public domain).

The mean ALP-photon decay length is given by [6]

$$\ell_{\rm ALP}(m_a, g_{a\gamma\gamma}, E_a) = \frac{\beta\gamma}{\Gamma_{a\gamma\gamma}} = \frac{64\pi}{g_{a\gamma\gamma}^2 m_a^3}\sqrt{\frac{E_a^2}{m_a^2} - 1}\,, \tag{3}$$

where $\Gamma_{a\gamma\gamma}$ is the decay width and we used the following relations of the β (called `betafactor` in the code) and γ factors:

$$\gamma = \frac{E_a}{m_a} = \frac{1}{\sqrt{1-\beta^2}}\,, \quad \beta^2 = 1 - \frac{m_a^2}{E_a^2}\,. \tag{4}$$

We now extend the discussion in Ref. [1] by visualising the geometry of the problem and adding more detailed discussion on the parameter transformation in the coordinate systems.

In Fig. 1, we show the ALP a emerging from SN1987A and decaying into two photons γ_1 and γ_2. The figure indicates the angles α_1 and α_2 used in the following. For convenience, let the x-axis be parallel to direction of the emergent ALP. We denote the ALP rest frame with “0” and the lab frame, i.e. the rest frame of the satellite with “L”. The Lorentz boost of the ALP 4-momentum from the rest into the lab frame is given by

$$p^\mu_{a,0} = \begin{pmatrix} m_a \\ 0 \\ 0 \\ 0 \end{pmatrix} \mapsto p^\mu_{a,\rm L} = \begin{pmatrix} \gamma\, m_a \\ \beta\gamma\, m_a \\ 0 \\ 0 \end{pmatrix}, \quad \text{with} \quad \Lambda = \begin{pmatrix} \gamma & \beta\gamma & 0 & 0 \\ \beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

The two decay photons are emitted back-to-back in the ALP rest frame and the transformed 4-momenta of photons “1” and “2” are hence

$$p^\mu_{1/2,0} = \frac{m_a}{2}\begin{pmatrix} 1 \\ \pm\cos\phi\sin\theta \\ \pm\sin\phi\sin\theta \\ \pm\cos\theta \end{pmatrix} \mapsto p^\mu_{1/2,\rm L} = \frac{\gamma\, m_a}{2}\begin{pmatrix} 1 \pm \beta\cos\phi\sin\theta \\ \beta \pm \cos\phi\sin\theta \\ \pm\frac{1}{\gamma}\sin\phi\sin\theta \\ \pm\frac{1}{\gamma}\cos\theta \end{pmatrix}. \tag{6}$$

$$\tag{7}$$

Consequently, the two photon energies E_1 and E_2 in the lab frame are

$$E_{1/2,\mathrm{L}} = \frac{\gamma\, m_a}{2}\left(1 \pm \beta\cos\phi\,\sin\theta\right) = \frac{E_a}{2}\left(1 \pm \beta\cos\phi\,\sin\theta\right), \tag{8}$$

while the photon angles α_1 and α_2 in the lab frame are defined geometrically via as the angle between the 3-momenta and the x-direction $\boldsymbol{e}_x$ such that

$$\cos\alpha_{1/2,\mathrm{L}} = \frac{\boldsymbol{e}_x\cdot\boldsymbol{p}_{1/2}}{\|\boldsymbol{e}_x\|\,\|\boldsymbol{p}_{1/2}\|} = \frac{E_a}{2}\,\frac{\beta\pm\cos\phi\,\sin\theta}{\|\boldsymbol{p}_{1/2}\|}\,. \tag{9}$$

