Assignment 1

Welcome to the first assignment of the lecture *Visual Computing*. **Please read all instructions carefully!** The goal of this exercise is to familiarize you with the programming language *Python* and the image processing library *OpenCV*. Submission is due on Wednesday, April 21th, 2021 at 8pm. Please submit your solutions via `read.mi.hs-rm.de`.

Aufgabe 1 (4 points)**.** Consider the three vectors

$$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}.$$

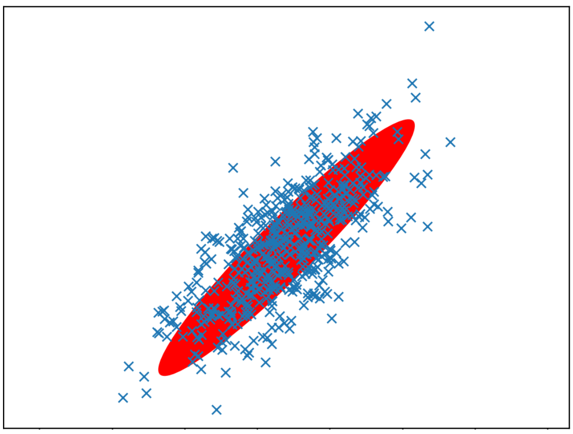
1. Determine the *inner product* $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^\top \mathbf{y}$ of $\mathbf{x}$ and $\mathbf{y}$.
2. Determine the *outer product* $\mathbf{x} \otimes \mathbf{y} = \mathbf{x}\mathbf{y}^\top$ of $\mathbf{x}$ and $\mathbf{y}$.
3. Determine $(\mathbf{x} \otimes \mathbf{y})\mathbf{z}$
4. What is the *rank* of $\mathbf{x} \otimes \mathbf{y}$?

Aufgabe 2 (4 points)**.** Show that for any nonzero vector $\mathbf{u} = (u_1, u_2, u_3)^\top \in \mathbb{R}^3$, the rank of the *skew-symmetric* matrix (sometimes also written as $\hat{\mathbf{u}}$ or $\mathbf{u}_\times$)

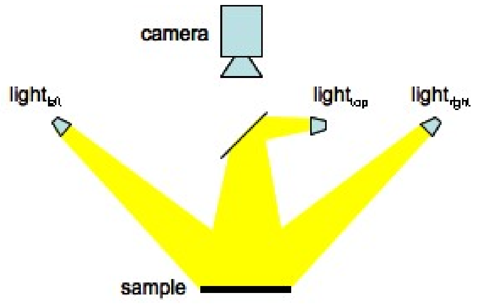
$$[\mathbf{u}]_\times = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

is always two. That is, the three row (or column) vectors span a two-dimensional subspace of $\mathbb{R}^3$. Hint: If we fix $\mathbf{u}$ the cross product $\mathbf{u} \times \mathbf{v}$ can be represented by linear map from $\mathbb{R}^3$ to $\mathbb{R}^3$ represented by the matrix $[\mathbf{u}]_\times$.

Aufgabe 3 (4 points)**.** Write a simple python programm `covmat.py` that uses `numpy` to compute the covariance matrix of the pointset that is given by the file `covtest.dat`. Your programm should also visualize the ellipse centered at the mean of the data which have the principal axes given by the eigenvectors and eigenvalues of the corvariance matrix (see figure below).



Aufgabe 4. (10 points) In industrial optical quality control, several images of an object are often taken under different lighting conditions, whereby the camera and object position are not changed. A typical setup in which three pictures of a test specimen are taken is shown in the adjacent figure. From the images I_{left}, I_{top}, I_{right} for example the images $I_{luminance}$, I_{bias}, $I_{brightness}$ can be calculated as follows:



$$I_{luminance} = \frac{I_{left} + I_{top} + I_{right}}{3}, \quad I_{bias} = I_{left} - I_{right}, \quad I_{brightness} = I_{top} - \frac{I_{left} + I_{right}}{2}$$

Write a *OpenCV* application that reads three images $I_{left}, I_{top}, I_{right}$, converts them to grayscale images and then calculates, displays and saves the three images $I_{luminance}, I_{bias}, I_{brightness}$. When making calculations, note that the values of all results must be in the range $[0, 255]$.

The following figures shows the grayscale images I_{left}, I_{top}, I_{right} and below the images $I_{luminance}, I_{bias}, I_{brightness}$. You can find the original pictures in the archive `cover.zip` on the web page of this lecture.