Calculating the angles in Eq. (9) requires some algebra, in particular we need that

$$\left(\frac{2}{E_a}\|\boldsymbol{p}_{1/2}\|\right)^2 = \left(\beta\pm\cos\phi\,\sin\theta\right)^2 + \frac{1}{\gamma^2}\left(\sin^2\phi\sin^2\theta+\cos^2\theta\right) \tag{10}$$

$$= \beta^2 \pm 2\beta\cos\phi\sin\theta + \cos^2\phi\sin^2\theta + (1-\beta^2)\left(\sin^2\phi\sin^2\theta+\cos^2\theta\right) \tag{11}$$

$$= \ldots = \left(1\pm\beta\cos\phi\sin\theta\right)^2 \tag{12}$$

$$\Rightarrow \frac{2}{E_a}\|\boldsymbol{p}_{1/2}\| = \left|1\pm\beta\cos\phi\sin\theta\right| = 1\pm\beta\cos\phi\sin\theta\,. \tag{13}$$

The absolute value in the last line can be dropped because the absolute values of β, $\cos\phi$, and $\sin\theta$ are all smaller than unity. The angles are therefore given by

$$\alpha_{1/2,\mathrm{L}} = \arccos\left(\frac{\beta\pm\cos\phi\,\sin\theta}{1\pm\beta\cos\phi\sin\theta}\right). \tag{14}$$

Equation (14) reveals additional rotational symmetry in the problem because only the product $\cos\phi\sin\theta$ is important. One could set e.g. $\theta = \pi$ to simplify our code.

These geometrical considerations are important as they influence the detected photon fluence. Since each ALP decays into two photons, the naïve photon fluence is twice the fluence computed from the axion spectrum in Eq. (1). To take into account the geometrical and experimental cuts, we follow the procedure outlined in Ref. [1]:

For each pair of ALP mass and ALP-photon coupling (m_a, $g_{a\gamma}$) we simulate 10^7 events. This number comes about because the naïve photon fluence in the parameter region of interest is of order $10^6\,\mathrm{cm}^{-2}$ while the limit is of order a few cm^{-2}. However, not all photons from the ALP decay reach the detector, which makes it necessary to perform the Monte Carlo simulation described below. The result of simulating the expected fluence is given by the function `expected_photon_fluence` in our code.

Draw a random ALP energies in the ALP rest frame. Each ALP is generated with a given energy (stored in `rng_ergs`), randomly sampled by interpreting the spectrum in Eq. (1) as a probability distribution. This allows us to numerically calculate the inverse of its cumulative distribution (CDF) These are named `inv_cdf_massless` and `inv_cdf` in the code. The former function is only calculated once and can be used for low axion masses. The latter function is computed again for each parameter pair (m_a, $g_{a\gamma}$).

Draw a random decay length. Similarly, we sample the decay length ℓ from its mean value given by Eq. (3), which is called `l_ALP` in the code. It follows an exponential distribution with `l_ALP` as the mean parameter. The code uses Python tools numpy.random to sample the decay length of each axion, which simply uses the analytical inverse of the CDF for the exponential distribution.

The first criterion is that the ALP has to decay before reaching the Earth, as almost no photons will reach the detector beyond this point (cf. discussion in Ref. [1]). On the other hand, if the ALP decays too close to SN1987A (the orange shell around SN1987A in Fig. 1), the photons will be absorbed or deviated by the turbulent environment. The radius of this region is about $R_{\mathrm{eff}} = 10^{10}\,\mathrm{m}$, while the distance to Earth/the satellite is $d_{\mathrm{SN}} = 51.4\,\mathrm{kpc}$. Only ALPs decaying within this range are considered further.

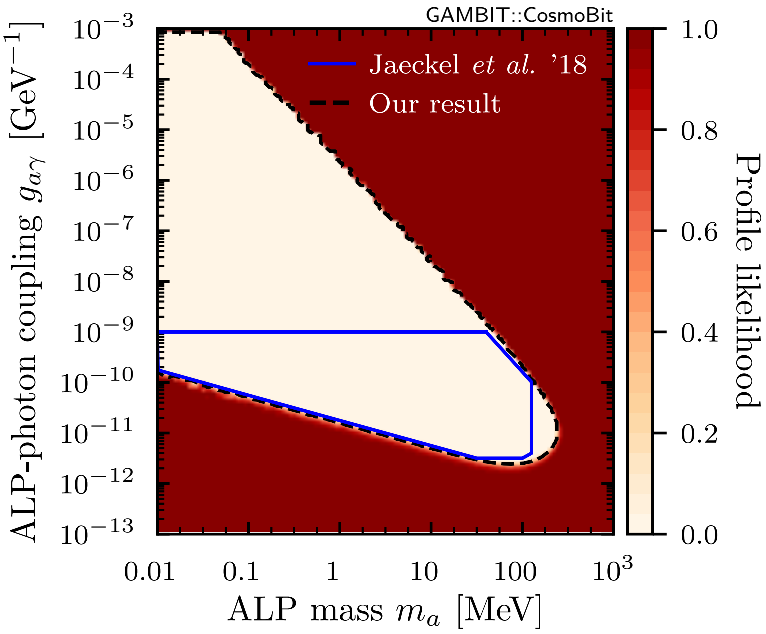

Figure 2: Our profile likelihood an exclusion region of m_a and $g_{a\gamma\gamma}$ (solid black line) compared to that of Ref. [1] (dashed blue line) at 3σ confidence limits.

Determine photon energies in the lab frame. We draw a random emission angle for one of the two photons in the ALP rest frame from uniform distributions $\mathscr{U}$, i.e. $\phi \sim \mathscr{U}(0, 2\pi)$ and $\cos\theta \sim \mathscr{U}(-1, 1)$. The angles of the other photon is the negative of the angles of the firsst photon since the two photons are emitted back-to-back. Then, for each of the two photons, we calculate the energies $E_{1/2,\mathrm{L}}$ according to Eq. (13) in the lab frame. We check that these energies are within the detector energy window of 25–100 MeV and discard these photons outside of that range.

Calculate the arrival time at the satellite. We first calculate the photon angles $\alpha_{1/2}$ in the lab frame according to Eq. (14). This allows us to compute the length of their path to the detector and hence their arrival time.

In the code, the variable `L21` and `L22` represent the distance travelled by the first and second photon after the decay, respectively. For each of them, the code then computes their arrival times `time1` and `time2` from the decay length `L1` and their decay anglse `angle1` and `angle2` (see [Sec. II.C 1] for details). The resulting arrival times are accepted if they are smaller than the duration of the measurement after the arrival of the first neutrino, which is 223 s.

Calculate the photon fluence. Once the above algorithm has run for all 10^7 ALPs in our simulation, we can determine ratio of axions that pass all the cuts. The number of photons arriving at the detector, `counts`, can simply be divided by the number of simulated ALPs, `number_alps`, to obtain the fraction of valid events for each ALP mass and ALP-photon coupling. Multiplying this fraction with the total ALP fluence computed by integrating Eq. (1) gives the expected photon fluence for each parameter point.

4 Results and Technical Details

The results are provided as `results/SN1987A_DecayFluence.dat`, we used a logarithmic grid of masses from 1 keV to 1 GeV with a step size of 0.05 dex and and ALP-photon couplings from $10^{-13}\,\mathrm{GeV}^{-1}$ to $10^{-3}\,\mathrm{GeV}^{-1}$ with the same step size. This results in a total of 24×10^3 grid points.

The results are shown in Fig. 2, where we compare the excluded region from our implementation with that of Ref. [1].[1] We can see that our code is in good agreement with the previous bounds on these ALPs; however, our results for the fluence are about a factor of 1.8 larger than Ref. [1], in line with what Ref. [8] finds. The range in the parameter space has been extended to explore a much larger region, which closes the gap between these limits and the Horizontal Branch star limits [e.g. 1].

Our code was developed in Python version 3.7.5), using the `numpy`, `scipy`, and `mpi4py` packages.

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[1] We warmly thank J. Jaeckel and P. C. Malta for discussions and letting us inspect their original code.